Strategic Incentives of Divestitures of Competing Conglomerates*

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Abstract

We provide an alternative theory of divestiture that relies on product-line complementarities and product market competition. We show that in equilibrium competing conglomerates with complementary product lines have incentives to divest and that such divestitures increase both the prices of all the products and the profits of the parent firms, but reduce total surplus. We further show that if the firms are able to coordinate their divestiture strategies, monopoly prices and profits can be achieved via a non-cooperative pricing game.

Key Words: Complement, substitute, divestiture, divisionalization.

JEL Classification Numbers: L10, L22, L40.

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1 Introduction

Since the early seventies, divestiture has been used as a major form of corporate restructuring in the United States. Divestiture involves the transfer of a part of a firm's business to a new owner, as opposed to the sale of the entire firm. The value of such divestitures reached over \$117 billion in the United States in 1996, accounting for $1,702$ transactions or 29 percent of all announced merger and acquisition activity.[1] Both the volume and nature of these transactions raise questions about the economics of organizations and about corporate strategies. What determines whether the complex bundle of business activities that comprises a corporation should be free-standing, independent enterprise or instead be a unit of a large firm? What determines whether a particular business unit should be divested, and, if so, how?

In this paper, we seek to provide some answers to these questions. More specifically, we provide an explanation for a class of divestitures where product-line complementarities exist between the divesting units. Casual empiricism suggests that such complementarities often exist. For example, one might think of Ford company producing oil filters, spark plugs, fenders and engines which are *perfect complements* in the production of Ford automobiles, and the resulting cars made by Ford are *imperfect substitutes* for the automobiles made by General motors. Recently, GM spun off Delphi (January, 1999), its auto part division; and Ford divested Visteon, the counterpart of Delphi in Ford(June, 2000)[2]. We argue that in the presence of competition, divestiture of the units supplying complementary products can increase the total value of a firm because of the changed competition structure and firms' strategic behaviors. To illustrate this

[1] See Mergerstat Review, page 29-32, 1997. According to the Review, divestitures, as a percentage of all announced merger and acquisition activity increased dramatically in the late 1960s and peaked in the mid-1970s, with 53 percent in 1976. During the 1980s, this percentage ranged from 35 percent to 45 percent. In the first half of the 1990s, divestitures declined in terms of percentage, but the number of divestitures and their total value have been increasing.

[2] Also, on September 20, 1995, AT&T Corp. announced that it would split into three independent firms with the first offering long-distance telephone and credit card services, the second supplying telecommunication equipment, and the third dealing in the computer businesses. These three businesses can be viewed as complementary. For details, see "AT&T's three-way split," *The Economist*, September 23rd, 1995. According to Mergerstat Review 1997, AT&T Corp. actually divested 6 units during the year of 1996. Many other telecom-equipment companies such as Sweden's Ericsson, Finland's Nokia, and the U.S.'s 3Com entered the computer market not long ago, but have now divested from the computer business.

Another example is shopping malls. We often observe that in-town shopping malls consist of many independently managed shops that usually sell ordinary complements. Different shopping malls compete by selling goods that are substitutes across malls. Furthermore, even if different stores in a mall offer similar products, consumers are not always sure which product they would prefer before visiting the stores. Their decisions are often based on expected prices. Therefore, the existence of shopping costs makes ordinary substitutes within a mall transaction complements (see Ayres, 1985, Stahl, 1987, and Beggs, 1994). Pashigian and Gould (1995) provide empirical evidence to show that positive agglomerate externalities exist when stores are located together in a mall.

point, we consider a simple environment in which there are two firms, each supplying a group of complements, with the products across the groups being imperfect substitutes. The firms' choices of divesting and pricing are modeled as a two-stage game. The duopolists simultaneously choose their divestiture strategies in the first stage of the game. That is, each parent firm decides whether to keep a subset of its product lines and sell the rest of its operations to independent entrepreneurs. In the second stage, the independent divisions (the parent firms and the divested divisions) compete by simultaneously setting prices. To highlight purely strategic motives in our analysis, we strip away other factors that might affect the firms' decisions to divest, such as agency problems, increasing returns to scale, and diseconomies of scope.

We characterize the subgame perfect equilibrium of the two-stage game and find that both firms have incentives to divest. This finding relies on the product-line complementarity and competition between the firms. In the absence of divestitures, two competing firms set prices of their products lower than monopoly prices that maximize the joint profits, since each firm ignores the positive externality of its pricing on the rival's profit arising from the substitutability of the products between the firms. After a firm divests into independent divisions, these divisions ignore the negative externality of their pricing on each other's profits due to the complementarity of the products among the divisions. By divesting, the firm creates own-group pricing externality to mitigate the opposite cross-group externality. When the degree of divestiture measured by the number of independent divisions is small, the own-group negative externality is smaller than the cross-group positive externality, which moves prices closer to monopoly price levels, leading to higher profits. This reasoning works for both firms. Moreover, this analysis can be extended to the case of more than two competing firms. We show that in equilibrium, the degree of divestiture is determined by severity of competition and the degree of complementarity of each firm's products. The more severe the competition among the firms, the higher the incentives to divest. Thus, exogenous factors such as deregulation policies, trade liberalization, and ease of entry can affect the competitive environment and trigger divestiture activities.

In this paper we also investigate the situation in which the firms are able to coordinate their divestiture decisions in the first stage and show that there exists a pair of divestiture strategies such that monopoly prices and profits are achieved in a non-cooperative pricing game in the second stage. By choosing the appropriate degree of divestitures, competing firms create an own-group negative externality that exactly offsets the cross-group externality. Such a coordinated number of divisions is greater than the non-cooperative number of divisions. This is due to another positive externality between the firms' choices of divisions. After divesting, a firm commits not to coordinate the pricing of its own divisions. This fat-cat strategy softens the second-stage competition, raises prices and thus benefits the rival firm. Coordination between the two firms internalizes the positive externality and thus results in more divisions than the non-cooperative equilibrium permits. With non-coordinated divestitures, prices move up but they are still below monopoly price levels. With coordinated divestitures, on the other hand, equilibrium prices are equal to monopoly price levels as if divisions of both firms are operated by a single agent. Through coordinated divestitures, the firms can achieve perfect collusion in pricing in the non-cooperative pricing game.

In the literature, a variety of other arguments have been put forward to explain divestiture. One argument is that divestiture can be rationalized as an institutional innovation in response to the loss due to agency problems in large corporations.[3] This theory is certainly useful in explaining the decentralization of the control of assets or business activities, but it does not explain why many divestitures often involve the units that had been previously acquired rather than those that were started from scratch by divesting firms (see Porter, 1987, Ravenscraft and Scherer, 1987, and Kaplan and Weisbach, 1992). Both Porter (1987) and Ravenscraft and Scherer (1987) interpret those sales as recognition of failures of acquisitions. The evidence assembled by Kaplan and Weisbach (1992) suggests that less than one-third of acquisitions that were later divested could be considered failures ex post. Our theory differs from those arguments by relating divestiture to the nature of competition and product-lines in the industry. We predict that increased competition tends to intensify divestiture activities related to complementary products.

This paper is closely related to the recent and growing literature on strategic divisionalization. Schwartz and Thompson (1986) show that the incumbent firm can forestall entry by setting up multiple rival divisions prior to entry. Corchon (1991), Polasky (1992), Gonzales-Maestro (1995), Baye, Crocker, and Ju (1996), and Yuan (1999) analyze the strategic incentives for firms to form independent divisions when competing in quantity in a homogeneous product market. They find that competing firms form multiple divisions in order to take a larger share of the market. When forming a division is costless, breakups lower prices and profits but increase social welfare[4]. Rysman (2001) extends those studies to the case where firms can sign contracts after franchising. Our paper analyzes a different competitive environment and identifies a different incentive for firms to break up. In our framework, firms with a set of complementary products set up independent divisions to soften the competition from competing firms. As a result, divestiture of this kind increases prices and profits and reduces social welfare. In addition, in order to ensure the existence of an interior equilibrium (i.e., a finite number of divisions), this literature requires a cost of forming a division. In our paper, although a cost of forming divisions will certainly reduce firms' incentives to set up divisions, it is not necessary for the existence of an interior equilibrium.

[3] See, for example, Hart and Moore (1990), Holmstrom and Milgrom (1991), Meyer, Milgrom and Roberts (1992).

[4] When divisionalization is costly, Baye, Crocker, and Ju (1996) show that breakups do not always increase social welfare and mergers to duopoly may increase social welfare.

Our analysis is related to, but distinct from, the merger literature. Salant, Switzer, and Reynolds (1983) show that a merger of a subgroup of firms supplying homogeneous product may result in a loss in profits for the merging firms. In a Cournot duopoly model with complements, Salop (1990) and Economides and Salop (1992) show that a merger reduces prices because it allows the coalition firm to absorb positive externalities. Our concern is with the opposite issue: competing firms' incentives to divest their units that supply complements, where the products between the two firms are imperfect substitutes.[5] Furthermore, unlike most of the merger literature, our analysis permits one to analyze the equilibrium consequences of these incentives in a noncooperative setting that allows all firms to divisionalize. The merger literature, in contrast, implicitly views merger as a cooperative game, where the set of firms that merge is exogenously selected.

Finally, our paper is related in spirit to Rey and Stiglitz (1995). Although they are mainly concerned with the effect of awarding exclusive territories to retailers who then engage in price competition, they also find that the manufactures have incentives to separate themselves from retailers in order to soften downstream competition.

The rest of the paper is organized as follows. The next section introduces a two-stage duopoly model with differentiated products. Each firm supplies a group of perfect complements and, across firms, the products are imperfect substitutes. Section 3 characterizes the equilibrium outcomes and provides the main results. Section 4 discusses possible extensions of the basic model. Section 5 concludes the paper.

2 The Model

Suppose that consumers demand a number of differentiated products, which are divided into two groups. Within each group the products are perfect complements and, across groups, they are imperfect substitutes. Let N_k denote the set of products in group k, $k = 1, 2$. The demand functions are given by

$$q_{ki} = D(p_k, p_l), \quad i \in N_k, \tag{1}$$

for $k, l = 1, 2$ and $l \neq k$, where p_{ki} and q_{ki} denote the price and quantity of product i in group k, respectively, and $p_k = \sum_{i=1}^{n_k} p_{ki}$ is the aggregate price of products in group k and n_k is the number of products in group k.

Notice that the demand system (1) is symmetric both within and between groups.[6] We make

[5] We address only the issue of strategic divestitures in our context, while the issue of mergers can be analyzed analogously in a setting where each complement is supplied by an independent firm.

[6] The symmetry of demands across groups is not crucial for our discussions below.

the following assumptions regarding the function $D(p_1,p_2)$[7]. Let $P = \{(p_1,p_2) \in R^2_+ | D(p_1,p_2) > 0,\ D(p_2,p_1) > 0\}$.

(A1) P is convex and bounded, $D(p_1,p_2)$ is twice continuously differentiable in P, $D_2(p_1,p_2) > 0$, and $D_1(p_1,p_2) + D_2(p_1,p_2) < 0$ for (p_1,p_2) in P.

Here $D_k(p_1,p_2)$ denotes the first-order derivative of $D(p_1,p_2)$ with respect to p_k, $k = 1,2$. The assumption that $D_2 > 0$ represents demand substitutability between the two groups of the products. $D_1 + D_2 < 0$ states that the effect of the aggregate price within the group on the demand (own-group effect) dominates the effect of the aggregate price from the other group (cross-group effect). To illustrate our analysis, we frequently use the linear form of demand functions

$$D(p_1,p_2) = \alpha - \beta p_1 + \gamma p_2, \tag{2}$$

where $\alpha > 0$, $\beta > \gamma > 0$. The ratio, γ/β, measures the degree of substitution between the two groups of products or the extent to which the own-group effect dominates the cross-group effect.[8]

There are two firms, 1 and 2. Firm 1 supplies all the goods in the first group and firm 2 offers all the complements in the second.[9] To focus on the strategic incentives, we assume that the production technology of each firm exhibits constant returns to scale and scope.[10] Furthermore, in order to simplify our presentation and emphasize the effects of the nature of demands on divisionalization, we set all the marginal costs equal to zero. This simplification does not affect the qualitative results in the paper.

We consider the subgame perfect equilibria of the following two-stage game with perfect information. In the first stage, the two firms simultaneously choose their restructuring strategies. In stage two, all independent firms compete by simultaneously setting prices. By restructuring, the parent firm keeps a subset of its product lines and sells the rest of its operations to independent entrepreneurs (not its rival firm). It should be noted that in our framework the restructuring strategies of the firms can be any of the following: divestiture, spin-off, breakup, or divisionalization, as long as the resulting firms independently choose their pricing strategies.

[7] These assumptions are standard in the literature on differentiated products. See Friedman (1977) and Deneckere and Davidson (1985).

[8] Beggs (1994) uses the same linear demand function to study effects of a merger in the context of shopping malls. There are only two complements within each group in his model.

[9] An alternative specification of the model is to assume that the total number of firms is $n_1 + n_2$ and each firm supplies one product. The issue of strategic incentives to merge can be addressed in this setup.

[10] The assumption of constant returns to scope implies that there is no operating synergy between different lines of businesses. Our model is motivated to address divestiture issues concerning conglomerates.

In the following analysis, we simply refer to a restructuring choice as divisionalization, namely, a firm setting up autonomous rival divisions.

We model divisionalization of a firm as a partition of its product space. Let m_k denote the number of cells in a partition of N_k and each cell in the partition is a division of firm k. Then, m_k is the number of divisions of firm k. For a pair of strategies (m_1, m_2), the profit function of division j in group k is

$$\pi_{kj}(p_{kj}, p_k, p_l; m_k, m_l) = p_{kj} D(p_k, p_l), \quad l \neq k \tag{3}$$

where, for notational simplicity, p_{kj} represents the aggregate price of the products in division j of group k, and $p_k = \sum_{j=1}^{m_k} p_{kj}$ is the aggregate price of products in group k. For $k = 1, 2$, let

$$\Pi_k(p_k, p_l; m_k, m_l) = \sum_{j=1}^{m_k} \pi_{kj}(p_{kj}, p_k, p_l; m_k, m_l) = p_k D(p_k, p_l) \tag{4}$$

A divisionalization (or divestiture) strategy can be viewed as a set of take-it-or-leave-it contracts signed between firm k (or the parent firm) and independent entrepreneurs. Each contract specifies a reserve price at which the parent firm is willing to sell its operations of a subset of its products. We assume that the contracts are restrictive so that no division can further divide or subcontract prior to the price decisions in the second stage of the game. It is then reasonable to assume that firm k sets a reserve price equal to the profit that division j can make in the second stage game and that each entrepreneur is indifferent about accepting the contract or rejecting it. Therefore, the total profit of firm k is given by (4).

The two-stage game can be solved via backward induction. In the second stage of the game, for any pair of strategies (m_1, m_2), each division j in group k chooses its price p_{kj} to maximize (3), given the price choices of other divisions within and across groups. In the first stage, firm k chooses a divisionalization strategy m_k to maximize (4), taking into account the divisionalization choice of the other firm and the equilibrium prices of the second stage game.

3 Analysis

In this section, we first introduce two benchmarks. One deals with competition between two firms without divestiture. The other is the joint profit maximization. We then characterize the equilibrium outcomes of the two-stage games, which can be solved by backward induction. For each set of divisions chosen by the two firms, the equilibrium of the second-stage pricing game is determined and comparative static properties of the equilibrium are discussed. Our main findings are then presented.

3.1 Two Benchmarks

In the first benchmark, two firms directly engage in Bertrand price competition without divestiture. That is, both m_1 and m_2 are equal to one. Given the simplification of the marginal costs, firm k has the following profit function

$$\Pi_k(p_k, p_l) = p_k D(p_k, p_l), \tag{5}$$

for $k, l = 1, 2$, $l \neq k$, and $p_k = \sum_{i=1}^{n_k} p_{ki}$. Notice that the profit functions depend only on the aggregate prices of the complements, p_k and p_l. Each firm only needs to decide on its aggregate price, and individual prices are indeterminate. Furthermore, the profit function of firm k increases with the price of the other firm since the products across groups are substitutes.

In equilibrium, firm k chooses p_k to maximize (5) given the aggregate price of the other group, p_l. The first-order conditions are

$$D(p_k, p_l) + p_k D_1(p_k, p_l) = 0, \tag{6}$$

for $k, l = 1, 2$ and $l \neq k$. The equilibrium price (p_1, p_2) is determined by (6). Given the symmetry of demands across groups, in equilibrium $p_1 = p_2$, which we denote by p^0. Later, we will provide sufficient conditions for the existence and uniqueness of the equilibrium for this game.

In the case of linear demand function (2), the best-reply functions determined by (6) are easily computed as

$$r_k(p_l) = \frac{\alpha + \gamma p_l}{2\beta}, \quad l \neq k,$$

which are linear and strictly increasing. Therefore, two prices are *strategic complements* in the sense of Bulow, Geanakoplos, and Klemperer (1985). The equilibrium price is

$$p^0 = \frac{\alpha}{2\beta - \gamma},$$

and the equilibrium profit for each firm is

$$\Pi_1^0 = \Pi_2^0 = \frac{\alpha^2 \beta}{(2\beta - \gamma)^2}.$$

In the second benchmark, the two firms do not divide, but collude by setting prices to maximize their joint profits:

$$\Pi(p_1, p_2) = p_1 D(p_1, p_2) + p_2 D(p_2, p_1). \tag{7}$$

Clearly, in this optimization problem only the aggregate prices matter. Assume for now that the global maximum is unique. Given the symmetry of $\Pi(p_1, p_2)$, the optimal aggregate prices are identical and denoted by p_M, which is determined by the first-order condition

$$D(p_M, p_M) + p_M D_1(p_M, p_M) + p_M D_2(p_M, p_M) = 0. \tag{8}$$

As compared to the first benchmark, the joint profit maximization internalizes the externality between the prices of the two groups. As a result, the monopoly price, p_M, is greater than the non-cooperative equilibrium price, p^0. This point can be clearly illustrated for the linear demand function in (2). In this case, $\Pi(p_1, p_2)$ is strictly concave. The monopoly price and joint profits are computed as

$$p_M = \frac{\alpha}{2(\beta - \gamma)}, \qquad \Pi_M = \frac{\alpha^2}{2(\beta - \gamma)}.$$

The monopoly price is greater than the duopoly price and monopoly profits are higher.

3.2 Second-Stage Pricing Game

We next analyze the equilibrium outcome of the second-stage pricing game. For a given pair of divisionalization strategies (m_1, m_2), the profit function (3) of division j in group k can be written as

$$\pi_{kj}(p_{kj}, p_k, p_l; m_k, m_l) = p_{kj} D(p_k, p_l), \tag{9}$$

for $k, l = 1, 2$, and $l \neq k$. Division j chooses its price p_{kj} to maximize (9), given the price choices of the other divisions both within and across groups. The first-order condition for an interior solution is

$$D(p_k, p_l) + p_{kj} D_1(p_k, p_l) = 0, \quad j = 1, ..., m_k, \tag{10}$$

for $k, l = 1, 2$, and $l \neq k$. Pure-strategy Nash equilibria are then determined by equations (10).

Notice that, by (10), the equilibrium prices of the divisions within the group are identical. Thus the equilibrium conditions (10) are equivalent to

$$m_k D(p_k, p_l) + p_k D_1(p_k, p_l) = 0, \tag{11}$$

for $k, l = 1, 2$, and $l \neq k$. Equations (11) determine the best-reply function for group k and hence the equilibrium aggregate prices.

Further assumptions on the demand function are stated to guarantee the existence and uniqueness of equilibrium in the price-setting game.

(A2) $D(p_1, p_2)$ is log-concave in p_1.

(A3) $D_{11}(p_1, p_2)+ \mid D_{12}(p_1, p_2) \mid \square\ 0$ for $(p_1, p_2) \in P$.

(A4) $D_2(p_1, p_2) + p_1 D_{12}(p_1, p_2) \geq 0$ for $(p_1, p_2) \in P$.

Here $D_{kl}(p_k, p_l)$ denotes the second-order derivative of $D(p_k, p_l)$ with respect to p_k and p_l, $k, l = 1, 2$. (A2) states that the demand function is log-concave in its own price. Since the cost function is linear in our model, this condition is sufficient for strict quasiconcavity of π_{kj} with respect to p_{kj}, which ensures the existence of a maximum (see Vives (1999), p. 149 and Calpin and Nalebuff (1991) for more detailed discussion). (A3) states that the difference between the magnitude of own-group effect and cross-group effect falls as the own-group price goes up. It guarantees the uniqueness of equilibrium given that we have constant marginal costs. (A4) is equivalent to assume that the marginal revenue of firm 1 declines with the price of firm 2, which implies that the best-reply functions determined by (11) are strictly increasing. Thus, the prices between the two groups are strategic complements. This is a common assumption in the literature on price competition with differentiated products. Some interesting comparative-static results can be obtained in this case.

Let $\hat{p}_1$ and $\hat{p}_2$ denote the equilibrium aggregate group prices, respectively, and $\hat{q}_k = D(\hat{p}_k, \hat{p}_l)$, for $k, l = 1, 2$, and $l \neq k$, denote the equilibrium quantities.

Notice that the above assumptions are satisfied for the linear demand function in (2). In this case, the best reply functions determined by (11) are

$$r_k(p_l) = \frac{m_k(\alpha + \gamma p_l)}{(1 + m_k)\beta}, \quad l \neq k,$$

which are linear and strictly increasing. Figure 1 illustrates the best-reply lines. As m_k increases, the best-reply line for group k shifts up, but the best-reply line for group l does not change. Therefore, equilibrium prices for both groups increase with m_k. These prices can be easily computed as

$$\hat{p}_k = \frac{m_k \alpha[(1 + m_l)\beta + m_l \gamma]}{(1 + m_k)(1 + m_l)\beta^2 - m_k m_l \gamma^2}, \quad l \neq k, \ k = 1, 2.$$

For the general form of demand functions, we have the following characterization and comparative statics.

Lemma 1: Suppose that (A1)-(A4) hold. Then, for each pair of (m_1, m_2), there exists a unique equilibrium in the price-setting game. Furthermore,

(a) the equilibrium prices are identical within a group;

(b) the aggregate price in group k, $\hat{p}_k$, increases with m_k for $k = 1, 2$;

(c) the aggregate price in group l, $\hat{p}_l$, increases with m_k for $k, l = 1, 2$ and $l \neq k$, if (A4) holds;

(d) the equilibrium quantity of each complement in group k, $\hat{q}_k$, decreases with m_k for $k = 1, 2$; and

(e) if $m_1 = m_2 = m$, then $\hat{p}_1 = \hat{p}_2$ increases with m and $\hat{q}_1 = \hat{q}_2$ decreases with m.

The proof of Lemma 1 is given in the Appendix. The intuition for the monotonicity of the aggregate group price with respect to the number of divisions in the group is the following. Since the products within the group are complementary, there is a *negative externality* among the prices of these products. In other words, an increase in the price of one product reduces the demand for the other products within the same group and hence decreases the profits of those products. If a firm is divided into independent divisions, divisions will not take this externality into account,and therefore tend to set prices higher. Given the weaken the competition (higher prices) after the divestiture of its rival, the other firm will respond to raise its price. This in turn provides an extra incentive for the divisions to raise their prices. Thus, the aggregate price of the complements within a group increases with its number of divisions.

¿From (11), an increase in m_k shifts up the best-reply curve of group k, but does not change the best-reply curve of group l. Given (A4), the best-reply curves for the two groups are upward-sloping. Therefore, both prices increase as m_k goes up (see Figure 1).

Lemma 1(d) states that the equilibrium quantity of each complement decreases with the number of divisions in that group. As the number of divisions in the other group goes up, however, the equilibrium quantity of the complement does not necessarily fall. It depends on the sizes of both m_1 and m_2. What we know is that when the firms divide symmetrically, i.e., $m_1 = m_2 = m$, the equilibrium quantity of each complement decreases as m increases.

3.3 Strategic Incentives to Divest

We now examine whether a divestiture improves the firms' profits. Given the characterization of the equilibrium in the second-stage pricing game, we can write the reduced-form profit function of firm k as

$$\hat{\Pi}_k(m_k, m_l) = \hat{p}_k D(\hat{p}_k, \hat{p}_l),$$

for $k, l = 1, 2$ and $l \neq k$, where $\hat{p_k}$ and $\hat{p_l}$ depend only on the numbers of divisions m_k and m_l.

In the rest of this section, we treat m_1 and m_2 as continuous variables. The profit of firm 1 depends on m_1 through the two prices. The derivative of firm 1's profit function with respect to m_1 can be computed as follows

$$\frac{\partial \hat{\Pi}_1(m_1, m_2)}{\partial m_1} = [D(\hat{p}_1, \hat{p}_2) + \hat{p}_1 D_1(\hat{p}_1, \hat{p}_2)] \frac{\partial \hat{p}_1}{\partial m_1} + \hat{p}_1 D_2(\hat{p}_1, \hat{p}_2) \frac{\partial \hat{p}_2}{\partial m_1}. \tag{12}$$

Using the first-order conditions (11), we can write the own-group effect of the price on the profit as

$$D(\hat{p}_1, \hat{p}_2) + \hat{p}_1 D_1(\hat{p}_1, \hat{p}_2) = \hat{p}_1 D_1(\hat{p}_1, \hat{p}_2)(m_1 - 1)/m_1, \tag{13}$$

which is negative for $m_1 > 1$. Therefore, by Lemma 1(b) and (c), an increase in m_1 has two effects on firm 1's profit. The first is the own-group effect: it increases the aggregate price of products in group 1, which reduces firm 1's profit due to the negative externality of prices within the group. The second is the cross-group effect: it increases the aggregate price of products in group 2, which increases firm 1's profit due to the positive externality of prices across groups. When m_1 is close to 1, the own-group effect is close to zero and the cross-group effect is positive. This means that a small degree of divestiture by firm 1 increases its total profit. If the degree of substitution between the two groups of products is high, then each firm has an incentive to divide unilaterally into multiple divisions.

Next, we determine the non-cooperative equilibrium of the first-stage division game, where the firms independently choose their numbers of divisions. Each firm chooses its number of divisions to maximize its profit, taking the other firm's number of divisions and the second stage pricing behavior as given. Using (12) and (13), we can write the first-order conditions for an interior solution as follows

$$D_1(\hat{p}_k, \hat{p}_l) \frac{m_k - 1}{m_k} \frac{\partial \hat{p}_k}{\partial m_k} + D_2(\hat{p}_k, \hat{p}_l) \frac{\partial \hat{p}_l}{\partial m_k} = 0 \tag{14}$$

for $k, l = 1, 2$ and $l \neq k$. The non-cooperative equilibrium is determined by (14). Since the reduced-form profit functions are symmetric in m_1 and m_2, which we denote by $V(m_1, m_2)$, i.e., $\hat{\Pi}_1(m_1, m_2) = V(m_1, m_2)$ and $\hat{\Pi}_2(m_2, m_1) = V(m_2, m_1)$, there exists a symmetric equilibrium. We denote by $\hat{m}$ the symmetric equilibrium number of divisions.

To determine the size of the equilibrium number of divisions, $\hat{m}$, we impose a further restriction on the demand function.

(A5) $D_{12}(p_1, p_2) + D_{22}(p_1, p_2)$ □ 0 for (p_1, p_2) in P.

Similar to (A3), (A5) states that the difference between the own-group effect and the cross-group effect does not increase as the cross-group price goes up. Both assumptions impose a limit on how the net effect varies with prices. (A3) and (A5) together imply that the degree of substitution at the symmetric price, $-D_2(p,p)/D_1(p,p)$, is non-increasing in price p. In the case of the linear demand function, (A5) is obviously satisfied. The following lemma illustrates the important properties of the reduced-form profit function when the firms divest symmetrically.

Lemma 2: Suppose that (A1)-(A3) and (A5) hold. Then the reduced-form profit function $V(m,m)$ is a single-peaked function of m and reaches the maximum at $m = m^*$, where

$$m^* = \frac{D_1(p_M, p_M)}{D_1(p_M, p_M) + D_2(p_M, p_M)}. \tag{15}$$

The proof of Lemma 2 is presented in the Appendix. Lemma 2 implies that a small degree of divestitures by both firms increase their profits, but too many divestitures can reduce their profits provided that m^* is less than n_1 and n_2. We now state our result on the non-cooperative equilibrium of divestiture.[11]

Proposition 1: Suppose that (A1)-(A5) hold. Then competing firms have unilateral incentives to divest into multiple divisions. The equilibrium number of divisions of a firm, $\hat{m}$, satisfies $1 < \hat{m} < m^*$.

The proof of Proposition 1 is presented in the Appendix. Unfortunately, the existence and uniqueness of the equilibrium in the division game require messy restrictions on the demand function involving third-order derivatives. Instead, we resort to assuming that equilibrium exists and is unique, and use the linear demand function in (2) to illustrate the existence and some

[11] It is worthy to point out that, if only one firm could divest, the equilibrium of two stage game is exactly the same as the conventional Stackelberg game. This results from the fact that the aggregate price of the firm with divestiture is a monotonically increasing function of the number of divisions (Lemma 1 (b)), and the best response function of the second firm in choosing price is the same. The proof is relatively easy. Here we will only provide an outline of the main argument. In the conventional Stackelberg game, the leading firm can always adopt the equilibrium price of the divesting firm in the two-stage game, and replicate the outcome of the two-stage game, because the best response function for the second firm in choosing price is the same in both games. Thus, Stackelberg outcome can be no worse than that of the two-stage game. Similarly, the divesting firm can always choose a number of divisions such that its aggregate price is the same as the leading firm in the Stackelberg game and therefore replicate the Stackelberg outcome. That is, the outcome of the two-stage game can be no worse than the Stackelberg game. Consequently, the Stackelberg game produces the same outcome as the two-stage game if only one firm is allowed to divest. We would like to thank a referee for raising this question.

properties of the unique equilibrium[12]. In this case, the equilibrium payoff from the pricing game can be computed as

$$V(m_1, m_2) = \frac{m_1 \alpha^2 \beta[(1+m_2)\beta + m_2\gamma]^2}{[(1+m_1)(1+m_2)\beta^2 - m_1 m_2 \gamma^2]^2}. \tag{16}$$

It can be easily verified that $V(m_1, m_2)$ increases strictly with m_2. Thus, there exists a positive externality between the firms' choices of divisions. The best-reply functions in the division game are $m_1 = R(m_2)$ and $m_2 = R(m_1)$, where

$$R(m) = \frac{(1+m)\beta^2}{(1+m)\beta^2 - m\gamma^2}, \tag{17}$$

which is strictly increasing and always greater than 1. Figure 2 illustrates the best-reply curves that cross at the point $(\hat{m}, \hat{m})$, where

$$\hat{m} = \frac{1}{\sqrt{1 - (\frac{\gamma}{\beta})^2}}.$$

It can be verified that $V(m_1, m_2)$ is a single-peaked function of m_1, which guarantees $(\hat{m}, \hat{m})$ to be a Nash equilibrium of the division game. Furthermore, the equilibrium is unique. It follows from Lemma 1 that the subgame perfect equilibrium of the two-stage game is unique and symmetric.

To illustrate Lemma 2 and Proposition 1, we can easily verify that $V(m, m)$ is single-peaked and reaches the maximum at

$$m^* = \frac{1}{1 - \frac{\gamma}{\beta}}.$$

Clearly, $\hat{m}$ is greater than 1, but less than m^*. Both $\hat{m}$ and m^* increase with γ/β, which measures the degree of substitution between the groups of products. As γ/β goes up, the competition between two groups intensifies. Competing firms respond by divesting into more and finer independent divisions.

Corollary 1: Given a linear demand, in the subgame perfect Nash equilibrium, each competing firm set up $\hat{m} = \frac{1}{\sqrt{1-(\frac{\gamma}{\beta})^2}}$ independent divisions. The equilibrium number of divisions increases with the degree of competition between firms (measured by $\frac{\gamma}{\beta}$).

[12] Rysman (2001) encounters a similar issue and resorts to a similar treatment.

We now discuss welfare implications of the equilibrium divestitures. Suppose that the demand system (1) is derived from a representative utility maximization subject to a budget constraint, where the utility function takes the following quasi-linear form

$$u = q_0 + U(q_{11}, ..., q_{1n_1}, q_{21}, ..., q_{2n_2}),$$

the good q_0 is a numeraire, and $U(\cdot)$ is a monotone increasing function. It follows that the consumers' surplus is

$$CS = U(q_{11}, ..., q_{1n_1}, q_{21}, ..., q_{2n_2}) - \sum_{k=1}^{2} \sum_{i=1}^{n_k} p_{ki} q_{ki},$$

and the total surplus is the consumer's surplus plus the firms' total profits, which is simply $TC = U(q_{11}, ..., q_{1n_1}, q_{21}, ..., q_{2n_2})$. From Lemmas 1 and 2 and Proposition 1, we know that, in equilibrium, divestiture of competing conglomerates moves the prices toward but below the profit maximizing levels and in turn raises firms' profits. Consequently, the demand falls, resulting in losses in both consumer and total surplus.

Proposition 2: Suppose that (A1)-(A5) hold. Then the equilibrium divestitures increase the firms' profits, and reduce the consumers' surplus and total surplus.

The implications on the social welfare consequences of divestitures are noteworthy. Conventional wisdom tells us that mergers of firms supplying homogeneous products or imperfect substitutes (with quantity competition) reduce competition, increase the prices of the products, and decrease consumer and total surplus. In our model, divestitures are motivated by product-line complementarities. Lemma 1 and Proposition 1 imply that such divestitures increase prices and lower quantities, and decrease both consumer and total surplus. Therefore, from the perspective of maximizing total surplus, divestitures involving complementary goods or services should be discouraged as much as mergers involving substitutes. In practice, however, competition authorities are often more concerned about mergers but not so much about divestitures. The possibility that competing firms can achieve tacit collusion in pricing through divestitures rather than mergers raises an interesting issue in antitrust law enforcement.

3.4 Coordinated divestitures

In this subsection, we consider the situation in which the firms are able to coordinate their divestiture decisions in the first stage. We show that there exists a pair of division numbers

(m_1, m_2) such that the joint profit maximizing prices (p_M, p_M) can be supported as a non-cooperative equilibrium outcome of the price-setting game given the divisions. This pair of division numbers turns out to be (m^*, m^*) defined in (15).

Indeed, comparing the equilibrium conditions (11) for the pricing game with the necessary condition (8) for the joint profit maximization problem, we find that the joint profit maximizing prices satisfy the Nash equilibrium conditions if $m_1 = m_2 = m^*$. Notice that $m^* = \epsilon_k(p_M, p_M)$, the number of divisions is equal to the own price elasticity of each group at monopoly prices (or 1 plus the cross elasticity). A positive degree of substitution between the two groups of products implies that $m^* > 1$. Since Lemma 1 provides sufficient conditions for the existence and uniqueness of the equilibrium, it follows that the joint profit maximizing prices can be supported as a unique equilibrium outcome of the pricing game if both firms break up into m^* number of independent divisions. In other words, when the firms are able to coordinate their choices of divisions, they can replicate the monopoly profit. This provides an alternative way for the firms to collude.

Proposition 3: Suppose that (A1)-(A3) hold and $\text{Min}\{n_1, n_2\} \geq m^*$. Then competing firms can achieve the maximum joint profit and corresponding profit-maximizing prices in a non-cooperative price-setting game by independent divisions, if competing firms each set the number of divisions $m_1 = m_2 = m^*$.

The logic behind Proposition 3 can be described as follows. Since the joint profit function (7) can be rewritten as the summation of the profit functions (9) over all independent divisions from both groups, the joint profit maximization problem can be equivalently solved by choosing prices $(p_{11}, ..., p_{1m_1})$ and $(p_{21}, ..., p_{2m_2})$. Notice that we can decompose the effect of an increase in price p_{1j} into the following three terms

$$\partial\Pi/\partial p_{1j} = [D(p_1, p_2) + p_{1j}D_1(p_1, p_2)] + \sum_{j' \neq j}^{m_1} p_{1j'}D_1(p_1, p_2) + p_2 D_2(p_2, p_1). \tag{18}$$

The first term in the square brackets represents the own-group effect of the price on the profit of division j in group 1, π_{1j}, which is same as the left-hand side of (10). The second term is the aggregate intra-group effect of price on the profits of the other divisions in the same group, $\pi_{1j'}$ for all j' different from j. It is negative since divisions within the same group supply complementary products. The third term is the aggregate inter-group effect of price on the profits of the divisions in the other group. Since products are imperfect substitutes across groups, the aggregate inter-group effect is positive.

Now, if there exists a number of divisions m such that the negative intra-group effect exactly offsets the positive inter-group effect, then the necessary condition for joint profit maximization is equivalent to the first-order condition (10). In other words, given the number of divisions m, the necessary conditions for joint profit maximization are identical to the first-order conditions for a non-cooperative equilibrium in the pricing game. This can be done by setting the second plus the third term on the left-hand side of (18) equal to zero. Imposing symmetry and substituting p_M for p_1 and p_2, we obtain

$$(m_1 - 1)(p_M/m_1)D_1(p_M, p_M) + p_M D_2(p_M, p_M) = 0. \tag{19}$$

The solution to (19) is $m_1 = m^*$. A similar argument determines $m_2 = m^*$. As a result, the monopoly price p_M satisfies (8) and (11), and hence consists of the unique solution to the joint profit maximization problem and of the equilibrium of the pricing game.

The driving force behind this finding is the commitment power of divestiture combined with the extended product space, which includes substitutes as well as complements. Prior to divestiture, the prices of products in each group are set coordinately. After divestiture, the firm credibly commits not to set prices of the group coordinately, therefore inducing less competition from the rival group. As a result, the prices and profits increase. This implies that price coordination by a group of firms supplying complements does not necessarily benefit the firms and harm consumers. In our model, it is the lack of coordination among the prices of complements that benefits the firms and makes consumers worse-off. In other words, firms have incentives to tie their own hands in order to induce a better (more profitable) response from their rival.[13]

The importance of incorporating both substitutability and complementarity within the same framework can be seen clearly in terms of externalities of each division's pricing decision. There is a *positive externality* between the prices across groups due to substitutability. Divestiture in the first-stage creates a *negative externality* of prices among independent divisions within a group. This negative externality can offset the positive externality. When the degree of divestiture is small, the positive externality dominates. When the degree of divestiture is large, the negative externality outweighs the positive one. At $m = m^*$, all externalities are neutralized and consequently the monopoly outcome is achieved.

We now understand why $\hat{m} \square m^*$. It is driven by the positive externality between the firms' choices of divisions. Given (A5), an increase in m_1 will increase firm 2's profit when both firms choose the same number of divisions. In the presence of such a positive externality, lack of coordination between the firms results in a smaller equilibrium number of divisions than the

[13] A similar commitment effect works when firms use other instruments such as most-favored-customer pricing policies (see Cooper 1986).

coordinated number of divisions.

What determines the size of the optimally coordinated number of firms? Notice that the ratio, $-D_2(p_M, p_M)/D_1(p_M, p_M)$, represents the degree of substitution between the two groups of the products. The following corollary provides a comparative-static result.

Corollary 2: The optimally coordinated number of divisions, m^*, increases with the degree of substitution between the two groups of the products.

In one extreme case where there are no substitute goods, m^* is equal to 1 and each firm should monopolize the supply of the complements and never divide. In the presence of competing substitutes, the firms have incentives to divide. As the relative degree of substitution between the two groups of complements increases, the positive externality increases and hence each firm should split into more divisions to increase the negative externality and mitigate the positive one. In the other extreme, if the number of complements in each group is small or the degree of substitution is large, further divestiture may not be possible. In this case, monopoly profit cannot be replicated and the firms prefer complete divestiture in which each product is supplied by an independent firm.

4 Extensions

In the previous section, we have discussed the effect of product market competition (or product differentiation) on divisionalization and shown that a higher degree of substitution between the two groups of products leads to a greater number of divisions in both groups. There are other factors that determine the coordinated and non-cooperative divisionalization strategies and the scopes of the firms. They include marginal costs, asymmetric demands, imperfect complements, and several groups of complements. In this section, we briefly discuss only two of these factors, imperfect complements and the number of groups of complements. For simplicity, we use linear demand functions.

a) Imperfect Complements

In our basic model, we have considered only perfect complements within the group. Our analysis can be extended to the case of imperfect complements. To illustrate, we consider an example with the following linear demand functions

$$q_{ki} = \alpha - \beta' p_{ki} - \beta \sum_{i' \neq i} p_{1i'} + \gamma p_l,$$

for $k, l = 1, 2,\ l \neq k,\ i = 1, 2, ..., n_k$, where $\alpha > 0$, $\beta^{'} \geq \beta > \gamma > 0$, and $p_k = \sum_{i=1}^{n_k} p_{ki}$. The assumption $\beta^{'} \geq \beta$ implies that product i and any other product within the group are imperfect complements and that the effect of the price p_{ki} on the demand for product i dominates the intra-group effect on the demand for any other product within the group. As before, $\beta > \gamma$ means that intra-group effects outweigh inter-group effects. The demand system is symmetric both within the group and between groups.

Notice that the demand function for product i in the first group can be written as

$$q_{1i} = -(\beta^{'} - \beta)p_{1i} + \alpha - \beta p_1 + \gamma p_2,$$

which consists of two parts, the first depending only on the individual price p_{1i} and the second being the same as the demand function in (2). Clearly, $\beta^{'} - \beta$ does not affect the demand externality arising from an increase in p_{1i}. Remember that the optimally coordinated number of divisions is determined by balancing the negative and positive externalities of the prices. Therefore, it is independent of $\beta^{'} - \beta$ and can be computed as $m^* = \beta/(\beta - \gamma)$. However, $\beta^{'} - \beta$ affects the Nash equilibrium number of divisions, $\hat{m}$. It can be easily calculated that $\hat{m}$ is determined by the following first-order condition

$$\hat{m}^2(\beta^2 - \gamma^2) - \beta^2 + 2(\beta^{'} - \beta)\hat{n}(\hat{m} - 1)/n = 0,$$

where n_1 and n_2 are assumed to be equal, for simplicity, and denoted by n. Notice that $\hat{m}$ decreases with $\beta^{'} - \beta$. When the demand elasticity with respect to own price increases (higher $\beta^{'}$), the incentives for each firm to divide fall because the second-stage equilibrium prices decrease. As a result, the equilibrium number of divisions decreases as $\beta^{'}$ increases.

b) Multiple Groups of Complements

Our analysis can be also extended to the case of many groups of complements, where the products are imperfect substitutes across groups. Let K be the number of groups, $K \geq 2$. The demand function for a complement in group k is linear and presented by

$$q_{ki} = \alpha - \beta p_k + \gamma \sum_{k' \neq k} p_{k'},$$

for $k = 1, ..., K$ and $i = 1, ..., n_k$, where $\alpha > 0$, $\beta > (K-1)\gamma > 0$, n_k is the number of complements in group k, and $p_k = \sum_{i=1}^{n_k} p_{ki}$ is the total prices of the complements in group k, $k = 1, ..., K$. The demand for other products is symmetric with the group and across groups.

We can compute the optimally coordinated number of divisions for each group and the non-cooperative equilibrium number of divisions as follows

$$m^* = \frac{\beta}{\beta - (K-1)\gamma},$$

$$\hat{m} = \frac{\sqrt{(K-2)^2\gamma^2 + 4(\beta+\gamma)(\beta-(K-1)\gamma)} - (K-2)\gamma}{2(\beta+\gamma)(\beta-(K-1)\gamma)}.$$

As before, the optimally coordinated number of divisions and the equilibrium number of divisions increase when the degree of substitution between the groups of the products increases. An interesting comparative-static result is that both m^* and $\hat{m}$ increase with K. As the number of groups of the complements goes up, competition across groups increases and hence the magnitude of the positive externalities among the prices across groups also increase. To mitigate the increased positive externalities, the firms need to create more negative externalities by dividing the firms into more independent divisions. Therefore, the optimally coordinated number of divisions as well as the non-cooperative equilibrium number of divisions increase.

5 Concluding Remarks

In this paper, we have argued that a firm that produces a group of complements can generate higher profits through divestitures when there is a competing firm supplying an imperfect substitute group of complements. By delegating pricing decisions to independent divisions, each firm credibly commits not to set the prices of the complements within the group coordinately and therefore reduces competition between the two firms. In other words, by divesting, the firms create a negative externality among the prices within the group that can offset some of the positive externality between the prices across the firms. Our analysis suggests that industry structure and size of firms are determined by the nature of heterogeneous products and strategic considerations of the firms, in addition to the well-known factors such as agency costs, coordination costs, economies of scale, and economies of scope. To what extent the firms' strategic incentives to divest or merge are significant is an interesting empirical question.

The welfare implications of divestiture in our model are significant. Firms supplying complements have incentives to divest when facing competition. Divestitures of this kind raise prices and reduce consumers' surplus. However, the profit gains are not large enough to compensate consumers' losses. As a result, the total surplus is reduced. This suggests that, when designing competition policies on divestiture, it is important to take into account the coexistence of substitutes and complements[14].

[14] Baye, Crocker and Ju (1996) show that merger to duopoly may enhance social welfare if firms can set up independent divisions and if there are fixed costs involved in setting up the divisions. The primary reason is that merger to duopoly can reduce the number of divisions in equilibrium and the costs associated with setting up

Another implication of our analysis is that divestitures motivated by product-line complementarities can be viewed as a response towards entry. Suppose that initially there is a monopoly that supplies a group of complementary goods or services. Clearly, in our context the monopolist does not have any incentives to divest its operations. When potential entrants enter the market and supply differentiated products, the monopolist has two possible responses. One is to compete directly against the entrants. If the entrants do not make enough profits to cover their entry costs, the entries are deterred. If the products offered by the entrants are differentiated enough from the incumbent's products, and if the entry costs are relatively small, the entries cannot be deterred. In this case, an optimal strategy for the incumbent firm is to divest some of its operations.[15] Our analysis suggests that the optimal number of divisions for the incumbent increases as more entrants enter the market.

It should be noted that our analysis is based upon a number of assumptions. One crucial assumption is that divisions cannot further divide before they choose their prices. This is reasonable in certain situations where parent firms still have major ownership control of the divisions, but do not make management decisions. Franchise contracts can be viewed as an example of such a divisionalization. In other situations, it can be difficult for firms or divisions to make this type of commitment. Divisions may then have incentives to divide further. This raises the issue of what determines a stable industry structure in the presence of both substitutes and complements. Further research along this line is needed.

Finally, we would like to point out that it will be a rewarding but challenging task to empirically test how divestiture of competing firms affect the degree of competition, even though our theory appears consistent with the finding that the accounting performance of the divesting firm improves after the divestiture, and that the announcement effects are positive (John and Ofek, 1995, for example).

divisions. In contrast, we assume costless divestiture and our welfare result is due to the coexistence of substitutes and complements in our model.

[15] In the case of imperfect substitutes, Judd (1985) shows that a multi-product incumbent may have incentives to give up some of its products to the entrants that already entered those markets. The opposite conclusion holds when products are complements instead of substitutes (see Hendricks, Piccione, and Tan, 1997). In our case, the incumbent of complements may want to sell some of its products to independent firms when facing the entrants that supply a competing group of complements.

Appendix

Proof of Lemma 1: The existence and uniqueness of the equilibrium follow from Friedman (1977) and Xivier Vives (1999). The symmetry of the equilibrium prices within the group follows from the necessary conditions (9). In what follows, we show that the equilibrium aggregate prices, $\hat{p}_1$ and $\hat{p}_2$, increase with m_1. Denoting $x_1 = (\hat{p}_1, \hat{p}_2)$ and $x_2 = (\hat{p}_2, \hat{p}_1)$ and applying standard comparative-static techniques to (11), we obtain

$$\partial \hat{p}_1 / \partial m_1 = -D(x_1)[(1+m_2)D_1(x_2) + \hat{p}_2 D_{11}(x_2)]/\Delta,$$

$$\partial \hat{p}_2 / \partial m_1 = D(x_1)[m_2 D_2(x_2) + \hat{p}_2 D_{21}(x_2)]/\Delta,$$

where

$$\Delta = [(1+m_1)D_1(x_1) + \hat{p}_1 D_{11}(x_1)][(1+m_2)D_1(x_2) + \hat{p}_2 D_{11}(x_2)]$$
$$-[m_1 D_2(x_1) + \hat{p}_1 D_{12}(x_1)][m_2 D_2(x_2) + \hat{p}_2 D_{21}(x_2)].$$

¿From (A1), we have $(1+m_i)D_1(x_i) < -m_i D_2(x_i) < 0$, and from (A3), $\hat{p}_i D_{11}(x_i) \Box - | \hat{p}_i D_{12}(x_i) | \Box 0$, for $i = 1, 2$. Thus, $\Delta > 0$. Then, $\partial \hat{p}_1 / \partial m_1 > 0$ follows from (A3) and $\partial \hat{p}_2 / \partial m_1 > 0$ follows from (A4). The proofs of statements (d) and (e) are analogous. Q.E.D.

Proof of Lemma 2: First notice that, by Lemma 1, when $m_1 = m_2 = m$ the equilibrium aggregate price for each group of the complements is symmetric, which we denote by $\hat{p}(m)$. Let $x = (\hat{p}(m), \hat{p}(m))$. Then,

$$\frac{dV(m,m)}{dm} = \frac{d\hat{p}(m)}{dm} \left(D(x) + \hat{p}(m) D_1(x) + \hat{p}(m) D_2(x) \right).$$

¿From the first-order conditions (11), $\hat{p}(m) = -mD(x)/D_1(x)$. It follows that

$$\frac{dV(m,m)}{dm} = mD(x) \frac{d\hat{p}(m)}{dm} \left(\frac{1-m}{m} - \frac{D_2(x)}{D_1(x)} \right).$$

(A5) implies that $-D_2(p,p)/D_1(p,p)$ is non-increasing with p, Lemma 1(e) implies that $\hat{p}(m)$ increases strictly with m, and $(1-m)/m$ strictly increases with m. It follows that $(1-m)/m - D_2(x)/D_1(x)$ strictly decreases with m and is equal to zero at $m = m^*$. Thus, $dV(m,m)/dm$ is positive for $m < m^*$ and negative for $m > m^*$. The claim follows. Q.E.D.

Proof of Proposition 1: Let $\hat{m}$ be the number of divisions in the symmetric equilibrium. We first show that $\hat{m} > 1$. Using (12) and (13), we obtain, for any m_2,

$$\frac{\partial \hat{\Pi}_1(m_1, m_2)}{\partial m_1} = \hat{p}_1 D_1(\hat{p}_1, \hat{p}_2) \frac{m_1 - 1}{m_1} \frac{\partial \hat{p}_1}{\partial m_1} + \hat{p}_1 D_2(\hat{p}_1, \hat{p}_2) \frac{\partial \hat{p}_2}{\partial m_1}.$$

Lemma 1(c) implies that $\partial \hat{p}_2 / \partial m_1 > 0$. The claim follows from the following inequality

$$\frac{\partial \hat{\Pi}_1(m_1, m_2)}{\partial m_1}|_{m_1=1} > 0.$$

Next, we show that $\hat{m} < m^*$. Suppose to the contrary that $\hat{m} \geq m^*$. By Lemma 1, when $m_1 = m_2 = m$ the equilibrium aggregate price for each group of the complements is symmetric, which we denote by $\hat{p}(m)$. Let $x = (\hat{p}(\hat{m}), \hat{p}(\hat{m}))$. It follows from (12) and (13) that, at $m_1 = m_2 = \hat{m}$,

$$\frac{\partial \hat{\Pi}_1(m_1, m_2)}{\partial m_1} = \hat{p}(\hat{m}) \left(\frac{\hat{m} - 1}{\hat{m}} D_1(x) \frac{\partial \hat{p}_1}{\partial m_1} + D_2(x) \frac{\partial \hat{p}_2}{\partial m_1} \right).$$

By the definition of m^*, $\hat{p}(m^*) = p_M$ and

$$\frac{m-1}{m} = -\frac{D_2(\hat{p}(m), \hat{p}(m))}{D_1(\hat{p}(m), \hat{p}(m))},$$

holds at $m = m^*$. The left-hand side of the above equation increases strictly in m, (A5) implies that $-D_2(p,p)/D_1(p,p)$ decreases in p, and Lemma 1(e) implies that $\hat{p}(m)$ increases in m. It follows that $-D_2(\hat{p}(m), \hat{p}(m))/D_1(\hat{p}(m), \hat{p}(m))$ decreases in m. Thus,

$$\frac{\hat{m}-1}{\hat{m}} \geq -\frac{D_2(\hat{p}(\hat{m}), \hat{p}(\hat{m}))}{D_1(\hat{p}(\hat{m}), \hat{p}(\hat{m}))}.$$

Therefore, at $m_1 = m_2 = \hat{m}$,

$$\begin{aligned} \frac{\partial \hat{\Pi}_1(m_1, m_2)}{\partial m_1} &\square \hat{p}(\hat{m}) D_2(x) \left(-\frac{\partial \hat{p}_1}{\partial m_1} + \frac{\partial \hat{p}_2}{\partial m_1} \right) \\ &= \frac{\hat{p}(\hat{m}) D_2 D [D_1 + \hat{m}(D_1 + D_2) + \hat{p}(\hat{m})(D_{11} + D_{12})]}{[(1+\hat{m})D_1 + \hat{p}(\hat{m}) D_{11}]^2 - [\hat{m} D_2 + \hat{p}(m) D_{12}]^2} \\ &< 0 \end{aligned}$$

where the last inequality follows from (A1) and (A3). This contradicts with the necessary condition for $\hat{m}$ to be an equilibrium strategy. The claim follows. Q.E.D.

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p_2

p_1

Figure 1: The Best-Reply Lines in the Price Game (the Linear Demand Functions)

m_2

m_1

Figure 2: The Best-Reply Curves in the Division Game (the Linear Demand Functions)