A Model of Collusion Timing

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Abstract

I develop a dynamic model of collusion, affording a substantial role to entry. Heterogeneous firms make collusion, entry, exit, and investment decisions within an evolving environment. The collusive agreement adopted is a rule of thumb, motivated by the details of collusion in the lysine market. It is found that an entrant will wait until it has a market share comparable to its competitors before agreeing to collude. Allowing for collusive possibilities tends to yield a less concentrated industry with reduced consumer welfare. The model characterises events in the lysine market and could provide insights into markets with the potential for collusion and entry, and uncertainty about the characteristics of potential entrants.

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1 Introduction

The emergence of a new competitor poses a serious challenge for any collusive arrangement. Careful examination of the experience of collusion in the market for lysine reveals that entry played a pivotal role. However, most theoretical models of collusion ignore the role of entry. In fact, most theoretical models allow no role for any firm actions that alter the collusive environment. This paper develops a dynamic model of collusion in which firms must respond to, and can influence, the evolution of their environment. Firms are free to make decisions about entry, exit, investment, and whether to collude. The model provides one rationale for the emergence of a price war upon entry, and a means for examining the timing of the decision to reinstigate collusion. It also facilitates a comparison between a world with and without collusive possibilities along the dimensions of industry structure and consumer and producer welfare.

The model is applied to the market for lysine, where the entry of a competitor dramatically altered the experience with collusion. Demand and cost estimates for the lysine market are used as parameters in the firm profit functions. Knowledge of the operation of collusion in that market is used to shape key assumptions of the model. In particular, the nature of the collusive agreement in the model reflects the simple rule of thumb that operated in the lysine market. However, the model has a more general applicability. It could provide lessons for any market with collusive possibilities, the potential for entry, and some uncertainty about the characteristics of potential entrants.

We might expect the response of incumbent firms to the entry of a competitor to depend critically on how serious a threat the entrant is expected to be. Incumbent firms may seek to drive out a feeble entrant, or simply ignore it. In contrast, incumbents may need to completely recast their behaviour upon the entry of a more substantial competitor. The model is designed to examine such a situation. Agreement of the recent entrant is required before a collusive arrangement can be settled. It was just this kind of entry that unsettled the lysine market. Hence, the lysine market provides an interesting examination of the model's performance. It should be noted, however, that due to the rich nature of the experience with collusion in the lysine market, we cannot hope to explain all the nuances of firm behaviour in that market. Rather, I characterise a key ingredient in the collusive experience in the lysine market.

I follow Fershtman and Pakes (2000) in developing a model with heterogeneous firms operating in a changing environment. Firms engage in repeated quantity competition in a market for a homogeneous product, subject to capacity constraints.[1] These capacity constraints determine the profits enjoyed by firms. Each period, firms can choose to indulge in investment spending aimed at increasing capacity, and thus allowing greater profit opportunities in the future. Firms are free to exit should their competitive environment become too hostile. Potential entrants join the fray if the competitive environment becomes conducive to them. Incumbent firms make decisions about whether to collude, taking these factors into account.

Every cartel must decide how much to produce and how to split the proceeds from collusion. Economic theory provides no clear guidance on this matter. The present model appeals to empirical observation by adopting the simple rule of thumb that operated in the market for lysine. The collusive agreement specifies that participating firms receive the market share they enjoyed at the time collusion was initiated and that these market shares are maintained throughout the agreement.[2]

This assumption reflects a coordination problem faced by the firms. The rule of thumb is only (at best) approximately optimal, but may be an easy rule on which to coordinate. This is a departure from

[1]In applying this model to the lysine market, I interpret capacity as the stock of interested customers at a firm's fingertips, and not as the stock of physical production capacity.

[2]In the recent vitamins price fixing cartel, according to industry experts, explicit market share rules of this nature applied in the markets for vitamins C, E, A, and B2 (Riboflavin). In addition, the realised market shares in most of the vitamin markets covered by the vitamin conspiracy are consistent with this rule of thumb.

most existing models of collusion, which assume the existence of an optimal collusive mechanism, or at the very least a collusive mechanism involving a high degree of rationality on the part of firms. That is, participating firms are assumed to devise complex conditional punishment and reward regimes to create the incentives necessary to sustain an optimal degree of collusion.

An additional motivation for this rule of thumb relates to the inherent uncertainty of incumbent firms about the characteristics of a new entrant. Incumbent firms will have only incomplete information about crucial attributes of an entrant, such as its marginal cost or production capacity. These variables are key elements in the determination of an optimal collusive arrangement. We might expect an entrant to build up market share over time as it advertises, customers learn about its product, and it develops a distribution network, but exactly how much of the market should it receive in an immediate agreement? Unless the entrant can credibly convey its private information, it may not be possible to negotiate an optimal agreement.

Stigler (1964) argues that fixing market shares is the most effective method of collusion because the incentive to secretly undercut prices is removed. In the absence of an optimal collusive mechanism, firms may gravitate toward an obvious market sharing arrangement that could serve as a focal point. In this context, a clear focal point is given by the existing market shares of the firms. Schelling (1960) presents experimental evidence suggesting that without communication, parties with coinciding or opposing interests will tend to coordinate on a focal point even if it yields an asymmetric or indeed "unfair" outcome. He argues that the status quo also provides a strong attraction in situations affording explicit communication. Schmalensee (1987, p.357) notes that if side payments are impossible and firm costs are asymmetric, colluding firms may wish to maintain market shares at their non-collusive levels, particularly if firms have imperfect information on their rivals' costs. In the next section, I briefly discuss the manifestations of this problem in the lysine market.

The crucial trade off for the entrant (or for a firm with a small capacity relative to its competitors) can then be summarised as follows. Should the entrant agree to collude today, it can enjoy collusive payoffs immediately, but will have a minor share in the collusive agreement for its working life. If the entrant waits, it obtains the reduced profits arising from the non-cooperative regime, but has the prospect of a potentially much larger share of the collusive regime in the future.

In most collusion models, the possibility of the secret undercutting of a cartel agreement is explicitly recognised by introducing incentive compatibility constraints. An implicit assumption is that if such incentive compatibility constraints break down, firms adopt non-cooperative behaviour such as we might observe in a one-shot Nash equilibrium. Levenstein (1997), in a study of the pre-World War I bromine industry, argues that the nature of the most severe price wars is more consistent with a bargaining and renegotiation process than with problems of imperfect monitoring and demand uncertainty. Similarly, in a study of collusion in the Indian tea industry in the Great Depressions, Gupta (1997) argues that price wars were a reflection of coordination and bargaining problems. In the current paper, the possibility of secret undercutting is not considered. Instead, the incentive compatibility constraints take a different guise. Firms can choose to break out of a cartel at any time if the terms of collusion are not favourable to them. Breaking out of a cartel invokes a punishment phase, reflecting a period of negotiation and reconciliation that must be endured before the prospects of collusion can again be entertained.[3] It is important to note that incentive compatibility constraints are not being abandoned here. Instead, deviations are detected immediately, rather than with a delay.

Several other assumptions play a central role in the model. First, there are convex adjustment costs to increasing output. Such costs could take the form of physical capacity constraints, advertising, customer

[3] Levenstein (1995) suggests that the low cost producer, Dow Chemical Company, broke out of the collusive agreement in the bromine industry, precipitating a price war, because it was dissatisfied with its output quota and sought to differentiate its product.

search, or distribution costs. Second, entry is assumed to interrupt the collusive agreement as firms must negotiate a new agreement that accounts for the entrant. This assumption follows naturally from our consideration of entrants of substance rather than fringe entrants. Relatedly, the entry of a competitor is assumed to interrupt any prevailing punishment between incumbent firms. The implications of this assumption are discussed in more detail in section 5.5. Finally, firms are able to collude on output, but not investment.[4] This restriction implicitly assumes that firms are better able to coordinate on output than investment. This may arise because output is easier to observe and verify, and the returns to investment are uncertain. This is particularly so if we interpret investment more broadly than simply investment in productive capacity.

The results suggest that if we permit entry and investment in an industry with collusive possibilities, we might expect a much richer set of firm behaviour. An entrant will tend to build up a market share comparable with its competitors before colluding. The reason is that, by waiting and foregoing the collusive profits in the short-run, the entrant can obtain a larger slice of the payoffs to collusion in the future. Relative to a world without collusion, if we permit collusion, we tend to obtain a less concentrated industry as potential entrants seek to benefit from the increased payoffs associated with collusion. Despite this result, we tend to observe higher prices and reduced consumer surplus if we allow collusive possibilities. The close interdependence found between collusion and market structure is supported by the empirical study of Symeonidis (2000). Using the controlled experiment presented by a change in competition policy in the UK in 1956, Symeonidis finds that the competitive regime has a significant impact on concentration, with a rise in price competition associated with an increase in concentration.

The rest of the paper is organised as follows. Below, I briefly discuss some of the related literature

[4]Fershtman and Pakes (2000) adopt this assumption in a similar context. Genesove and Mullin (2000) note that in the sugar refining cartel of the 1920s and 1930s, collusion encompassed more than simply quantity and price setting. Cartel members sought to homogenise business practices to make cheating more transparent.

on collusion. Section two describes the operation of collusion in the lysine market. This provides both a context for discussion and a real application for the model. Section three discusses the model. Section four sketches the computational algorithm used to solve the model. Results are presented in section five. The type of firm behaviour observed in equilibrium is described and compared to a model without the possibility of collusion. The model is simulated, generating industry and welfare statistics. I then examine the flavour of short term dynamics we might observe by simulating the model repeatedly, choosing as a starting point the industry structure prevailing in the lysine market at the time of a major entry into the market. Finally, I illustrate the implications of changing key parameters for the model's predictions. Avenues for refinement and further research are briefly discussed in section six. Concluding remarks are contained in section seven.

1.1 Related Literature

The collusion literature has highlighted the tension between the profitability of a collusive agreement and the enforcement of the agreement. Stigler (1964) discusses the industry conditions conducive to the enforcement of a cartel. Friedman (1971) demonstrates that the incentives for collusion can be maintained through a regime of punishments and rewards. The patience of the participating firms is crucial for determining the viability of collusion. The models of Green and Porter (1984) and Abreu, Pearce, and Stacchetti (1986) reveal that, if firms cannot perfectly monitor the behaviour of their rivals, periodic price wars may be necessary to maintain the incentives for collusion.

An assumption common to these models is that firms are symmetric and operate in an unchanging environment. However, introducing dynamic considerations and firm heterogeneity can dramatically alter the nature of collusion. Rotemberg and Saloner (1986) demonstrate that allowing demand to vary with time alters the nature of the collusive agreement that can be sustained.[5] A high level of demand

[5]Bagwell and Staiger (1997) and Haltiwanger and Harrington (1991) present interesting generalisations to the model of

results in an increased temptation to cheat on the collusive agreement, making collusion less effective in peak demand periods. In a collusive environment in which firm-specific cost shocks are hidden, Athey and Bagwell (2001) consider non-stationary equilibria in which firms use future market share favours to encourage optimal collusion. If firms are unable to grant future market share favours, then optimal collusion involves sacrificing productive efficiency.

A number of authors have considered collusion amongst asymmetric firms. Schmalensee (1987) argues that when firms have cost asymmetries, the nature of collusive outcomes depends strongly on the collusive mechanism employed. Harrington (1989) analyses a repeated price-setting game among firms with differing discount factors. Collusive profits are divided through a bargaining game, yielding greater profits to less patient firms. Similarly, Harrington (1991) examines the case of cost asymmetries in a repeated price-setting game, finding that low cost firms obtain greater market share and that the optimal collusive price responds differentially to changes in the costs of low-cost and high-cost firms. Compte, Jenny and Rey (2002) consider the problem of optimal collusion when firms face asymmetric capacity constraints, finding that collusion is more difficult to sustain with asymmetric firm capacities when aggregate capacity is limited.

Fershtman and Pakes (2000) develop a dynamic model of collusion with heterogeneous firms. In their collusion decisions, firms explicitly consider the entry, exit, and investment decisions of incumbents and potential competitors. It is found that collusion is particularly hard to sustain if one of the firms is likely to exit in the near future. Moreover, allowing for the possibility of collusion can have a dramatic impact on industry structure. The present paper differs in several respects, two of which are crucial for the flavour of the model's predictions. First, the nature of the collusive agreement is markedly different. Fershtman and Pakes assume that the terms of the collusive agreement are negotiated each period through a static

Rotemberg and Saloner (1986).

Nash bargaining game of perfect information. While the current paper assumes perfect information, the collusive agreement is designed to capture some of the informational asymmetries, especially with respect to a new entrant. The collusive agreement specifies that, for the life of the agreement, firms receive shares in the cartel profits based on their market shares at the time of the agreement. This reflects the inability of competing firms, and a recent entrant in particular, to credibly convey key information that would determine its market share under collusion. The implication is that a prospective entrant must establish itself through a price war before it can receive favourable terms from a collusive agreement. In contrast, in the model of Fershtman and Pakes, an entrant will often anticipate entering a comparatively benign industry in which collusion is maintained despite entry.

Second, in the current paper the flavour of punishment is quite different. In the model of Fershtman and Pakes, as in most of the literature on collusion, punishment is intended to deter deviation from the collusive agreement by any firm attempting to skim additional profits in the short term before its competitors can detect its deviant behaviour and coordinate a response. By its nature, this kind of punishment will not be observed in equilibrium. In the present model, punishment continues to discourage departure from a collusive agreement. However, if observed, it also conveys a general deterioration of the agreement. That is, a firm will willingly invoke the punishment regime if it no longer believes the collusive agreement is in its interests. As we shall see, this kind of punishment can be observed in equilibrium.

In a paper examining the sustainability of collusion with free entry, Friedman and Thisse (1994) consider some of the issues addressed in the present paper. They propose a collusive scheme in which an entrant joins a collusive industry initially as a junior partner with a lower production share than the incumbents and transitions gradually to a full partner with full production rights. They argue that such a scheme permits incumbent firms to earn positive rents in a free entry equilibrium. They find the existence of equilibria with this collusive scheme. As we shall see, in the present model a similar pattern of production growth for the entrant emerges endogenously as the entrant seeks to build market share prior to a collusive agreement.

Other authors have examined empirical applications of collusion models. Porter (1983) finds the existence of price wars in the Joint Executive Committee railroad cartel and concludes that the pattern of pricing behaviour is consistent with the Green and Porter model. Ellison (1994) reexamines this data set, contrasting the Green and Porter model with the model of Rotemberg and Saloner. Based on an examination of the recent international lysine price-fixing conspiracy, de Roos (2000) finds that imperfect monitoring models of price wars provide only limited guidance for firm behaviour. Genesove and Mullin (2000) examine the role of communication in the sugar refining cartel of the 1920s and 1930s. Collusion was aimed at standardising business practices to make cheating more transparent. Cheating did occur, but was met by only limited retaliation in contrast to the extreme punishments suggested by Abreu, Pearce, and Stacchetti (1986). Levenstein and Suslow (2002) survey a selection of the literature on cross-section studies and case studies of collusion, finding that bargaining problems and entry are issues faced by most cartels. Out of the 16 industries covered by the case studies they examine, the most common cause of cartel breakdown is firm entry and most cartels had to deal with entry at some time.

2 Collusion in the Market for Lysine

The market for lysine provides a unique opportunity to study the operation of an international cartel in a legal environment hostile to collusion. The original motivation for the current model arose from an examination of this market. In this section, I will provide a brief description of the operation of collusion in the lysine market.[6] This should serve the dual purposes of clarifying the nature of the model and

[6]For more detailed discussion of the operation of collusion in the market for lysine, see Connor (1998,2000) and de Roos (2000).

providing a real application. I will first describe some essential features of the lysine market, and then describe briefly the history of collusion in the lysine market.

Lysine is an essential amino acid for the lean muscle development of hogs and poultry. Being a chemical compound, it is a homogeneous product. There is a great deal of heterogeneity in the capacities, locations, and costs of firms in the lysine market. Prospective entrants face entry barriers arising from technological patents and the cost and length of time required to build a new lysine plant. Prices and volumes of lysine suppliers are contractual and not directly observable.

By the end of the 1980s, the lysine market was dominated by three Asian based firms; Ajinomoto and Kyowa Hakko of Japan, and Sewon of South Korea. Testimony by officials at Ajinomoto reveals that price fixing was a feature of this period (Connor, 2001). In 1989, Archer Daniels Midland (ADM) began construction of the world's largest lysine factory. ADM began production in February 1991, precipitating a severe price war. During the price war, Ajinomoto and Kyowa Hakko tried unsuccessfully to raise prices several times. Subsequently, ADM suggested the formation of a lysine trade association, with the first meeting taking place in June 1992. Lysine prices rose shortly afterwards. Thus, in the market for lysine, collusion was interrupted by the emergence of a large-scale entrant.

A cartel comprising the major firms operated with moderate success over the next year. However, no consensus was achieved on the operating mechanism of the cartel. Cooperation in the monitoring of sales and prices was scant. Firms were suspicious of rivals' costs and capacities. A second price war began in early 1993, and was resolved later that year. The character of the subsequent phase of collusion was considerably different. Uncertainty about costs and capacities was largely resolved. A centralised monitoring scheme was initiated. A system of global volume quotas was agreed to, based on the current market shares of firms. A compensation scheme involving intra-cartel sales operated for those firms not meeting their quotas.

The cartel operated successfully for a period of about two years before the FBI intervened in June 1995. The behaviour of prices following the breakup of the cartel is consistent with the existence of tacit collusion, with prices actually rising in 1996. Late in 1999, another large firm, a joint venture between Cargill and Degussa, entered the market. In this case, prices declined before the entry of Cargill and Degussa.

An interesting puzzle in this brief history is why collusion broke down at all when ADM entered the market. Incumbent firms had advanced warning of the construction of ADM's plant. Would it not have been better to simply let ADM into the collusive agreement immediately and thus prevent a price war? It is improbable that a price war arose from attempts by incumbents to scare the upstart ADM out of the market. The sheer size of ADM's plant seems to signal ADM's intentions of remaining in the market. Furthermore, the incumbent firms were the first to attempt to raise prices.

Two alternative, interrelated explanations offer themselves. First, firms may have been signalling in advance of a collusive market share agreement. This explanation requires that firms are uncertain about their rivals' capacities or costs or some other strategic variables, and expect the terms of the collusive agreement to depend on these factors. Hence, firms signal their low cost or high capacity in a price war prior to collusion. Second, the entrant may have wished to gain market share in advance of a collusive agreement. This presumes that the terms of the collusive agreement depend on the market shares of the participants prior to collusion.

I contend that these two explanations represent two sides to the same coin. Because of the high degree of uncertainty in the lysine market about marginal costs and production capacities, there were no obvious terms to set for a collusive agreement. Firms were unable to credibly convey their private information.[7] Hence, the only obvious focal point for an agreement was the existing market shares of the

[7] In fact, ADM conducted tours of its plant in June 1992 for Sewon and in April 1993 for Ajinomoto (Connor, 2000, p. 34).

firms. The actual collusive agreement specified that firms received market shares based on the market shares they enjoyed over the past year. These market shares were to be maintained over the life of the agreement. This kind of agreement naturally encourages any entrant to first build up market share before agreeing to collude. The fact that ADM appears to have been the principal stumbling block for collusion is consistent with this interpretation. In the model developed below, firms have perfect information about their rivals. However, the market sharing mechanism employed in the lysine market is adopted, imparting some of the flavour of a signalling game.

3 The Model

The model adapts the framework set out in Ericson and Pakes (1995). In this framework, firms solve a discrete time, infinite horizon problem involving endogenous entry, exit, and investment decisions. Each period, firms engage in price, quantity, or quality competition subject to constraints imposed by a set of firm-specific state variables. This process determines profits each period. In addition, firms can influence the vector of state variables that determines profit opportunities through investment spending.

The solution concept is subgame perfection and differs from a Markov-perfect Nash equilibrium in the sense of Maskin and Tirole (1988) because firms condition on the current competitive regime in addition to payoff-relevant states. Firms form perceptions about the distributions of the state variables, conditional on their actions. Firms choose optimal actions based on these perceptions. The realised conditional distributions of the state variables depend on the actions of all the firms. In equilibrium, these realised distributions accord with firms' perceptions.

Figure 1 describes the sequence of events taking place each period in the current model. There are three competitive regimes which yield different profits each period: a non-cooperative regime, a collusive regime, and a punishment regime. Profits in the non-cooperative and punishment regimes are determined by the Nash equilibrium in quantities to a one-shot capacity-constrained game. Profits in the collusive regime are determined by joint profit maximisation subject to capacity constraints and a market share constraint. The market share constraint specifies that firms must produce output in the ratio given by their ratio of capacities at the time collusion was instigated.[8] Hence, firms have an incentive to build up capacity prior to a collusive agreement.

The sequence of play each period is as follows. At the beginning of the period, collusion decisions are made. Play switches from the non-cooperative regime to the collusive regime if all firms wish to collude. It is assumed that any firm that is indifferent between colluding and not colluding will vote for collusion.[9] At any time, firms may choose to exit the collusive agreement, but this will invoke a punishment regime. In this punishment regime, firms behave as in the non-cooperative regime, except that there is no possibility of collusion. With fixed probability, γ, firms negotiate their way back to the non-cooperative regime. Hence, the expected length of the punishment phase is $1/\gamma$ periods. Once firms have negotiated their way out of the punishment regime, they then must achieve consensus to begin colluding. Following the collusion decision, incumbent firms decide whether to exit. Firms then make output decisions which determine profits. Finally, incumbent firms decide on investment spending and potential entrants decide whether to enter. Entrants take one period to set up operations and begin

[8]A more realistic market share constraint would specify production in the ratio of sales rather than capacity at the time of collusion. However, this would result in a considerably more computationally demanding problem without adding significantly to the flavour of the results. This market sharing rule is equivalent to that of Davidson and Deneckere (1990) except that Davidson and Deneckere specify that market shares become independent of capacity for firms able to unilaterally serve the entire market.

[9]More precisely, the computational algorithm specifies that firms compare the value to being in the collusive regime at the current state to the value of being in the non-cooperative regime at this state and “vote” accordingly, without conditioning on the votes of rivals. A unanimous preference for collusion ensures a transition to the collusive regime. In this way, the uninteresting equilibrium in which all firms decide not to collude each period is ruled out.

production in the next period.[10] If entry occurs, play switches to the non-cooperative regime in the next period. The dashed line in Figure 1 indicates that in section 5.5 a variation of the model is considered in which entry does not interrupt punishment.

3.1 The State Space

To make computation feasible, the state space is assumed to be discrete. The set of feasible capacities, Ω, can be mapped onto the set of positive integers. Several restrictions are made to the state space to make the problem more manageable. First, following Ericson and Pakes (1995), each firm's capacity can take on a finite set of values.[11] Second, payoffs are independent of the ordering of a firm's competitors. Hence, we do not have to consider different permutations of competitors' states. Finally, successive elements of Ω are assumed to increase exponentially rather than linearly as in Ericson and Pakes (1995). That is, the set of feasible capacities is given by $\Omega \equiv \left\{\tau g^k\right\}_{k=1}^{\bar{k}}$, where $\tau > 0$, $g > 1$, and $\bar{k}$ is in the set of positive integers.[12] τg and $\tau g^{\bar{k}}$ describe the minimum and maximum possible capacity, respectively. This assumption considerably restricts the state space because states that generate the same ratio of capacities amongst firms at the time of collusion are equivalent.[13]

Define $\omega_t = \{\omega_{i,t}\}_{i=1}^{n_t}$ to be the vector of capacities of the incumbent firms, where $\omega_{i,t} \in \Omega$, i indexes firms, t indexes time periods, and n_t is the number of active firms in period t. Similarly, define $\mu_t = \{\mu_{i,t}\}_{i=1}^{n_t}$ to be the vector of capacities at the time the last collusive agreement was negotiated, where

[10] This is unrealistically short for the lysine market. As discussed above, ADM spent approximately two years building their lysine production facility.

[11] Ericson and Pakes show there is a maximum state level that can be reached in equilibrium.

[12] In equation 2 below, this restricts the collusive state space, S^C, to states in which $\min_i\{\mu_{i,t}\} = \tau g$ because all other μ_t vectors share the common factor, g. For example, the vectors $\mu_t = \{\tau g^5, \tau g^4, \tau g^2\}$ and $\mu_t = \{\tau g^4, \tau g^3, \tau g^1\}$ are observationally equivalent.

[13] The assumption implies that there are some economies of scale in capacity generation. However, in the results discussed below, a value of $g = 1.2$ is chosen, implying relatively modest economies of scale.

$\mu_{i,t} \in \Omega$. With these definitions and restrictions, we can define the state space, conditional on being in each of the competitive regimes, as follows:

$$S^N = S^P \equiv \left\{\omega_t \middle| \text{ for } j > i, \omega_{i,t} \geq \omega_{j,t}\right\} \tag{1}$$

$$S^C \equiv \left\{\omega_t, \mu_t \middle| \text{ for } j > i, \omega_{i,t} \geq \omega_{j,t} \text{ and if } \omega_{i,t} = \omega_{j,t}, \text{ then } \mu_{i,t} \geq \mu_{j,t}\right\} \tag{2}$$

In the non-cooperative and punishment regimes, the state space is fully described by a weakly decreasing vector of capacities of the incumbent firms. In the collusive regime, a firm's share of the collusive profits depends on its capacity at the time the collusive agreement was negotiated. Equation 2 states that in S^C the vector of capacities, ω_t, is weakly decreasing, and, if neighbouring elements of ω_t are equal, then the corresponding elements of μ_t are weakly decreasing.

3.2 Profit Functions

Profits each period are determined by quantity competition for a homogeneous product. The inverse demand function is given by $P(Q_t) = a - bQ_t$, where Q_t is market output, and a and b are demand parameters. Current production decisions have no impact on state transition probabilities. Hence, the production game can be treated in isolation from the investment, exit, entry, and collusion decisions.

In the non-cooperative regime, the profit vector, $\pi^N(\omega_t) = \left\{\pi_i^N(\omega_t)\right\}_{i=1}^{n_t}$, is determined by the unique Nash equilibrium to a one-shot capacity-constrained quantity game. It is calculated recursively as follows.[14] Define $q_{i,t}$ to be firm i's output in period t. In the absence of capacity constraints, $q_{i,t} = \tilde{q} \equiv \frac{a-mc}{(n_t+1)b}$, where mc is the constant marginal cost of production. This is the Cournot-Nash equilibrium output for the unconstrained game. We introduce capacity constraints as follows. Starting with the smallest firm, we check to see if $\omega_{i,t} \geq \tilde{q}$. If so, then $q_{i,t} = \tilde{q}, i = 1, \ldots, n_t$. If $\omega_{i,t} < \tilde{q}$, then $q_{i,t} = \omega_{i,t}$, and we redefine $\tilde{q} \equiv \frac{a-mc-b\omega_{i,t}}{n_t b}$. We then verify whether for the next smallest firm, $\omega_{j,t} \geq \tilde{q}$, and so on.

[14] A proof that the algorithm described yields a Nash equilibrium is available from the author on request.

To calculate the collusive profit vector, $\pi^C(\omega_t, \mu_t) = \left\{\pi_i^C(\omega_t, \mu_t)\right\}_{i=1}^{n_t}$, let s_t be the vector of collusive shares with $s_{i,t} \equiv \mu_{i,t} / \sum_{j=1}^{n_t} \mu_{j,t}$. Then, in the absence of capacity constraints, $q_{i,t} = s_{i,t} \frac{a-mc}{2b}$, firm i's share of a monopolist's optimal output. To incorporate capacity constraints, we allocate excess production over capacity to the remaining firms according to their shares.

3.3 Investment

Let $\eta_{i,t} \in \{1, g\}$ represent the outcome of the firm's investment process. The probability of successful investment is an increasing, concave function of investment spending. Let $\nu_t \in \{1, g\}$ be the outcome of some exogenous process capturing developments in the industry. If we take ω to describe physical capacities, then an obvious interpretation for ν is the stochastic (industry-wide) decay of capacity. If ω represents the stock of interested customers for each firm, we could think of ν as an industry-wide demand shock. Then, the transition of firm i's capacity is governed by

$$\omega_{i,t+1} = \omega_{i,t} \frac{\eta_{i,t+1}}{\nu_{t+1}} \tag{3}$$

$$\eta_{i,t+1} = \begin{cases} g & \text{with probability } \frac{\alpha x_{i,t}}{1+\alpha x_{i,t}}, \\ 1 & \text{with probability } \frac{1}{1+\alpha x_{i,t}} \end{cases} \tag{4}$$

$$\nu_{t+1} = \begin{cases} g & \text{with probability } \delta, \\ 1 & \text{with probability } 1-\delta. \end{cases} \tag{5}$$

where $\alpha > 0$ is a constant determining the effectiveness of investment, and $x_{i,t}$ is firm i's investment expenditure in period t.

3.4 Entry and Exit

Each period, before firms engage in quantity competition, they have the option of exiting the industry. A firm which exits receives a pay-off of ϕ and takes no further part in the game. A firm will therefore exit if the expected discounted value of remaining in the market is less than ϕ.

Entry decisions are made concurrent with investment decisions. A single potential entrant observes an entry cost draw, x_e, from a uniform distribution $U\left(x_e^{min}, x_e^{max}\right)$ and then decides whether to enter. Should the firm decide to enter, it receives capacity ω_e and begins production in the following period. The entrant will choose to enter if the expected discounted value of entry at capacity ω_e is greater than or equal to the observed entry cost.

3.5 The Bellman Equations

A separate Bellman equation is defined for each of the three regimes.[15] The notation is defined as follows. The superscripts P,N, and C refer to the punishment, non-cooperative, and collusive regimes, respectively. Let $\Psi \in \{P, N, C\}$ denote the current operating regime. A negative subscript denotes omission of a single element. Thus, $\omega_{-i} \equiv (\omega_1, \ldots, \omega_{i-1}, \omega_{i+1}, \ldots, \omega_{n_t})$. In the characterisation of the state space, a semi-colon separates values of the same state variable for different firms, while a comma separates different state variables. For example, $V^C(\omega_j; \omega_{-j}, \mu_j; \mu_{-j})$ describes the value to firm j of operating in the collusive regime with capacity ω_j and capacity at collusion μ_j, while its competitors' capacities and capacities at collusion are given by ω_{-j} and μ_{-j}, respectively. Profits for firm j are given by $\pi^C(\omega_j; \omega_{-j}, \mu_j; \mu_{-j}) = \pi_j^C(\omega, \mu)$ and $\pi^N(\omega_j; \omega_{-j}) = \pi_j^N(\omega)$ under collusive and non-cooperative environments, respectively. Let $p(.|.)$ describe the conditional state transition probabilities. Finally, indicator functions describe transition between competitive regimes. $I^C(\omega) \in \{0, 1\}$ governs transition from the non-cooperative regime to the collusive regime, with

$$I^C(\omega) = 1 \Leftrightarrow V^C(\omega_j; \omega_{-j}, \omega_j; \omega_{-j}) \geq V^N(\omega_j; \omega_{-j}) \ \forall j; \tag{6}$$

[15] Alternatively, the system could be represented by a single Bellman equation by incorporating an additional state variable indicating the current regime of play.

and $I^P(\omega,\mu) \in \{0,1\}$ governs transition from the collusive regime to the punishment regime, with

$$I^P(\omega,\mu) = 1 \Leftrightarrow V^P(\omega_j;\omega_{-j}) > V^C(\omega_j;\omega_{-j},\mu_j;\mu_{-j}) \text{ for any } j. \tag{7}$$

The Bellman equations below describe the value to firm j for each (ω,μ) in the state space, for each of the regimes. Collusion decisions are based on these values.[16]

$$\begin{aligned}
V^P(\omega_j;\omega_{-j}) &= \max\Big\{\phi, \pi^N(\omega_j;\omega_{-j}) + \max_{x\geq 0}\big[-x \\
&\quad +\beta\sum_{\omega'}\left(\gamma V^N\left(\omega_j';\omega_{-j}'\right) + (1-\gamma)V^P\left(\omega_j';\omega_{-j}'\right)\right) \\
&\quad p\left(\omega_j'|\omega_j,x\right)p\left(\omega_{-j}'|\omega,\Psi=P\right)\big]\Big\} &(8)\\
V^N(\omega_j;\omega_{-j}) &= \max\Big\{\phi, \pi^N(\omega_j;\omega_{-j}) + \max_{x\geq 0}\big[-x \\
&\quad +\beta\sum_{\omega'}\left(I^C\left(\omega'\right)V^C\left(\omega_j';\omega_{-j}',\omega_j';\omega_{-j}'\right)\right. \\
&\quad \left.+\left(1-I^C\left(\omega'\right)\right)V^N\left(\omega_j';\omega_{-j}'\right)\right) \\
&\quad p\left(\omega_j'|\omega_j,x\right)p\left(\omega_{-j}'|\omega,\Psi=N\right)\big]\Big\} &(9)\\
V^C(\omega_j;\omega_{-j},\mu_j;\mu_{-j}) &= \max\Big\{\phi, \pi^C(\omega_j;\omega_{-j},\mu_j;\mu_{-j}) + \max_{x\geq 0}\big[-x \\
&\quad +\beta\sum_{\omega'}\left(I^P\left(\omega',\mu\right)V^P\left(\omega_j';\omega_{-j}'\right)\right. \\
&\quad \left.+\left(1-I^P\left(\omega',\mu\right)\right)V^C\left(\omega_j';\omega_{-j}',\mu_j;\mu_{-j}\right)\right) \\
&\quad p\left(\omega_j'|\omega_j,x\right)p\left(\omega_{-j}'|\omega,\mu,\Psi=C\right)\big]\Big\}, &(10)
\end{aligned}$$

In each of the regimes, the firm can choose to exit and receive ϕ, or receive the continuation value. The continuation value comprises current profits plus the expected discounted value of future returns. The simplest case is the punishment regime. Here, profits are given by the non-cooperative profit function. The continuation value depends on which competitive regime we are in next period, the firm's own

[16]The Bellman equations highlight the firm's investment decisions. To simplify exposition, equations 8 and 10 omit the role of entry in breaking punishment and collusion, respectively. The computational algorithm treats this issue more rigorously.

state next period, ω'_j, and the states of its competitors (including the entrant if entry occurs) next period, ω'_{-j}. The investment policy function, $x = x^P(\omega_j;\omega_{-j})$, depends on this continuation value. With probability γ, play switches to the non-cooperative regime. The distribution of the firm's own state in the next period, $p(\omega'_j|\omega_j, x)$, is determined by its level of investment this period, x, while the distribution of its competitors' states next period, $p(\omega'_{-j}|\omega, \Psi = P)$, depends on the investment, exit, and entry decisions made by its competitors this period.

In the non-cooperative regime, profits are given by the non-cooperative profit function. The continuation value again depends on next period's competitive regime, the firm's own state next period, and the states of its competitors next period. This determines the non-cooperative investment policy function, $x^N(\omega_j;\omega_{-j})$. The collusive policy function, $I^C(\omega)$, depends on the current vector of capacities. A value of one indicates that play switches to the collusive regime next period.

Finally, in the collusive regime, profits are given by the collusive profit function. Note that the collusive profit function (and therefore also the collusive value function) depends on both the vector of current capacities, and the vector of capacity shares at the time collusion took place. The punishment policy function, $I^P(\omega,\mu)$, and the investment policy function, $x^C(\omega_j;\omega_{-j},\mu_j;\mu_{-j})$, depend on this expanded state vector. If $I^P(\omega,\mu) = 1$, play switches to the punishment regime next period.

We are now in a position to discuss the entry decision. A single potential entrant observes an entry cost draw of x_e before deciding whether to enter. The entrant then spends the remainder of the period constructing a plant with capacity ω_e. It takes no part in the quantity game. In the following period, the entrant becomes an incumbent with capacity ω'_e with probability $p_e(\omega'_e)$. The stochastic nature of ω'_e arises from the uncertainty about industry wide developments during the period of plant construction. That is, $p_e(\omega_e) = 1-\delta$ and $p_e(\omega_e/g) = \delta$. The entry decision will depend on the current competitive regime because rival firms' investment decisions, and hence the state transition probabilities, depend on the competitive regime. Equations (11)-(13) summarise the conditions under which entry occurs for the punishment, non-cooperative, and collusive regimes, respectively.

$$
\begin{aligned}
\beta \sum_{\omega'} V^N\left(\omega'_e;\omega'_{-e}\right) p_e\left(\omega'_e\right) p\left(\omega'_{-e}|\omega,\Psi=P\right) &> x_e && (11)\\
\beta \sum_{\omega'} V^N\left(\omega'_e;\omega'_{-e}\right) p_e\left(\omega'_e\right) p\left(\omega'_{-e}|\omega\Psi=N\right) &> x_e && (12)\\
\beta \sum_{\omega'} V^N\left(\omega'_e;\omega'_{-e}\right) p_e\left(\omega'_e\right) p\left(\omega'_{-e}|\omega,\mu,\Psi=C\right) &> x_e. && (13)
\end{aligned}
$$

Let the resulting entry policies be given by $\chi_e^P\left(\omega_e;\omega_{-e}\right)$, $\chi_e^N\left(\omega_e;\omega_{-e}\right)$, and $\chi_e^C\left(\omega_e;\omega_{-e},\mu_{-e}\right)$ for the punishment, non-cooperative, and collusive regimes, respectively. Because the cost of entry, x_e, is random, these entry policies are probability measures reflecting the perceptions of the incumbent firms of the probability of entry at the time investment decisions are made, given the prevailing state vector and the distribution of x_e. In each regime, the entry condition depends on the non-cooperative value function because entry breaks any prevailing collusive agreement and interrupts punishment. Notice, however, that the transition probabilities of the incumbent firms depend on the investment decisions made in the prevailing regime.

4 Computational Algorithm

The reader can omit this section without loss of continuity. The computational algorithm used to solve for the equilibrium of the model is based on the method described in Pakes and McGuire (1994). The method involves iterative computation of the value and policy functions.[17] First, calculate the non-cooperative profits, $\pi^N(.)$, for each $\omega \in S^N$ and the collusive profits, $\pi^C(.)$, for each $(\omega,\mu) \in S^C$. Then, specify an initial value function and investment policy function for each of the competitive regimes,

[17]The computational algorithm, written in the C programming language, is available upon request.

$\left(V^{\Psi,0}(.), x^{\Psi,0}(.)\right)$, $\Psi \in \{N, P, C\}$. The superscripts refers to the competitive regime, and the iteration number, respectively. To complete one iteration, cycle through all the elements of the state space of the non-cooperative regime, obtaining updated value and policy functions. Then do the same for the punishment and collusive regimes. In equilibrium, the value and policy functions will not change from one iteration to the next.[18]

Consider first the calculations involved to obtain the iteration $k+1$ functions for the non-cooperative regime. We first determine the entry policy, $\chi_e^{N,k+1}(.)$ by testing equation (12) using $V^{N,k}(.)$ and $x^{N,k}(.)$. We then update the collusion policy function, $I^{C,k+1}(.)$, by verifying equation (6) using $V^{C,k}(.)$ and $V^{N,k}(.)$. The entry and collusion policies are then used to update the exit and investment policies. Firm j will exit if $V_j^{N,k}(\omega) < \phi$. If it exits, we can update j's investment and value functions by setting $x_j^{N,k+1}(\omega) = 0$ and $V_j^{N,k+1}(\omega) = \phi$. If it doesn't decide to exit, firm j's investment policy in iteration $k+1$ is given by

$$\begin{aligned} x_j^{N,k+1}(\omega) \quad = \quad & argmax_{x \geq 0} \Bigg[-x + \beta \sum_{\omega'} \Big(I^{C,k+1}(\omega') V^{C,k}(\omega'_j; \omega'_{-j}, \omega'_j; \omega'_{-j}) \\ & + \left(1 - I^{C,k+1}(\omega')\right) V^{N,k}(\omega'_j; \omega'_{-j}) \Big) \\ & p(\omega'_j | \omega_j, x)\, p\left(\omega'_{-j} | \omega_{-j}, x_{-j}^{N,k}, \chi_e^{N,k+1}\right) \Bigg]. \end{aligned} \tag{14}$$

[18]In practice, convergence is not exact for the model outlined. The difficulty lies in the discrete nature of the collusion and punishment decisions. However, we come arbitrarily close to convergence for every element of the non-cooperative and punishment regimes. Fluctuations in the value function for the collusive regime must then only occur for states that are not reached on the equilibrium path. With a minor refinement to the model, we can come arbitrarily close to perfect convergence. The refinement involves incorporating a stochastic element to the punishment decision. That is, each period while firms are colluding, each firm observes an i.i.d. "punishment cost" draw before deciding whether to break out of the cartel. The rationale for this assumption is that the returns to punishment will depend on privately observed, temporary, firm-specific shocks. Results based on this refinement, while similar, contain noticeable differences, and are not adopted in the results presented below. These results are available from the author upon request.

Notice that the distribution of states next period depends on the current iteration entry policies, $\chi_e^{N,k+1}(\omega)$, and the investment policies of competitors derived from the previous iteration, $x_{-j}^{N,k}(\omega)$. We can use $x_j^{N,k+1}(\omega)$ to update the value function for firm j for iteration $k+1$,

$$
\begin{aligned}
V_j^{N,k+1}(\omega) &= \pi_j^N(\omega) - x_j^{N,k+1} + \beta \sum_{\omega'} \Big(I^{C,k+1}(\omega') V^{C,k}(\omega'_j; \omega'_{-j}, \omega'_j; \omega'_{-j}) \\
&\quad + \left(1 - I^{C,k+1}(\omega')\right) V^{N,k}(\omega'_j; \omega'_{-j}) \Big) \\
&\quad p\left(\omega'_j | \omega_j, x_j^{N,k+1}\right) p\left(\omega'_{-j} | \omega_{-j}, x_{-j}^{N,k}, \chi_e^{N,k+1}\right). \qquad (15)
\end{aligned}
$$

A similar set of calculations is then performed for the punishment regime. Entry policy, $\chi_e^{P,k+1}(.)$, is determined by equation (11). Recall that entry causes play to switch to the non-cooperative regime. To explicitly account for this, let I^E and I^{NE} denote the events that entry does and does not occur, respectively. Firm j will exit if $V_j^{P,k}(\omega) < \phi$. If j does not exit, we calculate its iteration $k+1$ investment policy with

$$
\begin{aligned}
x_j^{P,k+1}(\omega) &= argmax_{x \geq 0} \Big[-x + \beta \sum_{\omega'} \Big[\chi_e^{P,k+1} V^{N,k}(\omega'_j; \omega'_{-j})\, p\left(\omega'_{-j} | \omega_{-j}, x_{-j}^{P,k}, I^E\right) \\
&\quad + \left(1 - \chi_e^{P,k+1}\right) \left(\gamma V^{N,k}(\omega'_j; \omega'_{-j}) + (1-\gamma) V^{P,k}(\omega'_j; \omega'_{-j}) \right) \\
&\quad p\left(\omega'_{-j} | \omega_{-j}, x_{-j}^{P,k}, I^{NE}\right) \Big]\, p\left(\omega'_j | \omega_j, x\right) \Big]. \qquad (16)
\end{aligned}
$$

We then use $x_j^{P,k+1}(\omega)$ to update the value function for firm j for iteration $k+1$,

$$
\begin{aligned}
V_j^{P,k+1}(\omega) &= \pi_j^N(\omega) - x_j^{P,k+1} + \beta \sum_{\omega'} \Big[\chi_e^{P,k+1} V^{N,k}(\omega'_j; \omega'_{-j})\, p\left(\omega'_{-j} | \omega_{-j}, x_{-j}^{P,k}, I^E\right) \\
&\quad + \left(1 - \chi_e^{P,k+1}\right) \left(\gamma V^{N,k}(\omega'_j; \omega'_{-j}) + (1-\gamma) V^{P,k}(\omega'_j; \omega'_{-j}) \right) \\
&\quad p\left(\omega'_{-j} | \omega_{-j}, x_{-j}^{P,k}, I^{NE}\right) \Big]\, p\left(\omega'_j | \omega_j, x_j^{P,k+1}\right). \qquad (17)
\end{aligned}
$$

Collusion entry policy, $\chi_e^{C,k+1}(.)$, is determined by equation (13). Again, entry causes play to switch to the non-cooperative regime. Firm j will exit if $V_j^{C,k}(\omega, \mu) < \phi$. If j does not exit, we calculate its iteration $k+1$ investment policy with

$$
\begin{aligned}
x_j^{C,k+1}(\omega,\mu) &= argmax_{x\geq 0}\Bigg[-x+\beta\sum_{\omega'}\Big[V^{N,k}\left(\omega'_j;\omega'_{-j}\right)p\left(\omega'_{-j}|\omega_{-j},x_{-j}^{C,k},I^E\right)\chi_e^{C,k+1} \\
&\quad +\left(I^{P,k+1}\left(\omega',\mu\right)V^{P,k}\left(\omega_j;\omega_{-j}\right)+\left(1-I^{P,k+1}\left(\omega',\mu\right)\right)V^{C,k}\left(\omega_{-}j;\omega'_{-j},\mu_j;\mu_{-j}\right)\right) \\
&\quad p\left(\omega'_{-j}|\omega_{-j},x_{-j}^{C,k},I^{NE}\right)\left(1-\chi_e^{C,k+1}\right)\Big]p\left(\omega'_j|\omega_j,x\right)\Bigg]. \qquad (18)
\end{aligned}
$$

We then use $x_j^{C,k+1}(\omega,\mu)$ to update the value function for firm j for iteration $k+1$,

$$
\begin{aligned}
V_j^{C,k+1}(\omega,\mu) &= \pi_j^C(\omega,\mu)-x_j^{C,k+1}+\beta\sum_{\omega'}\Big[V^{N,k}\left(\omega'_j;\omega'_{-j}\right)p\left(\omega'_{-j}|\omega_{-j},x_{-j}^{C,k},I^E\right)\chi_e^{C,k+1} \\
&\quad +\left(I^{P,k+1}\left(\omega',\mu\right)V^{P,k}\left(\omega_j;\omega_{-j}\right)+\left(1-I^{P,k+1}\left(\omega',\mu\right)\right)V^{C,k}\left(\omega_{-}j;\omega'_{-j},\mu_j;\mu_{-j}\right)\right) \\
&\quad p\left(\omega'_{-j}|\omega_{-j},x_{-j}^{C,k},I^{NE}\right)\left(1-\chi_e^{C,k+1}\right)\Big]p\left(\omega'_j|\omega_j,x_j^{C,k+1}\right). \qquad (19)
\end{aligned}
$$

Up to now, we have assumed that we know the extent of the state spaces, S^P, S^N, and S^C. However, the dimensions of the state spaces depend on the maximum allowable capacity of a firm, $\tau g^{\bar{k}}$, and the maximum number of firms, N. As in Pakes and McGuire (1994), the maximum allowable capacity is determined by the point at which a monopolist without prospects of entry would stop investing.[19] In Pakes and McGuire, the maximum number of firms is obtained by calculating the equilibrium for a restricted number of firms and increasing the number of firms until a potential entrant would no longer enter at any element of the state space. Due to computational limitations, I impose an upper limit of three

[19]This is in fact a mild restriction. Figure 5 shows that in the full model a monopolist does invest at the penultimate feasible capacity level for the baseline parameters chosen. This suggests that investment may continue at higher capacity levels if the maximum feasible capacity level were raised. However, the impact of this restriction is extremely minor, because entry occurs into a monopoly with probability 1.

firms.[20] I am unable to verify whether a 4^{th} firm would wish to enter.[21]

5 Results

The results below are broken into five sections. The first section describes a stripped-down model in which there is no possibility of collusion. In the next section, results using the full model are discussed, and compared with the model without collusion. Firm behaviour in equilibrium is characterised and the consequences for industry development are illustrated by simulating the model. In the third section, the model's results are compared with the experience in the lysine market. This exercise also highlights the type of short-term dynamics the model can produce. In the fourth section, the consequences of varying key parameters are examined. Finally, an alteration to the punishment regime is considered.

A list of the parameters used and their values for the base model is contained in Table 1. The static parameters, a, b, and mc, are based on the demand estimates and cost data obtained for the lysine market in de Roos (2000). A linear demand curve is estimated for lysine sold in the United States using monthly data for the period January 1991 through June 1996. The demand for lysine is a derived one, reflecting its use as a feed supplement for hogs and poultry. Demand shifters controlled for in estimation include the price of corn, the price of soybean meal, hog inventories, broiler quantities, the lysine feed

[20] As an example, with 3 firms and a set of feasible capacities, Ω, of dimension 20, the state space comprises 1,771 elements for the non-cooperative and punishment regimes and 1,955,401 elements for the collusive regime. With 4 firms and the same set of feasible capacities, the state space expands to 10,626 and 282,340,296 elements for the non-cooperative and collusive regimes, respectively.

[21] Informally, we could argue that with the base set of parameters chosen below, it is unlikely that the entry of a fourth firm would be observed on the equilibrium path. This is because the cost of the investment required to maintain a firm's capacity level in expectation is similar to the maximum collusive profits obtained in a four firm industry. With the high entry costs chosen in the base model and the painful initiation anticipated by a potential entrant (see section 5 below), entry by a fourth firm is unlikely.

efficiency, seasonal factors, and a trend. Of these variables, corn is a complementary product, soybean meal is a substitute for lysine and corn, and the lysine feed efficiency variable reflects the improved feed absorption rate of hogs over time following genetic improvement. The slope of the demand curve, b, is obtained directly from the demand estimates. The intercept, a, is calculated using the values of explanatory variables at the time of ADM's entry into the market.

The constant marginal cost, mc, is derived from the following parameterisation of ADM's cost function allowing for learning by doing.[22]

$$MC(q_{i,t}, u_{i,t}) = c_1 q_{i,t}^{c_2} \left[1 + c_3 e^{-c_4 \sum_{\tau=0}^{t} q_{i,t}}\right] + u_{i,t} \tag{20}$$

The parameters c_1 through c_4 in equation (20) are estimated by non-linear least squares. Data on marginal costs, fixed costs, production, and domestic sales for ADM are obtained from ADM production and sales reports over the period July 1991 to June 1995, available following the recent prosecution of several leading executives of the lysine manufacturers.[23] The restriction that $c_2 = 0$ cannot be rejected, and is therefore imposed in estimation. For the purposes of the current model, we set $mc = c_1$, reflecting constant marginal costs once learning has ceased.

The remaining parameters are chosen to produce a relatively concentrated industry. The high discount factor of 0.98 is chosen to reflect the monthly planning horizon that appeared to operate in the lysine market, and the monthly cost and demand data used for the estimation of the static parameters.

[22] Capacity utilisation is used in place of quantity in de Roos (2000) to allow for the possibility that marginal costs rise as the plant approaches capacity. Learning by doing is not incorporated in the dynamic model below, but is unlikely to have played a major role in the length and severity of the lysine price wars. The existence of learning by doing could contribute to a price war by providing an incentive for the learning firm to engage in additional production as a strategic investment to reduce future costs. However, ADM's learning concerned overcoming production teething troubles which actually restrained early production. These production difficulties appear to have been resolved within the first year of production.

[23] Specifically, these data are obtained from Exhibits 60-67, Transcript of *United States v. Michael D. Andreas et al.*, U.S. District Court, Northern District of Illinois, Eastern Division, No. 96 CR 762, July to September, 1998

Viewed as a monthly discount factor, it is in fact rather low, and reflects two concerns. First, using a lower discount factor considerably speeds the computational algorithm. Second, that we observe firms waiting to obtain equal market shares before agreeing to collude (see Section 5.2) becomes a more dramatic result with impatient firms. A relatively low entrant starting capacity of $\omega_e = \tau g^2 = 2.88$ was chosen to allow us to observe the dynamics of the entrant's growth. Entry costs were set sufficiently high to make the entry of a fourth firm implausible, and sufficiently low that entry still occurs with some frequency on the equilibrium path. The probability of renegotiation is arbitrarily set at 0.05, indicating an average length to the punishment phase of 20 months. The maximum number of firms was set at 3. This is not designed to mimic the market structure in the lysine market (where there were 5 major firms), but rather reflects the enormous computational burden of the state space.

5.1 A Model Without Collusion

The point of comparison is a model without the possibility of collusion. This is given by the punishment regime with $\gamma = 0$. Hence, there is no possibility of transition to either the non-cooperative or the collusive regimes. The reader can picture this in Figure 1 by focusing solely on the left column. This model is essentially the dynamic model of Pakes and McGuire (1994). A general characterisation of this model for a different static profit function and set of parameters can be found there.

Recall from equation 4 that the probability of successful investment is a monotonic transformation of the level of investment spending. In Figure 2, the probability of successful investment is shown for a single firm for a range of its own capacity levels (which increase as we move away from the reader) and a range of capacity levels of one of its rivals (which increase as we move from left to right). The capacity of firm i is indexed by k_i with $\omega_i = \tau g^{k_i}$. Four plots are drawn corresponding to different fixed values of the capacity of a third firm. The top left panel considers the case where the third firm is inactive, and there are only two firms in the market. A firm's investment is relatively insensitive to its rival's capacity, but there are diminishing returns to capacity expansion for any given rival capacity. With three firms in the market, the firm's investment diminishes with rival capacity.

In this model, entry is more likely to occur, the smaller and fewer in number the incumbent firms. Consequently, starting from a monopoly, we could envision a typical industry development as follows. Entry will tend to occur rapidly into a monopolistic industry. An entrant will invest heavily initially, and if this initial investment is successful, a sustained phase of duopoly is likely. The duopoly will be broken only if the firms have a bad sequence of investment realisations. A potential entrant may take advantage of this by entering while the incumbents are relatively small. We could then see a period of sustained competition between the three firms.

The industry is simulated for $1,000,000$ iterations and the distributions of several industry characteristics are collated. The initial state vector used was (4,0,0). That is, we start the simulations with a single firm with capacity of τg^4. Column 1 of Table 2 contains the results of these simulations. Columns 2 and 3 are discussed in sections 5.2 and 5.5, respectively. The industry contains the maximum number of firms (three) for about 3.4% of the periods, and is a duopoly for almost all the remaining periods. Firms charge a markup over marginal costs of about 74% on average. Entry and exit are highly correlated. The distribution of firm operating lives is skewed, reflecting initial difficulties faced by many firms. However, once entrenched, firms enjoy a long operating life on average.

5.2 The Full Model

5.2.1 Characteristics of Equilibrium

The interior of the contour plot of Figure 3 depicts the states in which firms decide to collude in the full model. This describes the elements of the state space in which all active firms wish to switch from the non-cooperative to the collusive regime. The axes index the capacities of three firms. A value of k corresponds to a capacity of τg^k, and a value of zero indicates that a firm is not participating in the market. Hence, the faces of the cube describe the states in which there are two active firms who decide to collude, while the interior of the cube depicts states in which there are three active firms who wish to collude.

Firms switch to the collusive regime when they have similar capacities. This is illustrated by the diagonal lines on each of the faces and within the cube. The smallest firm presents the greatest stumbling block to collusion because they obtain only a small share of the profits in a collusive agreement. Given the high discount factor assumed, smaller firms are prepared to suffer through a spell of non-cooperative profits while they invest heavily to later obtain a higher share of the collusive profits.

For the current set of parameters, punishment is not observed on the equilibrium path. In equilibrium, firms only collude with equal market shares, and, given that firms can never improve on equal market shares, there is no incentive to invoke punishment when collusive market shares are equal. As we shall see below, this result does not generalise to all possible parameter values. Off the equilibrium path, punishment is invoked if firms have collusive shares that are misaligned with their current shares of total market capacity. Punishment will occur over a greater range of capacity vectors the more unequal the collusive shares of the incumbent firms.

Figure 4 describes the elements of the state space in which entry occurs with positive probability both for the model without collusion, and for each of the competitive regimes of the model with collusion.[24] The axes index the capacity levels of each incumbent firm, with a value of 0 indicating that a firm is inactive and a positive value k indicating a capacity of τg^k. We need only two axes because there are at most three firms in the market. It can be seen that entry will always occur into a monopoly for both

[24]For the collusive regime, I examine only elements of the state space in which firms have equal shares of the cartel profits.

the model without collusion and all regimes of the model allowing collusion. In a duopoly, entry can be deterred through investment in capacity.[25] Entry is more likely in the model with collusion for a given state vector, reflecting the option value of collusion. Within the model of collusion, the likelihood of entry is approximately equal in each of the regimes. This is because we switch to the non-cooperative regime should entry occur. Any slight difference in entry behaviour, not captured in Figure 4, reflects differences in investment spending across regimes.

Investment behaviour changes dramatically once we permit collusion. Figures 5 and 6 depict investment behaviour in the non-cooperative and collusive regimes, respectively. In the non-cooperative regime, investment is tailored to speed the onset of collusion. In the top left panel, with only two firms in the market, the ridge along the diagonal indicates that a firm will abruptly reduce investment as soon as its capacity outstrips its rival. The mild hump in investment spending when both firms are relatively small reflects attempts to deter entry. In the remaining panels, with three firms operating, the ridge along the diagonal becomes softer, but we also observe a pronounced cliff around the point at which all firms have equal capacities, heralding a cessation of investment. For small levels of the firm's own capacity, investment is also more aggressive than in the corresponding state in the model without collusion, reflecting the incentive to speed up the attainment of collusive profits.

In the collusive regime, because there is no longer an incentive to equalise market shares, investment is comparatively insensitive to rival capacities. With two firms in the market, we again observe a mild hill in investment spending, corresponding to entry deterring behaviour. Compared to the model without

[25] In the model of Benoit and Krishna (1991), collusion tends to be easier to support with greater excess capacity, and hence firms seeking to deter entry may in fact invest less in capacity. In the current model, collusion is easier to support when firms have symmetric capacity rather than excess capacity. Hence, entry deterrence is a motive for investment in excess capacity. Indeed, firms have an additional motivation to invest in excess capacity because excess capacity acts as insurance against the possibility of an adverse technological shock.

collusion, investment spending is higher for low own-capacity states, reflecting the increased value of survival in the industry. This is emphasised for higher rival capacities. However, investment does tail off a little more dramatically as the firm's own capacity rises. This is because less capacity is required for insurance against adverse technological shocks, given the heavier investment observed at low capacity levels. Finally, investment in the punishment regime (not shown) follows similar, but less pronounced patterns to that of the non-cooperative regime. This is to be expected given that, with a positive renegotiation probability, γ, the value of operating in the punishment regime is effectively a weighted average of the values in the non-cooperative regime and the model without collusion.

5.2.2 Dynamics and Industry Statistics

Column 2 of Table 2 presents industry statistics for the model allowing for collusion. Industry statistics are generated in the same manner as the model without collusion. That is, beginning in the non-cooperative regime with a state vector of (4,0,0), the industry is simulated for 1,000,000 iterations, and industry characteristics are observed. There are typically a greater number of firms in the model with collusion, but each firm tends to produce less. This difference in industry structure is a reflection of the greater attractiveness of entry in the model with collusion. There is also a greater incidence of entry and exit in the model incorporating collusion. This, in turn is tied to the difference in industry structure in the two models. If collusion is not a possibility, we typically observe two firms in the market. With two firms in the market, the value of remaining in the market is greater and hence firms invest more heavily to avoid exit, resulting in a reduced incidence of exit. Exit by one firm is then often followed by the entry of another.

As we might expect, firms charge higher prices on average in the full model. Firm-level investment is similar, but industry-wide investment is greater if we allow collusion, owing to the greater average number of firms. The industry with collusion is characterised by lower consumer and total surplus.[26] Average producer surplus is very similar in the two models. Aggregate industry profits are weakly greater for any given state vector in the model allowing collusion, but the distribution of states observed in equilibrium is markedly different if collusion is possible. Both a less concentrated industry structure on average and greater industry-wide investment reduce the observed profitability in the full model relative to the model without collusion. Firms are longer lived and, in present value terms, earn greater lifetime profits in the industry without collusion. Intuitively, due to the option value of collusion, we might have expected the industry allowing collusion to be more profitable. It should be emphasised that, for *any* given state vector, *every* active firm would prefer to live in a world with collusive possibilities. Realised firm values are lower when we allow collusion because we tend to observe a different market structure over time. As noted above, with a less concentrated industry, producer surplus is spread among a greater number of firms in the industry allowing collusion. In addition, firms tend to be shorter lived, another reflection of the greater incidence of entry and exit. Finally, entrants earn particularly low profits in the early phases of their lives in the industry allowing collusion. This is because an entrant anticipates a non-collusive industry with three firms, in which it must invest heavily to build market share before it can enjoy collusive profits. With discounting, this painful initiation contributes significantly to the present value of an entrant's lifetime profits.

Table 3 presents some additional statistics on collusion and punishment, describing the length of time it takes firms to collude following the entry of a competitor, the length of the collusive regime, and the behaviour of prices when we switch competitive regimes. With the base parameters, collusion times tend to be relatively quick, but exhibit a large variance. When there are three firms in the market,

[26] This contrasts with the results of Fershtman and Pakes (2000). In their model, consumer surplus is higher if we allow collusion because the model allowing collusion generates a greater variety of products.

as is almost always the case following entry, the smallest firm will wish to build up market share prior to collusion. Collusion will then occur much later if the smallest firm experiences a bad sequence of investment outcomes, but is able to remain in the market.

The length of the collusive phase is calculated unconditionally, and by conditioning on the number of firms in the market when collusion was initiated. Collusion is long-lasting on average, but there is a great deal of variety in the success of collusion, reflecting the variety of states in which collusion is instigated.[27] With two firms in the market, the length of the collusive regime is very long on average because punishment is not observed on the equilibrium path and firms invest heavily whenever their capacities approach the region in which entry is attractive. When we begin the collusive regime with three firms, collusion tends to be even more stable. This is because there are no prospects for either punishment or entry while three firms operate in the market. Hence, collusion cannot be broken until one of the firms exits the market. Finally, as one might expect, prices jump substantially when we enter the collusive regime, and fall when collusion breaks down.

Table 4 presents a comparison of industry statistics, conditioning on each of the competitive regimes of the collusion model. As might be expected, the collusive regime yields higher average industry prices and producer surplus, and lower average consumer surplus. In the non-cooperative regime, producer surplus is almost completely eroded away amid a vigorous investment climate. The high degree of investment reflects two main factors. First, the smallest firms invest heavily to accelerate the onset of collusion. Second, the non-cooperative regime is invoked following entry, which will occur when active firms have small capacities and hence invest more heavily in a bid to ensure their survival. This

[27] We might expect collusion lengths to be shorter. However, the entry cost parameters, x_e, are chosen to limit the possibility that a fourth firm might wish to enter. With lower entry costs, we might expect the prevalence of entry to be greater, restricting the longevity of collusion. In addition, in section 5.5, price wars as a means to deter entry are considered, yielding a higher incidence of the punishment and non-cooperative regimes.

is reflected in the low average capacities observed in the non-cooperative regime. In turn, the greater incidence of entry and exit is a symptom of the smaller average size of firms when we enter the non-cooperative regime.

5.3 Comparison with the Lysine Market

The results of the previous section suggest the model is capable of explaining the type of collusion experienced in the lysine market. In particular, successful phases of collusion, and price wars following entry were observed in equilibrium. This section serves two purposes. First, it examines more closely the predictions of the model following an entry similar to the entry of ADM into the lysine market. The goal is not to replicate every nuance of firm behaviour in the lysine market, but rather to examine whether the flavour of events in the lysine market can be captured by the model.[28] Second, it illustrates the type of short term dynamics the model can generate.

The initial capacities of the incumbent firms are chosen to reflect the state of affairs after ADM entered. That is, a starting state vector of $(7,7,2)$ is chosen, meaning the incumbent firms have capacities of τg^7 and the entrant has capacity τg^2.[29] The model is then simulated 10,000 times for 100 periods each simulation. The time horizon is assumed to be monthly, reflecting the frequency with which lysine discussions were held, and the frequency of the data used for demand and cost estimates. Figures 7 and 8 present the results of these simulations.

Figure 7 depicts the market price in each period, averaged across the simulations. One standard

[28]It should be noted that the dynamic parameters are not estimated to fit the lysine market, but are only guided by the experience in that market. In fact, parameter choices are constrained by computational considerations which limit the size of the state space that can be handled.

[29]As discussed in note 1, we have in mind a more general notion of capacity than merely physical capacity. While ADM's initial physical capacity was large, and it did take advance steps to attract customers, its accumulation of global market share was gradual, reflecting constraints other than just physical capacity. Hence, ADM is accorded a relatively small initial capacity.

deviation error bands on the simulated price are also shown. A great deal of variety in the sample paths is masked. In the majority of simulations, there is a sustained price war before collusion is negotiated. Prices drop immediately after entry, and then rise when the collusive regime begins. The average market price begins to rise after only two periods. This is because in some simulations, exit occurs quickly, immediately followed by collusion by the remaining incumbents.[30]

Figure 8 shows for each period the fraction of simulations in each of the competitive regimes. It can be seen that we tend to enter the collusive regime in a larger fraction of periods over time, with the distribution settling down after about 30 periods. We enter the non-cooperative regime if there are a sequence of market-wide adverse investment outcomes and one of the firms exits the market. This provides the opportunity for the entry of a competitor while the remaining incumbents are relatively small.

A comparison with the discussion of Section 2 suggests that in principle the model can provide useful insights for the lysine market. However, there are some features that would be difficult to explain in the current model. First, the gradual nature of price movements in the lysine market associated with the onset of price wars and the beginning of collusion cannot be captured by the model. The smoothness of price movements in the lysine market reflects the existence of contracts and attempts to avoid arousing the suspicion of the anti-trust authorities, features not incorporated in the model. Second, the model assumes perfect information. This does not allow price wars that arise due to the existence of demand

[30]To control for this phenomenon, a subset of the simulations was chosen in which exit did not occur within the 100 periods of the simulation. The severity of the price war is then more apparent. The average price across the simulations falls for the first 4 months, and no collusion occurs for at least 5 months. The fraction of simulations with collusion then rises rapidly over time. However, the reader should notice the distortions induced by this selection exercise. An implication of this sample selection is that firms' investment activities were systematically more successful than anticipated and/or the decay of capacity was systematically overpredicted. However, the exercise does mimic ADM's successful entry into the market.

uncertainty and imperfect monitoring. de Roos (2000) notes that these factors may have contributed to the second price war in the lysine market.

5.4 Comparative Dynamics

In this section, I examine the consequences of varying some key parameters. Tables 5 and 6 summarise characteristics of industry equilibrium for four experiments. The results of the base model is reprinted in the first column of each table for reference. In each experiment, one of the parameters in Table 1 is separately raised and lowered.[31]

In the base case, the cost of entry, x_e, was drawn from the uniform distribution $U(15,20)$. In the first experiment entry costs are altered, with $x_e \sim U(16,21)$ in column 2 and $x_e \sim U(14,19)$ in column 3. As we might expect, the frequency of entry and exit is increased and a less concentrated industry results on average when entry barriers are lowered. The greater incidence of entry makes collusion more difficult to sustain with lower entry barriers. In addition, a reduction in entry barriers emboldens competitors to enter for higher incumbent capacity levels. A consequence of this is that, on average, it takes longer for an entrant to catch up to the incumbents before striking up a collusive agreement. Reflecting the reduced prevalence of collusion, we also observe lower average industry prices and higher average consumer surplus if entry costs are lowered. Individual firms produce less on average as entry costs are lowered, but industry production is comparatively insensitive to entry costs because of a counterbalancing change in the number of operating firms. Average producer surplus falls as entry costs are lowered because of the increased industry-wide investment required to sustain a more concentrated industry.

In the second experiment, the demand intercept is changed from 1.651 to 1.75 and 1.55 in columns

[31] In some of these experiments (specifically, if we decrease entry costs, increase the demand intercept, or increase the discount factor), we might expect the restriction to a maximum of three firms to be binding. The possibility of further entry could influence the statistics reported below, particularly by affecting the incidence of entry and the prevalence of collusion.

4 and 5, respectively, of Table 5. Notice that this is an equivalent experiment to altering the constant marginal cost parameter, except that the market price will differ by a constant. As we might expect, average industry concentration falls and average market price rises as the demand intercept is increased. Average firm production actually falls as demand is raised, but due to the change in industry structure, average industry-wide production rises. Average firm capacity is lower if we reduce the demand intercept, reflecting a reduced incentive to invest in a less profitable industry. However, firm capacity is also lower when we raise demand relative to the baseline case. This occurs because three firms are supported by the industry for the vast majority of the time. While there are three firms in the market, the incentive to invest in excess capacity is reduced.

If we raise the demand intercept, punishment is sometimes observed with two firms in the market. This occurs whenever one of the colluding firms has a sequence of adverse investment outcomes such that it becomes capacity constrained in the non-cooperative quantity game. In this situation, the larger firm produces more in the static Nash equilibrium than under the collusive agreement, and hence the punishment regime becomes more profitable for the larger firm. With the base set of parameters, the option value of collusion counteracts the short term profitability of the punishment regime and punishment is not observed. However, with the higher level of demand, entry is very likely in this scenario, thus reducing this option value to maintaining collusion. Finally, collusion occurs more quickly on average if we raise or lower the demand intercept relative to the baseline. When demand is expanded, investment is more aggressive for low capacity levels, thus accelerating convergence toward equal capacities. When demand is contracted, the entry of a third firm occurs only when the incumbents are very small, and hence firms equalise capacities relatively quickly.

In the third experiment, the discount factor is altered from the base value of 0.98 to 0.985 in column 2 and 0.97 in column 3 of table 6. The impact on industry structure is dramatic. More concentrated industries are associated with lower discount factors because firms have a reduced incentive to invest, as reflected in reduced investment spending for any given state vector. This is because they care less about future profitability. Notice that, because the distribution of states is altered, average investment per firm is actually similar in this experiment.However, average firm capacity falls considerably as the discount factor is reduced. The reason is that, with a lower discount factor, firms reduce investment substantially as their capacity rises, and hence spend a greater time with a low capacity, but moderate investment levels. Average capacity per firm also falls when we raise the discount factor, but this reflects the prevalence of three active firms in the market.

By changing the discount factor, we also alter the characteristics of collusion. When we increase the discount factor, punishment is observed on the equilibrium path. This occurs for the reasons described above in the experiment in which the demand intercept is raised. When we decrease the discount factor, punishment is observed in two situations. First, with three firms in the market, firms are no longer patient enough to wait until capacities are exactly equalised before colluding. Punishment will then occur in the future when capacities become misaligned with the market share allocations. Second, firms will enter the punishment regime when there are two firms in the market and the smallest firm is particularly small, even if collusive market shares are equal. This is because, by entering the punishment regime, the larger firm discourages investment by the smaller firm. Because its rival is so small, it can do so without sacrificing profits.

In the final experiment, the probability of renegotiation, γ, is changed from 0.05 to 0.1 and 0.025 in columns 4 and 5, respectively, of Table 6. For this experiment, a discount factor of 0.97 is retained, so comparison should be made with column 3 of the table.[32] We might expect a change in γ to have

[32]The effect of a similar change to the probability of renegotiation with the discount factor held at the baseline level of 0.98 was negligible. A more extreme experiment in which there is no punishment regime was also considered. In this experiment, the unilateral termination of the collusive regime by any cartel member leads play to the non-cooperative regime. As we might

two opposing effects on the prevalence of collusion. First, the average length of the punishment regime rises as γ falls. Second, as the severity of punishment rises, we might expect firms to alter their punishment policy and to tailor investment spending to avoid industry structures that are prone to punishment. The second effect dominates in this instance and the prevalence of punishment rises as we increase the probability of renegotiation.

5.5 A Variation to the Punishment Regime

In the main model presented above, we assumed that the entry of a competitor negates any punishment or negotiation phase of incumbent firms. Here, we briefly consider the alternative that entry does not disrupt the punishment phase.[33] This game could then be represented by Figure 1 with the broken line removed. We may wish to consider this variation in industries in which firms have access to a commitment technology. For example, in a legal environment hostile to collusion, if only a small minority of individuals in participating firms are cognisant of cartel activities, public announcements of price cuts may be difficult to reverse quickly. Alternatively, if potential entrants are imperfectly informed about the characteristics of the industry or of the incumbent firms, we could think of the punishment regime as a signalling game.[34] The results of this experiment are contained in Figure 9 and column 3 of Table 2.

Analogous to Figure 3, Figure 9 depicts states in which incumbent firms wish to collude. The diagonal lines in the interior and on the faces of the cube demonstrate that, as before, firms hold off their collusive venture until they have equal market shares. In contrast with the earlier result, we see that when

expect, this makes collusion more difficult to sustain, yielding approximately a doubling in the incidence of the non-cooperative regime. With the reduced commitment value attached to collusion in this experiment, firms were also willing to countenance less than equal collusive shares. Results of this experiment are available from the author on request.

[33] Earlier versions of this paper, available from the author, adopt this variation as the core version of the model.

[34] In the lysine industry, prices began their descent prior to ADM's physical entry, but after ADM announced its intention to enter.

there are two firms operating in the market and both are relatively small, they are willing to enter into a collusive agreement. This occurs for two reasons. First, when incumbent firms are relatively small, entry is likely in the near future. This can make collusion attractive even for the smallest firm, the principal stumbling block to an agreement. By colluding, firms obtain the collusive profits immediately and, because entry breaks the collusive regime, also envisage an equal share of collusive profits some time in the future.[35] This behaviour is not observed when the incumbent firms are larger because the probability of entry is markedly reduced. Second, incumbent firms may seek to deter entry. For a given state vector, the probability of entry is lower in the punishment regime. Incumbents could reduce the probability of entry by colluding and subsequently entering the punishment regime.

Column 3 of Table 2 presents industry statistics for this version of the model. Relative to the base model, firms spend a greater fraction of time in the punishment regime because the punishment regime is invoked to deter entry. A less concentrated industry with a lower market price emerges on average reflecting both the impact of entry deterrence and the reduced prevalence of collusion. Firms have higher average production and capacity owing to the lower fraction of time in which three firms are sustained in the industry. Finally, on average firms are longer lived and earn a higher discounted present value of profits.

[35] One could argue that if imminent entry were anticipated, the incentive to deviate from collusion would be strong. Firms do not deviate under the current modelling assumptions because any deviation is detected immediately and met with the punishment regime. Any temptation to deviate is mitigated by the prospect of future collusion with the same rivals. As we have seen, with the current set of parameters, firms are willing to forego current profits in order to secure favourable terms under collusion. It seems probable that firms would also forego the temptation to deviate from the cartel prior to entry in order to obtain collusive profits in the future. However, this issue should be reexamined if firms are significantly less patient.

6 Extensions

There are many possible avenues for extension of the current model. Policy experiments could be conducted through some fairly minor changes to the model. A simple way to incorporate the influence of the anti-trust authorities would be to include in the collusive regime a probability of detection, leading to pecuniary punishment. The detection probability could potentially be conditional on the level of prices or the rate of change of prices, although this will add to the computational complexity. There has been a recent surge in the number of successful international cartel prosecutions, with a contributing factor being a change in the amnesty program. An amnesty policy could be incorporated into the model by allowing any firm to blow the whistle on the collusive regime and avoid any punishment by the authorities.

Some more substantive enhancements to the model are also desirable. I will point out some features that appear particularly important based on consideration of the lysine market. First, a central prediction of the model is the tendency for firms to build up a market share comparable with their competitors before colluding. This prediction can only carry force if firms are on an equal footing. In the current model, firms differ only in their capacity constraints. A more realistic setting would allow for other asymmetries. Cost asymmetries are especially important. We might expect an entrant with a perceived relative cost disadvantage to pursue something less than a market share comparable to its competitors before caving in to the temptation of collusion. Second, an important element guiding firm behaviour in the lysine market was the expectation of future market growth. Altering the discount factor could represent a rough approximation for market growth. However, to account more precisely for growth necessitates an expansion of the state space. Third, an important assumption of the model is that the entrant must be a party to any collusive agreement. This assumption is relaxed by de Roos (2001) in a dynamic model of collusion with a competitive fringe applied to the market for vitamin C. More generally, we could consider collusion by any subset of the incumbent firms. This is a particularly thorny problem, even in the absence of dynamic considerations.[36]

7 Conclusions

This paper has developed a dynamic model of collusion that focuses on the role of entry. The model is sufficiently rich to allow a wide variety of behaviour in equilibrium. Successful collusion, price wars due to entry or punishment, and entry deterrence are all potential elements of equilibrium. Key assumptions of the model are motivated by recent empirical observations of collusion. The model could provide guidance for any market with collusive possibilities, the potential for entry, and some uncertainty about the characteristics of potential entrants. While the rule of thumb that operated in the lysine market plays a major role in the model, this rule of thumb has recently been observed in several other markets with a history of collusion, and we might expect the coordination and uncertainty arguments to be relatively general.

An important theme has been the link between collusion and market structure. On the one hand, the likelihood of collusion depends crucially on the current market structure. A central prediction of the model is that a firm entering a market characterised by collusion will tend to build up a market share comparable with its competitors before agreeing to collude. This was an important feature of the market for lysine, where collusion has recently played a dramatic role. On the other hand, the possibility of collusion in a particular industry has strong implications for the market structure that might evolve. We tend to observe a greater number of firms in industries affording collusive possibilities.[37]The results also highlight the intimate connection between the success of collusion and the prospects for entry. In market structures where entry is unlikely, collusion is more likely to be sustained.

[36] See, for example, Bernheim, Peleg and Whinston (1987).

[37] In the model of Fershtman and Pakes (2000), the impact of collusion on market structure is even stronger. The difference in results can be largely accounted for by the comparatively benign competitive environment faced by an entrant in their model.

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Table 1: Parameters Used for the Base Model

Parameter	Description	Value
a	Demand intercept	1.651
b	Demand slope	0.0857
mc	Marginal cost of production	0.527
α	Investment efficiency	1.2
ω_e	Entrant's starting capacity	2.88
x_e^{min}	Minimum entry cost	15.0
x_e^{max}	Maximum entry cost	20.0
ϕ	Scrap value of firm	1.0
τ	Minimum capacity parameter	2.0
g	Capacity growth factor	1.2
$\overline{k}$	Number of feasible capacity levels	18
β	Discount factor	0.98
δ	Capacity depreciation rate	0.4
γ	Renegotiation probability	0.05
N	Maximum number of firms	3

Table 2: Industry Statistics for Model With and Without Collusion

Industry Feature	Without Collusion	With Collusion	Punishment Variation
Periods with 0 firms	0.0%	0.0%	0.0%
Periods with 1 firm	0.10%	0.03%	0.04%
Periods with 2 firms	96.51%	51.06%	72.73%
Periods with 3 firms	3.39%	48.91%	27.24%
Periods with entry	0.16%	0.40%	0.24%
Periods with exit	0.15%	0.38%	0.23%
Periods in collusive regime	–	96.59%	77.40%
Periods in non-cooperative regime	–	3.41%	5.00%
Periods in punishment regime	–	0.00%	17.60%
Mean market price	0.915	1.081	1.050
	(0.047)	(0.045)	(0.077)
Mean investment per incumbent	0.619	0.612	0.611
	(0.352)	(0.341)	(0.338)
Mean firm production	4.223	2.671	3.085
	(0.428)	(0.553)	(0.739)
Mean firm capacity	6.672	6.378	6.866
	(2.748)	(2.822)	(2.973)
Mean one-firm concentration ratio	0.510	0.421	0.460
	(0.047)	(0.085)	(0.080)
Mean consumer surplus	3.170	1.905	2.140
	(0.370)	(0.359)	(0.598)
Mean producer surplus	2.025	2.079	2.173
	(0.899)	(1.370)	(1.151)
Mean total surplus	5.195	3.984	4.313
	(1.132)	(1.352)	(1.166)
Mean firm value	9.451	1.873	2.263
	(18.42)	(14.51)	(15.31)
Mean firm lifespan	1306.7	624.1	949.3
	(1609.7)	(842.4)	(1306.7)
Median firm lifespan	619	311	422

Standard deviations for observations are in parentheses ().

Table 3: Characteristics of Collusion and Punishment

	Mean	Standard Deviation
Periods taken to collude following entry		
	9.34	8.86
Length of collusive regime		
Beginning with 2 firms	156.5	253.1
Beginning with 3 firms	335.9	355.8
All cases	296.2	343.9
Price change at collusion		
	24.63%	15.25%
Price change on entry into industry with collusion		
	-19.11%	8.30%

Table 4: Industry Statistics for Different Regimes

Industry Feature	Collusive	Non-cooperative
Periods with 0 firms	0.0%	0.0%
Periods with 1 firm	0.02%	0.34%
Periods with 2 firms	52.27%	16.67%
Periods with 3 firms	47.70%	82.98%
Periods with entry	0.34%	2.13%
Periods with exit	0.30%	2.66%
Mean investment per incumbent	0.598	0.943
	(0.324)	(0.527)
Mean firm production	2.647	3.275
	(0.542)	(0.486)
Mean firm capacity	6.471	4.073
	(2.825)	(1.408)
Mean market price	1.089	0.858
	(0.006)	(0.078)
Mean one-firm concentration	0.422	0.393
	(0.085)	(0.091)
Mean consumer surplus	1.841	3.707
	(0.030)	(0.630)
Mean producer surplus	2.152	0.006
	(1.255)	(2.444)
Mean total surplus	3.994	3.713
	(1.268)	(2.825)

Standard deviations for observations are in parentheses ().

Table 5: Industry Statistics for Different Parameter Values

Industry Feature	Base Model	Experiments			
		Entry Costs		Demand	
		Higher	Lower	Higher	Lower
Periods with 1 firm	0.03%	0.04%	0.05%	0.03%	0.22%
Periods with 2 firms	51.06%	61.36%	33.35%	0.52%	94.71%
Periods with 3 firms	48.91%	38.60%	66.60%	99.45%	5.07%
Periods with entry	0.40%	0.32%	0.59%	0.55%	0.30%
Periods with exit	0.38%	0.31%	0.57%	0.53%	0.29%
Periods in collusive regime	96.59%	97.38%	94.78%	96.15%	98.38%
Periods in non-cooperative regime	3.41%	2.62%	5.22%	3.75%	1.62%
Periods in punishment regime	0.00%	0.00%	0.00%	0.10%	0.00%
Mean market price	1.081	1.083	1.077	1.129	1.038
	(0.045)	(0.039)	(0.056)	(0.051)	(0.022)
Mean investment per incumbent	0.612	0.614	0.607	0.616	0.616
	(0.341)	(0.352)	(0.332)	(0.376)	(0.366)
Mean firm production	2.671	2.776	2.513	2.418	2.919
	(0.553)	(0.555)	(0.514)	(0.217)	(0.292)
Mean firm capacity	6.378	6.375	6.214	5.581	5.855
	(2.822)	(2.714)	(2.954)	(2.216)	(2.342)
Mean one-firm concentration ratio	0.421	0.438	0.391	0.336	0.496
	(0.085)	(0.083)	(0.081)	(0.021)	(0.047)
Mean consumer surplus	1.905	1.888	1.941	2.262	1.535
	(0.359)	(0.309)	(0.449)	(0.442)	(0.138)
Mean producer surplus	2.079	2.152	1.947	2.398	1.737
	(1.370)	(1.341)	(1.479)	(1.585)	(1.168)
Mean total surplus	3.984	4.039	3.888	4.660	3.272
	(1.352)	(1.332)	(1.469)	(1.592)	(1.209)
Mean firm value	1.873	2.939	1.759	6.956	7.463
	(14.510)	(15.290)	(13.542)	(15.450)	(16.434)
Mean firm lifespan	624.1	736.6	450.2	544.6	691.5
	(842.4)	(960.7)	(588.2)	(633.7)	(809.6)
Mean periods to collude after entry	9.34	8.86	9.62	6.86	5.59
	(8.86)	(8.68)	(9.42)	(7.39)	(7.59)

Standard deviations for observations are in parentheses ().

Table 6: Industry Statistics for Different Parameter Values

Industry Feature	Base Model	Experiments			
		Discount Factor		Renegotiation Probability	
		Higher	Lower	Higher	Lower
Periods with 1 firm	0.03%	0.02%	0.31%	0.29%	0.33%
Periods with 2 firms	51.06%	0.46%	96.00%	97.02%	96.13%
Periods with 3 firms	48.91%	99.52%	3.69%	2.68%	3.54%
Periods with entry	0.40%	0.48%	0.38%	0.35%	0.41%
Periods with exit	0.38%	0.46%	0.37%	0.34%	0.40%
Periods in collusive regime	96.59%	96.35%	94.02%	86.08%	97.88%
Periods in non-cooperative regime	3.41%	3.54%	2.92%	7.05%	1.93%
Periods in punishment regime	0.00%	0.10%	3.05%	6.87%	0.19%
Mean market price	1.081	1.080	1.086	1.075	1.091
	(0.045)	(0.049)	(0.041)	(0.055)	(0.030)
Mean investment per incumbent	0.612	0.607	0.626	0.628	0.626
	(0.341)	(0.341)	(0.395)	(0.388)	(0.395)
Mean firm production	2.671	2.224	3.244	3.323	3.217
	(0.553)	(0.204)	(0.380)	(0.450)	(0.320)
Mean firm capacity	6.378	6.104	5.252	5.383	5.213
	(2.822)	(2.712)	(1.818)	(1.863)	(1.806)
Mean one-firm concentration ratio	0.421	0.335	0.505	0.508	0.504
	(0.085)	(0.019)	(0.050)	(0.050)	(0.049)
Mean consumer surplus	1.905	1.915	1.875	1.956	1.836
	(0.359)	(0.392)	(0.282)	(0.400)	(0.189)
Mean producer surplus	2.079	1.760	2.330	2.318	2.337
	(1.370)	(1.456)	(1.314)	(1.266)	(1.348)
Mean total surplus	3.984	3.676	4.205	4.274	4.173
	(1.352)	(1.441)	(1.418)	(1.379)	(1.446)
Mean firm value	1.873	5.130	6.575	5.750	6.514
	(14.510)	(16.349)	(13.766)	(13.821)	(14.104)
Mean firm lifespan	624.1	620.8	529.4	571.9	496.8
	(842.4)	(761.8)	(622.4)	(698.9)	(603.5)
Mean periods to collude after entry	9.34	7.43	4.67	4.96	4.78
	(8.86)	(8.15)	(6.59)	(7.06)	(6.79)

Standard deviations for observations are in parentheses ().

Figure 1: Sequence of Events

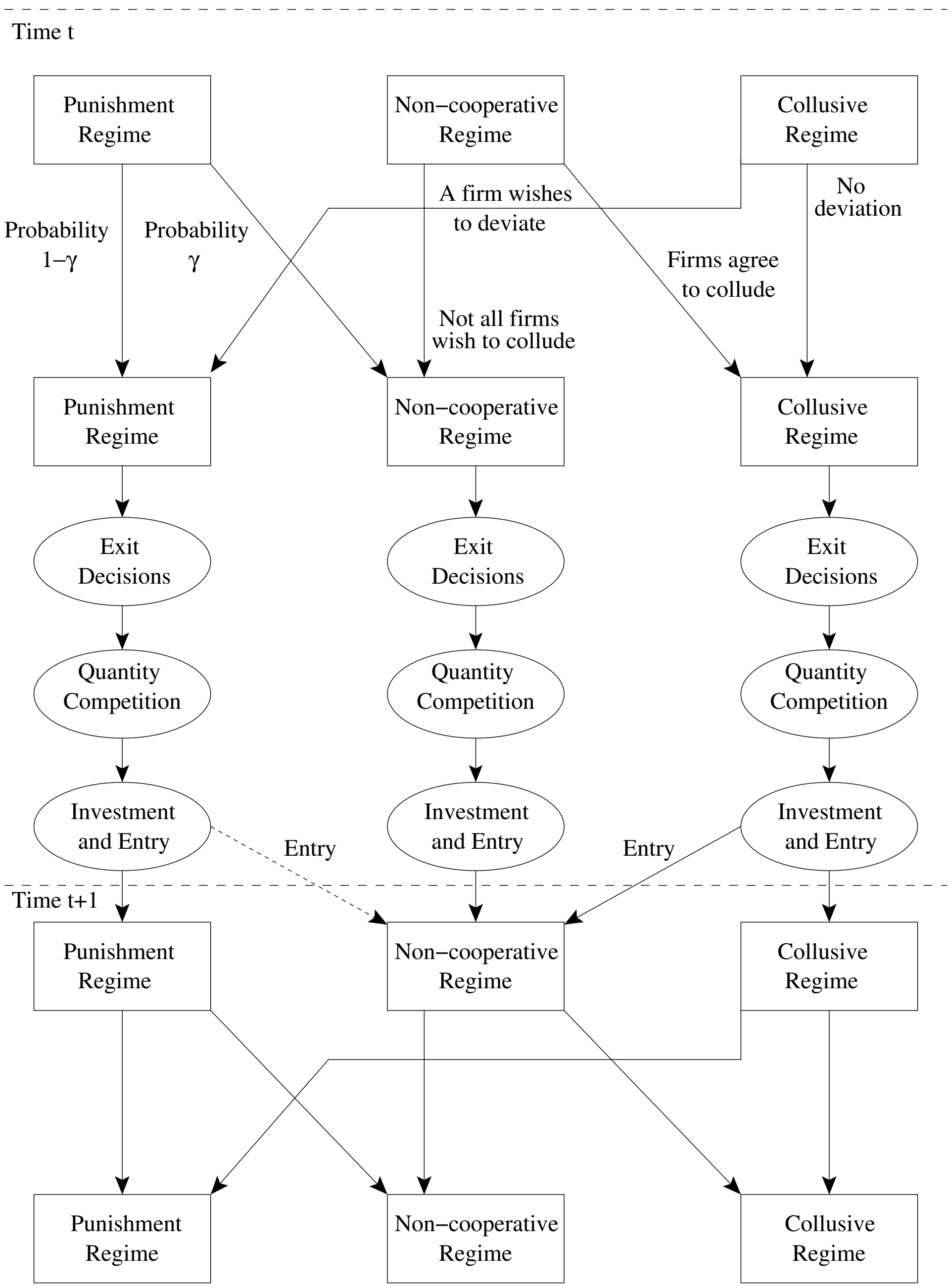

Figure 2: Investment in the Model without Collusion

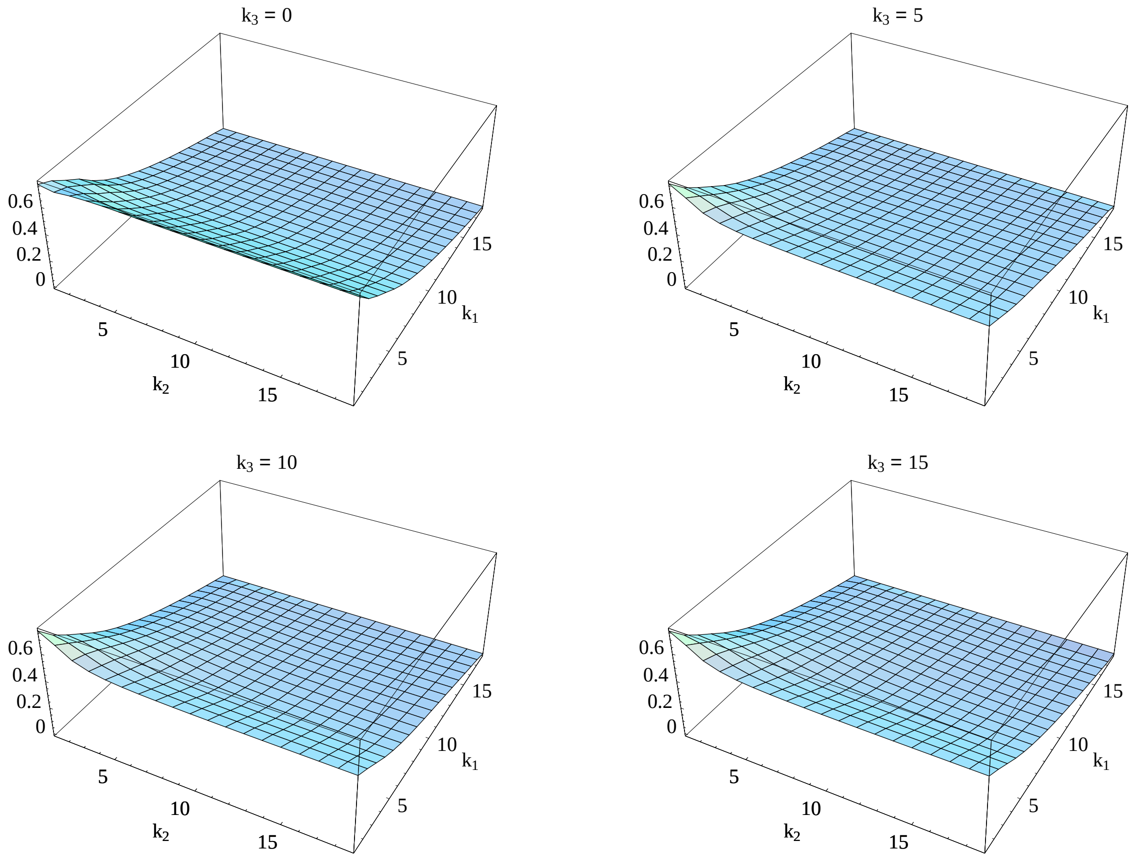

Figure 3: Collusion States

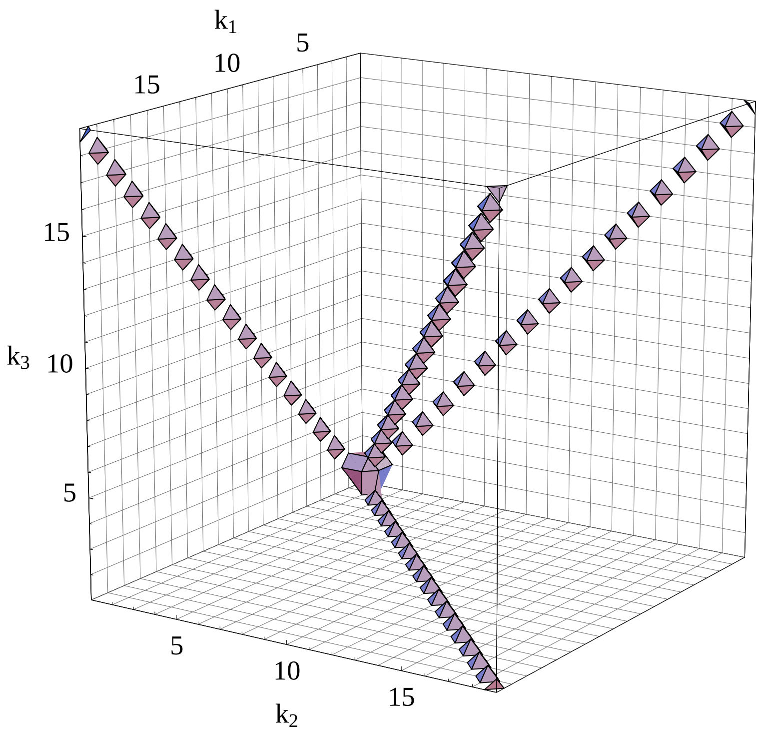

Figure 4: Entry Region by Model and Regime

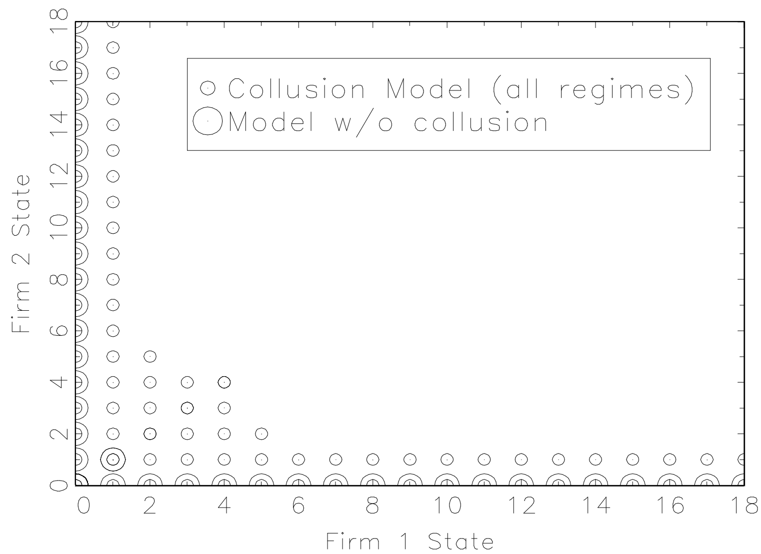

Figure 5: Investment in the Non-cooperative regime

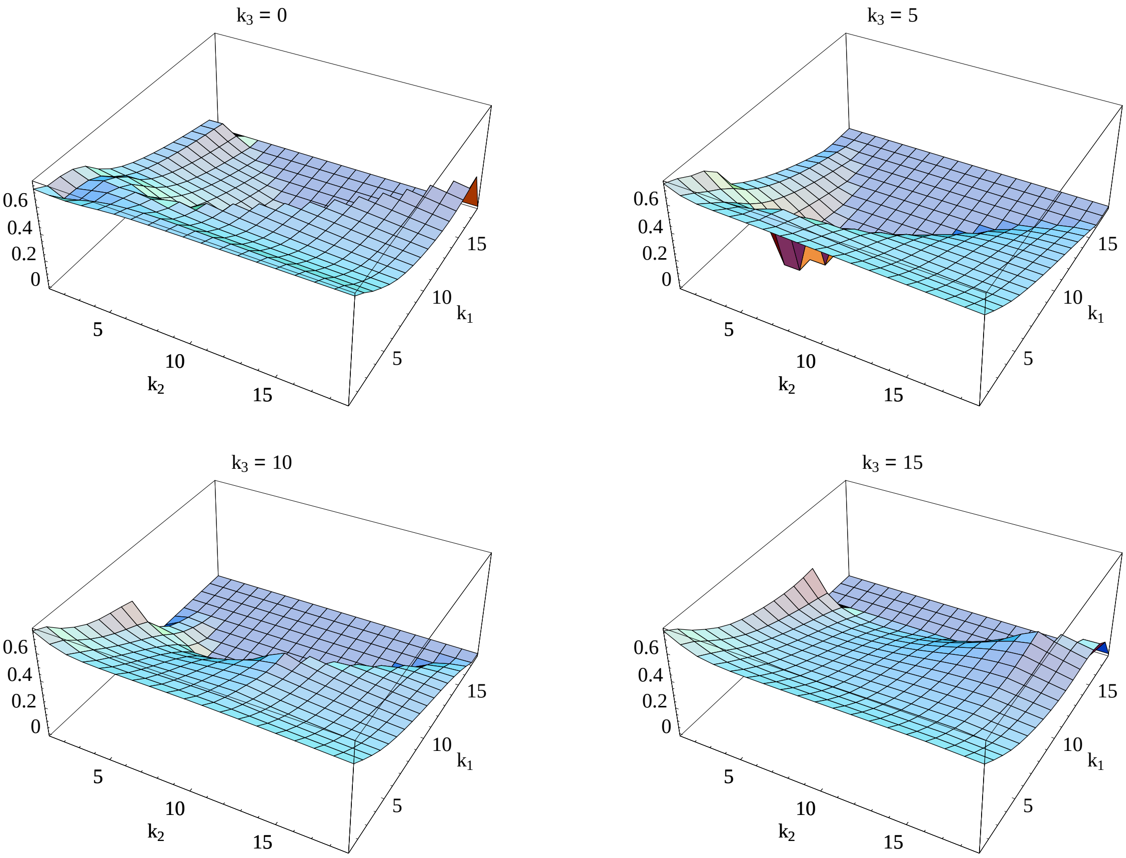

Figure 6: Investment in the Collusive regime

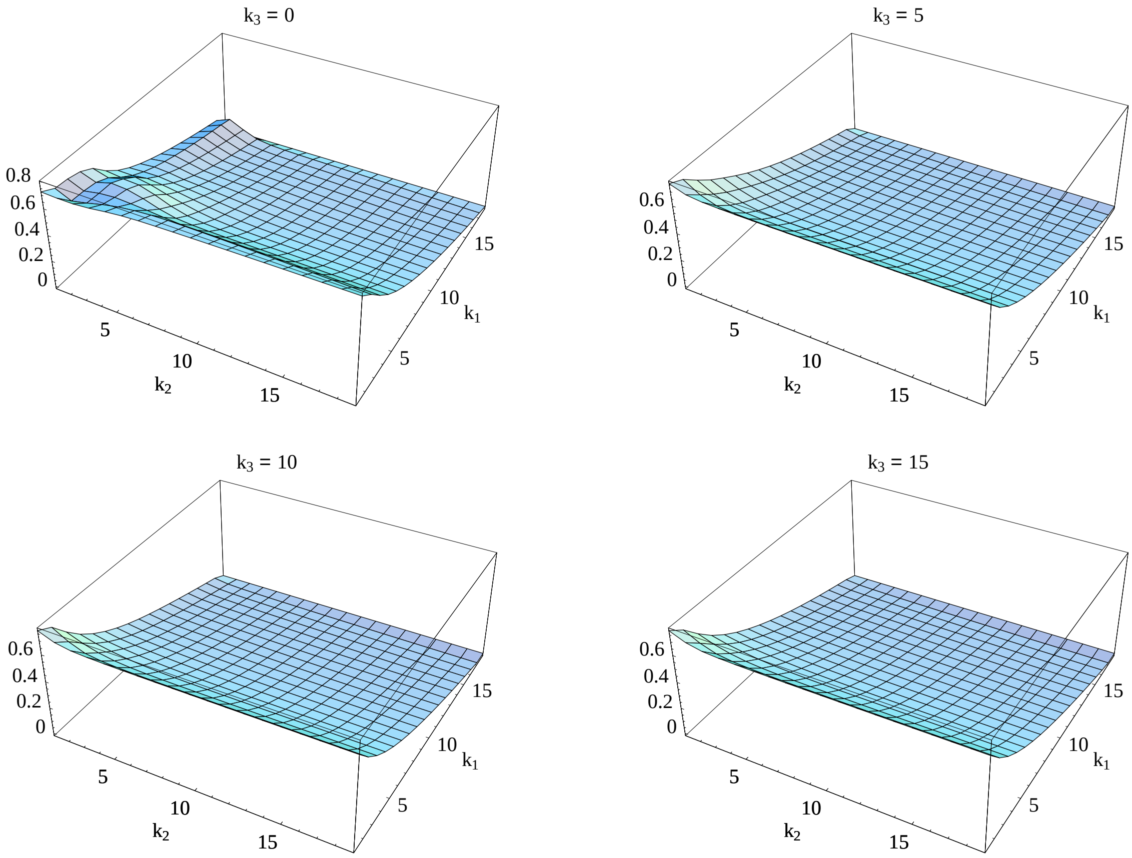

Figure 7: Mean Simulated Prices

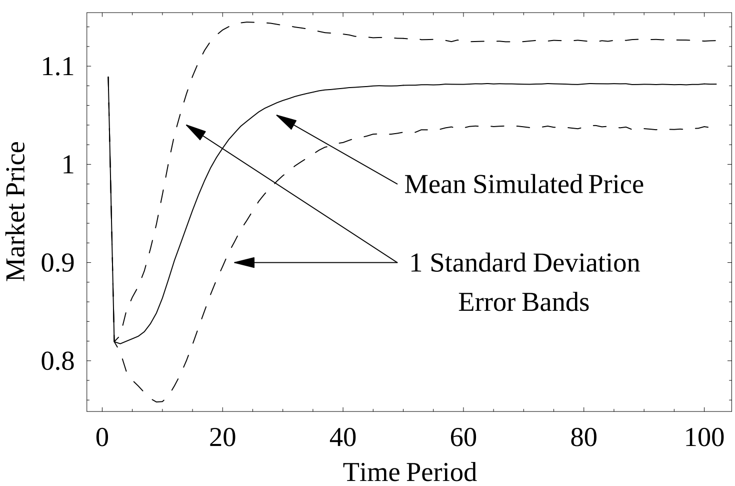

Figure 8: Fraction of Simulations in Each Regime

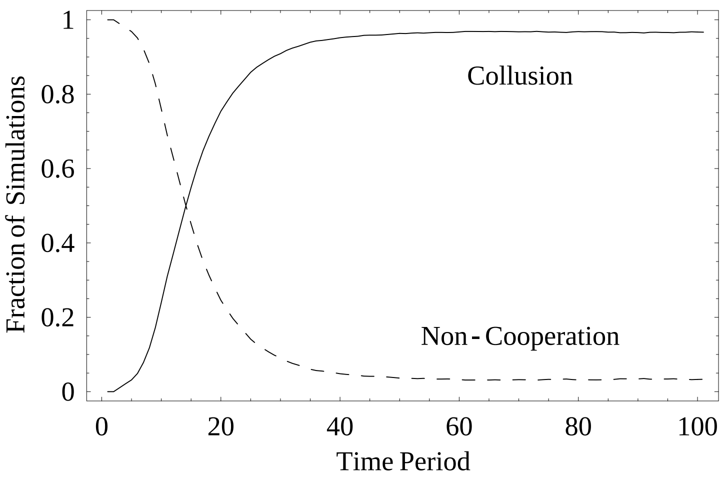

Figure 9: Collusion States with Punishment Refinement