Communication with Endogenous Investor Attention*

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Abstract

We develop a theoretical model to analyze how investors' information acquisition and processing costs and mandatory disclosure affect the incidence and effectiveness of firms' communication efforts whose main objective is to help investors better undersand mandatory reports. We find that the quality of mandatory disclosure can either substitute or complement firms' incentives to engage in communication efforts. For activities where firm-specific factors matter more, more accurate mandatory disclosure can discourage useful communication efforts, resulting in lower equilibrium use of information by investors and lower investment efficiency. For activities where firm-specific factors matter less, more accurate mandatory disclosure can encourage more communication efforts, leading to more efficient use of such information. We discuss the empirical and policy implications of our analyses.

1 Introduction

The paper examines analytically how and when investors' information processing costs and property of mandatory disclosure can affect firms' choice of communication efforts, whose primary objective is purportedly to help investors better understand the information in mandatory disclosure and achieve disclosure effectiveness.[1] Such communication efforts encompass a variety of voluntary disclosure activities firms can engage to increase investors' ability to assess, assimilate, and absorb the information in mandatory reports, including, for example, efforts to improve the readability of mandatory reports, to release voluntary disclosure (such as earnings forecasts), to hold conference calls, and to participate and disseminate information on social media, etc..[2]

Two oft-cited reasons for such efforts are investors' information acquisition and processing costs and the increasing complexity in mandatory disclosure.[3] While intuitive, it is unclear how these factors can explain the variations in such efforts among firms across industries of various technological and in the same industry sophistication with similar investor clientiles and subject to similar disclosure regulations. In addition, empirical evidence suggests conflicting views about whether firms' communication efforts are motivated to reduce investors' information overload or to take advantage of it (e.g., Li (2008), Guay, et al. (2016), Blanksespoor (2016)).[4] In addition, debate exists both among academia and policy makers

[1]See, for example, a report by Ernst & Young, LLP (2014) "Disclosure Effectiveness: what companies can do now."

[2]Bagnoli and Watts (2007) refer to the output of such disclosure activities as supplementary disclosure, although they restrict the content of supplementary disclosure to be the precision of the mandatory reports.

[3]Many critics, including a former SEC commissioner, Mr. Paredes, suggest that mandatory disclosure has become too complex and call for reduced disclosure complexity to minimize information overload on investors (Paredes (2003), KPMG (2011)). In response, the SEC had launched the Disclosure effectiveness initiatives in 2013, and issued several proposals aimed at "eliminating redundant, overlapping, outdated and superseded requirements". See https://www.sec.gov/spotlight/disclosure-effectiveness.shtml, and https://www.sec.gov/news/pressrelease/2016-141.html.

[4]A commonly used proxy for disclosure complexity is the readability index developed in linguistics (Li (2008)). Guay, et al. (2016) attribute the observed complexity to burdensome mandatory disclosure rules and accounting standards. They interpret the positive association between readability and firms' tendency

on what causes the observed disclosure complexity, whether it is a result of managerial obfuscation, or of overly burdensome disclosure regulations, or simply reflecting the complexity of the underlying business activities (e.g., Bushee, et al. (2016), Dryer, et al. (2016), Gerding (2016)). These seemingly conflicting views have made it difficult for standard setters to decide how to balance the need to provide more accurate information about the complex business transaction (with disclosure complexity as a possible by-product) with the concern for causing information overload on investors.

This paper contributes to these debates by examining the equilibrium supply of firms' communication efforts in a setting where investors are rational but have information processing costs. In doing so, we hope to shed light on several related issues. Specifically, what are the conditions that result in endogenous supply of communication efforts, by some firms but not all, when all investors have information processing costs? How effective are the communication efforts (i.e., do they actually achieve disclosure effectiveness)? How do such efforts vary by the nature of the underlying economic activities (i.e., do firms have more incentive to explain transactions regarding available-for-sale securities or regarding R&D and M&A activities)? And relatedly, how does the quality of mandatory reporting about these activities affect firms' incentives to engage in communication efforts (i.e., does more accurate, but presumably more complex, mandatory reports on M&A activities increase or decrease firms' efforts to explain these transactions)?

To address these questions, we consider a setting where the manager of a firm privately observes the quality of an economic activity (or equivalently an investment project) available to the firm's owner/investor (perhaps because the manager spent personal effort to identify and initiate the project in the first place).[5] A high (low) quality project has higher (lower)

to issue earnings forecast as evidence that well-intentioned managers increase communication efforts to help investors overcome information overload caused by regulatory disclosure complexity. Li (2008) finds a positive association between readability and firms' future performance, suggesting that the observed disclosure complexity can be the result of self-interested managers taking advantage of investors' information processing costs to obfuscate information. Blanksespoor (2016) finds that firms voluntarily increase quantitative footnote disclosure after the adoption of XBRL, suggesting that firms are reluctant to provide supplementary disclosure when investors' processing costs are high.

[5]Throughout the paper we use economic activity, transaction, and investment project interchangebly

chances of generating large payoffs and as a result requires large (small) investment by the investor. We assume the manager always prefers large investment, whereas the investor prefers to invest the correct scale according to the payoff of the project, based on the information presented to and understood by her. The types of economic activity we have in mind can vary widely, ranging from product development activities (such as R&D), capital investment activities (such as M&A transactions), or liquidity management activities (e.g., buying and selling financial securities).[6] We distinguish different categories of activities by how important firm-specific factors are in determining the quality of the activity. Firm-specific factors are likely to matter more for the quality of R&D activities (e.g., the ability to develop successful smartphones clearly differs widely between Apple and Microsoft), and arguably less in liquidity management activities such as transacting AHS securities.

The investor observes an accounting signal about the true payoff the project prepared from the firm's financial reporting system, for example, the balance sheet valuation for AFS securities, or the amount of R&D expenditure. The reporting system is set up according to the prevailing regulations and standards (e.g., GAAP), which determine how informativeness the signal is about the project's true payoff. However, the manager can, if they choose to, tamper the system to lower its informativeness before the signal is realized but after the manager observes the quality of his project. The manager can do so by abusing the discretion permitted in accounting standards or taking real actions (i.e., accrual or real management).

To help the investor better utilize the mandatory signal, the manager can engage in costly communication efforts (as discussed in the first paragraph) to improve the clarity of the firm's disclosure. Clarity is achieved when the disclosure has the ability to help the investor discern whether the manager has tampered/lowered the informativeness of the mandatory reporting system, or in language commonly used in the literature, to help the investor detect earnings management.[7] In our model, more communication efforts only stochastically help achieve

unless there is a chance of confusion.

[6]In the latter case, the investor's investment decision can be interpreted as the decision to continue as opposed to liquidate the firm's operations.

[7]In a completely different setting, Chen, et al. (2016) define clarity as the ability of a firm's disclosure to help users understand the precision of the reported signal. Thus, their notion of clarity is related to

clarity. An analogy to justify this assumption is what every teacher has experienced: he can spend as much time preparing a lecture to make it as clear as possible (in his own view), there is always a non-zero probability that some students find the lecture confusing (i.e., lack clarity). We focus on the cases where the manager with low quality project manipulates the reporting system and examine each manager's incentive to engage in efforts to improve clarity.[8]

We assume whether the firm's disclosure has clarity (e.g., whether the MD&A is readable or whether the manager is able to answer questions about the reasons for an M&A transaction during a conference call) is observable to the investor, who then decides whether to exert effort or pay attention to understand the disclosures (e.g., to actually read the MD&A or analyze the manager's response in the conference call). Without attention, the investor will never be able to detect earnings management. However, attention is costly to the investor as it consumes both time and mental energy.

Both the clarity of the firm's disclosure and the investor's attention are necessary conditions for effective communication (or equivalently disclosure effectiveness), defined as the realized outcome that the investor is indeed able to detect earnings management. This assumption builds on the insight from Dewatripont and Tirole (2005) that there is a moral hazard in team problem in achieving successful communication.[9] In addition, whether the disclosure effectiveness is achieved also depends on random factors outside both parties' control such as the fit between the manager's communication style and the investor's style or

the supplementary disclosure in Bagnoli and Watts (2007). In Bagnoli and Watts (2007), the content of the supplementary disclosure is the precision of the signal; therefore clarity is achieved when the manager chooses to issue supplementary disclosure. In all these studies, a common theme is that the manager can take costly actions to help investors better utilize the mandatory report.

[8] As will be clear later, the cases where high-quality manager manipulates the reporting system in equilibrium are theoretically possible but realistically uninteresting.

[9] Our model differs from theirs in several key aspects. In their model, communication efforts are simultaneously chosen by the sender and the receiver, there is no signal about the chosen effort and the sender does not have private information about his type. In our model, the communication efforts are sequential; the receiver observes noisy signals about the senders' communication effort; and senders have private information about their types.

skill in absorbing information). An intuitive example for this assumption is learning: while both a clearly presented and prepared lecture and the student's attention to the lecture are necessary for successful learning, the student's prior exposure and aptitude for learning certain subject matters is not always known for sure from the ex ante perspective.

We examine the equilibrium supply of communication and attention efforts by the manager and the investor, which collectively determine the investor's investment efficiency. We identify three types of equilibria, (approximately) ranked by the amount of effective communication in equilibrium: the strong communication equilibrium, the semi-strong communication equilibrium, and the weak communication equilibrium. In the strong communication equilibrium, only the high-quality manager engages in communication efforts. The investor always pays attention upon observing disclosure clarity, and will invest even when she fails to detect earnings management. In this equilibrium, the mere observation that a disclosure has clarity conveys information. In the semi-strong communication equilibrium, the high-quality manager always engages in communication efforts, and the low-quality manager and the investor randomize in their communication and attention efforts, respectively. Disclosure clarity still carries information content: the investor would invest upon observing disclosure clarity even when she fails to detect earnings management. The equilibrium amount of effective communication, however, is lower than in the strong communication equilibrium, because the investor doesn't always pay attention. In the weak-communciation equilibrium, no manager engages in communication efforts; and the investor either always pays attention and never does, depending on parameter values. Effective communication may still take place (because disclosure can obtain clarity by chance even without any communication efforts by the manager).

Clearly, the investment efficiency is the highest in the strong communication equilibrium because more information is successfully communicated to and utilized by the investor. However, a key result from our analysis is that for activities more dependent on firm-specific factors, the strong communication equilibrium exists only when the quality of mandatory reporting for such activities is relatively low. This implies a substitutive relationship between mandatory reporting and voluntary disclosure for this type of activities: when mandatory quality improves, firms reduce their voluntary communication efforts. Conversely, for more generic activities (whose payoffs do not depend much on firm-specific factors), more accurate mandatory reports is more likely to achieve the strong communication equilibrium. This implies a complementary relation between mandatory disclosure quality and voluntary communication efforts.

Our analyses produce both positive and normative implications. On the positive side, our results can help reconcile the different views on the effects of investors' information processing costs on firms' incentive to engage in communication efforts, and on the source of the disclosure complexity as measured by empirical researchers. To the extent that high quality standards necessarily entail more detailed and thus seemingly more complex disclosure, our analyses suggest that these views do not have to be mutually exclusive. We show that for certain activities, high quality standards do incrase managers' communication efforts (such as more earnings forecasts), consistent with Guay et al. (2016). On the other hand, our results also indicate that it is possible that high quality standards may crowd out managers' communication efforts, especially for activities that depend more on firm-specific factors.

Our analysis offers a potential explanation for cross-sectional variations in firms' communication efforts, as well as an alternative explanation for the findings in Guay et al. (2016). Guay et al. (2016) motivate their hypotheses based on the optimal-contracting perspective, which views the supply of public disclosure as driven by investors' demand for better monitoring. Guay et al. (2016) support this perspective by documenting a stronger relation between disclosure complexity and earnings forecasts for firms with more outside monitors. To the extent that investors who monitor are likely large institutional investors with relatively lower processing costs, our model also predicts a similar positive association without assuming optimal contracting or investor monitoring. Our explanation and the optimal contracting perspective are not mutually exclusive, although they do have (perhaps subtle) different implications for standard setting. The optimal contracting perspective does not predict how the quality of standards (which may be empirically difficult to distinguish from regulatory disclosure complexity) should differ by the nature of the underlying economic activities, where our explanation does.

Specifically, our analyses suggest that the optimal mandatory disclosure quality should be lower for activities more dependent on firm-specific factors such as R&D or M&A activities; this would provide more incentive for firms to engage in communication efforts to help investors understand the quality of these activities. Our analysis shows that when investors have no information processing costs, detailed mandatory disclosure about these activities would improve investment efficiency; however in the presence of information processing costs, too accurate and detailed mandatory disclosure can suppress managers' communication efforts, which would reduce the equilibrium investment efficiency as investors do not fully utilize the information in mandatory reports. Similarly, our model suggests that it's best to offer accurate disclosure about activities that are more generic in nature, for example, activities related to financial assets (for non-financial firms).

Our paper belongs and contributes to two broad literatures. The first is the literature aimed at understanding how investors' information processing costs affect stock market efficiency. The idea investors' information acquiring and processing costs can affect the informational efficiency of stock prices is well known in both theoretical literature (e.g., Grossman and Stiglize (1980) and empirical literature.[10] We contribute to this research field by formalizing the idea that information processing costs can also indirectly affect market efficiency, by affecting the supply of credible voluntary information by firms is relatively new and has not received much theoretical attention.[11] Similar idea is also discussed in the theoretical

[10] Earlier studies focus on investors' cost in acquiring information in exchange economies where information in price does not affect the payoff of the underlying securities (e.g., Verrecchia (1982), Diamond (1985)). Dow, Goldstein and Gumbel (2016) examine the role of information acquisition costs in a setting where information in stock prices can in turn affect firms' production (i.e., feedback effect). The rational inattention literature explicitly recognizes that investors have limited resources (time and energy) in processing information and explore its implication in financial markets (e.g., Sims (2006), Van Nieuwerburgh and Veldkemp (2009), Yang (2016)). A growing empirical literature in finance and accounting also document increasing evidence supporting the idea that investors' information processing costs can directly affect price efficiency by affecting the speed and extent to which existing public information is impounded in price. See Cohen and Frazzini (2008), DellaVigna and Pollet (2009), Hirshleifer, Lim and Teoh (2009), and Cohen and Lou (2012), among others.

[11] Empirical support for this idea can be found in recent empirical studies in accounting (e.g., Li (2008), Blanksespoor (2016), Guay et al. (2016), Bushee, et al. (2016)). However, as discussed earlier, consensus is

analyses of Hirshleifer and Teoh (2003). Our paper differs in that the investor in our model are rational, Bayesian, and does not suffer exogenously given psychological bias as modeled in Hirshleifer and Teoh (2003).

Our study also belongs to the literature exploring how managers' incentives affect the supply and credibility of firms' voluntary disclosure,[12] specifically to those focusing on the interaction between mandatory disclosure and voluntary disclosure (e.g., Dye (1990), Einhorn (2005)). It differs from the most prior literature in two ways: first is we explicitly consider the impact of investors' information process costs. Second is we focus on the type of disclosure whose objective is to help investor understand the hard information from financial reports. In this sense, our paper contributes to a burgeoning literature that examines situations where managers can take actions to help investors better utilize existing information (Bagnoli and Watts (2007), Bertomeu and Marinovic (2014) and Chen, et al. (2016)).[13]

The rest of the paper proceeds as follows. In section 2, we set up the model. Section 3 presents two benchmark cases against which our later results will be compared. We solve the

lacking among empiricists on whether information processing costs increase or decrease managers' incentives to voluntarily disclose more information.

[12] The literature on how managers' incentives affect the supply and credibility of voluntary disclosure is large, starting with the classic works by Grossman (1981) for hard, verifiable information (without credibility issue), introduced to the accounting literature by Verrecchia (1982), and Dye (1985); and Crawford and Sobel (1982) for soft, unverifiable information, applied to accounting research by Gigler (1994), Newman and Sansing (1994), Stocken (2000), and Bertemeou and Marinovic (2014), among others. Other strands of literature include costly state verification models (Townsend (1979)) and contracting models (Dye (1985), Gigler and Hemmer (1998)).

[13] Our paper is perhaps closest to Bagnoli and Watts (2007) who also examine the manager's incentive to engage in supplemental disclosure. Our model differs from theirs both in the setup and in many conclusions. In their model, investors have no information processing constraint and passively use the information to price the firm in secondary market. In our model, investors endogeneously choose whether to pay attention and use the information to make investment decisions. Bagnoli and Watts (2007) find that managers always engage in supplementary disclosure upon favorable mandatory reports, whereas we show this is the case only when parameters are such that strong-communication equilibrium exists. Further, perhaps most importantly, our framework can be used to perform welfare evaluations in terms of investment efficiency and therefore draw direct implications for security regulators and standard setters.

model and generate three types of equilibria in section 4. Section 5 discusses the implications of the paper and shows that our findings are robust to alternative assumptions of the model. We conclude in section 6.

2 Model Setup

2.1 Quality of firms' economic activities

Consider a representative firm operated by a manager who seeks investment from a single representative investor. The firm has only one project. We represent the final payoff/fundamental of the firm's project with a random variable θ with distribution function of $F(\theta)$. θ is not realized until the end of the game so no one (neither the manager nor the investor) observes the actual θ. It is common knowledge that $F(\theta)$ is a binary distribution function over a support $\{0, X > 0\}$, with $\Pr(\theta = X) = 1 - \Pr(\theta = 0) \in [0, 1]$.

To allow heterogeneity among firms, we assume the distribution function $F(\theta)$ differs across firms, and can be either $H(\theta)$ or $L(\theta)$ with $H(\theta)$ first-order stochastically dominating $L(\theta)$, i.e., $\Pr(\theta = X|H) - \Pr(\theta = X|L) = \alpha \geq 0$. Since θ determines the payoff of the project, the stochastic ordering of the distribution functions for θ allows us to interpret $F(\cdot)$ as representing the quality of the project, or equivalently, the quality of the manager's activities. Firms with $H(\theta)$ $(L(\theta))$ have high (low) quality project, because their projects are more (less) likely to result in larger payoffs.

We assume the manager privately observes whether $F(\cdot)$ is $H(\cdot)$ or $L(\cdot)$ because he executed the activities to identify and initiate the project in the first place. As such, we use H or L to denote the manager's type, and often say for instance, $F = H$, meaning that the manager is of type H. Since there is no conflict assumed between the firm and the manager, we use "firm" and "manager" interchangeably throughout the paper. The investor does not observe the firm's type, and has the prior that $\Pr(F = H) = 1 - \Pr(F = L) = \lambda \in (0, 1)$.

For notational ease and without loss of generality, we further normalize $\Pr(\theta = X|L) = 0$ and $\Pr(\theta = X|H) = \alpha > 0$. Thus, a higher α means that projects in H-type firms are more likely to achieve larger payoffs in the future (i.e., growth potential). More generally, α measures the importance of firm-specific characteristics in determining the quality of different underlying economic activities in the firm. This interpretation is useful in applying our setting to address reporting issues regarding different types of economic activities and transactions such as, for example, the reporting of activities related to product development, or activities related to buying and selling financial securities. Each type of activity constitutes one of the many projects in a firm's value-creation process. The quality of activities characterized by a large α ($\alpha \to 1$) depends more on firm-specific factors, whereas firm-specific factors play a smaller role in affecting the payoff structure of small α activities. For example, the ability of the management team at Trader Joe's and Whole Foods probably matters quite a bit in successfully implementing the strategy of providing affordable organic groceries. However, such ability arguably matters less in affecting the returns on the short-term securities they hold. Under this interpretation, our model can be applied to evaluating specific standards, e.g., SFAS 2 for R&D activities (a type of product development activities), or SFAS 115 for available-for-sale securities.

Lastly, it is worth noting that in our model managers know more about the likelihood of success of their own activities than outside investors, but they do not know for sure the final payoffs of their activities. Take for example the competing versions of self-driving cars currently being developed. It is likely that insiders know more (than outside investors) about their chances of success. It is unlikely that they know for sure the size of their products' market share in the future. The belief that their odds of success are better is soft information and difficult to be credibly communicated to outsiders. One way to do so is to engage in costly effort (voluntary communication activities) to help investors better analyze and understand the audited reports from the financial accounting system. However, as we will show, the firm's incentive to engage in such communication effort, and its effectiveness, depends critically on the quality of the financial accounting system.

2.2 Financial reporting and communication

We next describe the firm's reporting system and the communication game between the manager and the investor. Complying with regulations, the firm has installed a financial reporting system at the beginning of its operations. The system records and collects evidence from the firm's activities during the first stage (the reporting period). It then applies the recognition and measurement rules as prescribed by the Generally Accepted Accounting Principles (GAAP) and the prevailing regulations to report relevant evidence as a mandatory accounting signal $s \in \{s_l, s_h\}$ at the end of the first stage. When the firm applies the GAAP appropriately, the accounting signal can be informative of the final payoff θ with the following common knowledge statistical property:

$$\Pr(s = s_h \mid \theta = X) = \Pr(s = s_l \mid \theta = 0) = q \in \left(\frac{1}{2}, 1\right). \tag{1}$$

In other words, the reporting rules in GAAP produce a stochastic mapping from the fundamental θ to the signal s with a publicly known precision of q.

To reflect the notion that the manager may not always apply GAAP appropriately, we assume that after privately observing his type but before the accounting system produces the signal s, the manager has the opportunity to alter the mapping from θ to s.[14] To sharpen our analysis and highlight the main tension, we assume such an opportunity can be exploited costlessly for the L-type manager, whereas the cost is too high for the H-type manager to attempt such manipulation.[15] This assumption can be motivated by the idea that firms have different abilities in aligning their managers' incentives and such differences are difficult for outside investors to discern. It is also consistent with the conventional wisdom and a large body of empirical associations suggesting that low quality managers are more likely to exploit reporting opportunities to obfuscate information (e.g., Li (2008)).

To distinguish between the investor's knowledge about whether the mandatory accounting signal is manipulated, we use $\widehat{s} \in \{\widehat{s}_l, \widehat{s}_h\}$ to denote the manager's report (i.e., the signal observed by the investor), and s to denote the underlying signal generated from the unmanipulated mapping. Further, to simplify the exposition, we assume that when the L-type firm manipulates, he always chooses the mapping such that $\Pr(\hat{s} = \hat{s}_h | \theta = 0) = 1$.[16] This

[14] This type of discretion is also modeled in Chen, Hemmer and Zhang (2007).

[15] This assumption is for tractability. Secton 5.2 shows that our main results are not qualitatively affected if we assume that the cost for altering the reporting system is also low or zero for the H-type firm.

[16] This is without loss of generality, as we will show later that it is always optimal for the L-type firm to choose $\Pr(s_h|\theta = 0) = 1$ when he is allowed to choose any $\Pr(s_h|\theta = 0) = \widehat{q}$ to begin with.

implies that his disclosed mandatory signal will always be $\widehat{s}_h$, whereas the disclosed signal can be either $\widehat{s}_h$ or $\widehat{s}_l$ from a H-type manager. Furthermore, in equilibrium, the investor will rationally understand that the manager type is H conditional on observing $\widehat{s}_l$; without additional information, she will not be able to distinguish whether a $\widehat{s}_h$ is from a L-type firm or a H-type firm. Formally, for the H-type firm it is always the case that $\widehat{s} = s$. For the L-type firm, we denote its reporting strategy by $\Gamma \in \{0, 1\}$, where $\Gamma = 0$ if the manager manipulates the reporting mapping (i.e., $\Pr(\hat{s} = \hat{s}_h | \theta = X) = \Pr(\hat{s} = \hat{s}_h | \theta = 0) = 1$) and $\Gamma = 1$ if he does not manipulate the reporting mapping (i.e., $\Pr(\hat{s} = \hat{s}_h | \theta = X) = q$ and $\Pr(\hat{s} = \hat{s}_h | \theta = 0) = 1 - q$).

In addition to the mandatory financial reporting system, the firm can also engage in a variety of communication efforts to help the investor to assess, assimilate, and absorb the information in the mandatory report, including, for example, efforts to improve the readability of the report, to release voluntary disclosure (such as earnings forecasts), to hold conference calls, and to participate and disseminate information on social media. While we assume it is costly for the firm to engage in these communication efforts, they can *potentially* help the investor discern whether the firm has tampered with the mandatory reporting system, that is, the investor will be able to tell whether an observed $\widehat{s}_h$ is from an unmanipulated mapping of $\Pr(\hat{s} = \hat{s}_h | \theta = X) = \Pr(\hat{s} = \hat{s}_l | \theta = 0) = q$ or from a tampered mapping of $\Pr(\hat{s} = \hat{s}_h | \theta = X) = \Pr(\hat{s} = \hat{s}_h | \theta = 0) = 1$. We use the term *effective communication* to describe the event that the investor is able to discern which distribution generates the firm's disclosed signal $\hat{s}$. Note that effective communication does not completely remove the investor's uncertainty about the firm's final payoff θ. This is because unmanipulated signals are at best noisy indicators of the true payoff (i.e., $q < 1$).

To capture the gist in the age-old wisdom that it takes two to tango, we posit that the occurrence of effective communication depends on three factors. First, the firm can choose to engage in a costly communication effort, $e_M \in \{0, 1\}$, which in turn stochastically determines the clarity of the mandatory disclosure signal $\hat{s}$. We use $\mu \in \{0, 1\}$ to denote clarity. The disclosure is of high clarity (i.e., $\mu = 1$) if it enables the investor to see through the manager's manipulation with a strictly positive chance. Specifically, we assume that when the manager exerts communication effort ($e_M = 1$) the probability of a high clarity disclosure ($\mu = 1$) is bigger:

$$\Pr(\mu = 1 \mid e_M = 1) = \Pr(\mu = 0 \mid e_M = 0) = t \in \left(\frac{1}{2}, 1\right).$$

Note that exerting effort doesn't always lead to high clarity. Such uncertainty reflects the fact that the manager does not always have complete information about the preferences and sophistication level of the investor. Teachers are perhaps most familiar with this situation: the same lecture may work for some audience but not always for others. We assume the incremental cost for the manager to exert the communication effort is C_M. The investor does not observe the manager's communication effort choice e_M but instead observes clarity μ (i.e., μ is public information).

Second, after observing the realized disclosure clarity μ, the investor decides whether to exert an attention effort (i.e., spend time analyzing and understanding the manager's disclosure). We use $e_I \in \{0, 1\}$ to denote the investor's attention effort choice. Effective communication can be achieved only when the investor spends time and chooses $e_I = 1$, which entails an incremental private cost of C_I.

Finally, like the manager, the investor's attention effort can only result in effective communication (i.e., being able to discern if the mandatory disclosure mapping is tampered with) with probability $p \in (0, 1)$ which reflects either the level of the investor's sophistication (she may not know the difference between earnings and cash flows) or the level of complexity of the subject matter (for example, the subject matter is about highly technical derivative transactions). To put this assumption in a more intuitive setup, imagine that the attention effort is the post-lecture study a student is expected to perform. The student would not understand the lecture unless she studies afterwards, but with how much depth she can understand the material depends on her ability relative to the subject matter.

In summary, let $\eta \in \{0, 1\}$ denote whether effective communication is achieved: $\eta = 1$ if the communication is effective and $\eta = 0$ if it is not. Then, ex ante, the probability of effective communication conditional on the manager's communication effort choice e_M is given by:

$$\Pr(\eta = 1|e_M) = pe_I \Pr(\mu = 1|e_M).$$

Notice that, conditional on the realization of the disclosure clarity μ, the probability of effective communication is simply $\Pr(\eta = 1|\mu) = pe_I\mu$. That is, effective communication can only take place with positive probability if the disclosure is of high clarity, $\mu = 1$, and the investor pays attention, $e_I = 1$.

2.3 Timeline, players' objectives, and equilibrium

The timeline of the model is as follows:

$t = 0$: the manager privately observes his type. The L-type decides whether to manipulate the mandatory disclosure mapping, $\Gamma \in \{0, 1\}$;

$t = 1$: accounting signal $\{\widehat{s}_h, \widehat{s}_l\}$ are realized and disclosed. Both types choose communication effort $e_M \in \{0, 1\}$;

$t = 2$: disclosure clarity $\mu \in \{0, 1\}$ is realized and publicly observed;

$t = 3$: the investor chooses attention effort $e_I \in \{0, 1\}$;

$t = 4$: whether communication is effective (jointly determined by μ and e_I) is realized;

$t = 5$: the investor makes investment decisions $K \in \{0, X\}$ based on her information set $\Omega_I = \{\widehat{s}, \mu, \eta\}$;

$t = 6$: θ is realized and the game ends.

Let U_M denote the payoff for the manager/firm:

$$U_M \equiv \psi I(K = X) - e_M C_M.$$

$\psi > 0$ is the incremental private benefit the manager extracts when the investor invests (i.e., $K = X$) relative to the case when she doesn't invest (i.e., $K = 0$). U_M implies that, keeping everything else equal, the manager would like to increase the probability of the obtaining investment from the investor. Let Π denote the investor's payoff:

$$\Pi = \theta - (K - \theta)^2 - e_I C_I. \tag{2}$$

(2) indicates that the investor would like to invest the appropriate amount corresponding to the project's fundamental, i.e., more investment in projects with truly good fundamentals

($K = X$ for projects with $\theta = X$) and less in mediocre projects ($K = 0$ for projects with $\theta = 0$). Failing to match investment K with the fundamental θ results in a loss of X^2. For example, one may interpret θ as the uncertain demand for the firm's products and K as capacity decisions. While the investor benefits from a higher demand in general, profit is maximized only when the capacity decisions dovetail the realized demand.

The following defines the equilibrium concept in our model. Note that we use e_H (e_L) to denote the communication effort choice made by the H-type (L-type) firm.

Definition An equilibrium in our setting is characterized by a 5-tuple $\{\Gamma, e_L, e_H, e_I, K\}$, where $\Gamma \in \{0, 1\}$ is the L-type firm's manipulation strategy; $e_L : \{\hat{s}_l, \hat{s}_h\} \to \{0, 1\}$ is the L-type firm's communication effort strategy; $e_H : \{\hat{s}_l, \hat{s}_h\} \to \{0, 1\}$ is the H-type firm's communication effort strategy; $e_I : \{(\hat{s}, \mu)\} \to \{0, 1\}$ is the investor's attention effort strategy; $K : \{(\hat{s}, \mu, \eta)\} \to \{0, X\}$ is the investor's investment strategy; and they satisfy:

1. Conjecturing e_H, e_I and K, the L-type firm chooses Γ and e_L to maximize its expected payoff;
2. Conjecturing e_L, Γ, e_I and K, the H-type firm chooses e_H to maximize its expected payoff;
3. Conjecturing e_H, e_L and Γ, the investor chooses e_I and K to maximize her expected payoff.
4. All conjectures become true.

Furthermore, in order to avoid trivial equilibria, we impose two tie breaker conditions: First, if a L-type firm is indifferent between manipulating and not manipulating the mandatory reporting system, it manipulates. Second, if the investor is indifferent between investing ($K = X$) or not investing ($K = 0$), she invests.

3 Benchmark cases

To provide intuition and facilitate the reader' understanding of our main results, we start our analysis with two benchmark cases.

3.1 First best case

In the first best situation, the firm's type is publicly known and no mandatory disclosure manipulation is allowed (e.g., there exists a perfect audit). In this case, it is easy to show that the optimal investment decision is to not invest regardless of the accounting signal when it's from a L-type, due to our assumption that $\Pr(\theta = 0|L) = 1$, which makes the knowledge of the L-type a sufficient statistics for the mandatory accounting signal. For the H-type, the accounting signal and the prior about $\Pr(\theta|H)$ are both informative about θ. To ensure that the accounting signal is at least incrementally informative, we assume,

$$A1\text{:}\ \alpha \in (1-q, q) \, .$$

This assumption ensures that it is optimal for the investor to invest if and only if $s = s_h$. Otherwise, the investor's prior is so strong (either $\alpha \to 0$ or $\alpha \to 1$) that the accounting signal is not of any additional use.[17]

3.2 Mandatory disclosure manipulation without communication effort

Next we consider the case in which firms privately know their types and the L-type firm manipulates the mandatory disclosure mapping, but the firm doesn't exert the communication effort $e_M = 0$ and thus disclosure clarity is always low $\mu = 0$. The following lemma then shows the investor's optimal investment decision in this hypothetical scenario.

[17] This can be easily proved by noticing that $E[\Pi(K=1)|s_h, H] > E[\Pi(K=0)|s_h, H]$ iff $\Pr(X|s_h, H) > \Pr(0|s_h, H)$ which is guaranteed by $\alpha > 1-q$. Similarly, $E[\Pi(K=1)|s_l, H] < E[\Pi(K=0)|s_l, H]$ iff $\Pr(X|s_l, H) < \Pr(0|s_l, H)$ which is ensured by $\alpha < q$.

Lemma 0 (i) When $\alpha \in \left(\frac{1}{\lambda} - q, q\right)$, the investor optimally invests if and only if $\widehat{s} = \widehat{s}_h$; and (ii) when $\alpha \in \left(1 - q, \min\{\frac{1}{\lambda} - q, q\}\right)$, the investor does not invest regardless of $\widehat{s}$.

To prove Lemma 0, note that upon observing $\widehat{s} = \widehat{s}_l$, the investor for sure knows she is dealing with a H-type firm. Thus, in this case, the fact that $\alpha < q$ (which is part of Assumption A1) induces the investor not to invest. In contrast, upon observing $\widehat{s} = \widehat{s}_h$, the investor's posterior belief that the firm is of a H-type with $\theta = X$ is,

$$\Pr(\theta = X|\widehat{s}_h) = \frac{\Pr(\widehat{s}_h|X)\Pr(X)}{\Pr(\widehat{s}_h|H)\Pr(H) + \Pr(\widehat{s}_h|L)\Pr(L)} = \frac{\lambda\alpha q}{\lambda[(1-\alpha)(1-q)+\alpha q] + (1-\lambda)}$$

The denominator of the expression above utilizes the assumption that the L-type always reports $\widehat{s}_h$, $\Pr(\widehat{s}_h|L) = 1$. Under this posterior, one can show that, investing yields a higher expected payoff than not investing, $E[\Pi(K = X)|\widehat{s}_h] > E[\Pi(K = 0)|\widehat{s}_h]$, if and only if,

$$\Pr(\theta = X|\hat{s}_h) \geq \Pr(\theta = 0|\hat{s}_h).$$

It is straight-forward to show that $\Pr(\theta = X|\hat{s}_h) \geq \Pr(\theta = 0|\hat{s}_h)$ is satisfied if and only if $\alpha \geq \frac{1}{\lambda} - q$, with the equality holding if $\alpha = \frac{1}{\lambda} - q$ (implying that the investor is indifferent between investing and not investing). If α is larger than $\frac{1}{\lambda} - q$, the investor becomes more optimistic about the firm's type being H and hence strictly prefers to invest even though she is aware that with some probability the report $\widehat{s}_h$ results from a manipulated mapping.[18]

4 Informational role of disclosure clarity with mandatory reporting manipulation

In this section, we assume that firms privately know their types, and allow both mandatory disclosure manipulation by the L-type firm (and assume the tie-breaker that when indifferent, the L-type always manipulates) and communication efforts for both types. The communication stage adds an additional signal μ (i.e., whether the firm has produced high clarity disclosure) to the game. It also requires us to determine the firms' equilibrium communication effort choices and the investor's equilibrium attention effort choices. In this section,

[18] When $q < \frac{1}{2\lambda}$, or equivalently, $q < \frac{1}{\lambda} - q$, then the investor always doesn't invest.

we examine three types of equilibria: a strong communication equilibrium in which only the H-type firm engages in communication effort (i.e., $e_H = 1$) and in equilibrium it is possible that the investor reacts differently to the same reported $\hat{s}_h$ depending on the observed clarity μ; a weak communication equilibrium in which no manager engages in the communication effort; and a semi-strong communication equilibrium in which the H-type always exerts the communication effort but the L-type randomizes between effort and no effort.

4.1 Existence of a strong communication equilibrium

In this section, we construct an equilibrium where a H-type firm selects $e_H = 1$ and a L-type firm selects $e_L = 0$, such that the realized disclosure clarity μ could serve as a noisy signal to the investor. To the extent that a firm's communication effort choice depends on its type, henceforth, we refer to this equilibrium as a strong communication equilibrium. We note that the off-equilibrium belief issues do not arise in our setting as opposed to a pure sigaling game because both realizations of clarity $\mu \in \{0, 1\}$ lie on the equilibrium path. Lemma 1 confirms that the L-type firm always manipulates the mandatory disclosure mapping in equilibrium, and characterizes the investor's optimal investment strategy when she believes that the firm plays an effort choice strategy according to the strong communication equilibrium.

Lemma 1

Define $\Omega_I \equiv (\widehat{s}, \mu, \eta)$ as the investor's information set when she makes her investment decision. Assume that (3) and (4) below are satisfied,

$$\begin{aligned} \alpha &\in [\alpha_1, \min(\alpha_2, q)], \text{ where} \\ \alpha_1 &\equiv 1 - q + \frac{(1-\lambda)(1-t)}{\lambda t} \text{ and} \\ \alpha_2 &\equiv 1 - q + \frac{(1-\lambda)t}{\lambda(1-t)}, \end{aligned} \tag{3}$$

and

$$\lambda \geq \frac{1-t}{1+2qt-2t}. \tag{4}$$

Then (1) the L-type firm manipulates the mandatory disclosure mapping and always reports $\hat{s} = \widehat{s}_h$; and (2) the investor's optimal investment decision K^* is given below:

	$\Omega_I = (\widehat{s}, \mu, \eta)$	K^*	$Condition$
(a)	$(\widehat{s}_l, \cdot, \cdot)$	0	$if\ \alpha \in (1-q, q)$
(b)	$(\widehat{s}_h, \mu = 1, \eta = 1)$	X for H type only	$if\ \alpha \in (1-q, q)$
(c)	$(\widehat{s}_h, \mu = 1, \eta = 0)$	X	$if\ \alpha \geq \alpha_1 \equiv 1 - q + \frac{(1-\lambda)(1-t)}{\lambda t}$
(d)	$(\widehat{s}_h, \mu = 0, \eta = 0)$	0	$if\ \alpha \leq \alpha_2 \equiv 1 - q + \frac{(1-\lambda)t}{\lambda(1-t)}$

To understand the conditions in Lemma 1, first note that $t > 1/2$ implies $\alpha_2 > \alpha_1$. (4) is a necessary and sufficient condition for $q \geq \alpha_1$ which guarantees that (3) is not empty. Also notice that (3) implies $\alpha \in (1-q, q)$ for sure.

To start with, given our tie breaker, it is straight-forward to show that the L-type firm always manipulates. To see this, note that (3) implies that the investor withholds investment when observing $\hat{s} = s_l$ even if she believes the disclosure is made by a H-type firm. As such, a L-type firm is at least indifferent between reporting $\hat{s}_h$ and reporting $\hat{s}_l$ and may be strictly better off with reporting $\hat{s}_h$ if his manipulation is not revealed to the investor.

The investment rules for scenario (a) and (b) are easier to identify because the investor is dealing with a situation similar to that in the first best case where she knows the firm's type. Specifically, she knows the type is H in scenario (a) by understanding that the L-type firm always manipulates so only the H-type will ever disclose $\hat{s}_l$. She knows the type is H in scenario (d) because of the effective communication. Assumption (A1) earlier shows that her investment rule is to invest upon observing a $\hat{s}_h$ and not invest upon observing $\hat{s}_l$.

In scenario (c), investing yields a higher expected payoff than not investing if and only if,

$$\underbrace{\Pr\left(0|\hat{s} = \hat{s}_h, \mu = 1, \eta = 0\right)\left(X - 0\right)^2}_{\text{expected loss from overinvestment}} \leq \underbrace{\Pr\left(X|\hat{s} = \hat{s}_h, \mu = 1, \eta = 0\right)\left(0 - X\right)^2}_{\text{expected loss from underinvestment}}, \tag{5}$$

where,

$$\begin{aligned} \Pr\left(0|\hat{s} = \hat{s}_h, \mu = 1, \eta = 0\right) &= \frac{\lambda\left(1-\alpha\right)\left(1-q\right)\left(1-t\right) + \left(1-\lambda\right)t}{\lambda\alpha q\left(1-t\right) + \lambda\left(1-\alpha\right)\left(1-q\right)\left(1-t\right) + \left(1-\lambda\right)t} \\ \Pr\left(X|\hat{s} = \hat{s}_h, \mu = 1, \eta = 0\right) &= 1 - \Pr\left(0|\hat{s} = \hat{s}_h, \mu = 1, \eta = 0\right). \end{aligned}$$

Straight-forward algebra shows that (5) is true if and only if,

$$\alpha \geq \alpha_1 \equiv 1 - q + \frac{\left(1-\lambda\right)\left(1-t\right)}{\lambda t}.$$

Scenario (c) says that upon observing a favorable mandatory accounting report (i.e., $\hat{s} = \hat{s}_h$) and high clarity (i.e., $\mu = 1$), the investor invests even if effective communication is not achieved and thus she cannot discern the firm's type. This result is reflective a key feature of this model. That is, in our model, there are two channels through which the investor can be informed of the firm's fundamental. The first is a *direct* channel through the mandatory disclosure signal $\hat{s}$, while the second is an *indirect* one through the observed clarity μ. When the investor believes a H (L)-type firm selects communication effort $e_H = 1$ ($e_L = 0$), the realized disclosure clarity μ serves as an additional *indirect* noisy signal about the firm's type (on top of the *direct* signal $\hat{s}$) because $\mu = 1$ is more likely to obtain with a H-type firm than with a L-type firm (in the strong communication equilibrium). In this scenario, the mandatory signal ($\hat{s}_h$) and disclosure clarity ($\mu = 1$) reinforce each other in that they both point to a higher probability of a H-type firm with fundamental X. If the *ex ante* probability of fundamental X is sufficiently high (i.e., $\alpha \geq \alpha_1$), merely observing high clarity ($\mu = 1$) makes the investor sufficiently confident that she is dealing with a H-type firm with fundamental X.

In scenario (d), not investing achieves a higher expected payoff than investing if and only if

$$\underbrace{\Pr\left(X|\hat{s} = \hat{s}_h, \mu = 0, \eta = 0\right)(0 - X)^2}_{\text{expected loss from under-investment}} \leq \underbrace{\Pr\left(0|\hat{s} = \hat{s}_h, \mu = 0, \eta = 0\right)(X - 0)^2}_{\text{expected loss from over-investment}} \tag{6}$$

where,

$$\begin{aligned}
\Pr\left(X|\hat{s} = \hat{s}_h, \mu = 0, \eta = 0\right) &= \frac{\lambda\alpha q\,(1-t)}{\lambda\alpha q\,(1-t) + \lambda\,(1-\alpha)\,(1-q)\,(1-t) + (1-\lambda)\,t} \\
\Pr\left(0|\hat{s} = \hat{s}_h, \mu = 0, \eta = 0\right) &= 1 - \Pr\left(X|\hat{s} = \hat{s}_h, \mu = 0, \eta = 0\right).
\end{aligned}$$

Straight-forward algebra shows that the condition (6) is true if and only if

$$\alpha \leq \alpha_2 \equiv 1 - q + \frac{(1-\lambda)\,t}{\lambda\,(1-t)}.$$

In scenario (d) where the firm's mandatory disclosure is favorable (i.e., $\hat{s} = \hat{s}_h$) but is of low clarity (i.e., $\mu = 0$), the investor will not invest when effective communication is not achieved, unless α is very high ($\alpha > \alpha_2$). In this case, the *direct* channel ($s = s_h$) contradicts with the *indirect* channel ($\mu = 0$) which indicates a heightened probability that the firm is of the L-type. When the *ex ante* probability of fundamental X is sufficiently low (i.e., $\alpha \leq \alpha_2$), the *indirect* signal dominates the *direct* signal and the investor optimally withholds investment. As is obvious that (3) and (4) are necessary for the strong communication equilibrium to exist, for the reminder of the paper we will impose these as assumptions and focus on other factors that determine the existence of such an equilibrium.

The next lemma characterizes the firm's optimal communication effort choices when the investor is expected to play the investment strategy according to Lemma 1.

Lemma 2 If (7) below is satisfied

$$C_M \in [(1-p)(2t-1)\psi, (2t-1)\psi]. \tag{7}$$

and if the investor makes investment decisions according Lemma 1 and believes that a H-type firm selects $e_H = 1$ and a L-type firm selects $e_L = 0$,

(1) when the firm believes the investor exerts communication effort $e_I = 1$, the H-type firm optimally selects $e_H = 1$ and the L-type firm optimally selects $e_L = 0$;

(2) when the firm believes the investor exerts communication effort $e_I = 0$, both the H and L type firms select $e_H = e_L = 1$.

Proof [Please refer to the appendix.]

Lemma 2 sheds light on when different types of firms separate via their communication effort choices. Specifically, two important conditions need to be present. First, the investor must be expected to exert effort $e_I = 1$ rather than not. If the investor shirks and picks effort $e_I = 0$, then regardless of the firm's effort effective communication is not achieved and then Lemma 1 dictates that when the mandatory disclosure is favorable (i.e., $\hat{s}_h$ is observed), the investor would base her investment decision exclusively on the realized disclosure clarity and invest if and only if $\mu = 1$. Consequently, the expected payoff for a H-type firm would be the same as that for a L-type firm, making effort separation across types impossible. In contrast, when the investor exerts attention effort ($e_I = 1$), so that it is possible for her to discern managed vs. unmanaged mandatory disclosure, the expected payoff from exerting effort $e_M = 1$ is lower for a L-type firm because its type may be revealed and the investor withholds investment, thus rendering firm's communication effort separation possible. Second, the firm's incremental cost C_M for communication effort $e_M = 1$ must be within a medium range (i.e., $C_M \in [(1-p)(2t-1)X, (2t-1)X]$). Intuitively, if the cost is too low, a L-type firm would be induced to exert the effort $e_L = 1$ hoping effective communication is not attained. If the cost is too high, even a H-type firm doesn't find in its own best interest to engage in the effort.

Having characterized the firm's effort choice, we complete our construction of the strong communication equilibrium in Proposition 1.

Proposition 1

Define

$$\begin{aligned}\Sigma_1 &\equiv \Pr[H \mid \hat{s} = \hat{s}_h, \mu = 1] \\ &= \frac{\lambda t\,[\alpha q + (1-\alpha)(1-q)]}{\underbrace{\lambda t}_{\Pr(H)\Pr(\mu=1|H)} \left[\underbrace{\alpha q + (1-\alpha)(1-q)}_{\Pr(S_h|H)}\right] + \underbrace{(1-\lambda)(1-t)}_{\Pr(L)\Pr(s_h,\mu=1|L)}}.\end{aligned}$$

Then (1) a strong communication equilibrium exists if and only if

$$p(1-\Sigma_1)X^2 \geq C_I. \tag{8}$$

In this equilibrium, (i) the investor's investment decision follows Lemma 1; (ii) the investor exerts attention effort $e_I = 1$; and (iii) a H-type firm selects communication effort (i.e., $e_H = 1$) and a L-type firm does not exert effort ($e_L = 0$). (2)

$$\frac{\partial \Sigma_1}{\partial q} > 0 \text{ if and only if } \alpha > \frac{1}{2}.$$

Proof [Please refer to the appendix.]

As is evident in the proof, (8) part 1 of the proposition is the necessary and sufficient condition for the investor to optimally exert attention effort $e_I = 1$, when she observes $\hat{s} = \hat{s}_h$ and $\mu = 1$ and believes a H-type firm selects $e_H = 1$ and a L-type firm selects $e_L = 0$. If

(8) is satisfied, the investor's effort $e_I = 1$ becomes self-fulfilling and emerges as part of the equilibrium; otherwise, the manager's rational belief has to be that the investor selects effort $e_I = 0$ and hence both firm types will exert effort $e_M = 1$ according to Lemma 2, rendering the equilibrium impossible to exist. Rewriting the condition as $\Pr(L|\hat{s} = \hat{s}_h, \mu = 1) X^2 \geq \frac{C_I}{p}$, we can interpret the condition for the investor to exert attention effort $e_I = 1$ as requiring that the opportunity cost of investing in the L-type firm is larger than the cost of not paying attention.

Part 2 of the proposition shows that the left-hand-side (LHS) of (8) decreases with the mandatory disclosure quality q if and only if $\alpha > \frac{1}{2}$. In other words, when the firm-specific factor in identifying successful projects (α) is high, (8) is less likely to be satisfied when the mandatory reporting quality is high, and therefore the strong communication equilibrium is less likely to take place. To understand this result, note first that when the investor observes $\hat{s} = \hat{s}_h$ and $\mu = 1$ and believes a separation in firms' communication effort choices, her posterior assessment of the firm's type is captured by Σ_1. The higher Σ_1 is, the more confident the investor is in believing she is dealing with a H-type firm, the less incentives she has to exert attention effort $e_I = 1$. Second, how Σ_1 changes with respect to q depends on α. Intuitively, for a H-type firm, when α is above (below) $\frac{1}{2}$, a marginal increase in q increases (decreases) the probability for $\hat{s} = \hat{s}_h$. That is,

$$\begin{aligned}\frac{\partial \Pr[\hat{s} = \hat{s}_h \mid H]}{\partial q} &= \frac{\partial [\alpha q + (1-\alpha)(1-q)]}{\partial q} \\ &= 2\alpha - 1 \\ &> 0 \text{ if and only if } \alpha > \frac{1}{2}.\end{aligned}$$

As such, conditional on $\hat{s} = \hat{s}_h$ and $\mu = 1$, an increase in q enhances the posterior probability assessment for a H-type firm if and only if $\alpha > \frac{1}{2}$.

The significance of Proposition 1 lies in its implication for the relationship between the two aforementioned channels in conveying information regarding fundamental θ. In our paper, a firm could potentially convey information via two channels: *direct* versus *indirect*. q measures the quality of the *direct* channel conveying information via signal $\hat{s}$, while the *indirect* channel communicates information via the observed disclosure clarity μ. Proposition 1 suggests that the *direct* channel can be either a complement or a substitute for the *indirect* channel. Specifically, when α is large (small) enough, increasing the quality of the *direct* channel makes the *indirect* channel increasingly harder (easier) to satisfy. Corollary 1 formalizes this idea.

Corollary 1 Denote $\Sigma_1(q)$ as Σ_1 evaluated at q. When $\lambda \in \left(\frac{1-t}{1+qt-\frac{3}{2}t}, \frac{t}{q-\frac{1}{2}-qt+\frac{3}{2}t}\right)$,

1. $\alpha_1 < \frac{1}{2} < \min(q, \alpha_2)$;
2. if $\alpha \in \left(\frac{1}{2}, \min(q, \alpha_2)\right]$ and $C_I \in \left[p\left(1-\Sigma_1(q=1)\right)X^2, p\left(1-\Sigma_1\left(q=\frac{1}{2}\right)\right)X^2\right]$, then there exists a q_1 such that the strong communication equilibrium exists if and only if $q \leq q_1$;
3. if $\alpha \in \left[\alpha_1, \frac{1}{2}\right]$ and $C_I \in \left[p\left(1-\Sigma_1\left(q=\frac{1}{2}\right)\right)X^2, p\left(1-\Sigma_1(q=1)\right)X^2\right]$, then there exists a q_2 such that the strong communication equilibrium exists if and only if $q \geq q_2$.

Proof [Please refer to the appendix.]

Recall $\alpha \in [\alpha_1, \min(\alpha_2, q)]$ is a maintained assumption throughout the paper that ensures the validity of Lemma 1. The condition $\lambda \in \left(\frac{1-t}{1+qt-\frac{3}{2}t}, \frac{t}{q-\frac{1}{2}-qt+\frac{3}{2}t}\right)$ is to ensure that $\alpha = \frac{1}{2}$ lies in the interior of the interval $[\alpha_1, \min(\alpha_2, q)]$. When $\alpha \in \left(\frac{1}{2}, \min(q, \alpha_2)\right]$, $p(1-\Sigma_1)X^2$ is a decreasing function in q. If $C_I \in \left[p\left(1-\Sigma_1(q=1)\right)X^2, p\left(1-\Sigma_1\left(q=\frac{1}{2}\right)\right)X^2\right]$, since $p(1-\Sigma_1)X^2$ is continuous in q, there exists a threshold q_1 above which (8) is violated. That is, at $q = q_1$, marginally increasing the direct channel quality q renders the indirect channel via clarity μ infeasible. In this case, the direct channel is a substitute for the indirect channel. In contrast, with $\alpha \in \left[\alpha_1, \frac{1}{2}\right]$, $p(1-\Sigma_1)X^2$ is an increasing function in q. If $C_I \in \left[p\left(1-\Sigma_1\left(q=\frac{1}{2}\right)\right)X^2, p\left(1-\Sigma_1(q=1)\right)X^2\right]$, there exists a threshold q_2 below which (8) is violated. That is, at $q = q_2$, marginally increasing the direct channel quality makes it possible to use the indirect channel via clarity μ. In this case, the direct channel is a complement for the indirect channel.

4.2 Other Equilibria

In this game, in addition to the strong communication equilibrium, there are two other equilibria, a weak communication equilibrium (where both firm types select the same communication effort) and a semi-strong communication equilibrium (where at least one firm type randomizes between $e_M = 0$ and $e_M = 1$). Although these equilibria are not the focus of our analysis, we provide a description for completeness. We first characterize the weak communication equilibrium by stating a preliminary result in the following lemma:

Lemma 3 Assume that both firm types make the same communication effort decision, $e_L = e_H$. Define,

$$\Sigma_2 \equiv \Pr(H|\hat{s} = \hat{s}_h, e_L = e_H) = \frac{\lambda[\alpha q + (1-\alpha)(1-q)]}{\lambda[\alpha q + (1-\alpha)(1-q)] + (1-\lambda)}$$

The investor's optimal strategy is as follows:

I If $\alpha > \frac{1}{\lambda} - q$ and $pX^2(1-\Sigma_2) \geq C_I$, then the investor exerts attention effort $e_I = 1$ in state $(\hat{s} = \hat{s}_h, \mu = 1)$ and invests in states $(\hat{s} = \hat{s}_h, \mu = 0, \eta = 0)$, $(\hat{s} = \hat{s}_h, \mu = 1, \eta = 0)$ or when she identifies the H-type firm with $\hat{s} = \hat{s}_h$. She doesn't invest otherwise.

II If $\alpha > \frac{1}{\lambda} - q$ and $pX^2(1-\Sigma_2) \leq C_I$, then the investor does not exert attention effort $e_I = 0$ in state $(\hat{s} = \hat{s}_h, \mu = 1)$ and invests in states $(\hat{s} = \hat{s}_h, \mu = 0, \eta = 0)$ or $(\hat{s} = \widehat{s}_h, \mu = 1, \eta = 0)$. She doesn't invest otherwise.

III If $\alpha < \frac{1}{\lambda} - q$ and $pX^2\Sigma_2\frac{\alpha q - (1-\alpha)(1-q)}{\alpha q + (1-\alpha)(1-q)} \geq C_I$, then the investor exerts attention effort $e_I = 1$ in state $(\hat{s} = \hat{s}_h, \mu = 1)$ and invests if and only if she identifies the H-type firm with $\hat{s} = \hat{s}_h$.

IV If $\alpha < \frac{1}{\lambda} - q$ and $pX^2\Sigma_2\frac{\alpha q - (1-\alpha)(1-q)}{\alpha q + (1-\alpha)(1-q)} \leq C_I$, then the investor does not exert attention effort $e_I = 0$ in state $(\hat{s} = \hat{s}_h, \mu = 1)$ and never invests.

In all cases, the investor does not exert attention effort $e_I = 0$ upon observing $\hat{s} = \hat{s}_l$ or $\mu = 0$.

Proof [Please refer to the appendix.]

Lemma 3 states the conditions under which, assuming both firm types take the same communication effort choices, the investor is willing to invest after observing $\hat{s} = \hat{s}_h$ but failing to achieve effective communication. Since in such a equilibrium both firm types choose the same communication effort, the realization of clarity μ does not provide any additional information about the firm's type (hence the label weak communication equilibrium). Therefore, if effective communication is not achieved, the investor's investment decision is independent of μ and relies only on the reported mandatory signal. If $\alpha > \frac{1}{\lambda} - q$, then the observed $\hat{s} = \hat{s}_h$ is more likely to come from a H-type firm with fundamental X, and that makes the investment profitable in expectation. In contrast, when $\alpha < \frac{1}{\lambda} - q$, the investor's prior belief about fundamental X is so pessimistic that even with a favorable mandatory disclosure $\hat{s} = \hat{s}_h$ the investor may be unwilling to invest.

The following proposition characterizes the weak communication equilibria. In these equilibria both types exert low communication effort. A weak communication equilibrium always exists and cannot be refined away with the usual refinement criteria because there are no out-of-equilibrium outcomes.

Proposition 2 There exist four weak communication equilibria in which neither firm type exerts communication effort, $e_L = e_H = 0$. These equilibria differ only in the equilibrium strategies of the investor, which are as specified in the four cases stated in Lemma 3.

Proof [Please refer to the appendix.]

Notice that there is no equilibrium in which both types of firms exert high communication effort $e_H = e_L = 1$. Indeed, since both firms types exert the same effort, the realized clarity does not provide any additional information about the firm's type and thus the investor's investment decisions do not vary with μ holding everything else equal. As a result, the L-type firm doesn't find exerting high communication effort in its own interest as doing so would only incur the cost of effort and potentially increase the odds of being identified by the investor.

The following proposition formally characterizes the semi-strong communication equilibrium, which involves mixed strategies. In particular, the H-type firm exerts communication effort with certainty, and the L-type firm and the investor randomize their effort choices to make each other indifferent.

Proposition 3 For $\alpha \in [\alpha_d, \alpha_u]$ and $C_I \in [p(1-\Sigma_1)X^2, p(1-\Sigma_2)X^2]$, there exists a unique semi-strong communication equilibrium in which,

- the H-type firm selects $e_H = 1$ with certainty,
- the L-type firm mixes by choosing $e_L = 1$ with probability $\beta_L = \frac{C_I\lambda(\alpha q+(1-\alpha)(1-q))t}{(2t-1)((1-p)X^2-C_I)(1-\lambda)} - \frac{1-t}{2t-1}$,
- if the investor observes $\hat{s} = \hat{s}_l$, she chooses $e_I = 0$ and does not invest,
- if the investor observes $\hat{s} = \hat{s}_h$, she chooses $e_I = 1$ with probability $\gamma = \frac{(2t-1)\psi - C_M}{p(2t-1)\psi}$. She invests when she identifies the H-type firm (through effective communication) or when $(\mu = 1, \eta = 0)$. She doesn't invest otherwise.
- $\alpha_1 < \alpha_d < \frac{1}{\lambda} - q < \alpha_u < \alpha_2$ (see Appendix for the expressions for α_d, α_u).

Proof [Please refer to the appendix.]

Notice that, in the semi-strong communication equilibrium, the H-type firm always exerts the communication effort with a higher probability than the L-type firm does. Thus, in this case the realized clarity μ also conveys information regarding the firm's type (albeit to a less extent than in the strong communication equilibrium case, hence the label of "semi-strong").

5 Discussions

5.1 Effect of q on Investment Efficiency

In this section, we analyze the effects of mandatory reporting quality parameter q. We are particularly interested in the neighborhood around q_1 and q_2 (the values of q such that (8) is satisfied with equality). For expositional purposes and without loss of generality, we impose the condition $\max\left(\frac{1}{\lambda} - q_1, \frac{1}{\lambda} - q_2\right) < 1/2$ throughout this section. The next proposition characterizes all equilibria that exist in the small neighborhood surrounding q_1 and q_2

Proposition 4 1. When $\alpha > \frac{1}{2}$, there exist two equilibria in the small neighborhood of $q \in (q_1, q_1 + \varepsilon]$ with ε sufficiently small: Equilibrium I of Proposition 2 and the semi-strong communication equilibrium in Proposition 3; there exist two equilibria in the small neighborhood of $q \in [q_1 - \varepsilon, q_1]$ with ε sufficiently small: Equilibrium I of Proposition 2 and the strong communication equilibrium in Proposition 1.

2. When $\alpha < \frac{1}{2}$, there exist two equilibria in the small neighborhood of $q \in (q_2, q_2 + \varepsilon]$ with ε sufficiently small: Equilibrium I of Proposition 2 and the strong communication equilibrium in Proposition 1; there exist two equilibria in the small neighborhood of $[q_2 - \varepsilon, q_2]$ with ε sufficiently small: Equilibrium I of Proposition 2 and the semi-strong equilibrium in Proposition 3.

With q varying in the small neighborhoods described in the proposition, while the weak communication equilibrium I characterized in Proposition 2 always exists, the strong communication equilibrium that exists to the left (right) of q_1 (q_2) morphs into the semi-strong communication equilibrium characterized in Proposition 3 to the right (left) of q_1 (q_2), when $\alpha > \frac{1}{2}$ ($\alpha \leq \frac{1}{2}$). This result directly flows from the fact that Σ_1 in (8) is increasing (decreasing) in q if and only if $\alpha > \frac{1}{2}$.

Before evaluating the consequences of changing q, we first need to define a measure of welfare. To this end, we use the concept of Investment Efficiency (IE) as defined below.

Definition Investment Efficiency (IE) is the expected combined loss from a Type I error (a project with $\theta = X$ doesn't get funded) and a Type II error (a project with $\theta = 0$ gets funded).

Essentially, IE gauges the probability with which the investor's investment decision turns out to be incorrect, reflecting the amount of decision-useful information available to the investor. The shift in equilibria surrounding q_1 and q_2 as described in Proposition 4 points to the possibility of a discontinuous change in IE at those points, which is confirmed by the following Proposition.

Proposition 5 Impose the equilibrium selection criterion that the equilibrium with the highest IE dominates.

1. When $\alpha > \frac{1}{2}$, IE experiences of a discontinuous drop when q crosses q_1 from below;
2. When $\alpha < \frac{1}{2}$, IE experiences of a discontinuous jump when q crosses q_2 from below.

As is clear in the proof of Proposition 5, the strong communication equilibrium, if it exists, provides the most amount of information to the investor. Intuitively, in such an equilibrium, the fact that different types of the firm selects different effort levels makes the realized disclosure clarity μ a (noisy) signal about the firm's type. This in turn helps the investor makes a correct investment decision even if effective communication is not achieved (i.e., $\eta = 0$). However, as the strong communication equilibrium becomes untenable (i.e., when $q > q_1$ for the case of $\alpha > \frac{1}{2}$ and $q < q_2$ for the case of $\alpha < \frac{1}{2}$), either both the L-type firm and investor randomizes effort in the semi-strong communication equilibrium (resulting in reduced inference drawn from μ and decreased chance of effective communication) or μ ceases to be an informativeness signal of the firm's type in the weak communication equilibrium, rendering a discontinuous shift at the switching point q_1 and q_2.

The difference in the two parts of the proposition stems from whether the direct channel (via $\hat{s}$) and the indirect channel (via μ) are complements or substitutes. When they are substitutes ($\alpha > \frac{1}{2}$), improving the quality of the former channel (i.e., increasing q) makes the LHS of (8) increasingly small and the strong communication equilibrium harder to emerge. In contrast, when they are substitutes ($\alpha < \frac{1}{2}$), improving the quality of the former channel (i.e., increasing q) makes the LHS of (8) increasingly large and the strong communication equilibrium easier to emerge.

Proposition 5 could potentially inform financial reporting regulators in their rule making decisions that determine q. When $\alpha > \frac{1}{2}$, suppose that a regulator (say FASB) is in charge of picking a q within the small interval $[q_1 - \varepsilon, q_1 + \varepsilon]$ to maximize IE. Then, as an immediate implication of Proposition 5, such a regulator would optimally set $q = q_1$, implying that there exists a endogenous upper bound on the financial reporting quality. In contrast, when

$\alpha < \frac{1}{2}$, suppose the regulator can select any q within the small interval $[q_2 - \varepsilon, q_2 + \varepsilon]$ to maximize IE. Then, the regulator would optimally pick the highest possible value $q_2 + \varepsilon$.

5.2 Allowing H-type to manipulate mandatory disclosure mapping

So far we have imposed the assumption that only the L-type firm is able to manipulate the mandatory disclosure mapping. We now relax this assumption by affording this same discretion to the H-type firm as well. Proposition 6 specifies conditions under which our main results are robust toward this change in assumption. For expositional ease and without loss of generality, we continue require $\max\left(\frac{1}{\lambda} - q_1, \frac{1}{\lambda} - q_2\right) < 1/2$.

Proposition 6 With $\alpha > 1/2$ and $q \in [q_1 - \varepsilon, q_1 + \varepsilon]$, or $\alpha < 1/2$ and $q \in [q_2 - \varepsilon, q_2 + \varepsilon]$, when ε sufficiently small, there exists an equilibrium where the H-type firm chooses not to manipulate the mandatory disclosure mapping, if p is sufficiently big.

Intuitively, given the investor's belief that a firm must be L-type if it is caught manipulating the mandatory disclosure mapping, the H-type firm is reluctant to engage in manipulation as doing so would lead to no investment if it gets caught, when the probability of getting caught p is high. As such, the H-type firm is more willing to not manipulate and subsequently communicates its type via effort $e_H = 1$.

6 Conclusions

To be finished.

7 References

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8 Appendix

Proof of Lemma 0 Proved in the text, hence omitted.

Proof of Lemma 1

1. Suppose there exists an equilibrium where the L-type firm doesn't manipulate the mandatory disclosure information system. Then, the investor would have no incentive to exert the attention effort as there is no point of achieving effective disclosure. In addition, in this posited equilibrium, conditional on observing $\hat{s} = \hat{s}_l$, The investor's expected payoff from not investing is

$$E\left(\theta \mid s = s_l\right) - \frac{\lambda\alpha\left(1-q\right)}{\lambda\alpha\left(1-q\right) + \lambda\left(1-\alpha\right)q + \left(1-\lambda\right)\left(1-q\right)}X^2.$$

Similarly, her expected payoff from investing is

$$E\left(\theta \mid s = s_l\right) - \frac{\lambda\left(1-\alpha\right)q + \left(1-\lambda\right)\left(1-q\right)}{\lambda\alpha\left(1-q\right) + \lambda\left(1-\alpha\right)q + \left(1-\lambda\right)\left(1-q\right)}X^2.$$

Hence, the investor will not invest if and only if

$$\lambda\alpha\left(1-q\right) \leq \lambda\left(1-\alpha\right)q + \left(1-\lambda\right)\left(1-q\right)$$
$$\Leftrightarrow \alpha \leq q + \frac{1-\lambda}{\lambda}\left(1-q\right),$$

which is guaranteed by (3). Consequently, the L-type firm would at least weakly prefer to manipulate the information system because reporting $\hat{s} = \hat{s}_h$ may induce a positive probability of investment. Given our tie-breaker, then the L-type firm will deviate. A contradiction.

2. Note at this stage of the game, the investor's effort C_I has already been sunk and hence doesn't affect her subsequent investment decisions.

(a) When $\hat{s} = s_l$, the investor for sure knows the firm is of H-type. The investor's expected payoff from not investing is

$$E(\theta \mid \hat{s} = \hat{s}_l, H\text{-type}) - \frac{\alpha(1-q)}{\alpha(1-q) + (1-\alpha)q} X^2.$$

Similarly, her expected payoff from investing is

$$E(\theta \mid \hat{s} = \hat{s}_l, H\text{-type}) - \frac{(1-\alpha)q}{\alpha(1-q) + (1-\alpha)q} X^2.$$

Hence, the investor will not invest if and only if

$$\alpha(1-q) \leq (1-\alpha)q$$
$$\Leftrightarrow \alpha \leq q,$$

which is guaranteed by (3).

(b) If the investor identifies the firm as a L-type, obviously she will withhold investment. When the firm is revealed to be a H-type, The investor's expected payoff from not investing is

$$E(\theta \mid \hat{s} = \hat{s}_h, H\text{-type}) - \frac{\alpha q}{\alpha q + (1-\alpha)(1-q)} X^2.$$

Similarly, her expected payoff from investing is

$$E(\theta \mid \hat{s} = \hat{s}_h, H\text{-type}) - \frac{(1-\alpha)(1-q)}{\alpha q + (1-\alpha)(1-q)} X^2.$$

Hence, the investor will invest if and only if

$$\alpha q \geq (1-\alpha)(1-q)$$
$$\Leftrightarrow \alpha \geq 1-q,$$

which is guaranteed by (3).

(c) When $\hat{s} = \hat{s}_h$, $\mu = 1$ and $\eta = 0$, The investor's expected payoff from not investing is

$$E(\theta \mid \hat{s} = \hat{s}_h, \mu = 1, \eta = 0) - \frac{\lambda\alpha q t}{\lambda\alpha q t + \lambda(1-\alpha)(1-q)t + (1-\lambda)(1-t)} X^2.$$

Similarly, her expected payoff from investing is

$$E(\theta \mid \hat{s} = \hat{s}_h, \mu = 1, \eta = 0) - \frac{\lambda(1-\alpha)(1-q)t + (1-\lambda)(1-t)}{\lambda\alpha q t + \lambda(1-\alpha)(1-q)t + (1-\lambda)(1-t)} X^2.$$

Hence, the investor will invest if and only if

$$\lambda\alpha q t \geq \lambda(1-\alpha)(1-q)t + (1-\lambda)(1-t)$$
$$\Leftrightarrow \alpha \geq \alpha_1 \equiv 1 - q + \frac{(1-\lambda)(1-t)}{\lambda t},$$

which is guaranteed by (3).

(d) When $\hat{s} = \hat{s}_h$, $\mu = 0$ and $\eta = 0$, The investor's expected payoff from not investing is

$$E(\theta \mid \hat{s} = \hat{s}_h, \mu = 0, \eta = 0) - \frac{\lambda\alpha q(1-t)}{\lambda\alpha q(1-t) + \lambda(1-\alpha)(1-q)(1-t) + (1-\lambda)t} X^2.$$

Similarly, her expected payoff from investing is

$$E(\theta \mid \hat{s} = \hat{s}_h, \mu = 0, \eta = 0) - \frac{\lambda(1-\alpha)(1-q)(1-t) + (1-\lambda)t}{\lambda\alpha q(1-t) + \lambda(1-\alpha)(1-q)(1-t) + (1-\lambda)t} X^2.$$

Hence, the investor will not invest if and only if

$$\lambda(1-\alpha)(1-q)(1-t) + (1-\lambda)t \geq \lambda\alpha q(1-t)$$
$$\Leftrightarrow \alpha \leq \alpha_2 \equiv 1 - q + \frac{(1-\lambda)t}{\lambda(1-t)},$$

which is guaranteed by (3). Q.E.D.

Proof of Lemma 2 1. For a H-type firm, its expected payoff from exerting the communication effort $e_H = 1$ is $t\psi - C_M$ and that from shirking $e_H = 0$ is $(1-t)\psi$, reflecting that according to Lemma 1 the investor invests even with $\eta = 0$ as long as $\hat{s} = \hat{s}_h$ and $\mu = 1$. Thus, the H-type firm optimally selects $e_H = 1$ if and only if

$$t\psi - C_M \geq (1-t)\psi$$
$$\Leftrightarrow C_M \leq (2t-1)\psi,$$

which is guaranteed by (7). For a L-type firm, its expected payoff from selecting $e_L = 1$ is $t(1-p)\psi - C_M$ and that from selecting $e_L = 0$ is $(1-t)(1-p)\psi$, reflecting that according to Lemma 1 the investor withholds investment at seeing $\hat{s} = \hat{s}_h$ and $\mu = 0$. Thus, the L-type firm optimally selects $e_L = 0$ if and only if

$$t(1-p)\psi - C_M \leq (1-t)(1-p)\psi$$
$$\Leftrightarrow C_M \geq (1-p)(2t-1)\psi,$$

which is again guaranteed by (7).

2. When the investor selects $e_L = 0$, the investor will for sure not be able to achieve effective communication. As such, regardless of the firm's type, the firm's payoff from exerting effort or not is $t\psi - C_M$ ($(1-t)\psi$). Given (7), thus both types of the firm will exert the communication effort $e_H = e_L = 1$, contradicting the investor's conjecture stated in the lemma. Q.E.D.

Proof of Proposition 1 1. To establish the existence of an strong communication equilibrium, one only needs to make sure the investor has incentives to exert attention effort (i.e., $e_I = 1$) upon observing $\mu = 1$ and $\hat{s} = \hat{s}_h$. Denote Σ_1 as the conditional probability of a H-type firm conditional on $\mu = 1$ and $\hat{s} = \hat{s}_h$. Thus,

$$\Sigma_1 \equiv \Pr[H\text{-type} \mid \hat{s} = \hat{s}_h, \mu = 1]$$
$$= \frac{\lambda t[\alpha q + (1-\alpha)(1-q)]}{\lambda t[\alpha q + (1-\alpha)(1-q)] + (1-\lambda)(1-t)}.$$

The investor's expected payoff from exerting attention effort $e_I = 1$ is

$$E(\theta \mid \hat{s} = \hat{s}_h, \mu = 1) + p\left[-\Sigma_1 \frac{(1-\alpha)(1-q)}{(1-\alpha)(1-q) + \alpha q} X^2\right]$$
$$+ (1-p)\left[-\Sigma_1 \frac{(1-\alpha)(1-q)}{(1-\alpha)(1-q) + \alpha q} X^2 - (1-\Sigma_1) X^2\right] - C_I.$$

The second term reflects the fact that the investor conditional on $\eta = 1$ could invest on a H-type firm with fundamental $\theta = 0$ for which the expected loss is $\frac{(1-\alpha)(1-q)}{(1-\alpha)(1-q)+\alpha q} X^2$, where $\frac{(1-\alpha)(1-q)}{(1-\alpha)(1-q)+\alpha q}$ is $\Pr(\theta = 0 \mid \hat{s} = \hat{s}_h, H\text{-type})$. The third term reflects the fact that the investor conditional on $\eta = 0$ will invest upon observing $\mu = 1$. Her expected loss in this case is $-\Sigma_1 \frac{(1-\alpha)(1-q)}{(1-\alpha)(1-q)+\alpha q} X^2 - (1 - \Sigma_1) X^2$. The additional term $(1 - \Sigma_1) X^2$ corresponds to the expected loss when the firm turns out to be a L-type. In contrast, the investor's expected payoff from not exerting the attention effort (i.e., $e_I = 0$) is

$$E(\theta \mid \hat{s} = \hat{s}_h, \mu = 1) - \Sigma_1 \frac{(1-\alpha)(1-q)}{(1-\alpha)(1-q)+\alpha q} X^2 - (1 - \Sigma_1) X^2.$$

Here, the investor always invests and $\eta = 0$ for sure. Hence, the investor chooses the attention effort (i.e., $e_I = 1$) if and only if

$$\begin{gathered} E(\theta \mid \hat{s} = \hat{s}_h, \mu = 1) + p\left[-\Sigma_1 \frac{(1-\alpha)(1-q)}{(1-\alpha)(1-q)+\alpha q} X^2\right] \\ + (1-p)\left[-\Sigma_1 \frac{(1-\alpha)(1-q)}{(1-\alpha)(1-q)+\alpha q} X^2 - (1 - \Sigma_1) X^2\right] - C_I \\ \geq E(\theta \mid \hat{s} = \hat{s}_h, \mu = 1) - \Sigma_1 \frac{(1-\alpha)(1-q)}{(1-\alpha)(1-q)+\alpha q} X^2 - (1 - \Sigma_1) X^2 \Longrightarrow \\ p(1 - \Sigma_1) X^2 \geq C_I. \end{gathered}$$

2.

$$\begin{aligned} \frac{\partial \Sigma_1}{\partial q} &= \frac{\lambda(2\alpha - 1)(1-\lambda)(1-t)t}{\{\lambda t[\alpha q + (1-\alpha)(1-q)] + (1-\lambda)(1-t)\}^2} \\ &> 0 \text{ if and only if } \alpha > 1/2. \end{aligned}$$

Q.E.D.

Proof of Corollary 1 Proved in the text, hence omitted.

Proof of Lemma 3

Proceeding by backward induction, we first analyze the investor's investment decisions. Notice first that, given that both types of managers choose the same effort, μ is not informative about the manager's type. It can be proven that, as long as the investor sees a low signal report, $\hat{s_l}$, she does not invest. However, for brevity and to simplify the notation, we omit the proof. Assume the investor is in the information states $(\hat{s_h}, 1, ns)$ or $(\hat{s_h}, 0, ns)$, then the investor's payoff from investing is, $E[\theta|(\hat{s_h}, 1, ns)] - \frac{\lambda(1-\alpha)(1-q)+(1-\lambda)}{\lambda\alpha q+\lambda(1-\alpha)(1-q)+(1-\lambda)}X^2$, and the investor's payoff from not investing is, $E[\theta|(\hat{s_h}, 1, ns)] - \frac{\lambda\alpha q}{\lambda\alpha q+\lambda(1-\alpha)(1-q)+(1-\lambda)}X^2$. So, the investor wants to invest iff $\lambda\alpha q - (\lambda(1-\alpha)(1-q) + (1-\lambda)) > 0$, which can be reduced to $\alpha > \frac{1}{\lambda} - q$. Thus, the investor investment decisions can be summarized in the following table:

	Investment decision	
Information state	$\alpha > \frac{1}{\lambda} - q$	$\alpha < \frac{1}{\lambda} - q$
$(\hat{s_l}, ., .)$	0	0
$(\hat{s_h}, \mu = 1, L)$	0	0
$(\hat{s_h}, \mu = 1, H)$	X	X
$(\hat{s_h}, \mu = 1, ns)$	X	0
$(\hat{s_h}, \mu = 0, ns)$	X	0

We now take a step back and analyze the investor's effort decisions. We need to consider four informations states in which the investor makes an effort decision:$(\hat{s_h}, \mu = 0)$, $(\hat{s_h}, \mu = 1)$,$(\hat{s_l}, \mu = 0)$, and $(\hat{s_l}, \mu = 1)$. If $\mu = 0$, the investor does not exert effort, $e_I = 0$, because there is no chance of understanding the disclosure. Also, in state $(\hat{s_l}, \mu = 1)$, the investor does not exert effort because it makes no difference. Whether the investor understands the message or not, she does not invest. In all these information states, the weak communication equilibrium at $e_L = e_H = 0$ exists trivially: since the investor does not exert effort, there is no reason for the manager to exert effort either. So, the only information state we need to analyze is $(\hat{s_h}, \mu = 1)$. In this information state, the investor wants to know the type of the sender because it makes a difference in her investment decision. We need to consider several cases:

I Assume first that $\alpha > \frac{1}{\lambda} - q$. Denote the utility of the investor with investment K by $\Pi(K)$. The investor's program is,

$$\underset{e_I \in \{0,p\}}{Max} \; E[\Pi(K)]$$

In this program, the expected investor's utility can be detailed as follows:

$E[\Pi(K)] = e_I(\Pr(H|\hat{s_h}, \mu = 1)E[\Pi(X)|\hat{s_h}, \mu = 1, H] + \Pr(L|\hat{s_h}, \mu = 1)E[\Pi(0)|\hat{s_h}, \mu = 1, L])$

$+(1 - e_I)E[\Pi(X)|\hat{s_h}, \mu = 1, \eta = 0]$

Using,

$\Pr(H|(\hat{s_h}, 1)) = \frac{\lambda(\alpha q+(1-\alpha)(1-q))}{\lambda(\alpha q+(1-\alpha)(1-q))+(1-\lambda)} = \Sigma_2$

$E[\Pi(X)|(\hat{s_h}, 1, H)] = \frac{\alpha q X-(1-\alpha)(1-q)X^2}{\alpha q+(1-\alpha)(1-q)} - e_I C_I$

$E[\Pi(0)|(\hat{s_h}, 1, L)] = -e_I C_I$

$E[\Pi(X)|(\hat{s_h}, 1, \eta = 0)] = \frac{\lambda(\alpha q X+(1-\alpha)(1-q)(-X^2))+(1-\lambda)(-X^2)}{\lambda(\alpha q+(1-\alpha)(1-q))+(1-\lambda)} - e_I C_I$

We can express the program as,

$\underset{e_I \in \{0,p\}}{Max} \frac{\lambda(\alpha q X-(1-\alpha)(1-q)X^2)+(1-e_I)(1-\lambda)(-X^2)}{\lambda(\alpha q+(1-\alpha)(1-q))+(1-\lambda)} - e_I C_I$

The optimal effort for the investor after observing $(\hat{s_h}, \mu = 1)$ is $e_I = 1$ iff,

$\frac{\lambda(\alpha q X-(1-\alpha)(1-q)X^2)+(1-p)(1-\lambda)(-X^2)}{\lambda(\alpha q+(1-\alpha)(1-q))+(1-\lambda)} - C_I > \frac{\lambda(\alpha q X-(1-\alpha)(1-q)X^2)+(1-\lambda)(-X^2)}{\lambda(\alpha q+(1-\alpha)(1-q))+(1-\lambda)}$

which can be written as, $(1 - \Sigma_2)pX^2 > C_I$.

II Assume now that $\alpha < \frac{1}{\lambda} - q$. The investor's expected utility can now be expressed as,

$E[\Pi(K)] = e_I(\Pr(H|(\hat{s_h}, 1))E[\Pi(X)|(\hat{s_h}, 1, H)] + \Pr(L|(\hat{s_h}, 1))E[\Pi(0)|(\hat{s_h}, 1, L)])$

$+(1 - e_I)E[\Pi(0)|(\hat{s_h}, 1, \eta = 0)]$,

which reduces to,

$\underset{e_I \in \{0,p\}}{Max} \lambda \frac{\alpha q X-(e_I(1-\alpha)(1-q)+(1-e_I)\alpha q)X^2}{\lambda(\alpha q+(1-\alpha)(1-q))+(1-\lambda)} - C_I$.

In a similar way as in the previous case, one can obtain that the optimal effort for the investor is p iff $\Sigma_2 \frac{\alpha q-(1-\alpha)(1-q)}{\alpha q+(1-\alpha)(1-q)} pX^2 > C_I$.

Q.E.D.

Proof of Proposition 2

First we show that the strategy profile $e_H = e_L = 0$ is always an equilibrium. We need to explore four cases:

- ***Case:*** $\alpha > \frac{1}{\lambda} - q$ and $(1 - \Sigma_2)pX^2 < C_I$

  Since, $(1 - \Sigma_2)pX^2 < C_I$, the investor chooses low effort $e_I = 0$. Since the investor does not exert effort, there is no reason for the investor to exert effort either.

- ***Case:*** $\alpha > \frac{1}{\lambda} - q$ and $(1-\Sigma_2)pX^2 > C_I$

  Since, $(1-\Sigma_2)pX^2 > C_I$, the investor chooses high effort $e_I = p$. Also, since $\alpha > \frac{1}{\lambda} - q$ the investor invests if she does not understand the disclosure. For a weak communication equilibrium at the low effort we need to require the H-type to prefer a low effort, i.e., $X > X - C_M$, and the L-type to also prefer a low effort, i.e., $((1-t)(1-p)+t)X > (t(1-p)+(1-t))X - C_M$. The former condition is trivially satisfied, and the latter condition reduces to $-(2t-1)pX < C_M$ which is always satisfied. Therefore, this is an equilibrium.

- ***Case:*** $\alpha < \frac{1-\lambda q}{\lambda}$ and $\Sigma_2 \frac{\alpha q-(1-\alpha)(1-q)}{\alpha q+(1-\alpha)(1-q)} pX^2 < C_I$

  The investor chooses low effort $e_I = 0$. Since the investor does not exert effort, there is no reason for the manager to exert effort either. Therefore, this is an equilibrium.

- ***Case:*** $\alpha < \frac{1-\lambda q}{\lambda}$ and $\Sigma_2 \frac{\alpha q-(1-\alpha)(1-q)}{\alpha q+(1-\alpha)(1-q)} pX^2 > C_I$

  The investor chooses high effort $e_I = p$. Also, since $\alpha < \frac{1}{\lambda} - q$ the investor does not invest if she does not understand the disclosure. For a weak communication equilibrium at the low effort we need to require the H-type to prefer a low effort, i.e., $(1-t)pX > tpX - C_M$, and the L-type to also prefer a low effort, i.e., $0 > 0 - C_M$. The latter condition is trivially satisfied, and the former condition reduces to $(2t-1)pX < C_M$. Since this last condition is satisfied as long as $p < 1$, this is an equilibrium.

Now we show that there is no weak communication equilibrium in which both managers choose a high effort. As in the weak communication equilibrium at low effort, μ is not informative about the type because both types choose the same effort. Therefore, the updating expressions and the investment decisions are the same as in the low effort weak communication equilibrium. Thus, we can analyze the same cases:

- ***Case:*** $\alpha > \frac{1-\lambda q}{\lambda}$

  For a weak communication equilibrium at the high effort, we need to require the H-type to prefer a high effort. However, he obtains investment regardless of the effort he exerts and regardless of the clarity realization. Therefore, there is no point in exerting high effort and, hence, this is not an equilibrium.

- ***Case:*** $\alpha < \frac{1-\lambda q}{\lambda}$ and $\Sigma_2 \frac{\alpha q-(1-\alpha)(1-q)}{\alpha q+(1-\alpha)(1-q)} pX^2 < C_I$

  The investor chooses low effort $e_I = 0$. Since the investor does not invest if he does not understand, and the investor does not understand if it does not exert effort, there is no reason for the manager to exert effort. Therefore, this is not an equilibrium.

- ***Case:*** $\alpha < \frac{1-\lambda q}{\lambda}$ and $\Sigma_2 \frac{\alpha q-(1-\alpha)(1-q)}{\alpha q+(1-\alpha)(1-q)} pX^2 > C_I$

  The investor chooses high effort $e_I = p$. For a weak communication equilibrium at the high effort we need to require the L-type to prefer a high effort. However, the L-type manager does not obtain investment regardless of the outcome of his effort. Therefore, there is no point in exerting it.

Q.E.D.

Proof of Proposition 3

For this proof we need notation to denote the relevant mixed strategies. Let $\beta_M = Pr[e_M = 1|s_h]$ for $M \in \{H, L\}$ be the manager's effort strategy after observing. a signal s_h. Also, let $\gamma = Pr[e_I = 1|\hat{s_h}, \mu = 1]$, be the investor's effort strategy in the information state $(\hat{s_h}, \mu = 1)$. For simplicity, we also assume that, if the investor is indifferent between investing and not investing, he invests.

Analyzing the game with backward induction, we first analyze the investment decisions of the investor in the following information states:

- $(\hat{s_h}, \mu = 1, \eta = 0)$

  The investor compares his payoff from investing with that from not investing:

$$E[\Pi(X)|(\hat{s_h}, \mu = 1, \eta = 0)] = E[\theta|(\hat{s_h}, \mu = 1, \eta = 0)]$$
$$+\frac{-(\lambda(1-\alpha)(1-q)(\beta_H t+(1-\beta_H)(1-t))+(1-\lambda)(\beta_L t+(1-\beta_L)(1-t)))}{\lambda(\alpha q+(1-\alpha)(1-q))(\beta_H t+(1-\beta_H)(1-t))+(1-\lambda)(\beta_L t+(1-\beta_L)(1-t))} X^2,$$
$$E[\Pi(0)|(\hat{s_h}, \mu = 1, \eta = 0)] = E[\theta|(\hat{s_h}, \mu = 1, \eta = 0)]$$

$+\frac{-\lambda\alpha q(\beta_H t+(1-\beta_H)(1-t))}{\lambda(\alpha q+(1-\alpha)(1-q))(\beta_H t+(1-\beta_H)(1-t))+(1-\lambda)(\beta_L t+(1-\beta_L)(1-t))}X^2,$

$E[\Pi(X)|(\hat{s_h},\mu=1,\eta=0)]-E[\Pi(0)|(\hat{s_h},\mu=1,\eta=0)]$

$=\frac{\lambda(\alpha q-(1-\alpha)(1-q))(\beta_H t+(1-\beta_H)(1-t))-(1-\lambda)(\beta_L t+(1-\beta_L)(1-t))}{\lambda(\alpha q+(1-\alpha)(1-q))(\beta_H t+(1-\beta_H)(1-t))+(1-\lambda)(\beta_L t+(1-\beta_L)(1-t))}X^2.$

The investor wants to invest iff the last expression is positive. Which reduces to,

$\frac{\lambda}{1-\lambda}(\alpha+q-1)\geqslant\frac{\beta_L t+(1-\beta_L)(1-t)}{\beta_H t+(1-\beta_H)(1-t)}.$

- $(\hat{s_h},\mu=0,\eta=0)$

The investor compares his payoff from investing with that from not investing:

$E[\Pi(X)|(\hat{s_h},\mu=0,\eta=0)]=E[\theta|(\hat{s_h},\mu=0,\eta=0)]$

$+\frac{-(\lambda(1-\alpha)(1-q)+(1-\lambda)(\beta_H(1-t)+(1-\beta_H)t)X^2}{\lambda(\alpha q+(1-\alpha)(1-q))(\beta_H(1-t)+(1-\beta_H)t)+(1-\lambda)(\beta_L(1-t)+(1-\beta_L)t)},$

$E[\Pi(0)|(\hat{s_h},\mu=0,\eta=0)]=E[\theta|(\hat{s_h},\mu=0,\eta=0)]$

$+\frac{-\lambda\alpha q(\beta_H(1-t)+(1-\beta_H)t)}{\lambda(\alpha q+(1-\alpha)(1-q))(\beta_H(1-t)+(1-\beta_H)t)+(1-\lambda)(\beta_L(1-t)+(1-\beta_L)t)}X^2,$

$E[\Pi(X)|(\hat{s_h},\mu=0,\eta=0)]-E[\Pi(0)|(\hat{s_h},\mu=0,\eta=0)]$

$=\frac{\lambda(\alpha q-(1-\alpha)(1-q))(\beta_H(1-t)+(1-\beta_H)t)-(1-\lambda)(\beta_L(1-t)+(1-\beta_L)t)}{\lambda(\alpha q+(1-\alpha)(1-q))(\beta_H(1-t)+(1-\beta_H)t)+(1-\lambda)(\beta_L(1-t)+(1-\beta_L)t)}X^2.$

The investor wants to invest iff the last expression is positive. Which reduces to,

$\frac{\lambda}{1-\lambda}(\alpha+q-1)\geqslant\frac{\beta_L(1-t)+(1-\beta_L)t}{\beta_H(1-t)+(1-\beta_H)t}.$

In sum, the investment decisions are:

Information state	Investment
$(\hat{s}_l,.,.)$	0
$(\hat{s_h},\mu=1,L)$	0
$(\hat{s_h},\mu=1,H)$	X
$(\hat{s_h},\mu=1,\eta=0)$	ι_1
$(\hat{s_h},\mu=0,\eta=0)$	ι_0

where, $\iota_1=1$ if $\frac{\lambda}{1-\lambda}(\alpha+q-1)\geqslant\frac{\beta_L t+(1-\beta_L)(1-t)}{\beta_H t+(1-\beta_H)(1-t)}$ and $\iota_1=0$ if $\frac{\lambda}{1-\lambda}(\alpha+q-1)<\frac{\beta_L t+(1-\beta_L)(1-t)}{\beta_H t+(1-\beta_H)(1-t)}$; also, $\iota_0=1$ if $\frac{\lambda}{1-\lambda}(\alpha+q-1)\geqslant\frac{\beta_L(1-t)+(1-\beta_L)t}{\beta_H(1-t)+(1-\beta_H)t}$ and $\iota_0=0$ if $\frac{\lambda}{1-\lambda}(\alpha+q-1)<\frac{\beta_L(1-t)+(1-\beta_L)t}{\beta_H(1-t)+(1-\beta_H)t}$.

We now analyze the investor's effort decision. The investor's effort is zero, $e_I=0$, in the information states $(\hat{s}_l,0)$, $(\hat{s}_l,1)$ and $(\hat{s_h},0)$. However, in the information state $(\hat{s_h},\mu=1)$, the investor wants to know the type of the sender because it makes a difference in the decision. Assuming that the investor is in the information state $(\hat{s_h}, 1)$, we can derive the following updating expressions:

$\Pr(H|s_h, 1) = \frac{\lambda(\alpha q+(1-\alpha)(1-q))(\beta_H t+(1-\beta_H)(1-t))}{\lambda(\alpha q+(1-\alpha)(1-q))(\beta_H t+(1-\beta_H)(1-t))+(1-\lambda)(\beta_L t+(1-\beta_L)(1-t))} = \Sigma(\beta_L, \beta_H)$

$E[\Pi(X)|(\hat{s_h}, \mu = 1, H)] = \frac{\alpha q X-(1-\alpha)(1-q)X^2}{\alpha q+(1-\alpha)(1-q)} - C_I \frac{e_I}{p}$

$E[\Pi(0)|(\hat{s_h}, \mu = 1, L)] = -C_I \frac{e_I}{p}$

$E[\Pi(0)|(\hat{s_h}, \mu = 1, \eta = 0)] = \Sigma(\beta_L, \beta_H)\frac{\alpha q}{(\alpha q+(1-\alpha)(1-q))}(X - X^2) - C_I$

$E[\Pi(X)|(\hat{s_h}, \mu = 1, \eta = 0)] = \Sigma(\beta_L, \beta_H)\frac{\alpha q X-(1-\alpha)(1-q)X^2}{(\alpha q+(1-\alpha)(1-q))} + (1 - \Sigma(\beta_L, \beta_H))(-X^2) - C_I$

With the above expressions, we analyze the following cases:

- $\frac{\lambda}{1-\lambda}(\alpha + q - 1) \geqslant \frac{\beta_L t+(1-\beta_L)(1-t)}{\beta_H t+(1-\beta_H)(1-t)}$

In this case the investor invests at $(\hat{s_h}, \mu = 1, \eta = 0)$ and the expected investor's utility can be written as,

$e_I(\Sigma(\beta_L, \beta_H)E[\Pi(X)|(\hat{s_h}, \mu = 1, H)] + (1 - \Sigma(\beta_L, \beta_H))E[\Pi(0)|(\hat{s_h}, \mu = 1, L)])$
$+(1 - e_I)E[\Pi(X)|(\hat{s_h}, \mu = 1, \eta = 0)],$

which reduces to,

$\Sigma(\beta_L, \beta_H)\frac{\alpha q X-(1-\alpha)(1-q)X^2}{\alpha q+(1-\alpha)(1-q)} - (1 - e_I)(1 - \Sigma(\beta_L, \beta_H))X^2 - C_I \frac{e_I}{p}.$

Comparing the expected value of exerting effort p with that of not exerting effort, we can derive that the investor exerts effort p iff $(1 - \Sigma(\beta_L, \beta_H))pX^2 > C_I$.

- $\frac{\lambda}{1-\lambda}(\alpha + q - 1) < \frac{\beta_L t+(1-\beta_L)(1-t)}{\beta_H t+(1-\beta_H)(1-t)}$

In this case the investor does not invest at $(\hat{s_h}, \mu = 1, \eta = 0)$ and the expected investor's utility can be written as,

$e_I(\Sigma(\beta_L, \beta_H)E[\Pi(X)|(\hat{s_h}, \mu = 1, H)] + (1 - \Sigma(\beta_L, \beta_H))E[\Pi(0)|(\hat{s_h}, \mu = 1, L)])$
$+(1 - e_I)E[\Pi(0)|(\hat{s_h}, \mu = 1, \eta = 0)],$

which reduces to,

$e_I\Sigma(\beta_L, \beta_H)\frac{\alpha q X-(1-\alpha)(1-q)X^2}{\alpha q+(1-\alpha)(1-q)} + (1 - e_I)\Sigma(\beta_L, \beta_H)\frac{\alpha q}{(\alpha q+(1-\alpha)(1-q))}(X - X^2) - C_I \frac{e_I}{p}.$

Thus, the optimal effort for the investor is $e_I = p$ iff $\Sigma(\beta_L, \beta_H)\frac{\alpha q-(1-\alpha)(1-q)}{\alpha q+(1-\alpha)(1-q)}pX^2 > C_I$.

Now we explore the possibility of a mixed equilibrium in which the investor randomizes effort. To do that, we need to consider four cases:

- Investor invests at both $(\hat{s_h}, \mu = 0, \eta = 0)$ and $(\hat{s_h}, \mu = 1, \eta = 0)$.

The manager randomizes effort with $\beta_M = Pr[e_M = 1]$ for $M \in \{L, H\}$. The investor randomizes effort with $\gamma = Pr[e_I = p|(s_h, 1)]$, and uses pure investment strategies. Since the investor invests at both $(\hat{s_h}, \mu = 0, \eta = 0)$ and $(\hat{s_h}, \mu = 1, \eta = 0)$, we must have, $\frac{\lambda}{1-\lambda}(\alpha + q - 1) > \frac{\beta_L t + (1-\beta_L)(1-t)}{\beta_H t + (1-\beta_H)(1-t)}$ and $\frac{\lambda}{1-\lambda}(\alpha + q - 1) > \frac{\beta_L(1-t)+(1-\beta_L)t}{\beta_H(1-t)+(1-\beta_H)t}$.

Given $\hat{s} = \hat{s_h}$, the H-type manager can end up in any of these three states of the world: $(\hat{s_h}, \mu = 1, H)$, $(\hat{s_h}, \mu = 1, \eta = 0)$, and $(\hat{s_h}, \mu = 0, \eta = 0)$. He obtains investment X in all three of them. Therefore, the H-type cannot be indifferent. He prefers not to exert effort because he obtains the investment anyway. Thus,$\beta_H = 0$.

The L-type manager obtains investment only if the investor does not see through, i.e., in states $(\hat{s_h}, \mu = 1, \eta = 0)$ and $(\hat{s_h}, \mu = 0, \eta = 0)$. Assume that the investor exerts effort in state $(\hat{s_h}, 1)$ with probability $\gamma = Pr[e_I = p|(s_h, 1)]$, then the L-type manager's payoffs are:

$$
\begin{aligned}
E[U_L(e_H &= 1)] = (\Pr(\mu = 1|s_h, e_H = 1) \Pr(ns|\hat{s_h}, 1) \\
+ \Pr(\mu &= 0|s_h, e_H = 1))\psi - C_M = ((\gamma(1-p) + (1-\gamma))t + (1-t))\psi - C_M \\
E[U_L(e_L &= 1)] = (\Pr(\mu = 1|s_h, e_L = 1) \Pr(ns|\hat{s_h}, 1) \\
+ \Pr(\mu &= 0|s_h, e_L = 1))\psi = ((\gamma(1-p) + (1-\gamma))(1-t) + t)\psi
\end{aligned}
$$

Thus the bad manager can be made indifferent between exerting effort an not exerting effort by the investor as follows:

$$
\begin{aligned}
E[U_L(e_L &= 1)] = E[U_L(e_L = 0)] \\
((\gamma(1-p) + (1-\gamma))t + (1-t))\psi - C_M &= ((\gamma(1-p) + (1-\gamma))(1-t) + t)\psi
\end{aligned}
$$

However, the mixed strategy that solves this equation is negative, i.e., $\gamma = -\frac{C_M}{p(2t-1)\psi} < 0$. Therefore, this is not an equilibrium.

- Investor does not invest at $(\hat{s_h}, \mu = 0, \eta = 0)$ and invests at $(\hat{s_h}, \mu = 1, \eta = 0)$.

In this case we have that $\frac{\lambda}{1-\lambda}(\alpha+q-1) < \frac{\beta_L(1-t)+(1-\beta_L)t}{\beta_H(1-t)+(1-\beta_H)t}$ and $\frac{\lambda}{1-\lambda}(\alpha+q-1) \geqslant \frac{\beta_L t+(1-\beta_L)(1-t)}{\beta_H t+(1-\beta_H)(1-t)}$

Given $\hat{s}=\hat{s_h}$, the H-type manager can end up in any of these three states of the world: $(\hat{s_h},\mu=1,H)$, $(\hat{s_h},\mu=1,\eta=0)$, and $(\hat{s_h},\mu=0,\eta=0)$. However, he only obtains investment in the first two, i.e., if $\mu=1$. The H-type manager compares:

$$\begin{aligned} E[U_H(e_H &= 1)] = \Pr(\mu=1|s_h,e_H=1)\psi - C_M = tX - C_M \\ E[U_H(e_H &= 0)] = \Pr(\mu=1|s_h,e_H=0)\psi = (1-t)\psi \end{aligned}$$

Thus, the H-type manager cannot be made indifferent because his decision to exert effort is not affected by the investor's effort. The H-type manager exerts effort if $(2t-1)X > C_M$, which is one of our assumptions. Thus, $\beta_H=1$.

The L-type manager only obtains investment if $\mu=1$ and the investor does not see through. The manager's payoffs are:

$$\begin{aligned} E[U_L(e_L &= 1)] = (\Pr(ns|\hat{s_h},1)\Pr(\mu=1|s_h,e_L=1))\psi - C_M \\ &= ((\gamma(1-p)+(1-\gamma))t)\psi - C_M \\ E[U_L(e_L &= 0)] = (\Pr(ns|\hat{s_h},1)\Pr(\mu=1|s_h,e_L=0))\psi \\ &= ((\gamma(1-p)+(1-\gamma))(1-t))\psi \end{aligned}$$

Thus, the L-type manager can be made indifferent between exerting effort an not exerting effort by the investor as follows:

$$\begin{aligned} E[U_L(e_L &= 1)] = E[U_L(e_L=0)] \\ ((\gamma(1-p)+(1-\gamma))t)\psi - C_M &= ((\gamma(1-p)+(1-\gamma))(1-t))\psi \end{aligned}$$

The mixed strategy that solves this equation is $\gamma = \frac{(2t-1)\psi - C_M}{p(2t-1)\psi}$.

The L-type manager that makes the investor indifferent between exerting effort an not exerting effort is given by the expression:

$(1-\Sigma(\beta_L,\beta_H))pX^2 = C_I$. Plugging in $\beta_H = 1$ and solving for β_L we obtain, $\beta_L = \frac{C_I\lambda(\alpha q+(1-\alpha)(1-q))t}{(2t-1)(pX^2-C_I)(1-\lambda)} - \frac{1-t}{2t-1}$.

Plugging these values in the investment conditions we obtain

$\frac{\lambda}{1-\lambda}(\alpha+q-1) < \frac{\beta_L(1-t)+(1-\beta_L)t}{(1-t)}$ and $\frac{\lambda}{1-\lambda}(\alpha+q-1) > \frac{\beta_L t+(1-\beta_L)(1-t)}{t}$.

So, we have that $\beta_L = \frac{C_I\lambda(\alpha q+(1-\alpha)(1-q))t}{(2t-1)(pX^2-C_I)(1-\lambda)} - \frac{1-t}{2t-1}$, and $\beta_H = 1$ and $\gamma = \frac{(2t-1)X-C_M}{p(2t-1)X}$ constitute an equilibrium.

- The investor neither invest at $(\hat{s_h},\mu=0,\eta=0)$ nor at $(\hat{s_h},\mu=1,\eta=0)$.

In this case it must be true that, $\frac{\lambda}{1-\lambda}(\alpha+q-1) < \frac{\beta_L(1-t)+(1-\beta_L)t}{\beta_H(1-t)+(1-\beta_H)t}$ and $\frac{\lambda}{1-\lambda}(\alpha+q-1) < \frac{\beta_L t+(1-\beta_L)(1-t)}{\beta_H t+(1-\beta_H)(1-t)}$.

The H-type manager only obtains investment in the information state $(\hat{s_h},\mu=1,H)$. The H-type manager compares:

$$E[U_H(e_H = 1)] = \Pr(u|\mu=1)\Pr(\mu=1|s_h,e_H=1)\psi - C_M = \gamma pt\psi - C_M$$
$$E[U_H(e_H = 0)] = \Pr(u|\mu=1)\Pr(\mu=1|s_h,e_H=0)\psi = \gamma p(1-t)\psi$$

Thus, the H-type manager is indifferent if $\gamma pt\psi - C_M = \gamma p(1-t)\psi$. That is, $\gamma = \frac{C_M}{(2t-1)p\psi}$
.

The L-type manager never obtains investment. Therefore, he does not exert effort. That is, $\beta_L = 0$.

Now we need to calculate the strategy of the H-type manager that makes the investor indifferent. The investor is indifferent if,

$\Sigma(\beta_L,\beta_H)\frac{\alpha q-(1-\alpha)(1-q)}{\alpha q+(1-\alpha)(1-q)}pX^2 = C_I$. Setting $\beta_L = 0$ and solving for β_H we obtain,

$\beta_H = \frac{(1-\lambda)(1-t)C_I}{(2t-1)\lambda((\alpha+q-1)pX^2-(\alpha q+(1-\alpha)(1-q))C_I)} - \frac{1-t}{2t-1}$

Now we can check whether the conditions for the investment decisions are satisfied. Plugging in the values of the manager strategies we obtain that the condition $\frac{\lambda}{1-\lambda}(\alpha+q-1) < \frac{\beta_L t+(1-\beta_L)(1-t)}{\beta_H t+(1-\beta_H)(1-t)}$ is not satisfied. Therefore, this is not an equilibrium.

- Investor invests at $(\hat{s_h},\mu=0,\eta=0)$ but not at $(\hat{s_h},\mu=1,\eta=0)$.

In this case it must the that, $\frac{\lambda}{1-\lambda}(\alpha+q-1) < \frac{\beta_L(1-t)+(1-\beta_L)t}{\beta_H(1-t)+(1-\beta_H)t}$ and $\frac{\lambda}{1-\lambda}(\alpha+q-1) > \frac{\beta_L t+(1-\beta_L)(1-t)}{\beta_H t+(1-\beta_H)(1-t)}$.

The H-type manager compares:

$$\begin{aligned} E[U_H(e_H &= 1)] = (\Pr(u|\mu=1)\Pr(\mu=1|s_h, e_H=1) + \Pr(\mu=0|s_h, e_H=1))\psi - C_M \\ &= (\gamma pt + (1-t))\psi - C_M \\ E[U_H(e_H &= 0)] = (\Pr(u|\mu=1)\Pr(\mu=1|s_h, e_H=0) + \Pr(\mu=1|s_h, e_H=0))\psi \\ &= (\gamma p(1-t) + t)\psi \end{aligned}$$

However, $U_H(e_H = 1) = (\gamma pt + (1-t))\psi - C_M < (\gamma p(1-t) + t)\psi = U_H(e_H = 0)$ for all $\gamma \in [0,1]$. Therefore, the H-type takes low effort, $\beta_H = 0$. The L-type obtains investment only if $\mu = 0$. Therefore, the bad manager exerts low effort. That is, $\beta_L = 0$. So, this is not a mixed equilibrium.

Q.E.D.

Proof of Proposition 4 1. By Proposition 1, when $\alpha > 1/2$ and $q > q_1$, the strong communication equilibrium no longer exists. As to possible weak communication equilibria, $\alpha > \frac{1}{\lambda} - q_1$ rules out Equilibria III and IV in Proposition 2. In addition, though $pX^2(1-\Sigma_1) < C_I$ in this region of parameter values, since $\Sigma_2 < \Sigma_1$ and $pX^2(1-\Sigma_1)$ is continuous in q (and equals C_I at $q = q_1$), we can always find a small ε such that if $\alpha > 1/2$ and $q \in (q_1, q_1 + \varepsilon)$,

$$pX^2(1-\Sigma_2) > C_I,$$

implying the investor strictly prefers exerting attention effort $e_I = 1$. As such, the only weak communication equilibrium is Equilibrium I in Proposition 2. As to the semi-strong communication equilibrium, one can always find a pair of mixing probabilities such that both the investor and firm are indifferent between exerting effort and not exerting effort, when ε is sufficiently small. This implies such an equilibrium exists in this neighborhood. When $\alpha > 1/2$ and $q \in \left(\frac{1}{2}, q_1\right]$, Proposition 1 establishes the existence of the strong communication equilibrium. As to possible weak communication equilibria, $\alpha > \frac{1}{\lambda} - q_1$ rules out Equilibria III and IV in Proposition 2. In addition, when $\alpha > 1/2$ and $q \in \left(\frac{1}{2}, q_1\right]$,

$$pX^2(1-\Sigma_2) > pX^2(1-\Sigma_1) > C_I.$$

The first inequality is due to $\Sigma_2 < \Sigma_1$, while the second is due to the existence of the strong communication equilibrium. As such, the only weak communication equilibrium is Equilibrium I in Proposition 2. As to the semi-strong communication equilibrium, when the L-type firm mixes between $e_L = 0$ and $e_L = 1$ as in Proposition 3, the investor assesses a smaller posterior probability for a H-type firm upon observing $\mu = 1$ than under the strong communication equilibrium, implying the investor strictly prefers to exert the attention effort $e_I = 1$ and rendering the semi-strong communication equilibrium untenable.

2. This part of the proposition can be proved in a similar fashion as that for part 1, hence omitted. Q.E.D.

Proof of Proposition 5 The definition of IE and our assumption of a symmetric loss from a Type I and a Type II error implies that any comparison of the equilibrium can be based on the combined probability of these types of errors. The combined error probability under the strong communication equilibrium is

$$\lambda\alpha(1-q) + \lambda\alpha q(1-t) + \lambda(1-\alpha)(1-q)t + (1-\lambda)(1-t)(1-p). \tag{9}$$

The weak communication equilibrium generates a combined probability of

$$\lambda\alpha(1-q) + \lambda(1-\alpha)(1-q) + (1-\lambda)(1-t)(1-p) + (1-\lambda)t, \tag{10}$$

while the semi-strong communication equilibrium generates

$$\lambda\alpha(1-q) + \lambda\alpha q(1-t) + \lambda(1-\alpha)(1-q)t + (1-\lambda)(1-\beta_L)(1-t)(1-p)\gamma \tag{11}$$
$$+(1-\lambda)(1-\beta_L)(1-t)(1-\gamma) + (1-\lambda)\beta_L t(1-p)\gamma + (1-\lambda)\beta_L t(1-\gamma)$$

The discontinuous shifts in IE is immediately established by noting that(3) guarantees (9) $>$ $\max\{(10),(11)\}$ and the strong communication equilibrium only exists to the left (right) of q_1 (q_2) for the case of $\alpha > 1/2$ ($\alpha < 1/2$). Q.E.D.

Proof of Proposition 6 The expected payoff of manipulating the mandatory reporting system for the H-type firm is

$$(1-t)(1-p)\psi. \tag{12}$$

This expression is due to the fact that the investor believes she is dealing with a L-type firm once the firm is caught manipulating (hence, no investment) and that the probability of not getting caught is $(1-t)(1-p)$. In contrast, within the small interval $q \in [q_1 - \varepsilon, q_1 + \varepsilon]$ or $q \in [q_2 - \varepsilon, q_2 + \varepsilon]$, the expected payoff from not manipulating the system for the H-type firm under the strong and semi-strong communication equilibrium is

$$[\alpha q + (1-\alpha)(1-q)](t\psi - C_M), \tag{13}$$

reflecting the fact that the H-type firm would secure investment if and only if s_h and $\mu = 1$ are realized. Finally, under the weak communication equilibrium, the expected payoff from not manipulating the system for the H-type firm within those intervals is

$$[\alpha q + (1-\alpha)(1-q)]\psi, \tag{14}$$

reflecting the fact that investment always occurs to the H-type if and only if $s = s_h$. Plugging the highest allowable value of C_M under (7), i.e., $C_M = (2t-1)\psi$, into (13), we have

$$[\alpha q + (1-\alpha)(1-q)][t\psi - (1-p)(2t-1)\psi]. \tag{15}$$

Thus,

$$(12) < (15), \text{ if and only if,}$$
$$[\alpha q + (1-\alpha)(1-q)] > (1-p)t.$$

Obviously, a sufficient condition for the above inequality to hold is that p is sufficiently big. Finally,

$$(12) < (14), \text{ if and only if,}$$
$$[\alpha q + (1-\alpha)(1-q)] > (1-t)(1-p),$$

which is also satisfied with p sufficiently big. Q.E.D.