A Theory of Risk Disclosure

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1 Introduction

In the wake of the recent financial crisis, regulatory authorities are increasing the pressure on firms to disclose information about the riskiness of their cash flows. For example, the SEC approved rules that require firms to issue disclosures on compensation practices that could lead employees to take on excessive risks in 2009. Moreover, the FASB proposed an accounting standards update (Topic 285) in 2010. This update would require firms to disclose information about liquidity and interest rate risks, stating "users of financial statements overwhelmingly indicated that [. . .] understanding a reporting entity's exposures to risks that are inherent in financial instruments and the ways in which reporting entities manage these risks is integral to making informed decisions about capital allocation".[1] For there to be any value in the disclosure of information regarding risks it has to be the case that investors are uncertain of the riskiness of firms' cash flows. Generally, however, the existing theoretical literature assumes that the riskiness of cash flows is known, and focuses on the effects of disclosure regarding expected cash flows.[2] In this paper, we relax this assumption and analyze the capital market effects of risk disclosure.

In our model, investors are uncertain about the variance of a firm's cash flows. Before trading in the firm's shares, the firm discloses a noisy report about its risks and investors update their beliefs. To our knowledge, we are the first to provide a model of imperfect risk disclosures. The most similar study to ours is Jørgensen and Kirschenheiter (2003) which focuses on the discretionary disclosure of a perfect revelation of the variance. Jørgensen and Kirschenheiter find that mandatory disclosure requirements will increase firms' expected betas, as it forces some firms to suboptimally incur a disclosure cost.

We build our model around common assumptions in the disclosure literature; cash flows follow a normal distribution and investors maximize a negative exponential utility function.

[1]More recently, the Enhanced Disclosure Task Force issued an extensive report recommending several improvements in the risk disclosure of banks, claiming that "investors and other public stakeholders are demanding better access to risk information from banks; information that is more transparent, timely and comparable across institutions."

[2]See Verrecchia 2001 or Beyer, Cohen, Lys, and Walther 2010 for surveys of the disclosure literature.

Within this framework, we investigate cost of capital implications of the imperfect risk disclosure and highlight costs and benefits to mandatory risk disclosure requirements. Along the lines of Gron, Jørgensen, and Polson (2011) we first show that when price is endogenously derived from trade between investors with negative exponential utility functions, there is a "variance uncertainty" premium for dispersion in variance in addition to the standard risk premium. The intuition driving this result is that variance uncertainty creates "fat tails" in the distribution of cash flows. The disutility investors experience from risk increases at an increasing rate, analogous to the standard concavity result for the mean of cash flows. As a result, these investors are averse to "fat tailed" distributions. In the economics literature that studies risk attitudes, such an aversion is referred to as temperance (e.g., Crainich, Eeckhoudt, and Trannoy, 2013). This suggests that risk disclosure may reduce the cost of capital and provides an economic rationale for the FASB's statement that understanding a firm's riskiness is critical to efficient capital allocation. To investigate further, we derive a closed form expression for prices assuming a gamma distributed variance, and find that prices are a function of the mean of cash flows, the mean of the variance of cash flows, and the variance of the variance of cash flows.

We next investigate how price reacts to noisy signals of the variance, and find results that carry much of the intuition from the prior literature on mean-based disclosures. In particular, more precise signals receive greater weight, and signals receive greater weight when investors are more risk averse. Variance signals always reduce the variance uncertainty premium that investors place on the firm, since investors' perceived distribution over the variance narrows. However, the net effect on share price will depend on the realization of the variance signal, since, for example, a very high signal will increase investors' perception of the mean of the variance distribution. This is analogous to the classical result that mean-based signals reduce the risk premium that investors apply, but these signals' total effect on price depends on their realization relative to the market's prior.

From an ex ante perspective, risk disclosures reduce the variance uncertainty premium that investors impose on the firm, but have no impact on the risk premium (since the cash flow risk itself does not change in expectation). This implies that firms can reduce their cost of capital by making a commitment to disclose information concerning cash flow variance. Thus, we offer theoretical evidence in support of the Enhanced Disclosure Taskforce's statement that "by enhancing investors' understanding of banks' risk exposures and risk management practices, high-quality risk disclosures may reduce uncertainty premiums and contribute to broader financial stability." We perform comparative statics on this cost of capital reduction, and show that it increases in the prior uncertainty over the variance, investors' risk aversion, and the prior mean of the variance distribution.

Outside of Jørgensen and Kirschenheiter (2003) we are not aware of any theoretical work that examines the capital markets effects of risk disclosure. However, a subset of the accounting literature has considered the impact of disclosure about expected cash flows when investors face uncertainty over the variance of cash flows (Beyer 2009) or the precision of the accounting signal (Hughes and Pae 2004, Kirschenheiter and Melumad 2001, and Subramanyam 1995). This literature is based on the statistical literature on Bayesian updating with uncertain precisions and typically models information as true cash flows plus a noise term. Therefore, the disclosed report resembles an earnings announcement or the realization of historical cash flows. Investors in these models are able to use noisy signals of cash flows to indirectly update on the variance of cash flows or the precision of the firm's signal. In particular, when a firm discloses a signal that substantially deviates from the market's prior, investors infer that the variance of cash flows is high. Furthermore, the prior disclosure literature on uncertainty over the variance of cash flows or precision of firms' information exogenously specifies that prices are linear in the mean and/or in the expected variance of cash flows.[3] This assumption implies that the uncertainty over variances cannot have an

[3]See, for example, Beyer (2009), Hughes and Pae (2004), Kirschenheiter and Melumad (2001), Penno (1995), and Subramanyam (1995). Modeling variance disclosure as a direct signal regarding the variance demands a suitable nonnegative distribution for the variance, a conjugate prior for that distribution, and a utility function that yields a closed form solution with these distributions. Therefore, we believe that prior literature has assumed risk neutral or mean variance pricing for tractability purposes. While this is suitable for the settings these papers examine, our focus is on the pricing of variance uncertainty and the effect of

impact on price. We argue that disclosures of this type do not capture the direct disclosure of risks that regulators and investors demand.

Recent empirical work examines the information content of risk disclosure in firms' 10-K filings (for example, Bao and Datta 2014; Campbell, Chen, Dhaliwal, Lu, and Steele 2014; and Hope, Hu, and Lu 2014). Our paper offers a theoretical rationale for the existing findings that prices respond to risk disclosure. Furthermore, we predict that prices react more strongly to risk disclosure when prior uncertainty is high and that firms with greater uncertainty over the variance have increased incentives to disclose variance information. Our model also predicts that firms which commit to disclose information regarding their risks earn returns closer to the risk free rate, ceteris paribus. Campbell et al. (2014) offers empirical evidence for this result, finding that risk disclosure is associated with greater firm value.

We extend our study of risk disclosure to more general settings in order to generate additional empirical predictions regarding the value of risk disclosure and to better understand the role of regulation. First, we consider the interaction between disclosures regarding the mean and the variance. Our model suggests that mean and variance disclosures are substitutes. This suggests that regulations which mandate variance disclosure may have the unintended consequence of reducing voluntary mean disclosure. In a second application, we consider a setting where the firm sequentially acquires information regarding its risks. The firm can first learn and disclose one piece of variance information, and, based on that information, choose whether to continue information acquisition and disclosure. This process continues until the firm no longer wishes to acquire additional information. We show that in this setting, firms expend greater effort on learning when they receive bad, i.e., high variance, news. This implies that firms' financial statements will contain more information on risks when their cash flows are more risky. This is distinct from the result that greater *prior* uncertainty over the variance leads to more disclosure, and is a key difference between our results and those for mean-based disclosure with normal distributions. In line with our

risk disclosures.

predictions, Campbell et al. (2014) and Hope et al. (2014) find that firms with greater risks disclose more risk information in their 10-K's. Finally, we consider a multi-asset market. In this model, we show that prices continue to contain an additional risk premium for variance uncertainty over the common factor; however, the variance uncertainty premium for idiosyncratic risk vanishes as the economy grows large. We show that in order for disclosure to impact the cost of capital in this setting, it must contain information on systematic risk. Moreover, when disclosure contains information regarding the common risk factor, it reduces the risk discount of *all* firms and thus has positive externalities.

Our model is related to the literature on ambiguity aversion, which investigates uncertainty over the distribution of cash flows. A common assumption in this literature is that investors apply a discount to the expected cash flows either by operating under the most pessimistic distribution from a specified set of possible distributions (see Garlappi, Uppal, and Wang 2007; Gibloa and Schmeidler 1993; and Illeditsch 2011), or by applying a concave transformation to a specified set (see Caskey 2009). For example, Illeditsch (2011) considers the effect of ambiguity over the precision of a public signal on investors' portfolio allocation problem. Our paper supplements this literature by endogenously deriving the discount that investors apply when distributional uncertainty is over the variance of cash flows instead of assuming that such a discount exists and takes a particular form. This allows us to analyze what drives the discount and show that the discount may be reduced through risk disclosure.

Finally, our paper relates to the literature on estimation risk (Barry and Brown 1985; and Coles, Loewenstein, and Suay 1995). Barry and Brown (1985) examines the difference in betas that results when investors must estimate the mean and covariance matrix of returns. They find that betas are higher for firms with greater variance uncertainty. Barry and Brown (1985) does not directly model price formation, but rather assumes that returns are exogenous and that beta is the metric of importance when evaluating the effect of variance uncertainty. On the contrary, our multiasset model shows that beta alone is not sufficient to capture the effects of variance uncertainty on prices. Coles, Loewenstein, and Suay (1995) show that the CAPM does not hold in its traditional form when investors face estimation risk over the mean and variance of cash flows. Coles et al. (1995) assumes that an investor's expected utility is increasing in the mean and decreasing in the variance of cash flows, which, again, implies that uncertainty over the variance is not priced.

The remainder of the paper is organized as follows. Section 2 develops our core model, deriving prices under variance uncertainty, price responses to disclosure, and the effect of an ex ante commitment to disclosure on the cost of capital. Section 3 considers the interaction between mean and variance disclosure. Section 4 considers sequential learning and disclosure and highlights a key difference between our model and standard mean-based models of disclosure. Section 5 extends the single asset model to a multiple asset setting.

2 A Single Asset Model

2.1 Pricing

We consider a single period economy with two assets: a riskless asset with a price of 1, and a risky asset with a per-share price of P. There is a continuum of homogeneous risk-averse investors in the economy who have negative exponential utility, $u = -\exp[-\rho w]$, with risk aversion parameter ρ and terminal wealth w. The riskless bond has an unlimited supply while we normalize the per capita supply of the risky asset to 1. Conditional on the variance, the per-share payoff to the risky asset, $\tilde{x}$, is normally distributed with mean μ and variance $\tilde{V}$.[4] The distributional assumptions in our model differ from much of the prior literature on disclosure in that we assume that $\tilde{V}$ is unknown to investors and follows a gamma distribution. The gamma distribution is typically parameterized by a shape parameter a and a scale parameter b, with mean $\mu_V \equiv \frac{a}{b}$ and variance $\sigma_V^2 \equiv \frac{a}{b^2}$. In the main text, we parameterize the gamma distribution by its mean and variance in order to provide better intuition for our comparative statics. In particular, we assume that $\tilde{V}$ has the following

[4] We denote random variables with a tilde "~".

density function:

$$f(V) = \frac{\left(\frac{\mu_V}{\sigma_V^2}\right)^{\frac{\mu_V^2}{\sigma_V^2}} V^{\frac{\mu_V^2}{\sigma_V^2}-1} e^{-V\frac{\mu_V}{\sigma_V^2}}}{\Gamma\left(\frac{\mu_V^2}{\sigma_V^2}\right)} \text{ for } V \geq 0 \tag{1}$$

It is easily checked that $E\left(\tilde{V}\right) = \mu_V$ and $Var\left(\tilde{V}\right) = \sigma_V^2$.[5] We assume a gamma distribution for the variance as it has been widely used in the statistics literature.[6] Furthermore, as we will show, the gamma distribution yields a closed form solution for prices when combined with negative exponential utility.

Our assumption of an uncertain variance implies that the unconditional distribution of cash flows exhibits excess kurtosis (or "fat tails") relative to a normal distribution with a known variance.[7] This can be seen by computing excess kurtosis, defined as the fourth standardized moment minus 3 (where 3 is the kurtosis of a normal distribution):

$$\frac{E\left[\left(\tilde{V}-\mu\right)^4\right]}{\left(E\left[\left(\tilde{V}-\mu\right)^2\right]\right)^2} - 3 = 3\frac{\sigma_V^2}{\mu_V^2}. \tag{2}$$

This implies that the probability that cash flows take on extreme values is greater when uncertainty about the variance exists. Figure 1 compares a normal distribution with uncertain, gamma distributed variance to a normal distribution with known variance.

The intuition for many of our results is as follows: investors with negative exponential utility apply a discount to cash flow distributions that exhibit kurtosis as they have a distaste for extremely bad outcomes. This is generally true for utility functions that have a negative 4^{th} derivative (for example, power utility), or, in other words, utility functions The economics

[5] Characterizing the gamma distribution by its mean and variance creates the following restriction: $\mu_V = 0 \Longleftrightarrow \sigma_V^2 = 0$. This occurs because a zero mean implies the distribution is degenerate at zero.

[6] The inverse gamma is widely used as a conjugate prior for the variance of a normal distribution when signals are drawn from a normal-gamma distribution (see DeGroot 1970). We choose to examine the gamma distribution rather than the inverse gamma distribution as the moment generating function for an inverse gamma does not exist.

[7] While the fat tails that follow from the uncertain variance seemingly map to the empirical findings in Mandelbrot (1963) and Fama (1965), those studies suggest that stock *returns* exhibit fat tails whereas our result implies that cash flows themselves exhibit fat tails.

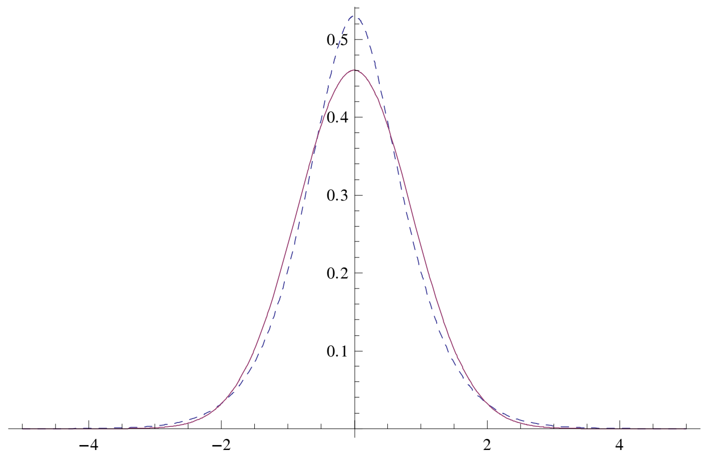

Figure 1: Dashed - Normal distribution with variance uncertainty; Solid - Normal distribution conditional on the variance equal to its expectation

literature on risk preferences refers to agents with such utility functions as "temperate" and refer to the negative ratio of the 4^{th} and 3^{rd} derivative, $-u^{''''}(x)/u^{'''}(x)$, as the coefficient of absolute temperance.[8] Note that we assume investors maximize a negative exponential utility where the coefficient of absolute temperance equals the coefficient of absolute risk aversion. Given that variance disclosure reduces the variance uncertainty, it also reduces the excess kurtosis (i.e., the fat tails phenomenon in figure 1), and, thus, the discount that investors apply.

We begin by deriving prices in the absence of disclosure. Several prior papers that feature uncertainty over the second moment have assumed either risk neutral or mean-variance utility (for example, Beyer 2009; Subramanyam 1995; and Kirschenheiter and Melumad 2001). Thus, these papers exogenously impose that prices do not have a component related to the uncertainty over variance. However, when investors have negative exponential utility, their certainty equivalents decrease in uncertainty over the variance (see Gron et al. 2011 for a general characterization of aversion to variance uncertainty). This is true for an arbitrary distribution of the variance and is a result of the preferences for temperance. In the appendix,

[8] See, for example, Eeckhoudt et al. (1996), Gollier and Pratt (1996), and Noussair et al. (2014). Noussair et al. (2014) also present experimental evidence that suggests that individuals are temperate.

we show that an investor's certainty equivalents given demand D reduces to the following:

$$CE(D,P) = \rho D(\mu - P) - \ln\left(E\left(e^{D^2\frac{\rho^2}{2}\tilde{V}}\right)\right). \tag{3}$$

Note that Jensen's inequality implies that $\ln\left(E\left(e^{D^2\frac{\rho^2}{2}\tilde{V}}\right)\right) > D^2\frac{\rho^2}{2}E\left(\tilde{V}\right)$. Thus, an investor's certainty equivalent is reduced when a mean preserving spread is applied to the variance. As a result, we should expect to find that prices decrease as uncertainty over the variance increases. Essentially, the reason for this result is that each additional point of variance becomes increasingly painful to an investor, as her expected utility is concave in the variance.[9] Hence, investors are averse to greater kurtosis or, temperate.

In order to provide the market clearing price, we have to assume that $\frac{1}{2}\rho^2\frac{\sigma_V^2}{\mu_V} < 1$. Technically, this condition is necessary because the gamma distribution is only defined over non-negative values.[10] Furthermore, the condition prevents a situation where investors have infinite negative utility when they hold their share of the per capita endowment; if the condition did not hold, no price would allow for the market to clear.[11] Proposition 1 shows that the concavity with respect to variance in the the investors' leads to an uncertainty premium in price.

Proposition 1 *Assume that* $\frac{1}{2}\rho^2\frac{\sigma_V^2}{\mu_V} < 1$. *The firm's price can be expressed as:*

$$\begin{aligned} P &= \mu - RP_0 - VUP_0 && (4)\\ \textit{where } RP_0 &= \rho\mu_V && (5)\\ \textit{and } VUP_0 &= \frac{\frac{1}{2}\rho^2\frac{\sigma_V^2}{\mu_V}}{1-\frac{1}{2}\rho^2\frac{\sigma_V^2}{\mu_V}}\rho\mu_V && (6)\end{aligned}$$

[9] It is easily seen that $\frac{\partial^2}{\partial V^2}E\left(-e^{-\rho\tilde{x}}\right) = \frac{\partial^2}{\partial V^2}\left(-e^{-\rho\mu-\frac{\rho^2}{2}V}\right) < 0$.

[10] The Gamma distribution with shape parameter a and a scale parameter b is only defined for $a > 0$ and $b > 0$. After developing an investor's certainty equivalent, we need $b > \frac{D^2\rho^2}{2}$ has to hold. That is, the scale parameter has to be sufficiently large or the equilibrium demand (that is, the shares per capita) need to be sufficiently small. We derive an investor's certainty equivalent with the standard parameterization in the proof to Proposition 1.

[11] If the per capita endowment were an arbitrary constant e rather than 1, the condition would become $\rho^2 e^2 < 2\frac{\mu_v}{\sigma_v^2}$. See the appendix for more details.

The price function in Proposition (1) reduces to the expected cash flow minus the standard risk premium minus an additional term for uncertainty over the variance which we label the "variance uncertainty premium" or VUP_0. We define the "total risk premium" as the sum of the risk premium and the variance uncertainty premium. Note that the variance uncertainty premium equals the risk premium multiplied by an inflation factor that is related to the excess kurtosis of the cash flow distribution. When there is no risk in the asset, i.e., $\mu_V = 0$, which implies $\sigma_V^2 = 0$, both the risk premium and the variance uncertainty premium vanish, and prices are equal to the mean. When there is no excess kurtosis in the asset, the variance uncertainty premium reduces to zero but the risk premium remains.

The inflation factor is increasing and convex in risk aversion and σ_V^2. Note that for a negative exponential utility function, the coefficient of absolute risk aversion, ρ, is equal to the coefficient of absolute temperance, $-u'''' (x) / u''' (x) = \rho$. Note further that while σ_V^2 increases excess kurtosis, μ_V decreases excess kurtosis. The variance uncertainty premium behaves accordingly. Intuitively, as ρ grows, investors become more averse to "low tail" events, and as σ_V^2 grows, the probability of these extreme events increases. Furthermore, an increase in μ_V increases the kurtosis of the underlying normal distribution and the premium that investors demand for this is embedded in the standard risk premium, $\rho\mu_V$. We discuss comparative statics in the following corollary:

Corollary 1 *Prices are (i) decreasing in the variance of the cash flow variance and risk aversion; (ii) decreasing in the mean of the variance for* $\mu_V > \rho\sigma_V^2$ *and increasing in the mean of the variance for* $\mu_V > \rho\sigma_V^2$*; (iii) decreasing in* μ_V *at an increasing rate; and (iv) uniformly decreasing in a location shift in the variance.*

The results in Corollary 1 (i) are straightforward given the previous discussion. The results in part (ii) are non-monotonic because, holding σ_V^2 constant, an increase in the expected variance, μ_V, has two effects. First, as usual, an increase in μ_V directly increases the risk premium. Second, however, it decreases the excess kurtosis which decreases the variance uncertainty premium. The direct effect on the risk premium dominates for $\mu_V > \rho\sigma_V^2$ such that the total risk premium is increasing in the expected variance. For $\mu_V < \rho\sigma_V^2$, the reduction in the variance uncertainty premium dominates, and price increases in μ_V. Note that for a distribution that only takes on positive values for $V \in [0, \infty)$ changing the mean while keeping the variance constant requires a change in the shape of the distribution. That is, μ_V is not a simple location parameter as it is for normal distributions, and thus does not uniformly reduce the investor's valuation of the distribution.[12] On the other hand, as we show in part (iv), a location shift of the form $\tilde{V}' = \tilde{V} + k$, for $k > 0$, strictly reduces prices as it only impacts the risk premium; this occurs because the location shift increases the distribution of $\tilde{V}$ in the sense of first order stochastic dominance.

To understand more generally how prices respond to shifts in the distribution, consider changes in the variance distribution in the sense of first and second order stochastic dominance (FSD and SSD respectively). We should expect that distributional shifts in $\tilde{V}$ in the sense of FSD reduce price, and distributional shifts in $\tilde{V}$ in the sense of SSD increase price. Ali (1975) derives the following necessary and sufficient conditions for first and second order stochastic dominance for the gamma distribution characterized by shape and rate a and b:[13]

$$\tilde{V}_1 \underset{FSD}{\succ} \tilde{V}_2 \text{ when } a_1 \geq a_2 \text{ and } b_1 \leq b_2 \text{ with one equality strict;} \tag{7}$$

$$\tilde{V}_1 \underset{SSD}{\succ} \tilde{V}_2 \text{ when } \frac{a_1}{a_2} \geq Max\left(1, \frac{b_1}{b_2}\right) \tag{8}$$

Expressing prices in terms of a and b, we find:

$$P = \mu - \frac{a}{b}\rho - \frac{\rho^2}{2b - \rho^2}\frac{a}{b}\rho \tag{9}$$

A shift in the distribution of $\tilde{V}$ in the sense of FSD involves either increasing a or decreasing b; in either case, price falls. A shift in the distribution of $\tilde{V}$ in the sense of SSD is achieved

[12] Increases in the mean holding the variance fixed reduce the degree of positive skew in the distribution.

[13] Note that Ali (1975) refers to the shape parameter as β and to the rate parameter as α whereas we refer to them as a and b, respectively. In particular, here and in the appendix we refer to a gamma distribution with PDF $\frac{b^a V^{a-1} e^{-xb}}{\Gamma(a)}$.

by either increasing b and increasing a by at least the same percentage, or by decreasing b and weakly increasing a. In either case, price increases as expected.[14]

Our results imply that empirically, both assets with higher variance and assets with more uncertainty about their variance should earn higher returns. This can act as a correlated omitted variable in empirical studies that consider the pricing of information. In the next section, we analyze how prices react to information announcements regarding their variance.

2.2 Risk Disclosure

This section introduces noisy disclosures of risk. Within this setting we derive the response coefficient to risk disclosure as an equivalent to the earnings response coefficient in the prior disclosure literature. The firm's price in eqn. (4) suggests that a disclosure of the firm's variance of cash flows affects price. To keep prices before and after the disclosure comparable the disclosure requires a likelihood function which has the gamma distribution as a conjugate prior such that prices before and after the signal have the same structural form. There exist two well recognized distributions which have this property: the Poisson distribution with unknown mean parameter, and the gamma distribution with known shape and unknown rate parameters (see Fink 1997). We employ the Poisson likelihood as the gamma likelihood does not yield analytically tractable results. Furthermore, the Poisson distribution has many desirable properties when combined with a gamma prior that resemble those of the standard combination of normal likelihood and normal prior.

We assume that the firm discloses signal $\tilde{S}$ which is equal to the mean of τ Poisson distributed random variables with mean $\tilde{V}$. Furthermore, the underlying signals are independent conditional on $\tilde{V}$. In the appendix we show that the mean of these signals is a sufficient statistic for their individual realizations. We can view the number of signals τ as a measure of the precision of $\tilde{S}$ since the variance of the signals (conditional on $\tilde{V}$) is decreas-

[14] The comparative static with respect to σ_V^2 in effect increases b while increasing a at the same rate. Eqn. (9) indicates that this increases prices only through its impact on the variance uncertainty premium.

ing in τ: $Var\left(\tilde{S}(\tau)|\tilde{V}\right) = Var\left(\tau^{-1}\Sigma_{i=1}^{\tau}\tilde{s}_i|\tilde{V}\right) = \tau^{-1}\tilde{V}$. Applying results from Bayesian statistics, one can show that $\tilde{V}|\tilde{S}$ is gamma distributed. In the following lemma, we express the conditional mean and variance in terms of the prior mean and variance, the signal, and the precision parameter.

Lemma 1 *The conditional mean and conditional variance of the variance distribution given the signal $\tilde{S}$ are equal to:*

$$E\left(\tilde{V}|\tilde{S}\right) = E\left(\tilde{V}\right) + \frac{Cov\left(\tilde{V},\tilde{S}\right)}{Var\left(\tilde{S}\right)}\left(\tilde{S} - E\left(\tilde{S}\right)\right) \text{ and} \tag{10}$$

$$Var\left(\tilde{V}|\tilde{S}\right) = Var\left(\tilde{V}\right) - \frac{Cov\left(\tilde{V},\tilde{S}\right)^2}{Var\left(\tilde{S}\right)} + \left(\frac{Cov\left(\tilde{V},\tilde{S}\right)}{Var\left(\tilde{S}\right)}\right)^2 \frac{\tilde{S} - \mu_V}{\tau} \tag{11}$$

where $Var\left(\tilde{V}\right) = Cov\left(\tilde{V},\tilde{S}\right) = \sigma_V^2$ *and* $Var\left(\tilde{S}\right) = \sigma_V^2 + \tau^{-1}\mu_V$.

As in the case of the normal prior and normal likelihood, Lemma 1 shows that the expected variance is linear in the signal and that the signal receives greater weight as precision τ increases. Furthermore, the coefficient on $\tilde{S}$ is equal to the regression coefficient $\frac{Cov(\tilde{V},\tilde{S})}{Var(\tilde{S})}$. If the signal is equal to its prior mean, μ_V, there is no updating on the mean, but the variance is reduced by $\frac{Cov(\tilde{V},\tilde{S})^2}{Var(\tilde{S})}$.

In a setting with normal distributions the conditional variance after receiving an information signal does not depend upon that signal's realization. However, Lemma 1 shows that the conditional uncertainty about the variance is not constant but, similar to the mean, is linearly increasing in the signal realization. This is a natural consequence of the fact that the distribution of the variance is constrained to be nonnegative. To demonstrate this point, consider what happens in the knife-edged case where investors receive a signal $\tilde{S} = 0$. As a zero mean has to imply a zero variance for a nonnegative distribution, all variance uncertainty disappears. Thus, low means tend to be associated with low variances for nonnegative

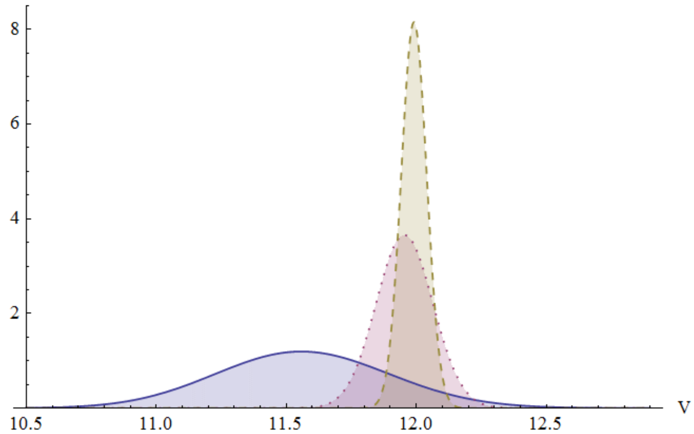

Figure 2: This figure depicts the variance distribution for $\tau = 100$, 1000, and 5000. The tighter distributions reflect greater τ.

distributions.[15] We will see that this feature results in differences between the results for mean disclosure and our results for risk disclosure.

Since the expected value of S is given by μ_V, it has to be the case that

$$E\left(Var\left(\tilde{V}|\tilde{S}\right)\right) = Var\left(\tilde{V}\right) - \frac{Cov\left(\tilde{V},\tilde{S}\right)^2}{Var\left(\tilde{S}\right)}. \tag{12}$$

In other words, the expected conditional variance is strictly lower than the unconditional variance and the difference is increasing in the precision of the signals. As $\tau \to \infty$, the expected conditional variance approaches 0. Figure 2 depicts the variance distribution as the number of signals increases, given that the mean of these signals is equal to their prior mean.[16] Although not apparent from the diagram, all three distributions have the same mean; the skewness of the gamma distribution obscures this fact. The tighter variance distributions correspond to cash flow distributions with less kurtosis.

Proposition 2 derives the firm's price conditional on the risk disclosure. Note that the necessary condition from Proposition (1), $\frac{1}{2}\rho^2\frac{\sigma_V^2}{\mu_V} < 1$, is relaxed after the risk disclosure and

[15] In the our model, there is both an increase in the amount of noise in the signal and in the assessed underlying variance distribution such that the conditional mean remains linear in the signal.

[16] Setting the mean of the signals equal to their prior mean isolates the uncertainty reduction effect of information from any effect due to a change in the posterior expectation of the variance distribution.

becomes $\frac{1}{2}\rho^2\frac{\sigma_V^2}{\mu_V} < 1 + \tau\frac{\sigma_V^2}{\mu_V}$.

Proposition 2 *Assume that* $\frac{1}{2}\rho^2\frac{\sigma_V^2}{\mu_V} < 1$. *The firm's price conditional on the risk disclosure can be expressed as:*

$$P\left(\tilde{S}\right) = \mu - RP_0 - \phi(\tau) VUP_0 - \alpha(\tau)\left(\tilde{S} - E\left(\tilde{S}\right)\right), \tag{13}$$

$$\text{where } \phi(\tau) = \frac{1 - \frac{1}{2}\rho^2\frac{\sigma_V^2}{\mu_V}}{1 - \frac{1}{2}\rho^2\frac{\sigma_V^2}{\mu_V} + \tau\frac{\sigma_V^2}{\mu_V}} \tag{14}$$

$$\text{and } \alpha(\tau) = \rho\frac{\tau\frac{\sigma_V^2}{\mu_V}}{1 - \frac{1}{2}\rho^2\frac{\sigma_V^2}{\mu_V} + \tau\frac{\sigma_V^2}{\mu_V}}. \tag{15}$$

To demonstrate the effect of a disclosure on price, consider the price reaction to a signal that is equal to its prior mean. While the risk premium does not change (due to the constant expected variance), the variance premium is multiplied by a factor which is less than one and decreasing in τ. Intuitively, even when a risk disclosure has no mean effect, it reduces the ex post uncertainty about the variance in proportion to its precision, and thus increases price. In the limit, as $\tau \to \infty$, the variance uncertainty premium disappears.

Next, consider the price reaction to a signal that deviates from the prior mean. Similar to the result for normal distributions, price responds linearly to the deviation of a signal from its prior mean. Furthermore, the strength of the linear response is increasing in the precision τ (it is easily checked that $\alpha'(\tau) > 0$). However, note that for mean disclosures with normal distributions, the signal's *realization* impacts the mean, but has no effect on the risk premium. In contrast, for risk disclosures the realization of the signal itself affects both the risk premium and the variance uncertainty premium. To illustrate, we rewrite $P\left(\tilde{S}\right)$ from Proposition (2) to absorb the term $\alpha(\tau)\left(\tilde{S} - E\left(\tilde{S}\right)\right)$ into the risk and variance uncertainty premia:

$$P\left(\tilde{S}\right) = \mu - \rho E\left(\tilde{V}|\tilde{S}\right) - \frac{\frac{1}{2}\rho^2\frac{\sigma_V^2}{\mu_V}}{1 - \frac{1}{2}\rho^2\frac{\sigma_V^2}{\mu_V} + \tau\frac{\sigma_V^2}{\mu_V}}\rho E\left(\tilde{V}|\tilde{S}\right). \tag{16}$$

Note that $\tilde{S}$ affects both the risk premium and the variance premium through its impact on $E\left(\tilde{V}|\tilde{S}\right)$. This arises because the uncertainty over the variance is increasing in the signal's realization. Further, note that the price response to the signal $\alpha(\tau)$ is increasing in risk aversion ρ (higher risk aversion increases the importance of changes in the expected variance) and increasing in the ratio of the prior variance to the prior mean $\frac{\sigma_V^2}{\mu_V}$. An increase in $\frac{\sigma_V^2}{\mu_V}$ increases the signal to noise ratio of $\tilde{S}$. That is, the total amount of uncertainty to be resolved divided by the expected noise in the signal, $\frac{Var(\tilde{V})}{E(Var(\tilde{S}|\tilde{V}))} = \frac{1}{\tau}\frac{Var(\tilde{V})}{E(\tilde{V})} = \frac{1}{\tau}\frac{\sigma_V^2}{\mu_V}$ increases in $\frac{\sigma_V^2}{\mu_V}$. The expected noise in the signal, $E\left(Var\left(\tilde{S}|\tilde{V}\right)\right)$, is increasing in μ_V due to the correlation between mean and variance. Intuitively, when the expected cash flow variance is high, we expect to receive a noisier signal. On the other hand, when σ_V^2 is higher, there is more uncertainty to be resolved, that is, the prior precision is lower.

2.3 Cost of Capital Effects

The discussion of Proposition (2) suggests that there is a price response even when the signal merely confirms the prior expectations. This implies that risk disclosure should have an effect on the firm's expected cost of capital. As an extension to the last section, we now examine the *ex ante* effects of risk disclosure, i.e., the impact on the cost of capital. Following prior literature, we define the cost of capital as the discount that is applied to price relative to expected cash flows, that is, $E[\tilde{x}] - E(P|\Theta)$, where Θ is all information available to the market. Taking the expectation of price conditional on the signal, we find that:

$$E[\tilde{x}] - E\left(P\left(\tilde{S}\right)\right) = RP_0 + \phi(\tau)\, VUP_0. \tag{17}$$

The cost of capital is a decreasing function of τ since $\phi'(\tau) < 0$. Moreover, the effect of τ on the cost of capital is increasing in risk aversion and increasing in the ratio of the prior variance to the prior mean of the variance distribution, $\frac{\sigma_V^2}{\mu_V}$. Intuitively, the value of reducing uncertainty over the variance is greater when the price responsiveness to the signal is larger.

As one would expect, $E\left(P\left(\tilde{S}\right)\right)$ is decreasing in τ at a decreasing rate, such that as

$\tau \to \infty$, the variance uncertainty premium disappears and expected price is exactly equal to the mean less a discount for the prior expectation of risk. Concavity of the price effect in τ is a desirable attribute, as it suggests that firms which act to maximize their expected price in general will not find it optimal to fully acquire and/or disclose risk information even if the cost of doing so is linear. Our results therefore imply that firms acquire great benefit from acquiring and disclosing at least some variance information. Moreover, our results provide a theoretical rationale for the regulatory efforts of the SEC and FASB if legal mandates provide a mechanism for firms to commit to disclosure.

The following corollary summarizes the cost of capital effects of risk disclosure.

Corollary 2 *A firm's cost of capital is decreasing at a decreasing rate in the precision* τ *of its disclosure. The effect of* τ *on the cost of capital is increasing in the ratio* $\frac{\sigma_V^2}{\mu_V^2}$ *and increasing in risk aversion* ρ*.*

While our model captures disclosure about the uncertain variance of cash flows, a related issue is disclosure with an uncertain precision in a setting with a constant cash flow variance (as in Subramanyam, 1996). A random disclosure precision implies that from the investors' perspective there is uncertainty about the conditional cash flow variance. Corollary 2 suggests that, ex ante, it is beneficial for firms to commit to a constant disclosure precision because it reduces the cost of capital.

3 The Interaction between Mean and Variance Disclosure

In this section, we consider whether firms face trade-offs between disclosing mean information and disclosing variance information. First, consider the knife-edged case where a firm discloses a perfect mean signal of its cash flows. In this situation, there is no residual uncertainty about cash flows such that there is no role for risk disclosure. This suggests that mean disclosure, even when it is not perfect, may act as a substitute for risk disclosures. The obvious way to formally model this phenomenon is to assume that the firm discloses a mean signal $\tilde{x}+\tilde{\varepsilon}$, where $\tilde{\varepsilon}|\tilde{V}_{\varepsilon} \sim N\left(0, \tilde{V}_{\varepsilon}\right)$. Unfortunately, there does not exist a well known conjugate prior for a normal distribution with a gamma distributed variance; instead, one would have to assume an *inverse* gamma distributed variance, which would lead to the nonexistence of a closed form for expected utility.[17]

In order to solve this problem, we assume that the firm can scale down its variance at a cost. While this abstracts away from statistical details, it captures the essence of mean disclosure. In particular, assume that a firm jointly chooses a level of mean disclosure, ω, and a level of variance disclosure, τ. For a given level of mean disclosure, $\omega \in [0,1]$, the firm's cash flows are distributed as:

$$\tilde{x} \sim N\left(\mu, (1-\omega)\tilde{V}\right), \tag{18}$$

where $\tilde{V}$ follows the same set of distributional assumptions that we made previously. Furthermore, assume there is a weakly convex, increasing cost for mean disclosure, $c_M(\omega)$, and a weakly convex, increasing cost function for variance disclosure, $c_V(\tau)$. Finally, assume the firm acts to maximize price. Then, the firm solves:

$$\max_{\tau,\varpi} \mu-\rho(1-\omega)\mu_V-\frac{\frac{1}{2}\rho^2(1-\omega)\frac{\sigma_V^2}{\mu_V}}{1-\frac{1}{2}\rho^2\frac{\sigma_V^2}{\mu_V}(1-\omega)+\tau(1-\omega)\frac{\sigma_V^2}{\mu_V}}\rho(1-\omega)\mu_V-c_M(\omega)-c_V(\tau). \tag{19}$$

Proposition 3 confirms the intuition from the knife-edged case of full disclosure:

Proposition 3 *Mean and variance disclosure are substitutes in the following sense: mean disclosure ω increases when variance disclosure becomes more costly, and variance disclosure*

[17] It is possible to model a disclosure that contains pure mean information by assuming that cash flows are equal to an uncertain mean term plus an uncertain variance term, and letting the disclosure equal the uncertain mean term plus noise. Then, mean disclosure does not contain any information on the variance and there is no interaction between mean and variance disclosure. Here, we consider the more realistic case where mean disclosure is a noisy realization of cash flows that contains variance information as well.

τ increases when mean disclosure becomes more costly.

In our model, mean disclosure reveals some of the uncertainty over the variance. Thus, the benefits to variance disclosure are much greater when it is very costly for the firm to generate or disclose mean information. By the same reasoning, variance disclosure reduces the benefit to mean disclosure, and mean disclosure is more profitable when variance disclosure is costly.

Proposition 3 suggests that legal mandates which require firms to disclose more variance information lead to a reduction in mean disclosure. Furthermore, after taking costs into consideration, regulation can only harm the firm's price.

Corollary 3 *Suppose that a regulator sets the mandatory level of variance disclosure equal to τ^R. If τ^R is greater than the firm's optimal choice in the absence of disclosure, τ, then the firm will respond by reducing mean disclosure.*

Corollary 3 suggests that regulatory actions trade off several costs and benefits. Regulations may give firms a mechanism to commit to providing disclosure ex ante, but they may force firms away from their optimal levels of mean and variance disclosure.

4 Sequential Information Acquisition

Equation (10) shows that the conditional variance in our model is increasing in the value of the disclosed signal, S. This is a consequence of the correlation between mean and variance for non-normal distributions. Intuitively, the uncertainty about the underlying variance distribution is perceived to be higher when the mean is higher. As a result, the higher the signal investors receive, the greater their residual uncertainty. This is in contrast to the standard result for normal distributions in which the conditional variance is constant; in that case, any signal realization leaves the same amount of residual uncertainty. This implies that if firms disclose preliminary information that suggests risks are high, they will have strong incentives to disclose further information regarding their risks.

To formally model a firm's incentives to learn and disclose information, we develop a model where a firm sequentially acquires information and can choose whether to continue or stop its information acquisition. Specifically, we assume that the firm can pay a cost k to receive a variance signal s_1. After receiving the first signal, the firm chooses whether or not to acquire an additional signal at an additional cost k. The firm can then pay yet another k to receive a third signal, and so on. Any information the firm gathers must be disclosed truthfully. The difference between these assumptions and our previous assumptions is that the firm observes the realization of the τ^{th} signal before choosing whether to disclose the $(\tau+1)^{th}$ signal. The firm can acquire at most T signals where T is an arbitrarily large integer. Our setup is very similar to the classic statistical problem of sequential sampling (see DeGroot 1970, ch. 12) where the decision problem is bounded at time T.

In the standard disclosure model with normal distributions, this change of assumption would have no impact on the results. To see this, consider the set up of our model with no uncertainty over the variance of cash flows, and consider the sequential collection of mean signals $\tilde{m}_\tau$. In particular, let $\tilde{m}_\tau = \tilde{x} + \tilde{\varepsilon}_\tau$ where $\tilde{\varepsilon}_\tau \sim N(0,\eta)$, $Cov(\tilde{\varepsilon}_i, \tilde{\varepsilon}_j) = 0\ \forall i,j$, and $Cov(\tilde{\varepsilon}_\tau, \tilde{x}) = 0$. Let $\tilde{M}_\tau$ be the mean of the first τ signals, and let V equal the known variance of cash flows. Then, after collecting $\tau - 1$ signals with mean $\tilde{M}_{\tau-1}$, the expected benefit from receiving another signal is:

$$E\left(P\left(\tilde{M}_\tau\right)|\tilde{M}_{\tau-1}\right) - P\left(\tilde{M}_{\tau-1}\right) = \delta, \tag{20}$$

where $\delta = \rho\left(\frac{1}{\tau\eta^{-1}+V^{-1}} - \frac{1}{(\tau+1)\eta^{-1}+V^{-1}}\right)$. Since the benefit is not a function of $\tilde{M}_{\tau-1}$, the decision to acquire an additional signal never depends on signal realizations. Thus, whether the firm is able to observe signal realizations prior to choosing its optimal level of information acquisition is irrelevant.

Returning to the setting of risk disclosure, suppose that a firm receives a low variance, signal. Then, due to correlation between mean and variance, residual uncertainty is low, and the firm has a reduced incentive to learn an additional signal. Additionally, when choosing whether to acquire an additional signal, the firm must take into account that there exists an option value to continuing. An option value arises since if the signal in the next period is sufficiently high, the firm will find it optimal to acquire yet another signal. Since the continuation decision is a function of the random signal realizations, the number of signals a firm acquires before stopping, which we refer to as $\tilde{\varsigma}$, is a random variable.

Let $\tilde{\Gamma}_\tau$ be the sum of the first τ signals received. In the appendix we show that the immediate benefits to learning an additional signal after acquiring τ signals, $E\left(P\left(\tilde{\Gamma}_{\tau+1}\right)|\tilde{\Gamma}_\tau\right) - P\left(\tilde{\Gamma}_\tau\right)$, can be written as $\tilde{\Gamma}_\tau f(\tau)$ for a positive function $f(\tau)$ with $f'(\tau) < 0$. Furthermore, let C_τ be the event such that the firm continues in period τ. The event C_{T-1} is equivalent to $\tilde{\Gamma}_{T-1} \in [\frac{k}{f(T-1)}, \infty)$. The continuation value in period $T-2$, the first period in which there exists a real option, thus takes the following form:

$$\tilde{\Gamma}_{T-2} f(T-2) + \Pr\left(C_{T-1}|\tilde{\Gamma}_{T-2}\right) E\left(P\left(\tilde{\Gamma}_T\right) - P\left(\tilde{\Gamma}_{T-1}\right) - k|C_{T-1}, \tilde{\Gamma}_{T-2}\right), \tag{21}$$

Clearly, the immediate benefit $\tilde{\Gamma}_{T-2} f(T-2)$ increases in $\tilde{\Gamma}_{T-2}$. In the appendix, we show a higher signal $\tilde{\Gamma}_{T-2}$ increases the distribution of $\tilde{\Gamma}_{T-1}$ in the sense of first order stochastic dominance, and that both the probability of continuing next period and the expected benefit to continuing are increasing in $\tilde{\Gamma}_{T-1}$. Together, these results imply that the continuation value also increases in $\tilde{\Gamma}_{T-2}$. Continuing via backwards induction, we prove the following proposition:

Proposition 4 *The firm continues in any given period τ if the sum of the signals it has learned and disclosed thus far, $\tilde{\Gamma}_\tau$, belongs to an interval of the form $[c_\tau, \infty)$. This implies that after receiving high variance news, the firm is more likely to continue acquiring and disclosing additional signals. When the underlying variance $\tilde{V}$ is higher, the expected number of signals disclosed, $E\left(\tilde{\varsigma}|\tilde{V}\right)$, is greater.*

Proposition 4 suggests that empirically, we should observe that firms with high variances disclose more information on their risk. Furthermore, it predicts that increased risk disclosures follow economy-wide increases in average risk such as the 2008 financial crisis. This result is distinct from our previous finding that σ_V^2 increased the impact of disclosure on prices; here we find that the true variance, in addition to the degree of variance uncertainty, impacts the amount of variance information a firm discloses.

As discussed above, in a setting with known cash flow variance but uncertain precision of disclosure (Subramanyam, 1996) leads to an uncertain conditional cash flow variance. The results of this section suggest that in such a setting, investors have an increased incentive to acquire information. In equilibrium, this would suggest a relatively constant conditional variance. For example, a conservative disclosure policy that provides more precise information about a firm's downside risk may lead to increased incentives to acquire information about a firm's upside potential.

5 Multiple Asset Extension

In this section, we consider an extension of our model to a multi-asset setting. Our single asset model leaves open the question of whether risk disclosures impact the cost of capital when investors hold diversified portfolios. Furthermore, we would like to address whether there exist positive externalities to risk disclosure. In order to address these questions, we develop a factor model where both idiosyncratic and systematic variances are unknown.

Assume that there are N firms in the economy with a per capita supply of $\frac{1}{N}$ whose cash flows $\tilde{x}_i$ are equal to a common factor loading plus idiosyncratic noise, i.e., $\tilde{x}_i = \beta_i \tilde{F} + \tilde{\varepsilon}_i$ where $\tilde{F}$ and $\tilde{\varepsilon}_i$ are independent. Furthermore, assume that both the common factor and the idiosyncratic components of cash flows have variance uncertainty: $\tilde{F}|\tilde{V}_F \sim N\left(\mu_F, \tilde{V}_F\right)$ and $\tilde{\varepsilon}_i|\tilde{V}_i \sim N\left(\mu_{\varepsilon_i}, \tilde{V}_i\right)$ where $\tilde{V}_F \sim Gamma\left(\frac{\mu_F^2}{\sigma_F^2}, \frac{\mu_F}{\sigma_F^2}\right)$ and $\tilde{V}_i \sim Gamma\left(\frac{\mu_{V_i}^2}{\sigma_{V_i}^2}, \frac{\mu_{V_i}}{\sigma_{V_i}^2}\right)$. Finally, assume that the uncertain variance distributions are independent. Without loss of generality, we scale the factor such that the average beta is one: $\bar{\beta} \equiv \frac{1}{N}\sum_{i=1}^{N}\beta_i = 1$.

By assuming that the per capita supply of each asset is equal to $\frac{1}{N}$, the total risk in the economy remains constant when we vary the number of firms in the economy. There are two possible interpretations of this assumption. First, we could argue that as the economy grows, the numerator of per capita supply shrinks. In this case, we are letting each firm become an arbitrarily small portion of the economy while keeping the total size of the economy the same. An alternative interpretation is that the investor base grows with the economy, and hence the per capita supply decreases because the denominator grows (see Lambert, Leuz, and Verrecchia 2007 for an in depth discussion of this issue). In either case, in the limit, each individual investor holds an arbitrarily small amount of any given asset.

Let $\mu_i = E(\tilde{x}_i) = \beta_i \mu_F + \mu_{\varepsilon_i}$. Then, as a baseline, the price of firm k in the standard case when there is no variance uncertainty, that is, $\tilde{V} = \mu_{V_k}$ with certainty, equals:

$$P_k = \mu_k - \frac{\rho \mu_F}{N}\left(\beta_k^2 + N\beta_k\right) - \frac{\rho \mu_{V_k}}{N}. \tag{22}$$

We define the risk premium associated with the systematic component as $RP_S \equiv \frac{\rho \mu_{V_F}}{N}\left(\beta_k^2 + N\beta_k\right)$ and the risk premium associated with the idiosyncratic component as $RP_I \equiv \frac{\rho \mu_{V_k}}{N}$. As $N \to \infty$, we find that $RP_S \to \rho \beta_k \mu_{V_F}$ and $RP_I \to 0$, i.e., the idiosyncratic risk premium vanishes and the systematic risk premium converges to risk aversion times the covariance of the firm's cash flows with the common factor. Next, we perform an analysis similar to the single asset case to derive prices under variance uncertainty.

Proposition 5 *Assume that* $\frac{\rho^2}{2}\frac{\sigma_{V_F}^2}{\mu_{V_F}} < 1$ *and* $\frac{\rho^2}{2}\frac{\sigma_{V_k}^2}{\mu_{V_k}} < N^2$.[18] *Then, the price of the* k^{th} *asset is equal to the mean less the systematic and idiosyncratic risk premia less systematic and*

[18] These conditions mirror the condition from the single asset case and imply that investors are willing to hold shares at any finite price.

idiosyncratic variance uncertainty premia:

$$P_k = \mu_k - RP_S - RP_I - VUP_S - VUP_I \tag{23}$$

$$\text{where } VUP_S = \frac{\frac{1}{2}\rho^2\frac{\sigma^2_{V_F}}{\mu_{V_F}}}{1-\frac{1}{2}\rho^2\frac{\sigma^2_{V_F}}{\mu_{V_F}}} RP_S \tag{24}$$

$$\text{and } VUP_I = \frac{\frac{1}{2}\rho^2\frac{\sigma^2_{V_k}}{\mu_{V_k}}}{N^2-\frac{1}{2}\rho^2\frac{\sigma^2_{V_k}}{\mu_{V_k}}} RP_I. \tag{25}$$

In essence, the systematic and idiosyncratic components of cash flows are valued separately and price is equal to the sum of their values. Thus, there exist variance uncertainty premia for both components. When $\beta_k > 0$, as in the single asset case, the systematic variance uncertainty premium is increasing in $\sigma^2_{V_F}$, ambiguous with respect to μ_{V_F}, but decreasing for first order stochastic dominant shifts in the factor variance. As $\sigma^2_{V_F} \to 0$ and $\sigma^2_{V_k} \to 0$, price converges to the baseline price with no variance uncertainty. Prices continue to be quadratic in the factor loading. While the factor loading leads to a fourth order effect on variance uncertainty, the effect on price is only linear due to the hedgeability of systematic variance uncertainty. For $\beta_k < 0$, all of the comparative statics are reversed, since the firm serves as a hedge.

The portion of the variance uncertainty premium associated with idiosyncratic risk is identical to that in the single asset case if the endowment had been $\frac{1}{N}$. As such, it carries the same intuition as in the single asset case for finite N. We summarize these results in the following corollary:

Corollary 4 *As $\sigma^2_{V_F} \to 0$ and $\sigma^2_{V_k} \to 0$, the price of firm k converges to the baseline price with no variance uncertainty. For $\beta_k > 0$, the price of firm k is decreasing in the variance of the idiosyncratic and systematic variances, $\sigma^2_{V_F}$ and $\sigma^2_{V_k}$, is decreasing in location shifts in the idiosyncratic or systematic variances, and is decreasing in the risk aversion ρ. For $\beta_k < 0$, the comparative statics on the systematic component are reversed.*

Next, consider what happens as the number of assets in the economy approaches infinity. Recall that as supply per capita equals $\frac{1}{N}$, the total risk in the economy is constant in N. Thus, when we let $N \to \infty$ in (23), the idiosyncratic component vanishes, and price becomes:

$$\lim_{N\to\infty} P_k = \mu_k - \rho\mu_{V_F}\beta_k - \frac{\frac{1}{2}\rho^2 \frac{\sigma^2_{V_F}}{\mu_{V_F}}}{1 - \frac{1}{2}\rho^2 \frac{\sigma^2_{V_F}}{\mu_{V_F}}} \rho\mu_{V_F}\beta_k \tag{26}$$

Similar to a setting with known variance, all idiosyncratic risk factors are not priced. Furthermore, investors are willing to pay a premium for firms whose cash flows are negatively correlated with the market portfolio.

5.1 Systematic and Idiosyncratic Disclosure

Equation (26) implies that, in a large economy, idiosyncratic disclosure will have no impact on the cost of capital. Only disclosure that contains information on the systematic factor will affect the cost of capital. For demonstration purposes, suppose that each firm issues a unique signal that can be broken down into two components, a systematic component and an idiosyncratic component. Mathematically, we assume this implies that the firm discloses two signals of the sort we discussed in the single asset case:

$$\begin{aligned} \tilde{S}_{kI} &= \sum_{j=1}^{\tau_I} s_{jIk};\ s_{jIk} \sim Poisson\left(\tilde{V}_k\right) \\ \tilde{S}_{kF} &= \sum_{i=1}^{\tau_F} s_{jFk}; s_{jFk} \sim Poisson\left(\tilde{V}_F\right) \end{aligned} \tag{27}$$

where the s_{jIk}'s and s_{jFk}'s are mutually independent conditional on $\tilde{V}_k$ and $\tilde{V}_F$.[19] Let $\tilde{\mathbf{S}}_I = \left(\tilde{S}_{1I}, ..., \tilde{S}_{NI}\right)$ be the vector of firms' idiosyncratic signals and $\tilde{\mathbf{S}}_F = \left(\tilde{S}_{1F}, ..., \tilde{S}_{NF}\right)$ be

[19] Mathematically, modeling variance disclosure by multiple firms in our setup is not straightforward since the systematic components of firms' disclosures likely overlap. In particular, multiple firms may aggregate signals that contain the same $\tilde{s}_{ik}$. Nevertheless, it is intuitive that firms' information disclosures can be jointly used to assess the uncertain variance of the factor, and thus we assume that investors can tease apart the novel information in a firm's disclosures.

the vector of firms systematic signals. Then, as $N \to \infty$, the k^{th} firm's price conditional on the signals is equal to:

$$P_k|\tilde{\mathbf{S}}_I, \tilde{\mathbf{S}}_F = \mu_k - RP_S - \phi\left(\boldsymbol{\tau}'\mathbf{1}\right) VUP_S - \alpha\left(\boldsymbol{\tau}'\mathbf{1}\right)\left(\bar{S}_F - E\left(\bar{S}_F\right)\right), \tag{28}$$

where $\bar{S}_F = \left(\boldsymbol{\tau}'\mathbf{1}\right)^{-1}\boldsymbol{\tau}'\tilde{\mathbf{S}}_F$, $\phi\left(\boldsymbol{\tau}'\mathbf{1}\right) = \left(1 - \frac{1}{2}\rho^2\frac{\sigma^2_{V_F}}{\mu_{V_F}}\right)\left(1 - \frac{1}{2}\rho^2\frac{\sigma^2_{V_F}}{\mu_{V_F}} + \frac{\sigma^2_{V_F}}{\mu_{V_F}}\boldsymbol{\tau}'\mathbf{1}\right)^{-1}$, and $\alpha\left(\boldsymbol{\tau}'\mathbf{1}\right) = \rho\beta_k\frac{\sigma^2_{V_F}}{\mu_{V_F}}\boldsymbol{\tau}'\mathbf{1}\left(1 - \frac{1}{2}\rho^2\frac{\sigma^2_{V_F}}{\mu_{V_F}} + \frac{\sigma^2_{V_F}}{\mu_{V_F}}\boldsymbol{\tau}'\mathbf{1}\right)^{-1}$. This implies the cost of capital is equal to:

$$\mu_k - E\left(P_k|\tilde{S}_{kF}\right) = RP_S - \phi\left(\boldsymbol{\tau}'\mathbf{1}\right) VUP_S. \tag{29}$$

The market uses a weighted average $\bar{S}_F = \left(\boldsymbol{\tau}'\mathbf{1}\right)^{-1}\boldsymbol{\tau}'\tilde{\mathbf{S}}_F$ of the firms' signals when updating on the factor variance, and takes into account the total precision of all disclosures, $\boldsymbol{\tau}'\mathbf{1}$. The cost of capital is decreasing in τ_{iF}, but idiosyncratic disclosure τ_{ik} plays no role. As long as disclosure contains some information regarding the common factor, the results from all of our single asset models will continue to hold in an economy with multiple assets.

Similar to the results found by Admati and Pfleiderer (2000) for mean disclosure, risk disclosure that has a systematic component generates a positive externality in that it reduces uncertainty for all firms in the economy and reduces the aggregate cost of capital. This provides another potential rationale for the attention regulatory authorities have been giving to mandated risk disclosures.

6 Conclusion

In recent years, the SEC and FASB have taken action to increase firms' risk disclosure. However, the theoretical literature on disclosure has offered sparse evidence on the effect of risk disclosure on prices. In particular, it has either assumed away pricing of variance uncertainty, or has focused on the strategic disclosure setting with perfect disclosure of variance uncertainty. In this paper, we address how risk disclosure is priced from an ex ante point of view, which allows us to highlight several costs and benefits of mandated risk disclosure.

We begin by deriving prices when investors have negative exponential utility and cash flows are normal conditional on a gamma distributed variance. We show that investors penalize firms with variance uncertainty. We next consider how an ex ante commitment to risk disclosure can affect a firm's cost of capital. Analogous to the standard result for mean-based disclosure, risk disclosure reduces investors perceived riskiness of the variance distribution. This leads to a decrease in the cost of capital. Technically, this results from that fact that variance uncertainty implies that the cash flow distribution exhibits positive excess kurtosis and that investors with a negative exponential utility are kurtosis-averse (that is, they are "temperate"). Risk disclosure reduces the excess kurtosis and, thus, increases expected price.

We next model a firm's joint choice of the precisions of mean and risk disclosures. We find mean disclosure and risk disclosure are substitutes, and thus, mandated increases in risk disclosure can lead to decreases in mean disclosure. To further explore the differences between mean and risk disclosure, we endogenize the acquisition of information by the firm, and show that firms tend to acquire and disclose more information after receiving high variance signals. This arises due to the correlation between mean and variance for non-negative distributions. Finally, we show that the results of mean disclosure in a multi-asset environment carry over to risk disclosure.

While our model highlights several potential costs and benefits of regulating risk disclosure, it is but a first step toward developing regulatory recommendations. It should be clear from our model that many of the same trade-offs that have been highlighted for mean disclosure (see, for example, Beyer et al. 2010) hold for risk disclosure, and that regulators must take into consideration the multitude of effects that mandated disclosure may have.

Furthermore, a notable contribution of our paper is that it provides a set of tools for examining risk disclosure that can be applied to other research settings. In particular, we have derived expected utility given negative exponential utility functions with a well known distribution over the variance. This could be applied to study a setting in which investors have private information. We have also abstracted completely from agency issues by assuming the firm exogenously makes decisions to maximize prices. Future research could extend our results on variance uncertainty to study agency problems.

Unfortunately, our study shows that the expected utility of an investor with a negative exponential utility is relatively complex. This naturally limits the ability to answer several questions with our setting. However, future research could be based on a mean-variance-kurtosis utility function and a state contingent variance with a binary state.[20] Consistent with our results, one can show that variance uncertainty increases kurtosis more generally. For example, consider two possible lotteries, A and B, whose payoff distributions depend on whether the state is 1 or 2. Lottery A pays off $\varepsilon_1 \sim N(0, \sigma_1^2)$ in state 1 and $\varepsilon_2 \sim N(0, \sigma_2^2)$ in state 2. Whereas lottery B pays off 0 in state 1 and $\varepsilon_1 + \varepsilon_2$ in state 2. When $\sigma_1^2 = \sigma_2^2$, lottery A does not exxhibit excess kurtosis but lottery B, by "grouping" the risks, does. Noussair et al. (2014) show that agents with a positive coefficient of absolute temperance, $-u''''(x)/u'''(x)$, prefer lottery A over B. In this setting, risk disclosure is equivalent to a signal regarding the state of nature.

[20] A mean-variance-kurtosis utility function is consistent with the fourth-order development of the Arrow-Pratt expression for the risk premium (see Le Courtois, 2012).

7 Appendix

Proof of Proposition 1:

For simplicity, we first find price for a gamma distribution parameterized by shape and scale, a and b. We then convert this to price for a gamma characterized by μ_V and σ_V^2 by using the fact that $a = \frac{\mu_V^2}{\sigma_V^2}$ and $b = \frac{\mu_V}{\sigma_V^2}$. Investors solve

$$\begin{aligned} & \arg\max_D E\left(E\left(u(D(\tilde{x}-P))|\tilde{V}\right)\right) \\ = & \arg\max_D E\left(-e^{-\rho D(\mu-P)+D^2\frac{\rho^2}{2}\tilde{V}}\right) \\ = & \arg\max_D -e^{-\rho D(\mu-P)}E\left(e^{D^2\frac{\rho^2}{2}\tilde{V}}\right). \end{aligned} \tag{30}$$

where the simplifications follow from the law of iterated expectations and the MGF of a normal distribution. After applying a monotone transformation, this is equal to:

$$\arg\max_D \rho D(\mu-P) - \ln\left(E\left(e^{D^2\frac{\rho^2}{2}\tilde{V}}\right)\right). \tag{31}$$

Solving for the latter term, we find:

$$\begin{aligned} E\left(e^{\frac{D^2\rho^2}{2}\tilde{V}}\right) &= \int_0^\infty e^{\frac{D^2\rho^2}{2}x}\frac{x^{a-1}e^{-xb}}{b^{-a}\Gamma(a)}dx \\ &= \int_0^\infty \left(\frac{b-\frac{D^2\rho^2}{2}}{b}\right)^{-a}\frac{x^{a-1}e^{-x\left(b-\frac{D^2\rho^2}{2}\right)}}{\left(b-\frac{D^2\rho^2}{2}\right)^{-a}\Gamma(a)}dx \\ &= \left(\frac{b-\frac{D^2\rho^2}{2}}{b}\right)^{-a}\underbrace{\int_0^\infty \frac{x^{a-1}e^{-x\left(b-\frac{D^2\rho^2}{2}\right)}}{\left(b-\frac{D^2\rho^2}{2}\right)^{-a}\Gamma(a)}dx}_{\text{Gamma PDF}} \\ &= \left(1-\frac{D^2\rho^2}{2b}\right)^{-a}. \end{aligned} \tag{32}$$

Note that the integral of the gamma PDF only exists for $b > \frac{D^2\rho^2}{2}$. Otherwise, the integral is equal to negative infinity. We conjecture an equilibrium where $b > \frac{D^2\rho^2}{2}$ holds. Then, in equilibrium, since all investors are homogenous, all investors hold the per capita endowment of 1 share. Thus, given our assumption $b > \frac{\rho^2}{2} \iff \rho^2\sigma_V^2 < 2\mu_V^2$, the conjecture is verified. Each investor's first order condition is:

$$\frac{\partial}{\partial D}\left(\rho D(\mu - P) + a\ln\left(1 - \frac{D^2\rho^2}{2b}\right)\right) = 0 \tag{33}$$
$$\Longrightarrow \quad (\mu - P)\left(b - \frac{1}{2}\rho^2 D^2\right) = a\rho D.$$

Again, in equilibrium, $D = 1$ since all investors are homogeneous. This implies

$$P = \mu - \frac{a\rho}{b - \frac{1}{2}\rho^2} \tag{34}$$

Substituting $a = \frac{\mu_V^2}{\sigma_V^2}$ and $b = \frac{\mu_V}{\sigma_V^2}$, we get:

$$\begin{aligned} P &= \mu - \frac{1}{1 - \frac{1}{2}\rho^2\frac{\sigma_V^2}{\mu_V}}\rho\mu_V \\ &= \mu - \rho\mu_V - \frac{\frac{1}{2}\rho^2\frac{\sigma_V^2}{\mu_V}}{1 - \frac{1}{2}\rho^2\frac{\sigma_V^2}{\mu_V}}\rho\mu_V \end{aligned} \tag{35}$$

Proof of Corollary 1:

The result that prices are decreasing in σ_V^2 and ρ can easily be seen by taking the respective derivatives of (4). The derivative with respect to μ_V is equal to:

$$-4\rho\frac{\mu_V}{(2\mu_V - \rho^2\sigma_V^2)^2}\left(\mu_V - \rho^2\sigma_V^2\right) \tag{36}$$

which has the same sign as $\mu_V - \rho^2\sigma_V^2$. Finally, to show that prices uniformly decrease in a location shift in the variance, note that, repeating the analysis for the proof of Proposition

1 yields:

$$\arg\max_D E\left(E\left(u(D(\tilde{x}-P))|\tilde{V}\right)\right) = \arg\max_D E\left(-e^{-\rho D(\mu-P)+D^2\frac{\rho^2}{2}\left(\tilde{V}+k\right)}\right)$$
$$= \arg\max_D -e^{-\rho D(\mu-P)+\frac{D^2\rho^2}{2}k} E\left(e^{D^2\frac{\rho^2}{2}\tilde{V}}\right). \quad (37)$$

such that investor's FOC is:

$$\frac{\partial}{\partial D}\left(\rho D(\mu-P)+\frac{D^2\rho^2}{2}k+a\ln\left(1-\frac{D^2\rho^2}{2b}\right)\right)=0, \quad (38)$$

which implies

$$P=\mu-k\rho-\frac{2a\rho}{2b-\rho^2}=\mu-(k+\mu_V)\rho-\frac{\frac{1}{2}\rho^2\frac{\sigma_V^2}{\mu_V}}{1-\frac{1}{2}\rho^2\frac{\sigma_V^2}{\mu_V}}\rho\mu_V. \quad (39)$$

Proof of Lemma 1:

The well known result from Bayesian statistics is couched in terms of the shape and scale parameters, a and b (see, e.g., DeGroot (1970) pg. 164). Let $\tilde{V} \sim Gamma(a,b)$ and $\tilde{s}_i \sim Poisson\left(\tilde{V}\right)$ where $\tilde{s}_1,...,\tilde{s}_\tau$ are independent conditional on $\tilde{V}$. Then,

$$\tilde{V}|\tilde{s}_1,...,\tilde{s}_\tau \sim Gamma\left(a+\sum_{i=1}^{\tau}\tilde{s}_i, b+\tau\right). \quad (40)$$

We next show that the mean of the signals is a sufficient statistic for their individual realizations by using the Fisher-Neyman factorization theorem:

$$\begin{aligned}
f(s_1,...,s_\tau,V) &= f(s_1,...,s_\tau|V)\,f(V) \\
&= \prod_{i=1}^{\tau} f(s_i|V)\,f(V) = \prod_{i=1}^{\tau}\frac{V^{s_i}e^{-V}}{s_i!}\frac{V^{a-1}e^{-Vb}}{b^{-a}\Gamma(a)} \\
&= \frac{V^{\Sigma_{i=1}^{\tau}s_i}e^{-V}}{\prod_{i=1}^{\tau}s_i!}\frac{V^{a-1}e^{-Vb}}{b^{-a}\Gamma(a)} = \frac{1}{\prod_{i=1}^{\tau}s_i!}\frac{V^{a-1+\Sigma_{i=1}^{\tau}s_i}e^{-V(b+1)}}{b^{-a}\Gamma(a)} \\
&\equiv h(s_1,...,s_\tau)\,g\left(\Sigma_{i=1}^{\tau}s_i,V\right).
\end{aligned} \tag{41}$$

Proving the lemma is simply a matter of using these results and performing algebraic manipulations.

$$\begin{aligned}
E\left(\tilde{V}|\tilde{S}\right) &= E\left(\tilde{V}|\tilde{s}_1,...,\tilde{s}_\tau\right) = \frac{a+\sum_{i=1}^{\tau}\tilde{s}_i}{b+\tau} = \frac{a+\tau\tilde{S}}{b+\tau} \\
&= \frac{\tau\tilde{S}\sigma_V^2+\mu_V^2}{\tau\sigma_V^2+\mu_V} = \mu_V + \frac{\sigma_V^2}{\sigma_V^2+\tau^{-1}\mu_V}\left(\tilde{S}-\mu_V\right) \text{ and} \\
Var\left(\tilde{V}|\tilde{S}\right) &= Var\left(\tilde{V}|\tilde{s}_1,...,\tilde{s}_\tau\right) = \frac{a+\sum_{i=1}^{\tau}\tilde{s}_i}{(b+\tau)^2} = \frac{a+\tau\tilde{S}}{(b+\tau)^2} \\
&= \frac{\frac{\mu_V^2}{\sigma_V^2}+\tau\tilde{S}}{\left(\frac{\mu_V}{\sigma_V^2}+\tau\right)^2} = \frac{\left(\tilde{S}\tau\sigma_V^2+\mu_V^2\right)\sigma_V^2}{(\tau\sigma_V^2+\mu_V)^2}.
\end{aligned} \tag{42}$$

Note that:

$$\begin{aligned}
Cov\left(\tilde{V},\tilde{S}\right) &= E\left(\tilde{V}\tilde{S}\right) - E\left(\tilde{V}\right)E\left(\tilde{S}\right) \\
&= E\left(\tilde{V}E\left(\tilde{S}|\tilde{V}\right)\right) - \mu_V^2 \\
&= E\left(\tilde{V}^2\right) - \mu_V^2 = \sigma_V^2 \text{ and} \\
Var\left(\tilde{S}\right) &= Var\left(E\left(\tilde{S}|\tilde{V}\right)\right) + E\left(Var\left(\tilde{S}|\tilde{V}\right)\right) \\
&= \sigma_V^2 + \tau^{-1}\mu_V.
\end{aligned} \tag{43}$$

Hence, we can alternatively write:

$$E\left(\tilde{V}|\tilde{S}\right) = E\left(\tilde{V}\right) + \frac{Cov\left(\tilde{V},\tilde{S}\right)}{Var\left(\tilde{S}\right)}\left(\tilde{S} - E\left(\tilde{S}\right)\right) \text{ and} \tag{44}$$

$$Var\left(\tilde{V}|\tilde{S}\right) = Var\left(\tilde{V}\right) - \frac{Cov\left(\tilde{V},\tilde{S}\right)^2}{Var\left(\tilde{S}\right)} + \left(\frac{Cov\left(\tilde{V},\tilde{S}\right)}{Var\left(\tilde{S}\right)}\right)^2 \frac{\tilde{S} - \mu_V}{\tau}.$$

Proof of Proposition 2:

Proving the proposition is simply a matter of plugging the results from the lemma into (4):

$$\begin{aligned} P|S &= \mu - \rho E\left(\tilde{V}|\tilde{S}\right) - \frac{\frac{1}{2}\rho^2 \frac{\sigma_V^2}{\tau\sigma_V^2+\mu_V}}{1 - \frac{1}{2}\rho^2 \frac{\sigma_V^2}{\tau\sigma_V^2+\mu_V}} \rho E\left(\tilde{V}|\tilde{S}\right) \\ &= \mu - \rho E\left(\tilde{V}|\tilde{S}\right) - \frac{\frac{1}{2}\rho^2 \frac{\sigma_V^2}{\mu_V}}{1 - \frac{1}{2}\rho^2 \frac{\sigma_V^2}{\mu_V} + \tau \frac{\sigma_V^2}{\mu_V}} \rho E\left(\tilde{V}|\tilde{S}\right). \end{aligned} \tag{45}$$

Proof of Proposition 3:

Define $g(\omega,\tau)$ to be the maximand in eqn. (19). It is easily checked that the price component of this expression is concave in ω and τ. Combining this with the assumption that the cost functions are weakly convex, the maximization problem has a unique solution (ω^*,τ^*). Consider an increase in the cost function for risk disclosure, $c_\tau(\tau) + k_V\tau$, and an increase in the cost function for mean disclosure, $c_M(\varpi) + k_M\varpi$. In the interesting case that this solution is interior, applying the implicit function theorem we find:

$$\begin{pmatrix} \frac{\partial\omega^*}{\partial k_M} & \frac{\partial\omega^*}{\partial k_V} \\ \frac{\partial\tau^*}{\partial k_M} & \frac{\partial\tau^*}{\partial k_V} \end{pmatrix} = -\begin{pmatrix} \frac{\partial^2 g}{\partial\omega^2} & \frac{\partial g}{\partial\omega\partial\tau} \\ \frac{\partial g}{\partial\omega\partial\tau} & \frac{\partial^2 g}{\partial\tau^2} \end{pmatrix}. \tag{46}$$

It can be shown that $\frac{\partial g(\omega,\tau)}{\partial\tau\partial\omega} = \frac{\partial}{\partial\tau\partial\omega}\left(\mu - \rho(1-\omega)\mu_V - \frac{\frac{1}{2}\rho^2(1-\omega)\frac{\sigma_V^2}{\mu_V}}{1-\frac{1}{2}\rho^2\frac{\sigma_V^2}{\mu_V}(1-\omega)+\tau(1-\omega)\frac{\sigma_V^2}{\mu_V}}\rho(1-\omega)\mu_V\right) = -2\rho^3\sigma_V^4\mu_V\frac{(1-\omega)^2\left(6\mu_V-\rho^2\sigma_V^2+2\tau\sigma_V^2(1-\omega)+\omega\rho^2\sigma_V^2\right)}{\left(2\mu_V-\rho^2\sigma_V^2+2\tau\sigma_V^2(1-\omega)+\omega\rho^2\sigma_V^2\right)^3} < 0.$

Proof of Corollary 3:

The corollary is easily shown by noting that for $\tau^R > \tau^*$, ω^* now solves $\left[\frac{\partial g}{\partial \omega^*}\right]_{\tau=\tau^R} = 0$ rather than $\left[\frac{\partial g}{\partial \omega^*}\right]_{\tau=\tau^*} = 0$; using $\frac{\partial g(\omega,\tau)}{\partial \tau \partial \omega} < 0$ implies this leads to a lower optimum.

Proof of Proposition 4:

We start by establishing two useful lemmas. Let $\tilde{\Gamma}_\tau$ be the sum of all signals received up until the τ^{th} signal.

Lemma 2 *The immediate benefit to acquiring an additional signal,* $E\left(P\left(\tilde{\Gamma}_{\tau+1}\right)|\tilde{\Gamma}_\tau\right) - P\left(\tilde{\Gamma}_\tau\right)$ *takes the form* $\tilde{\Gamma}_\tau f(\tau)$ *where* $f(\tau) > 0$ *and* $f'(\tau) < 0$.

Proof of Lemma 2:

The firm chooses to learn an additional signal $\tilde{s}_{\tau+1}$ when its expectation of the change in price given the additional signal is greater than the cost of acquiring an additional signal:

$$E\left(P\left(\tilde{\Gamma}_{\tau+1}\right)|\tilde{\Gamma}_\tau - P\left(\tilde{\Gamma}_\tau\right)\right) > k \tag{47}$$

Writing out the left hand side and applying the law of iterated expectations, we find:

$$\begin{aligned}
& E\left(P\left(\tilde{\Gamma}_{\tau+1}\right)|\tilde{\Gamma}_\tau - P_\tau\right)|\tilde{\Gamma}_\tau && (48)\\
= \; & -E\left(E\left(\tilde{V}|\tilde{\Gamma}_{\tau+1}\right)|\tilde{\Gamma}_\tau\right)\rho - \frac{\frac{1}{2}\rho^2 \frac{E\left(Var\left(V|\tilde{\Gamma}_\tau, s_{\tau+1}\right)|\tilde{\Gamma}_\tau\right)}{E\left(V|\tilde{\Gamma}_\tau\right)}}{1 - \frac{1}{2}\rho^2 \frac{E\left(Var\left(V|\tilde{\Gamma}_\tau, s_{\tau+1}\right)|\tilde{\Gamma}_\tau\right)}{E(V|S_\tau)}} E\left(E\left(V|\tilde{\Gamma}_{\tau+1}\right)|\tilde{\Gamma}_\tau\right)\rho \\
& + E\left(\tilde{V}|\tilde{\Gamma}_\tau\right)\rho + \frac{\frac{1}{2}\rho^2 \frac{Var\left(V|\tilde{\Gamma}_\tau\right)}{E\left(V|\tilde{\Gamma}_\tau\right)}}{1 - \frac{1}{2}\rho^2 \frac{Var\left(V|\tilde{\Gamma}_\tau\right)}{E\left(V|\tilde{\Gamma}_\tau\right)}} E\left(\tilde{V}|\tilde{\Gamma}_\tau\right)\rho \\
= \; & E\left(\tilde{V}|\tilde{\Gamma}_\tau\right)\rho\left(\frac{\frac{1}{2}\rho^2 \frac{Var\left(V|\tilde{\Gamma}_\tau\right)}{E\left(V|\tilde{\Gamma}_\tau\right)}}{1 - \frac{1}{2}\rho^2 \frac{Var\left(V|\tilde{\Gamma}_\tau\right)}{E\left(V|\tilde{\Gamma}_\tau\right)}} - \frac{\frac{1}{2}\rho^2 \frac{E\left(Var\left(V|\tilde{\Gamma}_\tau, s_{\tau+1}\right)|\tilde{\Gamma}_\tau\right)}{E\left(V|\tilde{\Gamma}_\tau\right)}}{1 - \frac{1}{2}\rho^2 \frac{E\left(Var\left(V|\tilde{\Gamma}_\tau, s_{\tau+1}\right)|\tilde{\Gamma}_\tau\right)}{E\left(V|\tilde{\Gamma}_\tau\right)}}\right) \\
= \; & E\left(\tilde{V}|\tilde{\Gamma}_\tau\right)\rho\left(\frac{\frac{1}{2}\rho^2 \frac{\sigma_V^2}{\mu_V}}{1 - \frac{1}{2}\rho^2 \frac{\sigma_V^2}{\mu_V} + \tau\frac{\sigma_V^2}{\mu_V}} - \frac{\frac{1}{2}\rho^2 \frac{\sigma_V^2}{\mu_V}}{1 - \frac{1}{2}\rho^2 \frac{\sigma_V^2}{\mu_V} + (\tau+1)\frac{\sigma_V^2}{\mu_V}}\right) && (49)\\
\equiv \; & \tilde{\Gamma}_\tau f(\tau). && (50)
\end{aligned}$$

where we used the fact that

$$E\left(Var\left(\tilde{V}|\tilde{\Gamma}_{\tau+1}\right)|\tilde{\Gamma}_\tau\right) = E\left(\frac{\left(\left(\tilde{\Gamma}_\tau+\tilde{s}_{\tau+1}\right)\sigma_V^2+\mu_V^2\right)\sigma_V^2}{\left(\left(\tau+1\right)\sigma_V^2+\mu_V\right)^2}|\tilde{\Gamma}_\tau\right) \tag{51}$$

$$= \frac{\left(\left(\tilde{\Gamma}_\tau+\frac{\tilde{\Gamma}_\tau\sigma_V^2+\mu_V^2}{\tau\sigma_V^2+\mu_V}\right)\sigma_V^2+\mu_V^2\right)\sigma_V^2}{\left(\left(\tau+1\right)\sigma_V^2+\mu_V\right)^2}$$

$$= E\left(\tilde{V}|\tilde{\Gamma}_\tau\right)\frac{\sigma_V^2}{\left(\tau+1\right)\sigma_V^2+\mu_V}.$$

We next prove the following lemma which will be used several times.

Lemma 3 *An increase in $\tilde{\Gamma}_{\tau-1}$ increases the distribution of $\tilde{\Gamma}_\tau|\tilde{\Gamma}_{\tau-1}$ in the sense of first order stochastic dominance.*

Proof of Lemma 3:

Define $F_\tau\left(\cdot|\tilde{\Gamma}_{\tau-1}\right)$ to be the CDF of the τ^{th} signal $\tilde{s}_\tau$ conditional on the sum of the first $\tau-1$ signals. We start by showing $\frac{\partial}{\partial\tilde{\Gamma}_{\tau-1}}F_\tau\left(\cdot|\tilde{\Gamma}_{\tau-1}\right)<0$.

$$F_\tau\left(c|\tilde{\Gamma}_{\tau-1}\right) = \Pr\left(\tilde{s}_\tau<c|\tilde{\Gamma}_{\tau-1}\right) \tag{52}$$

$$= \int \Pr\left(\tilde{s}_\tau<c|V\right)\Pr\left(V|\tilde{\Gamma}_{\tau-1}\right)dV \tag{53}$$

Recall the following general result: if some random variable $\tilde{x}$ leads to an increase in the conditional distribution $f\left(\cdot|\tilde{x}\right)$ in the sense of FSD, and $u'<0$, then $\frac{\partial}{\partial x}\int u'\left(\cdot\right)f\left(\cdot|\tilde{x}\right)<0$. This can be shown using integration by parts. Furthermore, recall that $\tilde{\Gamma}_{\tau-1}$ increases the distribution of $\tilde{V}$ in the sense of FSD, and that $\frac{\partial}{\partial\tilde{V}}\Pr\left(\tilde{s}_\tau<c|\tilde{V}\right)<0$. This implies

$\frac{\partial}{\partial \tilde{\Gamma}_{\tau-1}} F_\tau \left(\cdot | \tilde{\Gamma}_{\tau-1} \right) < 0$. Combining these results:

$$\begin{aligned} \Pr\left(\tilde{\Gamma}_\tau \leq c|\tilde{\Gamma}_{\tau-1}\right) &= \Pr\left(\tilde{\Gamma}_{\tau-1} + \tilde{s}_\tau \leq c|\tilde{\Gamma}_{\tau-1}\right) && (54) \\ &= \Pr\left(\tilde{s}_\tau \leq c - \tilde{\Gamma}_{\tau-1}|\tilde{\Gamma}_{\tau-1}\right) && (55) \\ &= F_\tau\left(g\left(\tilde{\Gamma}_{\tau-1}\right)|\tilde{\Gamma}_{\tau-1}\right) && (56) \\ \text{where } g\left(\tilde{\Gamma}_{\tau-1}\right) &\equiv c - \tilde{\Gamma}_{\tau-1} && (57) \end{aligned}$$

Using the chain rule,

$$\frac{d}{d\tilde{\Gamma}_{\tau-1}} \Pr\left(\tilde{\Gamma}_\tau < c|\tilde{\Gamma}_{\tau-1}\right) = \frac{\partial F_\tau\left(g\left(\tilde{\Gamma}_{\tau-1}\right)|\tilde{\Gamma}_{\tau-1}\right)}{\partial \tilde{\Gamma}_{\tau-1}} + \frac{\partial F_\tau}{\partial g} \frac{\partial g\left(\tilde{\Gamma}_{\tau-1}\right)}{\tilde{\partial}\tilde{\Gamma}_{\tau-1}} < 0. \tag{58}$$

To simplify notation, call the event of continuing in period τ, C_τ. Our strategy in proving the proposition is as follows. We start by considering the last period with the option to learn, $T-1$. We show that the value to learning additional information is of the form $\tilde{\Gamma}_{T-1} f(T-1) - k$, and thus is increasing in $\tilde{\Gamma}_{T-1}$. We then look at the previous period, and show that the expected benefit to learning is equal to $\tilde{\Gamma}_{T-2} f(T-2) + \text{OptVal}\left(\tilde{\Gamma}_{T-1}\right) - k$. We show that the option value has the property that it is increasing in $\tilde{\Gamma}_{T-1}$. This process can then be repeated iteratively.

Consider terminal period $T-1$. The value to continuing is:

$$ContVal_{T-1} = \tilde{\Gamma}_{T-1} f(T-1) - k. \tag{59}$$

There is clearly no option value, as the next period is the terminal period. In period $T-2$, the firm's continuation value is:

$$\begin{aligned}
ContVal_{T-2} &= \Pr\left(C_{T-1}|\tilde{\Gamma}_{T-2}\right) E\left(P\left(\tilde{\Gamma}_T\right) - P_{T-2} - 2k|C_{T-1}, \tilde{\Gamma}_{T-2}\right) && (60)\\
&\quad + \left(1 - \Pr\left(C_{T-1}|\tilde{\Gamma}_{T-2}\right)\right) E\left(P\left(\tilde{\Gamma}_{T-1}\right) - P_{T-2} - k|C^C_{T-1}, \tilde{\Gamma}_{T-2}\right) && (61)\\
&= \tilde{\Gamma}_{T-2} f(T-2) - k + OptVal\left(T-2, \tilde{\Gamma}_{T-2}\right), && (62)
\end{aligned}$$

where $OptVal\left(T-2, \tilde{\Gamma}_{T-2}\right) = \Pr\left(C_{T-1}|\tilde{\Gamma}_{T-2}\right) E\left(P\left(\tilde{\Gamma}_T\right) - P\left(\tilde{\Gamma}_{T-1}\right) - k|C_{T-1}, \tilde{\Gamma}_{T-2}\right)$. This can be seen by adding and subtracting $\Pr\left(C_{T-1}|\tilde{\Gamma}_{T-2}\right) E\left(P\left(\tilde{\Gamma}_{T-1}\right) - P_{T-2} - k|C_{T-1}, \tilde{\Gamma}_{T-2}\right)$ and simplifying.

The option value is nonnegative since the firm would never choose to continue unless doing so yielded positive returns in expectation. We next show that option value is increasing in $\tilde{\Gamma}_{T-2}$. Clearly, $\Pr\left(C_{T-1}|\tilde{\Gamma}_{T-2}\right)$ is increasing in $\tilde{\Gamma}_{T-2}$. Furthermore, by the smoothing law:

$$\begin{aligned}
E\left(P\left(\tilde{\Gamma}_T\right) - P\left(\tilde{\Gamma}_{T-1}\right) |\tilde{\Gamma}_{T-1} \geq c, \tilde{\Gamma}_{T-2}\right) &= E\left(E\left(P\left(\tilde{\Gamma}_T\right) - P\left(\tilde{\Gamma}_{T-1}\right) |\tilde{\Gamma}_{T-1}\right) |\tilde{\Gamma}_{T-1} \geq c, \tilde{\Gamma}_{T-2}\right)\\
&= f(T-1) E\left(\tilde{\Gamma}_{T-1}|\tilde{\Gamma}_{T-1} \geq c, \tilde{\Gamma}_{T-2}\right).
\end{aligned}$$

Next, we show that this expectation is increasing in $\tilde{\Gamma}_{T-2}$. The argument is nontrivial since FSD is not a sufficient condition for *truncated* expectations to be increasing (see, e.g., Shaked and Shanthikumar (2006)).

Lemma 4 $\frac{\partial}{\partial \tilde{\Gamma}_{T-2}} E\left(\tilde{\Gamma}_{T-1}|\tilde{\Gamma}_{T-1} \geq c, \tilde{\Gamma}_{T-2}\right) > 0.$

Proof of Lemma 4:

Note that

$$\begin{aligned}
E\left(\tilde{\Gamma}_{T-1}|\tilde{\Gamma}_{T-1} \geq c, \tilde{\Gamma}_{T-2}\right) &= E\left(E\left(\tilde{\Gamma}_{T-1}|\tilde{\Gamma}_{T-1} \geq c, \tilde{\Gamma}_{T-2}, \tilde{V}\right) |\tilde{\Gamma}_{T-1} \geq c, \tilde{\Gamma}_{T-2}\right)\\
&\equiv E\left[h^*\left(\tilde{\Gamma}_{T-2}, \tilde{V}\right) |\tilde{\Gamma}_{T-1} \geq c, \tilde{\Gamma}_{T-2}\right],\\
\text{where } h^*\left(\tilde{\Gamma}_{T-2}, \tilde{V}\right) &= E\left(\tilde{\Gamma}_{T-1}|\tilde{\Gamma}_{T-1} \geq c, \tilde{\Gamma}_{T-2}, \tilde{V}\right).
\end{aligned}$$

We first show that $h^*\left(\tilde{\Gamma}_{T-2}, \tilde{V}\right)$ is increasing in $\tilde{\Gamma}_{T-2}$ and $\tilde{V}$. Combining this with the fact that $\tilde{\Gamma}_{T-2}$ shifts the distribution of $\tilde{V}$ in the sense of FSD and using an integration by parts argument as was done previously, we will have proved the result.

Note that $\tilde{V}$ and $\tilde{\Gamma}_{T-2}$ shift the distribution of $\tilde{\Gamma}_{T-1}|\tilde{\Gamma}_{T-2}, \tilde{V}$ upwards in the sense of monotone likelihood ratio dominance. To see this, suppose $\Gamma_{t-1} > \Gamma'_{t-1}$. For MLR dominance to hold, we must show the following increases in V and Γ_{t-2}:

$$\begin{aligned}
\frac{\Pr\left(\Gamma_{t-1}|\Gamma_{t-2}, V\right)}{\Pr\left(\Gamma'_{t-1}|\Gamma_{t-2}, V\right)} &= \frac{\frac{e^{-V} V^{\left(\Gamma_{t-1}-\Gamma_{t-2}\right)}}{\left(\Gamma_{t-1}-\Gamma_{t-2}\right)!}}{\frac{e^{-V} V^{\left(\Gamma'_{t-1}-\Gamma_{t-2}\right)}}{\left(\Gamma'_{t-1}-\Gamma_{t-2}\right)!}} \\
&= \frac{\left(\Gamma'_{t-1}-\Gamma_{t-2}\right)!}{\left(\Gamma_{t-1}-\Gamma_{t-2}\right)!} V^{\left(\Gamma_{t-1}-\Gamma'_{t-1}\right)}.
\end{aligned}$$

This is clearly increasing in V, and is increasing in Γ_{t-2} since $\frac{\left(\Gamma'_{t-1}-\Gamma_{t-2}\right)!}{\left(\Gamma_{t-1}-\Gamma_{t-2}\right)!} = \left(\prod_{i=1}^{\Gamma'_{t-1}-\Gamma_{t-1}} \left(\Gamma'_{t-1}-\Gamma_{t-2}+i\right)\right)^{-1}$ increases in Γ_{t-2}.

MLR dominance implies that truncated expectations are increasing, that is, if z increases the expectation of $\tilde{x}$ in the sense of MLR, then $\forall T$, $\frac{\partial}{\partial z} E\left[\tilde{x}|\tilde{x} > T, z\right]$ (see chapter 1 of Shaked and Shanthikumar (2006) for a proof in the continuous case; a proof in the discrete case follows similarly).

Finalizing the proof of Proposition 4:

Thus, we have that the option value is increasing in $\tilde{\Gamma}_{T-2}$, and hence the continuation value in time $T-2$ is increasing in $\tilde{\Gamma}_{T-2}$. Backing up one more step to show how the inductive argument would continue, consider the continuation value at time $T-3$. This is given by:

$$ContVal_{T-3} = \tilde{\Gamma}_{T-3} f\left(T-3\right) - k + OptVal\left(T-3\right), \tag{63}$$

where

$$OptVal\,(T-3) = \Pr\left(C_{T-2}|\tilde{\Gamma}_{T-3}\right) E\left(ContVal_{T-2}|\tilde{\Gamma}_{T-3}\right) \tag{64}$$
$$= \Pr\left(C_{T-2}|\tilde{\Gamma}_{T-3}\right) E\left(\tilde{\Gamma}_{T-2} f\,(T-2) + \Pr\left(C_{T-1}|\tilde{\Gamma}_{T-2}\right) ContVal_{T-1}|\tilde{\Gamma}_{T-3}\right).$$

We know that $\frac{\partial}{\partial \tilde{\Gamma}_{T-3}} \Pr\left(C_{T-2}|\tilde{\Gamma}_{T-3}\right) > 0$. In addition, $\frac{\partial}{\partial \tilde{\Gamma}_{T-2}} ContVal\left(\tilde{\Gamma}_{T-2}\right) > 0$; combined with the first lemma that says increases in $\tilde{\Gamma}_{T-3}$ increase the distribution of $\tilde{\Gamma}_{T-2}$ in the sense of first order stochastic dominance, this implies that an increase in $\tilde{\Gamma}_{T-3}$ increases the expected continuation value. Continuing by induction, this shows that the continuation value in any period τ is increasing in $\tilde{\Gamma}_{\tau}$, and thus, the firm continues for values of $\tilde{\Gamma}_{\tau} \in [c_{\tau}, \infty)$ for some $c_{\tau} \geq 0$.

Finally, we show that $E\left(\tilde{\varsigma}|\tilde{V}\right)$ is increasing in $\tilde{V}$. Note that:

$$E\left(\tilde{\varsigma}|\tilde{V}\right) = \sum_{i=1}^{T} \Pr\left(\tilde{\varsigma} \geq i|\tilde{V}\right) \tag{65}$$
$$= \sum_{\tilde{s}_1 > c_1} \sum_{\tilde{s}_2 > c_2 - \tilde{s}_1} \cdots \sum_{s_i > c_i - \tilde{\Gamma}_{i-1}} \Pr\left(\tilde{s}_1|\tilde{V}\right) \cdots \Pr\left(\tilde{s}_2|\tilde{V}\right) \tag{66}$$
$$= \sum_{\tilde{s}_1 > c_1} \Pr\left(\tilde{s}_1|\tilde{V}\right) \sum_{\tilde{s}_2 > c_2 - \tilde{s}_1} \Pr\left(\tilde{s}_2|\tilde{V}\right) \cdots \sum_{s_i > c_i - \tilde{\Gamma}_{i-1}} \Pr\left(\tilde{s}_2|\tilde{V}\right). \tag{67}$$

Since $\tilde{V}$ increases the distribution of $\tilde{s}_i$ in the sense of first order stochastic dominance, for any $q > 0$, $\frac{\partial}{\partial V} \sum_{\tilde{s}_i > q} \Pr\left(\tilde{s}_i|\tilde{V}\right) > 0$. This implies the result.

Proof of Proposition 5:

We prove the proposition in terms of a_i and b_i and then transform the result into the parameterization in terms of μ_V and σ_V^2. The proof is very similar to the case for a single asset. First, apply the MGF of a normal distribution, we get:

$$
\begin{aligned}
& E\left(E\left(U(D\,(x-P))|\tilde{V}\right)\right) \\
= & -e^{-\sum\limits_{i=1}^{N}\rho D_i(\mu_i-P_i)} E\left(e^{\frac{\rho^2\tilde{V}_F}{2}\left(\sum\limits_{i=1}^{N}D_i^2\beta_i^2+2\sum\limits_{i=1}^{N}\sum\limits_{j=i+1}^{N}\beta_i\beta_j D_i D_j\right)}\right)\prod_{i=1}^{N}E\left(e^{\frac{\rho^2}{2}D_i^2\tilde{V}_i}\right). \qquad (68)
\end{aligned}
$$

Applying monotone transformations, and then using the MGF of the gamma distribution, we find:

$$
\begin{aligned}
& \arg\max_{\tilde{D}} E\left(E\left(U(D\,(x-P))|\tilde{V}\right)\right) \qquad (69) \\
= & \arg\max_{\tilde{D}} \sum_{i=1}^{N}\rho D_i\left(\mu_i-P_i\right)-\ln E\left(e^{\frac{\rho^2\tilde{V}_F}{2}\left(\sum\limits_{i=1}^{N}D_i^2\beta_i^2+2\sum\limits_{i=1}^{N}\sum\limits_{j=i+1}^{N}\beta_i\beta_j D_i D_j\right)}\right)-\sum_{i=1}^{N}\ln E\left(e^{\frac{\rho^2}{2}D_i^2\tilde{V}_i}\right) \\
= & \arg\max_{\tilde{D}} \sum_{i=1}^{N}\rho D_i\left(\mu_i-P_i\right)+a_F\ln\left(1-\frac{\rho^2}{2b_F}\left(\sum_{i=1}^{N}D_i^2\beta_i^2+2\sum_{i=1}^{N}\sum_{j=i+1}^{N}\beta_i\beta_j D_i D_j\right)\right) \\
& +\sum_{i=1}^{N}a_i\ln\left(1-\frac{\rho^2 D_i^2}{2b_i}\right).
\end{aligned}
$$

Note that, as in the single asset case, a conjecture and verify approach and the assumption that $\frac{\rho^2}{N^2}\left(\sum\limits_{i=1}^{N}\beta_i^2+2\sum\limits_{i=1}^{N}\sum\limits_{j=i+1}^{N}\beta_i\beta_j\right)<2b_F$ ensures that expected utility exists. The first order conditions for the k^{th} demand imply:

$$
\rho\left(\mu_k-P_k\right)+a_F\frac{-\frac{\rho^2}{b_F}D_k\beta_k^2-\frac{\rho^2}{b_F}\sum\limits_{i=1}^{N}D_i\beta_i\beta_j}{1-\frac{\rho^2}{2b_F}\left(\sum\limits_{i=1}^{N}D_i^2\beta_i^2+2\sum\limits_{i=1}^{N}\sum\limits_{j=i+1}^{N}\beta_i\beta_j D_i D_j\right)}+a_k\frac{-\frac{\rho^2 D_k}{b_k}}{1-\frac{\rho^2 D_k^2}{2b_k}}=0. \qquad (70)
$$

In equilibrium, we know that all demands are $\frac{1}{N}$. Plugging this in and solving, we get

$$\begin{aligned} P_k &= \mu_k - \frac{a_F\rho\left(\beta_k^2 + \sum_{i=1}^{N}\beta_i\beta_k\right)}{Nb_F - \frac{\rho^2}{2N}\left(\sum_{i=1}^{N}\beta_i^2 + 2\sum_{i=1}^{N}\sum_{j=i+1}^{N}\beta_i\beta_j\right)} - \frac{a_k\rho}{Nb_k - \frac{\rho^2}{2N}} \\ &= \mu_k - \frac{a_F\rho\left(\beta_k^2 + N\beta_k\right)}{Nb_F - N\frac{\rho^2}{2}} - \frac{a_k\rho}{Nb_k - \frac{\rho^2}{2N}}. \end{aligned} \tag{71}$$

After substituting $a_k = \frac{\mu_{V_k}^2}{\sigma_{V_k}^2}$, $b_k = \frac{\mu_{V_k}}{\sigma_{V_k}^2}$, $a_F = \frac{\mu_{V_F}^2}{\sigma_{V_F}^2}$, and $b_F = \frac{\mu_{V_F}}{\sigma_{V_F}^2}$, and performing algebraic manipulations, we find:

$$\begin{aligned} P_k &= \mu_k - \frac{\frac{\mu_{V_F}^2}{\sigma_{V_F}^2}\rho\left(\beta_k^2 + N\beta_k\right)}{N\frac{\mu_{V_F}}{\sigma_{V_F}^2} - N\frac{\rho^2}{2}} - \frac{\frac{\mu_{V_i}^2}{\sigma_{V_i}^2}\rho}{N\frac{\mu_{Vi}}{\sigma_{V_i}^2} - \frac{\rho^2}{2N}} \\ &= \mu_k - \frac{\rho\mu_{V_F}}{N}\left(\beta_k^2 + N\beta_k\right) - \frac{\rho\mu_{Vk}}{N} \\ &\quad - \frac{\frac{1}{2}\rho^2 N\frac{\sigma_{V_F}^2}{\mu_{V_F}}}{N - \frac{1}{2}\rho^2 N\frac{\sigma_{V_F}^2}{\mu_{V_F}}}\frac{\rho\mu_{V_F}}{N}\left(\beta_k^2 + N\beta_k\right) - \frac{\frac{1}{2}\rho^2\frac{\sigma_{V_k}^2}{\mu_{V_k}}}{N^2 - \frac{1}{2}\rho^2\frac{\sigma_{V_k}^2}{\mu_{V_k}}}\frac{\rho\mu_{V_k}}{N}. \end{aligned} \tag{72}$$

It can be shown that a first order stochastic dominant shift in $\tilde{V}_F$ reduces the price of all firms by adding $k_F > 0$ to $\tilde{V}_F$ as was done in the proof of Proposition 1.

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