Patent Litigation when Innovation is Cumulative*

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Abstract

This paper studies the effect of litigation as a way to enforce patents when firms hold private information. Patent protection granted by courts affect the entry, settlement and litigation decisions of future innovations. The model is broadly consistent with recent empirical evidence. We show that higher protection might be detrimental to the patentholder since it reduces entry of infringers that would otherwise license the patent. We argue that this is more likely to be the case for large improvements or large litigation costs. Finally, we compare the effects of Preemptive Injunctive Relief on innovation and litigation.

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1 Introduction

Patents are imperfect assignments of property rights on innovation. The uncertainty surrounding the innovation process usually does not allow a precise specification of the range of innovations protected, provoking extensive infringement of these rights. These unavoidable disputes have made of litigation an integral part of the patent system, with important consequences for the incentives to innovate and patent.

The costs of patent enforcement are substantial. In the U.S., from October 1996 to September 1997, 1,530 lawsuits involving patents were reported, although the number of disputes is much higher.[1] The associated legal costs can represent as much as 25% of firms' basic R&D expenses. For this reason recent debates have emphasized the observation that the supervision of patent applications is not very rigorous, and as a result, litigation is mainly used to obtain a judicial definition of the boundaries of the protection that firms obtain.

An important dimension of a patent is its *breadth* or *scope*, understood as the range of competing products and processes that are covered by the patent. Although breadth is considered an essential component, the ambiguity of the concept has not allowed a general formulation. Most of the literature assumes that the infringement of a patent's breadth is evident.[2] Such a setup makes the existence of litigation irrelevant. In reality, the boundaries of the protection associated with a patent are rather blurry, and the process through which courts assess whether an invention infringes an existing

[1] This data was published by the U.S. Patent and Trademark Office (1997). It does not take into account patent disputes that are settled before filing the suit. Lerner(1995) estimates using a sample of 530 patents awarded to firms in the biotechnology sector in Massachusetts that 6% of them were involved in a litigation process.

[2] Two remarkable exceptions are Waterson (1990) and Aoki and Hu (1999).

patent or not is subject to uncertainty. The existence of courts and their rulings can be interpreted as a way to aggregate the relevant features of a patent. In other words, the breadth of a patent is endogenously determined by the "litigation technology."

When innovation is cumulative, Green and Scotchmer (1995), Chang (1995) and O'Donoghue et al (1998) show that the optimal patent must provide claims over future research. Innovations represent permanent improvements over the stock of knowledge, but firms can only partially appropriate the value of the improvement through the sale of their products, since future producers will compete away the profits. By protecting the patentees against future innovation, the life of their monopoly power is extended. One of main concerns of those papers is that in general more protection benefits the patentholder, but it reduces the incentives for future research to be undertaken. Therefore, the design of the optimal patent must counterbalance these effects.

These papers usually interpret patent breadth as the minimum size of the following invention that will be allowed and consider it a policy variable. One of the contributions of this paper is to show that the way courts enforce patents defines endogenously not only the decisions to enter of future inventors but also the licensing and litigation choices. In particular, the real protection that a patent grants to an innovator depends on the quality of the invention, and it is for bigger inventions that more patent breadth can reduce profits.

Previous models of patent litigation, such as Meurer (1989) and Aoki and Hu (1999) deal with the case of a pure imitator. However, when the protection is related to the use that future innovators will make of the patented research, the definition of breadth becomes particularly difficult. While we do not attempt to provide a formal definition of patent breadth, we relate it to the probability that the patentholder succeeds in court. Following the legal arguments presented by Merges and Nelson (1990), this probability also depends on the size of future improvements. Consequently, patents provide less chances in court against better infringers, reducing the protection. With this caveat, we argue that patent breadth is related to the size of the future innovator.

We assume that firms have private information about the quality of their inventions and courts might make mistakes when judging the conformity of an innovation with the existing patents. For this reason some patent disputes are settled in court.

The legal procedure helps reveal some of the private information. As a result, there is some litigation despite it is costly and firms behave optimally.[3] Moreover, the cost of litigation deters entry of some competitors that otherwise would try to innovate, while in other situations it gives incentives to reach a settlement between the patentholder and the potential infringers.

We also show that there is an ambiguous relationship between our interpretation of patent protection and the probability of a patent being litigated, which seems to conform with empirical evidence provided by Lanjouw and Schankerman (1997) and Lerner (1994).

In our framework, increasing the protection of the patentee does not necessarily imply an increase in the profits that he can obtain. The reason is that an important share of the revenue of the patentholder originates from future research. More protection implies that the patentee will have more chances in court, and so he can demand higher licensing fees. On the other hand, with more protection fewer firms are willing to build on that patent, reducing the licensing proceeds. When protection is small, the first effect dominates, but as protection increases and the number of potential licensees decreases, the second effect becomes dominant.

This second effect is particularly important for innovators with a more valuable

[3] Lerner (1995) estimates the direct legal costs to be almost \$1 billion every year.

patent. The reason is that the value of the patent is related to the compensation that the infringer will be forced to pay if courts rule against her, and therefore, it becomes an additional barrier to entry, contributing to the real protection of the patentee. This *excessive* protection discourages more small improvements that would benefit the patentholder through licensing. An interesting policy implication of this setup is that valuable inventions should obtain weaker protection, allowing the patentholder to commit to license the patent to future entrants.

It is important to emphasize that Green and Scotchmer (1995) also show that protection might hurt the patentholder when the negotiation with a future innovator is conducted before the investment occurs. The result in this paper does not rely on this assumption, but rather on the existence of patent litigation. We also show that this effect can occur when legal costs are large, since they are in fact a barrier to entry.

Finally, we study the effects of a Preliminary Injunctive Relief. This is a legal motion that prevents the infringer to produce while the dispute is decided in court. One of the implications of this motion is the change in the decisions to litigate. We show that in this case patent protection is independent of the size of the original invention since the infringer is not required to compensate the patentholder for the foregone profits. Compared to the previous case, this legal motion seems to benefit relatively more innovators with more valuable patents, as opposed to the original intention of the law.

The model is presented in Sections 2 and 3. Section 4 compares the results with a setup without litigation. Section 5 considers the case of Preemptive Injunctive Relief and section 6 concludes. All the results are proven in the Appendix.

2 The Model

The model consists of a consumer and two firms producing in the same market. The products are ordered in a quality ladder. Initially, the first firm, the *patentee* (or *patentholder*) holds a patent for a good of quality v, which is common knowledge to both the consumer and the other producer.

A second firm, denoted as the *infringer*, obtains an idea to improve the good by an amount Δ. Although obtaining this idea is free, the firm cannot collect profits unless a quantity c is invested to transform the idea into an operating invention.[4] The size of the improvement Δ is private information that the infringer holds. The patentee knows only that Δ is drawn from the distribution $\Phi(\Delta) = 1 - e^{-\Delta}$ with support in $\Re_+$. The corresponding density function will be denoted as $\phi(\Delta) = e^{-\Delta}$.

A unique consumer buys at most one unit of the good. The utility of consuming a good of quality v at a price p is $u(v) = v - p$. Therefore, if we denote p_p the price set by the patentholder and p_i the price of the infringer, the utility obtained will be $u(v) = v - p_p$ and $u(v + \Delta) = v + \Delta - p_i$ respectively. For simplicity we assume that marginal cost of production is 0 so that the firm that sells obtains profits $\pi_n = p_n$, where $n = p, i$. The other firm obtains profits equal to 0.

Firms are price competitors, so that the consumer buys from the patentee if

$$v - p_p \geq \max\{v + \Delta - p_i, 0\},$$

since the consumer can always guarantee a utility of 0 by not buying. As a consequence, if only the *patentee* produces, monopoly profits become $\pi_p^M(v) = v$, while if only the infringer does $\pi_i^M(v, \Delta) = v + \Delta$. Finally, if both firms compete, duopoly profits are

[4] These assumptions, as well as the structure of the product market, are similar to those used by O'Donoghue, et al. (1998).

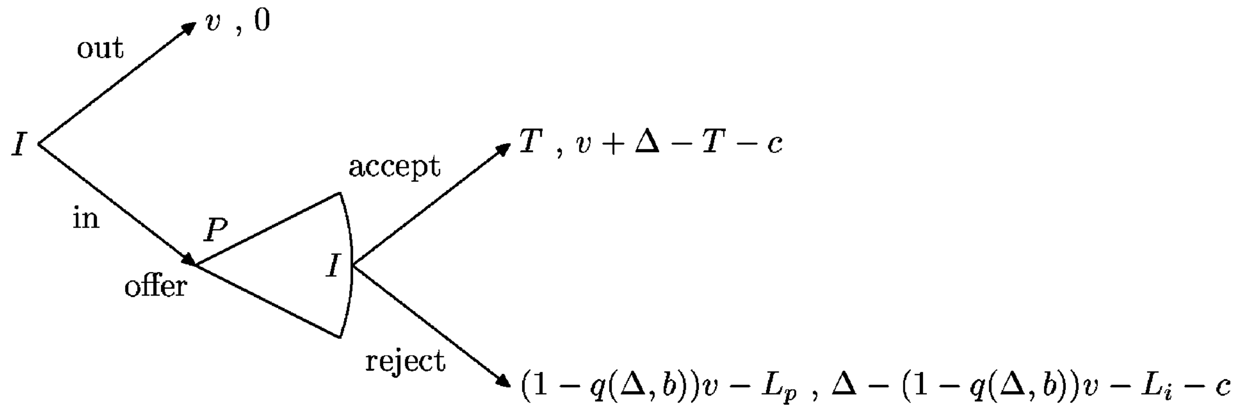

Figure 1: Structure of the game

$\pi_p^D(v, \Delta) = 0$ and $\pi_i^D(v, \Delta) = \Delta$, since the infringer will always undercut the price of the patentholder.[5]

The patent system is modelled in the following way: The patent is granted to the first inventor. If the potential competitor produces an innovation, the patentee can try to reach a settlement or litigate the infringement. As we will see later, the result of the litigation process will depend on the difference, Δ, between the quality of the new product and that of the patented one, and on how favorable to the patentee are the statutes and their interpretation by courts.

The structure and timing of the game are shown in *Figure 1.* First, the infringer needs to decide whether or not to turn the idea into an invention by incurring the irreversible cost c. If there is no entry the patentee will be the only firm to produce, obtaining profits of v.

Upon infringer's entry, the patentee may try to negotiate a settlement with her.

[5]The model can be easily reinterpreted as incremental cost reducing innovations. In that case, the good is homogeneous, but the patentee has a production cost of c_p, while the infringer reduces it to $c_i = c_p - \Delta$. All the results of the paper follow under this specification.

The fact that Δ is private information restricts the kind of settlement schemes that the patentee might implement. We assume that this negotiation process will take the form of a take-it-or-leave-it offer that the patentholder makes. A licensing fee T is offered to the infringer in order to avoid litigation.[6] In any event, if the competitor is allowed to use the invention, the patentholder will be better off by not producing, since competition lowers the profits of the licensee and consequently the value of a license.[7]

Therefore, profits if the offer is accepted are

$$\pi_i^s(\Delta, v, T) = \Delta + v - T - c, \tag{1}$$

$$\pi_p^s(\Delta, v, T) = T \tag{2}$$

for the infringer and the patentholder respectively.

We assume that the patentholder can commit to litigate if the settlement offer is rejected. The implications of this assumption are studied at the end of this section. Thus, if the settlement offer is rejected both firms will produce while the legal process is resolved in court. As a result of Bertrand competition, profits for the patentholder will be 0 while the infringer will obtain profits of Δ.

[6] Llobet (2001), using standard mechanism design arguments, shows that a flat fee is indeed the optimal contract that a patentholder can establish with the potential infringer, among the combinations of fees and positive royalties on sales.

However, if the patentholder were not competing in the same market as the infringer, royalties could play a role in screening different innovators.

[7] In this model monopoly profits do not cause any welfare loss. For this reason, collusive agreements will always be optimal from a social point of view, since they provide more incentives for research to be undertaken. The results of the paper are not affected as long as there is a minimal collusion. See Green and Scotchmer (1995) and Chang (1995) for an extensive treatment of the relationship between antitrust policy and innovation.

During the legal process, courts observe a signal of the true quality of the alternative invention.[8] This observation allows them to decide with a certain probability q that the new invention is not infringing the original patent. This probability is assumed to depend on the quality of the alternative invention, Δ and a parameter $b \geq 0$ related to how favorable to the patentee are courts in their rulings. Therefore, $q(\Delta, b)$ is increasing in Δ and decreasing in b.

The parameter b can be understood as the extent to which the *Doctrine of Equivalents* or the *Reverse Doctrine of Equivalents* are applied.[9] The different application of these principles is already used in practice: pioneer inventions enjoy a more favorable interpretation for the patentee of these doctrines than subsequent improvements.

Another dimension in which courts affect the implementation of patent breadth is a consequence of the uncertainty of future innovation. At the time of application it is difficult to establish the range of innovations covered. As Merger and Nelson (1990) write:

> "it is difficult to resolve issues like these [future research] when a patent is filed; at that point, no one knows what future developments will follow or how difficult it will be to achieve them. Thus there is an argument for granting a broad set of claims for pioneering inventions. Since the inventor

[8]The model implicitly assumes that although consumers observe Δ, courts can only obtain a noisy signal of this value. Alternatively, it is enough to assume that consumers are risk neutral with respect to the quality of the good they consume.

[9]The Doctrine of Equivalents states that "if two devices do the same work in substantially the same way, and accomplish the same result, they are the same, even though they differ in name, form and shape" (US Supreme Court, *Graver Tank & Mfg. Co v Linde Air Prods.*, 1950).

The Reverse Doctrine of Equivalents considers that innovations that are substantially different do not infringe the patent in which they are based.

> may have enabled a broad new range of applications, courts reason, it is unfair to limit her to the precise embodiment through which she discovered the broader principle claimed."

Narrowing these claims is left to courts. For example, Merges and Nelson (1990) mention the Selden Patent on the automobile that courts later narrowed considerably. In the same direction operates the *Written description requirement* and the possibility to amend claims to the application when a new product is introduced. However, if the inventor re-files the specification, it cannot claim what the first filing enabled but failed to describe.

The Doctrine of Equivalents also provides a clear interpretation of the effect of Δ on the probability $q(\Delta, b)$. Again, according to Merges and Nelson (1990),

> "the more significant the technological advance represented by the allegedly infringing device, the less willing the courts should be to find it an equivalent of the patentee's device".

Although most of the results are true for more general functions $q(\Delta, b)$, in order to provide more intuition we choose a logistic specification of the following form

$$q(\Delta, b) = \frac{1}{1 + b\exp(-\Delta)}.$$

Notice that $q(\Delta, 0) = 1$, and therefore $b = 0$ is equivalent to not having a patent, while $\lim_{b\to\infty} q(\Delta, b) = 0$ so that for b sufficiently large the firm is awarded almost infinite protection.

The *Litigation Technology* involves considerable legal expenses. These litigation costs are denoted as L_p and L_i for the patentee and the infringer respectively. As studied in Llobet (2001), the model can easily accommodate the fact that courts allocate these legal costs, imposing compensations from the loser to the winner.

If the infringer loses in the procedure, which occurs with probability $1-q(\Delta, b)$, she will have to compensate the patentee for the foregone profits, $\pi_p^M(v) = v$. Otherwise, she can keep the earnings from production. In any case, litigation costs L_i will be paid. Therefore, the expected return for the infringer when going to court is,

$$\pi_i^l(\Delta, v, b) = \Delta - (1 - q(\Delta, b))v - L_i - c, \tag{3}$$

where these profits are increasing in Δ and decreasing in b.

In the legal process, the patentee will succeed with probability $1 - q(\Delta, b)$ resulting in a compensation for the lost profits from production, v, net of litigation costs, L_p, or

$$\pi_p^l(\Delta, v, b) = (1 - q(\Delta, b))\, v - L_p. \tag{4}$$

We will denote σ_i the strategy played by the infringer. It consists of two elements: the decision to enter and the acceptance or rejection of the offer. An infringer will accept the settlement if $\pi_i^s(\Delta, v, T) \geq \pi_i^l(\Delta, v, b)$. Otherwise, she will choose to fight the patentee in court. In a previous stage, if the infringer expects to be offered a licensing fee T upon entry, she will decide to pay the development cost c if the expected revenue after either licensing or litigation is bigger than those costs,[10] or in other words, if

$$\max\left\{\pi_i^s(\Delta, v, T), \pi_i^l(\Delta, v, b)\right\} \geq 0. \tag{5}$$

Given a licensing fee T and a size of invention Δ, the strategy used by an infringer, σ_i, can be characterized in the following way. Let's denote $\Delta_s(T)$ the size of invention for which litigation provides the same profits as a settlement offer T. Because $q(\Delta, b)$

[10]This structure is substantially different from Bebchuk (1984) because entry in our case is endogenous. This assumption is particularly relevant in this context, since the profits that the patentee obtains depend on the arrival of future research, which in turn is affected by the legal enforcement of patents.

is increasing in Δ we know that those infringers with $\Delta > \Delta_s$ will go to court. Hence, we can define implicitly $\Delta_s(T)$ from (1) and (3) as

$$T = (2 - q(\Delta_s, b))\, v + L_i. \tag{6}$$

Moreover, there will be a threshold $\underline{\Delta}$ so that firms with a quality below it will not enter. From equation (5), $\underline{\Delta}$ is obtained as

$$\pi_i^s(\underline{\Delta}, v, T) = \underline{\Delta} + v - T - c = 0. \tag{7}$$

If $\underline{\Delta} < \Delta_s(T)$, all infringers with $\Delta \in [\underline{\Delta}, \Delta_s(T)]$ will accept the settlement offer. Therefore, σ_i can be defined as,

$$\sigma_i = \begin{cases} (\text{out, accept}) & \text{if } \Delta < \underline{\Delta}, \\ (\text{in, accept}) & \text{if } \Delta \in [\underline{\Delta}, \Delta_s(T)], \\ (\text{in, reject}) & \text{if } \Delta > \Delta_s(T). \end{cases}$$

Despite the fact that the patentee does not know the quality of the infringer, he has some beliefs about it. It is easy to see that the profits accrued by an infringer according to (5) are strictly increasing in Δ, meaning that if a certain competitor enters, all others with a higher quality will also be interested in entering. For this reason, beliefs will take the form of a minimum size of competitor $\widetilde{\underline{\Delta}}$ that the patentee expects to face.[11] Conditional on that belief, the patentee will offer the contract that maximizes expected

[11] Notice that the decisions of the patent holder depend on the beliefs about $\underline{\Delta}$ but court decisions about infringement are only a function of the observed Δ and b. This assumption can be justified by the limited information they might have about v and that only the infringer observes by being an active competitor in the market.

profits according to,

$$W(v,b,L_i,L_p|\underset{\sim}{\Delta}) = \max_T \int_{\Delta_s(T)} \left[(1-q(\Delta,b))\,v - L_p\right] \frac{\phi(\Delta)}{1-\Phi\left(\underset{\sim}{\Delta}\right)} d\Delta + \tag{8}$$
$$+T\frac{\Phi(\Delta_s(T)) - \Phi\left(\underset{\sim}{\Delta}\right)}{1-\Phi\left(\underset{\sim}{\Delta}\right)},$$

That is, contingent on $\Delta > \underset{\sim}{\Delta}$, if the infringer has a quality of innovation below Δ_s she will settle, making a payment of T. Otherwise, they will go to court, where the legal costs will be incurred, and the patentholder will be compensated with probability $1 - q(\Delta, b)$ for the foregone profits v. Equation (6) defines Δ_s as a function of T. In other words, changing the offer affects the size of the invention for which the infringer is indifferent between litigation and settlement.

Of course, for a licensing contract T to be part of a Sequential Equilibrium, the patentee must hold beliefs consistent with the strategy played by the infringer. The next definition states the corresponding requirements.

Definition 1 *A Pure Strategy Sequential Equilibrium of the Litigation Game given a patentholder with invention v will be a strategy profile (T^*, σ_i^*) supported by beliefs $\underset{\sim}{\Delta}$ such that,*

(i) T^* maximizes expected profits for the patentholder in (8) given (correct) beliefs $\underset{\sim}{\Delta}$; and

(ii) For all Δ, $\sigma_i^*(\Delta)$ maximizes expected profits for the infringer given T^*.

Accordingly, the next proposition shows that this game has a unique sequential equilibrium.

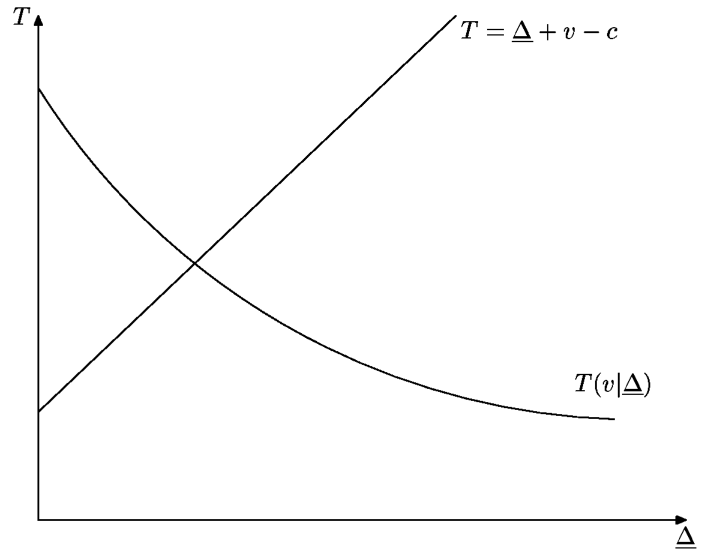

Figure 2: The conditions that define the *Sequential Equilibrium* of the game.

Proposition 1 *The Litigation game has a unique Sequential Equilibrium in pure strategies.*

The characterization of this unique sequential equilibrium is fairly simple. From the problem stated in (8) we obtain for any belief $\widetilde{\underline{\Delta}}$ the optimal licensing fee $T(v, b, L_i, L_p|\widetilde{\underline{\Delta}})$. The second condition comes from equation (7) that describes how the decision of entry by the infringer is determined. Using *Definition 1,* the Sequential Equilibrium can be obtained as the T^* and $\underline{\Delta}^*$ that satisfy the conditions,

$$T^* = T(v, b, L_i, L_p|\underline{\Delta}^*), \tag{9}$$

$$\underline{\Delta}^* = T^* - v + c. \tag{10}$$

It can be shown from equation (8) that T is weakly decreasing in $\widetilde{\underline{\Delta}}$ and as a result there can be at most a pair $(T^*, \underline{\Delta}^*)$ satisfying these two constraints. Of course, it could be the case that $T(v, L|0) < v - c$ so that these curves do not cross. In this case, it is easy to verify that $\underline{\Delta}^* = 0$ and $T = v - c$ constitutes a Sequential Equilibrium.[12]

The next result explains in this case how the licensing offer depends on the parameters of the model and the legal environment.

Lemma 2 *For any belief $\underline{\widetilde{\Delta}}$, the optimal licensing agreement consists of*

(i) $\Delta_s(v, b, L_i, L_p, \underline{\widetilde{\Delta}})$ *characterized by*

$$\Delta_s(v, b, L_i, L_p, \underline{\widetilde{\Delta}}) = -\ln\left(-\frac{1}{b} - \frac{v - \sqrt{v^2 + 4v\left(1 + be^{-\underline{\widetilde{\Delta}}}\right)(v + L_i + L_p)}}{2\,(v + L_i + L_p)\,b}\right)$$

if $b > \frac{v+L_i+L_p}{ve^{-\underline{\widetilde{\Delta}}}}$. Δ_s *is increasing in* L_p, L_i *and* $\underline{\widetilde{\Delta}}$, *decreasing in* v *and only decreasing in* b *if*

$$b \leq \frac{5v + 4L_i + 4L_p}{ve^{-\underline{\widetilde{\Delta}}}}.$$

Moreover, there will be no litigation if $b > \frac{v+L_i+L_p}{ve^{-\underline{\widetilde{\Delta}}}}$.

(ii) $T(v, b, L_i, L_p, \underline{\widetilde{\Delta}})$ *increasing in* v *and* b *and decreasing in* L_p *and* $\underline{\widetilde{\Delta}}$,

(iii) $W(v, b, L_i, L_p|\underline{\widetilde{\Delta}})$ *strictly increasing in* v, b *and* L_i *and decreasing in* L_p.

The effect of most parameters is expected. Higher legal costs for the patentee decrease litigation, and by weakening his position they also decrease the licensing payment T. Similarly, if the patentee believes that the conditional distribution of infringers is better – $\underline{\widetilde{\Delta}}$ increases –, he will decide to litigate less often, resulting in a lower licensing fee T.

[12] The existence of a unique pure strategy equilibrium is independent of the function $q(\Delta, b)$ or $\Phi(\Delta)$ postulated. It only relies on the fact that Δ has full support. Nevertheless, even if this condition is not met, an equilibrium will in general exist, although in mixed strategies.

The effect of b on the threshold Δ_s is particularly interesting. For intermediate values of b, Δ_s is minimum while it tends to infinity when b is very low and increases in b when b is large. The reason for the first is that when protection is very low and the patentee has few chances in court the offer T will be sufficiently low to prevent litigation. For the second, notice that more protection allows the patentee to demand higher settlement offers and infringers are less inclined to litigate.

The previous lemma also characterizes the expected profits for the patentee given any belief $\widetilde{\underline{\Delta}}$. More protection or a bigger improvement increase profits for the patent holder as long as the distribution of future entrants remains constant.

However, the results in *Lemma 2* do not take into account that changes in the legal structure can affect the equilibrium entry decision of infringers. It is expected that a more pro-patent legislation might discourage competitors to invest. The next proposition characterizes the effects in the sequential equilibrium in *Definition 1* as a result to changes in the main parameters of the model. These effects can be explained as shifts in the curves displayed in *Figure 2*.

Proposition 3 *In the unique pure strategy Sequential Equilibrium of the Litigation game,*

(i) The marginal infringer corresponds to

$$\Delta_s^* = -\ln\left(\frac{1}{b}\left(\frac{L_i + c - \underline{\Delta}^*}{v + L_i + c - \underline{\Delta}^*}\right)\right).$$

There exists a b^* such that Δ_s^* is decreasing in b if $b < b^*$ and increasing otherwise. It is also increasing in c.

(ii) $\underline{\Delta}^*$ is increasing in v and b and decreasing in L_p. $\underline{\Delta}^*$ is also concave in b.

(iii) T^* is increasing in v and b and decreasing in L_p and c.

From the previous proposition we can derive predictions for the model corresponding to the probability of entry, litigation and settlement that we would expect. In particular, the probability of entry can be defined as $1-\Phi\left(\underline{\Delta}^*\right)$, the probability of litigation is $1-\Phi\left(\Delta_s^*\right)$ and the probability of settlement is $\Phi\left(\Delta_s^*\right)-\Phi\left(\underline{\Delta}^*\right)$.

The effect of b on entry depends on two opposing forces. On one hand, a higher b decreases the chances of an infringer in court, and therefore, reduces entry. On the other, tougher courts allow the patentee to ask for higher licensing payments, for the same level of litigation, giving more incentives for entry to occur. The first effect, however, dominates, and entry in equilibrium is always decreasing in b.

This proposition shows that in equilibrium the effects of b on Δ_s described in Lemma 2 are preserved. For low values of b, increases in protection improve considerably the chances of the patentee in court, and as a result it makes litigation more appealing. However, when b raises and the chances of the patentholder are high, increases in protection do not make an important difference in the probability to win, and together with a high licensing payment and the possibility to avoid legal costs, justify a lower probability of litigation. In the same direction, the probability of entry decreases, resulting in a better distribution of possible entrants.

In recent empirical studies, some attention has been devoted to the relationship between breadth of a patent and the probability of being litigated. While most of the papers, such as Lerner(1994) suggest that broader patents are litigated more often, other results presented by Lanjouw and Schankerman (1997) predict the opposite relationship. This paper provides an explanation for this ambiguous result.

At this point it is important to emphasize the role of the assumption that the patentee can commit to litigate the entrant if the offer is rejected. Leaving aside the reputational issues discussed by Choi (1998), it is clear that this might not be optimal even if the expected value ex-ante of going to court is positive. When the offer is rejected the patentee learns that $\Delta > \Delta_s^*$, meaning that the chances in court decrease. The lower is the licensing payment rejected, the higher will be Δ_s. Nalebuff (1987) studies a related model where this option is explicitly considered. His results suggest that the option of not going to court will only be relevant when v is low or when legal costs are very high. Moreover, because T is increasing in b the option of not going to court might be relevant for low values of protection.

The previous proposition also shows that increases in the protection are to some extent equivalent to decreases in the legal costs that the patentee incurs, L_p. That is, both are considered barriers to entry to future competitors. The quality of the patented invention has a similar effect. A higher v implies that the infringer has to pay a higher compensation to the patentholder if she loses in court, reducing entry.

Empirical evidence supports these results. Lerner (1995) shows that firms, especially small ones, tend to direct their research or patenting activities away from sectors where big firms are established. These firms are likely to have smaller legal costs and an important number of patents.

Legal costs have also an ambiguous effect on the amount of litigation we expect to see. Higher costs make litigation less appealing, and a lower licensing fee is expected in order to avoid it. However, as in the case of b, a lower licensing fee induces more entry, reducing the incentives to offer a settlement. The resulting effect on litigation will depend on which force prevails.

Finally, technological parameters do not affect the decisions to litigate, meaning that a higher cost of invention c induces less and better firms to enter, decreasing the interest of the patentee in litigating.

3 The Incentives to Innovate

In the previous sections we have analyzed which are the effects of the legal environment in the incentives for other firms to engage in R&D when there is an innovator already in the market. However, the legal structure has also an impact on the incentives for inventions to be patented. One might expect that more favorable courts or lower litigation costs would increase the profits from innovation. As we show in this section, this will not always be the case.

The reason for this result is that firms obtain profits not only from their inventions but also from the future stream of profits that posterior uses of their ideas generate, usually through licensing. Therefore, two forces must be taken into account. A more favorable environment allows the patentee to accrue profits for a longer period of time. However, this same change alters the incentives for other firms to produce, affecting the licensing revenues that the patentee can collect due to infringement.

For simplicity, we assume that all innovators face the same technology. Therefore, the patentee incurs in a constant research cost of c to implement the idea v. Moreover, a patent needs to be purchased at a price f. Ex-ante profits from patenting will be denoted as W_0 and obtained as

$$W_0\left(v, b, L_i, L_p\right) = \Phi\left(\underline{\Delta}^*\right) v + \left(1 - \Phi\left(\underline{\Delta}^*\right)\right) W\left(v, b, L_i, L_p | \underline{\Delta}^*\right) - c - f, \qquad (11)$$

where $\underline{\Delta}^*$ is the infringer indifferent between entering and staying out, derived from the sequential equilibrium of the game.

The different forces described in the previous section make difficult to obtain clear predictions in most cases and for this reason we rely on numerical examples. These examples show, that W_0 might be in general an increasing or decreasing function of b. In particular, for low values of v and L_i the value of holding the patent increases when more protection is awarded. In other cases, associated with high values of v or L_i more protection harms the patentholder.

The reason for these two differentiated effects is that both v and L_i represent barriers to entry. As emphasized before, a higher v corresponds to a bigger compensation that the infringer has to pay if he loses in court. Similarly, a higher L_i corresponds to a bigger interest in accepting the settlement offer, and as a consequence, a bigger T that in turn reduces entry.

To provide some intuition for these numerical results, consider the following simplifying assumption. Suppose that $q(\Delta, b)$ is independent of Δ, and it corresponds to $q(b)$, decreasing in b. In this case, all infringers have the same expected loses from going to court, and as a result if the marginal entrant (firm with invention of size $\underline{\Delta}^*$) does not litigate, all the rest will decide to settle as well. Therefore, in equilibrium we will have no litigation and as a result the outcome would be equivalent to not having private information on Δ. For this reason we can compute the effect of b on W_0 as

$$\frac{\partial W_0}{\partial b} = -(T^* - v)\,\phi(\underline{\Delta}^*)\,\frac{\partial \underline{\Delta}^*}{\partial b} + (1 - \Phi(\underline{\Delta}^*))\,\frac{\partial \underline{\Delta}^*}{\partial b}.$$

The trade-offs of more protection are the following. The first term, negative because $T > v$, implies that more protection deters entry from firms that through licensing could increase profits for the patentholder. The second term, measures the fact that more protection increases the probability that the patentholder can maintain his monopoly power, with obvious positive effects on profits.

Given that $\underline{\Delta}^* = (1 - q(b))\,v + L_i + c$ and the exponential distribution for ideas, we can rewrite the expression as

$$\frac{\partial W_0}{\partial b} = e^{-\underline{\Delta}^*}\left((1 - q(b))\,v + L_i - 1\right) q'(b)\,v,$$

that will negative if b, v or L_i are large.

In the more general specification we are considering, the effect of b on W_0 will be

$$\begin{aligned}\frac{\partial W_0}{\partial b} &= -(T^* - v)\phi(\underline{\Delta}^*)\frac{\partial \underline{\Delta}^*}{\partial b} + (1 - \Phi(\underline{\Delta}^*))\frac{\partial W}{\partial b} \\ &= e^{-\underline{\Delta}^*}\left((1 - q(\Delta_s, b))v + L_i - 1\right)\frac{\partial q}{\partial b}v - v\int_{\Delta_s^*}\left(\frac{\partial q}{\partial b}(\Delta, b) - \frac{\partial q}{\partial b}(\Delta_s, b)\right)e^{-\Delta}d\Delta.\end{aligned}$$

Compared to the simple case outlined before, the first term has the same interpretation as before. The additional term takes into account that the effect of b on the probability of winning differs depending on Δ. In order to evaluate whether increases in v or L_i lead to a positive or negative effect of b on W_0 the analysis is more complicated, because of the effects on Δ_s that these changes represent. However, in general, the effect of v and L_i on Δ_s are second order compared with the ones described before and they usually operate in the same direction. In particular, notice that $\frac{\partial q}{\partial \Delta \partial b}$ is positive for large values of Δ and therefore, the last term is positive when litigation involves a small proportion of infringers, which occurs when v and L_i are large.

Therefore, the results indicate that when the firm already obtains a substantial protection from the size of the invention, v, or legal costs are high, more protection decreases profits for the patentee by reducing entry of future innovators. Moreover, if we take into account that, as argued before, courts award systematically more protection to better inventions, this result is even more likely to occur.

4 The Importance of Litigation

The existence of litigation changes the incentives for firms to invent and patent, and also importantly, for new competitors to appear. There are two reasons for litigation to arise in this model. First, the fact that protection is not absolute, and instead, it depends on how different is the new invention from the already patented one. The second reason is the private information that the infringer holds about the quality of her invention. Without any of these features intellectual property rights could be perfectly enforced.

In Llobet (2001) we show that if protection is absolute the infringer is hold-up, since the patentee cannot commit not to litigate any infringer. As a result there might be no further innovation. Although this extreme result relies on the assumption that all the bargaining power is in hands of the patentee, severe degrees of hold-up will occur as long as he has part of this power.[13]

The second reason for litigation to arise is the existence of private information. Suppose that, as in Chang (1995), Δ is public information, observable by the patentee and courts. For simplicity, patent breadth could be understood in this case – and in the same way that Chang (1995) does – as a threshold value that with some abuse of notation we will call b. That is, legislation sets a threshold b on the minimum size of the new invention that does not infringe the patent. It is easy to see that two cases might arise. If $b < c$, breadth becomes irrelevant. That is, the infringer will only enter if Δ is bigger than c, so that it makes positive profits. If $b > c$, all inventors with $b \geq \Delta \geq c$, will not enter because the patentee cannot commit to offer a license even though it is mutually beneficial. Notice that the patentee will earn v when the competitor does not produce. However, if $\Delta > b$, he can license his invention to the competitor, and earn at most v. Hence, profits become,

$$W_0(v, b) = v\Phi(\max\{b, c\}) + (1 - \Phi(\max\{b, c\}))\, T$$

for $T \leq v$. It is obvious that in this case ex-ante profits for the patentee are weakly increasing in the protection offered. This result differs from the case with private

[13] Hall and Ziedonis (2001) argue that cross-licensing is a way to reduce this hold-up.

information as discussed in the previous section. It also shows that the mechanism that operates in this model is different from the one present in Green and Scotchmer (1995).

Finally, if instead we assume that even though there is no private information courts are probabilistic, the results resemble those studied in the previous section, as long as $q(\Delta, b) = q(b)$. That is, protection does not necessarily make the patentholder better off. However, in equilibrium there is no litigation.

To summarize, we observe that litigation has several important implications. It affects the optimal licensing agreement that the patentee will offer, and by providing more incentives for future innovation to arise, it might increase growth. It also affects the recommendations that we can make regarding the use of breadth to provide incentives to innovate.

5 Preliminary Injunctive Relief

One of the assumptions that we have maintained throughout the paper is that while the legal procedure is resolved, both firms compete in the production of the good. However, in recent years, as it has been documented by Lanjouw and Lerner (1996), a legal motion known as Preliminary Injunctive Relief (PIR), has become popular. This motion allows the patentee to stop the other firm from producing until a verdict is reached. While the profits that the patentee gets in this case are higher, it also implies that the infringer must be compensated if courts rule against him.[14] Therefore the final effect is a priory ambiguous, and it might depend on the distribution of Δ. In this

[14] The effect of this motion is quite relevant. The legal process can take up to three years to be completed, compared to a patent that often does not reach its statutory life.

section we study the equilibrium of the game when this motion is filed and we discuss which patentees benefit from it.

Under the PIR, the patentee will still be the only producer, obtaining profits of v. After production, the legal process will take place and if the infringer wins, which occurs with probability $q(\Delta, b)$, he will be compensated for the lost profits. Hence, the function π_p^l and π_i^l become,

$$\begin{aligned} \pi_p^l(\Delta, v, b) &= v - q(\Delta, b)\Delta - L_p, \\ \pi_i^l(\Delta, v, b) &= q(\Delta, b)\Delta - L_i - c. \end{aligned}$$

As before we study the optimal licensing fee T that the patentee can offer. This fee determines which infringers will go to court and those that will settle. The threshold value, denoted $\Delta_s(T)$ as before, is obtained from

$$\pi_i^l(\Delta_s, v, b) = q(\Delta_s, b)\Delta_s - L_i - c = v + \Delta_s - T - c = \pi_i^s(\Delta_s, v, T),$$

and as a result,

$$T = (1 - q(\Delta_s, b))\,\Delta_s + v + L_i. \tag{12}$$

It is important to notice that as opposed to the previous case it is not necessarily true that all $\Delta > \Delta_s(T)$ prefer litigation to settlement, and this division relies on the condition that

$$\Delta \frac{\partial q}{\partial \Delta}(\Delta, b) + q(\Delta, b) > 1 \text{ for } \Delta > \Delta_s.$$

We will verify later that the equilibrium we consider satisfies this assumption. Profits correspond to $\pi_p^l(\Delta, v, b)$ if the infringer does not accept the settlement, and to the licensing fee T otherwise. That is,

$$\begin{aligned} W\left(v, b, L_i, L_p | \widetilde{\underline{\Delta}}\right) &= \max_T \int_{\Delta_s(T)} [v - q(\Delta, b)\Delta - L_p] \frac{\phi(\Delta)}{1 - \Phi(\widetilde{\underline{\Delta}})} d\Delta + \\ &\quad + \left[\frac{\Phi(\Delta_s(T)) - \Phi(\widetilde{\underline{\Delta}})}{1 - \Phi(\widetilde{\underline{\Delta}})}\right] T, \end{aligned} \tag{13}$$

Compared to the case in (8) the only relevant difference is that here the patentee produces, obtaining profits of v and the infringer will have to be compensated with probability $q(\Delta, b)$ if she wins in court, with an amount corresponding to Δ.

The existence and uniqueness of the equilibrium of this problem can be obtained in a similar way as the one described in *Definition 1* and its main characteristics are presented in the next proposition.

Proposition 4 *In the unique pure strategy Sequential Equilibrium of the Litigation game,*

(i) $\underline{\Delta}^*$ *is increasing in* c*, decreasing in* L_p *and independent of* v*,*

(ii) Δ_s^* *is increasing in* c*, increasing in* L_p *and* L_i *and independent of* v*,*

(iii) T^* *is increasing in* v *and decreasing in* c *and* L_p *and* L_i*.*

In general the results are similar to the case studied in the previous sections. The main difference is that implications about b are not so clear-cut, although numerical examples show that they are qualitatively similar to the previous case.

When a PIR is requested, entry and litigation do not depend on v. This is an important difference with respect to our benchmark case, because here better inventions do not deter more entry. To see it, notice that the profits that the infringer expects to obtain if she accepts the settlement are

$$\pi_i^s(\Delta, b, T^*) = \Delta - (1 - q(\Delta_s^*, b))\Delta_s^* - L_i - c,$$

independent of v. Obviously, this feature makes the optimal transfer T^*, according to equation (12), linear in v.

If we define the ex-ante profits that the patentee obtains as in Section 3, the linearity of T in v and the fact that neither $\underline{\Delta}^*$ nor Δ_s are affected by v are enough to show that W_0 is linear in v. That is,

$$\begin{aligned} W_0(v, b, L_i, L_p) &= \Phi(\underline{\Delta}^*)v + (1 - \Phi(\underline{\Delta}^*))\, W(v, b, L_i, L_p|\underline{\Delta}^*) - c - f \\ &= v + W_0(0, b, L_i, L_p). \end{aligned}$$

When comparing this case with the situation in which PIR is not requested, the results are quite different. Here the profits that firms obtain increase linearly in v. This result means that the capacity that the patentee has to capture profits from future innovation is independent of the quality of the original invention as opposed to the case where PIR is not available. There, innovators with small improvements could capture a relatively higher share of future research, by inducing more entry and obtaining the corresponding licensing fees.

6 Concluding Remarks

The goal of this paper was to study how the enforcement of Intellectual Property Rights affects the decisions to innovate. While other papers, such as Aoki and Hu (1999), have considered the effect of litigation on the value of patents, they usually study the trade-off that imitation constitutes between adding competition and the incentives to innovate. Nevertheless, patents also affect the decisions of future innovators. This paper tries to address this issue.

We have presented a model of patent settlement that takes explicitly into account the consequences of litigation, or the threat of using it. The results show that with private information on the quality of new ideas there is in equilibrium a certain amount of litigation, together with the settlement of low quality innovations.

We stress the fact that the protection that patents grant is intimately related to the way in which it is enforced by courts. The outcome of the legal procedure is used by the patentee as a threat to obtain better terms in the negotiation with the infringer. However, more protection has a downside. The patentee cannot commit beforehand to offer a particular license, and therefore, as courts become more favorable to patentees, only the best infringers will enter. Since an important part of the value of a patent stems from licensing to future innovators more protection does not necessarily mean more profits for the patentee. This effect is particularly important on inventions that have a high value, because they already deter more entry.

By explicitly modeling the legal institutions, the model allows us to compare different legal systems. In particular, we study the consequences of a legal motion that has become popular in recent years, the *Preemptive Injunctive Relief.* Its use seems to benefit relatively more better inventors by increasing their licensing revenue.

The answers provided in this paper are far from complete. We show that the effects of patent protection and litigation costs are more complex than what is usually considered. However, not much has been said about the socially optimal amount of protection. Some preliminary numerical results show that the Preemptive Injunctive Relief should be granted to innovations that have a higher market value, together with more protection to the corresponding patentees.

Finally, in this paper we have abstracted from the dynamic environment associated to the negotiation among the parties, and for simplicity we have collapsed all the process in a take-it-or-leave-it offer. An interesting extension would include the study of the dynamic decisions undertaken by firms. In particular, the output market can be understood as a way to obtain noisy signals of the quality of the improvement. As a result, the decisions of the patentee to produce, the timing of the licensing agreements and trials, as well as the decisions taken by courts will depend on this signal extraction mechanism. The speed of the trials is also an interesting and unexplored policy variable that will affect the use and optimality of the Preemptive Injunctive Relief.

A Appendix: Proofs

Here we reproduce the proofs of all the results in the paper.

Proposition 1

Proof. In the text. The only case not considered corresponds to $\underline{\Delta} = \infty$. Trivially, this will constitute an equilibrium with no entry. ■

Lemma 2

Proof. From equation (8) it is more convenient to derive with respect to Δ_s. The corresponding first order condition with respect to Δ_s is,

$$(v + L_i + L_p)\phi(\Delta_s) - \frac{\partial q}{\partial \Delta}(\Delta_s, b)v\left[\Phi(\Delta_s) - \Phi(\widetilde{\underline{\Delta}})\right] \leq 0, \tag{14}$$

with equality if $\Delta_s = \widetilde{\underline{\Delta}}$. This case can be ruled out because $\phi(\Delta) > 0$ for all Δ. Replacing ϕ and q by their expressions we obtain the optimal Δ_s. It can be verified that the second derivative is negative. Using the implicit function theorem we obtain that Δ_s is increasing in L_i, L_p and $\widetilde{\underline{\Delta}}$. We can also show that

$$\frac{v - \sqrt{v^2 + 4v\left(1 + be^{-\widetilde{\underline{\Delta}}}\right)(v + L_i + L_p)}}{2(v + L_i + L_p)b} = \frac{1 - \sqrt{1 + 4\left(1 + be^{-\widetilde{\underline{\Delta}}}\right)\frac{(v+L_i+L_p)}{v}}}{2\frac{v+L_i+L_p}{v}b}$$

is decreasing in v, since $\frac{v+L_i+L_p}{v}$ is increasing in v, proving that Δ_s is decreasing in v.

With respect to b notice that

$$-\frac{1}{b} - \frac{v - \sqrt{v^2 + 4v\left(1 + be^{-\widetilde{\underline{\Delta}}}\right)(v + L_i + L_p)}}{2(v + L_i + L_p)b}$$

is positive if $b \geq \frac{v+L_i+L_p}{ve^{-\underline{\Delta}}}$ – and hence, $\Delta_s < \infty$ – and that this function is convex and reaches a maximum for $b = \frac{5v+L_i+4L_p}{ve^{-\underline{\Delta}}}$.

The results for v, L_p and $\widetilde{\underline{\Delta}}$ in (i) follow from the definition of T and the results on Δ_s. Moreover, notice that

$$T = 2\left(1 + \frac{v+L_i+L_p}{v - \sqrt{v^2 + 4v\left(1+be^{-\widetilde{\underline{\Delta}}}\right)(v+L_i+L_p)}}\right) v + L_i$$

and therefore $\frac{\partial T}{\partial b} > 0$.

Part (iii) can be directly obtained from (8) using the Envelope Theorem. ■

Proposition 3

Proof. As proven before, the unique pure strategy Sequential Equilibrium is determined by the T^* and $\underline{\Delta}^*$ such that the following two conditions are satisfied:

$$T^* = T(v, b, L_i, L_p | \underline{\Delta}^*), \tag{15}$$

$$\underline{\Delta}^* = T^* - v + c. \tag{16}$$

Replacing Δ_s in $\underline{\Delta}^*$ we obtain that the equilibrium $\underline{\Delta}^*$ satisfies.

$$\underline{\Delta}^* = \left(1 + 2\frac{v+L_i+L_p}{v - \sqrt{v^2 + 4v\left(1+be^{-\underline{\Delta}^*}\right)(v+L_i+L_p)}}\right) v + L_i + c.$$

Suppose towards a contradiction that b increases but $\underline{\Delta}^*$ decreases. This decrease requires $be^{-\underline{\Delta}^*}$ to fall, but this is inconsistent with the previous assumption. From that we can obtain that $be^{-\underline{\Delta}^*}$ is increasing in b which is enough to prove that T increases with b.

To show that $\underline{\Delta}^*$ is concave in b we use the previous expression and do it by contradiction. That is, suppose that $\underline{\Delta}^*(b)$ in the right-hand side is a convex function of b. Therefore, if we name $\Psi(b) = \sqrt{v^2 + 4v\left(1 + be^{-\underline{\Delta}^*}\right)(v + L_i + L_p)}$ then

$$\begin{aligned}
\Psi'(b) &= \frac{4v(v + L_i + L_p)}{2\Psi(b)} \frac{\partial}{\partial b}\left(be^{-\underline{\Delta}^*}\right) > 0 \\
\Psi''(b) &= -\frac{\Psi'(b)^2}{\Psi(b)} + \frac{4v(v + L_i + L_p)}{2\Psi(b)} \frac{\partial}{\partial b^2}\left(be^{-\underline{\Delta}^*}\right) < 0
\end{aligned}$$

where the last inequality is derived from the assumption on concavity of $\underline{\Delta}^*$. As a result, we can compute

$$\frac{d\underline{\Delta}^*}{db} = v\frac{v + L_i + L_p}{(v - \Psi(b))^2}\Psi''(b) - 2v\frac{v + L_i + L_p}{(v - \Psi(b))^3}\Psi'(b) < 0$$

contradicting our premise.

Now, to show that there is a value b^* such that only for $b < b^*$, Δ_s^* is decreasing in b, given the previous lemma, it is enough to show that for all b for which $\frac{d\Delta_s^*}{db} = 0$, the second derivative is positive.

From $\Delta_s^* = -\ln\left(\frac{1}{b}\left(\frac{L_i + c - \underline{\Delta}^*}{v + L_i + c - \underline{\Delta}^*}\right)\right)$ the previous condition implies that the second derivative of $F(\underline{\Delta}^*, b) = \frac{1}{b}\left(\frac{L_i + c - \underline{\Delta}^*}{v + L_i + c - \underline{\Delta}^*}\right)$ is negative. Notice that

$$\frac{dF}{db} = \left(-\frac{1}{b} - \frac{v}{(L_i + c - \underline{\Delta}^*)(v + L_i + c - \underline{\Delta}^*)}\frac{\partial\underline{\Delta}^*}{\partial b}\right)F(\underline{\Delta}^*, b)$$

and

$$\left.\frac{d^2F}{db^2}\right|_{\frac{dF}{db}=0} = \left(-\frac{2(L_i + c - \underline{\Delta}^*)}{b^2 v} - \frac{v}{(L_i + c - \underline{\Delta}^*)(v + L_i + c - \underline{\Delta}^*)}\frac{\partial^2\underline{\Delta}^*}{\partial b^2}\right)F(\underline{\Delta}^*, b)$$

which proves the result, given that $F(\underline{\Delta}, b) < 0$.

To show that T^* is decreasing in L_p we do it again by contradiction. Suppose that for some parameters T increases in L_p. This implies that $\underline{\Delta}^*$ must also increase. Since Δ_s is increasing in L_p and $\underline{\Delta}$, then Δ_s^* must also increase, leading to a lower T, which contradicts our premise. The same steps prove that T^* and $\underline{\Delta}$ are increasing in v.

Finally, suppose that for some parameters $\underline{\Delta}^*$ is decreasing in c. This means that Δ_s must decrease and from (16) that T is also decreasing in c. However, a lower T

implies that Δ_s must increase, leading to a contradiction. Therefore, Δ_s increases in c. ■

Proposition 4

Proof. The proof is almost identical to the one in Proposition 3. In particular, the first order condition for the problem in (13) is

$$(\Delta_s + L_i + L_p)\,\phi(\Delta_s) - \left(\frac{\partial q}{\partial \Delta}\Delta_s + q(\Delta_s, b) - 1\right)\left(\Phi(\Delta_s) - \Phi\left(\underline{\widetilde{\Delta}}\right)\right) \leq 0$$

with equality if $\Delta_s > \underline{\widetilde{\Delta}}$. Obviously, either Δ_s is infinite or $\frac{\partial q}{\partial \Delta}\Delta_s + q(\Delta_s, b) - 1 > 0$. Otherwise, the previous derivative would be positive. Therefore, the requirement imposed in the text is satisfied.

There is no closed-form solution for Δ_s but from the previous condition we can observe that it is increasing in L_i, L_p and $\underline{\widetilde{\Delta}}$ and independent of v.

Moreover, the second condition for the equilibrium, on $\underline{\Delta}$, is defined as

$$\underline{\Delta} = (1 - q(\Delta_s, b))\,\Delta_s + c + L_i.$$

As a result, $\underline{\Delta}^*$ is independent of v and $\frac{\partial T}{\partial v} = 1$. Clearly

$$\frac{\partial \underline{\Delta}^*}{\partial \Delta_s} = -\frac{\partial q}{\partial \Delta}\Delta_s - q(\Delta_s, b) + 1 < 0.$$

The same arguments used in Proposition 3 prove the other results. ■

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