Capital Market Imperfections and Forms of Foreign Operations

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REVISED
10 February 2003

Abstract

We develop a model of foreign operations to determine a parent firm’s choice between wholly owned subsidiaries and joint ventures (and the degree of equity participation in the latter case). We introduce capital market imperfections into the model to capture a positive effect of the firm’s net worth on investment; this effect plays an important role in determining organization forms because the investment level affects the optimal form, which determines technology transfer. We show that parent firms with strong technological advantages can be controlling shareholders even if they take minority positions in ventures with local firms.

keywords: foreign direct investment; joint ventures; technology transfer; ownership share; capital market imperfections

JEL classification: F23; G32; L22

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1 Introduction

The traditional explanation for foreign direct investment (FDI) rests on the existence of proprietary intangible assets. Since arm’s-length transfers of intangibles between firms are prone to market failures, it is widely recognized that FDI is a crucial channel of international technology diffusion. However, existing FDI theories are not well suited for understanding observed variation in ownership patterns, that is, a parent firm’s ownership share and its effect on technology transfer. In this paper, we introduce capital market imperfections into a model and investigate how financial constraints affect organization forms of foreign operations. The model explains: (1) the choice between wholly owned subsidiaries and joint ventures; (2) the ownership share in a joint venture; and (3) the relationship between ownership share and control.

When capital markets are efficient, firms investing abroad can choose the optimal level of capital investment. In practice, it is difficult for small parent firms to undertake a large project because of capital market imperfections, such as a higher cost of external funds (as compared to internal funds). In the case of FDI through acquisitions, it is also difficult for small parent firms to acquire large local firms when capital markets are inefficient. Thus, if capital markets are inefficient, an increase in investors’ net worth can stimulate the level of capital investment. Empirical studies such as Fazzari, Hubbard, and Petersen (1988) support the positive relationship between investment spending and internal funds. Bernanke and Gertler (1989) ascribe this relationship theoretically to an asymmetry of information between borrowers and lenders, and explain how net worth stimulates capital investment in a business cycle model.

Several authors consider this relationship in open economies. Gertler and Rogoff (1990) explain how capital market imperfections mute North-South capital flows. Froot and Stein (1991) consider FDI through acquisitions, and show that a depreciation of the dollar increases the relative wealth position of foreigners and hence stimulates FDI (acquisition of U.S. assets) into the United States. Their FDI
model does not allow for joint ventures and the degree of equity participation. However, if net worth plays an important role in determining investment, it may affect forms of foreign operations in the presence of capital market imperfections.

When the minimum efficient scale for foreign operations is too large, parent firms often choose joint ventures or equity participation rather than wholly owned subsidiaries. For example, in the telecommunications industry, where proprietary technology and a large amount of capital are necessary, a few leading companies such as Britain’s Vodafone and Japan’s NTT DoCoMo are investing all over the world for the third generation mobile phone through mergers and acquisitions. However, the degree of equity participation varies across firms; NTT DoCoMo adopts a strategy of taking minority stakes in its local partners, while Vodafone prefers majority ownership. In this paper, we show that the degree of equity participation depends on parent firms’ net worth in a formal model. (We also show that the level of technology matters and how the interaction between technology and net worth affects forms of foreign operations.)

Note that, for the success of a foreign project, not only proprietary assets of the parent firm but also country-specific assets, which are owned by local firms, such as goodwill, a license or permit from the authorities, and local distribution channels, are essential. As Gomes-Casseres (1989) and Hennart (1991) argue, joint ventures are chosen when parent firms need local partners’ assets and when it is costly to obtain them through full acquisitions. Thus, unlike wholly owned subsidiaries and greenfield investment, joint ventures involve partners, and hence the distribution of ownership shares (which is assumed to be the distribution of profit shares in this paper) affects the degree of technology transfer from the parent firms. In this paper, we present a simple model of FDI where the form of organization and the degree of technology transfer are simultaneously and endogenously determined, and provide a rationale for “partial” ownership.

Some recent studies consider contractual arrangements with local partners. For example, Horstmann and Markusen (1996) focus on the form of foreign sales op-
eration; they examine the multinational firm’s choice between establishing a direct sales branch and making a contract with a local agent (and the length of contract if the firm initially chooses an agency contract). Since they focus on sales operation, they do not consider the multinational firm’s proprietary technology and its transfer, which play an important role in the traditional explanation for FDI. Nakamura and Xie (1998) and Dasgupta and Tao (1998), which investigate the optimality of joint ventures, consider how organization forms or ownership structures affect the firm’s incentive in settings of contractual incompleteness. However, none of these papers consider the effect of net worth, which matters when there are informational asymmetries between borrowers and lenders as noted above. (We believe that borrowers are better informed than lenders in the actual open economies.)

Hence, in this paper, we consider both the effect of ownership on the incentive to transfer technology (Nakamura and Xie (1998), Dasgupta and Tao (1998)) and the effect of net worth on capital investment (Bernanke and Gertler (1989), Gertler and Rogoff (1990)) to investigate forms of foreign operations. We show that ownership structures play an important role in foreign operations without assuming contractual incompleteness.

The model of joint ventures also sheds some light on the relationship between the degree of equity participation and control over joint ventures. The model shows that the parent firm with strong technological advantages can be the controlling shareholder even if it takes minority stakes in the joint venture. This result is consistent with the empirical results in Hennart (1991) and Blomstrom and Zejan (1991); their results suggest that technologically oriented firms do not necessarily choose full ownership although the traditional FDI theory implies that such firms prefer full ownership because their proprietary knowledge is poorly protected. (In practice, NTT DoCoMo, which has strong technological advantages, takes minority positions in its foreign subsidiaries.)

The remainder of the paper is organized as follows. Section 2 presents a basic model of investment under asymmetric information, and shows how the parent firm’s
net worth affects the level of capital investment. We first consider the case of wholly owned subsidiaries in Section 3, and then allow for joint operations in Section 4. We indicate conditions under which the parent firm and the local firm agree on the joint-venture partnership. In Section 5, we define the controlling shareholder in the model of joint ventures. We show the minimum ownership share required to control a joint venture, and demonstrate that firms with technological advantages can control joint ventures even if they take minority stakes. Section 6 summarizes the main results of the paper.

2 A model of investment under asymmetric information

We use a two-period, open-economy model of imperfect capital markets developed by Gertler and Rogoff (1990). The model assumes that lenders and borrowers are risk neutral, and illustrates how the borrower’s private information decreases investment. We suppose that a local firm is located in a small country, which we call the host country. The local firm is endowed with \( w_L \) (net worth) and an indivisible project in period 1. First-period investment at \( k \) level in the project yields a random second-period output \( Y \) distributed as follows:

\[
Y = \begin{cases} 
A > 0 & \text{with probability } F(k) \\
0 & \text{with probability } 1 - F(k).
\end{cases}
\]

We assume that \( F' > 0, F'' < 0, F(0) = 0, F(\infty) = 1 \) and \( (1 + r)/A < F'(0) < \infty \), where \( r > 0 \) is the fixed world interest rate. In this setting, investment \( k \) raises the expected output of the project \( AF(k) \) as more traditional production functions imply. The risk-neutral local firm maximizes the expected profit in period 2 by choosing \( k \). Note that, if necessary, the firm will borrow to finance capital investment as we explain below.

The information structure is as follows. When a firm borrows, lenders can observe the borrower’s first-period endowment, gross borrowing and second-period output,
but not first-period investment $k$. It implies that the borrower may secretly lend abroad, and that the terms of the loan cannot be indexed to $k$. Note that, although $k$ is unverifiable, the function $F(\ )$ is common knowledge.

To invest $k$ larger than the initial endowment $w_L$, the local firm needs external funds in period 1. The owner of the project (local firm) faces the finance constraint:

$$w_L + b \geq k,$$

(2)

where $b$ is gross foreign borrowing. In return for this, the borrower issues a state-contingent security in period 1. Since there are no other variables the payment can be indexed to, the second-period payment depends on $Y$ (which is verifiable); a security pays $R$ if $Y = A$, and 0 if $Y = 0$. Since no payment would be possible in the bad state, the payment in the good state must be large enough. That is, the payment $R$ must satisfy

$$RF(k) \geq (1 + r)b.$$

(3)

The expected profit of the local firm is given by

$$\Pi_L(k) = AF(k) - \gamma - RF(k) + (1 + r)(w_L + b - k),$$

(4)

where $\gamma$ is the operating cost in period 2 and $RF(k)$ is the expected second-period payment shown in (3). The last term describes the return from risk-free investments; this implies that firms may secretly invest borrowed funds abroad rather than at home. (In the Appendix, we demonstrate that $w_L + b = k$ holds.)

The borrower (local firm) chooses $k$ to maximize the expected profit (4) for a given $R$ that satisfies (3). Hence, from (4), we obtain the first-order condition

$$(A - R)F'(k) = 1 + r.$$

(5)

This equation, which we interpret as an incentive-compatibility constraint, is drawn
as the downward-sloping curve IC in Figure 1. Note that \( R = 0 \) implies that there is no informational asymmetry. We denote the first-best level of investment by \( k^* \), which satisfies \( AF'(k) = 1 + r \). The curve IC demonstrates that \( k = k^* \) at \( R = 0 \) and \( k < k^* \) as long as \( R > 0 \); this curve implies the moral hazard problem (underinvestment) induced by the borrower’s private information. In the Appendix, we examine the borrower’s problem formally. We show that \( w_L < k < k^* \) (i.e., \( b > 0 \)) if \( w_L < k^* \), and that (2) and (3) are binding. We assume throughout this paper that no investors can finance their optimal investment levels without loans (i.e., \( w_L < k^* \)).

From (2) and (3), which are binding, we obtain

\[
R = \frac{1 + r}{F(k)}(k - w_L). \tag{6}
\]

This equation is the constraint that risk-neutral lenders must receive the market rate of return and is drawn as the curve \( MR^L \) in Figure 1. From (6), we obtain \( dR/dk > 0 \).

The solution of the local firm’s problem, a pair \((k, R)\), must satisfy (5) and (6). Let \( k_L \) denote the investment level \( k \) that satisfies both (5) and (6). We obtain \( k_L \) from the intersection of IC and \( MR^L \) in Figure 1. The figure shows that \( k_L \) is an increasing function of the borrower’s net worth \( w_L \). The result can be confirmed by differentiating (5) and (6) as follows.

\[
\frac{dk_L}{dw_L} = \frac{1}{D}(1 + r)F'(k) > 0,
\]

where

\[
D = (1 + r)F'(k) \left\{ 1 - \frac{F'(k)}{F(k)/k} \left( 1 - \frac{w_L}{k} \right) \right\} - (A - R)F(k)F''(k) > 0.
\]

Thus, a rise in \( w_L \) shifts \( MR^L \) downward (leaves IC unchanged) and hence increases \( k_L \) in Figure 1.
We assume throughout the paper that a project makes a positive expected profit even if the firm’s net worth \( w_L \) is 0 (i.e., \( AF(k_L) - \gamma - (1 + r)k_L > 0 \) for any \( w_L \geq 0 \)). Therefore, from (3) and (4), \( \Pi_L(k_L) > 0 \) holds for any \( w_L \geq 0 \).

3 Wholly owned subsidiaries

In Section 2, we suppose that the local firm owns and manages a project. In practice, foreign firms, which have technological advantages, often acquire the project (or the firm that owns it) to expand their business in the local market. In this section, we consider foreign ownership. The model developed here allows a foreign firm to acquire the local firm’s project.

The potential acquirer, which can use advanced technology no local firms have, is called the MNE. We formulate the MNE’s technology as follows: if the MNE manages the project, the operation cost in period 2 is given by \( \gamma - C(s) \), where \( C'(s) > 0, C''(s) < 0, C(0) = 0, \lim_{s \to 0} C'(s) = \infty \) and \( s \) is the MNE’s unverifiable input (effort) to transfer technology.

Here, as in Teece (1977), we distinguish two technology forms: technology embodied in physical items (such as plants and blueprints) and unembodied knowledge. The former is easily transferred to others through transactions of technical specifications. (In this model, this technology is represented by the production function of the project (1).) However, the latter is technology or information that relates to methods of operation, quality control, and other manufacturing procedures; the effective conveyance of these methods requires training and personnel exchange.\(^{11}\) Note that these activities are (at least partially) unverifiable because the effectiveness of them depends on effort expended by the MNE. For example, even if the number of hours of training and the number of personnel exchanged can be verifiable (written in the contract), it is difficult to verify the quality of trainers. Hence, in this paper, we assume that the cost of technology transfer (which is represented by the MNE’s effort \( s \)) is unverifiable. This implies that the MNE must bear the whole cost of technology transfer and hence the MNE’s incentive affects the level of
technology transfer.

In this model, a wholly owned subsidiary is described as follows. We assume that the MNE is risk neutral and endowed with \( w_M \). To establish a wholly owned subsidiary, the MNE acquires the local firm’s project in period 1. Then, the MNE chooses the levels of investment \( k \) and unverifiable input \( s \) to maximize its expected profit.\(^{12}\)

The MNE’s expected profit from acquiring a project is

\[
\Pi_M^{WOS}(k, s) = AF(k) - (\gamma - C(s)) - RF(k) + (1 + r)(w_M + b - k - P) - s, \tag{7}
\]

where \( P \) represents the price of the project (payment to the local firm) in period 1. We assume that input \( s \) can be expressed in terms of the good in period 2. As in Section 2, the borrower (the MNE in this case) must satisfy the constraint (3) and the finance constraint

\[
w_M + b \geq k + P, \tag{8}
\]

which implies that the MNE must finance investment \( k \) and the cost of acquisition \( P \). From these two constraints, which are binding as shown in the Appendix, we obtain

\[
R = \frac{1 + r}{F(k)}(k + P - w_M). \tag{9}
\]

This equation is the constraint that risk-neutral lenders must receive the market rate of return (as in Section 2), and is drawn as MR\(^{WOS}\) in Figure 1. The curve MR\(^{WOS}\) is upward sloping as MR\(^L\). In Figure 1, MR\(^{WOS}\) intersects the horizontal axis at \( k = w_M - P \). (In the figure, \( w_M - P \) is larger than \( w_L \). However, as we will see, this is not always the case.)

To complete the model, we must consider a price at which the local firm is willing
to give up the project. The price \( P \) must satisfy
\[
(P + w_L)(1 + r) \geq \Pi_L(k_L).
\] (10)

From (3) and (4), the lowest price \( P \) that satisfies (10) can be written as
\[
P = \frac{AF(k_L) - \gamma - (1 + r)k_L}{1 + r}.
\] (11)

Note that \( P \) given in (11) is positive (because \( AF(k_L) - \gamma - (1 + r)k_L > 0 \) for any \( w_L \geq 0 \) by assumption) and an increasing function of \( w_L \) (because \( dk_L/dw_L > 0 \) and \( AF(k) - (1 + r)k \) is an increasing function of \( k \in (0, k^*) \)). We assume that the MNE pays this lowest possible price.\(^{13}\)

The MNE chooses \( s \) to maximize (7). Then, we obtain \( C'(s^*) = 1 \), where \( s^* \) is the first-best level of \( s \). Thus, the MNE chooses the first-best \( s^* \) in a wholly owned subsidiary because the MNE can obtain the surplus. As in Section 2, the solution of the MNE’s problem must satisfy the incentive-compatibility constraint (5), which is drawn as IC, and the constraint given in (9), which is drawn as MR\(^{WOS} \) in Figure 1. Let \( k_M \) denote the investment level \( k \) that satisfies both (5) and (9). We obtain \( k_M \) from the intersection of IC and MR\(^{WOS} \) in Figure 1. Since the level of investment is an increasing function of the borrower’s net worth (as shown in Section 2), whether \( k_M \) is larger than \( k_L \) depends on the relative net worth. Note that the MNE’s net worth after acquisitions can be written as \( w_M - P \). Hence, \( k_M \geq k_L \) holds if \( w_M - P \geq w_L \).

Finally, we examine whether this foreign operation is feasible. (Note that the MNE does not necessarily purchase the project if the price is too high.) The MNE chooses to acquire the local firm’s project if
\[
\Pi_M^{WOS}(k_M, s^*) \geq (1 + r)w_M,
\] (12)

where \((1 + r)w_M \) is the MNE’s minimum income. From (3), (7) and (11), the
inequality (12) can be rewritten as

\[ AF(k_M) - \gamma + C(s^*) - (1 + r)k_M - s^* \geq AF(k_L) - \gamma - (1 + r)k_L. \] (13)

This inequality implies that the expected benefit from the operation is greater than or equal to the price of the project. If (13) holds, the MNE acquires the project to establish a wholly owned subsidiary. Since \( AF(k) - (1+r)k \) is an increasing function of \( k \in (0,k^*) \) and the level of investment is an increasing function of the owner’s net worth, (13) holds if \( w_M/w_L \) is large enough. Specifically, without technology transfer, a wholly owned subsidiary is feasible only if the net worth of the MNE is greater than that of the local firm because of the cost of acquisitions. The result can be stated in the following proposition:

**Proposition 1** If the MNE’s net worth is much larger than the local firm’s net worth (if \( w_M/w_L \) is large enough), the MNE establishes a wholly owned subsidiary.

This proposition implies that firms with a large amount of internal funds can establish their subsidiaries by acquiring local firms because they mitigate the moral hazard problem induced by the borrower’s private information.

### 4 Joint ventures

In Section 3, we have considered wholly owned subsidiaries to study the MNE’s overseas investment. The model developed in Section 3 implies that the MNE owns 100 percent of the local firm’s stock for foreign operations. In practice, however, many firms cooperate with local firms to start foreign operations by forming joint ventures or by acquiring a substantial equity interest in local partners. In this section, to examine these joint operations, we develop a model of joint ventures.

Joint ventures between the MNE and the local firm have the following features: (1) the local firm can take advantage of the MNE’s advanced technology although the level of technology transfer depends on the MNE’s incentive (this implies that
the MNE’s input $s$ is less than the first-best level $s^*$; (2) joint ventures reduce the financial burden of the two firms because each firm benefits from its partner’s net worth and the cost of acquisitions for the MNE becomes smaller. In short, joint ventures can reduce the MNE’s financial burden and provide the local firm with both advanced technology and funds. In the following, using a model of joint ventures with these features, we investigate the conditions under which the MNE and the local firm agree on the joint-venture partnership.

Here, we suppose that the joint-venture partnership is a contractual arrangement for sharing the project ownership. In the model developed below, for simplicity, we assume that the ownership share (the percentage of ownership) is the same as the profit share. Accordingly, if the MNE takes 60 percent of the ownership, for example, the MNE obtains 60 percent of the joint venture’s total profit. We assume that the joint venture has $w$, and that this initial endowment (net worth) is the sum of the two firm’s endowments (i.e., $w \equiv w_L + w_M < k^*$) and fixed. We treat the MNE’s ownership share $\alpha \equiv w_M / w$ as a parameter. By using this joint-venture contract, the MNE does not have to purchase the project by paying $P$ for foreign operations; instead, the MNE has to give up $(1 - \alpha)$ of the profit. Then, the MNE’s expected profit from the joint venture is given by

$$\Pi_{JV}^M(k, s) = \alpha [AF(k) - \gamma + C(s) - RF(k) + (1 + r)(w + b - k)] - s. \quad (14)$$

The local firm’s expected profit from the joint venture is

$$\Pi_{JV}^L(k, s) = (1 - \alpha) [AF(k) - \gamma + C(s) - RF(k) + (1 + r)(w + b - k)]. \quad (15)$$

Note that, since the MNE’s input $s$ is unverifiable, it is impossible to write a contract for cost sharing; the MNE independently determines the level of $s$ although the input benefits the local firm as well. Then, from (14), we obtain

$$\alpha C'(s_{JV}) = 1, \quad (16)$$
where $s_{JV}$ denotes the level of $s$ determined by the MNE for the joint venture. A rise in $\alpha$ increases $s_{JV}$. Since $\alpha \in (0, 1)$, we obtain $0 < s_{JV} < s^*$ and $\lim_{\alpha \to 1} s_{JV} = s^*$.

Thus, the level of technology transfer to the joint venture (measured by $s_{JV}$) is an increasing function of the MNE’s share ($\alpha$).

The MNE’s problem with respect to $k$ is consistent with the local firm’s problem. Note that, in (14) and (15), the joint venture’s expected total profit

$$\Pi_{JV}(k) \equiv AF(k) - \gamma + C(s) - RF(k) + (1 + r)(w + b - k)$$

is given in the square bracket. This implies that, when the MNE and the local firm choose $k$ to maximize their expected profits, there are no conflicts between them. As in Section 2, the joint venture must satisfy the constraint (3) and the finance constraint $w + b \geq k$. From these two constraints, which are binding as shown in the Appendix, we obtain

$$R = \frac{1 + r}{F(k)}(k - w). \quad (17)$$

This equation is the constraint that lenders must receive the market rate of return (as in Section 2), and is drawn as $MR_{JV}$ in Figure 1. The curve $MR_{JV}$ is upward sloping and intersects the horizontal axis at $k = w$. As illustrated in Figure 1, $w_M - P < w$ and $w_L < w$ by definition.

As in the analysis above, the solution of the joint venture’s problem must satisfy the incentive-compatibility constraint (5) and the constraint given in (17). Let $k_{JV}$ denote the investment level $k$ that satisfies both (5) and (17). We obtain $k_{JV}$ from the intersection of IC and $MR_{JV}$ in Figure 1. Note that the level of investment is an increasing function of the borrower’s net worth as shown in Section 2. Here, $k_{JV}$ is larger than $k_M$ and $k_L$ as shown in Figure 1 (because $w_M - P < w$ and $w_L < w$).

We now examine whether this partnership is feasible. This joint venture is feasible only if both the MNE and the local firm agree on a joint-venture contract. When the two firms agree on a joint-venture contract, the terms of the contract must
satisfy the following conditions: (M) the MNE’s expected profit from a joint venture \((\Pi^M_{JV})\) is larger than the expected profits from other feasible organization forms; (L) the local firm’s expected profit from a joint venture \((\Pi^L_{JV})\) is not smaller than the expected profits from other feasible organization forms. The condition (M) can be divided into three classes as follows: (M1) \(\Pi^M_{JV}(k_{JV}, s_{JV}) > (1 + r)w_M\) when wholly owned subsidiaries are not allowed in the host country; (M2) \(\Pi^M_{JV}(k_{JV}, s_{JV}) > \Pi^WOS_M(k_M, s^*) \geq (1 + r)w_M\); (M3) \(\Pi^M_{JV}(k_{JV}, s_{JV}) > (1 + r)w_M > \Pi^WOS_M(k_M, s^*)\).

The condition (L) can be divided into two classes: (L1) \(\Pi^L_{JV}(k_{JV}, s_{JV}) \geq \Pi^L_L(k_L)\); (L2) \(\Pi^L_{JV}(k_{JV}, s_{JV}) \geq (1 + r)(P + w_L)\).

We first examine the MNE’s decision in the case where wholly owned subsidiaries are not allowed. The following proposition states that (M1) holds for any \(\alpha\).

**Proposition 2** When wholly owned subsidiaries are prohibited, the MNE chooses the joint-venture contract instead of lending abroad.

**Proof:** The right-hand side of inequality (M1) represents the MNE’s minimum income (the MNE earns \((1 + r)w_M\) by lending abroad). From (3) and (14), the condition (M1) can be rewritten as \(AF(k_{JV}) - \gamma + C(s_{JV}) - (1 + r)k_{JV} - s_{JV}/\alpha > 0\). Note that \(C(s_{JV}) - s_{JV}/\alpha > 0\) for any \(\alpha\) (because \(\lim_{s \to 0} C'(s) = \infty\)) and \(k_{JV} > k_L\). Hence, we obtain \(AF(k_{JV}) - \gamma + C(s_{JV}) - (1 + r)k_{JV} - s_{JV}/\alpha > AF(k_L) - \gamma - (1 + r)k_L\).

The right-hand side (which is equal to \((1 + r)P\)) is positive by assumption.

Thus, the MNE prefers a joint venture regardless of its share because it increases efficiency through technology transfer (which is represented by \(C(s_{JV}) - s_{JV}/\alpha > 0\)) although, as we will see, the local firm may not sign the joint-venture contract.

Next we examine the MNE’s decision in the case where wholly owned subsidiaries are allowed. The following proposition shows that there exists a share \(\alpha\) (the ratio of \(w_M\) to \(w\)) such that (M2) holds.

**Proposition 3** The MNE’s expected profit from a joint venture can be higher than the expected profit from a wholly owned subsidiary even if the wholly owned subsidiary
is an economically feasible organization form (i.e., the expected profit from the wholly owned subsidiary is greater than or equal to the minimum income).

**Proof:** First, we examine $\Pi_{M}^{WOS}(k_M, s^*) \geq (1 + r)w_M$. This inequality, which can be rewritten as (13), holds if $w_M$ is large enough (see Proposition 1). Let $\alpha^e$ denote the value of $\alpha$ (i.e., the ratio of $w_M$ to $w$) that satisfies (13) with equality. Second, we examine $\Pi_{M}^{JV}(k_{JV}, s_{JV}) > \Pi_{M}^{WOS}(k_M, s^*)$. By subtracting $(1 + r)w_M$ from both sides, we obtain $\Pi_{M}^{JV}(k_{JV}, s_{JV}) - (1 + r)w_M > \Pi_{M}^{WOS}(k_M, s^*) - (1 + r)w_M$. From (3) and (14), the left-hand side can be rewritten as $\alpha[AF(k_{JV}) - \gamma + C(s_{JV}) - (1 + r)k_{JV}] - s_{JV}$, which is strictly positive for any $\alpha$ (see Proposition 2). At $\alpha = \alpha^e$, the right-hand side is equal to zero and hence $\Pi_{M}^{JV}(k_{JV}, s_{JV}) - (1 + r)w_M > \Pi_{M}^{WOS}(k_M, s^*) - (1 + r)w_M = 0$ holds. Therefore, (M2) holds for $\alpha^e$. Note that the same argument is applicable to some slightly larger $\alpha$.14

Thus, the MNE prefers a joint venture for some $\alpha$ even if it is possible to acquire the project because the partnership reduces the financial burden and this benefit can outweigh the cost of joint ownership. The next proposition states that (M3) holds for $\alpha \in (0, \alpha^e)$.

**Proposition 4** The MNE’s expected profit from a joint venture is higher than the expected profit from a wholly owned subsidiary when the wholly owned subsidiary is an economically infeasible (unprofitable) organization form.

**Proof:** For $\alpha \in (0, \alpha^e)$, $(1 + r)w_M > \Pi_{M}^{WOS}(k_M, s^*)$ from Proposition 1 while $\Pi_{M}^{JV}(k_{JV}, s_{JV}) > (1 + r)w_M$ from Proposition 2. Hence, (M3) holds for $\alpha \in (0, \alpha^e)$.

Thus, because of technology transfer, the joint-venture contract is desirable for the MNE that cannot acquire a project.

Finally, we examine the local firm’s decision. The condition (L1) implies that the local firm’s expected profit from a joint venture is not less than that of independent
operation. From (3), (4) and (15), (L1) can be rewritten as

\[
(1 - \alpha) [AF(k_{JV}) - \gamma + C(s_{JV}) - (1 + r)k_{JV}] + (1 + r)w_L \\
\geq AF(k_L) - \gamma - (1 + r)k_L + (1 + r)w_L. \tag{18}
\]

Note that \( \lim_{\alpha \to 0} \Pi^J_L(k_{JV}, s_{JV}) = \Pi_L(k_L) \) because \( \lim_{\alpha \to 0} k_{JV} = k_L \) and \( \lim_{\alpha \to 0} s_{JV} = 0 \). The right-hand side of (18) can be rewritten as \((1 + r)(P + w_L)\) from (11). Hence, (L1) is identical to (L2), which implies that the local firm’s expected profit from a joint venture is not less than the profit from selling the project.\( ^{15} \)

Then, we obtain the following:

**Proposition 5** There exists a share \( \alpha \) such that the local firm agrees on the joint-venture contract.

**Proof:** The right-hand side of (18) (or \( \Pi_L(k_L) \)) is a decreasing function of \( \alpha \) (since \( dk_L/dw_L > 0 \) and \( w_L = (1 - \alpha)w \)). By differentiating the left-hand side, we obtain

\[
d\Pi^J_L/\alpha = -[AF(k_{JV}) - \gamma + C(s_{JV}) - (1 + r)(k_{JV} - w)] + (1 - \alpha)C'\partial s_{JV}/\partial \alpha,\]

\( \lim_{\alpha \to 0} d\Pi^J_L/\alpha = \infty \), and \( \lim_{\alpha \to 1} d\Pi^J_L/\alpha = 0 < \lim_{\alpha \to 1} \Pi_L(k_L) \) (since \( AF(k_{JV}) - \gamma - (1 + r)k_{JV} > 0 \) for any \( w_L \geq 0 \) by assumption).

From these results, \( \Pi^J_L(\alpha) > \Pi_L(\alpha) \) holds for sufficiently small values of \( \alpha \).

To simplify the analysis, we suppose \( C(s) = s^\delta \), where \( 0 < \delta < 1 \). Then, \( d^2\Pi^J_L/\alpha^2 < 0 \) holds for any \( \alpha \) as depicted in Figure 2 because \( \partial s_{JV}/\partial \alpha > 0 \) and \( \partial^2 s_{JV}/\partial \alpha^2 < 0 \) from (16). Let \( \overline{\alpha} \) denote \( \alpha \in (0, 1) \) such that \( \Pi^J_L = \Pi_L \) (i.e., (18) holds with equality). In this case, there is a unique \( \overline{\alpha} \), and \( \Pi^J_L \geq \Pi_L \) if \( \alpha \leq \overline{\alpha} \) and \( \Pi^J_L < \Pi_L \) if \( \alpha > \overline{\alpha} \). This reflects the fact that the marginal effect of \( s \) is very large for a small \( \alpha \) and decreases as \( \alpha \) rises (the profit share for the local firm also decreases as \( \alpha \) rises).

Hence, the local firm can benefit from joint ownership as long as the MNE’s share is not too large. Note that the maximum acceptable share \( \overline{\alpha} \) depends positively on the impact of the MNE’s input \( s \) (\( C(s_{JV}) \)).

Propositions 2 and 5 immediately imply the following:
Proposition 6  When wholly owned subsidiaries are not allowed, the two firms agree on the joint-venture contract for any \( \alpha \in (0, \overline{\alpha}] \).

This proposition suggests that the host country’s policy (such as local ownership requirement) can protect the interests of local firms by forcing foreigners to choose joint ventures (see (L2)). Nevertheless, even without the local ownership policy, the MNE chooses joint ventures under certain conditions; Propositions 3-5 imply that it can be optimal for the MNE and the local firm to choose joint ventures.

Proposition 7  When wholly owned subsidiaries are allowed, there exists a share \( \alpha \) such that the two firms agree on the joint-venture contract.

As shown in Figure 3, the two firms agree on the contract when the MNE’s share is small; then the MNE can benefit from the local firm’s funds and the local firm can take advantage of the MNE’s technology. Note that the result can be divided into two cases: (a) if \( \alpha^e \leq \overline{\alpha} \), the two firms agree on the joint-venture contract at \( \alpha^e \) (and \( \alpha \) that is slightly larger than \( \alpha^e \) if \( \alpha^e < \overline{\alpha} \)) because (M2) and (L1) hold, and they agree on the contract also for any \( \alpha \in (0, \alpha^e) \) because (M3) and (L1) hold; (b) if \( \overline{\alpha} < \alpha^e \), the two firms agree on the joint-venture contract for any \( \alpha \in (0, \overline{\alpha}] \) because (M3) and (L1) hold. The implication of this will be discussed in Section 5.

We have treated the MNE’s share \( \alpha \) as an exogenous variable (i.e., we have assumed that each firm cannot choose its own share freely) and examined whether two firms agree on the joint-venture contract for a given \( \alpha \). In the model, firms cannot choose their net worth although they can choose the level of investment by borrowing. In practice, however, firms can choose the percentage of ownership in joint ventures. For example, without internal funds, a parent firm can increase its share \( \alpha \) by borrowing. In the following, we will examine whether either shareholder (the MNE or the local firm) will increase its stake in the joint venture by borrowing.

First, we examine whether the MNE will increase \( \alpha \) when \( w_L \) is given and whether the local firm will accept the MNE’s offer. Suppose that the MNE increases the share by borrowing \( v \). Then, \( \alpha = (w_M + v)/(w + v) \), where \( w \) is fixed as before. Thus, \( \alpha \) is
an increasing function of \( v \). In this case, the finance constraint for the joint venture is \( w + b \geq k + v \) and the constraint (17) is replaced by \( RF(k) = (1 + r)(k + v - w) \). Then, in Figure 1, an increase in \( v \) shifts MR\textsubscript{JV} to the left and hence \( k_{JV} \) decreases. At the same time, however, a rise in \( \alpha \) has a positive effect on \( s_{JV} \). Because of the negative effect of a decrease in \( k_{JV} \) and the positive effect of an increase in \( s_{JV} \) on the joint venture’s total expected profit, it is not clear whether a rise in \( \alpha \) (by borrowing) has a net positive impact. Note that the local firm will accept the MNE’s offer only if it has a net positive impact. This implies that, only if \( \alpha \) is small enough (because \( \partial s_{JV} / \partial \alpha > 0 \) and \( \lim_{s \to 0} C'(s) = \infty \)), the local firm will accept the MNE’s offer, and the MNE is able to increase its stake in the joint venture by borrowing. Note also that the MNE wishes to increase \( \alpha \) only if the negative effect of a decrease in \( k_{JV} \) is small enough (when \( \alpha \) is small enough, the effect of increases in the profit share and the input \( s_{JV} \) becomes larger).

Second, we examine whether the local firm will increase the share \((1 - \alpha)\) by borrowing when \( w_M \) is given and whether the MNE will accept the local firm’s offer. As in the case of the MNE, an increase in its share decreases \( k_{JV} \) by changing the finance constraint for the joint venture. Moreover, it decreases \( s_{JV} \) in this case by decreasing \( \alpha \). Hence, a rise in \((1 - \alpha)\) decreases the joint venture’s total expected profit. Then, the MNE will never accept the local firm’s offer. Note that the local firm wishes to increase its share only if the negative effect of a decrease in \( s_{JV} \) is small enough (i.e., \((1 - \alpha)\) is small enough).

These results show that: (1) only if the share is very small, the MNE (the local firm) wishes to increase the share by borrowing; (2) the local firm may accept the MNE’s offer because it positively affects the total expected profit through technology transfer while the MNE will never accept the local firm’s offer.

5 Control and ownership

In practice, many foreign investors are minority shareholders in their joint ventures with local firms. Does this mean that parent firms have less control over the joint
ventures than local firms? (The answer is probably no.) In this section, we examine whether the MNE with a minority share can control the joint venture, and investigate the minimum ownership share required to control the joint venture. In the following, using the model developed above, we will show that, even if the MNE is the minority shareholder (and then the local firm is the majority shareholder), it is possible for the MNE to control the joint venture.

First, we discuss the term “control” in our model. If the MNE (the local firm) has control of the joint venture, the MNE (the local firm) has the power to make all the important decisions about the joint venture’s operation. In the model developed above, since there are no conflicts between the MNE and the local firm, having “control” is insignificant for both firms ($k$ is chosen by both firms to maximize $\Pi^{JV}(k)$; $s$ is unverifiable; and the profit share is equal to the percentage of ownership). In practice, however, several factors may create a conflict between partner firms, and hence they wish to have control of the joint venture. For example, when two firms establish a joint venture, each firm wishes to have the right to decide the new company name, the location of headquarters, and the board of directors. Thus, even if the firms agree on input levels (such as $k$) and profit shares, they will try to have substantial control of the joint venture. Here we do not explicitly consider these causes of conflicts, but we assume that there are terms of the contract, which may create a conflict between the MNE and the local firm before they start the operation, other than the input levels and the profit share. In other words, we assume that the terms of the joint-venture contract cannot be spelt out in full detail and that only the input $k$ and the profit share $\alpha$ can be specified in the contract.

Now we can define the controlling shareholder in the model. The controlling shareholder is a firm that has the right to decide unwritten terms and conditions of a joint-venture contract. It implies that the controlling shareholder has the residual rights of control, which are discussed in the literature on contractual incompleteness (see, for example, Grossman and Hart (1986)). In standard models of incomplete contracts, the owner obtains the residual rights of control; however, in our model,
because of joint ownership, the owner does not necessarily have the residual rights. Joint-venture models such as Svejnar and Smith (1984) and Nakamura and Xie (1998) (which use the Nash bargaining solution) consider the parent firm’s ownership share or profit share; however, these models explain the firm’s share by treating the firm’s bargaining power as an exogenous variable. Hence, some questions such as what gives control and bargaining power to the firm remain to be answered. In this section, we will make clear under what circumstances the parent firm has greater bargaining power, and more specifically the minimum ownership share required to control a joint venture, by focusing on threat points endogenously determined.

In the joint-venture model, we define a threat point as the maximum possible gain without the joint-venture contract. If the MNE is able to purchase the project (i.e., if the MNE’s net worth is large enough), the MNE’s threat point is the expected profit from the wholly owned subsidiary ($\Pi_{WOS}^{M}(k_{M}, s^*)$); if not, it is the minimum income from lending abroad ($((1 + r)w_{M})$). Thus, the MNE’s threat point is larger when the MNE is rich enough as seen in Proposition 1. Suppose that there is a conflict between the MNE and the local firm in period 1 (after they agree on a joint-venture contract). Then, the MNE becomes more patient when it is feasible to establish a wholly owned subsidiary (by purchasing the project from the local partner opposing the MNE’s policy) than when it is not. This patience (the larger threat point) gives greater bargaining power to the MNE. On the other hand, the local firm’s threat point is the expected profit from independent operation. Thus, without the joint-venture contract, the local firm inevitably would have the minimum income $\Pi_{L}(k_{L})$. Hence, in this model, it is fair to say that the MNE is the controlling shareholder if it is profitable for the MNE to purchase the project; more precisely, the MNE has control of the joint venture if the inequality (M2) holds. Note that, as discussed in Section 4, (L1) does not necessarily hold; as shown in Figure 3, only if $\alpha^e \leq \bar{\alpha}$, the joint venture controlled by the MNE exists (because the local firm never accepts the MNE’s share larger than $\bar{\alpha}$). Thus, (only) in Figure 3(a), there are two joint-venture types: the joint venture for $\alpha \in (0, \alpha^e)$ and that for $\alpha$ equal to
\( \alpha^e \) or larger. The former is characterized by (M3) and the latter satisfies (M2). We denote the latter (joint ventures controlled by the MNE) by \( JV_M \).

The discussion above suggests that \( \alpha^e \) is the minimum ownership share required to control a joint venture. We now show that the level of the MNE’s technology (the degree of technological advantages) determines \( \alpha^e \). From (13), we know that \( \alpha^e \) (which satisfies (13) with equality) is small when \( C(s^*) - s^* \) is large. Note that \( C(s^*) - s^* \) represents the impact of the MNE’s cost-reducing technology on the wholly owned subsidiary. On the other hand, from (18), we know that \( \overline{\alpha} \) (which satisfies (18) with equality) is large when \( C(s_{JV}) \) is large. Note that \( C(s_{JV}) \) represents the impact of the MNE’s cost-reducing technology on the joint venture. Accordingly, if the impact of the MNE’s cost-reducing technology is large enough, then \( \alpha^e \leq \overline{\alpha} \) holds as in Figure 3(a).

This result sheds some light on the relationship between technology and ownership. Although the conventional theory suggests that MNEs prefer full ownership to protect their proprietary assets, empirical support for this view is weak; several measures of R&D fail to have a significant impact on ownership choice (see Hen-nart (1991) and Blomstrom and Zejan (1991)). Our model provides an explanation for this. As shown in Figure 3, the level of technology has a positive effect on the probability of full ownership by decreasing \( \alpha^e \); it also has a positive effect on the probability of joint ownership by increasing \( \overline{\alpha} \) (see the case (b)). Thus, our model suggests that advanced technology does not necessarily lead to full ownership. We summarize the discussion in this section as follows.

**Proposition 8** If the impact of the MNE’s cost-reducing technology is large enough \( (\alpha^e \leq \overline{\alpha}) \), there exists an ownership share \( \alpha \) such that the two firms agree to establish the joint venture controlled by the MNE.

Thus, a \( JV_M \) can be established only if the MNE has strong technological advantages; then, the MNE with \( w_M \) that satisfies \( \alpha \geq \alpha^e \) can be the controlling shareholder. This is due to the fact that advanced technology reduces the MNE’s financial burden, and then the MNE becomes more patient. It is important to note that the MNE
does not have to be the majority shareholder to control the joint venture because \( \alpha \) can be smaller than 0.5.

In the real world, it seems likely that investors from developed countries are richer and use more advanced technology than local firms in developing countries (in contrast, local firms in developed countries often have advanced technology and foreigners acquire them to utilize their technology). If this is the case, many joint ventures in developing countries will be controlled by foreigners; then the local-ownership policy will give the bargaining power to the local firms (by lowering the MNEs’ threat point) although it has a negative effect on technology transfer.

6 Conclusions

Many scholars believe that firms with proprietary technology start foreign operations, and that FDI is an important vehicle for the transfer of technology. To examine these assertions in the conventional literature, we have developed a formal model of foreign operations by introducing explicitly the incentive to transfer technology and the percentage of ownership. Our model focuses on the impact of imperfect capital markets because the cost of external funds, which is higher than that of internal funds, affects the level of investment in the real world.

Our main conclusions are: (1) the firm’s net worth is a key determinant of the ownership structure of foreign subsidiaries, and joint ventures are established when the parent firm’s net worth is relatively small; (2) the firm with strong technological advantages can be the controlling shareholder of minority owned ventures. Thus, the model explains not only forms of foreign operations but also the relationship between control and ownership. Although the traditional FDI theory implies that technologically-oriented firms prefer large ownership shares, there seem to be no simple relationship between technological advantages and ownership shares in the real world as shown in Hennart (1991) and Blomstrom and Zejan (1991). Our second result can help us understand why parent firms sometimes take minority positions and reconcile the observation and the theory of FDI.
Notes

1 For this explanation for FDI and a rationale for the existence of multinational enterprises, see Hymer (1960), Caves (1971), Buckley and Casson (1976), Dunning (1977) and Magee (1977).

2 Froot and Stein (1991) present a model in which relative wealth affects FDI, and find evidence of the implied relationship between the value of the dollar and FDI flows into the United States. A recent study by Stevens (1998), however, suggests that the empirical support for the connection between exchange rates and FDI is weak.

3 Blomstrom and Zejan (1991) investigate the effects of the market size and the firm size on the ownership structure of foreign operations.

4 See, for example, Financial Times, December 1, 2000.

5 Neven and Siotis (1996) state that technological sourcing is an important motive behind FDI from the United States and Japan to Europe. Thus, advanced technology of local firms can be an important determinant of FDI. In this paper, however, we focus on the local asset that can be transferred with low transaction costs (e.g., a permit from the authorities, land, facilities).

6 For formal models of international joint ventures, see Svejnar and Smith (1984), Marjit, Broll, and Mallick (1995), Al-Saadon and Das (1996), Mukherjee and Sengupta (2001), and Roy Chowdhury and Roy Chowdhury (2001). Most of them focus on the effect of host government policies on joint ventures in developing countries. In our model, however, the country’s policy and level of development do not play any role. We focus on the organization structure chosen by two firms from different countries.

7 Ramachandran (1993) empirically examines the effect of ownership structures on the incentives of parent firms to transfer technology, and confirms that the equity share positively affects the incentive to transfer technology.

8 As in the traditional FDI theory of Hymer (1960), we suppose (1) a potential direct investor with some monopoly power (at least in the home country) arising from
proprietary technology, and (2) a local partner who owns market-specific assets that play an important role in the success of horizontal FDI. Without these two features, our model describes joint ventures between two domestic firms.

9 None of the results below would change if payments were positive in the bad state. As in Gertler and Rogoff (1990), if the borrower has verifiable incomes after the realization of output, the payment in the bad state can be positive. However, Gertler and Rogoff show that the level of investment $k$ depends only on the lifetime wealth (the endowment $w_L$ in this model) and that a positive repayment in the bad state does not matter.

10 The slope of MR$^L$ is $dR/dk = \{1-[1-(w_L/k)]F'(k)/(F(k)/k)\}(1+r)/F(k)$. Note that $0 < 1 - (w_L/k) < 1$ (since $w_L < k$ as shown in the Appendix) and $0 < F'(k)/(F(k)/k) < 1$ (since $F(k)$ is strictly concave).

11 Ramachandran (1993) uses the number of foreign personnel sent to the host country (per technology transfer agreement) and that of workers trained abroad as measures of costs of technology transfer.

12 Throughout this paper, we assume that the local firm owns a project and the MNE can purchase it. In addition, we assume that the MNE can choose the level of unverifiable input $s$; this reflects the fact that foreign firms transfer unembodied knowledge to local firms (as in the traditional FDI theory). Alternatively, we could assume that the MNE owns a project and the local firm with input $s$ can purchase it (i.e., the two firms simply exchange their positions); then, we could examine the MNE’s choice between selling the project (which can be interpreted as technology embodied in physical items) to the local firm and establishing a subsidiary (or a joint venture, as we will see in Section 4). In this case, selling the project can be interpreted as licensing while our qualitative results (the effect of net worth and the optimality of joint ventures) would not be affected by this change. I am indebted to an anonymous referee for the discussion above.

13 This implies that the MNE has the ex ante bargaining power because there are many potential takeover targets.
\[ \Pi_M^{JV}(k_{JV}, s_{JV}) > (1 + r)w_M \] for any \( \alpha \) while \( \Pi_M^{WOS}(k_M, s^*) > (1 + r)w_M \) for \( \alpha \in (\alpha^e, 1) \). Since \( \lim_{\alpha \to \alpha^e} \Pi_M^{WOS}(k_M, s^*) = (1 + r)w_M \), (M2) holds for a slightly larger \( \alpha \).

15 As we have assumed, if the MNE has all the ex ante bargaining power, the project price \( P \) is the lowest value as in (11). If not, \( P \) will be higher under Nash bargaining; then the price depends positively on the local firm’s bargaining power, the MNE’s net worth \( w_M \), and the technology level \( C(s^*) - s^* \). In this case (even if they split the surplus evenly), the local firm prefers selling the project rather than independent operation (i.e., \( \Pi_L(k_L) < (1 + r)(P + w_L) \)), and (L1) and (L2) are no longer identical. However, as long as \( P \) is not too large, (as we will see) there exists a share \( \alpha \) such that the local firm agrees on the joint-venture contract (i.e., \( \Pi_L^{JV} > (1 + r)(P + w_L) \)) because \( \lim_{\alpha \to 0} d\Pi_L^{JV}/d\alpha = \infty \) as depicted in Figure 2. Hence, the qualitative nature of the results is preserved.

16 Note that, in our specification, if \( \delta \) rises, then both \( C(s_{JV}) \) and \( C(s^*) - s^* \) increase.

**Acknowledgements**

I would like to thank Kenzo Abe, Tadashi Inoue, Hiroshi Ohta and two anonymous referees for helpful comments. This research was partially supported by Japan Society for the Promotion of Science, Grant-in-Aid for Encouragement of Young Scientists 12730018.
Appendix

As in Gertler and Rogoff (1990), we consider the borrower’s formal problem. We prove that, if \( w_L < k^* \), (2) and (3) are binding and \( w_L < k < k^* \) holds. The borrower solves:

\[
\max_{k,b,R} \text{(4)} \quad \text{subject to (2), (3), and (5)}.
\]

The Lagrangian for the borrower’s problem is

\[
\mathcal{L} = AF(k) - \gamma - RF(k) + (1 + r)(w_L + b - k) \\
+ \lambda(w_L + b - k) + \psi[RF(k) - (1 + r)b] + \mu[(A - R)F'(k) - (1 + r)].
\]

Here, \( \lambda \) is the Lagrange multiplier on the constraint (2), \( \psi \) that on (3), and \( \mu \) that on (5). The Kuhn-Tucker necessary conditions can be written as

\[
\lambda = \psi RF'(k) + \mu (A - R) F''(k),
\]

\[
(1 + r)(1 - \psi) + \lambda = 0,
\]

\[
F(k)(\psi - 1) = \mu F'(k),
\]

\[
\lambda(w_L + b - k) = 0,
\]

\[
\psi[RF(k) - (1 + r)b] = 0,
\]

where (i), (ii), and (iii) are the first-order conditions for \( k \), \( b \), and \( R \), respectively, and (iv) and (v) are complementary slackness conditions.
**Proposition A1.** (2) is binding.

Proof: We prove $\lambda > 0$ by contradiction. Suppose $\lambda = 0$. Then, $\psi = 1$ from (ii). This implies that, from (i), $\mu$ must be positive (since $F' > 0$ and $F'' < 0$). However, this means that (iii) cannot be satisfied unless $F'' = 0$, which is ruled out. We see that the assumption $\lambda = 0$ leads to a contradiction. From (iv), $\lambda > 0$ implies that (2) is binding.

**Proposition A2.** (3) is binding.

Proof: Since $\lambda > 0$, from (ii), $\psi > 1$, which implies that (3) is binding from (v).

**Proposition A3.** $k < k^*$

Proof: We prove $k < k^*$ by contradiction. Suppose $k \geq k^*$. From (2), $b > 0$ (since $w_L < k^*$ by assumption). Then, from (3), $R > 0$. From (5), this implies $k < k^*$, which leads to a contradiction.

**Proposition A4.** $w_L < k$

Proof: Since $k < k^*$, from (5), $R > 0$. This implies $b > 0$ from (3) and $b = k - w_L > 0$ from (2).
References


