

Strategic Timing of IPO and Disclosure - a Dynamic Model of Multiple Firms*

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May 2015

Preliminary Draft

Abstract

We study a dynamic game between multiple firms who decide when to disclose their private information and sell the firm (IPO) or a project. Firms privately learn their type and are uncertain as to a common factor to all the firms. The common factor follows a stochastic mean-reverting process and is revealed only following an IPO. We characterize the unique symmetric threshold equilibrium and show that there is always a positive amount of delay in going public. Firms consider the trade-off between the direct costs of delaying the IPO and the value of the real option from delaying the IPO, which stems from potentially learning the common factor. The model predicts that the number of expected IPOs in the second period is increasing in the realization of the common factor in the first period, so that we expect clustering of IPOs following a successful IPO. We suggest several empirical predictions regarding firm equilibrium strategies and the timing of IPOs.

*We are grateful to seminar participants at Baruch, Carnegie Mellon, Columbia, NYU, and Warwick.

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1 Introduction

In 2014, U.S. public equity markets saw more initial public offerings (IPOs) than in any year since the 2000 dot-com boom. The recent wave of IPOs has been especially interesting given the initial difficulty the market had in evaluating firms in new industries, particularly social media and cloud computing. As one commentator noted during the 100% price increase on the initial day of trading for LinkedIn: "New internet companies based on new and innovative technologies are more difficult to value."¹ In new industries with uncertain fundamentals, firms that had received higher than expected valuations led to further, more immediate public offerings by other firms within the same industry, whereas firms who received less favorable valuations delayed the IPO plans of other similar firms. For example, consider the pioneer firm to go public in the new social media industry, Facebook. The price fall that ensued Facebook's IPO allegedly pushed back the offering of Twitter for several months. Twitter went public only when the market was better able to assess Facebook's value, in a very favorable way, which resulted in a tremendous price increase around the IPO. The ability to see the market sentiment before going public provide firms an advantage in choosing their disclosure time.²

We seek to study such behavior in a strategic game of disclosure/IPO by multiple firms, in which each firm strategically chooses when to disclose its private information and go public (or sell a project). In particular, we study the following three-period multi-firm/entrepreneur setting. The value of each firm/project is determined by two components: an idiosyncratic component, which we refer to as the firm's type, and a common component which affects all firms in the industry/economy who consider an IPO. The first ingredient of our model is that, given all else equal, each firm's manager/entrepreneur wants to sell the firm/project as early as possible. This assumption could reflect that delaying IPO leads to, for example: forgoing profitable investment and expansion opportunities, potential loss of market power relative to competitors and hence reduced payoff, the costs of debt that is used to finance projects or operations, or even the tendency of a firm's idiosyncratic component to mean-revert. To

¹"Wall Street 'mispriced' LinkedIn's IPO." *Financial Times*, March 30, 2011.

²Several other firms, such as Kayak, have been reported to delay their IPO dates specifically because of the market reaction to the Facebook IPO. See "Did IPO damage Facebook brand?", *CBS Money Watch*, June 6, 2012.

capture this time preference, we assume that firms/managers discount the payoff of future payoff from selling the firm/project. The second ingredient of our model pertains to the common factor, which can capture the state of nature, the state of the economy/industry, or market sentiment. The state of nature is assumed to follow a mean-reverting stochastic process. The state variable can be common to firms only within a specific industry, or to all firms.³ Bessembinder et al. (1995) found that all the markets they examined are characterized by mean-reversion, where there is substantial variation across industries in terms of the reversal rates. The state variable can also be thought of as reversal of macroeconomic shocks, as evidenced by Bloom (2009) and Bloom et al. (2014).

The state of nature is not observable, unless at least one of the firms goes public. As part of the IPO process, the market learns and forms an opinion about the new technology or the market conditions (captured by the state of nature) and reveals this information through the pricing of the IPO. If no firm goes public, the state of nature is not revealed, e.g., had Facebook not gone public in May 2012, there would have been a much greater uncertainty about the market's perception of the value and potential of the social media industry.

The mean reverting nature of the common factor gives rise to a real option from delaying the IPO in the first period. In case a firm delays its IPO and another firm goes public, the state of nature in the first period is revealed. If the realization of the state of nature in the first period is sufficiently low the firm is better off further delaying the IPO to the third period. The reason is that while the state is expected to be low also in the second period, it is expected to further revert to the mean in the third period, i.e. the state is expected to increase between the second and the third period. However, if the realization of the state of nature in the first period is sufficiently high, the firm finds it more profitable to go public in the second period. When deciding whether to IPO in the first period, the firm considers the trade-off between the direct costs of delaying the IPO and the benefit from the value of the real option from delaying the IPO⁴. The firm considers the probability that the other firms

³There is evidence that firms in different industries have different market sentiments, and that IPOs within an industry share similar one-day returns and similar average returns. For example, technology IPOs performed very well in 2014, whereas bank IPOs often failed to meet their price range. See "Bank IPO Falls Short of Target Price Range," *Wall Street Journal*, September 24, 2014.

⁴When firms go through an IPO they are required to disclose information, as part of the IPO prospectus. We will be using IPO and disclosure interchangeably throughout the paper.

will disclose and IPO in the first period; if no other firm goes public in the first period, the state of nature will not be revealed and the option value will not be realized. This introduces strategic interaction between firms, as the disclosure/IPO strategy of one firm affects the payoff and the optimal strategy of the other firms.

We analyze the above setting and show that there exists a unique symmetric equilibrium. In equilibrium, each firm follows a threshold strategy in each period. In particular, each firm goes public in the first period if and only if the realization of its idiosyncratic component (hereafter the firm's type) is sufficiently high.⁵ If there was no IPO by any firm in the first period then the first-period state of nature is not revealed, and hence, all firms go public in the second period (as the game ends in the third period). If at least one firm went public in the first period then a firm that did not IPO in the first period goes public in the second period only if the realization of the first-period state was sufficiently high. The realization of the first-period state of nature below which a firm delays the IPO in the second period is lower the higher the firm type is. Low-type firms are thus comparatively more inclined to delay the IPO not only in the first period, but in the second period as well. The reason is two-fold: (i) the cost of delay due to the discount is comparatively lower for low-type firms, and (ii) the value of the real option from delaying the IPO in the first period is decreasing in a firm's type.

Several interesting insights and empirical predictions emerge from this analysis. There is always a positive amount of delay of IPO in equilibrium, where sufficiently high type firms do not delay. In general, the model predicts that the higher a firm's type, the earlier it will disclose and go public, as higher type firms exhibit higher discounting costs and lower value of the real option from delaying the IPO.

If no firm went public in the first period, we expect clustering of IPOs in the second period.⁶ If there was at least one IPO in the first period, the expected number of IPOs in the second period is increasing in the first-period realization of the state. That is, we

⁵In an extension of the model we study a setting in which firm type is bounded from above and show that for this setting the symmetric equilibrium may not be unique. In particular, we show that, for sufficiently low discount factors, there may also exist equilibria in which one firm always goes public in the first period and the other firms never IPO in the first period.

⁶For simplicity, we study a three period setting, in which firms that did not IPO by the end of the second period, do so in the third and last period. In a setting with more than three periods, there will be some delay of IPOs in the second period, even if no firm went public in the first period.

expect clustering of IPOs in the second period following a successful IPO in the first period, and fewer IPOs (or none) if the state of nature in the first period turned out to be low. We find that the threshold level in each period is decreasing in the discount factor, since for higher discount factor delaying the IPO is more costly and also the value of the real option from delaying is lower. The variance of the state of nature affects the first-period threshold but does not affect the second-period threshold. In particular, an increase in the variance of the state of nature increases the option value from delaying the IPO due to the increased volatility in the realization of the state, and hence increases the first-period threshold. However, the second-period threshold is unaffected by the variance of the state since firms/entrepreneurs are risk-neutral and at the second period there is no real option from delaying the IPO. The reversal rate of the state variable has a less straightforward effect on the threshold levels in both the first and the second period. For low levels of reversal rate both periods' thresholds are increasing in the reversal rate where for higher levels both period's thresholds are decreasing in the reversal rate. The intuition is as follows. When the reversal is full (that is, when state of nature is *iid* over time), the value of the real option from learning the realization of the state in the first period is zero, since the states in the second and third periods are independent of the first period's state. At the other extreme, as the reversal rate goes to zero, the process of the state variable converges to a random walk. In such case, there is no reversal and the only value the real option may have is when the realized state is sufficiently low, such that the overall value of the firm is negative, and hence delaying a negative payoff is beneficial.

The problem we investigate is practically relevant as the strategic timing of IPOs is a veritable concern among firms. Indeed, firms typically delay their offering dates due to unfavorable market sentiment (e.g., the case of Virtu who delayed its IPO due to dissatisfaction over flash-trading⁷). Numerous empirical papers also provide evidence of the strategic timing of IPOs, e.g., Lougran, Ritter, and Rydqvist (1994), Lerner (1994), Pagan, Panetta, and Zingales (1998) which document that IPO volume is higher following increase in market

⁷"For Virtu IPO, Book Prompts a Delay." The Wall Street Journal, April 3, 2014. The timing is a serious concern for firms: "Analysts said Virtu had little choice but to postpone the offering. 'The timing couldn't be worse,' said Pat Healy, CEO of Issuer Advisory Group LLC, which advises companies on going public."

valuations.

The two papers perhaps most closely related to our study are Persons and Warther (1997) and Alti (2005). Persons and Warther (2005) develop a model of financial innovation among several firms who may move sequentially. Each firm observes the noisy cash flow returns of firms who have already adopted the innovation and accordingly decides whether to adopt the innovation. They generate "booms" in the adoption of the new technology, as each additional firm that adopts the innovation may lead to another firm's subsequent adoption. However, a fundamental assumption in their model is that it is common knowledge which firms benefit the most from the adoption of the technological innovation, and correspondingly, the firms adopt the technology in a predetermined order, beginning with the firm that benefits the most. This would be equivalent to the model here where each firm's idiosyncratic component was commonly known, the state of nature does not follow a mean-reversion process and adoption of the innovation increases the precision of the beliefs about the profitability of the innovation. Likewise, Alti (2005) develops a model of information spillover in an IPO setting, where information asymmetry decreases following an IPO, which consequently lowers the cost of going public for the other firms. The cost of going public is due to adverse pricing by the market in a second price auction in the presence of informed trader. The common component among firms is the cash flow generated in the period of IPO, which is assumed to be identical to all firms (and not mean-reverting). The support of per-period cash flow, however, is assumed to be binary and unchanging.

Several other papers consider optimal IPO timing and IPO waves. Pastor and Veronesi (2003) model the strategic timing of an IPO as an inventor who faces a problem analogous to an American call option. The inventor can exercise the option to capitalize on abnormal profits, but sacrifices the possibility that market conditions may worsen to cover the initial investment. As here, their model relies heavily on market conditions for the timing of the IPO, however, our model incorporates strategic interaction between firms that affects the timing of IPOs. A number of other papers look at the strategic timing of IPOs. He (2007) considers a game between investment banks and investors to generate high first day returns during periods of high IPO volume. Chemmanur and Fulghieri (1999) models IPO timing as a trade-off between selling the firm to a risk-averse venture capitalist at a discount or through

the loss in informational advantage from going public. Benninga, Helmantel, and Sarig (2005) model the decision to go public as a trade-off between diversification and the private benefits of control. They generate IPO waves during periods when expected cash flows are high. Our model differs from these three as they are all single-firm models, whereas we are principally interested in the strategic interaction between firms and the resulting clustering effects. Indeed, a multi-firm setting of IPOs where firms' strategies are interdependent has not been examined in the context of IPO waves in the extant literature.

Our model varies from the literature on dynamic voluntary disclosure (e.g., Dye and Sridhar (1995), Acharya, DeMarzo, and Kremer (2011), Guttman, Kremer, and Skrzypacz (2014)) in three ways: (i) in our setting there is no uncertainty about whether the firm/entrepreneur is endowed with private information, (ii) in our setting the entrepreneur is only concerned with the firm's value in the period of disclosure and IPO, and (iii) in our setting there are multiple firms/entrepreneurs whose decisions are interrelated.⁸

The following section presents the setting of the model and section three analyzes the equilibrium. Section four examines comparative statics and offers empirical predictions. Section five studies an extension of the model in which the support of the firms' type is bounded from above and the final section concludes. Proofs are relegated to the Appendix, unless otherwise stated.

2 Model Setup

We study a setting with three periods, $t \in \{1, 2, 3\}$, and $N \geq 2$ firms. A firm's value is a function of an idiosyncratic component and the value of a common factor. Prior to $t = 1$, each firm's manager/entrepreneur privately observes the idiosyncratic component of her firm's value or project, θ_i , which is the realization of a random variable $\tilde{\theta}$ with a cumulative density $G(\theta)$ and probability density function $g(\theta)$. We will often refer to θ_i as the type of firm i . The support of θ is $[0, \infty)$ and $g(\theta)$ is positive over the entire support of θ .⁹ For all $i \neq j$ the idiosyncratic components, θ_i and θ_j , are independent. We constrain

⁸The latter feature is present in Dye and Sridhar (1995).

⁹We later study the case in which the support of θ is bounded from above, i.e., $\theta \in [0, \bar{\theta}]$ and show that the symmetric equilibrium that we characterize in the current section still holds. However, when the support

θ to be non-negative since this simplifies the analysis, however, the results would not be qualitatively affected with negative firm values¹⁰. Firms' managers/owners are assumed to be risk-neutral. Every firm manager must IPO the firm (or sell the project) in one of the periods, while as part of the IPO the manager discloses the private type, θ_i . Disclosure of the type is credible and costless. The managers are assumed to maximize the firm's market price at the time of disclosure. For example, the manager/owner may want to sell (IPO) the firm and needs to make a disclosure at the selling time (IPO). The firm's price at the time of the IPO depends on investors' beliefs about both the idiosyncratic component, θ_i , as well as on the state of nature at the time of the IPO, which is denoted by s_t . The market price at time τ of firm i that discloses θ_i at $t = \tau$ equals investors' expectation of $\theta_i + s_\tau$ given all the available information at $t = \tau$, which we denote by Ω_τ . Every firm's manager has a time preference (discount) which is denoted by r , such that the expected utility of the owner/manager of firm i from going public and disclosing θ_i at $t = \tau$ is given by:

$$u_{i,\tau} = \frac{E(\theta_i + s_\tau | \Omega_\tau)}{(1+r)^{\tau-1}}.$$

Discounting is meant to capture the costs associated with delaying the sale of a project or shares. Such cost could be due, for example: costs of debt, the cost from forgoing investment and acquisition opportunities due to lack of financing, and the decrease in profitability due to increase in competition. The state of nature in each period, s_t , is ex-ante unobserved, however, upon IPO by at least one of the firms, all firms learn s_t at the end of the period in which an IPO took place¹¹. We assume that the state of nature follows a mean-reverting AR(1) process of the form:

$$s_t = \gamma s_{t-1} + \varepsilon_t,$$

of θ is bounded from above, there exists another equilibrium in which one firm always discloses at $t = 1$ and all the other firms do not disclose (or follow a disclosure threshold).

¹⁰With negative values, firms would be compelled to delay disclosure since discounting works to improve the firm's payoff. We eliminate this case to not confound the results.

¹¹A possible extension is to assume that the market obtains a noisy signal of the state of nature when a disclosure is made. The more firms disclose the higher the precision of the market inference about the state of nature.

We believe this is a realistic assumption that should not make a qualitative difference. However, it will complicate the analysis as the indifference condition will have to take into account all the permutations of types that will disclose at $t = 1$ and the corresponding option value - which depends on the variance of the signal about s_1 , which in turn depends on the number of firms that disclose at $t = 1$.

where $\gamma \in (0, 1)$ and $\varepsilon_t \sim N(0, \sigma^2)$ with a cumulative distribution function $F(\cdot)$ and density function $f(\cdot)$ ¹². The initial state is given by $s_0 = 0$, and so the first period's state is given by $s_1 = \tilde{\varepsilon}_1$. Hence, the state of the economy in the first period is simply a mean-zero error term.

The mean-reversion property of the state of nature, which is one of the central assumptions in our model, is taken exogenously. However, both the empirical and theoretical literature provide ample support for mean reversion of both specific stock returns (e.g., Fama and French (1988) and Poterba and Summers (1988)) and of macroeconomic measures, such as stock market indices (e.g., Richards (1997)). Mean reversion can be motivated by fully rational settings (e.g., Cecchetti, Lam and Nelson 1990) and high-order beliefs in an overlapping generation (as in Allen, Morris, and Shin (2006))¹³, or by behavioral explanations such as investors sentiment and limits to arbitrage (e.g., Baker and Wurgler (2006)). Mean reversion of the state of nature in our setting can also be motivated by dynamic competition in the market that affects the common factor. For example, when the state of nature, which may represent the perceived profitability of the relevant technology, is high in the first period, firms have an incentive to increase their activity in this market/technology, which in return will decrease the profitability in this market. A symmetric argument applies to a low state of nature.

The sequence of events in the game is as follows: Prior to $t = 1$ all managers/firms privately observe the idiosyncratic component of their firm value, θ_i . In $t = 1$, each firm decides whether to IPO in this period. Firms make their decisions simultaneously. If at least one firm made an IPO the state of nature at $t = 1$, s_1 , is publicly observed and firms that disclosed and IPO receive their market valuation. Those firm managers receive their corresponding payoff and the remainder of the game is irrelevant for them. At period $t = 2$, all firms that did not IPO at $t = 1$ make a disclosure/IPO decision. If at least one firm IPO

¹²Alternatively, we could have the variance of the error decreasing in each period to reflect the market's ability to better evaluate the firm in later periods. This would not affect the results since firms are assumed to be risk neutral and only the variance level in the first period, which affects the value of the real option from delaying disclosure, is consequential.

¹³Mean reversion due to high-order beliefs in an Allen, Morris and Shin setting is as follows. Since in the first period the private signals are underweighting in the price formation it gives rise to a biased investors beliefs about the intrinsic value. As time goes by, on expectation, this bias decreases and the price converges to the unbiased mean.

at $t = 2$ the realization of the state of nature, s_2 is publicly revealed. The market valuation of firms that IPO at $t = 2$ is determined and manager's of those firms receive their payoff. Finally, at $t = 3$, which is the last period of the game, all firms that have not yet gone through IPO must do so and those firm's managers obtain their payoff. The timeline of a generic period is given in Figure 1.

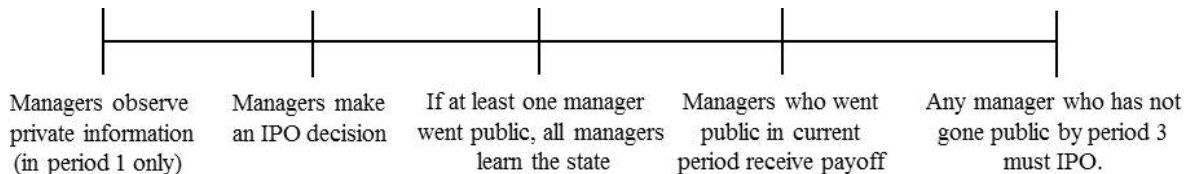


Figure 1 – Sequence of the stage game.

We assume that all firms are ex-ante homogeneous, that is, all firms have the same distribution of idiosyncratic component of value, θ_i , the same discount rate, r , and that the common factor, s_t , affects all firm's market value in the same way. The following section analyzes the equilibrium of the above reporting game.

3 Equilibrium

Before we derive the equilibrium of our setting, note that in a two-period (rather than three-period) version of our model, all firms IPO at the beginning of the game. The reason is that in the first period, none of the managers have any information about s_1 , and hence, the expected value of s_2 (which in this case is the last period) is zero. As such, the expected payoff from delaying the IPO is $\frac{\theta_i}{1+r}$, which is lower than the expected payoff from IPO at $t = 1$ (which is θ_i)¹⁴.

We conjecture a symmetric threshold equilibrium, in which each firm IPO in the first period if and only if its type, θ_i , is greater than a threshold θ_1^* , where θ_1^* is a function of

¹⁴Note that also in a single firm setting with more than two periods, the firm is better off disclosing at $t = 1$ than deferring disclosure, since no information about the state of nature, s_t , will be revealed before the firm discloses.

all the parameters of the model (the distributions of the types, the distribution of the state of nature and the degree of mean-reversion, the number of firms, and the discount factors). At $t = 2$, if firm $j \neq i$ went public at $t = 1$, firm i IPO if and only if $\theta_i > \theta_2^*(s_i)$. Note that if there were no IPOs at $t = 1$, then this reduces to the two-period setting mentioned above, and hence, all firms IPO in $t = 2$. Given that there is positive probability of IPO by at least one other firm in the first period, firm i has a real option from delaying the IPO at $t = 1$, hoping to observe s_1 at the end of period 1. Upon observing the state of nature, s_1 , for sufficiently negative realizations of the state of the economy, the firm rather delay the IPO until $t = 3$, as the state of nature follows a mean-reverting process, such that the state of nature is expected to increase towards zero at $t = 3$.

In light of the above behavior in period 2, firms at $t = 1$ have to take into consideration the trade-off between the benefit from the above real option and the cost of delaying the IPO. The cost of delaying, due to the discount factor r , increases in the firm's type, θ_i . Moreover, as we show below, the value of the real option from delaying the IPO at $t = 1$ is decreasing in the firm's type, θ_i . As such, both of the above effects work in the same direction. That is, any firm follows a threshold strategy at $t = 1$ such that, for realizations of θ_i that are sufficiently high, the manager prefers to IPO at $t = 1$, whereas for lower realizations the manager is better off delaying the IPO at $t = 1$. We solve for the unique symmetric threshold equilibrium. We start by deriving the IPO policy in the second period and then analyze the first period's decision.

3.1 Period 2

As indicated above, if no firm went public at $t = 1$, all firms IPO at $t = 2$.

Given an IPO by at least one firm at $t = 1$ and the realization of s_1 , firm i of type θ_i is indifferent between going public and delaying the IPO at $t = 2$ if and only if the following indifference condition holds:

$$\frac{\theta_i + E(s_2|s_1)}{1+r} = \frac{\theta_i + E(s_3|s_1)}{(1+r)^2}.$$

The above has a unique solution. The unique optimal strategy in $t = 2$, which we denote by

$\theta_2^*(s_1)$, is as follows.

Lemma 1 *In any equilibrium, the strategy of firm i that did not IPO at $t = 1$ is as follows. If no firm went public at $t = 1$, firm i goes public at $t = 2$. If at least one firm went public at $t = 1$ (and hence s_1 is observed) firm i follows a threshold strategy at $t = 2$ such that it goes public if and only if¹⁵*

$$\theta_i \geq \theta_2^*(s_1) \equiv -s_1 ((1+r) - \gamma) \left(\frac{\gamma}{r}\right). \quad (1)$$

Having observed the market condition in the first period, s_1 , firms will delay the IPO only for sufficiently negative values of s_1 . Note that for all $s_1 \geq 0$, all managers that did not IPO at $t = 1$ will IPO at $t = 2$, as both effects (discounting and the reversal of the state of nature) work in the same direction - not to delay IPO. When the realization of s_1 is negative (or in general lower than the mean of s) the mean-reversion property of s implies that s_3 is expected to be higher than both s_1 and s_2 , which provides an incentive to delay the IPO to $t = 3$. However, delaying the IPO is costly due to discounting, and hence, the manager's IPO threshold at $t = 2$ resolves the trade-off between these two effects.

To further the intuition for the threshold at $t = 2$, it is useful to consider extreme parameter values. For $\gamma = 1$, such that the state of nature follows a random walk, the manager goes public at $t = 2$ if and only if $\theta + s_1 > 0$. On the contrary, when $\gamma = 0$, such that s_1 and s_2 are independent, the manager goes public immediately. For extreme values of the discount rate it is easy to see that for $r = 0$ firms IPO at $t = 2$ if and only if $\gamma s_1 > 0$, or equivalently $s_1 > 0$, as the only effect in place is the reversal of the state of nature. As the discount rate goes to infinity, all firms IPO immediately. We investigate comparative statics formally in section 4.

Next, we analyze the equilibrium behavior at $t = 1$.

¹⁵An alternative way to think about the disclosure strategy is to take θ is given and to specify the realizations of s_1 for which the firm will and will not disclose at $t = 2$. This approach yields that for a given θ_i firm i discloses at $t = 2$ if and only if $s_1 < s_1^*(\theta_i) \equiv -\frac{\theta_i}{((1+r)-\gamma)(\frac{\gamma}{r})}$.

3.2 Period 1 and the option value from delayed disclosure

We conjecture a threshold strategy at $t = 1$ such that firm i goes public in the first period if and only if $\theta_i \geq \theta_1^*$. Recall that if the manager of firm i goes public in $t = 1$, her expected payoff is $\theta_i + E[s_1] = \theta_i$. If manager i does not IPO at $t = 1$, then her payoff depends on whether at least one other firm goes public at $t = 1$. If there were no IPOs at $t = 1$, firm i (as well as all other firms) will IPO at $t = 2$ and will obtain an expected payoff of $\frac{E(\theta_i + s_2)}{1+r} = \frac{\theta_i}{1+r}$. If at least one firm went through IPO at $t = 1$, then firm i IPO at $t = 2$ if and only if $\theta_i > \theta_2^*(s_1)$, in which case, the expected payoff is $\frac{E(\theta_i + s_2 | \theta_i > \theta_2^*(s_1))}{1+r}$. Otherwise, firm i will delay the IPO to $t = 3$, in which case the expected payoff is $\frac{E(\theta_i + s_3 | \theta_i < \theta_2^*(s_1))}{(1+r)^2}$. In summary, the expected payoff of manager i from delaying the IPO at $t = 1$ is:

$$\Pr(ND_{j \neq i}^1) \left(\frac{\theta_i}{1+r} \right) + (1 - \Pr(ND_{j \neq i}^1)) \left(\begin{array}{l} \Pr(D_i^2) E[\text{payoff at } t = 2 | \theta_i, D_i^2] \\ + \Pr(ND_i^2) E[\text{payoff at } t = 3 | \theta_i, ND_i^2] \end{array} \right), \quad (2)$$

where $\Pr(ND_{j \neq i}^1)$ is the probability that no IPO is made by any other firm at $t = 1$, D_i^2 (ND_i^2) indicates that firm i goes public (does not IPO) at $t = 2$, and $\Pr(D_i^2)$ ($\Pr(ND_i^2)$) is the probability that firm i , which did not IPO at $t = 1$, will IPO (not IPO) at $t = 2$.

We analyze a symmetric equilibrium of $N \geq 2$ firms whose types θ_j are independent, so the ex-ante probability of IPO is identical to all firms. Consequently, the probability that no IPO is made at $t = 1$ by any other firm is $\Pr(ND_{j \neq i}^1) = [G(\theta_1^*)]^{N-1}$. The probability that firm i with type θ_i that did not IPO at $t = 1$ will IPO at $t = 2$, given that s_1 was revealed, is the probability that the realization of s_1 will be sufficiently high, such that (1) holds. That is, for any given θ_i the firm will IPO at $t = 2$ if and only if $s_1 > s_1^*(\theta_i) \equiv -\frac{\theta_i}{((1+r)-\gamma)(\frac{\gamma}{r})}$. The probability of such an event is $F\left(\frac{\theta_i}{((1+r)-\gamma)(\frac{\gamma}{r})}\right)$. Substituting the above into the expected

payoff of the manager of firm i from not going public at $t = 1$, given in (2), yields:

$$\begin{aligned}
& [G(\theta_1^*)]^{N-1} \left(\frac{\theta_1^*}{1+r} \right) \\
& + \left(1 - [G(\theta_1^*)]^{N-1} \right) \left(\begin{aligned} & F \left(\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)} \right) E[\text{payoff at } t = 2 | \theta_i, D_i^2] \\ & + \left(1 - F \left(\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)} \right) \right) E[\text{payoff at } t = 3 | \theta_i, ND_i^2] \end{aligned} \right). \tag{3}
\end{aligned}$$

Note that unlike the threshold in $t = 2$, which depends on the manager's type and the realization of s_1 , the IPO threshold of the first period, θ_1^* , depends only on the firm's type, θ_i (and all the other parameters of the model).

In order to derive and analyze the equilibrium it is useful to define and characterize the properties of the manager's real option from delaying IPO at $t = 1$. The option value arises from the manager's opportunity to determine his IPO decision at $t = 2$ based on the observed value of s_1 (whenever at least one other manager IPO at $t = 1$). As Lemma 1 prescribes, the manager prefers to take advantage of the real option and to delay IPO at $t = 2$ only for sufficiently low values of θ and s_1 . To capture the option value that stems from not going public at $t = 1$ we first express the expected payoff of a type θ_i manager who is not strategic and always IPO at $t = 2$. We denote the expected payoff of such non-strategic manager by $NS(\theta_i)$, which is given by:

$$NS(\theta_i) \equiv E[\text{Payoff if IPO at } t = 2] = E \left[\frac{\theta_i + s_2}{1+r} \right] = \frac{\theta_i}{1+r}.$$

The expected payoff of a type θ_i manager that never goes public at $t = 1$ but is strategic at $t = 2$, which we denote by $S(\theta_i)$ (where S stands for strategic), is given by:

$$S(\theta_i) \equiv E[\text{Payoff if follows IPO strategy } \theta_2^* \text{ at } t = 2].$$

Finally, we define the option value as the increase in the expected payoff of a manager who does not IPO in $t = 1$ from being strategic in $t = 2$, relative to always IPO in $t = 2$. The

option value, which we denote by $V_2(\theta_i)$ is given by:

$$\begin{aligned} V_2(\theta_i) &\equiv S(\theta_i) - NS(\theta_i) \\ &= \Pr(s_2 < s_2^*(\theta_i)) E \left[\frac{\theta_i + s_3}{(1+r)^2} - \frac{\theta_i + s_2}{1+r} \mid s_1 < s_2^*(\theta_i) \right]. \end{aligned}$$

The following Lemma describes a fairly intuitive property of the option value, which is very useful in showing existence and uniqueness of the symmetric threshold equilibrium.

Lemma 2 *The option value is decreasing in θ_i , i.e.,*

$$\frac{\partial V_2(\theta_i)}{\partial \theta_i} < 0.$$

Intuitively, the option value is decreasing in θ due to two effects. The first is that the discounting is comparatively more punitive for higher type firms, and hence, delaying disclosure is relatively more costly for high type firms. The second, and more salient effect, is that the likelihood of taking advantage of the real option in period 2 is decreasing in θ , even conditional on the state having been observed by that point. The reason for this can be seen from Lemma 1; the manager at time $t = 2$ only delays the IPO for sufficiently negative realizations of s_1 . Moreover, higher θ firms require even lower realizations of s_1 in order to find it profitable to delay the IPO until $t = 3$. As such, the likelihood of obtaining a sufficiently low realization of s_1 such that the manager take advantage of the real option and delay the IPO at $t = 2$ is decreasing in his type, θ . So both of the above effects point at a decreasing real option as a function of the firm's type, θ . The proof of the Lemma provides a full and formal analysis.

Having established that the option value from delaying the IPO is decreasing in θ , and given that the the cost of delaying the IPO (due to discounting) is increasing in θ for any given strategy of the other firms, we can conclude that

Corollary 1 *In any equilibrium, any firm's optimal strategy is characterized by an IPO threshold in both $t = 1$ and $t = 2$.*

We next solve for, and analyze, the symmetric equilibrium, in which all firms follow the same strategy. We show that there is a unique symmetric equilibrium. While our main focus is the symmetric equilibrium in the setting with an unbounded support, we study in section 5 an extension of the model in which the support of firm's type is bounded from above, i.e., $\theta \in [0, \bar{\theta}]$. For this setting, the unique symmetric equilibrium still always exists, however, for sufficiently low discount factors we show the existence of another equilibrium, in which one firm always goes public at $t = 1$ and all the other firms never IPO at $t = 1$. Such an equilibrium does not exist in our main setting in which the support of firm's type is unbounded from above.

In a symmetric equilibrium, each manager's best response to all other managers' strategies, who play a threshold strategy θ_1^* , is consequently given by θ_1^* . The $t = 1$ threshold level of all firms is such that each manager of the threshold type, θ_1^* , is indifferent between going public and not going public at $t = 1$. Therefore, the threshold level is the type for which θ_1^* equals the expected payoff from not going public at $t = 1$, given in equation (3).

Lemma 3 *The threshold at $t = 1$ is given by the solution to the following indifference condition of the manager at $t = 1$:*

$$\theta_1^* = [G(\theta_1^*)]^{N-1} \left(\frac{\theta_1^*}{1+r} \right) + \left(1 - [G(\theta_1^*)]^{N-1} \right) \left[F \left(\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)} \right) \frac{\theta_1^*}{1+r} + \frac{1}{1+r} \gamma \sigma^2 f \left(-\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)} \right) + \left(1 - F \left(\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)} \right) \right) \frac{\theta_1^*}{(1+r)^2} - \frac{1}{(1+r)^2} \gamma^2 \sigma^2 f \left(-\frac{\theta_1^*}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)} \right) \right]. \quad (4)$$

3.3 Equilibrium

In this part we establish that there exists a unique equilibrium in which all firms follow the same threshold strategy. We refer to this equilibrium as the symmetric equilibrium. Using Lemmas 1 – 3, we show existence and uniqueness of a symmetric threshold equilibrium. Lemmas 1 and 3 tie down the IPO thresholds in a symmetric equilibrium. We use Lemma 2 to show that this equilibrium exists – any firm whose value is above the threshold indeed finds it optimal to go public at $t = 1$, given the discounting costs and since the option value

is decreasing in θ . Moreover, we show that the threshold characterized by Lemma 1 and Lemma 3 is the unique threshold level in the symmetric equilibrium.

Theorem 1 *There exists a unique symmetric strategy in which firm i , $i = 1, 2, \dots, N$, uses the following IPO threshold strategy:*

- (i) *Firm i goes public at $t = 1$ if and only if $\theta_i \geq \theta_1^*$, where θ_1^* is given by the solution to (4);*
- (ii) *if any other firm went public at $t = 1$ the firm i goes public at $t = 2$ if and only if $\theta_i \geq \theta_2^*(s_1) \equiv -s_1((1+r) - \gamma) \left(\frac{\gamma}{r}\right)$ (when firm i did not IPO at $t = 1$);*
- (iii) *if no IPO was made by any firm at $t = 1$ firm i goes at $t = 2$ for all θ_i .*

Proof. Given that the IPO strategy of firm i at $t = 2$ does not depend on beliefs about θ_j , the IPO strategy at $t = 2$ is given by (ii) and (iii) (note that if no other firm went public at $t = 1$ we are back to a two-period setting, in which all firms IPO immediately as they can). Under the assumption of existence of a threshold equilibrium, any IPO threshold at $t = 1$ should satisfy the first period's indifference condition in (4).

At $t = 2$ the firm will IPO if and only if the expected payoff from IPO is higher than if it delays the IPO, i.e., it will IPO if $\frac{\theta_i + E(s_2|s_1)}{1+r} \geq \frac{\theta_i + E(s_3|s_1)}{(1+r)^2}$, which holds for all $\theta_i > \theta_2^*(s_1) = -s_1((1+r) - \gamma) \left(\frac{\gamma}{r}\right)$. Therefore, no type has an incentive to deviate at $t = 2$.

Next, we show that no type has an incentive to deviate at $t = 1$. Assume that type $\theta_i = \theta_1^*$ is indifferent between going public and delaying the IPO at $t = 1$. To show that all types higher (lower) than θ_1^* strictly prefer to IPO (not to IPO) note that the marginal loss in delaying the IPO for higher (lower) θ_i is greater (smaller) due to discounting, i.e. discounting is more pronounced for higher θ_i 's. In addition, the marginal benefit from delaying the IPO (captured by the option value) is lower (higher) for higher θ_i , as shown in Lemma 2. Hence, no type has an incentive to deviate at $t = 1$.

Next, we show uniqueness of a symmetric IPO threshold. Assume by contradiction that there are two values of θ_1^* : θ_L and θ_H where $\theta_H > \theta_L$, that are consistent with a symmetric equilibrium. If all firms move from θ_L to θ_H , the probability that the other managers will IPO decreases, which in turn increases any manager's incentive to IPO. That is, it decreases the best response IPO threshold. However, this contradicts the assumption of the existence of a higher threshold θ_H . A similar argument follows for a lower IPO threshold. More

formally, the manager's indifference condition at $t = 1$ is given by:

$$\begin{aligned}
\theta_i &= \Pr (ND_{j \neq i}^1) \left(\frac{\theta_i}{1+r} \right) \\
&\quad + (1 - \Pr (ND_{j \neq i}^1)) \left[\begin{array}{l} \Pr (D_i^2) E [\text{payoff at } t = 2 | \theta_i, \text{ and IPO at } t = 2] \\ + \Pr (ND_i^2) E [\text{payoff at } t = 3 | \theta_i, \text{ and delay IPO at } t = 2] \end{array} \right] \\
&= \Pr (ND_{j \neq i}^1) \left(\frac{\theta_i}{1+r} \right) + (1 - \Pr (ND_{j \neq i}^1)) \left(\Pr (D_i^2) \frac{\theta_i}{1+r} + \Pr (ND_i^2) \left(\frac{\theta_i}{1+r} + V_2(\theta_i) \right) \right) \\
&= \frac{\theta_i}{1+r} + (1 - \Pr (ND_{j \neq i}^1)) \Pr (ND_i^2) V_2(\theta_i).
\end{aligned}$$

If the IPO threshold of firm $j \neq i$ increases to θ_H , it has no effect on the option value (conditional on getting to $t = 2$ when firm j went public at $t = 1$ and firm i 's type is $\theta_i > \theta_2^*$), however the probability of this event decreases as the threshold of firm i increases. As such, the right hand side of the above indifference condition decreases, which implies that, in order for firm i to be indifferent at $t = 1$, the IPO threshold of firm i at $t = 1$ must decrease as well – in contradiction to the assumption of the increased IPO threshold. ■

Note that in obtaining the above results we imposed no restriction regarding the distributions of θ , and for tractability we assumed that ε_t is normally distributed (one can show that the above Theorem holds for other distributions of the noise term, including distributions with bounded support such as a uniform distribution).

In the next section we provide comparative statics and empirical predictions that come out of the symmetric equilibrium.

4 Comparative Statics and Empirical Predictions

The first immediate prediction of the model is that firms with higher type, θ , go public earlier than firms with lower type. Another immediate prediction is that following a “successful” IPO in the first period, in which the state of nature turned out to be relatively high, we expect clustering of IPOs. Our particular and stylized setting assumes that the distribution of the innovation in the state of nature, ε , is symmetric, which implies that all firms will IPO following a state of nature that is above the mean. However, under a more general

distribution of the innovation in the state of nature, higher realizations of the state of nature in the first period increase the expected number of firms that will go public in the second period. The following Corollary summarizes these immediate predictions of the model.

Corollary 2 *In the unique symmetric equilibrium:*

- *The higher a firm's type is the earlier it will disclose and go public.*
- *The expected number of IPOs in the second period, following an IPO in the first period is increasing in the realization of the state of nature in the first period.*

Next, we analyze how the equilibrium is affected by the various parameters. In particular, we generate empirical predictions with respect to changes in the following parameters of the model: the discount factor, the rate of mean reversion, and the variance of the error term. We start by studying the effect of the parameters on the disclosure threshold in the second period and then study the effect on the first period's disclosure threshold.

4.1 Comparative Statics for θ_2^*

We begin the analysis with the second period's equilibrium threshold, $\theta_2^*(s_1)$. Note that the threshold of the second period, which is the unique best response at $t = 2$, is independent of the other firm's characteristics. So the analysis of this part is independent of whether the firms are homogeneous or not and the specific characteristics of all the other firms.

Recall that the IPO threshold at $t = 2$, given that there was at least one IPO at $t = 1$ (and hence s_1 is observed) is given by:

$$\theta_2^*(s_1) = -s_1 \left((1+r) - \gamma \right) \left(\frac{\gamma}{r} \right).$$

We will keep everything constant (including s_1) and see how the threshold at $t = 2$ is affected by changes in: (i) firm i 's manager discount factor, r ; (ii) the extent of persistence in the state of nature, γ (where lower γ implies higher mean-reversion); and (iii) the variance of the shock to the state of nature, σ_ε .

Taking the derivative of the threshold with respect to the discount factor, r , yields:

$$\frac{\partial}{\partial r} \theta_2^*(s_1) = \frac{\partial}{\partial r} \left(-s_1 ((1+r) - \gamma) \left(\frac{\gamma}{r} \right) \right) = -\frac{1}{r^2} \gamma s_1 (\gamma - 1) < 0.$$

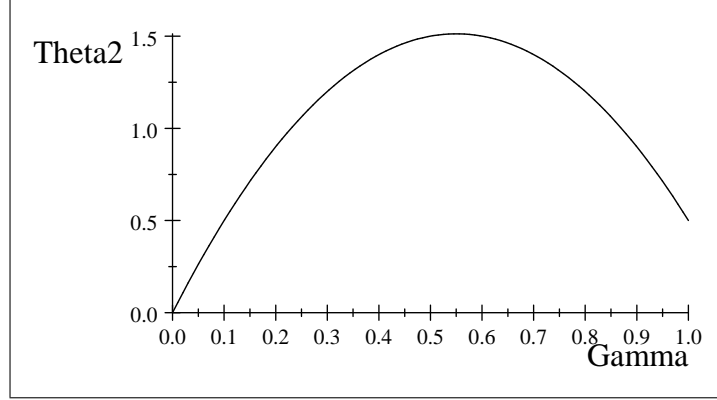
Note that $\frac{\partial}{\partial r} \theta_2^*(s_1) < 0$ since $\gamma \in (0, 1)$ and at the threshold we have $s_1 < 0$. The fact that the threshold level at $t = 2$ is decreasing in r is very intuitive. To see that, recall that at the IPO threshold $\theta_2^*(s_1)$, at which the manager is indifferent between going public and delaying the IPO, it must be that $\theta + s_1 > 0$ (otherwise the manager would strictly prefer to delay the IPO to $t = 3$). Since the expected payoff of the threshold type is positive, an increase in the discount factor increases the cost from delaying the IPO, and hence, decreases the IPO threshold (equivalently, for a given θ the threshold level of s_1 is lower).

Next we analyze the effect of the extent of mean-reversion of the state of nature, γ , on the IPO threshold at $t = 2$. While the mathematical derivation of this effect is straight forward, the intuition for the result is a little more complex.

Taking the derivative of the second period's threshold with respect to γ , yields:

$$\begin{aligned} \frac{\partial}{\partial \gamma} \theta_2^*(s_1) &= \frac{\partial}{\partial \gamma} \left(-s_1 ((1+r) - \gamma) \left(\frac{\gamma}{r} \right) \right) \\ &= -\frac{1}{r} s_1 (r - 2\gamma + 1) = \left\{ \begin{array}{ll} > 0 & \text{for } \gamma < \frac{r+1}{2} \\ 0 & \text{for } \gamma = \frac{r+1}{2} \\ < 0 & \text{otherwise} \end{array} \right\}. \end{aligned}$$

The direction of the effect of changes in γ on the threshold $\theta_2^*(s_1)$ varies with the level of γ . To illustrate the effect of γ on $\theta_2^*(s_1)$, the figure below plots $\frac{\partial}{\partial \gamma} \theta_2^*(s_1)$ as a function of γ using parameter values $r = 0.1$ and $s = -\frac{1}{2}$.



The effect of γ on $\theta_2^*(s_1)$, for $r = 0.1$ and $s = -\frac{1}{2}$

To get better intuition for the above result, it might be useful to consider separately the effect of the idiosyncratic component, θ , and the common factor, s_1 , on the incentive to IPO or delay the IPO at $t = 2$. Since $\theta_i > 0$ and is constant over time, it always provides an incentive not to delay the IPO due to discounting. This incentive increases in θ . The incentive due to the state of nature, which is more complex, is determined by two effects: (i) the mean-reverting feature of the state of nature (characterized by γ) which provides incentive to delay the IPO for low realizations of s_1 ; and (ii) the discount factor. For $\gamma = 0$ the realizations of the state of nature are *iid* and s_2 and s_3 are independent of s_1 . Hence, since $E(s_2|s_1) = 0$ there is no benefit from delaying the IPO. As such, for $\gamma = 0$ all managers IPO at $t = 2$ (if they did not IPO already at $t = 1$). As γ increases from $\gamma = 0$ and the mean-reversion effect is no longer perfect, s_1 becomes more informative about s_2 and s_3 . Hence for negative values of s_1 the value from delaying the IPO increases in γ . However, there is a second, mitigating, effect that stems from the fact that the mean-reversion of s_3 decreases as γ increases - which decreases the benefit from delaying the IPO at $t = 2$ for a given $\{\theta_i, s_1\}$. For sufficiently low γ the former effect dominates and the option value increases in γ , and hence, the IPO threshold increases in γ . As γ further increases, the second effect becomes relatively more pronounced, such that from one point and on the option value decreases in γ . As γ approaches one, the process of the state converges to a random-walk and there is no mean-reversal. Hence, the part of the option value that stems from mean-reversion of low realizations of s_1 disappears, and the only reason the option is still valuable is that when the expected firm value ($\theta + s$) is negative, there is a benefit from delaying a negative payoff.

Finally, $\theta_2^*(s_1)$ is independent of the variance in the noise of the state of nature, σ^2 , and independent for the distribution of θ (recall that θ is assumed to have a positive support), conditional on the state of nature s_1 being revealed in $t = 1$. The threshold level in $t = 2$ is consequently unaffected by changes in σ^2 .

4.2 Comparative Statics for θ_1^*

The comparative statics for the first period threshold level, θ_1^* , are slightly less intuitive, however, the analysis of $\theta_2^*(s_1)$ serves as a useful guide. We start with the effect of γ on θ_1^* :

Claim 1 *The effect of the rate of mean-reversion, γ , on the IPO threshold at $t = 1$, θ_1^* , is similar to its effect on the second period's threshold, $\theta_2^*(s_1)$. Specifically,*

$$\frac{\partial \theta_1^*}{\partial \gamma} = \begin{cases} 0 & \text{for } \gamma = \frac{r+1}{2} \\ > 0 & \text{for } \gamma < \frac{r+1}{2} \\ < 0 & \text{otherwise} \end{cases}.$$

Recall that $\frac{\partial \theta_2^*}{\partial \gamma} = \begin{cases} 0 & \text{for } \gamma = \frac{r+1}{2} \\ > 0 & \text{for } \gamma < \frac{r+1}{2} \\ < 0 & \text{otherwise} \end{cases}$. Let's assume by contradiction that $\frac{\partial \theta_1^*}{\partial \gamma} < 0$

for $\gamma < \frac{r+1}{2}$. An increase in γ affects the expected option value from not going public at $t = 1$ in several ways. First, conditional on another firm going public at $t = 1$, the threshold at $t = 2$ increases in γ , which consequently increases the expected value of the option. Moreover, under the contradictory assumption, the probability that the other firm IPO at $t = 1$ increases in γ , and hence the probability of taking advantage of the option value at $t = 2$ also increases in γ . Overall, the expected option value increases. The manager thus has a stronger incentive not to IPO at $t = 1$, which contradicts the assumption that θ_1^* is decreasing in γ . A symmetric argument applies for the case of $\gamma > \frac{r+1}{2}$. A more formal proof is included in the Appendix. The intuition for the non-monotonicity of the disclosure threshold in γ follows similar arguments to our discussion in the analysis of the comparative statics for the second period's IPO threshold.

Next we analyze the effect of the discount factor, r , on the first period's threshold. Similar to the second period's threshold, the first period threshold is also decreasing in the discount rate:

$$\frac{\partial \theta_1^*}{\partial r} < 0.$$

From the comparative statics for θ_2^* , we know that $\frac{\partial \theta_2^*(s_1)}{\partial r} < 0$, i.e., for a given level of θ_i the manager is more likely to IPO in the second period for higher values of r , and hence, is less likely to take advantage of the real option. In addition, a higher r increases the manager's cost from delaying the IPO. Both effects lead to a stronger incentive to IPO at $t = 1$. This results in a lower IPO threshold at $t = 1$ for higher values of r .

Finally, we consider the effect of the variance of the periodic innovation in the state of nature, σ , on the first period's threshold.

Claim 2

$$\frac{\partial \theta_1^*}{\partial \sigma} > 0.$$

That is, the IPO threshold increases in the variance of the state of nature, s , i.e., a higher variance induces less IPO in the first period.

The intuition for this result is that an increase in volatility increases the value of the option, and hence, induces less IPO in the first period. This implies that the threshold of the first period is increasing in the variance, σ . While this is intuitive the proof (which is delegated to the appendix) requires few steps.

5 Extensions

5.1 Bounded Support - Symmetric and Non-Symmetric Equilibria

In this subsection we show that when the support of θ is bounded from above and the discount rate is sufficiently low, there exists, in addition to the symmetric equilibrium which we characterized in Theorem 1, an equilibrium in which only one firm always discloses at $t = 1$ and the others always delay. We define this special asymmetric threshold equilibrium as the "asymmetric" equilibrium:

Definition 1 Define the asymmetric equilibrium as one where the first period threshold for player $j \neq i$ is $\theta_{1,j}^* = \underline{\theta}$, and the first period threshold for all other players is $\theta_{1,-j}^* = \bar{\theta}$.

We further divide the support of the discount rate, r , into three regions. $(0, r^L)$, (r^L, r^H) and (r^H, ∞) , where:

Definition 2 r^H is such that given that at least one other firm discloses at $t = 1$ (so that s_1 is revealed for sure) upon observing the lowest type, $\theta = 0$, firm i is indifferent between disclosing and not disclosing at $t = 1$.

r^L is such that given that at least one other firm discloses at $t = 1$ (so that s_1 is revealed for sure) a firm with the highest type, $\theta = \bar{\theta}$, is indifferent between disclosing and not disclosing at $t = 1$.

We now show the existence of the discount rate thresholds that define the set of equilibria in the given regions of r :

Proposition 1 The set of equilibria for each of the above regions of the discount factor are as follows:

1. For $r \in (r^H, \infty)$ the unique equilibrium is the symmetric equilibrium in which all firms disclose at $t = 1$, i.e., $\theta_1^* = \underline{\theta}$.
2. For $r \in (r^L, r^H)$ the unique equilibrium is the symmetric equilibrium defined in Theorem 1, in which all firms disclose at $t = 1$ if and only if their type is greater than the interior disclosure threshold, θ_1^* .
3. For $r \in (0, r^L)$: there exist both the symmetric equilibrium with interior disclosure threshold as well as N asymmetric equilibria.

The intuition for the proof is relatively straightforward. For $r \in (r^H, \infty)$ any firm always prefers to disclose at $t = 1$, as even if the lowest type, $\theta = 0$, knows for certainty that s_1 will be revealed, the discounting is too severe to justify delay of disclosure. For $r \in (0, r^L)$ any firm that believes that at least one other firm will disclose is better off not disclosing over disclosing at $t = 1$. To show the existence of the *asymmetric equilibrium* assume that one

firm, firm i , always discloses at $t = 1$. The best response of all other firms is not to disclose at $t = 1$. Now, given that the probability that any other firm will disclose at $t = 1$ is zero, it is optimal for firm i to disclose at $t = 1$. So for $r \in (0, r^L)$ there exist N *asymmetric* equilibria such that in each one of them a single firm always discloses at $t = 1$ and all the other firms do not disclose at $t = 1$. Finally, for $r \in (r^L, r^H)$ there are sufficiently high types that will disclose at $t = 1$ even if they are certain that s_1 will be observed. Hence, there is always a positive probability that at least one firm will disclose at $t = 1$. Let's assume by contradiction that there exists an *asymmetric equilibrium* in which firm i always discloses. Then, there exists a disclosure threshold, such that any other firm discloses if and only if its type is lower than this threshold. This, however implies that there is a positive probability that a firm other than firm i will disclose at $t = 1$. As such, if the realized type of firm i is sufficiently low, the discount effect can be arbitrarily low and the value of the real option is strictly positive. Therefore, firm i will disclose for sufficiently low types - in contradiction to the assumption that firm i does not disclose.

6 Conclusion

In this study we have developed a model to help shed light on the strategic interaction between firms who decide to disclose information and sell shares or a project. We have shown that the unique equilibrium is in threshold strategies where all players follow identical strategies. The primary implication of this result is that, in the presence of other firms and common uncertainty, there is always a positive amount of delay of IPOs in equilibrium.

Several extensions can be considered for future work. We have considered only cases in which the disclosure of the firm's value is verifiable and non-manipulable. A possibly interesting study would be to relax this assumption, in which case firm managers can engage in costly manipulation of the firm's value. We have also assumed that the firm's type (idiosyncratic component) is constant over time. A potentially interesting research question is to investigate a model where the firm's value also follows a stochastic process. Lastly, our model can be extended to a continuous time setting with finite number of firms. We conjecture that in a continuous time setting there exists an equilibrium in which each firm's

delay of the IPO is decreasing in the firm's type and the more negative the revealed state of nature is, the more firms delay their IPOs. As such, the continuous time setting seem to share the main characteristics of our discrete time model.

7 Appendix

Proof of Lemma 1. By the second period indifference condition, we have:

$$\begin{aligned}
 \frac{\theta_i + E(s_2|s_1)}{1+r} &= \frac{\theta_i + E(s_3|s_1)}{(1+r)^2} \\
 \frac{\theta_i + \gamma s_1}{1+r} &= \frac{\theta_i + \gamma^2 s_1}{(1+r)^2} \\
 \theta_i + \gamma s_1 &= \frac{\theta_i + \gamma^2 s_1}{1+r} \\
 \theta_i \left(\frac{r}{1+r} \right) &= \frac{\gamma^2 s_1}{1+r} - \gamma s_{t-1} = s_1 \left(\frac{\gamma}{1+r} - 1 \right) \gamma \\
 \theta_2^*(s_1) &= -s_1 \left((1+r) - \gamma \right) \left(\frac{\gamma}{r} \right).
 \end{aligned}$$

■

Proof of Lemma 2. The option value is equal to the likelihood that the firm which did not disclose at $t = 1$ chooses not to disclose at $t = 2$ times the increase in expected payoff due to the delay in the disclosure, which is

$$V_2(\theta_i) = S(\theta_i) - NS(\theta_i) = \Pr(S < s_2^*(\theta_i)) E \left[\frac{\theta_i + s_3}{(1+r)^2} - \frac{\theta_i + s_2}{1+r} \mid s_1 < s_2^*(\theta_i) \right], \quad (5)$$

where $s_2^*(\theta_i)$ is the value of s_1 such that the agent is indifferent between disclosing and not disclosing at $t = 2$. From equation (1), we have:

$$s_2^*(\theta_i) = -\frac{\theta_i}{((1+r) - \gamma) \left(\frac{\gamma}{r} \right)}$$

Note that

$$\frac{\partial s_2^*(\theta_i)}{\partial \theta_i} < 0,$$

which implies that also

$$\frac{\partial \Pr(S < s_2^*(\theta_i))}{\partial \theta_i} < 0.$$

The derivative of the option value with respect to θ_i is:

$$\begin{aligned} \frac{\partial}{\partial \theta_i} V_2(\theta_i) &= \frac{\partial}{\partial \theta_i} \left[\Pr(S < s_2^*(\theta_i)) E \left[\frac{\theta_i + s_3}{(1+r)^2} - \frac{\theta_i + s_2}{1+r} \mid s_1 < s_2^*(\theta_i) \right] \right] \\ &= \frac{\partial}{\partial \theta_i} \left[F(s_2^*(\theta_i)) \cdot \left(\frac{1}{F(s_2^*(\theta_i))} \int_{-\infty}^{s_2^*(\theta_i)} \left(\frac{\theta_i + E(s_3|s_1)}{(1+r)^2} - \frac{\theta_i + E(s_2|s_1)}{1+r} \right) f(s_1) ds_1 \right) \right] \end{aligned}$$

Plugging in $E(s_2|s_1) = \int_{-\infty}^{\infty} (\gamma s_1 + \varepsilon_2) f(\varepsilon_2) d\varepsilon_2$ and

$E(s_3|s_1) = \int_{-\infty}^{\infty} (\gamma \int_{-\infty}^{\infty} (\gamma s_1 + \varepsilon_2) f(\varepsilon_2) d\varepsilon_2 + \varepsilon_3) f(\varepsilon_3) d\varepsilon_3$, yields:

$$\begin{aligned} \frac{\partial}{\partial \theta_i} V_2(\theta_i) &= \frac{\partial}{\partial \theta_i} \int_{-\infty}^{s_2^*(\theta_i)} \left[\frac{\theta_i + \int_{-\infty}^{\infty} (\gamma \int_{-\infty}^{\infty} (\gamma s_1 + \varepsilon_2) f(\varepsilon_2) d\varepsilon_2 + \varepsilon_3) f(\varepsilon_3) d\varepsilon_3}{(1+r)^2} - \frac{\theta_i + \int_{-\infty}^{\infty} (\gamma s_1 + \varepsilon_2) f(\varepsilon_2) d\varepsilon_2}{1+r} \right] f(s_1) ds_1 \\ &= \frac{\partial}{\partial \theta_i} \int_{-\infty}^{s_2^*(\theta_i)} \left[\frac{\theta_i + \gamma^2 s_1}{(1+r)^2} - \frac{\theta_i + \gamma s_1}{1+r} \right] f(s_1) ds_1 \end{aligned}$$

Recall that $s_2^*(\theta_i)$ is the value of s_1 such that a firm of type θ_i is indifferent between disclosing in $t = 2$ or $t = 3$ upon the realization of s_1 in the beginning of $t = 2$. Hence, by definition, we have that $\frac{\theta_i + \gamma^2 s_1}{(1+r)^2} - \frac{\theta_i + \gamma s_1}{1+r} > 0$ for all $s < s_2^*(\theta_i)$ (i.e. it is more profitable to wait until $t = 3$ for even worse/more negative realizations of s_1). A marginal increase in θ_i thus has two effects. First, we see immediately that $\frac{\partial}{\partial \theta_i} \left(\frac{\theta_i + \gamma^2 s_1}{(1+r)^2} - \frac{\theta_i + \gamma s_1}{1+r} \right) = \frac{1}{1+r} \left(\frac{1}{1+r} - 1 \right) < 0$ since $r > 0$. Moreover, $s_2^*(\theta_i)$ is decreasing in θ_i (i.e. the s_1 required for a higher θ_i to be indifferent must be even more negative), and thus the interval over which we integrate is truncated as θ_i increases. Hence, the integral $\int_{-\infty}^{s_2^*(\theta_i)} \left[\frac{\theta_i + \gamma^2 s_1}{(1+r)^2} - \frac{\theta_i + \gamma s_1}{1+r} \right] f(s_1) ds_1$ is decreasing in θ_i .

This can also be explicitly shown. Using Leibniz's rule, we have

$$\begin{aligned}
& \frac{\partial}{\partial \theta_i} \int_{-\infty}^{s_2^*(\theta_i)} \left[\frac{\theta_i + \gamma^2 s_1}{(1+r)^2} - \frac{\theta_i + \gamma s_1}{1+r} \right] f(s_1) ds_1 \\
&= \int_{-\infty}^{s_2^*(\theta_i)} \frac{\partial}{\partial \theta_i} \left[\frac{\theta_i + \gamma^2 s_1}{(1+r)^2} - \frac{\theta_i + \gamma s_1}{1+r} \right] f(s_1) ds_1 + \frac{\partial s_2^*(\theta_i)}{\partial \theta_i} \left[\frac{\theta_i + \gamma^2 s_2^*(\theta_i)}{(1+r)^2} - \frac{\theta_i + \gamma s_2^*(\theta_i)}{1+r} \right] f(s_2^*) \\
&= \int_{-\infty}^{s_2^*(\theta_i)} \left[\frac{1}{(1+r)^2} - \frac{1}{1+r} \right] f(s_1) ds_1 \\
&\quad + \frac{\partial \left(-\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)} \right)}{\partial \theta_i} \left[\frac{\theta_i + \gamma^2 \left(-\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)} \right)}{(1+r)^2} - \frac{\theta_i + \gamma \left(-\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)} \right)}{1+r} \right] \frac{1}{\sigma\sqrt{2\pi}} \exp \left[-\frac{(s_2^*)^2}{2\sigma^2} \right] \\
&= \int_{-\infty}^{s_2^*(\theta_i)} \left[\frac{1}{(1+r)^2} - \frac{1}{1+r} \right] f(s_1) ds_1 - \frac{r}{\gamma(r-\gamma+1)} [0] \frac{1}{\sigma\sqrt{2\pi}} \exp \left[-\frac{\left(-\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)} \right)^2}{2\sigma^2} \right] \\
&= \int_{-\infty}^{s_2^*(\theta_i)} \frac{-r}{(1+r)^2} f(s_1) ds_1 \\
&< 0.
\end{aligned}$$

Note that $s_1 = \varepsilon_1$ and we define the integral in terms of s_1 rather than ε_1 for presentational ease. ■

Proof of Lemma 3. Starting from (2), given our disclosure threshold in $t = 2$, (2) becomes:

$$\begin{aligned}
& [G(\theta_1^*)]^{N-1} \left(\frac{\theta_i}{1+r} \right) + \left(1 - [G(\theta_1^*)]^{N-1} \right) \\
& \cdot \left[\begin{array}{l} \Pr \left[\theta_i > -s_1 ((1+r) - \gamma) \left(\frac{\gamma}{r} \right) \right] E \left[\frac{\theta_i + s_2}{1+r} \mid \theta_i > -s_1 ((1+r) - \gamma) \left(\frac{\gamma}{r} \right) \right] \\ + \Pr \left[\theta_i \leq -s_1 ((1+r) - \gamma) \left(\frac{\gamma}{r} \right) \right] E \left[\frac{\theta_i + s_3}{(1+r)^2} \mid \theta_i \leq -s_1 ((1+r) - \gamma) \left(\frac{\gamma}{r} \right) \right] \end{array} \right]. \quad (6)
\end{aligned}$$

Note that in any point in time, the agent knows the value of her θ . Next, we calculate each of the terms above:

$$\Pr \left[\theta_i > -s_1 ((1+r) - \gamma) \left(\frac{\gamma}{r} \right) \right] = \Pr \left[s_1 > -\frac{\theta_i}{((1+r) - \gamma) \left(\frac{\gamma}{r} \right)} \right] = F \left(\frac{\theta_i}{((1+r) - \gamma) \left(\frac{\gamma}{r} \right)} \right).$$

And:

$$E \left[\frac{\theta_i + s_2}{1+r} \mid \theta_i > -s_1 ((1+r) - \gamma) \left(\frac{\gamma}{r} \right) \right] = E \left[\frac{\theta_i + s_2}{1+r} \mid s_1 > -\frac{\theta_i}{((1+r) - \gamma) \left(\frac{\gamma}{r} \right)} \right].$$

Which becomes:

$$\begin{aligned} & \frac{1}{F \left(\frac{\theta_i}{((1+r) - \gamma) \left(\frac{\gamma}{r} \right)} \right)} \int_{-\frac{\theta_i}{((1+r) - \gamma) \left(\frac{\gamma}{r} \right)}}^{\infty} \frac{\theta_i + E(s_2 | s_1)}{1+r} f(s_1) ds_1 \\ &= \frac{\theta_i}{1+r} + \frac{1}{1+r} \frac{1}{F \left(\frac{\theta_i}{((1+r) - \gamma) \left(\frac{\gamma}{r} \right)} \right)} \int_{-\frac{\theta_i}{((1+r) - \gamma) \left(\frac{\gamma}{r} \right)}}^{\infty} [E(s_2 | s_1)] f(s_1) ds_1 \\ &= \frac{\theta_i}{1+r} + \frac{1}{1+r} \frac{1}{F \left(\frac{\theta_i}{((1+r) - \gamma) \left(\frac{\gamma}{r} \right)} \right)} \int_{-\frac{\theta_i}{((1+r) - \gamma) \left(\frac{\gamma}{r} \right)}}^{\infty} \left[\int_{-\infty}^{\infty} (\gamma s_1 + \varepsilon_2) f(\varepsilon_2) d\varepsilon_2 \right] f(s_1) ds_1 \\ &= \frac{\theta_i}{1+r} + \frac{1}{1+r} \frac{1}{F \left(\frac{\theta_i}{((1+r) - \gamma) \left(\frac{\gamma}{r} \right)} \right)} \int_{-\frac{\theta_i}{((1+r) - \gamma) \left(\frac{\gamma}{r} \right)}}^{\infty} \gamma s_1 f(s_1) ds_1 \\ &= \frac{\theta_i}{1+r} + \frac{1}{1+r} \gamma E \left[s_1 \mid s_1 > -\frac{\theta_i}{((1+r) - \gamma) \left(\frac{\gamma}{r} \right)} \right]. \end{aligned}$$

Recall that the formula for the expectation of the truncated normal distribution where $x \sim N(\mu_x, \sigma^2)$ is¹⁶:

$$E(x | x \in [a, b]) = \mu_x - \sigma^2 \frac{f(b) - f(a)}{F(b) - F(a)}.$$

Using the above formula, we have:

$$\begin{aligned} E \left[\frac{\theta_i + s_2}{1+r} \mid \theta_i > -s_1 ((1+r) - \gamma) \left(\frac{\gamma}{r} \right) \right] &= \frac{\theta_i}{1+r} + \frac{1}{1+r} \gamma \left(0 - \sigma_\varepsilon^2 \frac{-f\left(-\frac{\theta_i}{((1+r) - \gamma) \left(\frac{\gamma}{r} \right)}\right)}{1 - F\left(-\frac{\theta_i}{((1+r) - \gamma) \left(\frac{\gamma}{r} \right)}\right)} \right) \\ &= \frac{\theta_i}{1+r} + \frac{1}{1+r} \gamma \left(\sigma_\varepsilon^2 \frac{f\left(-\frac{\theta_i}{((1+r) - \gamma) \left(\frac{\gamma}{r} \right)}\right)}{F\left(\frac{\theta_i}{((1+r) - \gamma) \left(\frac{\gamma}{r} \right)}\right)} \right). \end{aligned}$$

¹⁶For $a = -\infty$ we have

$$E(x | x < b) = \mu_x - \sigma^2 \frac{f(b)}{F(b)}$$

Finally:

$$\begin{aligned}
E \left[\frac{\theta_i + s_3}{(1+r)^2} \middle| \theta_i \leq -s_1 ((1+r) - \gamma) \left(\frac{\gamma}{r} \right) \right] &= \frac{\theta_i}{(1+r)^2} + \frac{1}{(1+r)^2} \gamma^2 E \left[s_1 \middle| s_1 < -\frac{\theta_i}{((1+r) - \gamma) \left(\frac{\gamma}{r} \right)} \right] \\
&= \frac{\theta_i}{(1+r)^2} + \frac{1}{(1+r)^2} \gamma^2 \left(-\sigma_\varepsilon^2 \frac{f\left(-\frac{\theta_i}{((1+r) - \gamma) \left(\frac{\gamma}{r} \right)}\right) - 0}{F\left(-\frac{\theta_i}{((1+r) - \gamma) \left(\frac{\gamma}{r} \right)}\right) - 0} \right) \\
&= \frac{\theta_i}{(1+r)^2} + \frac{1}{(1+r)^2} \gamma^2 \left(-\sigma_\varepsilon^2 \frac{f\left(-\frac{\theta_i}{((1+r) - \gamma) \left(\frac{\gamma}{r} \right)}\right)}{\left(1 - F\left(\frac{\theta_i}{((1+r) - \gamma) \left(\frac{\gamma}{r} \right)}\right)\right)} \right).
\end{aligned}$$

Plugging this back to (2):

$$\begin{aligned}
& [G(\theta_1^*)]^{N-1} \left(\frac{\theta_i}{1+r} \right) \tag{7} \\
& + \left(1 - [G(\theta_1^*)]^{N-1}\right) \left[\begin{aligned} & F\left(\frac{\theta_i}{((1+r) - \gamma) \left(\frac{\gamma}{r} \right)}\right) \left(\frac{\theta_i}{1+r} + \frac{1}{1+r} \gamma \left(\sigma_\varepsilon^2 \frac{f\left(-\frac{\theta_i}{((1+r) - \gamma) \left(\frac{\gamma}{r} \right)}\right)}{F\left(\frac{\theta_i}{((1+r) - \gamma) \left(\frac{\gamma}{r} \right)}\right)} \right) \right) \\ & + \left(1 - F\left(\frac{\theta_i}{((1+r) - \gamma) \left(\frac{\gamma}{r} \right)}\right)\right) \left(\frac{\theta_i}{(1+r)^2} + \frac{1}{(1+r)^2} \gamma^2 \left(-\sigma_\varepsilon^2 \frac{f\left(-\frac{\theta_i}{((1+r) - \gamma) \left(\frac{\gamma}{r} \right)}\right)}{\left(1 - F\left(\frac{\theta_i}{((1+r) - \gamma) \left(\frac{\gamma}{r} \right)}\right)}\right) \right) \right) \end{aligned} \right] \\
& = [G(\theta_1^*)]^{N-1} \left(\frac{\theta_i}{1+r} \right) \tag{8} \\
& + \left(1 - [G(\theta_1^*)]^{N-1}\right) \left[\begin{aligned} & F\left(\frac{\theta_i}{((1+r) - \gamma) \left(\frac{\gamma}{r} \right)}\right) \frac{\theta_i}{1+r} + \frac{1}{1+r} \gamma \sigma_\varepsilon^2 f\left(-\frac{\theta_i}{((1+r) - \gamma) \left(\frac{\gamma}{r} \right)}\right) \\ & + \left(1 - F\left(\frac{\theta_i}{((1+r) - \gamma) \left(\frac{\gamma}{r} \right)}\right)\right) \frac{\theta_i}{(1+r)^2} - \frac{1}{(1+r)^2} \gamma \sigma_\varepsilon^2 f\left(-\frac{\theta_i}{((1+r) - \gamma) \left(\frac{\gamma}{r} \right)}\right) \end{aligned} \right]
\end{aligned}$$

The disclosure threshold for $t = 1$, θ_1^* , is such that the agent is indifferent between disclosing at $t = 1$ and obtaining $\theta_1^* + E[s_1] = \theta_1^*$ and the expected payoff from not disclosing at $t = 1$, given in (8). So the candidate for a disclosure threshold is the solution to:

$$\begin{aligned}
\theta_1^* &= [G(\theta_1^*)]^{N-1} \left(\frac{\theta_1^*}{1+r} \right) \\
& + \left(1 - [G(\theta_1^*)]^{N-1}\right) \left[\begin{aligned} & F\left(\frac{\theta_1^*}{((1+r) - \gamma) \left(\frac{\gamma}{r} \right)}\right) \frac{\theta_1^*}{1+r} + \frac{1}{1+r} \gamma \sigma_\varepsilon^2 f\left(-\frac{\theta_1^*}{((1+r) - \gamma) \left(\frac{\gamma}{r} \right)}\right) \\ & + \left(1 - F\left(\frac{\theta_1^*}{((1+r) - \gamma) \left(\frac{\gamma}{r} \right)}\right)\right) \frac{\theta_1^*}{(1+r)^2} - \frac{1}{(1+r)^2} \gamma \sigma_\varepsilon^2 f\left(-\frac{\theta_1^*}{((1+r) - \gamma) \left(\frac{\gamma}{r} \right)}\right) \end{aligned} \right]
\end{aligned}$$

■

Proof of Claim 1. From Lemma 2 we know that

$$V_2(\theta_i) = \int_{-\infty}^{s_2^*(\theta_i)} \left[\frac{\theta_i + \gamma^2 s_1}{(1+r)^2} - \frac{\theta_i + \gamma s_1}{1+r} \right] f(s_1) ds_1.$$

Since the discount rate is held constant, the first period threshold changes in γ according to the change in the option value and the change in θ_2^* . Taking the derivative of $V_2(\theta_i)$ with respect to γ and substituting $s_2^*(\theta_i) = -\frac{\theta_i r}{\gamma(1+r)-\gamma^2}$ we get

$$\begin{aligned} & \frac{\partial}{\partial \gamma} \int_{-\infty}^{s_2^*(\theta_i)} \left[\frac{\theta_i + \gamma^2 s_1}{(1+r)^2} - \frac{\theta_i + \gamma s_1}{1+r} \right] f(s_1) ds_1 \\ &= \int_{-\infty}^{s_2^*(\theta_i)} \frac{\partial}{\partial \gamma} \left[\frac{\theta_i + \gamma^2 s_1}{(1+r)^2} - \frac{\theta_i + \gamma s_1}{1+r} \right] f(s_1) ds_1 + \frac{\partial s_2^*(\theta_i)}{\partial \gamma} \left[\frac{\theta_i + \gamma^2 s_2^*(\theta_i)}{(1+r)^2} - \frac{\theta_i + \gamma s_2^*(\theta_i)}{1+r} \right] \\ &= \int_{-\infty}^{s_2^*(\theta_i)} \frac{\partial}{\partial \gamma} \left[\frac{\theta_i + \gamma^2 s_1}{(1+r)^2} - \frac{\theta_i + \gamma s_1}{1+r} \right] f(s_1) ds_1 + \frac{\partial s_2^*(\theta_i)}{\partial \gamma} \left[\frac{\theta_i - \frac{\theta_i r}{\frac{1}{\gamma}(1+r)-1}}{(1+r)^2} - \frac{\theta_i - \frac{\theta_i r}{(1+r)-\gamma}}{1+r} \right] \\ \frac{\partial}{\partial \gamma} V_2(\theta_i) &= \int_{-\infty}^{s_2^*(\theta_i)} \left[\frac{2\gamma s_1}{(1+r)^2} - \frac{s_1}{1+r} \right] f(s_1) ds_1 \\ &\quad + \left[\theta_i r (\gamma(1+r) - \gamma^2)^{-2} (1+r - 2\gamma) \right] \left[\frac{\theta_i - \frac{\theta_i r}{\frac{1}{\gamma}(1+r)-1}}{(1+r)^2} - \frac{\theta_i - \frac{\theta_i r}{(1+r)-\gamma}}{1+r} \right] \\ &= \int_{-\infty}^{s_2^*(\theta_i)} \left[\frac{2\gamma s_1}{(1+r)^2} - \frac{s_1}{1+r} \right] f(s_1) ds_1 + \left[\theta_i r (\gamma(1+r) - \gamma^2)^{-2} (1+r - 2\gamma) \right] [0] \\ &= \int_{-\infty}^{s_2^*(\theta_i)} \left[\frac{2\gamma s_1}{(1+r)^2} - \frac{s_1}{1+r} \right] f(s_1) ds_1 \end{aligned}$$

Next we show how the sign of $\frac{\partial \theta_1^*}{\partial \gamma}$ depends on the value of γ .

First note that for $\gamma = \frac{r+1}{2}$,

$$\begin{aligned} \frac{\partial}{\partial \gamma} V_2(\theta_i) &= \int_{-\infty}^{s_2^*(\theta_i)} \left[\frac{2\frac{r+1}{2}s_1}{(1+r)^2} - \frac{s_1}{1+r} \right] f(s_1) ds_1 \\ &= \int_{-\infty}^{s_2^*(\theta_i)} \left[\frac{s_1}{(1+r)} - \frac{s_1}{1+r} \right] f(s_1) ds_1 = 0. \end{aligned}$$

For $\gamma > \frac{r+1}{2}$, we have that:

$$\int_{-\infty}^{s_2^*(\theta_i)} \left[\frac{2\gamma s_1}{(1+r)^2} - \frac{s_1}{(1+r)} \right] f(s_1) ds_1 < 0.$$

And finally, for $\gamma < \frac{r+1}{2}$:

$$\int_{-\infty}^{s_2^*(\theta_i)} \left[\frac{2\gamma s_1}{(1+r)^2} - \frac{s_1}{(1+r)} \right] f(s_1) ds_1 > 0.$$

Since θ_2^* follows the same direction as the change in the option value, the behavior of θ_1^* can be characterized by the above. For example, for $\gamma < \frac{r+1}{2}$, since $\frac{\partial \theta_2^*}{\partial \gamma} > 0$ and $\frac{\partial}{\partial \gamma} V_2(\theta_i) > 0$, then $\frac{\partial \theta_1^*}{\partial \gamma} > 0$. I.e. since the option value increases in $\gamma < \frac{r+1}{2}$, the period 1 threshold will increase since it waiting becomes more valuable, while the cost of waiting, r , remains the same. Likewise, since the second period threshold increases in $\gamma < \frac{r+1}{2}$, the likelihood of taking advantage of the real option is increasing for fixed s_1 , thus making the real option more valuable, resulting in an increased period one threshold for fixed r . Both of these effects work in the same direction and hence the θ_1^* is increasing in $\gamma < \frac{r+1}{2}$. A similar argument applies for $\gamma > \frac{r+1}{2}$ and $\gamma = \frac{r+1}{2}$. ■

Proof of Claim 2. Recall that the disclosure threshold in the second period, $\theta_2^*(s_1)$, is independent of σ . In addition, for any θ the manager will disclose for any $s_1 > \mu_s = 0$. So, the manager will take advantage of the real option only for sufficiently low realizations of s_1 , which are all lower than the mean of s_1 .

An increase in σ , increases the probability that a manager that does not disclose at $t = 1$ will take advantage of the real option (and delay disclosure to $t = 3$). This however, is not sufficient to increase the incentive to delay disclosure at $t = 1$. A sufficient argument for the comparative static is to keep the threshold at $t = 1$ constant and to show that following an increase in σ the manager is no longer indifferent between disclosing and not disclosing for $\theta = \theta_1^*$ but rather strictly prefers not to disclose at $t = 1$.

A type θ_1^* will disclose at $t = 2$ either if the other manager did not disclose at $t = 1$ or if s_1 is lower than a threshold $s_2^*(\theta_i) = -\frac{\theta_i}{((1+r)-\gamma)\left(\frac{\gamma}{r}\right)}$. So the value from delaying disclosure comes only from realizations $s_1 < s_2^*(\theta_i) < 0$. First, note that following an increase in σ

the probability of a realization of $s_1 < s_2^*(\theta_i)$ increases, i.e., $\frac{\partial \Pr(s_1 < s_2^*(\theta_i))}{\partial \sigma} > 0$. Second, the expected value from delaying disclosure decreases in s_1 .

There exists a value s' such that for all $s_1 < s'$ the probability of such an s_1 increases in σ . If $s' > s_2^*(\theta_i)$ that completes the proof. For $s' < s_2^*(\theta_i)$, following an increase in σ the probability of $s_1 < s'$ increase where $\Pr(s_1 \in (s', s_2^*(\theta_i)))$ decreases. It can be shown that we can “shift” mass from realization $s_1 < s'$ to realization $(s_1 \in (s', s_2^*(\theta_i)))$ under the high variance distribution such that pdf for all $(s_1 \in (s', s_2^*(\theta_i)))$ will be identical to the distribution with the low variance. Note that any such shift decreases the expected value from delaying disclosure at $t = 1$. Since the cumulative distribution for $s_1 < s_2^*(\theta_i)$ is higher under the high variance distribution, following this “shifting procedure” for any $s_1 < s'$ the pdf under the new distribution is still higher than under the low variance distribution (since the overall mass for $s_1 < s_2^*(\theta_i)$ is higher for the high variance distribution). This implies that the option value under the high variance distribution is strictly higher than under the low variance distribution. ■

Proof of Proposition 1. Assume that in the case of indifference, the firm discloses. Note that when $r = 0$, we have no interior solution. The only equilibria are asymmetric equilibria. It is easy to show that these are equilibria and that no interior equilibrium exists—in any equilibrium in which firm j discloses with positive probability, type $\bar{\theta}_i$ is better off waiting with probability 1, as this gives her strictly higher expected utility over disclosing when $r = 0$. Note that there always exists an $r > 0$ in which we have the asymmetric equilibria.

Setting $G(\theta_{1,j}^*) = 0$, we have from Lemma 3 that, as $r \rightarrow 0$,

$$\begin{aligned}
& \lim_{r \rightarrow 0} G(\theta_j^*) \left(\frac{\theta_i^*}{1+r} \right) \\
& + (1 - G(\theta_j^*)) \left[F \left(\frac{\theta_i^*}{((1+r)-\gamma)(\frac{\gamma}{r})} \right) \frac{\theta_i^*}{1+r} + \frac{1}{1+r} \gamma \sigma^2 f \left(-\frac{\theta_i^*}{((1+r)-\gamma)(\frac{\gamma}{r})} \right) \right. \\
& \left. + \left(1 - F \left(\frac{\theta_i^*}{((1+r)-\gamma)(\frac{\gamma}{r})} \right) \right) \frac{\theta_i^*}{(1+r)^2} - \frac{1}{(1+r)^2} \gamma^2 \sigma^2 f \left(-\frac{\theta_i^*}{((1+r)-\gamma)(\frac{\gamma}{r})} \right) \right] \\
& = F(0) \frac{\theta_i^*}{1+r} + \frac{1}{1+r} \gamma \sigma^2 f \left(-\frac{r}{\gamma} \cdot \frac{\theta_i^*}{((1+r)-\gamma)} \right) \\
& + (1 - F(0)) \frac{\theta_i^*}{(1+r)^2} - \frac{1}{(1+r)^2} \gamma^2 \sigma^2 f \left(-\frac{r}{\gamma} \cdot \frac{\theta_i^*}{((1+r)-\gamma)} \right). \\
& = F(0) \theta_i^* + \gamma \sigma^2 f(0) + (1 - F(0)) \theta_i^* - \gamma^2 \sigma^2 f(0) \\
& = \theta_i^* + \gamma \sigma^2 f(0) - \gamma^2 \sigma^2 f(0) \\
& = \theta_i^* + \sigma^2 f(0) (\gamma - \gamma^2) \\
& = \theta_i^* + \sigma^2 \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{\mu}{2\sigma^2}} \gamma (1 - \gamma) \\
& = \theta_i^* + \frac{\sigma}{e \sqrt{2\pi}} \gamma (1 - \gamma).
\end{aligned}$$

Since $\gamma \in (0, 1)$ and $\sigma > 0$, the benefit of waiting in the limit is strictly positive. Hence, for all $\sigma > 0$ and $\gamma \in (0, 1)$, we can find r sufficiently close to zero such that an asymmetric equilibrium can be supported when $G(\theta_{1,j}^*) = 0$. Recall that the upper bound of the asymmetric equilibria is denoted by r^L . Now for any $r > r^L$, type $\bar{\theta}_{-j}$ still finds disclosure profitable even when $G(\theta_{1,j}^*) = 0$, and hence the asymmetric equilibria do not exist for $r > r^L$. Finally, as $r \rightarrow \infty$, the payoff from waiting to disclose goes to zero. For θ with bounded support, we can find an $r < \infty$ such that $\theta_i^* \leq \bar{\theta}$ when $G(\theta_j^*) = 0$. Denote the maximum r that supports this equilibrium as r^H :

$$r^H = \max_r \left\{ \begin{aligned} & \Pr \left(s_1 > \frac{r}{\gamma} \cdot \frac{-\theta_i}{((1+r)-\gamma)} \right) E \left[\frac{\theta_i + s_2}{1+r} \mid s_1 > \frac{r}{\gamma} \cdot \frac{-\theta_i}{((1+r)-\gamma)} \right] \\ & + \left(\Pr \left(s_1 < \frac{r}{\gamma} \cdot \frac{-\theta_i}{((1+r)-\gamma)} \right) \right) E \left[\frac{\theta_i + s_3}{(1+r)^2} \mid s_1 \leq \frac{r}{\gamma} \cdot \frac{-\theta_i}{((1+r)-\gamma)} \right] < \bar{\theta} \end{aligned} \right\}.$$

Which we know exists by Theorem 1. ■

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