Resale Price Maintenance in an Oligopoly with Uncertain Demand

Hao Wang[*]

China Center for Economic Research

Peking University

Abstract

This paper considers a model where oligopolistic manufacturers sell to consumers through competitive retailers who face demand uncertainty. The setup is a symmetric duopoly with product differentiation and a perfectly competitive retail market. In the game with unconstrained price competition, there is price dispersion in the retail market. In the game with resale price maintenance (RPM), manufacturers' profits are higher, retail inventories are at least as high, and total surplus is at least as high. The effect of RPM on consumer surplus is ambiguous. RPM also precludes possible coordination failure between the manufacturers.

Key Words: Demand uncertainty, Oligopoly, RPM

JEL Classification Code: L1, L4, M3

[*] Corresponding Address: China Center for Economic Research, Peking University, Beijing, 100871, P.R.China. Telephone: (86)(10)6275-8934. Fax: (86)(10)6275-1474. E-mail: hwang@ccer.edu.cn

I. Introduction

This paper considers a market where oligopolistic manufacturers sell to consumers through retailers who face demand uncertainty. The setup is a symmetric duopoly with product differentiation and perfectly competitive retailers. Retail prices are decided before the demand uncertainty resolves. The games defined in this paper differ from those in previous works by modeling strategic interaction between manufacturers. We characterize the subgame perfect equilibrium of the game with unconstrained price competition. This paper also considers the game with resale price maintenance (RPM) where manufacturers set retail prices. As compared to the game with unconstrained price competition, the effects of RPM on manufacturers' profits, retail inventories, social surplus and consumer surplus are discussed.

Generally, RPM refers to any attempt of manufacturers to control the prices at which their products must be resold by retailers or distributors. RPM through agreements between manufacturer and retailers is per se illegal in the U.S. and many other industrialized countries. However, under the Colgate Doctrine[1] established by the Supreme Court in 1919, manufacturers can legally recommend prices and refuse selling to retailers who discount those prices, as long as they do so unilaterally. For example, "Minimum Advertised Price[2] (MAP)" program is a form of price maintenance, which could yield the same market outcome as RPM. But MAP is broadly used today. RPM has been a contentious topic in the industrial organization literature for decades. One line of justifications is the services theories represented by Telser [1960], Mathewson and Winter [1984], Marvel and McCafferty [1984], and Winter [1993]. The services theories assume that the demand for a manufacturer's product depends on some informational services provided

[1] United States vs. Colgate & Co., 250 U.S. 300 (1919).

[2] Under MAP programs, retailers seeking cooperative advertising funds from a manufacturer have to observe the manufacturer's minimum advertised price in their advertisements. Failure of observing that would result in suspension of cooperative advertising funding offered by the manufacturer

by retailers. RPM enables the retailers to capture the demand generated by the services and thus provides optimal incentive for them to invest in those services. RPM hence improves social efficiency by internalizing the negative externality caused by insufficient retail services. However, new theory is needed to justify the use of RPM for products that do not need extensive sale services. Deneckere, Marvel and Peck [1996, 1997] explain RPM uses from another standpoint. They find that a manufacturer facing uncertain demand has an incentive to support adequate retail inventories by preventing the emergence of discount retailers. They analyze a model with a monopolistic manufacturer selling to competitive retailers in a market where demand is uncertain. They find that with RPM, a monopolistic manufacturer can generate higher wholesale demand and make more profits.

The results reported by Deneckere, Marvel and Peck [1996], [1997] are impressive. However, their analyses are based on a model with a single manufacturer. In the real life, RPM or its equivalent programs are more frequently observed in markets with upstream competition. The strategic interaction among manufacturers significantly complicates the games played in those markets and it is unclear whether the incentives that work in a monopoly are still there in an oligopoly. Some issues, especially some anti-trust issues, cannot be addressed satisfactorily without a theory capable of extending those results on RPM to markets where manufacturers have to compete with one another. The literature contains some studies on vertical restraints in oligopoly. For example, Rey and Stiglitz [1988] argue that exclusive territories reduce inter-manufacturer competition because manufacturers would perceive less elastic demand. Gal-Or [1991] considers a three-stage game where upstream firms choose a form of pricing at the first stage. She finds if duopolistic manufacturers choose to use vertical restraints, they choose franchise fee pricing rather than RPM. RPM eliminates double marginalization, which may not be desirable when competitive pressure is high.

This paper will consider RPM in a duopoly with demand uncertainty, where wholesale prices are chosen before the demand uncertainty resolves. The competition among manufacturers adds another layer of complication to the model considered by Deneckere, Marvel and Peck [1996]. In the monopoly model, the retailers' behaviors are fully determined by the manufacturer's wholesale price. The manufacturer virtually faces a decision-making problem. In an oligopoly, manufacturers have to anticipate not only the strategic interaction among retailers, but also their rivals' strategies before choosing their own strategies. It would be interesting to see how RPM works under such a strategic situation. If demand is uncertain, as the price dispersion found in a monopoly (Prescott [1975], Bryant [1980], Eden [1990]), this paper finds that in a duopoly, the entire set of retail prices of each manufacturer's product is still solely determined by that manufacturer's wholesale price. This finding makes the model tractable. Nevertheless, retail inventories are now determined by both manufacturers' wholesale prices. This paper also finds that RPM encourages retailers to stock greater inventories, even when the manufacturers have to compete with one another. The manufacturers thus enhance the retailers' expected sale and make more profits. Under the assumption that the demand is either high or low, the game without RPM can result in two types of symmetric equilibria: low wholesale price equilibrium and high wholesale price equilibrium. And it is possible for the game to have the two types of equilibria simultaneously. In that case, the high wholesale price equilibrium results in higher manufacturer profits but lower consumer surplus. This can be regarded as a type of coordination between the manufacturers. We find that if RPM is allowed, the possible coordination failure is precluded. Our model also shows that RPM enhances the social welfare by facilitating greater expected sales to consumers. RPM transfers some benefits from consumers to manufacturers, because average retail prices are not lower under RPM. Consumers as a whole can be better off if significantly more consumers are served under RPM. Otherwise they would be worse off with RPM.

This paper will advance as follows. Section II depicts the model and the definitions of the games. Sections III and IV analyze and solve for the subgame perfect equilibria of the games. The welfare issues are addressed in section V. Section VI discusses an anti-trust case about the use of

Minimum Advertised Price (MAP) programs in prerecorded music market. Some summary remarks are given in section VII. Appendix provides proofs for the propositions in Section III.

II. The Model

The market and the games

This is a symmetric model with duopolistic manufacturers and competitive retailers. The two manufacturers are denoted by 0 and 1. They produce horizontally differentiated, non-storable products. The manufacturers have zero production costs[3]. The possible physical specifications of the goods are normalized to interval [0,1]. Assume the specification of manufacturer 0's product is 0 and that of manufacturer 1's product is 1. The consumers have unit demand. It is common knowledge that the consumers' most preferred specifications are evenly distributed along [0, 1]. But each particular consumer's taste is private information. If a consumer's most preferred specification is x, the consumer's utility from consuming manufacturer 0's product is $1-tx$ and the utility from consuming manufacturer 1's product is $1-t(1-x)$. Parameter t represents the degree of product differentiation.

The market demand is uncertain and there are n demand states. The measure of active consumers θ is a random variable and $prob\{\theta=\theta_i\}=\alpha_{i+1}-\alpha_i$, for $i\in\{1,2,...,n\}$, where $0\le\theta_1<\theta_2<...<\theta_n=1$ and $0=\alpha_1<\alpha_2<...<\alpha_n<\alpha_{n+1}=1$. We can define *demand pocket* i as the incremental demand in state i over state $i-1$. Therefore we have n demand pockets denoted as $1,2,...,n$. The measure of demand pocket i is $\theta_i-\theta_{i-1}$ (let $\theta_0=0$). The demand pocket i's probability of entering the market is $1-\alpha_i$ because $prob\{\theta\ge\theta_i\}=1-\alpha_i$. The

[3] The targeted commodities include some high technology devices, information products, and medicines, whose marginal production costs are very low.

consumers' probabilities of entering the market are independent to their preferences over the product specifications.

The retail market is perfectly competitive.[4] The retailers have zero dealing costs. Unsold retail inventories cannot be returned to the manufacturers. Without loss of generality, we adopt the convention that every retailer carries only one manufacturer's product and charges a single price.[5] We define two games depending on whether RPM is allowed:

The niche competition game: *First, the two manufacturers simultaneously announce their wholesale prices.* [6] *Second, the retailers order inventories from the manufacturers and decide the*

---

[4] This paper studies the markets where manufacturers have to sell their products through many independent retailers. In other markets, manufacturers may wish to set up exclusive territories in order to restrict downstream competition, especially when the fixed cost of setting up an outlet is high or the products need some retail services. On the other hand, though our model is borrowed from the "linear-city" of Hotelling [1929], we actually assume the manufacturers' products have physical but not spatial differentiation. Consumers do not incur travel costs in order to reach a retailer. Rather, "distance" in our model represents the difference between the manufacturer's product specification and the consumers' ideal product specification. For these reasons, we assume competitive retailers.

[5] As long as the retailers are perfectly competitive, every single unit of retail inventory with a price tag on it should yield zero expected profit for the retailer who carries that unit. Assuming a retailer can carry both manufacturers' products and charge multiple prices would not affect the price-inventory configuration in equilibrium. Therefore it would not change any result of this paper except requiring more notations. In the literature of exclusive dealing, there are arguments allowing market power and profits on the retail level, for examples, Lin [1990], O'brien and Shaffer [1993], and Babrielsen and Sorgard [1999]. In those models, exclusive dealing "dampens" the upstream competition and increases the manufacturers' profits. In our model, the competitive retailers do not have the market power to effectively play against the manufacturers.

[6] In our model, the manufacturers do not have incentive to use non-linear prices. Non-linear pricing is attractive only if it helps to extract more surplus from the buyers. This happens when the buyers have

retail prices. Third, the demand uncertainty resolves and the consumers come to the market. The consumers enter the market sequentially in random order. Each active consumer chooses a retailer to purchase such that his/her utility is maximized.

The RPM game: *First, the two manufacturers simultaneously announce their wholesale prices and retail prices. Second, the retailers order inventories from the manufacturers. Third, the demand uncertainty resolves and the consumers come to the market. The consumers enter the market sequentially in random order. Each active consumer chooses a retailer to purchase such that his/her utility is maximized. Since a unique price is charged for each manufacturer's product, we assume the ratios of sale to inventory are identical for all retailers selling the same product.*

A note on the model with certain demand

As a benchmark, consider the price competition game played by two vertically integrated manufacturers, where the market demand is certain. Suppose there is a continuum of consumers with measure of 1. Denote the prices as p_0 and p_1. Let $p_0 \leq 1$ and $p_1 \leq 1$ since the consumers' reservation prices are not greater than 1. There are three possible types of symmetric equilibria for this game, depending on the degree to which the manufacturers' products are differentiated.

1. *Typical Oligopoly*: If $t < \frac{2}{3}$, the equilibrium prices are $p_0^* = p_1^* = t$ (See Tirole [1988] for details). In this equilibrium, the manufacturers compete at the margin and all active consumers receive positive consumer surplus. A result that can apply to all games defined in this paper is

---

market power or positive consumer surplus (under linear price). But in our model, competitive retailers make zero profit, which means linear pricing is enough to extract the most from the retailers. On the other hand, though the retailers charge different prices in equilibrium, they are homogenous *ex ante* and all make the same zero profit *ex post*. Hence there is no adverse selection problem that warrants second-degree price discrimination.

Lemma 2.1 *If the retail prices* p_0 *and* p_1 *satisfy* $|p_0 - p_1| \le t$ *, then all consumers are served by the manufacturers if and only if* $p_0 + p_1 \le 2 - t$ *.*

Proof: The consumer that is least likely to purchase the product is the "marginal consumer" who is indifferent between purchasing from either manufacturer. Denoting the marginal consumer's most preferred specification as x, it should satisfy $p_0 + tx = p_1 + t(1-x)$. Hence $x = \frac{p_1 - p_0}{2t} + \frac{1}{2}$. But $0 \le x \le 1$ if and only if $|p_0 - p_1| \le t$. The marginal consumer would purchase the product if and only if $(\frac{p_1 - p_0}{2t} + \frac{1}{2})t + p_0 \le 1$, which is equivalent to $p_0 + p_1 \le 2 - t$. *Q.E.D.*

2. *Restricted oligopoly*: If $\frac{2}{3} \le t \le 1$, the equilibrium prices are $p_0^* = p_1^* = 1 - \frac{t}{2}$. In this case the manufacturers still interact with each other, but the competition is limited. Given $p_0 = 1 - \frac{t}{2}$, manufacturer 1 faces a demand curve that has a kink point at $1 - \frac{t}{2}$: If $p_1 < 1 - \frac{t}{2}$, the manufacturers compete at the margin since $p_0 + p_1 < 2 - t$ (Lemma 2.1). Thus the demand function would be $x_1 = \frac{p_0 - p_1}{2t} + \frac{1}{2}$. If $p_1 > 1 - \frac{t}{2}$, the manufacturers do not compete with each other since $p_0 + p_1 > 2 - t$. Thus the demand function would be $x_1 = \frac{1 - p_1}{t}$. Both manufacturers choose price $1 - \frac{t}{2}$ in equilibrium when $\frac{2}{3} < t \le 1$, and the marginal consumer receives zero surplus.

3. *Monopoly*: If $t>1$, the equilibrium prices are $p_0^* = p_1^* = \frac{1}{2}$. The market is segmented into two monopoly markets.

III. The Niche Competition Game

In this section we will find the subgame perfect Nash equilibrium of the niche competition game. We will solve the retailers' subgame first, then solve the manufacturers' problems. In order to focus on the oligopolistic situations, we assume $0 \le t < \frac{2}{3}$ from now on.

The retailers' subgame

In the niche competition game, the wholesale prices, denoted as w_0 and w_1, are exogenous to the retailers. Lemma 3.1 characterizes the equilibrium of the retailers' subgame. An important characteristic of the equilibrium configuration of the niche competition game derived from Lemma 3.1 is presented in Proposition 3.2.

Lemma 3.1 *In the niche competition game, the retailers' subgame has a unique equilibrium configuration of prices and quantities. In that equilibrium configuration, there is a group of retailers catering to each demand pocket. The retailers catering to demand pocket i charge retail price of $\frac{w_0}{1-\alpha_i}$ or $\frac{w_1}{1-\alpha_i}$, and stock the pre-sale total inventories (denoted as I_0^i and I_1^i) that are exactly enough to serve demand pocket i at those retail prices.* [7]

[7] Inventories I_0^i and I_1^i can be uniquely determined given w_0 and w_1. But explicitly specifying them is very complex for arbitrary w_0 and w_1. Essentially, we find the measure of consumers in demand pocket 1 who can purchase from retailers at prices w_0 and w_1. Next, we find the measure of consumers in demand

Proof: According to the rules of the game and the price configuration depicted in the lemma, for any $i \in \{1,2,...,n\}$, the retailers charging retail price $\frac{w_0}{1-\alpha_i}$ or $\frac{w_1}{1-\alpha_i}$ can sell their inventories with probability of $1-\alpha_i$ (when $\theta \geq \theta_i$). So they earn zero profits. This price configuration is an equilibrium because any deviation by any single retailer would not be profitable: raising its price leads to a loss because of lower probability of selling; lowering its price also leads to a loss because of less revenue from sale.

Now we show that any departure from the configuration stated in the lemma cannot be an equilibrium. *First*, apparently no retailer would charge retail price lower than the wholesale price w_0 (or w_1) in equilibrium. *Second*, if the retail inventory at w_0 (or w_1) is not I_0^1 (or I_1^1), the configuration cannot be an equilibrium because: if the total retail inventory at w_0 (or w_1) is greater than I_0^1 (or I_1^1), the retailers cannot break even because its probability of selling is less than $1-\alpha_1 = 1$; if the total retail inventory at w_0 (or w_1) is less than I_0^1 (or I_1^1), the residual demand can be profitably served by a retailer charging $w_0 + \varepsilon$ (for small ε). *Third*, any retailer charging a price between $\frac{w_0}{1-\alpha_1}$ and $\frac{w_0}{1-\alpha_2}$ (or a price between $\frac{w_1}{1-\alpha_1}$ and $\frac{w_1}{1-\alpha_2}$) cannot break even, because they can only sell their inventories with probability of $1-\alpha_2$ (note that demand pocket 1 is served by retailers charging w_0 and w_1). Iterating this reasoning, we can see that for any $i \in \{2,3,...,n\}$, if the retail inventory at $\frac{w_0}{1-\alpha_i}$ (or $\frac{w_1}{1-\alpha_i}$) is not I_0^i (or I_1^i), the configuration cannot be an equilibrium. And any retailer charging a price between $\frac{w_0}{1-\alpha_i}$ and

pockets 1 and 2 who are not able to purchase at prices w_0 and w_1, but wish to purchase at prices $w_0/(1-\alpha_1)$ and $w_1/(1-\alpha_1)$. Iterating this procedure yields I_0^i and I_1^i. We omit the details.

$\frac{w_0}{1-\alpha_{i+1}}$ (or between $\frac{w_1}{1-\alpha_i}$ and $\frac{w_1}{1-\alpha_{i+1}}$) cannot break even. Thus the configuration stated in the lemma is the unique equilibrium configuration of the retailers' subgame. *Q.E.D.*

Proposition 3.2 *In the niche competition game, the retail prices of a manufacturer's product only depend on the wholesale price of that manufacturer.*

Introducing the competition by manufacturers into the model complicates the retailers' subgame because the retailers now face inter-brand competition as well as intra-brand competition. It can be shown that if the manufacturers were selling through imperfectly competitive retailers, the retail prices of each manufacturer's product would be influenced by both manufacturers' wholesale prices. That is because the retailers can choose among a range of profitable markups over wholesale prices, with the markups depending on the inter-brand competition. However, Proposition 3.2 points out that when the retail market is perfectly competitive, the whole set of retail prices of a manufacturer's product is solely determined by the manufacturer's wholesale price. It is not affected by its rival's pricing, because the retail prices are determined by zero profit conditions, hence determined by the corresponding wholesale price and probabilities of selling. The retail price configuration in this oligopoly game is generated by the intra-brand competition in the same manner as that in a monopoly. But notice that the retailers' inventories depend on both manufacturers' wholesale prices. The inter-brand competition does influence the retailers' inventory decisions.

The manufacturers' problem

Each manufacturer, anticipating the ensuing retailer's subgame, chooses its optimal reacting wholesale price w_0 or w_1 to maximize its profit. The manufacturers' profits are simply their wholesale revenues since the production is costless. Unfortunately, even with the relief brought by Proposition 3.2, solving the manufacturers' problem in a general model is still exceptionally difficult because too many situations need to be discussed. We will use a simplified model, where the demand is either low or high to illustrate the ideas. The measure of active consumers is 0.5 in the low demand and 1 in the high demand. Assume $prob(\theta = 0.5) = q$ and $prob(\theta = 1) = 1 - q$. Hence we have two demand pockets, denoted by 1 and 2. Each has the size of 0.5. Demand pocket 1 and 2 enter the market with probability of 1 and $1 - q$ respectively.

We only consider the symmetric equilibria in order to compare them to the equilibrium of the RPM game, which is always symmetric. The following approach is used to locate the equilibria of the niche competition game. First, assume the symmetric equilibrium wholesale prices are in a certain region so that the manufacturers' profit functions can be identified. Second, solve for the corresponding candidate equilibrium prices. At last, characterize the necessary and sufficient conditions under which the manufacturers cannot profitably deviate from the candidate equilibrium. There are many potential types of equilibrium. In particular, for each demand pocket, competition can be characterized as "typical oligopoly" where the marginal consumer receives surplus; "restricted oligopoly" where the marginal consumer receives no surplus yet the demand pocket is fully served; "monopoly" where the demand pocket is partially served; and the case in which the demand pocket is not served at all. Fortunately, it turns out that only three types of equilibria can actually occur, as shown by Lemma 3.3. Please see the appendix for proof.

Lemma 3.3 *In the niche competition game, the equilibrium can only take three types:*

Type I. $w_0^*, w_1^* \in (0, (1 - \frac{t}{2})(1 - q))$.

Type II. $w_0^* = w_1^* = (1 - \frac{t}{2})(1 - q)$.

Type III. $w_0^*, w_1^* \in (1 - q, 1 - \frac{t}{2})$, *(when* $t < 2q$*).*

Types I, II and III equilibria are characterized in the following Propositions 3.4, 3.5 and 3.6 respectively. In type I equilibrium, the manufacturers compete at the margin in both demand pockets and all active consumers receive positive surplus. In type II equilibrium, both demand pockets are fully served but the competition between the manufacturers is limited in demand pocket 2. It can be regarded as the boundary situation of type I equilibrium. In type III equilibrium, demand pocket 1 is fully served, but demand pocket 2 is not served. The proofs of the three propositions are put in the appendix.

Proposition 3.4 *In the niche competition game, the prices*

$$w_0^* = w_1^* = \frac{2-2q}{2-q}t \tag{1}$$

are the equilibrium wholesale prices of the game if and only if one of the following two conditions is satisfied

(a). $0 < q \le \frac{12-4\sqrt{2}}{7} (\approx 0.906)$ *and* $0 \le t \le \frac{4-2q}{6-q}$,

(b). $\frac{12-4\sqrt{2}}{7} < q < 1$ *and* $0 \le t \le \frac{(2-2q)(2-q)}{4-3q}$.

All active consumers are served in this equilibrium. The corresponding equilibrium profits of the manufacturers are

$$\pi_0^* = \pi_1^* = \frac{1-q}{2-q}t\,. \tag{2}$$

Conditions (a) and (b) show that if parameters t and q are relatively small, it is likely for all active consumers to be served even when demand is high. A small t implies low degree of product differentiation, which means that a manufacturer can easily attract more customers by lowering its price. A small q implies high likelihood for demand pocket 2 to enter the market, which also encourages the manufacturers to serve demand pocket 2. The gray area in Figure 1, called area 1, illustrates the set of parameters (q,t) satisfying condition (a) or (b).

(Figure 1 here)

Proposition 3.5 *In the niche competition game, the prices*

$$w_0^* = w_1^* = (1-\frac{t}{2})(1-q) \tag{3}$$

are the equilibrium wholesale prices of the game if and only if

(c). $\frac{4-2q}{6-q} \le t \le \frac{2-2q}{7-4\sqrt{2}-q}$.

All active consumers are served in this equilibrium. The corresponding equilibrium profits of the manufacturers are

$$\pi_0^* = \pi_1^* = \frac{1}{2}(1-\frac{t}{2})(1-q)\,. \tag{4}$$

The gray area in Figure 2, called area 2, shows the set of parameters (q,t) that satisfy condition (c).

(Figure 2 here)

Proposition 3.6 *In the niche competition game, the prices*

$$w_0^* = w_1^* = t \tag{5}$$

are the equilibrium wholesale prices of the game if and only if one of the following two conditions is satisfied

(d). $0 \le t \le \frac{-1+\sqrt{3}}{2} (\approx 0.366)$ *and* $\frac{1-(3-\sqrt{3})t}{1-t} \le q < 1$,

(e). $\frac{-1+\sqrt{3}}{2} < t < \frac{2}{3}$ *and* $1-\frac{2t^2}{1+2t} \le q < 1$.

Only demand pocket 1 is served in this equilibrium. The corresponding equilibrium profits of the manufacturers are

$$\pi_0^* = \pi_1^* = \frac{t}{4}. \tag{6}$$

Type III equilibrium is likely to occur with relatively great parameter t and q, which imply that the product differentiation is significant and demand pocket 2 is unlikely to enter the market. In order to have demand pocket 2 served, the manufacturers have to offer very low wholesale prices, which may reduce the profits they could earn from demand pocket 1. Under certain parameter settings, the manufacturers would rather give up demand pocket 2 and serve pocket 1 only. From the perspective of the retailers, they will not carry inventories for demand pocket 2 because they would have to sell the products at prices higher than the consumers' reservation prices in order to break even. The retailers' inventory decisions negatively affect the manufacturers' profits. This is somewhat similar to the vertical externality of the service theories. For instance, see Winter [1993]. Type III equilibrium is Pareto inefficient because some consumers are not served, though they have positive evaluations on the products while the production being costless. Figure 3 shows the set of parameters (q,t) satisfy condition (d) or (e), called area 3, and thus supports type III equilibrium.

(Figure 3 here)

In most cases, the game has a unique symmetric equilibrium. Under some parameter settings, both manufacturers may offer low wholesale prices in equilibrium and serve all consumers even when the demand is high. For some other parameters, the manufacturers may both offer high prices and only prepare for the low demand state. The market would stock out if the demand turns out to be high. But under some parameters, it is possible for the two situations to happen simultaneously, which means the game would have multiple equilibria. For example: the parameters $q = 0.83$ and $t = 0.4$ satisfy both conditions (a) and (e). By Proposition 3.4 and 3.6, we have two symmetric equilibria for this game: wholesale prices $w_0^* = w_1^* = 0.116$ with profits

$\pi_0^* = \pi_1^* = 0.058$, and wholesale prices $w_0^* = w_1^* = 0.4$ with profits $\pi_0^* = \pi_1^* = 0.1$. When there are multiple equilibria, a manufacturer would charge a high price if the rival charges a high price, and charge a low price if the rival charges a low price.[8] The manufacturers are better off at the high wholesale price equilibrium where only demand pocket 1 is served. But it is Pareto inefficient because some consumers may not be served. The possibility of multiple equilibria is jointly caused by the pricing externality at the manufacture level and the demand uncertainty.

IV. The RPM Game

The retailers' subgame

In the RPM game, the manufacturers specify both wholesale prices and retail prices. The retailers only decide how much inventory to order. Because the retail market is perfectly competitive, the retailers should make zero economic profits. The following lemma characterizes the equilibrium of the retailers' subgame.

Lemma 4.1 *In the equilibrium of the retailers' subgame in the RPM game, the ratio of a retailer's inventory to its expected sale equals the manufacturer's retail price markup.*

Proof: Without loss of generality, consider manufacturer 0 only. Denote: w_0— wholesale price of manufacturer 0, r_0— retail price of manufacturer 0, d_0— expected sale of a retailer dealing manufacturer 0's product, i_0— inventory of the retailer, π_0^r— expected profit of the

[8] Deviating from either equilibrium by either manufacturer would be unprofitable. For instance, from the high price equilibrium, deviating to a lower price enables a manufacturer to serve demand pocket 2. But this gain is dominated by the profit loss from pocket 1. The issue is rather subtle: starting at the high-price equilibrium, a manufacturer's optimal deviation to a low wholesale price that serves both demand pockets is different from the wholesale prices in the low-price equilibrium.

retailer. Since the retail market is competitive and the retailers are risk-neutral, the expected profit of the retailer in equilibrium is $\pi_0^r = r_0 \cdot d_0 - w_0 \cdot i_0 = 0$, which implies $\frac{i_0}{d_0} = \frac{r_0}{w_0}$. *Q.E.D.*

The manufacturers' problem

The next lemma is about the manufacturers' profits. It is easy to prove if we notice that the retailers earn zero profits and there is no production or dealing cost. We omit the detail.

Lemma 4.2 *Each manufacturer's profit equals the total expected revenue of its retailers.*

The retail prices and retail inventories determine the retailers' revenues, which equal the manufacturers' profits. The manufacturers can first optimally choose the retail prices. Given the retail prices, the manufacturers can stimulate optimal retail inventories by choosing the right wholesale prices. Denote the retail prices of the manufacturers as r_0 and r_1. According to Lemma 4.2, the manufacturers' profits are

$$\pi_0 = \sum_{i=1}^{n} r_0 \left(\frac{r_1 - r_0}{2t} + \frac{1}{2}\right)(1-\alpha_i)(\theta_i - \theta_{i-1}) = r_0 \left(\frac{r_1 - r_0}{2t} + \frac{1}{2}\right) \sum_{i=1}^{n} (1-\alpha_i)(\theta_i - \theta_{i-1})$$

$$\pi_1 = \sum_{i=1}^{n} r_1 \left(\frac{r_0 - r_1}{2t} + \frac{1}{2}\right)(1-\alpha_i)(\theta_i - \theta_{i-1}) = r_1 \left(\frac{r_0 - r_1}{2t} + \frac{1}{2}\right) \sum_{i=1}^{n} (1-\alpha_i)(\theta_i - \theta_{i-1})$$

Notice that $\sum_{i=1}^{n} (1-\alpha_i)(\theta_i - \theta_{i-1})$ is the expected measure of active consumers in the market. The equilibrium of the RPM game can be characterized by the following proposition. We omit the proof because it is essentially the same as that of the vertically intergraded situation.

Proposition 4.4 *There exists a unique Nash equilibrium for the RPM game. The equilibrium retail prices are*

$$r_0^* = r_1^* = t.$$

All active consumers are served in this equilibrium. The corresponding equilibrium profits of the manufacturers are

$$\pi_0^{r*} = \pi_1^{r*} = \frac{t}{2}\sum_{i=1}^{n}(1-\alpha_i)(\theta_i - \theta_{i-1}).$$

Note that the equilibrium retail prices of the RPM game are the same as that of the niche competition game when only demand pocket 1 is served. In a RPM game, it is impossible to have the situation of multiple equilibria. Compared to the high wholesale price equilibrium of the niche competition game, RPM allows the manufacturers to earn even more profits by serving more consumers. Apply Proposition 4.4 on the simplified model defined in Section III, we have

$$\pi_0^{r*} = \pi_1^{r*} = \frac{2-q}{4}t. \tag{7}$$

If we modify the game by letting the manufacturers to decide whether to use RPM, it can be shown that the equilibrium specified in Proposition 4.4 is still an equilibrium of the new game. Starting from the equilibrium above, if manufacturer 1 now stops using RPM and only offers a wholesale price w_1, its profit would be

$$\pi_1 = \frac{1}{2}w_1(\frac{r_0^* - w_1}{2t} + \frac{1}{2}) + \frac{1}{2}w_1(\frac{r_0^* - \frac{w_1}{1-q}}{2t} + \frac{1}{2})$$

$$= \frac{1}{2}w_1(\frac{t - w_1}{2t} + \frac{1}{2}) + \frac{1}{2}w_1(\frac{t - \frac{w_1}{1-q}}{2t} + \frac{1}{2})$$

The first order condition gives the optimal wholesale price $w_1 = \frac{2t(1-q)}{2-q}$. The corresponding profit is $\pi_1 = \frac{1-q}{2-q}t$, which is smaller than its original profit $\pi_1^{r*} = \frac{2-q}{4}t$. Thus the deviation is unprofitable. Similar result can be reached even if we have many demand pockets. Hence

Proposition 4.4 still gives an equilibrium of the new game. This result justifies the modeling that both manufacturers use RPM in the RPM game.

V. Welfare Effects

In this section we study the welfare effects of RPM in the oligopolistic market with demand uncertainty. Consider the effects of RPM on the manufacturers' profits first.

Proposition 5.1 *In the symmetric equilibria of the games, the manufacturers' profits are strictly higher in the RPM game than that in the niche competition game.*

Proof: We have explicitly identified the equilibrium profits for both games. The manufacturers' profits in the equilibrium of the RPM game are $\pi^{RPM} = \frac{2-q}{4}t$ by (7). In the niche competition game, there are three situations. First, in type I equilibrium, the manufacturers' profits are $\pi^{niche} = \frac{1-q}{2-q}t$ by (2). We have $\pi^{RPM} = \frac{2-q}{4}t > \frac{1-q}{2-q}t = \pi^{niche}$ if and only if $q > 0$. Second, in type II equilibrium, the manufacturers' profits are $\pi^{niche} = \frac{1}{2}(1-\frac{t}{2})(1-q)$ by (4). But $\pi^{RPM} = \frac{2-q}{4}t > \frac{1}{2}(1-\frac{t}{2})(1-q) = \pi^{niche}$ if and only if $t > \frac{2-2q}{3-2q}$, which is true if condition (c) is satisfied because $\frac{2-2q}{3-2q} < \frac{4-2q}{6-q}$. Third, in type III equilibrium, the manufacturers' profits are $\pi^{niche} = \frac{t}{4}$ by (6). But $\pi^{RPM} = \frac{2-q}{4}t > \frac{t}{4} = \pi^{niche}$ if and only if $q < 1$. Summing up, the manufacturers' profits in symmetric equilibrium are strictly higher in the RPM game. *Q.E.D.*

With RPM, manufacturers ensure that all active consumers are charged the same optimal prices that maximize their profits. The price distortion in the retail market is avoided. Proposition

5.1 also shows that in an oligopoly, the upstream competition does not take away the advantage of RPM on manufacturers' profits. Our next proposition is on the total inventory of the retailers. Recall that in the niche competition game, we may have an equilibrium that only demand pocket 1 is served and no retail inventory is ordered for demand pocket 2. But in the RPM game, the retailers always stock enough inventories for all potential consumers. We thus immediately have

Proposition 5.2 *In the symmetric equilibria of the games, the total inventory of the retailers in the RPM game is as high as, or strictly higher than that in the niche competition game.*

Since the production is costless, the social welfare from this market (includes the expected consumer surplus and the producer surplus) equals the expected consumer benefits from the consumption, which can be measured by the expected consumption quantity (notice that a demand pocket is either served in full or not served at all in the equilibrium of the niche competition game). We have shown that the expected consumption quantity in the RPM game is not lower than that in the niche competition game: If only demand pocket 1 is served in the niche competition game, the expected consumption quantity is strictly higher in the RPM game. Otherwise they are the same in both games. We therefore have

Proposition 5.3 *In the symmetric equilibria of the games, the social welfare in the RPM game is as high as, or strictly higher than that in the niche competition game.*

Proposition 5.3 is logically related with Proposition 5.2, because the consumption quantity depends on the retail inventories. RPM can improve the social welfare because it encourages the retailers to stock greater inventories and thus facilitates greater expected sale to the consumers. At last we observe the consumer surplus, which is the benefit from consumption net of the consumers' payments, or equivalently, the social welfare net of the manufacturers' profits.

Proposition 5.4 *In the symmetric equilibria of the games, if both demand pockets are served in the niche competition game, the consumer surplus is higher in the niche competition game; If only demand pocket 1 is served in the niche competition game, the consumer surplus is higher in the RPM game.*

Proof: Denote π as the total profit of the manufacturers, τ as the consumer surplus, and ω as the social welfare. Then $\omega = \pi + \tau$, or $\tau = \omega - \pi$. First, if both demand pockets are served in the niche competition game, the social welfare ω is identical in both games. But the total profit of the manufacturers π is lower in the niche competition game (Proposition 5.1). So the consumer surplus τ is higher in the niche competition game. Second, if only demand pocket 1 is served in the niche competition game, the equilibrium retail prices are the same in both games, which are t. So the consumer surplus of demand pocket 1 is the same for both games. But the consumer surplus of demand pocket 2 is positive in the RPM game, but is zero in the niche competition game. So the consumer surplus τ is higher in the RPM game. *Q.E.D.*

Proposition 5.4 shows that RPM favors the manufacturers more than the consumers. The gain of the manufacturers comes not only from the efficiency gain brought by RPM, but also from the consumer surplus. If RPM encourages greater wholesale demand from the retailers, the efficiency gain can make not only the manufacturers but also the consumers better off. Otherwise, the manufacturers can still earn more profits, but the consumers are liable to be worse off.

VI. Application: Prerecorded Music Market in the United States

The Minimum Advertised Price (MAP) programs[9] of the major distributors of prerecorded music in the United States provide an illustration of RPM use in an oligopoly. In the United

[9] The FTC file No. 971 0070 "Five Consent Agreements Concerning the Market for Prerecorded Music in the United States." (http://www.ftc.gov/os/2000/05/index.htm).

States, five distributors, Sony Music Distribution, Universal Music & Video Distribution, BMG Distribution, Warner-Elektra-Atlantic Corporation and EMI Music Distribution account for approximately 85% of the industry's $13.7 billion sales. The Federal Trade Commission (FTC) recently (May 2000) found that the MAP programs adopted by these five companies violated Section V of the Federal Trade Commission Act. Those companies adopted much stricter MAP programs between late 1995 and 1996. The programs prevent the retailers from advertising prices below the distributors' minimum advertised prices by denying the cooperative advertising funds to any retailer that promotes discounted prices, even in advertisements funded solely by the retailers. The advertisements are broadly defined and include even in-store displays. The penalty to violating the MAP provisions is that, failure to stick to the provisions for any particular music title would subject the retailer to a suspension of all cooperative advertising funding offered by the distributor for about 60 to 90 days. This ensures that even the most aggressive retailer would stop advertising prices below the MAP, which makes the MAP programs essentially equivalent to RPM. This market fits our model pretty well:

1. As the FTC found, the sale of music CDs does not require extensive sale services. Thus the services theories cannot explain the use of MAP in this market.

2. Each of the major distributors has substantial market power. Their copyrighted products are well differentiated from each other. Thus this is a typical oligopoly with horizontal product differentiations.

3. The marginal production costs of music CDs are almost zero.

4. Potential music retailers do not face significant entry barriers. It is reasonable to treat the retail market as perfectly competitive.

5. The market values of music CDs are not storable because most music titles are fashion goods. People's valuations on them are significantly lower after the enthusiasm fades.

6. The consumers can readily be modeled as having unit demand.

7. The demand is uncertain: It is difficult to predict the demand toward a certain music title.[10]

Also notice that as Footnote 5 states, our model applies perfectly even if each music retailer carries all distributors' products, as long as the retail market is perfectly competitive. The FTC argues that the distributors can preclude retail price competition through the stricter MAP programs and thus eventually increase their own wholesale prices. This intuition is not necessarily true because it overlooks the strategic interactions among the distributors and retailers in a new market mechanism. Manufacturers can always raise the retail prices by increasing their wholesale prices. The question is why they are interested in controlling the retail prices. Our model shows that it is perfectly possible that the wholesale price is strictly lower under the RPM game.[11] It is not generally true that the MAP leads to higher wholesale prices.

People may be concerned about the fact that consumers have to pay higher prices under the MAP programs. While this is likely to be the case, they may neglect the important effect of the MAP programs on the quantity of sales. The judgment on the MAP programs would be fairer if we notice that MAP may facilitate greater expected sales to consumers and thus result in higher consumer surplus. As music dealer Joan C. Bradley from Northeast One Stop, Inc. argued,

"... They (mass merchants) also discriminate against all music except the top sellers. When less than top sellers stop being sold, future generations will be deprived of new music and older

---

[10] In the music CD industry, the distributors often buy back the unsold copies from the retailers. This violates an assumption of this paper. However, our model still applies as long as the retailers have to incur some notable costs for unsold CDs, for example, costs associated with organizing, displaying and shipping.

[11] If only demand pocket 1 is served in the niche competition game, the retail prices are the same in both the niche competition game and the RPM game. In this case, the wholesale price is strictly low in the RPM game. Notice that more sales occur in the RPM game. A relevant result is by Gal-Or [1991], who finds RPM may lead to lower retail prices in an oligopoly, because it eliminates double marginalization.

generations will be deprived of catalog product (including classical, jazz and their favorite tunes) not carried by the mass merchants and not promoted with loss-leader prices."[12]

The "mass merchants", or discount stores, charge relatively low retail prices for the music titles and cover their costs by selling quickly. Those discount stores cannot profitably deal the catalog products at low prices because the sales of those items are slow and uncertain. Some catalog items have to be sold at very high prices under the niche competition, because the retailers may have to confront the costs associated with unsold copies. But the high prices may suffocate the demand and destroy the business. The retailers under the MAP programs are able to carry the catalog products because the discounters will not undercut them.

The MAP programs are likely to improve the social welfare in the prerecorded music market because they encourage the retailers to stock greater inventories and serve more consumers. To estimate its effect on the consumer surplus, it is important to observe whether those programs can stimulate significantly higher wholesale demand for inventories from the retailers. The MAP programs are beneficial to consumers if considerably greater wholesale demand is observed. When one judges the welfare effects of MAP, it is not adequate to just look at the prices, which could be misleading.

VII. Concluding Remarks

This paper tries to extend the results found by Deneckere, Marvel and Peck [1996] to a market where manufacturers face competition from rivals. One might conjecture that the competition by manufacturers may take away the advantages of RPM, because the competition tends to drive the prices down and thus discourage manufacturers from imposing RPM. But this paper shows that may not be the case. Manufacturers still have incentive to impose RPM even when there is upstream competition. It is interesting to see that RPM sometimes rules out the

[12] Public comments to FTC file No. 971 0070 (http://www.ftc.gov/os/2000/05/index.htm).

possible coordination failure among manufacturers, which makes the market outcome more predictable and favorable from the perspective of the manufacturers. RPM can enhance social welfare by encouraging retailers to stock greater inventories and thus facilitating greater expected sales to consumers. Though the economy as a whole can benefit from it, consumers could be strictly worse off with RPM. Only when the efficiency gain is significant, RPM can make not only manufacturers but also consumers better off.

One might also think that the competition by manufacturers should cause very complicated interactions in the niche competition game, since there are many different retail prices in the market and the interaction among manufacturers and retailers could be exceptionally complex. But we show it could be analyzed tractably. Part of the reason is that with oligopolistic manufacturers and competitive retailers, the entire set of retail prices of a product is still solely determined by its manufacturer's wholesale price, although the retail inventories depend on all manufacturers' wholesale prices. With similar methodology, we may be able to study the effect of RPM on other types of wholesale markets, for example, a market with monopolistically competitive manufacturers.

Notice that most of our works are devoted to solving the niche competition game. The demand uncertainty complicates the unconstrained price competition, and sometimes makes the market outcome inefficient. On the other hand, it is relatively easy to show that the vertically integrated outcome can be achieved with RPM. And the outcome under RPM tends to be more efficient. However, some other vertical relationships, for instance, return policies, quantity forcing, or exclusive dealership, may attain this goal too.

Appendix

Following are the proofs of Lemma 3.3 and Proposition 3.4, 3.5 and 3.6 of Section III about the niche competition game.

If $t < 2q$, the following price ranges need to be considered separately in order to locate the equilibrium of the game. Note each price range is either an open interval or a point.

A. $w_0^*, w_1^* \in (0, (1-\frac{t}{2})(1-q))$. This is akin to the "typical oligopoly".

B. $w_0^* = w_1^* = (1-\frac{t}{2})(1-q)$. This is akin to the "restricted oligopoly".

C. $w_0^*, w_1^* \in ((1-\frac{t}{2})(1-q), 1-q)$. Demand pocket 1 is fully served. Pocket 2 is partially served.

D. $w_0^* = w_1^* = 1-q$. This is the boundary situation of C and E.

E. $w_0^*, w_1^* \in (1-q, 1-\frac{t}{2})$. Demand pocket 1 is fully served and pocket 2 is not served.

F. $w_0^* = w_1^* = 1-\frac{t}{2}$. "Restricted oligopoly" in demand pocket 1. Pocket 2 is not served.

G. $w_0^*, w_1^* \in (1-\frac{t}{2}, 1)$. Demand pocket 1 is partially served and pocket 2 is not served.

If $t \geq 2q$, the price ranges that need to be considered separately are

A', B'. (Same as A or B).

C'. $w_0^*, w_1^* \in ((1-\frac{t}{2})(1-q), 1-\frac{t}{2})$. Demand pocket 1 is fully served. Pocket 2 is partially served.

D'. $w_0^* = w_1^* = 1-\frac{t}{2}$. "Restricted oligopoly" in demand pocket 1. Pocket 2 is partially served.

E'. $w_0^*, w_1^* \in (1-\frac{t}{2}, 1-q)$. Both demand pockets are partially served.

F'. $w_0^* = w_1^* = 1-q$. This is the boundary situation of E' and G'.

G'. $w_0^*, w_1^* \in (1-q, 1)$. Demand pocket 1 is partially served. Pocket 2 is not served.

Fortunately, not all these situations can happen in equilibrium. Lemma 3.3 claims the situations that can occur in equilibrium are A (A'), B (B') and E. Situation A, B and E leads to type I, II and III equilibrium respectively. It is easy to see situations F, G, F' and G' cannot happen: same to the "typical oligopoly" of Section II, when the manufacturers just compete in one demand pocket and $t < \frac{2}{3}$, the equilibrium wholesale price is t and the whole demand pocket 1 is served. Two claims are presented at the end of this appendix. Claim 1 precludes situations C, C', D' and E'. Claim 2 precludes situation D. Lemma 3.3 is reached automatically after all the propositions and claims are proven.

Proof of Proposition 3.4: Suppose the manufacturers' wholesale prices satisfy $w_0, w_1 \in (0, (1-\frac{t}{2})(1-q))$. By Lemma 2.1, demand pocket 2 is fully served since $\frac{w_0}{1-q} + \frac{w_1}{1-q} < 2-t$. So is pocket 1 of course. For manufacturer 1, the demand for its products from demand pocket 1 is $\frac{1}{2}(\frac{w_0 - w_1}{2t} + \frac{1}{2})$ and the demand from pocket 2 is $\frac{1}{2}(\frac{w_0 - w_1}{2t(1-q)} + \frac{1}{2})$. Thus manufacturer 1's profit function is $\pi_1 = \frac{1}{2} w_1 (\frac{w_0 - w_1}{2t} + \frac{1}{2}) + \frac{1}{2} w_1 (\frac{w_0 - w_1}{2t(1-q)} + \frac{1}{2})$. The first order condition is $w_1 = \frac{1}{2} w_0 + \frac{1-q}{2-q} t$. Similarly, we have $w_0 = \frac{1}{2} w_1 + \frac{1-q}{2-q} t$ from manufacturer 0's problem. Solve for the candidate equilibrium from them $w_0^* = w_1^* = \frac{2-2q}{2-q} t$. The manufacturers' profits are $\pi_0^* = \pi_1^* = \frac{1-q}{2-q} t$. A necessary condition for $w_0^* = w_1^* = \frac{2-2q}{2-q} t$ to be the equilibrium can be obtained by observing $w_0^*, w_1^* \in (0, (1-\frac{t}{2})(1-q))$, which leads to

$$t \le \frac{4-2q}{6-q}, \text{ or equivalently } q \le \frac{4-6t}{2-t}. \tag{8}$$

To ensure prices $w_0^* = w_1^* = \frac{2-2q}{2-q}t$ are really the equilibrium of the game, we need to guarantee that neither manufacturer has incentive to deviate them. Without loss of generality, we only examine whether manufacturer 1 can profitably deviate.

First, we see whether manufacturer 1 can profitably deviate to $w_1 \in [(1-\frac{t}{2})(1-q), 1-q]$. We have two cases to consider. 1. $w_0^* + w_1 > 2-t$. The manufacturers do not compete in either demand pocket. The profit function is $\pi_1 = \frac{1}{2}w_1(\frac{1-w_1}{t}) + \frac{1}{2}w_1(\frac{1}{t} - \frac{w_1}{t(1-q)})$. It can be showed that manufacturer 1 cannot do better than that when $w_1 = 2-t-w_0^*$ (by checking the first order derivative). So we enter the second case. 2. $w_0^* + w_1 \le 2-t$. If the manufacturers compete at the margin in demand pocket 2, we have showed that the optimal wholesale price is $w_1^* = \frac{2-2q}{2-q}t$. Now we suppose they compete only in demand pocket 1. Manufacturer 1's customers from demand pocket 2, denoted as x, should satisfy $\frac{w_1}{1-q} + tx = 1$, which implies $x = \frac{1}{t} - \frac{w_1}{t(1-q)}$. So the profit function is $\pi_1 = \frac{1}{2}w_1(\frac{w_0^* - w_1}{2t} + \frac{1}{2}) + \frac{1}{2}w_1(\frac{1}{t} - \frac{w_1}{t(1-q)})$. It can be showed that manufacturer 1 cannot do better than that when $w_1 = (2-t)(1-q) - w_0^*$, which yields a lower profit than $\pi_1^* = \frac{1-q}{2-q}t$. Thus the deviation is not profitable.

Second, we see if manufacturer 1 wishes to deviate to $w_1 > 1-q$ and serve demand pocket 1 only. Again we have two cases: 1. If $w_0^* + w_1 > 2-t$, the manufacturers do not compete even in demand pocket 1. It can be showed that manufacturer 1's profit is always lower than that when $w_1 = 2-t-w_0^*$. So we enter the second case. 2. If $w_0^* + w_1 \le 2-t$ Manufacturer 1's profit function is $\pi_1 = \frac{1}{2} w_1 (\frac{w_0^* - w_1}{2t} + \frac{1}{2})$. The first order condition is $w_1^{**} = \frac{1}{2} w_0^* + \frac{t}{2}$. Substituting $w_0^* = \frac{2-2q}{2-q} t$ into it, we have $w_1^{**} = \frac{4-3q}{4-2q} t \le 2 - t - w_0^*$. If $\frac{4-3q}{4-2q} t \le 1-q$, or equivalently

$$t \le \frac{2(1-q)(2-q)}{4-3q}, \qquad (9)$$

manufacturer 1's optimal wholesale price cannot possibly be greater than $1-q$. If $t > \frac{2(1-q)(2-q)}{4-3q}$, manufacturer 1's profit from the deviation is

$$\pi_1^{**} = \frac{1}{2} w_1^{**} (\frac{w_0^* - w_1^{**}}{2t} + \frac{1}{2}) = \frac{1}{2} \cdot \frac{4-3q}{4-2q} \cdot t \cdot (\frac{\frac{2-2q}{2-q} t - \frac{4-3q}{4-2q} t}{2t} + \frac{1}{2}) = \frac{(4-3q)^2}{16(2-q)^2} t.$$

This is not greater than its original profit if and only if $\frac{(4-3q)^2}{16(2-q)^2} t \le \frac{1-q}{2-q} t$, or equivalently

$$q \le \frac{12 - 4\sqrt{2}}{7} \approx 0.906. \qquad (10)$$

Summing up, prices $w_0^* = w_1^* = \frac{2-2q}{2-q} t$ are the equilibrium wholesale prices of the game if and only if (8) and (9), or, (8) and (10) are satisfied. That is equivalent to

(a). $0 < q \le \frac{12 - 4\sqrt{2}}{7}$ and $0 \le t \le \frac{4-2q}{6-q}$, OR

(b). $\frac{12 - 4\sqrt{2}}{7} < q < 1$ and $0 \le t \le \frac{(2-2q)(2-q)}{4-3q}$. *Q.E.D.*

***Proof of Proposition 3.5*:** Suppose $w_0^* = w_1^* = (1-\frac{t}{2})(1-q)$. At those wholesale prices, the profits of the manufacturers are $\pi_0^* = \pi_1^* = \frac{1}{2}(1-\frac{t}{2})(1-q)$. Now we check what conditions are needed for $w_1^* = (1-\frac{t}{2})(1-q)$ to be optimal for manufacturer 1, given $w_0^* = (1-\frac{t}{2})(1-q)$.

First, consider any deviation price $w_1 < (1-\frac{t}{2})(1-q)$, which implies the manufacturers compete at the margin in demand pocket 2 (Lemma 2.1). Manufacturer 1's profit is $\pi_1 = \frac{1}{2}w_1(\frac{w_0^* - w_1}{2t} + \frac{1}{2}) + \frac{1}{2}w_1(\frac{w_0^* - w_1}{2t(1-q)} + \frac{1}{2})$. The first order condition is $w_1 = \frac{1}{2}w_0^* + \frac{1-q}{2-q}t$. Substituting $w_0^* = (1-\frac{t}{2})(1-q)$ into it, we have $w_1^{**} = \frac{1}{2}(1-\frac{t}{2})(1-q) + \frac{1-q}{2-q}t$. This is less than $(1-\frac{t}{2})(1-q)$ if and only if $t < \frac{4-2q}{6-q}$. Otherwise if

$$t \geq \frac{4-2q}{6-q}, \tag{11}$$

it can be showed that manufacturer 1 cannot do better than that when $w_1 = (1-\frac{t}{2})(1-q)$. Thus the condition for manufacturer 1 not to deviate to $w_1 < (1-\frac{t}{2})(1-q)$ is (11).

Second, consider deviation to $w_1 \in ((1-\frac{t}{2})(1-q), 1-q]$, which implies demand pocket 2 is served in partial. We have two cases: 1. If $w_0^* + w_1 > 2-t$, the manufacturers do not compete even in demand pocket 1. The profit function is $\pi_1 = \frac{1}{2}w_1(\frac{1-w_1}{t}) + \frac{1}{2}w_1(\frac{1}{t} - \frac{w_1}{t(1-q)})$. It can be showed that manufacturer 1 cannot do better than that when $w_1 = 2-t-w_0^*$. So we enter the second case. 2. If $w_0^* + w_1 \leq 2-t$, the manufacturers compete in and only in demand pocket 1.

The profit function is $\pi_1 = \frac{1}{2}w_1(\frac{w_0^* - w_1}{2t} + \frac{1}{2}) + \frac{1}{2}w_1(\frac{1}{t} - \frac{w_1}{t(1-q)})$. It can be showed that manufacturer 1 cannot do better than that when $w_1 = (1-\frac{t}{2})(1-q)$. Hence deviating to $w_1 \in ((1-\frac{t}{2})(1-q), 1-q]$ cannot possibly be an advantage.

Third, consider deviation to the range of $w_1 > 1-q$, which implies manufacturer 1 now serves demand pocket 1 only. Consider two cases: 1. If $w_0^* + w_1 > 2-t$, the manufacturers do not compete even in demand pocket 1. It can be showed that manufacturer 1 cannot do better than that when $w_1 = 2 - t - w_0^*$. So we enter the second case. 2. If $w_0^* + w_1 \le 2-t$, the profit function is $\pi_1 = \frac{1}{2}w_1(\frac{w_0^* - w_1}{2t} + \frac{1}{2})$. The first order condition gives $w_1^{**} = \frac{1}{2}w_0^* + \frac{t}{2}$. Substituting $w_0^* = (1-\frac{t}{2})(1-q)$ into it, we have $w_1^{**} = \frac{1}{2}\cdot(1-\frac{t}{2})(1-q) + \frac{t}{2} < 2 - t - w_0^*$. This is manufacturer 1's optimal price in $w_1 > 1-q$ only if $\frac{1}{2}\cdot(1-\frac{t}{2})(1-q) + \frac{t}{2} > 1-q$, which requires $t > \frac{2-2q}{1+q}$. Substitute w_1^{**} to the profit function, we have $\pi_1^{**} = \frac{(2-2q+t+qt)^2}{64t}$.

This is not better than manufacturer 1's original profits $\pi_1^* = \frac{1}{2}(1-\frac{t}{2})(1-q)$ if and only if $\pi_1^{**} = \frac{(2-2q+t+qt)^2}{64t} \le \frac{1}{2}(1-\frac{t}{2})(1-q)$, which can be simplified to $t \le \frac{2-2q}{7-4\sqrt{2}-q}$.

Therefore manufacturer 1 cannot profitably deviate to $w_1 > 1-q$ if $t \le \frac{2-2q}{1+q}$ or $\frac{2-2q}{1+q} < t \le \frac{2-2q}{7-4\sqrt{2}-q}$, which is equivalent to

$$t \le Max\{\frac{2-2q}{1+q}, \frac{2-2q}{7-4\sqrt{2}-q}\}, \tag{12}$$

Both $\frac{2-2q}{1+q}$ and $\frac{2-2q}{7-4\sqrt{2}-q}$ are greater than $\frac{2}{3}$ when $q < \frac{1}{2}$. Thus (12) is not binding when $q < \frac{1}{2}$. If $q \ge \frac{1}{2}$, we have $\frac{2-2q}{1+q} < \frac{2-2q}{7-4\sqrt{2}-q}$. Thus (12) is equivalent to

$$t \le \frac{2-2q}{7-4\sqrt{2}-q}. \tag{13}$$

Summing up, the conditions for $w_0^* = w_1^* = (1-\frac{t}{2})(1-q)$ to be the equilibrium wholesale prices of the game are (11) and (13), which can be written as

(c). $\frac{4-2q}{6-q} \le t \le \frac{2-2q}{7-4\sqrt{2}-q}$. *Q.E.D.*

Proof of Proposition 3.6: Suppose $w_0, w_1 \in (1-q, 1-\frac{t}{2})$. Only demand pocket 1 is served. The manufacturers' profit functions are $\pi_0 = \frac{1}{2}w_0(\frac{w_1-w_0}{2t}+\frac{1}{2})$ and $\pi_1 = \frac{1}{2}w_1(\frac{w_0-w_1}{2t}+\frac{1}{2})$. It is easy to solve for the candidate equilibrium as $w_0^* = w_1^* = t$. The equilibrium profits are $\pi_0^* = \pi_1^* = \frac{t}{4}$. Note a necessary condition for $w_0^* = w_1^* = t$ to be a type III equilibrium is $1-q < t < 2q$.

When $t < \frac{2}{3}$ and $1-q < t < 2q$, the manufacturers cannot profitably deviate to $w_1 \ge 1-\frac{t}{2}$, or $w_1 = 1-q$. The proofs are the same as that with vertical integrated manufacturers. Now we check if one of the manufacturers, say 1, has incentive to deviate to $w_1 < 1-q < t$ and serve both demand pockets. Given $w_0^* = t$, manufacturer 1 faces demand of $(\frac{t-w_1}{2t}+\frac{1}{2})$ from demand pocket 1, and demand from pocket 2, denoted as x, satisfying $\frac{w_1}{1-q}+tx=1$ if $x<1$, and $x=1$ otherwise, i.e., $x=\begin{cases}\frac{1}{t}-\frac{w_1}{t(1-q)} & if \quad w_1\geq(1-q)(1-t)\\ 1 & if \quad w_1<(1-q)(1-t)\end{cases}$.

If $w_1\geq(1-q)(1-t)$, the profit function is $\pi_1=\frac{1}{2}w_1(\frac{t-w_1}{2t}+\frac{1}{2})+\frac{1}{2}w_1(\frac{1}{t}-\frac{w_1}{t(1-q)})$. The first order condition is $w_1=\frac{(1-q)(t+1)}{3-q}$. Note $\frac{(1-q)(t+1)}{3-q}<1-q$ constantly holds. Denote w_1^{**} as manufacturer 1's optimal price below $1-q$. We have two possible cases:

(1). If $\frac{(1-q)(t+1)}{3-q}\geq(1-q)(1-t)$, or equivalently $q\geq\frac{2-4t}{1-t}$, the optimal wholesale price is $w_1^{**}=\frac{(1-q)(t+1)}{3-q}$. Substituting it to the profit function, we have $\pi_1^{**}=\frac{(1-q)(1+t)^2}{4t(3-q)}$. This profit is not greater than its original profit $\frac{t}{4}$ if and only if $q\geq1-\frac{2t^2}{1+2t}$. Thus $w_0^*=w_1^*=t$ gives the equilibrium of the game if $q\geq\frac{2-4t}{1-t}$ and $q\geq1-\frac{2t^2}{1+2t}$, which is equivalent to

$$q\geq\begin{cases}1-\frac{2t^2}{1+2t}, & if \quad t\in(\frac{-1+\sqrt{3}}{2},\frac{2}{3}]\\ \frac{2-4t}{1-t}, & if \quad t\in(0,\frac{-1+\sqrt{3}}{2}]\end{cases}. \tag{14}$$

(2). If $\frac{(1-q)(t+1)}{3-q}<(1-q)(1-t)$, or equivalently, $q<\frac{2-4t}{1-t}$, we would have $w_1^{**}=(1-q)(1-t)$. The manufacturer's profit with the deviation would be

$$\pi_1^{**}=\frac{1}{2}w_1^{**}(\frac{t-w_1^{**}}{2t}+\frac{1}{2})+\frac{1}{2}w_1^{**}=\frac{1}{4t}(1-q)(1-t)(5t-qt+q-1).$$

This is not greater than the original profit $\frac{t}{4}$ if and only if $q \le \frac{1-(3+\sqrt{3})t}{1-t}$ or $q \ge \frac{1-(3-\sqrt{3})t}{1-t}$. But $q \le \frac{1-(3+\sqrt{3})t}{1-t}$ conflicts with the condition $t > 1-q$. Thus for $w_0^* = w_1^* = t$ to be an equilibrium, we only need $q < \frac{2-4t}{1-t}$ and $q \ge \frac{1-(3-\sqrt{3})t}{1-t}$, which imply

$$\frac{1-(3-\sqrt{3})t}{1-t} \le q \le \frac{2-4t}{1-t}, \quad \text{and} \quad t \le \frac{-1+\sqrt{3}}{2}. \tag{15}$$

The condition ((14) or (15)) can be summarized as

(d). $0 \le t \le \frac{-1+\sqrt{3}}{2}$ and $\frac{1-(3-\sqrt{3})t}{1-t} \le q < 1$, OR

(e). $\frac{-1+\sqrt{3}}{2} < t \le \frac{2}{3}$ and $1 - \frac{2t^2}{1+2t} \le q < 1$,

One can check that condition $1-q < t < 2q$ is satisfied in (d) and (e). Thus they are the sufficient and necessary conditions for $w_0^* = w_1^* = t$ to be the equilibrium prices of the game. *Q.E.D.*

Claim 1: *In the niche competition game, the equilibrium wholesale prices cannot possibly satisfy* $w_0^*, w_1^* \in ((1-\frac{t}{2})(1-q), 1-q)$.

Proof: Suppose wholesale prices $w_0, w_1 \in ((1-\frac{t}{2})(1-q), 1-q)$. Consider three cases. 1. If $w_0 + w_1 \le 2-t$, the manufacturers compete in and only in demand pocket 1. Manufacturer 1's profit function is $\pi_1 = \frac{1}{2} w_1 (\frac{w_0 - w_1}{2t} + \frac{1}{2}) + \frac{1}{2} w_1 (\frac{1}{t} - \frac{w_1}{t(1-q)})$. It can be showed that $\frac{\partial \pi_1}{\partial w_1} < 0$ whenever $w_0 + w_1 \le 2-t$ and $w_0, w_1 \in ((1-\frac{t}{2})(1-q), 1-q)$. So this case cannot contain an equilibrium. 2. If $w_0 + w_1 > 2-t$, the manufacturers do not compete even in demand pocket 1.

Manufacturer 1's profit function is $\pi_1 = \frac{1}{2} w_1 (\frac{1-w_1}{t}) + \frac{1}{2} w_1 (\frac{1}{t} - \frac{w_1}{t(1-q)})$. The first order condition is $w_1^* = \frac{1-q}{2-q}$. Similarly, we have $w_0^* = \frac{1-q}{2-q}$. But $w_0^* + w_1^* > 2-t$ if and only if $t > \frac{2}{2-q} > 1$, which is impossible. Therefore it is impossible to have equilibrium prices $w_0^*, w_1^* \in ((1-\frac{t}{2})(1-q), 1-q)$. *Q.E.D.*

Claim 2: *The equilibrium of the niche competition game cannot be* $w_0^* = w_1^* = 1-q$.

Proof: At wholesale prices $w_0^* = w_1^* = 1-q$, only demand pocket 1 is served. The equilibrium wholesale prices would be t if only demand pocket 1 is served (Proposition 3.6). Thus $w_0^* = w_1^* = 1-q$ is not an equilibrium when $t \neq 1-q$. If $t = 1-q$, it can be showed that lowering the price from $1-q$ and thus serving demand pocket 2 enables a manufacturer to earn more profit (the opportunity to serve demand pocket 2 destroys the first order conditions at t). Hence $w_0^* = w_1^* = 1-q$ cannot be an equilibrium of the game. *Q.E.D.*

Acknowledgement: I am grateful to Professor James Peck for his help in writing this paper. I am in debt to Professor Howard Marvel and an anonymous referee, whose suggestions have greatly improved this paper. I also thank Professor Andrew Ching, Dan Levin, Justin Lin, and participants of the Midwest Economic Theory meeting and seminar at Peking University for helpful comments. All errors are mine of course.

References

Anthony, S.F., 1998, 13th Annual Advanced ALI-ABA Course of Study Vertical Issues in Federal Antitrust Law.

Bryant, J., 1980, Competitive Equilibrium with Price Setting Firms and Stochastic Demand, International Economic Review 21, 619-626.

Butz, D.A., 1997, Vertical Price Controls with Uncertain Demand, Journal of Law & Economics 40, 433-459.

Carlton, D., 1986, The Rigidity of Prices, American Economic Review 76, 637-658.

Dana, J.D. Jr., 1999, Equilibrium Price Dispersion under Demand Uncertainty: The Roles of Costly Capacity and Market Structure, Rand Journal of Economics 30, 632-660.

Deneckere, R., H.P. Marvel and J. Peck, 1996, Demand Uncertainty, Inventories, and Resale Price Maintenance, Quarterly Journal of Economics 111, 885-913.

Deneckere, R., H.P. Marvel and J. Peck, 1997, Demand Uncertainty and Price Maintenance: Markdowns as Destructive Competition, American Economic Review 87, 619-641.

Eden, B., 1990, Marginal Cost Pricing When Spot Markets Are Complete, Journal of Political Economy 98, 1293-1306.

Gabrielsen, T.S. and L. Sorgard, 1999, Exclusive versus Common Dealership, Southern Economic Journal 66, 353-366.

Gal-Or, E., 1991, Duopolistic Vertical Restraints, European Economic Review 35, 1237-1253.

Hotelling, H., 1929, Stability in Competition, Economic Journal 30, 41-57.

Lin, Y.J., 1990, The Dampening-Of-Competition Effect of Exclusive Dealing, Journal of Industrial Economics 39, 209-223.

Marvel, H. and S. McCafferty, 1984, Resale price maintenance and quality certification, Rand Journal of Economics 15, 346-359.

Mathewson, G.F. and A. Winter, 1984, An Economic Theory of Vertical Restraints, Rand Journal of Economics 15, 27-38.

O'Brien, D.P. and G. Shaffer, 1993, On the Dampening-of-Competition Effect of Exclusive Dealing, Journal of Industrial Economics 41, 215-221.

Prescott, E.C., 1975, Efficiency of the Natural Rate, Journal of Political Economy, 83, 1229-1236.

Rey, P. and J. Stiglitz, 1988, Vertical Restraints and Producers' Competition, European Economic Review 32, 561-568.

Spengler, J., 1950, Vertical Integration and Antitrust Policy, Journal of Political Economy 58, 347-352.

Telser, L.G., 1960, Why Should Manufacturers Want Fair Trade? Journal of Law & Economics 3, 86-105.

Tirole, J., 1988, The Theory of Industrial Organization, Massachusetts Institute of Technology, 277-287.

Winter, R.A., 1993, Vertical Control and Price versus Nonprice Competition. Quarterly Journal of Economics 108, 61-76.

Figures

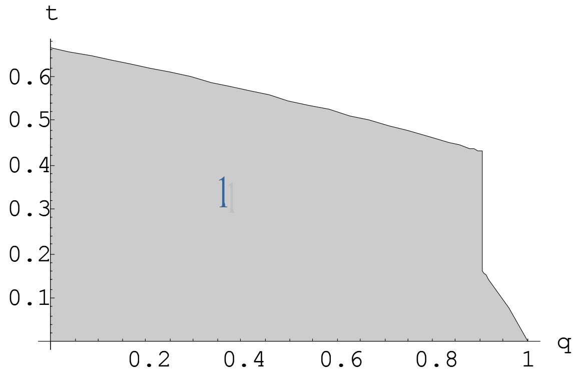

Figure 1: Parameters supporting type I equilibrium

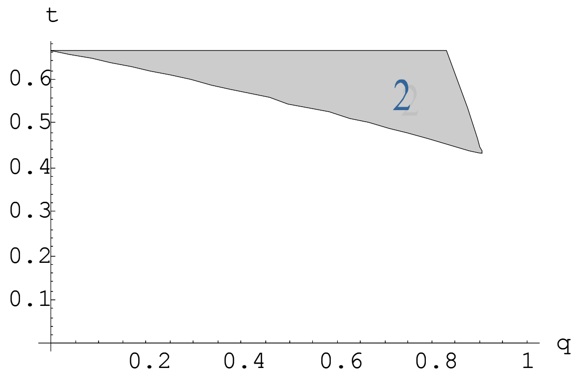

Figure 2: Parameters supporting type II equilibrium

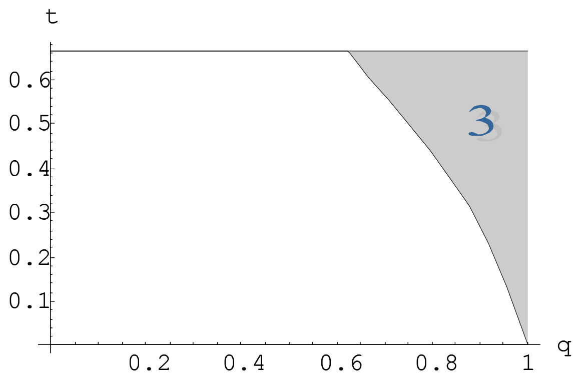

Figure 3: Parameters supporting type III equilibrium