Abstract

This paper demonstrates that higher category prices and higher advertising are consistent with markets where low-priced private labels have become more important. In our model, the private label is a version of a national brand without the perceived quality enhancement provided by advertising. The unadvertised private label allows a monopolistic channel to price discriminate between customers who want advertised brands and those who do not. This can lead to either increases or decreases in average category prices. When advertising creative and media costs are high, the model predicts that increased private label availability leads to higher average category prices.

Key Words: Private labels, store brand, retailer brand, national brand, second-degree price discrimination

JEL Classification Codes: D21, L22
1 Introduction

Many retail categories have witnessed the rapid growth of low-priced private label versions of products.¹ These products are priced lower than nationally advertised brands and frequently offer equivalent quality (Hinloopen and Martin 1997, Connor and Peterson 1997). A recent publication by the European Commission notes that “The role of the retailer brand has changed, particularly in the food sector. The original market position of these food brands was a low price/lower quality alternative to manufacturer brands; but... they have been repositioned, their quality improved and are increasingly associated with new product launches.”² This has been facilitated in large part by the increased willingness of national brand manufacturers to supply quality-equivalent private labels.

In spite of the growth of these low-priced alternatives to advertised brands, the average price level in many categories has increased. In fact, recent empirical research has shown that in many categories where “quality-equivalent” private labels have grown, average market prices have increased and not decreased as one might expect (e.g. Connor and Peterson 1992).

We propose a simple model to explain these observations based on the idea that a private label is a version of a national brand without the perceived quality enhancement provided by advertising. The unadvertised private label allows a channel with monopoly power to discriminate between customers who want advertised brands and those that do not. We show that this can lead to increases or decreases in average category prices. However, in a context where advertising is costly, the model predicts that increased availability of private label will lead to higher (and not lower) category prices.

A starting point for our analysis is the observation that some consumers are willing to pay more for advertised products. One potential explanation for this premium is the objective familiarity effect identified in the behavioral literature (Wilson 1979, Obermiller 1985). Consumers are observed to exhibit more positive attitudes towards objects with which they are familiar and advertising familiarizes consumers with brands. Other arguments to explain why consumers might pay more for an advertised product include enhancing product value by creating pleasurable associations with consuming the product (Cafferata and Tybout

¹From negligible levels in the 1950s, private label brand sales represented by the mid-1990s approximately 18 percent of the U.S. retail market (Liesse 1993a, 1993b).
²A recent study for the U.S. Private Label Manufacturers Association (PLMA) found that 60% of consumers surveyed believe that private label products are the same as manufacturers’ brands when it comes to the overall quality of the products, taste, availability, freshness, guarantee of satisfaction, clarity of labeling, and the quality of packaging among other attributes.
Eaton and White (2001) also show how the image created for a product (through advertising) can provide value to consumers by allowing them to signal information about themselves in a social context. The empirical literature also suggests that the premium some consumers are willing to pay for a product is closely related to how aggressively it is advertised. Connor and Peterson (1992) study hundreds of grocery items and find that prices are higher in categories with higher levels of advertising by the national brands. In addition, Connor and Peterson find that the “gap” between national brands and private labels is higher, the higher are category advertising levels.

A focus of advertising research has been to understand how advertising leads to increased willingness to pay; however, a second interesting effect of advertising activity is that of creating customer heterogeneity in terms of willingness to pay. Because not all consumers respond to advertising by increasing their willingness to pay, a situation where some consumers are willing to pay more than others for advertised products is created: advertising creates a market with psychologically-based vertical differentiation.3

When a channel has market power, the creation of psychologically-based differentiation creates an incentive to launch a second unadvertised version of the same product. Such a product provides the channel with the opportunity to offer two choices that are better tailored to the needs of the market. The notion of private labels as a vehicle for price discrimination is not new. It was first mentioned by Wolinsky (1987) where private labels allow a manufacturer to charge higher prices to those consumers who prefer its product while offering unlabeled products for those consumers who are not willing to pay the premium. The addition of an unadvertised version of the name-brand product to the product line is a form of second degree price discrimination by which consumers self select to the product that provides the most utility (Maskin and Riley 1984).

Our objective is to extend this idea by examining how the addition of an unadvertised brand affects average category prices when the channel makes a strategic decision (advertising) that endogenously determines the level of premium that a segment of consumers are willing to pay.4 The paper is thus related to the work of Varian (2000) and Shapiro and Varian (1998, 1999) regarding the versioning of information goods. The fundamental idea is

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3 We distinguish here between vertically differentiated markets where differences in valuation are based on evaluation of the physical product (Shaked and Sutton 1982).

4 Similar to Tremblay and Polasky (2002), the model captures the effect of national brand advertising on a specific segment in terms of their willingness to pay for the national brand.
that a firm or channel can increase its profits by designing versions of its products tailored to the needs of different customers.

The paper proceeds as follows. In the next section, we present a simple model to analyze the impact that the addition of an unadvertised quality equivalent brand has on category pricing. In the third section, we present the results of our analysis and also examine the impact that the versioning of typical consumer products has on total welfare. Finally, in the fourth section, we conclude and discuss the limitations of our analysis.

## 2 The Model

The model consists of a monopolistic channel that faces a unitary market consisting of two segments of consumers. The first are brand seekers (denoted by superscript \(b\)) and the others are product seekers (denoted by superscript \(p\)). A fraction \(\lambda\) of the market are brand seekers and \(1-\lambda\) are product seekers. Brand seekers are willing to pay a premium for the national brand that is related to \(A\), the advertising effort implemented by the channel. In contrast, product seekers’ willingness to pay is unaffected by advertising. The cost of advertising is assumed to be quadratic \(\gamma A^2\) and \(\gamma\) is a cost parameter that reflects the relative expense of advertising. Three key assumptions underlie the model we propose:

A1. Private labels are identical to advertised brands except that brand seekers are willing to pay more for advertised brands.

A2. The monopolistic channel is the sole supplier of products that are quality equivalent. In other words, we assume that only the channel that produces the national brand is capable of producing brands that are quality equivalent.\(^5\)

A3. The launching, stocking and management of a private label is associated with a fixed cost that is unrelated to volume, \(k_g\).

The game consists of three stages. In the first stage, the channel, which already offers an advertised product, decides whether to add an unadvertised version of the product to its line. After the product line decision, the channel sets \(A\), the level of advertising for the advertised brand. In the final stage, the manufacturer sets the prices for the product in its line (\(p_n\) for

\(^5\)The model does not preclude the sale of lower quality products in the same category. Implicitly, the model is focused on those consumers who restrict their purchases to products that are of “national brand” quality.
the national brand and $p_g$ for the unadvertised brand). Sales and profits are realized in the channel based on the quantities purchased by each of the two segments in the market (brand and product seekers). To simplify the analysis without loss of generality, we normalize the marginal cost of both the national brand and the private label to 0.

When only the national brand is available, we characterize demand from the two segments in the market as follows. The demand from brand seekers is linear in the price charged and advertising increases the level of demand at any price, $q_b = A + 1 - p_n$. For product seekers, demand is simply a function of the price, $q_p = 1 - p_n$. The absence of $A$ in the demand function from product seekers reflects the fact that they are only interested in the physical characteristics of the product (for product seekers, no increase in willingness to pay is created by advertising).

When the channel offers both the national brand and an unadvertised brand, demand from brand seekers is unchanged (they continue to buy the national brand). Conversely, product seekers move to the unadvertised brand (because in equilibrium $p_n > p_g$) and their demand is given by $q_p = 1 - p_g$.

The structure described above leads to the following objective function for the channel when only the national brand is available:

$$\pi = p_n (\lambda (A + 1 - p_n) + (1 - \lambda)(1 - p_n)) - \gamma A^2 \text{ when } p_n < 1$$
$$\pi = p_n (\lambda (A + 1 - p_n) - \gamma A^2 \text{ when } p_n > 1$$

When the channel adds an unadvertised brand to its product line, the objective function is:

$$\pi = p_n (\lambda (A + 1 - p_n) + (1 - \lambda)(1 - p_g)) - \gamma A^2 - k_g$$

In the following section, we present the optimal pricing strategy for the channel under both scenarios. We then use these results to derive average category pricing.

### 3 Results

#### 3.1 Equilibrium Advertising and Pricing Policy as a Function of Product Line

When the channel only distributes one brand, equation 1 is maximized with respect to $A$ and $p_n$. First, we derive the optimal price as a function of $A$. This expression is then used to solve for the optimal level of advertising. In the allowable zone for $\gamma$ (i.e. where second
order conditions are satisfied), the reduced-form profit function has two local maxima when \( \lambda < \frac{1}{2} \). Thus, when \( \lambda < \frac{1}{2} \), the profits at each maxima are compared to determine the optimal policy, \((A, p_n)\). The results are summarized in Lemma 1 (the complete solution is found in the appendix).

**Lemma 1**

1. When \( \lambda < \frac{1}{2} \) and \( \gamma > \frac{\lambda}{4(1-\lambda)} \), the channel serves both segments with \((A, p_n)\) equal to \((0, \frac{1}{2})\). When \( \gamma \in \left(\frac{\lambda}{4}, \frac{\lambda}{4(1-\lambda)}\right) \), the optimal strategy \((A, p_n)\) is \(\left(\frac{\lambda}{4\gamma-\lambda}, \frac{2\gamma}{4\gamma-\lambda}\right)\) and product seekers are not served.

2. When \( \lambda > \frac{1}{2} \) and \( \gamma > \frac{1}{2} \), the channel serves both segments with \((A, p_n)\) equal to \(\left(-\frac{1+2\lambda}{\lambda+4\gamma}, 2\gamma\left(-\frac{1+2\lambda}{\lambda+4\gamma}\right)\right)\). When \( \gamma \in \left(\frac{1}{4}, \frac{1}{2} \lambda\right) \), the optimal strategy \((A, p_n)\) is \(\left(\frac{\lambda}{4\gamma-\lambda}, \frac{2\gamma}{4\gamma-\lambda}\right)\) and product seekers are not served.

In order to clarify the implications of Lemma 1, we illustrate the strategies associated with different regions of the parameter space in Figure 1.

*Figure 1*

The figure underlines the primary disadvantage of selling a single product to a market where advertising creates heterogeneity. Whenever advertising is inexpensive, the channel chooses to serve only brand seekers (the channel capitalizes on a high willingness to pay created with inexpensive advertising). As noted in Shapiro and Varian (1999), without the ability to offer distinct versions of a product, the best strategy is frequently to offer only the high end product at a premium price and leave the low willingness to pay consumers unserved. As shown in Figure 1, this is precisely what happens when the cost of advertising is low (Cases 1 and 2). Case 3 relates to a situation where the fraction of product seekers in the market is high and advertising is sufficiently expensive to make advertising (even for a single brand) infeasible.

Case 4 is the most interesting as it relates to a situation where the channel serves both segments with a single brand. The optimal advertising and pricing for the channel is a compromise between the need to advertise and charge brand seekers a high price and the need to maintain a low price to maintain demand from product seekers. Case 4 is perhaps the
most important for a typical consumer product as it relates to situations where a) heterogeneous consumers buy the same product and b) the cost to inform brand seekers about the psychological benefits of the national brand are high. We now consider the situation where the channel launches a second unadvertised brand.

When the channel distributes two brands, equation 2 is maximized with respect to \( A, p_n \) and \( p_g \). As before, prices are set after the level of advertising has been chosen. The results are summarized in Lemma 2.

**Lemma 2** When the channel sells both an advertised and an unadvertised brand, the optimal strategy \((A, p_n, p_g)\) is \( \left( \frac{\lambda}{\lambda+4\gamma}, 2\frac{\gamma}{\lambda+4\gamma}, \frac{1}{2} \right) \). The advertising level chosen in the two brand scenario is greater than or equal to the advertising level chosen when only the national brand is sold.

When the channel offers two brands to the market, two effects are unambiguous. First, the advertising level chosen in the two brand scenario is greater than or equal to the advertising level chosen when only one brand is available. With two brands, the channel has greater ability to extract surplus from brand seekers and this increases the channel’s incentive to advertise. Second, the quantities sold to the product seeker segment are strictly higher (except for Case 3). When only the national brand is sold, the channel sets prices higher than \( \frac{1}{2} \) in order to capitalize on the higher willingness to pay of brand seekers.

Our focus is not on the equilibrium product-line decision of the channel. The variable profit when two brands are sold is strictly higher. Whether or not the channel chooses to extend its product line depends on the magnitude of the fixed cost, \( k_g \), required to launch. It is useful to note that the relative fixed cost of introducing a private label has decreased because there is more volume over which to spread the fixed costs (channel concentration has increased substantially).\(^6\) Furthermore, the costs of packaging changes and inventory management have dropped due to computerization. This may explain the recent growth of quality-equivalent private labels.

\(^6\)In the last 5 years, there has been significant consolidation in the retail channels. Examples of consolidation in the food sector include the acquisition of Monoprix by Promodes in France (August 1999), the November 1988 takeover of Provigo by Loblaws in Canada (2 of Canada’s four largest grocery retailers); and the acquisition by Ahold (a Dutch food retailer) of Giant Food (May 1998) and U.S. Foodservice (March 2000) in the U.S. References for more than 20 major mergers in the U.S., Canada and Europe are available from the authors.
3.2 A Comparison of Average Category Prices

We now consider the fundamental issue of our analysis. Can the increased availability of private labels lead to an increase in average category prices and when does this happen? Following our discussion above, we focus on Case 4. The average price when only a national brand is sold is the equilibrium price identified in Lemma 1. The average price when both the national brand and the private label are sold is based on the demand weighted price i.e.,

$$\bar{p}_{n+g} = \frac{d_b p_b + d_p p_p}{d_b + d_p}$$. Simple calculations show that

$$\bar{p}_{n+g} = \frac{16\lambda^2 + \lambda^2 - 8\gamma + 16\gamma^2}{2(4\gamma - \lambda + 4\gamma)(4\gamma - \lambda)}$$. Comparison of the average prices leads to Proposition 1.

**Proposition 1** When $\lambda \geq 0.83757$ and $\gamma > \gamma^* = -\frac{1}{8} \left(1 + \sqrt{1 + 4\lambda^3 - 4\lambda}\right) \frac{\lambda}{\lambda^2 - 1}$, average category prices increase when the channel adds an unadvertised brand to its product line. Otherwise average category prices decrease.

The finding highlights the conditions needed for the addition of private label to lead to an increase in average category prices. First, the market must be composed primarily of brand seekers i.e., $\lambda \geq 0.83757$ (the third root of a cubic equation shown in the appendix). Because product seekers move to a lower priced product when an unadvertised brand is introduced, their behavior induces a downward effect on category prices. For an increase in average category prices, the price increase on the national brand (weighted by the fraction of the market that are brand seekers) must be sufficient to counter this. Second, advertising must be sufficiently costly for a price increase to occur. The intuition for this finding is that when advertising is inexpensive, the motivation for a channel with single brand is to focus more on the profit that can be earned from brand seekers. The channel will choose high advertising levels and charge high prices (ultimately when $\gamma < \frac{1}{2} \lambda^2$, product seekers are not served at all). However, when advertising is costly, the profit that can be earned from product seekers is relatively more important. Here, the need to maintain profit seeker sales keeps prices low.

As a result, when an unadvertised low-priced brand is introduced, overall category prices increase. Interestingly, these conditions appear to parallel actual conditions in many consumer categories. First, the fraction of the market that is actually interested in quality equivalent private label is relatively small. As noted in footnote 1, private label sales average around 18% of retail sales in the US market; however, this figure consists of the total basket of private

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7 Cases 1 and 2 obviously lead to decreases in average category prices (the price charged to brand seekers does not change and product seekers buy at $p_b = \frac{1}{2}$ which is strictly less than $p_n$). Case 3 is degenerate in the sense that the advertised brand is not advertised.
label products including those of lower quality. The market share of quality-equivalent private labels is obviously significantly less than 18%. Second, the cost of advertising has increased significantly because of media fragmentation and increasingly heterogeneous viewing habits amongst consumers. As a result, the model provides a straightforward explanation for why average prices in many categories have increased even though the most significant structural change has been the growth of low-priced alternatives to advertised brands.

3.3 Welfare Implications of adding an Unadvertised Brand to the Product Line

A fundamental assumption of our model is that advertising itself is a source of utility for the segment of the market that responds to advertising. From an economic perspective, this is similar to the depiction of advertising as a complementary good (Becker and Murphy 1993): the main effect of advertising is to raise the marginal utility of the advertised good. This assumption is critical to welfare effects: welfare-reducing effects of advertising are often based on the advertising of one firm nullifying the costly advertising efforts of a competitor. Because of the monopoly structure, our model does not capture this source of reduced welfare.8

Because advertising creates surplus in this model, it is natural to expect that when advertising increases after the introduction of an unadvertised brand, welfare should be higher. This expectation is further strengthened by the fact that when two brands are sold, the quantity sold to product seekers (and hence the welfare created by product seeker consumption) is strictly higher.

In order to focus on the structural effects of private label introduction, we normalize the cost to the channel of introducing an unadvertised brand, $k_g$, to 0. The zone of welfare reduction is minimized with this assumption; however, it will allow us to highlight the conditions where reductions in total welfare are most likely.

To calculate total welfare under each scenario, we add channel profits to the consumer surplus of each segment weighted by each segment’s importance. The profit expressions follow from the equilibrium values shown in Lemmas 1 and 2. Because the demand functions have a slope of one, consumer surplus is the area of a triangle with a height of $q_s$ and a base of $q_s$, i.e., $\frac{q_s^2}{2}$, where $q_s$ is the quantity sold in each segment.

8 The monopoly structure is nevertheless well suited to explaining empirical observations from markets where a dominant manufacturer has market power.
As before, we focus on Case 4. Straightforward calculations lead to the following expressions for welfare without and with a private label:

\[
W_n = -\frac{\frac{1}{2} - 4\gamma^2 - 8\lambda^4\gamma - 8\lambda^2\gamma^2 - \lambda^4 - \lambda^3 + 2\lambda^5 + 10\lambda^2\gamma}{(\lambda - 4\gamma)^2 \lambda^2}
\]  
\[
W_{n+g} = -\frac{\frac{1}{8} - 48\gamma^2 - 3\lambda^2 + 3\lambda^3 + 24\lambda\gamma - 16\lambda^2\gamma}{(\lambda - 4\gamma)^2}
\]

Comparison of these two expressions leads to Proposition 2.

**Proposition 2** When \(\gamma > \gamma^{**} = \frac{2\lambda^3 + 5\lambda^2 + \lambda\sqrt{\lambda(8\lambda + 4\lambda^3 + 5\lambda^2 - 4)}}{4(1 + 3\lambda)}\), total welfare is strictly reduced by the introduction of an unadvertised brand.

Proposition 2 shows that when the costs of advertising are sufficiently high, the introduction of a private label leads to reductions in total welfare. The limit, \(\gamma^{**}\) is strictly higher than the limit for both segments to be served under national brand distribution, \(\gamma^*\). This implies that when \(\gamma \in (\gamma^*, \gamma^{**})\), total welfare is increased by the launch of an unadvertised brand but it decreases at values of \(\gamma > \gamma^{**}\).

These findings echo the observations of Varian (2000). Total welfare can be reduced by versioning when both segments are served under the distribution of a solitary brand with uniform pricing. The reason that total welfare drops when \(\gamma\) is high is that without the unadvertised brand, advertising is primarily used to increase demand from brand seekers (the channel uses a low price to retain demand from product seekers and advertising increases demand from the brand seeker segment). In Case 4 when the private label is introduced, the price of the national brand rises significantly. As a result, even though advertising levels are higher, the quantity sold to brand seekers is strictly lower. When advertising costs are sufficiently high, the reduction in the quantity sold to brand seekers leads to a reduction in total welfare.

A further observation is that when \(\lambda \gtrapprox 0.855\), the region where average category prices increases is also a region of lower total welfare. Even for values of \(\lambda \in (0.8375, 0.855)\), only at values marginally higher than the limit \(\gamma^*\) do increases in average category prices and increases in total welfare occur simultaneously. For the majority of situations where average category prices increase with the introduction of an unadvertised brand, reductions in total welfare are also observed.

\(^9\)In Cases 1, 2 and 3, total welfare increases because the quantity sold to one of the two segments does not change and the quantity sold to the remaining segment increases.
Surprisingly, even when advertising levels increase substantially, there are regions where the discriminatory pricing practiced by the monopolistic channel leads to reductions in total welfare. Interestingly, these conditions are highly correlated with situations where the introduction of an unadvertised brand leads to higher category pricing.

Do the above findings imply that regulation is needed to manage the growth of private labels? Probably not. As shown in the analysis, in three of the four cases in Figure 1, the introduction of private label leads to increases in total welfare (primarily because of higher consumption amongst product seekers). In addition, our findings rely on the fact that only the monopolistic channel supplies quality-equivalent private label. Conversely, the model does show that total welfare may be reduced when higher sales of private labels are correlated with higher average category prices. This is especially true when the primary suppliers of quality-equivalent private label are national brand manufacturers. The model thus highlights a second reason why increased product variety may not be welfare increasing. The usual reason is that the cost of providing variety may not be sufficient to justify the increase in surplus (Salop 1979). In our model, we consider conditions where the cost of increased variety is negligible (the marginal cost and $k_g$ are zero). Nevertheless, welfare reductions can occur when increased variety allows a monopolistic seller to limit the quantity provided to a segment that has high willingness to pay.

4 Conclusion and Extensions

The objective of the paper has been to explain why in many consumer goods categories, higher average prices and higher national brand advertising have been witnessed concurrently with significant growth of quality-equivalent unadvertised brands. Because the unadvertised private label is nothing but a version of the national brand without the premium generated by national brand advertising, it is an effective vehicle for a channel with market power to capitalize on both the heterogeneity in willingness to pay and the increase in willingness to pay generated by brand advertising.

In conditions where the fraction of product seekers is relatively small (i.e., the majority of consumers do prefer national brands ceteris paribus) and advertising is relatively expensive, the introduction of an unadvertised brand naturally leads to an overall increase in category pricing. In these conditions, decreases in total welfare are generally observed. Despite the fact that the introduction of a private label leads to higher levels of value-creating advertising,
discriminatory pricing behavior on the part of the channel more than eliminates the gain created by advertising.

One limitation of our analysis is that we have assumed decision-making by an integrated channel. In general, channels for typical consumer goods have at least two levels (manufacturers and retailers). Moreover, the decision to launch an unadvertised version of an existing product is invariably taken independently by one member of the channel. Empirically, it is usually the retailer (most unadvertised brands are private labels available only at specific retailers); however, there are categories where the manufacturers themselves market unadvertised alternatives.\(^{10}\) Given the decentralized nature of retail channels, the benefits of discriminatory pricing provided by unadvertised versions of national brands accrue to both channel members. There is thus a coordination problem that may be a barrier to the growth of unadvertised brands.\(^{11}\) In addition, it is unclear how the relative benefits of private labels accrue to channel members. In general, manufacturers finance the cost of national brand advertising but retailers finance the cost of private label packaging development and inventory management. This raises additional questions that provide a basis for continued investigation.

\(^{10}\)This is common in the mainstream beer business of North America. Both Molson and Labatt sell the Carling and Wildcat brands respectively, which are unadvertised versions of the mainstream brands.

\(^{11}\)According to the Coase theorem, the manufacturer and the retailer will bargain to share the fixed costs of a private label introduction if the transactions costs of bargaining are sufficiently low (Coase 1960). This implies that the provision of private labels will be inefficient when transactions costs are high (as might be expected in a highly fragmented retail market). This provides an additional argument why the incidence of private labels should be higher in more concentrated retail channels.
Appendix

Proof of Lemma 1

When both segments are served

The channel maximizes \( \pi = p_n (\lambda (A + 1 - p_n) + (1 - \lambda) (1 - p_n)) - \gamma A^2 \) with respect to \( A \) and \( p_n \).

\[
\frac{\partial \pi}{\partial p_n} = \lambda A + 2\lambda - 2\lambda p_n - 1 = 0 \Rightarrow p_n = \frac{1}{2} \frac{\lambda A + 2\lambda - 1}{\lambda}
\]

However \( p_n > 1 \) or the demand function for product seekers does not apply. In other words, \( A < \frac{1}{\lambda} \). Assume \( A < \frac{1}{\lambda} \Rightarrow \frac{\partial \pi}{\partial A} = \frac{1}{2} \lambda A + \lambda - \frac{1}{2} - 2\gamma A = 0 \Rightarrow A = \frac{1 + 2\gamma}{\lambda - 4\gamma} \). The optimal price and profit \( \pi \) is obtained by substituting the optimal values into equation 1, \( \pi = \frac{\gamma - \lambda^2 + \lambda^3}{\lambda - 4\gamma} \). Also \( A < \frac{1}{\lambda} \) implies that \( \gamma > \frac{1}{2} \lambda^2 \) (otherwise the channel will serve only brand seekers). A further constraint is \( A = \frac{1 + 2\gamma}{\lambda - 4\gamma} > 0 \). This implies that \( \lambda > \frac{1}{2} \) (because the denominator is always positive when second order conditions are satisfied). This implies that when both segments are served and \( \lambda < \frac{1}{2} \), the optimal advertising is \( A = 0 \) and \( p_n = \frac{1}{2} \).

When only brand seekers are served

The channel maximizes \( \pi = p_n (\lambda (A + 1 - p_n) - \gamma A^2) \) with respect to \( A \) and \( p_n \).

\[
\frac{\partial \pi}{\partial p_n} = \lambda A + \lambda - 2\lambda p_n = 0 \Rightarrow p_n = \frac{1}{2} A + \frac{1}{2}
\]

We substitute the optimal price and optimize with respect to \( A \). This implies that \( \frac{\partial \pi}{\partial A} = \frac{1}{2} A + \frac{1}{2} \lambda - \frac{1}{2} \lambda c - 2\gamma A = 0 \Rightarrow A = \frac{\lambda}{4\gamma - \lambda}, p_n = \frac{2\gamma}{4\gamma - \lambda} \). We then substitute to obtain the optimal profit \( \pi = \frac{\gamma}{4\gamma - \lambda} \). When \( \lambda > \frac{1}{2} \), following the analysis above, the channel will serve only brand seekers when \( \gamma \in (\frac{1}{4} \lambda, \frac{1}{2} \lambda^2) \). However, when \( \lambda < \frac{1}{2} \), the profit from serving both segments with an unadvertised brand must be compared to the profit from only serving brand seekers and advertising i.e., \( \pi = \lambda \frac{\gamma}{4\gamma - \lambda} \). Straightforward calculations show that the optimal price and profit when serving both segments with an unadvertised brand are \( p_n = \frac{1}{2} \) and \( \pi = \frac{1}{4} \). The channel only serves brand seekers when \( \frac{1}{4} < \lambda \frac{\gamma}{4\gamma - \lambda} \Rightarrow \gamma < \frac{\lambda}{4(1 - \lambda)} \).

Proof of Lemma 2

The objective function for the channel is \( \pi = p_n (\lambda (A + 1 - p_n) + p_g (1 - \lambda)(1 - p_g) - \gamma A^2) \).

First we maximize with respect to \( p_g \). \( \frac{\partial \pi}{\partial p_g} = 1 - 2p = 0 \Rightarrow p = \frac{1}{2} \). This implies that \( q = \frac{1}{2} \), next we maximize with respect to \( p_n \).

\[
\frac{\partial \pi}{\partial p_n} = \lambda A + \lambda - 2\lambda p_n = 0 \Rightarrow p_n = \frac{1}{2} A + \frac{1}{2}
\]
We then substitute this expression into the objective function and maximize with respect to $A$. 

$$\frac{\partial \pi}{\partial A} = \frac{1}{2}A\lambda + \frac{1}{2}\lambda - 2A\gamma = 0 \Rightarrow A = \frac{\lambda}{4\gamma - \lambda}$$

This implies an equilibrium with $A = \frac{\lambda}{4\gamma - \lambda}, p_n = \frac{2\lambda}{4\gamma - \lambda}$ and $p_g = \frac{1}{2}$. Demand from brand seekers is $q_b = -2\lambda \frac{\gamma}{\lambda - 4\gamma}$. The channel earns profits of $\pi = \frac{\gamma \lambda^2 + \lambda^3}{\lambda (4\gamma - \lambda)}$.

**Proof of Proposition 1**

Following the text, $\mathbf{\Pi}_{n+g} = \frac{d\pi + rp_g}{db + dp_g}$. Using the results of Lemma 2, this implies that $\mathbf{\Pi}_{n+g} = \frac{1}{\pi} \frac{16\lambda^2 + \lambda^3 - 8\lambda + 16\gamma^2}{(4\lambda\gamma - \lambda + 4\gamma)(4\gamma - \lambda)}$. We restrict our attention to Case 4 ($\lambda > \frac{1}{2}$ and $\gamma > \frac{1}{2}\lambda^2$). Here $p_n$ (the price when only the national brand is available) is $2\gamma \frac{2\lambda - 1}{(4\gamma - \lambda)^2}$. $\mathbf{\Pi}_{n+g} > p_n$ when $\frac{16\lambda^2 + \lambda^3 - 8\lambda + 16\gamma^2}{2(4\lambda\gamma - \lambda + 4\gamma)(4\gamma - \lambda)} > 0 \Rightarrow \frac{16\lambda^2 + \lambda^3 - 8\lambda + 16\gamma^2}{2(4\lambda\gamma - \lambda + 4\gamma)(4\gamma - \lambda)} > 0$. Rearranging, this implies that $\frac{16\lambda^2 + \lambda^3 - 8\lambda + 16\gamma^2}{2(4\lambda\gamma - \lambda + 4\gamma)(4\gamma - \lambda)} > 0$. The second order conditions imply that the denominator is positive, so the numerator is an upward facing parabola in $\gamma$ with two roots:

$$\gamma_1 = \frac{1}{8} \left(-1 + \sqrt{(1 + 4\lambda^3 - 4\lambda)}\right) \frac{\lambda}{\lambda^2 - 1}$$

$$\gamma_2 = \frac{1}{8} \left(1 + \sqrt{(1 + 4\lambda^3 - 4\lambda)}\right) \frac{\lambda}{\lambda^2 - 1}$$

Only the second root is relevant because the first root is less than $\frac{1}{2}\lambda^2$, the limit for both segments to be served when only the national brand is sold. In order for the roots to be real, $1 + 4\lambda^3 - 4\lambda > 0$. This is a cubic equation with three real roots: $-1.1072, 0.26959$ and $0.83757$. Because of the constraint that $\lambda > \frac{1}{2}$ (in case 4), $1 + 4\lambda^3 - 4\lambda > 0$ in the relevant range when $\lambda \gtrapprox 0.83757$.

**Proof of Proposition 2**

Following Lemma 1, in Case 4, the quantities sold to each segment are $q_b = \frac{\lambda^2 - \lambda + 2\gamma}{(4\gamma - \lambda)^2}$ and $q_g = \frac{\lambda^2 - 2\gamma}{(4\gamma - \lambda)^2}$. Following the discussion in the text, $W_n = \pi + \frac{\lambda q_b^2 + (1 - \lambda)q_g^2}{2}$. Substituting, we obtain $W_n = -\frac{1}{2} - 4\gamma^2 - 8\lambda^4\gamma - 8\lambda^2\gamma^2 - \lambda^4 - 2\lambda^3 + 10\lambda^2$. Similarly, $W_{n+g} = -\frac{1}{8} - 48\gamma^2 - 3\lambda^4 + 33\lambda^3 + 24\lambda^2 - 16\lambda^3$. Let $\Delta = W_n - W_{n+g}$. Then $\Delta = -\frac{1}{8} - 16\gamma^2 - 16\lambda^4\gamma - 32\lambda^2\gamma^2 - \lambda^4 - 4\lambda^3 + 5\lambda^2 + 40\lambda^2\gamma + 48\lambda^3\gamma^2 - 24\lambda^3\gamma$. This is an upward facing parabola in $\gamma$ with two roots:

$$\gamma_1 = \frac{2\lambda^3 + 5\lambda^2 - \lambda\sqrt{\lambda}}{4 (1 + 3\lambda)} \sqrt{(8\lambda + 4\lambda^3 + 5\lambda^2 - 4)}$$

$$\gamma_2 = \frac{2\lambda^3 + 5\lambda^2 + \lambda\sqrt{\lambda}}{4 (1 + 3\lambda)} \sqrt{(8\lambda + 4\lambda^3 + 5\lambda^2 - 4)}$$

Only the second root is relevant because $\gamma_1 < \frac{1}{2}\lambda^2$. As a result, when $\gamma > \gamma_2$, $W_n > W_{n+g}$ and total welfare is reduced by the introduction of an unadvertised second brand. Note
that $\gamma_2 > \gamma^*$ because $\gamma_2 - \gamma^* = -\frac{1}{4} \lambda \frac{4\lambda^2 - 3\lambda - \sqrt{\lambda} \sqrt{(8\lambda + 4\lambda^3 + 5\lambda^2 - 4)}}{1 + 3\lambda} > 0$. This implies that $4\lambda^2 - 3\lambda - \sqrt{\lambda} \sqrt{(8\lambda + 4\lambda^3 + 5\lambda^2 - 4)} < 0$. In the allowable range, when $\lambda \in \left(\frac{1}{2}, \frac{3}{4}\right)$, the first two terms are negative so the expression is trivially negative. When $\lambda \in \left(\frac{3}{4}, 1\right)$, assume the expression is positive then: $4\lambda^2 - 3\lambda > \sqrt{\lambda} \sqrt{(8\lambda + 4\lambda^3 + 5\lambda^2 - 4)}$. This implies that $4\lambda^2 - 11\lambda + 4 > 0$ (square both sides and simplify). But this implies that $\lambda > \frac{11}{8} + \frac{1}{8} \sqrt{57} \approx 2.3187$ or $\lambda < \frac{11}{8} - \frac{1}{8} \sqrt{57} \approx 0.43127$ which is a contradiction. A similar calculation can be used to show that $\gamma_2 < \gamma^{**}$ for all $\lambda \geq 0.855$. 


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References


[8] Liesse, J., 1993a, Private-Label nightmare: Big name marketers are being stalked by high-quality store brands, Advertising Age 64 (15), 1-4.


Figure 1: Summary of Parametric Conditions when only 1 Product is Sold

<table>
<thead>
<tr>
<th>Cost of Advertising</th>
<th>( \lambda &lt; 1/2 )</th>
<th>( \lambda &gt; 1/2 )</th>
</tr>
</thead>
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<tr>
<td>Low</td>
<td>Case 1</td>
<td>Case 2</td>
</tr>
<tr>
<td></td>
<td>serve only brand seekers</td>
<td>serve only brand seekers</td>
</tr>
<tr>
<td>High</td>
<td>Case 3</td>
<td>Case 4</td>
</tr>
<tr>
<td></td>
<td>serve both segments (no advertising)</td>
<td>serve both segments (+ve advertising)</td>
</tr>
</tbody>
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