Income Smoothing as Rational Equilibrium Behavior? A Second Look

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Abstract

In this paper I revisit the issue of real income smoothing in the setting used by Lambert (1984). I demonstrate that the particular effect identified in his paper is actually an error: under his assumptions there is no input driven equilibrium income smoothing of the type he suggests. There are, however, several other drivers of equilibrium behavior ignored in that paper. In this paper I identify those and for the particular model structure show that when all effects are considered together there is little support for the suggestion that second-best earnings generally is being smoothed through the equilibrium behavior

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1 INTRODUCTION

One of the earliest formal results in the Accounting Literature on (real) equilibrium earnings management is that of income smoothing provided by Lambert (1984). In a multi-period setting where the optimal first-best strategy is to implement the same expected earnings (i.e., "action") in every sub-period, the deviation in equilibrium behavior under the optimal second-best multi-period contract is not just a matter of lowering the effort level as in the similar one-period model variant, but also a matter of conditionality: in the second-best, future actions, in this class of models, generally depend on past earnings realizations. Lambert (1984) aimed to provide if not a general proof then a strong suggestion that such interdependencies would likely lead to less volatile earnings as low actions would follow high outcomes (and vice versa) and thus sub-period earnings would be mean-reverting, thereby depressing the aggregate volatility of earnings.

The fact that the result forwarded by Lambert (1984) has survived and been a key reference for over more than three decades may be attributable to the seemingly straightforward idea(s) and the intuition behind the result. Specifically the idea that when a manager learns that "things" are on course to be better than initially expected, and thus that his total expected compensation and utility exceed his initial expectations, this manager may start to value leisure more relative to additional future compensation. Consequently, he may therefore choose to pull back a bit on future effort, causing the above-mentioned mean reversion. Because this does make some intuitive sense, the presence of negative auto-correlation in second-best earnings remains not only generally accepted as being valid from a formal theoretical perspective, but also continues to be frequently cited by, in particular, empirical papers investigating issues related to managerial incentives for managing earnings.

It should be noted that Lambert (1984) is careful to point out that negative serial correlation between realized outcomes and future efforts leads to smoother earnings (in expectation) only if earnings is defined as the aggregate output of several periods (two in his case). While that is arguably more consistent with sacrificing earnings smoothness on the altar of low balance sheet volatility, that point is not a focus of this paper, nor is it something I address directly. Yet, while Lambert (1984) makes no attempt to extend the correlation result to alternate preference representations, he does argue that real (and perhaps also accounting) income-smoothing is a natural if not general property of the second-best to the point where the behavior should be considered empirically relevant. The results and insights provided in this paper makes clear that this line of thinking is neither complete nor correct.

While my analysis (coincidentally) does expose the error(s) contained in Lambert (1984) , the overall purpose here is to give a more detailed understanding of all earnings-related properties that can reasonably be predicted by a second-best agency model of the specific type explored by Lambert (1983 & 1984). In doing so, I make several points that should significantly change the status-quo thinking on this issue. As the starting point, I first establish that the proof of the Proposition in Lambert (1984) is incorrect for a number of reasons. Perhaps most significantly, Lambert (1984) implicitly over-constrains the problem in such a way that one of the key effects that potentially *does* lead to the behavior suggested, is disallowed from the set of feasible solutions and is therefore absent from his analysis.[1] This particular effect, which I refer to as the "intertemporal incentive effect" in this paper, consists of inducing outcome contingent variations in *future* (costly) workload to reduce the costly variations in future pay needed to incentivize current efforts.[2]

To establish "smoothing" as part of second-best equilibrium behavior, Lambert's (1984) proof instead relies on wealth effects resulting from "memory" in the optimal contract. However, for this particular class of multi-period full-commitment models, the cost of providing incentives in any given sub-period is independent of updates to the agent's expected utility during the contracting horizon if (and only if) the agent has a power utility function where the power is one half. Therefore, as I also show, absent the above-mentioned intertemporal incentive effect, the optimal current period action for the Lambert (1984) preference specifi-

[1] This problem actually originates in Lambert (1983). Specifically equation (9) on p. 445.

[2] As I show, partially rewarding (penalizing) the agent for good (bad) outcomes using reduced (increased) future workload is always optimal in this type of model.

cation, is actually independent of prior outcomes even if the current period's compensation is not.

Because the particular model formulation used in Lambert (1984) actually represents the case in which wealth-effect driven real earnings management does *not* take place, it also provides the cleanest setting for characterizing the real source of equilibrium demand for outcome contingent effort choice in this class of models: lowering the cost of implementing *prior* periods' actions, i.e., the intertemporal incentive effect. Specifically, the cost of having to work harder/less hard in the future represents a penalty/reward to the agent that provides current incentive just as getting a smaller/bigger bonus in the future does. As I show, splitting current incentives between variations in future compensation and variation in future (costly) work-loads is always efficient regardless of the specifics of the principal's and the agent's respective utility functions.[3] As such I am able to establish generally a version of the gross effect suggested by Lambert (1984): the derivative of current effort with respect to past performance is strictly less when innovations in expected utility are caused by prior moral hazard problems than when they are unrelated to prior moral hazard.

While the intertemporal incentive effect is one-directional and thus always in favor of the behavior suggested by Lambert (1984), the equilibrium relation between current actions and past results is, however, determined jointly by *both* the wealth and the intertemporal incentive effect. Absent the latter, wealth-effects drive the relation between past outcomes and present efforts, but there is no particular natural prediction to be had here. For agents with utility functions for which aversion to risk, properly defined, decreases in wealth, the basic incentive is to make equilibrium effort an increasing function of past outcomes whereas the opposite is obviously the case when risk aversion is advancing in past outcomes. On top of that, this is conditional on the principal being risk neutral. With a risk-averse principal, the equilibrium implications of wealth effects, while clearly central here, become even more intractable.

[3] Within the class of models with time additive preferences where the agent's cost of effort is denominated in utiles.

Lastly, regardless of the (net) equilibrium relation between current actions and past outcomes, equilibrium actions in this type of model are in general a function of time: expected second-best effort is declining period-by-period and expected income is therefore also declining over time. This general effect of moral hazard on the time series properties of earning is also missing from Lambert (1984) which instead suggests that if actions are not allowed to be outcome dependent, they would actually be *constant* over time. To the contrary, I show that the time-dependent decline in expected earnings is robust to the specification of the agent's preferences. More importantly, it is generally at odds with standard definitions of, motives for, or causes of income smoothing even in cases where the agent's preferences are such that the behavior suggested by Lambert (1984) actually is part of the equilibrium.

The remainder of this paper proceeds as follows. In the next section, the model and the notation used here are laid out. A formal result on the nature of wealth effects in the additively-separable model formulation is provided in section 3. Section 4 then identifies the unrelated and previously ignored features of the model that do make the time-series behavior of the second-best deviate from that of the first-best. Concluding remarks are contained in section 5.

2 MODEL

In the interest of familiarity and comparability, I mainly adapt the notation of Holmström (1979) with the added structure necessary to accommodate the specific assumptions of Lambert (1984). Accordingly, a risk-neutral principal, who values his end of horizon aggregate residual by the linear function $v(y) = y$, contracts with a risk- and effort-averse agent for $T > 1$ (sub-) periods. The objective of doing so is for the agent to favorably impact the period $t \in \{1, ..., T\}$ cash-flow probability distribution $f(x_t|a_t)$, where $x_t \in X$ is the realized (and immediately observed) cash-flow for period t, and $a_t \in A \subseteq R$ is the effort committed by the agent at the start of period t. The contract specifies the compensation paid to the agent at the end of each period $t \in \{1, ..., T\}$ as a function of everything observed up to that point in time. Let $\overrightarrow{x}_t$ denote the vector of realized cash flows up to and including period t. The agent's period t compensation then is denoted as $s_t\left(\overrightarrow{x}_{t-1}, x_t\right)$, where x_t then, obviously, denotes the outcome realized at the end of period t, and $\overrightarrow{x}_0 = \emptyset$.

The agent is assumed to be risk-averse and have time additive preferences for consumption of the form $u\left(\{s_t\}_{t=1}^{T}\right) = \sum_{t=1}^{T} u\left(s_t\left(\overrightarrow{x}_t\right)\right)$. Similarly, his (convexly increasing) cost of all efforts exerted at the start of each sub-period t are time additive as well and thus takes the form $c\left(\{a_t\}_{t=1}^{T}\right) = \sum_{t=1}^{T} c\left(a_t\left(\overrightarrow{x}_{t-1}\right)\right)$. Both parties are assumed able to fully commit to the contract agreed prior to the start of period one (hereafter with a slight abuse of notation denoted period $t = 0$).[4] At the time of contracting the agent has outside opportunities worth $\underline{U}$ utiles should he not accept the long-run contract offered by the principal. As is always assumed in this particular class of models, the principal has free access to any needed liquidity. The agent, in contrast, has no personal means of intertemporal consumption transfers, and, thus, can neither borrow nor save privately: all income physically received (i.e., paid which is different here from what is actually earned) by the agent by the close of period t therefore goes towards creating utility for that period and that period alone. For simplicity, I ignore any discounting as implications are largely trivial here. Finally, as in Lambert (1983 & 1984), the first-order approach is assumed to be valid with the standard implications for the differentiability of $f\left(x_t|a_t\right)$ and $c\left(\cdot\right)$ with respect to a_t.

3 BENCHMARK

The objective of this section is to identify the particular preference representation for which a current risk-premium is independent of any wealth effects caused by partially compensating past results in the current period. Initially I focus on the relation between x_{T-1} and $a_T\left(\overrightarrow{x}_{T-2}, x_{T-1}\right)$ for some fixed $\overrightarrow{x}_{T-2}$. Let λ be the multiplier on the agent's participation constraint, μ_1 be the multiplier on the first period incentive compatibility constraint,

[4] For notational convenience I'll use the convention that $\overrightarrow{x}_0 \equiv x_0 \equiv \emptyset$.

$\mu_t\left(\overrightarrow{x}_{t-1}\right)$ As is known from the initial literature detailing the solution to this class of models based on the first-order approach,[5] when the principal is risk-neutral, the optimal period T contract must satisfy

$$\frac{1}{u'\left(s_T\left(\overrightarrow{x}_{T-1}, x_T\right)\right)} = \lambda_T\left(\overrightarrow{x}_{T-1}\right) + \mu_T\left(\overrightarrow{x}_{T-1}\right)\frac{f_a\left(x_T|a_T\left(\overrightarrow{x}_{T-1}\right)\right)}{f\left(x_T|a_T\left(\overrightarrow{x}_{T-1}\right)\right)}, \tag{1}$$

where

$$\lambda_T\left(\overrightarrow{x}_{T-1}\right) \equiv \lambda + \mu_1\frac{f_a\left(x_1|a_1\right)}{f\left(x_1|a_1\right)} + \sum_{t=2}^{T}\lambda_{t-1}\left(\overrightarrow{x}_{t-1}\right) + \mu_t\left(\overrightarrow{x}_{t-1}\right)\frac{f_a\left(x_t|a_t\left(\overrightarrow{x}_{t-1}\right)\right)}{f\left(x_t|a_t\left(\overrightarrow{x}_{t-1}\right)\right)}$$

is non-decreasing in each $x_t \in \overrightarrow{x}_{T-1}$ by the $MLRC$ and the fact that each $\mu_t > 0\ \forall t = 1, ..., T$, which in this case follows directly from Jewitt's (1988) Lemma 1, since the principal here is assumed risk-neutral.

Denote by $z\left(\cdot\right)$ a function such that $z\left(1/u'\left(\cdot\right)\right) \equiv u\left(\cdot\right).$ Then I can write

$$E_{x_T}u\left(s_T\left(\overrightarrow{x}_{T-1}, x_T\right)\right) = E_{x_T}\left[z\left(\lambda_T\left(\overrightarrow{x}_{T-1}\right) + \mu_T\left(\overrightarrow{x}_{T-1}\right)\frac{f_a\left(x_T|a_T\left(\overrightarrow{x}_{T-1}\right)\right)}{f\left(x_T|a_T\left(\overrightarrow{x}_{T-1}\right)\right)}\right)\right].$$

where then $E_{x_T}\left[u\left(s_T\left(\overrightarrow{x}_{T-1}, x_T\right)\right)\right]$ is increasing in all elements of $\overrightarrow{x}_{T-1}$ since as it is easily verified, $z\left(\cdot\right)$ is a strictly increasing differentiable function of its argument. Accordingly,

$$s_T\left(\overrightarrow{x}_{T-1}, x_T\right) = h\left(E_{x_T}\left[u\left(s_T\left(\overrightarrow{x}_{T-1}, x_T\right)\right)\right] + \Phi_T\left(\overrightarrow{x}_{T-1}, x_T\right)\right)$$

where $h\left(\cdot\right)$ is the agent's inverse utility function and $\Phi_T\left(\overrightarrow{x}_{T-1}, x_T\right)$ is the realization of a utile-denominated risky gamble in x_T, $\widetilde{\Phi}_T\left(\overrightarrow{x}_{T-1}, x_T\right)$, that is actuarially fair in *utility*-space. With the additively separable representation here where the agent trades off cost of effort against expected utiles it is straightforward to verify that a given $\widetilde{\Phi}_T\left(\overrightarrow{x}_{T-1}, x_T\right)$ implements the same $a_T\left(\widetilde{\Phi}_T\left(\overrightarrow{x}_{T-1}, x_T\right)\right)$ independent of $E_{x_T}\left[u\left(s_T\left(\overrightarrow{x}_{T-1}, x_T\right)\right)\right] \equiv \underline{U}_T\left(\overrightarrow{x}_{T-1}\right).$

So consider now some risky contract, $\widetilde{s}_T\left(\overrightarrow{x}_{T-2}, x_{T-1}\right)$, that implements $\widetilde{a}_T\left(\overrightarrow{x}_{T-2}, x_{T-1}\right)$

[5] See Lambert (1983).

as a best response by the agent with a_T unobservable to the principal. In particular, consider the excess cost to the principal of implementing $\widetilde{a}_T\left(\overrightarrow{x}_{T-2}, x_{T-1}\right)$ using the contract $\widetilde{s}_T\left(\overrightarrow{x}_{T-1}, x_T\right)$ over the cost of implementing $\widetilde{a}_T\left(\overrightarrow{x}_{T-2}, x_{T-1}\right)$ had a_T been jointly observable so the principal would be able to simply instruct the agent to implement $\widetilde{a}_T\left(\overrightarrow{x}_{T-2}, x_{T-1}\right)$. Denote this cost differential $\Delta_T\left(\underline{U}_T\left(\overrightarrow{x}_{T-1}\right)\right)$. In other words,

$$\begin{aligned}\Delta_T\left(\underline{U}_T\left(\overrightarrow{x}_{T-1}\right)\right) &\equiv E_{x_T}\left[h\left(\underline{U}_T\left(\overrightarrow{x}_{T-1}\right)+\widetilde{\Phi}_T\left(\overrightarrow{x}_{T-1}, x_T\right)\right)\right] \\ &\quad -h\left(\underline{U}_T\left(\overrightarrow{x}_{T-1}\right)\right),\end{aligned}$$

so that

$$\begin{aligned}\partial\Delta_T\left(\underline{U}_T\left(\overrightarrow{x}_{T-1}\right)\right)/\partial\underline{U}_T\left(\overrightarrow{x}_{T-1}\right) &= E_{x_T}\left[h'\left(\underline{U}_T\left(\overrightarrow{x}_{T-1}\right)+\widetilde{\Phi}_T\left(\overrightarrow{x}_{T-1}, x_T\right)\right)\right] \\ &\quad -h'\left(\underline{U}_T\left(\overrightarrow{x}_{T-1}\right)\right).\end{aligned}$$

Then I can state the following (adapted from Hemmer (2016a)):

Lemma 1 $\partial\Delta_T\left(\underline{U}_T\left(\overrightarrow{x}_{T-1}\right)\right)/\partial\underline{U}_T\left(\overrightarrow{x}_{T-1}\right) = 0$ *if and only if* $u(\cdot) = \alpha + \beta\sqrt{\cdot}$, *with* $\beta > 0$.

Proof.

It follows directly from Jensen's inequality that the above derivative is zero iff $h'(\cdot)$ is linear in its argument. Straightforward integration reveals then that this is the case iff $h(\cdot)$ is quadratic and thus iff $u(\cdot) = \alpha + \beta\sqrt{\cdot}$.

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In other words, the cost of providing incentives in a multi-period setting like this is independent of updates to the agent's expected wealth if and only if that agent has a power utility function where the power is one half.

4 EQUILIBRIUM CAUSES OF SERIAL CORRELATION

The purpose of this section is to dissect the difference between the first- and second-best behavior in such a way as to isolate and identify the nature of the three unique causes of second-best serial correlation present in this model formulation: wealth-effects, intertemporal incentive effects and horizon effects. Because wealth effects are the focal point of Lambert (1984), I start, in the next sub-section, by establishing that for the model as specified, the particular case of a risk neutral principal and an agent with square-root preferences is actually the special case where wealth effects are not present in the model. This, in turn, helps provide the simplicity that allows me to cleanly identify the other two effects that are always present here.

4.1 Wealth Effects

For simplicity and for ease of comparison with Lambert (1984), for the remainder of this paper I will concentrate of a simple two period version of the model introduced in the prior section. The general problem can then be stated as choosing $\{s_1(x_1), s_2(x_1, x_2), a_1, a_2(x_1)\}$ to

$$max \quad \int \left\{ x_1 - s_1(x_1) + \int (x_2 - s_2(x_1, x_2)) f(x_2, a_2(x_1)) dx_2 \right\} f(x_1, a_1) dx_1 \tag{PP}$$

$$s.t. \quad \int \left\{ u(s_1(x_1)) + \int u(s_2(x_1, x_2)) f(x_2, a_2(x_1)) dx_2 - c(a_2(x_1)) \right\} f(x_1, a_1) dx_1$$
$$-c(a_1) - \underline{U} \tag{IRP}$$

$$\int \left\{ u(s_1(x_1)) + \int u(s_2(x_1, x_2)) f(x_2, a_2(x_1)) dx_2 - c(a_2(x_1)) \right\} f_a(x_1, a_1) dx_1$$
$$-c'(a_1) = 0 \tag{IC1P}$$

$$\int u(s_2(x_1, x_2)) f_a(x_2, a_2(x_1)) dx_2 - c'(a_2(x_1)) = 0 \tag{IC2P}$$

The contract/actions combination that maximizes this Lagrangian must satisfy the standard Kuhn-Tucker first-order conditions, which for $s_1(x_1), s_2(x_1, x_2), a_1$ and $a_2(x_1)$ according to Lambert (1983) are:

$$\frac{1}{u'(s_1(x_1))} = \lambda + \mu_1 \frac{f_a(x_1, a_1)}{f(x_1, a_1)}, \tag{2}$$

$$\frac{1}{u'(s_2(x_1, x_2))} = \lambda + \mu_1 \frac{f_a(x_1, a_1)}{f(x_1, a_1)} + \mu_2(x_1) \frac{f_a(x_2, a_2(x_1))}{f(x_2, a_2(x_1))}, \tag{3}$$

$$\begin{aligned} & \int \left\{ x_1 - s_1(x_1) + \int (x_2 - s_2(x_1, x_2)) f(x_2, a_2(x_1)) dx_2 \right\} f_a(x_1, a_1) dx_1 \\ + \quad & \mu_1 \left[\int \left\{ u(s_1(x_1)) + \int u(s_2(x_1, x_2)) f(x_2, a_2(x_1)) dx_2 - c(a_2(x_1)) \right\} f_{aa}(x_1, a_1) dx_1 \right. \\ & \left. - c''(a_1) \right] = 0, \end{aligned} \tag{4}$$

$$\begin{aligned} & \int (x_2 - s_2(x_1, x_2)) f_a(x_2, a_2(x_1)) dx_2 \\ + \quad & \frac{\mu_2(x_1)}{f(x_1, a_1)} \left[\int u(s_2(x_1, x_2)) f_{aa}(x_2, a_2(x_1)) dx_2 - c''(a_2(x_1)) \right] = 0, \end{aligned} \tag{5}$$

where λ, μ_1 and $\mu_2(x_1)$ are, again, the standard Lagrange multipliers on the IR- and first- and second-period IC-constraints respectively. Note that the third of these conditions (w.r.t. a_1) does not reference the IR-constraint part of the Lagrangian. This is, according to Lambert (1983), because the first-order approach is assumed valid and that part of the derivative therefore must equal zero.[6] The same is given as the reason why the last condition references *neither* (IRP) or $(IC1P)$. Further note that the second and the fourth of the above conditions, that is (3) and (5), are the exclusive ingredients in the proof of equilibrium income smoothing offered by Lambert (1984).

To identify the link between past outcomes and future actions implied by this set of conditions, it is useful, as well as instructive, to consider a slightly different problem. Specifically, let $\{\lambda_1^*, \mu_1^*, \mu_2^*(x_1), a_1^*, a_2^*(x_1)\}$ denote the values of the parameters that solve the principal's problem as captured by (PP) and consider then an alternate situation where the principal does *not* face a first period moral hazard problem but where the optimal first period action

[6] I will return to the fallacy of this in section 4.2.

as well as the structure and nature of the second period problem remain intact. Specifically, assume:

Assumption: Suppose *i*) a_1 is observable, *ii*) $f_a(x_1, a_1) = 0$ for $a_1 > a_1^*$, and *iii*) that $s_1(x_1)$ and $s_2(x_1, x_2)$ are exogenously restricted to take the form of (2) and (3) respectively with $\mu_1 = \mu_1^*$.

This alternate problem then consists of choosing $\{k, \phi_2(x_1), w(x_2), a_1, a_2(x_1)\}$ to

$$max \quad \int \left\{ x_1 - s_1(x_1) + \int (x_2 - s_2(x_1, x_2)) f(x_2, a_2(x_1)) dx_2 \right\} f(x_1, a_1) dx_1 \tag{AP}$$

$$s.t. \int \left\{ u(s_1(x_1)) + \int u(s_2(x_1, x_2)) f(x_2, a_2(x_1)) dx_2 - c(a_2(x_1)) \right\} f(x_1, a_1) dx_1 - c(a_1) = \underline{U}$$

$$- \quad c(a_1) = \underline{U} \tag{IRA}$$

$$\int u(s_2(x_1, x_2)) f_a(x_2, a_2(x_1)) dx_2 - c'(a_2(x_1)) = 0 \tag{IC2A}$$

$$\frac{1}{u'(s_1(x_1))} = k + \mu_1^* \frac{f_a(x_1, a_1)}{f(x_1, a_1)} \tag{CO1A}$$

$$\frac{1}{u'(s_2(x_1, x_2))} = k + \mu_1^* \frac{f_a(x_1, a_1)}{f(x_1, a_1)} + \phi_2(x_1) w(x_2) \tag{CO2A}$$

Let λ and $\mu_2(x_1)$ represent the Lagrange multipliers on (IRA) and the $(IC2A)$ constraints respectively. It is then straight-forward to verify that here $k = \lambda$ and $\phi_2(x_1) w(x_2) = \mu_2(x_1) f_a(x_2, a_2(x_1)) / f(x_2, a_2(x_1))$ so that the structure of the (constrained) optimal contracts here are the same as for (PP).

The purpose of the alternate problem represented by (AP) is that it provides a means to address the following question: *if* the second period contract is irrelevant for the first period solution but the agent's second period compensation *does* depend on the first period's realized outcome (by fiat here, but none the less), what then would be the relation between second period second best action and first period realized outcome? Let $\left\{ \widetilde{a}_1, \widetilde{a}_2(x_1), \widetilde{\lambda}, \widetilde{\mu}_2(x_1) \right\}$ denote the parameter values that solves the alternate problem represented by (AP). The answer then is:

Proposition 1 *For* $u(y) = 2\sqrt{y}$, $d\widetilde{a}_2(x_1)/dx_1 = 0$.

Proof.

Clearly, the solution to the alternate problem has $a_1 = a_1^*$. Since $\widetilde{a}_2(x_1)$ is part of the solution to the Principal's overall alternate problem, given there are no first-period incentive considerations it must also be the solution to the following second-period effort-allocation problem:

$$\begin{aligned}
\max_{a_2(x_1)} \quad & \int_{x_1}\int_{x_2} x_2 - \left(\frac{\lambda}{2} + \mu_1^* \frac{f_a(x_1,a_1^*)}{f(x_1,a_1^*)} + \mu_2(x_1)\frac{f_a(x_2,a_2(x_1))}{f(x_2,a_2(x_1))}\right)^2 f(x_2|a_2(x_1))\,dx_2 f(x_1|a_1^*)\,dx_1 \\
s.t. \quad & \int_{x_1} c(a_2(x_1)) f(x_1|a_1^*)\,dx_1 = E\left[c(\widetilde{a}_2(x_1))\right], \\
& \mu_2(x_1) = c'(a_2(x_1)) \Big/ \int \frac{f_a(x_2,a_2(x_1))^2}{f(x_2,a_2(x_1))}dx_2,
\end{aligned}$$

where the second constraint is simply the second period incentive compatibility constraint rewritten using the properties of the agent's assumed utility function here. That is, the expected cost of the potentially state-contingent second-best second-period effort procured will be allocated over states such that the expected output minus the expected cost of the second-best second-period effort procured minus the expected cost of implementing the second-best second-period effort is maximized. Because randomization doesn't pay, here any relation between past outcomes and current actions must be the exclusive result of compensating past outcomes differentially in the current period. Further note, that both the cost of effort, $c(a_2)$, and the gross value of effort to the principal, $E[x_2|a_2]$, are independent of the agent's expected utility from consumption, $E[u(s_2(x_1,x_2))|x_1]$. Thus, the optimal period 2 contract will introduce effort-randomization based on x_1 if and only if the second-best cost-differential of implementing a particular a_2, $\Delta_2(\underline{U}_2(x_1))$, depends on $E[u(s_2(x_1,x_2))|x_1]$. Lemma 1 then concludes the proof.

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The point here is that absent first period incentive considerations, even if the second-period compensation paid to the agent *does* depend on the first period's outcome, the equilibrium second-period action does *not* when the agent has square-root preferences over consumption levels. This is significant for a number of reasons. First note that the first-order condition for the second period action choice of the alternate program (AP) is

$$\begin{aligned}
&\int (x_2 - s_2(x_1,x_2)) f_a(x_2,a_2(x_1))\,dx_2 \\
&+\frac{\mu_2(x_1)}{f(x_1,a_1^*)}\left[\int u(s_2(x_1,x_2)) f_{aa}(x_2,a_2(x_1))\,dx_2 - c''(a_2(x_1))\right] = 0
\end{aligned}$$

and thus identical to the one provided by Lambert (1983 & 1984) for the original problem (PP). And because so is the structure of the second period contract used here, the relation between $\mu_2(x_1)$ and $a_2(x_1)$ is identical as well. Accordingly, all the steps of the proof offered by Lambert (1984) can be replicated here and, if done, yield the same (false) conclusion that $d\widetilde{a}_2(x_1)/dx_1 < 0$. This proof is thus invalid here and given the parts of the two problems the proof relies on are structurally identical, it is invalid in the case of the original problem as well.

Before proceeding, then, it may be useful to point out one of the logical fallacies of this proof (I will return to others later). Lambert (1984) argues based on his expression $(A7)$ which is the same as the second constraint in the proof of Proposition 1 above, that $\mu_2(x_1)$ only depends on x_1 insofar $a_2(x_1)$ does. This is, of course, also a not so subtle hint that $a_2(x_1)$ here does not depend on x_1 unless $\mu_2(x_1)$ does. And unlike the chicken and the egg there is actually a defined logical sequence to the present problem. Recall that $\mu_2(x_1)$ is chosen at $t = 0$ as the part of the optimal contract that provides output-based variation in compensation and thus effort-incentives for the agent. The agent implements $a_2(x_1)$ subsequently as the agent's optimal response to the optimal contract at that later time. Clearly, then, here the agent's subsequent choice of second-period action depends on x_1 only to the extent $\mu_2(x_1)$ does. This implies conceptually that if $\mu_2(x_1)$ does not depend directly on x_1, neither will $a_2(x_1)$ which is exactly what is established by Proposition 1. Technically, then, when taking the partial derivative of (5) with respect to x_1 the derivative of $\mu_2(x_1)$ with respect to x_1 cannot be taken to be zero as part of a proof to establish that the derivative of $a_2(x_1)$ with respect to x_1 is not.

The bottom line is that in the case of the square root preference representation there are no wealth effects, and if one considers the optimal period 2 action entirely independent of its impact on the incentives for the period 1 action, which is the purpose here of the alternate program, (AP) , there is no demand for outcome contingent effort-randomization in this particular set-up. This is of course not true in general. As long as the principal remains risk-neutral, the nature of the wealth-effects depend directly on the functional form of $h'(\cdot)$. For example, staying within the power class, it is easily verified that for $\gamma \in (1/2, 1)$, $h'(U)$ is concave while the opposite is just as easily verified to be the case for $\gamma \in (0, 1/2)$. In the former case the opposite behavior from that proposed by Lambert (1984) is the effect of responding to past realizations while the effect is as suggested in the latter case. But the direction of the wealth effect does not even have to be the same across wealth-levels: for the case where $u(w) = -e^{2w^{1/2}}$, for example, where the agent exhibits decreasing *relative* risk-aversion, $h'(u)$ is convex for relatively low values of u but concave for relatively high ones.

4.2 Intertemporal Incentive Effects

While the second-period wealth-effects generated by the first period risk-sharing can go either way, optimal second period actions always depend on the nature of the first period incentive problem. In particular, it turns out, the more severe the first period moral hazard problem is, the more valuable it is to condition the second period action on realized first period outcome. As demonstrated above, absent a first-period moral hazard problem the optimal second-period action here is invariant to exogenously mandated wealth permutations generated by first period outcomes when the agent has a square root utility function. When the very same wealth permutation arises endogenously due to a first-period moral hazard problem, however, otherwise inefficient second-period effort variations emerge in equilibrium as a means of lowering the cost of providing first-period incentives

To see this, consider again the original problem represented by (PP). First note that for the square root case, by (2) and (3) here

$$u(s_1(x_1)) = \int u(s_2(x_1, x_2)) f(x_2, a_2(x_1)) \, dx_2.$$

Also note, that if we simply were to exogenously restrict attention to sharing rules that satisfy (2) and (3) and solve only for the optimal actions (along with the corresponding multiplier values) we would identify the same (second-best) solution as obtains from (PP). Following this 20/20 hindsight approach $(IC1P)$ can be re-expressed simply as

$$\int \{2u(s_1(x_1)) - c(a_2(x_1))\} f_a(x_1, a_1)dx_1 - c'(a_1) = 0.$$

The significance of this is, of course, that variations in second period actions that are dictated by first period outcomes impact the agent's first period incentives through variations in second-period costs and is a direct substitute for compensation-variations tied to first period outcome realizations. In particular, using this version of $(IC1P)$, the derivatives of the the Lagrangian with respect to first- and second-period effort become

$$\begin{aligned} & \int \left\{ x_1 - s_1(x_1) + \int (x_2 - s_2(x_1, x_2)) f(x_2, a_2(x_1)) dx_2 \right\} f_a(x_1, a_1)dx_1, \\ + \quad & \mu_1 \left[\int \{2u(s_1(x_1)) - c(a_2(x_1))\} f_{aa}(x_1, a_1)dx_1 - c''(a_1) \right] = 0, \end{aligned} \tag{6}$$

and

$$\begin{aligned} & \int (x_2 - s_2(x_1, x_2)) f_a(x_2, a_2(x_1)) dx_2 \\ & -\mu_1 \int c'(a_2(x_1)) f_a(x_1, a_1)dx_1 \\ & + \frac{\mu_2(x_1)}{f(x_1, a_1^*)} \left[\int u(s_2(x_1, x_2)) f_{aa}(x_2, a_2(x_1)) dx_2 - c''(a_2(x_1)) \right] = 0 \end{aligned} \tag{7}$$

The main point here is that the second period can only be viewed in isolation when there is no first period incentive problem; when $(IC1P)$ does not bind. If it is binding the choice of second period action as a function of first-period outcome plays a direct role in resolving the *first*-period incentive problem and $(IC1P)$ thus cannot be ignored by the principal when choosing $a_2(x_1)$ as suggested by (5).

The flaw behind the missing term in equation (9) and $(A4)$ in Lambert (1983) and

Lambert (1984) respectively, i.e., the second line of (7) above, is the argument referenced above that it is the validity of the first-order approach as reflected by $(IC2P)$ that makes this term equal to zero. This represents a fundamental misunderstanding of the vastly different choice problems facing the agent and the principal. $(IC1P)$ and $(IC2P)$ represent the agent's choice problem *after* the principal has chosen the structure of the contract. The principal's choice problem, in contrast, is to craft a deal that both attracts and appropriately incentivizes the agent. To see this clearly, consider the principal's problem of choosing an incentive compatible a_1. In its most general form $(IC1P)$ can here be written as:

$$\frac{\partial E\left[u\left(s_1\left(x_1\right)\right)\right]}{\partial a_1}+\frac{\partial E\left[u\left(s_2\left(x_1,x_2\right)\right)\right]}{\partial a_1}-c'\left(a_1\right)-\frac{\partial E\left[c\left(a_2\left(x_1\right)\right)\right]}{\partial a_1}=0. \tag{8}$$

Because the optimal contracts always must satisfy (2) and (3), $u(\cdot)=\sqrt{\cdot}$ implies, as is well known, that the agent's utility from consumption under the optimal contract is additively separable in x_1 and x_2 as well. Accordingly, in the *agent's* choice problem,

$$\frac{\partial E\left[u\left(s_2\left(x_1,x_2\right)\right)\right]}{\partial a_1}$$

is independent of $f\left(x_2,a_2(x_1)\right)$. But then, of course, in the *principal's* choice problem,

$$\begin{aligned}&\frac{\partial\left[\frac{\partial E\left[u\left(s_2\left(x_1,x_2\right)\right)\right]}{\partial a_1}\right]}{\partial a_2\left(x_1\right)}=\\&\frac{\partial\int\int u\left(s_2\left(x_1,x_2\right)\right)f\left(x_2,a_2(x_1)\right)dx_2 f_a(x_1,a_1)dx_1}{\partial a_2\left(x_1\right)}=0\qquad\forall x_1,\end{aligned}$$

while obviously $\partial\int c\left(a_2\left(x_1\right)\right)f_a(x_1,a_1)dx_1/\partial a_2\left(x_1\right)$ is *not*.

To crisply identify the effect of incentivizing first period action via variations in second period actions consider for simplicity (and wlog) the case where $c\left(a_t\right)=a_t$.[7] Then using (IRP), $(IC1P)$ and $(IC2P)$ along with (2) and (3), I can easily calculate

[7] See Rogerson (1985).

$$\lambda_1 = c(a_1) + E\left[c\left(a_2\left(x_1\right)\right)\right] + \underline{U}, \tag{9}$$

$$\mu_1 = \frac{1 + \int c\left(a_2\left(x_1\right)\right) f_a(x_1, a_1) dx_1}{\int \left(\frac{f_a(x_1,a_1)}{f(x_1,a_1)}\right)^2 f(x_1, a_1) dx_1} \tag{10}$$

and

$$\mu_2\left(x_1\right) = \frac{2}{\int \left(\frac{f_a(x_2,a_2(x_1))}{f(x_2,a_2(x_1))}\right)^2 f\left(x_2, a_2(x_1)\right) dx_2}. \tag{11}$$

Again, it is immediately clear from (11) that there is no *second*-period demand for some outcome-contingent variation in the second-period action here. The sole reason why second-period effort may depend on first-period outcome is though the impact of $a_2\left(x_1\right)$ on μ_1. It is also immediately obvious from (10) that if $a_2^*\left(x_1\right)$ *does* depend on x_1, $E\left[a_2^*\left(x_1\right)\right]$ necessarily is smaller for positive than for negative values of $f_a(x_1, a_1)$, consistent with the Proposition in Lambert (1984), since this lowers the cost of incentivizing *first*-period effort as captured by μ_1. This, of course, comes at the expense of second period second-best efficiency, so the optimality of conditioning second period effort on first period output depends on the net of these effects. Let $X_1^+ = \{x_1 | f_a(x_1, a_1) \geq 0\}$ and $X_1^- = \{x_1 | f_a(x_1, a_1) < 0\}$. For the model as specified we then have

Proposition 2 *For* $u\left(y\right) = \sqrt{y}$, $E_{X_1^+}\left[a_2\left(x_1\right)\right] < E_{X_1^-}\left[a_2\left(x_1\right)\right]$.

Proof.

Start by solving for the optimal a_1 and a_2 when the latter exogenously is restricted not to depend on x_1. Consider then to add a variation, $\delta_2\left(x_1\right)$, to the μ_2 thus obtained that is strictly positive for X_- and strictly negative for X_+ and with $\delta_2\left(X_1^+\right) f\left(X_1^+ | a_1\right) + \delta_2\left(X_1^-\right) f\left(X_1^- | a_1\right) = 0$. It is then easily verified that the derivative of the solution with respect to this variation evaluated at $\delta_2\left(x_1\right) = 0$ yields a strict improvement as there is no effect on the second period while the effect on numerator of (10), and thus the risk-premium needed to incentivize a_1^*, is strictly negative. The cost of effort-randomization is increasing convexly from zero, while the marginal benefit of a lower first-period risk premium is decreasing towards zero. Thus there is strict value to introduce a strictly positive amount of such a variation in the agent's contract. Finally note that for any $x_1 \in X_1^-$ there can be no value to set $a_2\left(x_1\right) < a_2\left(\widehat{x}_1\right)$ if $\widehat{x}_1 \in X_1^+$ since doing so introduces a costly variation in second-period effort while at the same time increasing the cost of incentivizing first period effort.

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The effect documented here is a somewhat general one in the additively separable preference specification: variations in compensation and variations in future workload are clearly substitutes when it comes to providing incentives.

4.3 Horizon Effects

The third and final second-best force that shapes the time-series properties of earnings is time itself, or, remaining time to be precise. As should be obvious from (1) and the discussion throughout, the time-additive preference structure makes it optimal for the principal to spread current period's incentive risk over remaining periods. Intuitively, then, the more periods left, the closer the solution is to the first-best while the fewer, the closer it is to the standard one-period second-best. As per the argument in the previous section, certainly the last period is worse (in expectation) than the one-period second-best due to the use of otherwise inefficient effort-randomization designed to aide incentive provision in prior periods. The effect of this is that (expected) effort decreases over time at an increasing rate. Although Lambert (1983) does show that commitment is valuable here in the sense the more periods that are covered by a contract the better, the link to the time series properties of output appears to be missed by Lambert (1984) as well.

To highlight this I will again use as a benchmark the case where second period action cannot depend on first period outcome but only be a function of time. Eliminating the term in the numerator of (10) that is the source of second period outcome-dependence and substituting into the objective function, the principal's constrained problem can here then be expressed as choosing a_1 and a_2 to maximize

$$E[x_1] + E[x_2] - \frac{\lambda^2}{2} - \int \frac{\mu_1^2}{2}\left(\frac{f_a(x_1,a_1)}{f(x_1,a_1)}\right)^2 f(x_1,a_1)dx_1$$
$$-\left\{\int \left(\frac{\mu_2(x_1)}{2}\right)^2 \int \left(\frac{f_a(x_2,a_2(x_1))}{f(x_2,a_2(x_1))}\right)^2 f(x_2,a_2(x_1))\,dx_2\right\} f(x_1,a_1)dx_1 \qquad (12)$$

or

$$E[x_1] + E[x_2] - \frac{\lambda^2}{2} - \frac{1}{2\int \left(\frac{f_a(x_1,a_1)}{f(x_1,a_1)}\right)^2 f(x_1,a_1)dx_1}$$
$$-\frac{1}{\int \left(\frac{f_a(x_2,a_2)}{f(x_2,a_2)}\right)^2 f(x_2,a_2)\,dx_2}$$

Let $\overline{a}_1^*$ and $\overline{a}_2^*$ denote the solution to (12). Since everything is symmetric in this formulation except for the "2" in the denominator of the term representing the cost of first-period incentives, it is clear here that $\overline{a}_1^* > \overline{a}_2^*$. This in turn implies that expected output is also decreasing over time here.

To establish that expected second-best effort is indeed decreasing over time, then, consider the difference between this solution and the solution to the unrestricted problem, a_1^* and $a_2^*(x_1)$. Again, following the result of the prior section, second-period effort-randomization lowers the cost of first-period incentives but increases the (expected) marginal cost of second period effort. As a result, we have $a_1^* > \overline{a}_1^* > \overline{a}_2^* > E[a_2^*(x_1)]$. That this relation generalizes to other utility functions than $u(y) = \sqrt{y}$ is established by the final proposition.

Proposition 3 $a_1^* > E[a_2^*(x_1)]$.

Proof.

The result follows almost directly from the proceeding discussion. To sketch a formal proof, consider two different problems: one where the principal contracts with the agent for two periods but where there is only an action to be taken/a moral hazard problem in the first and one otherwise identical problem that only entails taking an action/a moral hazard problem in the second period. Because the solution to the first problem spreads the implementation risk over the two remaining periods while the solution to the second problem can only allocate risk to that remaining period, the marginal cost to the principal of eliciting first-period effort in the first problem is strictly less than that of eliciting second-period effort in the second.

Next consider the full two period problem. First note that the wealth and intertemporal incentive effects always (weakly) increase the average marginal cost of eliciting second period effort. Since the optimal contract always transfers first period risk to the second period and since that is always (weakly) inefficient from the perspective of the second period, the result follows.

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While expected income thus is going to be declining over time, the model arguably also predicts that income volatility will be changing too. The effect that the shrinking remaining horizon has on income volatility is not guaranteed to be in one or the other direction, however. Clearly for the class of production functions identified by Jewitt (1988) for which the first-order approach is valid and of which the Gamma specification is a member, volatility is mechanically linked to expected output and is therefore also guaranteed to fall over time. For less natural specifications supportive of the first order approach such as those identified by LiCalzi and Spaeter (2003), all that can be said is that volatility will change over time but it is conceivable that the direction of the change itself will be changing (once) over time. For the "weighing of two distribution" specification as per Hart and Holmström (1987) the effect on volatility depends on the relative volatility of the two distributions in question, as well as their correlation. Only in the special cases where the production-function is of the Laplace-Normal type of Hemmer (2013) is the volatility guaranteed to be constant over time even as the expected income declines. None of this appears consistent with income smoothing behavior, however.

5 CONCLUSION

In this paper I explore the various ways the properties of earnings may be affected by agency problems in the early formulation of the multi-period model first proposed and analyzed by Lambert (1983 & 1984) . The enduring key insight in respect to earnings properties from his analysis is that second-best income (at least for the particular preference structure employed by Lambert (1984)) is "managed" in equilibrium in a way that results in smoother earnings, appropriately defined. I demonstrate that this result is false: under the assumptions of his model, explicit and implicit, there is no equilibrium relation between past income and future actions. Because the particular setting is actually a knife-edge case, it is clear that when such relations exist in this formulation, they are entirely due to wealth-effects in the agent's utility function. Such effects, however, can go either way: they just as plausibly lead to smoother as to less smooth income regardless of how one chooses to define "income smoothness."

I then proceed to use this benchmark case of no relation to identify other, generally ignored, implications for the time-series behavior of income predicted by this type of model. First, I show that when effort is not implicitly (and sub-optimally) required to be compensated in the period/state where it is exerted output-contingent variation in future effort is optimally used to incentivize current effort. This "intertemporal incentive effect," which is separate from the wealth effect does push the solution in the direction of making future effort inversely correlated with current output. The net of this and the wealth effect, however, is what determines the equilibrium relation which thus can go either way. Second, I show expected effort and, thus, income is going to be declining over time. This "horizon effect" is separate from the other two and is not output contingent. Since effort generally also impact the volatility of income, however, this effect is hard to reconcile with standard notions of smoothing behavior as well.

6 REFERENCES

References

[1] Hart, O. and B. Holmström, 1987, The Theory of Contracts. In *Advances in Economic Theory, Fifth World Congress*, edited by T. Bewley. Cambridge: Cambridge University Press.

[2] Hemmer, T., 2016, On the Non-Monotone Relation between Risk and "Pay-Performance-Sensitivity" in Optimal Incentive Contracts: Theory and Empirical Implications. *Working Paper.*

[3] Hemmer, T., 2016, Optimal Convex Dynamic Contracts in Standard Agencies. *Working Paper.*

[4] Holmström, B., 1979, Moral Hazard and Observability. *The Bell Journal of Economics* 10, 74-91.

[5] Jewitt, I., 1988, Justifying the First-Order Approach to Principal-Agent Problems. *Econometrica* 56, 1177-1190.

[6] Lambert, R., 1983, Long-Term Contracts and Moral Hazard. *The Bell Journal of Economics* 14, 441-152.

[7] Lambert, R., 1984, Income Smoothing as Rational Equilibrium Behavior. *The Accounting Review* 59, 604-618.

[8] LiCalzi, M. and S. Spaeter, 2003, Distributions for the first-order approach to principal-agent problems. *Economic Theory* 21, 167-173.