Non Cooperative Networks in Oligopolies *

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22nd January 2004

Abstract

In an oligopoly, prior to competing in the market, firms have an opportunity to pick up externalities from other firms by setting links. The links formation defines an industrial network. We study the incentives for firms to form links and the effect of this links formation on the architecture of the resulting networks. Our analysis shows that equilibrium networks differ dramatically depending on the nature of market competition (Cournot or Bertrand). More precisely, in the case of Cournot oligopoly, we should expect to see networks in which each firm obtains access to externalities of either all firms or no firm. In the case of Bertrand oligopoly, we should expect to see networks in which one firm derives benefits from externalities of all other firms while the latter get no externality. We also present some results on the architecture of socially efficient networks.

JEL Classification Number: C70, L13, L20.

Key Words: Networks, Oligopolies, Information Externalities.

*We are especially grateful to two anonymous referees and the Editor for helpful comments. Moreover, we thank participants at ESEM 2002 Venice, and seminar participants at GREQAM 2003, for comments.

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1 Introduction

Empirical studies have emphasized the role played by the access to information of competitors in the competitive strength of the firms (Cohen, Levinthal, 1990). This information can concern various aspects, such as markets, products, or technologies. They often constitute non rival and non free resources. More precisely, information which a firm is endowed with can be acquired by another firm in forming a link, regardless of how many firms have already formed a link with this firm. However, in establishing and maintaining a link with another firm in order to get externality flows, a firm must incur costs, in terms of effort, time, and money. The notion of link setting is very large; it is not restricted to entering into communication with another firm, but includes all the resources devoted by a firm to pick up externalities produced by another firm. Thus, it concerns the hiring of a worker of another firm who embodies the required information, or the reorganization of the firm to enhance her absorptive capacity of these externalities, among others.

In fact, two types of links are distinguished in the existing literature. In the first instance, firms form pair-wise collaborative links (Jackson, Wolinski (1996)) with other firms, which involve a commitment of resources on the part of collaborators. This first instance has been extensively developed by scholars (Kranton, Minehart (2001), Goyal, Joshi (2002)). These works are part of a wider literature relating to group formation and cooperation, which has been a central concern in economics and game theory in particular (Yi, 1997, 1998, Bloch, 1995). The traditional approach to these issues has been in terms of coalitions. The network approach differs from the coalition approach in studying two-players or bilateral relationships.

As for the second instance concerning the types of links, it has received much less attention in spite of his growing empirical evidence. In this second instance, links setting is non cooperative in the sense that a firm can establish a link with another firm and get information or resources from the latter without seeking her permission (Bala, Goyal, 2000 [a], Bala, Goyal, 2000 [b]). Activities relating to links setting in this second instance are numerous and vary greatly in their form. These activities include, among others, reading of industry trade press or patents literature, talking with technology vendors, sales representatives or industry experts, visiting the commercial trade fairs and analyzing the competitors' products. In this latter case, the setting of a link with a competitor consists of acquiring a competitor's product in order to learn about the characteristics and the manufacturing process of this product, by using techniques like tearing down and reverse-engineering of the product. The ultimate goal of this activity for a firm is to improve the quality and to lower the cost of her own product as a result of the information acquired about competitor's advantage (T.M. van Ryn, 2000).

All these activities form a large part of what Prescott and Gibbons (1993) define as competitive intelligence activities. The significant growth in intelligence activities directed at competitors can be traced to the publication in 1980 of Michael Porter's Competitive Strategy: Techniques for Analysing Industries and Competitors [1].

In this paper, we concentrate on this second instance. More specifically, we assume a set of firms potentially able to operate on a set of different markets given their technology and organizational capabilities. We choose to focus our analysis on one these markets, supposed to be defined as an (endogenous) asymmetric homogeneous oligopoly market.

We assume that each firm operating on this market can form links with the other firms operating on this particular market or more generally on the set of the different markets under consideration. These links generate cost economies, through externality flows. Thus the set of links of the different firms, present on this particular market, with other firms, located on this market or on the set of the different markets, defines a network and induces a distribution of costs across the firms in the industry.

Given these costs, firms then compete in the market. We examine two alternative approaches about firms' behaviors on this market and mechanisms by which individual consumers' demands are allocated among competing firms: the Cournot approach and the Bertrand approach.

We focus on the characterization of the structure of the equilibrium networks. Literally, a given network is said to be an equilibrium network in the following way: holding the links of the other firms to be constant, no firm has an incentive to break a link or links she has already set or to establish new links with other firms present on the market or on the set of

[1] Several studies stress that these activities are significant and can be very effective. For instance, a poll at 53 large U.S. and multinational companies found that four out of five (83 percent) of the companies surveyed have either an in-house competitive intelligence capability or use both inside and outside sources (SCIP, 2003). Clay D. Smith (2003, Chicago Business, 2003) asserts that Andersen's client realized a full 40 % reduction in costs thanks to the reviewing of competitors' production techniques

the different markets.

Formally, our model can be described as a two-stage game. In stage I, each firm decides which links to establish with competitors. We suppose that the results of stage I, i.e. the set of links formed by the firms, are perfectly observable among competitors when firms make decisions about quantities (Cournot) or prices (Bertrand) in stage II. Given these links, in stage II, the firms choose the quantities they produce (Cournot) or the prices they set (Bertrand) simultaneously. In the spirit of equilibrium analysis, a natural conjecture is that the second stage output (Cournot) or price (Bertrand) choices will be those of a Cournot Equilibrium or Bertrand Equilibrium for the prevailing network architecture of the industry. We assume that firms are aware of this fact.

This assumption has the following implication: in stage I, when a firm makes a decision about her links, she takes into account not only the quantity (or price) the new configuration of links induces her to produce, but also the quantities (or prices) this new configuration of links induces her competitors to produce (or choose). In other words, we suppose each firm knows the Cournot (or Bertrand) Nash equilibrium associated with every configuration of links and keeps these equilibria in mind in taking her decision in stage I. Hence, the equilibrium networks can be described as equilibria in the network subgame. Then, the equilibria profiles can be seen as subgame perfect equilibria since they are induced by Nash equilibria in every subgame of the original game.

Our results concerning equilibrium networks differ dramatically depending on the nature of market competition (Cournot or Bertrand) and on the costs of forming links. More precisely, in the case of Cournot oligopoly, if the cost of links setting is low enough, we should expect to see networks in which each firm picks up externalities of all other firms. Moreover, if the cost of links setting is high enough, we should expect to see networks in which each firm gets no resources from their competitors. In this respect, we obtain the same results as Goyal and Joshi (2003). Furthermore, in our model, there exists cost of links setting such that, in equilibrium networks, some firms pick up externalities of all other firms while the others obtain no resources from their competitors. This result differs from GJ since they find that there exist equilibria such that firms with links are only linked to each other and do not have any links with firms without links. Thus in our model, firms' heterogeneity concerning the resources obtained is stronger than in GJ model. Lastly, we show that in the efficient networks, firms have formed links either with all other firms or with no firm.

In the case of Bertrand oligopoly, when the cost of setting a link is not too high, we should expect to see networks in which one firm gets externalities from all other firms while the latter obtain no externality from their competitors. This result differs dramatically from those of GJ. Indeed, in their model, the empty network is the only equilibrium network. Moreover, by contrast to the Cournot model, we show that there exist conditions such that in the Bertrand Model, some network architectures are efficient but are not equilibrium.

The rest of this paper is organized as follows. Section 1 describes the basic model while sections 2 and 3 present the results. In section 2, we study Cournot oligopolies and in section 3, we analyze Bertrand oligopolies.

2 The Model

2.1 Definitions

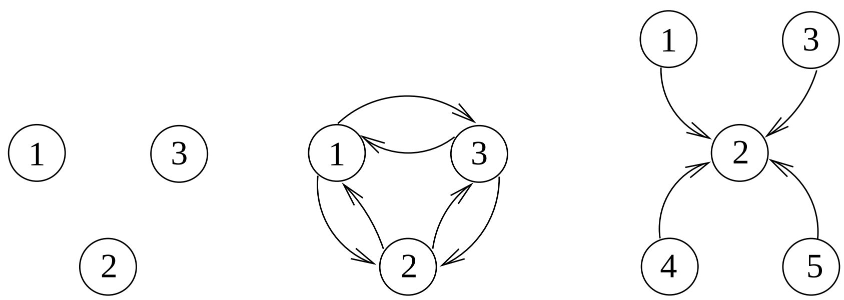
Let $N = \{1, \ldots, i, j, \ \ldots \ , n\}$, with $n \geq 3$, denote by a finite set of *ex ante* identical firms. For any $i, j \in N$, $g_{i,j} = 1$ means that firm i has formed a directed link with firm j, $g_{i,j} = 0$ otherwise. Hence, we denote by $\mathbf{g}_i = (g_{i,1}, \ldots, \ g_{i,i-1}, 0, \ g_{i,i+1} \ldots, g_{i,j}, \ldots, g_{i,n})$ the directed links vector of firm i.

A directed network $\mathbf{g} = \{(g_{i,j})_{i \in N, j \in N}\}$ is a formal description of the directed links that exist between the firms. Let $\mathcal{G}$ denote the set of all directed networks without loops (since we assume that a firm i can not form a link with herself).

We assume that the link $g_{i,j} = 1$ enables firm i to access j's information, but not *vice versa*. In other words, we study a one way flow model of resources. Let $N_i(\mathbf{g}) = \{j \mid g_{i,j} = 1\}$ be the set of firms j such that i gets externalities from j. Let $n_i(\mathbf{g})$ be the cardinal of $N_i(\mathbf{g})$. We will frequently refer to all firms other than some given firm i as firm i's opponents and denote them by $-i$. Let us denote by $n_{-i}(\mathbf{g}) = \sum_{j \neq i} n_j(\mathbf{g})$ the number of links in the network $\mathbf{g}$ excluding those links originating from firm i. $n_{-i}(\mathbf{g})$ can be interpreted as the number of externalities which benefit all firms except firm i.

We now define the main network configurations that are extensively used in our model. A network $\mathbf{g}$ is complete if for every pair of firms i and j, there is a link from i to j. We denote the complete network by $\mathbf{g}^c$. A network $\mathbf{g}$ is an inward-pointing star if there is one and only one firm i such that i has formed a link with each firm j, and every $j \neq i$ has formed no link (see Figure 1). We denote the inward-pointing stars by $\mathbf{g}^s$. A network $\mathbf{g}$ is empty if no firm has formed a link. We denote by $\mathbf{g}^e$ the empty network.

Figure 1: Networks Architecture



(a) Empty Network (b) Complete Network (c) Inward-Pointing Star

2.2 Links Setting and Cost Reduction

Firms first choose their links, then observe the resulting network and compete in the market. Hence there are two stages in the game. More precisely, in stage I, each firm decides which links to establish with other firms. A link setting is an investment in externality search about technology, products or markets. The externalities obtained can be interpreted as externality flows. Recall that we assume that these flows are one-way.

We will suppose that setting a link requires a fix investment cost given by $\delta > 0$. We assume that firms initially are symmetric with zero fix cost and identical cost functions. We suppose that the purpose of establishing a link is to reduce variable production cost. More specifically, we assume that the marginal cost function and the average variable cost function of a firm i have the same following form:

$$c_i(n_i(\mathbf{g})) = \gamma_0 - \gamma n_i(\mathbf{g}), \tag{1}$$

where $\gamma_0 > 0$, $\gamma > 0$ and such that $\gamma_0 > \gamma(n-1)$. A network $\mathbf{g}$ induces an average variable cost for each firm.

2.3 Network and Market Behavior

We suppose the result of stage I, i.e. the set of links formed by the firms, is perfectly observable among competitors when firms make decisions about quantities (Cournot) or prices (Bertrand) in stage II. Given these links, in stage II, the firms choose the quantities they produce (Cournot) or the prices they set (Bertrand) simultaneously.

Let q_i be the quantity produced by firm i and p be the price on the market. We study the textbook example of a market with homogeneous product and with price or quantity competition. More specifically, for reasons of tractability, we assume the inverse demand function in the product market is linear:

$$p = \alpha - \sum_{i \in N} q_i, \ \ \alpha > 0. \tag{2}$$

2.4 Equilibrium Networks

A network $\mathbf{g} \in \mathcal{G}$ is said to be in equilibrium if, holding the set of links formed by the other firms to be constant, any firm that is linked to another firm in $\mathbf{g} \in \mathcal{G}$ has an incentive to maintain this link. Moreover, any firm that is not linked to another firm in $\mathbf{g} \in \mathcal{G}$ has no incentive to form a link with this firm.

Let $\mathbf{g}'$ be a network where i is the only firm who does not have the same links as in $\mathbf{g}$. Let us define $\Pi_i(n_i(\mathbf{g}), n_{-i}(\mathbf{g}))$ the net profit of firm $i \in N$. A network $\mathbf{g}$ is an equilibrium network if, for all i, we have:

$$\Pi_i(n_i(\mathbf{g}), n_{-i}(\mathbf{g})) \geq \Pi_i(n_i(\mathbf{g}'), n_{-i}(\mathbf{g}')), \ \ \forall \mathbf{g}' \in \mathcal{G}. \tag{3}$$

3 Non Cooperative Networks and Cournot Oligopoly

In non cooperative networks and Cournot oligopoly each firm first chooses the links she sets, then chooses the quantity to be produced. Given any network $\mathbf{g} \in \mathcal{G}$, the Cournot equilibrium quantity of a firm i, called q_i^*, is:

$$q_i^*(n_i(\mathbf{g}),n_{-i}(\mathbf{g})) = \frac{(\alpha - \gamma_0) + n\gamma n_i(\mathbf{g}) - \gamma \sum_{j \neq i} n_j(\mathbf{g})}{n+1}. \tag{4}$$

We assume that:

$$(\alpha - \gamma_0) - (n-1)^2 \gamma > 0, \tag{5}$$

in order to ensure that each firm produces a strictly positive quantity. We observe that the equilibrium quantity of firm $i \in N$ depends (positively) on her proper links and (negatively) on the links of other firms. The equilibrium profit is:

$$\Pi_i(n_i(\mathbf{g}),n_{-i}(\mathbf{g})) = (q_i^*(n_i(\mathbf{g}),n_{-i}(\mathbf{g})))^2 - \delta n_i(\mathbf{g}), \ \forall i \in N. \tag{6}$$

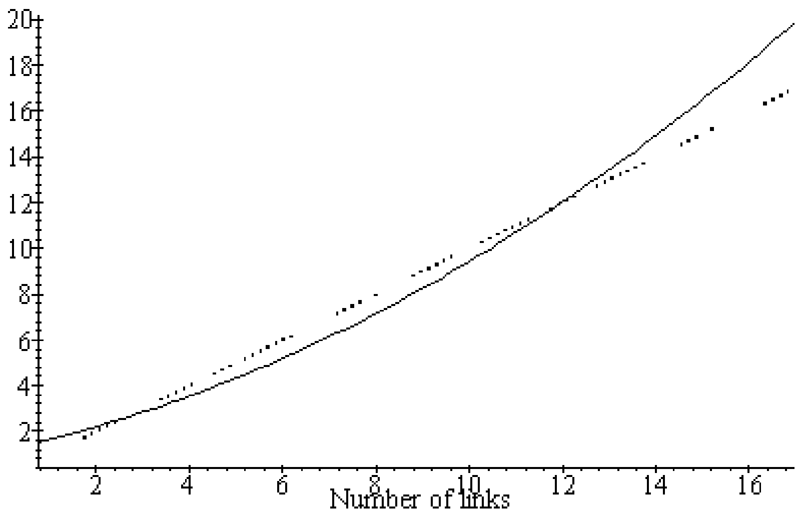
We now characterize equilibrium networks under quantity competition.

Proposition 1 *Suppose there is quantity competition among firms.*

1. *if* $\delta < n\gamma \frac{2(\alpha-\gamma_0)-n\gamma(n-3)}{(n+1)^2}$, *then* $\mathbf{g}^c$ *is the unique equilibrium network;*
2. *if* $\delta > n\gamma \frac{2(\alpha-\gamma_0)+n\gamma(n-1)}{(n+1)^2}$, *then* $\mathbf{g}^e$ *is the unique equilibrium network,*
3. *if* $\delta \in (n\gamma \frac{2(\alpha-\gamma_0)-n\gamma(n-3)}{(n+1)^2}, n\gamma \frac{2(\alpha-\gamma_0)+n\gamma(n-1)}{(n+1)^2})$, *then in an equilibrium network* g^*, *there are* x *firms* i, $x \in \{1, ..., n-1\}$, *such that* $n_i(g^*) = n-1$,*and* $n - x$ *firms* j *such that* $n_j(g^*) = 0$.

The proof of proposition 1 is given in the appendix. It can be easily interpreted. Firstly, in Figure 2, the solid graph describes the gross profit (profit gross of the cost of forming links) of firm i in Cournot game, and the dashed line represents the total cost incurred by i from forming links, setting $\alpha = 200$, $\gamma_0 = 50$, $\gamma = 0.2$, $n = 100$, $n_{-i}(\mathbf{g}) = 200$, and $\delta = 1$. From this figure, it appears that the difference between the gross profit and the total cost of forming links (that is, the net profit) is maximized either at $n_i(\mathbf{g}) = 0$ or at $n_i(\mathbf{g}) = n - 1$. This result

Figure 2: Gross Profit and Total Cost of Links



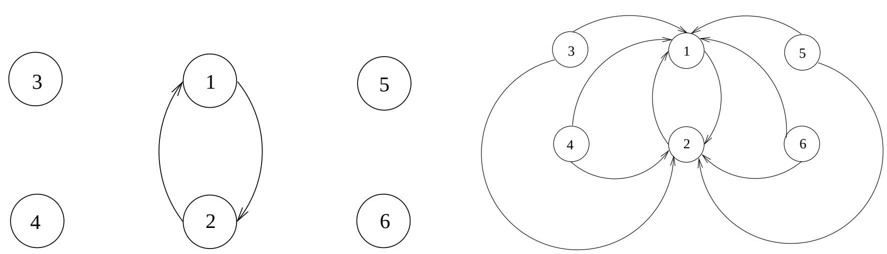
stems from the fact that in a Cournot market structure, the gross profit function is convex with respect to the number of links. It implies that if there exists heterogeneity among the firms, then some of them obtain resources from all firms while the others get no resources from competitors.

Secondly, the third part of our proposition can be explained in the following way. Straightforward calculations show that if the number of firms having set $n-1$ links increases, then the net profits of firms having set $n-1$ links decrease faster than the profits of firms having set no link. To put it another way, the difference in profits between the firms who have set $n-1$ links and the other firms is decreasing with the number of firms having set $n-1$ links. Then, we can understand that in some cases there exist firms k and $k+1$ such that if firms $i = \{1, ..., k-1\}$, we have $\Pi_k(n-1, n_{-i}) > \Pi_k(0, n_{-i})$, and $\Pi_{k+1}(n-1, n_{-i} + n - 1) < \Pi_k(0, n_{-i} + n - 1)$. We give an example of such a case below.

Example 1 *Assume* $\alpha = 13000$, $\gamma_0 = 100$, $\gamma = 2$, $n = 20$, $\delta = 2300$. *Moreover, suppose a network* $\mathbf{g}$ *divided in two parts of sixteen and four firms respectively. In the first part the firms obtain resources from everyone, while in the second part the firms get only their own resources. We check that* $\mathbf{g}$ *is an equilibrium network.*

This result suggests that even if firms initially are symmetric with respect to their cost function, *ex post*, in equilibrium, significant asymmetries may appear between firms, with respect to the number of links and profit. These asymmetries also appear in the model of GJ but they are less stringent. More precisely, in the two models, in equilibrium, we can have two groups of firms: one group where firms have links, another group where firms have no links. However, in the model of GJ, in equilibrium, a firm may obtain access to the information of k firms, whereas in our setting a firm gets access either to the information of all other firms or to the information of no other firm (see Figure 3). Furthermore, in our model, the level of the cost of a link under which this link is formed is higher than in the model of GJ, *ceteris paribus*, and the complete network is an equilibrium network for higher costs of link formation. These differences in results can be explained in the following way: in the model of GJ when a firm i establishes a link with a firm j, she reduces not only her own cost but also the cost of her partner. This last effect increases the quantity equilibrium of firm j, which lowers the quantity equilibrium and profit of firm i. In our setting, the formation of a link between a firm i and a firm j has no effect on the cost of firm j and the negative externality of the formation of a link does not exist anymore.

Figure 3: Stable network in GJ vs our Equilibrium Network



(a) two firms form links in GJ (b) two firms form links in our model

Efficient Networks: For any network $\mathbf{g}$, aggregate welfare $W(\mathbf{g})$ is defined as the sum of consumer surplus and aggregate profits of the n firms. We shall say that a network $\mathbf{g}^*$ is efficient if $W(\mathbf{g}^*) \geq W(\mathbf{g})$ for all $\mathbf{g} \in \mathcal{G}$. The aggregate welfare under quantity competition is defined as follows:

$$W(\mathbf{g}) = \frac{1}{2}Q^2(\mathbf{g}) + \sum_{i \in N} q_i^2(\mathbf{g}), \tag{7}$$

where $Q(\mathbf{g}) = \sum_{i \in N} q_i(\mathbf{g})$.

In the following, it is helpful to write $W(\mathbf{g})$ as $W(\mathbf{g}) = W(n_i(\mathbf{g}), n_{-i}(\mathbf{g}))$.

Proposition 2 *A network* $\mathbf{g}$ *is an efficient network if* $n_i(\mathbf{g}) \in \{0, n-1\}$*, for all* $i \in N$*. Moreover,*

1. *If the cost of setting up a link,* δ*, is low, then the complete network is the unique efficient network;*
2. *If the cost of setting up a link,* δ*, is high, then the empty network is the unique efficient network.*

The proof is given in the appendix. We note that if the cost of links setting is low, we obtain the same result as GJ. But, unlike the GJ model, our setting allows us to characterize efficient networks when the cost of links setting is not low. More precisely, we show that the architecture of equilibrium networks and efficient networks are identical: the networks where firms have formed either 0 link or $n-1$ links.

The analysis of the relationship between equilibrium networks and efficient networks turns out to be very complicated. However, through the following example, it appears that equilibrium networks and efficient networks do not always coincide.

Example 2 *Assume* $\alpha = 13000$*,* $\gamma_0 = 100$*,* $\gamma = 2$*,* $n = 20$*, and* $\delta = 10000$*, then the empty network is an equilibrium network, but is not an efficient network.*

4 Non Cooperative Networks and Bertrand Oligopoly

We follow the Bertrand assumption that firms compete in prices at each point in the market. Let $D(p) = \alpha - p$ be the demand function. We suppose that the demand faced by each firm i, if she sets the price p_i, is given by:

$$d_i(p_i) = \begin{cases} D(p_i) & \text{if } p_i < p_j, \forall j, \\ \dfrac{D(p_i)}{k} & \text{if } p_i \leq p_j, \forall j, \text{ with equality for } k \text{ firms,} \\ 0 & \text{if } p_i > p_j \text{ for some } j \neq i. \end{cases}$$

The total net profit of firm i is given by:

$$\Pi_i(n_i(\mathbf{g}), n_{-i}(\mathbf{g})) = d_i(p_i)(p_i - c_i(n_i(\mathbf{g}))) - \delta n_i(\mathbf{g})$$

We now characterize the equilibrium networks under Bertrand competition.

Since we wish to compare equilibrium networks architectures of Bertrand and Cournot oligopolies under the same conditions, we assume as in the previous section that inequality (5) is verified.

Lemma 1 *In equilibrium, there is at most one firm who forms links.*

The proof is given in the appendix. Let ϵ be the smallest possible monetary denomination (we assume that ϵ converges to 0)

Lemma 2 *Suppose that there is one and only one firm (say ℓ) that forms links. Then, the Bertand equilibrium market price is given by $p = \gamma_0 - \epsilon$.*

This lemma follows from the fact that if only firm ℓ forms a link, then this firm has the lowest marginal cost, and hence sets her price at $p_\ell = \gamma_0 - \epsilon$ in order to undercut all other firms.

Proposition 3

1. *if $\delta > \gamma(\alpha - \gamma_0)$, then the empty network, $\mathbf{g}^e$, is the unique equilibrium network;*
2. *if $\delta < \gamma(\alpha - \gamma_0)$, then the inward pointing stars, $\mathbf{g}^s$, are the unique equilibrium networks.*

The proof is given in the appendix. It is interesting to compare our result with that of GJ who under a similar specification of demand and average variable cost derive the empty network as the only equilibrium network. This difference in our result can be explained by the one-way property of externality flows. Indeed, in the model of GJ, where externality flows are two-way, the star networks can not be equilibrium networks for the following reason: in the star, all firms have formed links and get the same externalities. Hence, their profits gross of the cost of forming links are null and their net profits are negative. Therefore each firm has an incentive to sever her links and star networks are not equilibrium networks. In our model, firm i, at the centre of the inward-pointing star, is the only firm who obtains externalities of the others. Then her cost is the lowest, so she can set a price greater than her average variable cost and get a positive return on her investment in the formation of links. Moreover, by contrast to the result of GJ (or the standard Bertrand oligopoly model), our framework allows for positive profits.

We note that in Cournot and Bertrand oligopolies, in equilibrium, each firm obtains resources from either 0 firm or $n-1$ firms. However, if δ is sufficiently low, then in equilibrium, in Bertrand oligopoly, there is at most one firm who has formed links whereas in Cournot oligopoly, there is at least one firm who has formed links. It appears that the competition in quantity makes easier the setting of links than the price competition.

Efficient Networks: we now examine efficient networks. Our proposition builds upon the following lemma.

Lemma 3 *If* $\mathbf{g}$ *is an efficient network, then in* $\mathbf{g}$ *there are at most two firms who set links.*

The proof is given in the appendix.[2]

Proposition 4

1. *if* $\delta < \frac{1}{2}\gamma^2$*, then the inward-pointing stars are equilibrium networks but are not efficient networks. If the inward-pointing stars are efficient, then they are equilibrium networks;*
2. *If the empty network is an equilibrium network, then it is an efficient network.*

The proof is given in the appendix. This result shows that the relation between equilibrium networks and efficient networks are quite complicated. It is noteworthy that there exist conditions such that we observe a conflict between stability and efficiency in networks.

[2]The arguments in the proof are closed to those used in the proof of Proposition 3.4 in GJ (2003).

5 Concluding Remarks

This paper has developed a simple model of networks formation for studying incentives of firms in oligopolies to form links with other firms. A link between a firm i and a firm j is interpreted as a way for i to access to information hold by j. Our interest has been in the interaction between the incentive of firms to get other firms' information and the nature of the competition. We have characterized a set of equilibrium networks. We have shown that few architectures can appear in equilibrium: the empty network, the complete network and networks where some firms obtain access to information of all other firms while the latter get no information of other firms. An important finding of our study is that even if firms are initially symmetric with respect to their cost function, *ex post*, in equilibrium, significant asymmetries appear between firms: some firms have many links while other firms have no links. This situation generates different levels of cost and competitiveness between firms.

This work is close to the work of GJ, since these authors also study the formation of bilateral links between firms to reduce cost in oligopolies. There is however a substantial difference: GJ analyze situations where a link between two firms only can be formed if both firms agree to, whereas we suppose that a firm can get access to the information of another firm without the consent of the latter. This difference translates into the results obtained. In particular, in our setting, under Bertrand competition, the empty network is no more the only equilibrium network. Indeed, we have shown that a structure where one firm obtains the information of all other firms while the latter obtain no information of other firms also can be an equilibrium network. Under Cournot competition, asymmetries of firms with respect to their cost appear in our model, like in GJ model. More precisely, in the two models, in equilibrium, we can have two groups of firms : one group with links, the other without links. Nevertheless, in our model, asymmetries of firms are much stronger, since in equilibrium a given firm has an incentive to get access either to the information of all other firms or to the information of no other firm. Furthermore, in our model, the level of the cost of a link under which this link is formed is higher than in the model of GJ, and the complete network is an equilibrium network for higher costs of link formation.

Differences of results obtained in the two models suggest that the equilibrium architectures of networks are sensitive to the way information can be produced or accessed to. A natural conjecture is that in industries where the information is explicit or already created, it is possible and easier for a firm to get access to the information of another firm without the consent of the latter. In this case, our model is more relevant, and we might observe more links between firms (*ceteris paribus*).

This work can be extended in several directions. For instance, we have assumed that externality flows are non transitive. It would be interesting to examine cases where externality flows from one firm to another are transitive or decay with the distance between the firms in the network. Moreover, we have assumed that firms produced homogenous goods. The consequences of the introduction of heterogeneous goods upon our results could be questioned.

Appendix

Proof of proposition 1: This proof is in three parts. Firstly, we show that in an equilibrium network, $\mathbf{g}^*$, for all firms i, we have $n_i(\mathbf{g}^*) \in \{0, n-1\}$. The third part of the proposition results from this first part of the proof. Secondly, we show that if $\delta < n\gamma\frac{2(\alpha-\gamma_0-\gamma)-n\gamma(n-3)}{(n+1)^2}$, then the complete network, $\mathbf{g}^c$, is the unique equilibrium network. Thirdly, we show that if $\delta > n\gamma\frac{2(\alpha-\gamma_0)+n(n-1)\gamma}{(n+1)^2}$, then the empty network, $\mathbf{g}^e$, is the unique equilibrium network.

1. Let $\mathbf{g}^*$ be an equilibrium network, where there exists at least a firm i such that $n_i(\mathbf{g}^*) \in \{2, ..., n-2\}$. We assume $n_i(\mathbf{g}^*) = k$. We show that if i does not have any incentive to delete a link, then she has an interest to form a link (and conversely).

   We know that, in an equilibrium network, a firm never has an incentive to sever a link. So, we have:

   $$\Pi_i(k, n_{-i}(\mathbf{g}^*)) - \Pi_i(k-1, n_{-i}(\mathbf{g}^*)) > 0,$$

   that is, $\delta < n\gamma\frac{2(\alpha-\gamma_0-\gamma n_{-i}(\mathbf{g}^*)+n\gamma k)-n\gamma}{(n+1)^2} = A$.

   Likewise, in an equilibrium network a firm never has an incentive to set a link. So, we have:

   $$\Pi_i(k, n_{-i}(\mathbf{g}^*)) - \Pi_i(k+1, n_{-i}(\mathbf{g}^*)) > 0,$$

   that is, $\delta > n\gamma\frac{n\gamma(1+2x)-2(\gamma n_{-i}(\mathbf{g}^*)-\alpha+\gamma_0)}{(n+1)^2} = B$. Consequently, we must have $A - B > 0$.

But, since

$$A - B = -\frac{2n^2\gamma^2}{(n+1)^2},$$

we have a contradiction.

Hence, if there exists a firm i who has formed k links, $k \in \{1, ..., n-2\}$, in a network $\mathbf{g}^*$**,** then $\mathbf{g}^*$ is not an equilibrium network. So, in an equilibrium network, each firm has formed either 0 link or $n-1$ links.

2. In the second part of the proposition, we show that if $\delta < n\gamma\frac{2(\alpha-\gamma_0-\gamma)}{(n+1)^2} - (n\gamma)^2\frac{(n-3)}{(n+1)^2}$, then the complete network, $\mathbf{g}^c$, is the only equilibrium network.

   We first show that $\mathbf{g}^c$ is an equilibrium network if the above condition is satisfied. Then we prove that no other network is an equilibrium network.

   (a) We establish that no firm has an incentive to break links in $\mathbf{g}^c$. We have:

   $$\Pi_i(n-1, (n-1)^2) - \Pi_i(n-1-k, (n-1)^2) > 0,$$

   that is,

   $$\delta < n\gamma\frac{2(n^2\gamma + \alpha - \gamma_0 - \gamma(n-1)^2 - n\gamma) - n\gamma k}{(n+1)^2}.$$

   If this inequality is verified for $k = n-1$ then this inequality is verified for all k. Hence, we have:

   $$\delta < n\gamma\frac{2(\alpha - \gamma_0 - \gamma) - n\gamma(n-3)}{(n+1)^2}.$$

   The result follows.

   (b) We show that a network $\mathbf{g} \neq \mathbf{g}^c$ is not an equilibrium network. We have shown in the first part of this proof that a network, $\mathbf{g}$, where there exists a firm i such that $n_i(\mathbf{g}) \notin \{0, n-1\}$, can not be an equilibrium network. Hence, it remains to show that a network $\mathbf{g}$, where there exists at least a firm i such that $n_i(\mathbf{g}) = 0$, is not an equilibrium network. To establish a contradiction, assume an equilibrium network, $\mathbf{g}^*$, such that there exist at least a firm i such that $n_i(\mathbf{g}^*) = 0$. As $\mathbf{g}^*$ is an equilibrium network, we have:

   $$\Pi_i(0, n_{-i}(\mathbf{g}^*)) - \Pi_i(k, n_{-i}(\mathbf{g}^*)) > 0,$$

that is,

$$\delta > n\gamma \frac{2(\alpha - \gamma_0 - \gamma n_{-i}(\mathbf{g}^*)) + n\gamma k}{(n+1)^2}.$$

Therefore, there exists $n_{-i}(\mathbf{g}^*)$ such that firm i does not have any incentive to form links whatever the number of these links. Then, we have:

$$\delta > n\gamma \frac{2(\alpha - \gamma_0 - \gamma) - n\gamma(n-3)}{(n+1)^2}.$$

A contradiction.

3. In the last part of the proposition, we show that, if $\delta > n\gamma \frac{2(\alpha-\gamma_0)}{(n+1)^2} + (n\gamma)^2 \frac{(n-1)}{(n+1)^2}$, then the empty network, $\mathbf{g}^e$, is the only equilibrium network. We first show that $\mathbf{g}^e$ is an equilibrium network, then we prove that no other network is an equilibrium network.

   (a) We first establish that no firm has an incentive to set links in $\mathbf{g}^e$. We have:

   $$\Pi_i(0,0) - \Pi_i(k,0) > 0,$$

   that is,

   $$\delta > n\gamma \frac{2\alpha - 2\gamma_0 + n\gamma k}{(n+1)^2}.$$

   If this inequality is verified for $k = n-1$, then this inequality is verified for all k. Hence, we have:

   $$\delta > n\gamma \frac{2(\alpha - \gamma_0) + n\gamma(n-1)}{(n+1)^2}.$$

   The result follows.

   (b) We now show that if $\delta > n\gamma \frac{2(\alpha-\gamma_0)+n\gamma(n-1)}{(n+1)^2}$, then no other network than the empty network is an equilibrium network. We have shown in the first part of this proof that a network, $\mathbf{g}$, where there exists a firm i such that $n_i(\mathbf{g}) \notin \{0, n-1\}$, can not be an equilibrium network. Hence, we must show that a network $\mathbf{g}$, where there exists at least a firm i such that $n_i(\mathbf{g}) = n-1$, is not an equilibrium network. To establish a contradiction, assume an equilibrium network, $\mathbf{g}^*$, where there exists at least a firm i such that $n_i(\mathbf{g}^*) = n-1$. We have:

   $$\Pi_i(n-1, n_{-i}(\mathbf{g}^*)) - \Pi_i(0, n_{-i}(\mathbf{g}^*)) > 0,$$

that is,

$$\delta < n\gamma \frac{2(\alpha - \gamma_0 - \gamma n_{-i}(\mathbf{g}^*)) + n\gamma k}{(n+1)^2}$$

Therefore, there exists $n_{-i}(\mathbf{g}^*)$ such that firm i does not have any incentive to sever links, whatever the number of the deleted links. Then, we have:

$$\delta < n\gamma \frac{2(\alpha - \gamma_0) + n\gamma(n-1)}{(n+1)^2},$$

a contradiction.

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Proof of proposition 2: This proof is in four parts. Firstly, we show that in an efficient network $\mathbf{g}$, there is at most one firm i which sets $n_i(\mathbf{g}) \in \{1, ..., n-2\}$ links. Secondly, we show that there does not exist any firm such that $n_i(\mathbf{g}) \in \{1, ..., n-2\}$. Thirdly, we show that there exists δ such that the complete network is an efficient network. Lastly, we show that there exists δ such that the empty network is efficient.

1. Straightforward calculations show that $Q(n_i(\mathbf{g}), n_{-i}(\mathbf{g})) = Q(n_i\ (\mathbf{g}) + k,\ n_{-i}(\mathbf{g}) - k)$. That is, the aggregate product is not modify if a firm i sets k more links and firms $j \neq i$ delete k of their links. Hence, if the distribution of the links among firms is modified then the price and the total revenue of the firms do not change. Therefore, the variation of the aggregate welfare only depends on the variation of the sum of the total costs. Assume i and j such that $n_i(\mathbf{g}) \geq n_j(\mathbf{g})$. We now show that the sum of the total costs decreases if i forms k links while j deletes k links. Indeed, in that case, the variation of the total costs is $-2\left(k + n_i(\mathbf{g}) - n_j(\mathbf{g})\right)\gamma^2 k < 0$, for all k. It follows that in an efficient network $\mathbf{g}$, there is at most one firm i which sets $n_i(\mathbf{g}) \in \{1, ..., n-2\}$ links.

2. We now show that a network $\mathbf{g}$ where a firm i has set $n_i(\mathbf{g}) \in \{1, ..., n-2\}$ is not efficient. To introduce a contradiction, suppose that there exists an efficient network $\mathbf{g}$ where i has set $n_i(\mathbf{g}) \in \{1, ..., n-2\}$ links. Let us assume that $y \in N/\{i\}$ agents have formed $(n-1)$ links. Let $k \in \{1, ..., n-1-n_i(\mathbf{g})\}$ be the number of links that i adds. Since $\mathbf{g}$ is efficient, we have $W(n_i(\mathbf{g}), n_{-i}(\mathbf{g})) > W(n_i(\mathbf{g}) + k, n_{-i}(\mathbf{g}))$, that implies

$$\delta > \frac{\gamma}{(n+1)^2}((n+2)(\alpha-\gamma_0)+\gamma(y(3-n-2n^2) + n_i(\mathbf{g})(2n-1+2n^2)+k(n+n^2-\frac{1}{2}))) \quad (8)$$

Let $t \in \{1,...,n_i(\mathbf{g})\}$ be the number of links that i deletes. Since $\mathbf{g}$ is efficient, we have $W(n_i(\mathbf{g}), n_{-i}(\mathbf{g})) > W(n_i(\mathbf{g})-t, n_{-i}(\mathbf{g}))$, that implies:

$$\delta < \frac{\gamma}{(n+1)^2}((n+2)(\alpha-\gamma_0)+\gamma(t(-n+\frac{1}{2}-n^2) + n_i(\mathbf{g})(2n^2+2n-1)+y(-2n^2+3-n))) \quad (9)$$

Inequalities (8) and (9) imply a contradiction. The result follows.

3. We now show that if the cost of setting links is small ($\delta \to 0$), then the complete network is the unique efficient network. As $\delta \to 0$, it is sufficient to show that the gross social welfare reaches its unique maximum if the network is complete.

   We know by the previous part of the proposition that in an efficient network firms form 0 or $n-1$ links. We now show that the social welfare is increasing with the number of firms forming $n-1$ links. Assume that there are y firms who have formed $n-1$ links, and $n-y$ firms who have formed no link. Assume that one of the latter firms forms $n-1$ links. Then the social welfare difference between the second situation and the first situation is:

$$\frac{\gamma}{(n+1)^2}(n^2+n-2)(\alpha-\gamma_0)+\gamma(2n+n^4-\frac{1}{2}-n^3-\frac{3n^2}{2}) + \gamma y(4n-2n^3+n^2-3).$$

   This expression is strictly positive since $\alpha-\gamma_0 > \gamma(n-1)^2$ and $n \geq 3$. The result follows.

4. It is obvious that if the cost of setting links is sufficiently high (for instance $\delta \to \infty$), then the empty network is the unique efficient network.

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Proof of lemma 1: To establish a contradiction, suppose that in an equilibrium network, $\mathbf{g}$, there exist at least two firms i and j who have formed links. Without loss of generality, assume that $n_j(\mathbf{g}) \geq n_i(\mathbf{g})$. As, $c_j(n_j(\mathbf{g})) \leq c_i(n_i(\mathbf{g}))$, gross profit (profit minus fixed costs) of firm i are null. Hence, we get:

$$\Pi_i(n_i(\mathbf{g}),n_{-i}(\mathbf{g})) = -\delta \sum_{j \in N \setminus \{i\}} g_{i,j}.$$

Since firm j has an average variable cost $c_j(n_j(\mathbf{g})) < \gamma_0$, firm i does not produce if she has set no link. Therefore, we get:

$$\Pi_i(0, n_{-\mathbf{i}}(\mathbf{g})) = 0.$$

It follows:

$$\Pi_i(0, n_{-i}(\mathbf{g})) - \Pi_i(n_i(\mathbf{g}),n_{-i}(\mathbf{g})) = \delta \sum_{j \in N \setminus \{i\}} g_{i,j} > 0,$$

since, by assumption, $\sum_{j \in N \setminus \{i\}} g_{i,j} \geq 1$. A contradiction. Hence, in equilibrium, there is at most one firm, say ℓ, who has established links, whereas all other firms have formed no links.

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Proof of proposition 3: We know by lemma 1 that, in equilibrium, there is at most one firm who has formed links.

We show that in an equilibrium network, $\mathbf{g}^*$, the number of links established by this firm is 0 if $\delta > \gamma(\alpha - \gamma_0)$ while this number is $n-1$ if $\delta < \gamma(\alpha - \gamma_0)$.

Suppose that firm ℓ has set $n_\ell(\mathbf{g}^*)$ links, $n_\ell(\mathbf{g}^*) \in \{1, \ldots, n-1\}$, whereas the other firms have established no links. We know by lemma 2 that the market price is $p = \gamma_0 - \epsilon$. Then, the net profit of ℓ is:

$$\Pi_\ell(n_\ell(\mathbf{g}^*), n_{-\ell}(\mathbf{g}^*)) = n_\ell(\mathbf{g}^*)(\gamma(\alpha - \gamma_0 + \epsilon) - \delta) - \epsilon(\alpha - \gamma_0 + \epsilon).$$

We can distinguish two cases:

1. if $\delta > \gamma(\alpha - \gamma_0)$, then $\Pi_\ell(n_\ell(\mathbf{g}^*), n_{-\ell}(\mathbf{g}^*))$ is decreasing with $n_\ell(\mathbf{g}^*)$. This implies that firm ℓ has no incentive to form links. Then, the empty network is the only equilibrium network;

2. if $\delta < \gamma(\alpha - \gamma_0)$, then $\Pi_\ell(n_\ell(\mathbf{g}^*), n_{-\ell}(\mathbf{g}^*))$ is increasing with $n_\ell(\mathbf{g}^*)$. This implies that firm ℓ has an incentive to form $n-1$ links. Then, the inward-pointing star networks are the only equilibrium networks.

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Proof of lemma 3: We show that in an efficient network $\mathbf{g}$, there are at most two firms, say ℓ and ℓ', who have formed links. Indeed, suppose that other firms, say i, j, ... form links (call this network $\mathbf{g'}$) and assume that $n_\ell(\mathbf{g}) \geq n_{\ell'}(\mathbf{g})$, $n_{\ell'}(\mathbf{g}) \geq n_i(\mathbf{g})$ for all $i \in N \setminus \{\ell\}$. By the same reasoning as in lemma 2, we show that the price equilibrium is $\gamma_0 - \gamma n_{\ell'}(\mathbf{g}) - \epsilon$ if $n_\ell(\mathbf{g}) > n_{\ell'}(\mathbf{g})$ and is $\gamma_0 - \gamma n_{\ell'}(\mathbf{g})$ if $n_\ell(\mathbf{g}) = n_{\ell'}(\mathbf{g})$. In these two cases the social welfare function respectively is:

$$\begin{aligned} W_1(n_\ell(\mathbf{g}), n_{-\ell}(\mathbf{g})) &= \frac{(\alpha - \gamma_0 + \gamma n_{\ell'}(\mathbf{g}) + \epsilon)^2}{2} + (\gamma n_\ell(\mathbf{g}) - \gamma n_{\ell'}(\mathbf{g}) \\ & -\epsilon)\,(\alpha - \gamma_0 + \gamma n_{\ell'}(\mathbf{g}) + \epsilon) - \delta \sum_{k \in N} n_k(\mathbf{g}) \end{aligned}$$

$$W_2(n_\ell(\mathbf{g}), n_{-\ell}(\mathbf{g})) = \frac{[\alpha - \gamma_0 + \gamma n_\ell(\mathbf{g})]^2}{2} - \delta \sum_{k \in N} n_k(\mathbf{g}),$$

We have $W_1(n_\ell(\mathbf{g}), n_{-\ell}(\mathbf{g})) - W_1(n_\ell(\mathbf{g}'), n_{-\ell}(\mathbf{g}')) = W_2(n_\ell(\mathbf{g}), n_{-\ell}(\mathbf{g})) - W_2(n_\ell(\mathbf{g}'), n_{-\ell}(\mathbf{g}')) = \delta \sum_{k \neq \ell, \ell'} n_k$. The result follows.

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Proof of proposition 4: By lemma 3, we know that there are only two firms, say ℓ and ℓ', who have formed links (in the following, $n_{-\ell}(\mathbf{g}) = n_{\ell'}(\mathbf{g})$). Moreover:

1. Suppose $n - 1 > n_\ell > n'_\ell$. $W_1(n_\ell(\mathbf{g}) + 1,\, n_{\ell'}(\mathbf{g})) - W_1(n_\ell(\mathbf{g}),\, n_{\ell'}(\mathbf{g})) > 0$ if and only if $\delta < \gamma(\alpha - \gamma_0 + \gamma n_{\ell'}(\mathbf{g}))$ (I_1);
2. Suppose $n - 1 > n_\ell > n'_\ell$. $W_1(n_\ell(\mathbf{g}), n_{\ell'}(\mathbf{g}) + 1) - W_1(n_\ell(\mathbf{g}), n_{\ell'}(\mathbf{g})) > 0$ if and only if $\delta < \gamma^2(2(n_\ell(\mathbf{g}) - n_{\ell'}(\mathbf{g})) - 1)$ (I_2);
3. Suppose $n - 1 > n_\ell = n'_\ell$. $W_2(n_\ell(\mathbf{g}) + 1, n_{\ell'}(\mathbf{g}) + 1) - W_2(n_\ell(\mathbf{g}), n_{\ell'}(\mathbf{g})) > 0$ if and only if $\delta < \frac{\gamma}{4}\,(2\gamma n_\ell(\mathbf{g}) + \gamma + 2(\alpha - \gamma_0))$ (I_3).

We now prove the two parts of the proposition successively.

1. We show that if $\delta < \frac{1}{2}\gamma^2$, then the inward-pointing star, $\mathbf{g}^s$, is an equilibrium network but is not an efficient network. Indeed, if $\delta < \frac{1}{2}\gamma^2$, then $\delta < \gamma(\alpha - \gamma_0)$ by (5) and the inward-pointing star is an equilibrium network (from proposition 3). Moreover, if $\delta < \frac{1}{2}\gamma^2$, then

$\delta < \frac{1}{2}\gamma^2(2(n_\ell(\mathbf{g}) - n_{\ell'}(\mathbf{g})) - 1)$ and $W_1(n_\ell(\mathbf{g}), n_{\ell'}(\mathbf{g}) + 1) - W_1(n_\ell(\mathbf{g}), n_{\ell'}(\mathbf{g})) > 0$ by (I_2). It follows that the inward-pointing star is not an efficient network.

Then, we show that if the inward-pointing star, $\mathbf{g}^s$, is an efficient network, then it is an equilibrium network. Indeed, if the inward-pointing star is an efficient network, then it is obvious that $\delta < \gamma(\alpha - \gamma_0)$ from (I_1). Moreover, if $\delta < \gamma(\alpha - \gamma_0)$, then the inward-pointing star is an equilibrium network (from proposition 3).

2. Lastly, we show that if the empty network $\mathbf{g}^e$ is an equilibrium network, then $\mathbf{g}^e$ is an efficient network. Assume $\mathbf{g}^e$ is an equilibrium network. We proceed in three steps.

   Firstly, we show that $\mathbf{g}^e$ is the greatest social welfare network among networks where $n_\ell(\mathbf{g}) = n_{\ell'}(\mathbf{g})$. We know by proposition 3, that if $\mathbf{g}^e$ is an equilibrium network, then $\delta > \gamma(\alpha - \gamma_0)$. If $\delta > \gamma(\alpha - \gamma_0)$ then $\delta > \frac{\gamma}{4}(2\gamma n_\ell(\mathbf{g}) + \gamma + 2(\alpha - \gamma_0))$ by inequality (5). It follows that $W_2(n_\ell(\mathbf{g}) + 1, n_{\ell'}(\mathbf{g}) + 1) - W_2(n_\ell(\mathbf{g}), n_{\ell'}(\mathbf{g})) < 0$ by (I_3).

   Secondly, we show that networks where $n_{\ell'}(\mathbf{g}) \neq 0$ are not the greatest social welfare networks among networks where $n_\ell(\mathbf{g}) \neq n_{\ell'}(\mathbf{g})$. Indeed, if $\delta > \gamma(\alpha - \gamma_0)$, then $\delta > \gamma^2(2(n_\ell(\mathbf{g}) + n_{\ell'}(\mathbf{g})) - 1)$ by inequality (5) and $W_1(n_\ell(\mathbf{g}), n_{\ell'}(\mathbf{g}) + 1) - W_1(n_\ell(\mathbf{g}), n_{\ell'}(\mathbf{g})) < 0$ by (I_2). The result follows.

   Thirdly, we compare $W_2(0,0)$ with $W_1(n_\ell(g), 0)$. We obtain that $W_2(0,0) - W_1(n_\ell(g), 0) > 0$, if $\delta > \gamma(\alpha - \gamma_0)$. The result follows.

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