Idiosyncratic Shocks in an Asymmetric Cournot Oligopoly

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March 2004

Abstract

Several authors have studied the impact of a marginal cost variation in the Cournot model on consumers' surplus, firms' profits and social welfare. We unify and extend the results when all marginal costs change simultaneously. Hence we bring together two strands of literature, respectively on mean-preserving and subgroup common shocks. The effect of any shock is decomposed into two parts: an average impact and a heterogeneity effect. The former improves welfare if the inverse demand function is concave enough and the market is concentrated enough, the later when the diffusion of the shock is negatively correlated with the market share distribution. Finally, we apply our decomposition to various special cases.

Keywords: Cournot, Welfare, Asymmetric costs, Industry shock.

JEL **Classification:** L13, L4.

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1 Introduction

Competition is often pictured as a race. In a boat race peaceful weather conditions do not allow the best teams to prove their superiority.[1] Let the weather deteriorate and the weakest teams are left far behind. If the business environment is very favorable, many firms can enjoy reasonable market shares and comfortable profits. But let production costs suddenly increase or demand unexpectedly falls and only those among the fittest firms will get through the storm. Does this mean that some firms might wish for gloomy market conditions? Do consumers' interests coincide with firms' ones? If not, what is the overall effect on total surplus?

In this paper we study the impact of idiosyncratic shocks on a Cournot oligopoly. All firms are affected by a shock on their marginal cost of production (which can also be interpreted in terms of a demand shock) but the effect on each firm may be different. The consequences of such a shock on consumers' surplus, individual firms' profits, industry aggregate profit and welfare are determined. The focus is on the short term consequences and we exclude entry or exit by firms which would be more important in a long term perspective.

Several authors have studied the impact of a variation of marginal costs on equilibrium. Seade (1985) provides a general framework but mainly focuses on the symmetric case. Lahiri and Ono (1988) and Zhao (2001) show that a reduction of the marginal cost of a firm with a small market share can reduce welfare. They only analyse a shock that affects a single firm. By contrast, Kimmel (1992) analyses a shock that affects all firms in the same way.

[1] In the America's Cup 2000 (on the Hauraki Gulf, an expanse of water just north of Auckland that is noted for its strong winds), the race has had to be called off three times due to too little wind in the best-of-nine final series between the Italian and New-Zealand teams. New-Zealand won 5-0.

He specifies conditions (on the market shares and the shape of the demand function) under which the profit of a firm increases. Linnemer (2003) looks at the case where all firms but one experience a marginal cost increase. He finds conditions for welfare to increase. More generally, when firms have heterogeneous production functions, an industry-wide shock might affect them differently.

In another line of research, Bergstrom and Varian (1985), Salant and Shaffer (1999), and Long and Soubeyran (2001) study, in a sense, the welfare impact of such a shock.[2] They restrict, however, their analyses to the case where the mean of the individual variation is zero. They show that welfare increases if and only if the variance of marginal costs increases.

This paper unifies and extends these two strands of literature by analyzing idiosyncratic shocks: all firms are affected and the average shock is not zero. We show that the effects of any shock can be decomposed into two parts. First, an average shock, *à la* Seade (1985), which hits all firms in the same way. Second, a mean preserving shock, *à la* Bergstrom and Varian (1985), which affects firms differently. The average impact depends only on the market structure before the shock (market share, elasticity of the slope of the demand function, and Herfindahl index) while the heterogeneity effect only depends on how the marginal costs vary (in particular on how the variance of the marginal costs is affected). One application of the paper is the comparison of unit and ad valorem taxes in the spirit of Anderson, de Palma, and Kreider (2001a,b).

The paper is organized as follows: section 2 presents the basic assumptions. Section 3 studies the impact of a shock on the individual profit, section 4 analyses the impact on both the aggregate profit welfare. Section 5 concludes.

[2]In a recent paper, Symeonidis (2003) extends this line of research to model quality heterogeneity.

2 Assumptions and notation

Consider n ($n > 1$) firms competing *à la* Cournot. Firm i chooses its production $q_i \geq 0$. Let $Q = \sum_{i=1}^{n} q_i$ denote the total production. Firms face an inverse demand function, $P(Q)$, with $P'(Q) < 0$ (as long as $P(Q) > 0$). The constant marginal cost of firm i is denoted c_i. Let $\overline{c} = \frac{1}{n}\sum_{i=1}^{n} c_i$ be the average marginal cost. In the presence of a shock w, the marginal cost becomes $c_i + \gamma_i w$ where γ_i reflects the idiosyncrasy of the shock. The parameter γ_i can be positive or negative. Let $\overline{\gamma} = \frac{1}{n}\sum_{i=1}^{n} \gamma_i$ be the average sensitivity. Without loss of generality, it is assumed that $\overline{\gamma} \geq 0$. As firms have different production functions, the same variation of the vector of the input prices leads to different changes in their marginal cost values.

Note that the shock could also be interpreted in terms of a demand shock: indeed, assume that each firm faces a different price $P_i(Q)$, the assumption could be that $P_i(Q) = P(Q) - \gamma_i w$.

Throughout the paper it is assumed that the marginal cost variations under consideration are such that no firm is driven out of the market.

Assumption 1. *All firms produce a strictly positive quantity in equilibrium.*

The elasticity $\Theta(Q) = \frac{P''(Q)Q}{P'(Q)}$ of the slope of the inverse demand function plays a crucial role in the results of the paper. To insure the existence of an equilibrium it cannot be too negative.

Assumption 2. $\Theta(Q) \geq -2$.

If $\Theta = 0$, demand is linear. If the inverse demand function is concave (resp. convex), then $\Theta > 0$ (resp. < 0). In the case of constant elasticity of demand: $P(Q) = aQ^{-\varepsilon}$, then $\Theta = -(1+\varepsilon)$. Finally the family of constant-Θ functions is easily computed. Let

$\theta \neq -1$, and $P(Q) = \max\left\{a + \frac{b}{1+\theta} Q^{1+\theta}; 0\right\}$, then $\Theta(Q) = \theta$ (as long as $P > 0$). Finally, if $P(Q) = \max\{a + b\text{Log}(Q); 0\}$, then $\Theta(Q) = -1$.

Under assumption 2 *existence and uniqueness of the Cournot-Nash equilibrium are guaranteed.*[3] Moreover, this assumption implies that the equilibrium is characterized by the first order conditions.[4] In several papers, it is only assumed that $\Theta \geq -1$ but with constant marginal costs, this condition can be weakened to include the class of constant elasticity demands for which $-2 < \Theta < -1$.[5] In Kimmel (1992) it is assumed that $\Theta \geq -(n+1)$. This assumption is, however, not always compatible with second order conditions (see footnote 4). Finally, in Anderson and Renault (2003) it is shown that $P(Q)$ is $\rho-$concave if and only if $\Theta \geq \rho - 1$. Therefore Θ is a measure of the degree of concavity of the inverse demand function.

We use the following convention: for any variable X, X^* denotes the equilibrium value of the function for $w = 0$, while the function itself is denoted, in equilibrium, $X^*(.)$ or $X^*(w)$. For example, $\Theta^* = \Theta(Q^*(0))$.

In the following lemma the equilibrium properties of the Cournot-Nash equilibrium are briefly reviewed.

Lemma 1. *The Cournot-Nash industry output, $Q^*(w)$, is the solution of:*

$$nP(Q^*(w)) + Q^*(w) P'(Q^*(w)) = n\bar{c} + n\bar{\gamma}w \tag{1}$$

[3] See Seade (1980), Novshek (1985), Shapiro (1989) pages 334-343, Gaudet and Salant (1991b), Amir (1996) and Amir and Lambson (2000) for more general results.

[4] Indeed, the second order condition of firm i is: $2 + \Theta(Q) q_i/Q > 0$.

[5] In Lahiri and Ono (1988), Farrell and Shapiro (1990), Gaudet and Salant (1991a) and Linnemer (2003), it is assumed that $\Theta > -1$. In Zhao (2001) demand is linear, therefore $\Theta = 0$.

the Cournot-Nash production of firm i and the market share of firm i are:

$$q_i^*(w) = \frac{Q^*(w)}{n} + \frac{\overline{c} - c_i + (\overline{\gamma} - \gamma_i)\, w}{-P'(Q^*(w))}, \text{ and } s_i^*(w) = \frac{q_i^*(w)}{Q^*(w)} = \frac{1}{n} + \frac{\overline{c} - c_i + (\overline{\gamma} - \gamma_i)\, w}{n\,(P(Q^*(w)) - \overline{c} - \overline{\gamma} w)},$$

the Cournot-Nash profit of competitor i is: $\pi_i^*(w) = (-P'(Q^*(w)))\,(q_i^*(w))^2$.

Proof. The best reply function of firm i is the solution of:

$$P'(Q)\, q_i + P(Q) = c_i + \gamma_i w \tag{2}$$

Then the usual trick is to sum over all i these first order conditions which leads to:

$$P'(Q)\, Q + nP(Q) = n\overline{c} + n\overline{\gamma} w,$$

which defines in a unique way the total quantity $Q^*(w)$ produced at the Cournot equilibrium. Indeed, let $f(Q) = P'(Q)\, Q + nP(Q)$, $f'(Q) = P'(Q)\,(n + 1 + \Theta(Q))$, therefore assumption 2 insures that f is a strictly decreasing function of Q. Under the assumptions that $f(0) > n\overline{c} + n\overline{\gamma} w$ and $f(+\infty) < n\overline{c} + n\overline{\gamma} w$ the equation $f(Q) = n\overline{c} + n\overline{\gamma} w$ has a unique solution.

Next, using the first order condition, the quantity produced by firm i is:[6]

$$q_i^*(w) = \frac{P(Q^*(w)) - c_i - \gamma_i w}{-P'(Q^*(w))} = \frac{\overline{c} - c_i + (\overline{\gamma} - \gamma_i)\, w}{-P'(Q^*(w))} + \frac{Q^*(w)}{n}, \text{ and}$$

the market share of firm i is: $s_i^*(w) = \dfrac{q_i^*(w)}{Q^*(w)} = \dfrac{1}{n} + \dfrac{\overline{c} - c_i + (\overline{\gamma} - \gamma_i)\, w}{n\,(P(Q^*(w)) - \overline{c} - \overline{\gamma} w)}$. $\square$

The aim of the paper is to study the effect of a variation of w on consumers' surplus, individual and aggregate profits as well as on welfare. Let $S^*(w) = \int_0^{Q^*(w)} P(u)du -$

[6] For the second order condition, see footnote 4.

$P(Q^*(w))\,Q^*(w)$ denote the equilibrium consumers' surplus, $\Pi^*(w) = \sum_{i=1}^{n} \pi_i^*(w)$ the aggregate profit of the industry, and finally, $W^*(w) = S^*(w) + \Pi^*(w)$ the equilibrium welfare. The impact of idiosyncratic shocks is analyzed through the signs of $\partial S^*/\partial w$, $\partial \pi_i^*/\partial w$, $\partial \Pi^*/\partial w$, and $\partial W^*/\partial w$, these derivatives being evaluated at $w = 0$.[7] In technical terms, we study the directional derivative (in all directions) of $S^*(.)$, $\pi_i^*(.)$, $\Pi^*(.)$, and $W^*(.)$.

Using our notations, it is useful to link our model to previous works: in Bergstrom and Varian (1985) it is assumed that $\overline{\gamma} = 0$ (a mean preserving shock). In Seade (1985) as well as in Kimmel (1992), for all i, $\gamma_i = \overline{\gamma}$ (an average shock). Moreover they only study the impact of the shock on individual profit but neither on the industry profit nor on total welfare. In Lahiri and Ono (1988) the shock affects only one firm: $\gamma_i \neq 0$ and for $j \neq i$ $\gamma_j = 0$. In Zhao (2001) demand is restricted to be linear. He studies in details the one firm shock and extends it to a subgroup firm shock: for all $i \in T$, $\gamma_i = 1$ and for $j \notin T$, $\gamma_j = 0$. Finally, in Linnemer (2003) demand is not restricted to be linear and T includes all firms but one.

3 Impact of a shock on consumers and firms

In this section, the effects of a variation of w on consumers' surplus and on firms' profits are studied. The simplest result is on consumers' surplus.

Proposition 1. *The equilibrium quantity Q^* (and therefore consumers' surplus) is not affected if $\overline{\gamma} = 0$, while it decreases if $\overline{\gamma} > 0$.*

[7]This is without loss of generality. If $w > 0$, let $c_i' = c_i + \gamma_i w$, and let us consider the marginal cost of firm i as: $c_i' + \gamma_i w'$, then the derivative with respect to w is equivalent to the derivative with respect to w'.

Proof. Differentiating equation (1) with respect to w yields, for $w = 0$, to

$$P'(Q^*) \frac{\partial Q^*}{\partial w} \times \left[n + 1 + \frac{P''(Q^*) Q^*}{P'(Q^*)} \right] = n\overline{\gamma} \text{ that is } P'(Q^*) \frac{\partial Q^*}{\partial w} = \frac{n\overline{\gamma}}{n + 1 + \Theta^*} \tag{3}$$

As $S^*(w) = \int_0^{Q^*(w)} P(u)\,du - P(Q^*(w))\,Q^*(w)$, it follows that

$$\frac{\partial S^*}{\partial w} = -Q^* \frac{\partial Q^*}{\partial w} P'(Q^*) = \frac{-n\overline{\gamma}Q^*}{n + 1 + \Theta^*} \tag{4}$$

□

The variation of consumers' surplus depends only on the variation of $Q^*(.)$ which is affected by $\overline{\gamma}$ but not by the distribution of the γ_i.

The impact of a variation of w on the profits of a firm is detailed in proposition 2 which links together the mean-preserving literature *à la* Bergstrom and Varian (1985) and the average shock papers *à la* Seade (1985) and Kimmel (1992).

Proposition 2. *The equilibrium profit of a firm increases with w if and only if*

$$\frac{n\overline{\gamma}}{(n + 1 + \Theta^*)} \left[\Theta^* s_i^* - \frac{2}{n} (1 + \Theta^*) \right] + 2(\overline{\gamma} - \gamma_i) > 0 \tag{5}$$

Proof. By differentiating the first order condition of a firm i (equation 2) with respect to w and using (3) it follows that

$$P'(Q^*) \frac{\partial q_i^*}{\partial w} = \frac{n(\gamma_i - \overline{\gamma}) + (1 + \Theta^*)\gamma_i - n\overline{\gamma}\Theta^* s_i^*}{n + 1 + \Theta^*} \tag{6}$$

Using the expression of π_i^* given in lemma 1, it comes that

$$\frac{\partial \pi_i^*}{\partial w} = \frac{\partial Q^*}{\partial w} \left(-P''(Q^*)\right) (q_i^*)^2 + 2\left(-P'(Q^*)\right) \frac{\partial q_i^*}{\partial w} q_i^*$$

Using (3) and (6) , it follows that

$$\frac{\partial \pi_i^*}{\partial w} = q_i^* \left[2(\overline{\gamma} - \gamma_i) + \frac{n\overline{\gamma}}{n + 1 + \Theta^*} \left(\Theta^* s_i^* - \frac{2}{n}(1 + \Theta^*) \right) \right] \tag{7}$$

$\square$

The impact of the shock is thus decomposed into two parts. First, there is an average impact: firm i's profit varies as if its individual shock were exactly the average shock $\gamma_i = \overline{\gamma}$. The sign of this average variation depends on the sign of $\left[\Theta^* s_i^* - \frac{2}{n}(1+\Theta^*)\right]$ and it is studied in corollary 3.1. Second, as the shock does not hit all firms in the same way, firm i may or not benefit from the heterogeneity. The sign of this individual (or heterogeneity) effect is given by the sign of $(\overline{\gamma} - \gamma_i)$ which is positive if firm i is less affected by the shock than the average and negative otherwise as summed up in corollary 3.2.

Corollary 3.1. (Seade (1985) and Kimmel (1992)) *If for all i, $\gamma_i = \overline{\gamma}$, only the average effect is present and* $\frac{\partial \pi_i^*}{\partial w} > 0 \Leftrightarrow \Theta^* s_i^* > \frac{2}{n}(1+\Theta^*)$. *More precisely,* $\frac{\partial \pi_i^*}{\partial w} > 0 \Leftrightarrow$ ($n \geq 3$, $\Theta^* > \frac{2}{n-2}$ *and* $s_i^* > \widetilde{s} = \frac{2}{n}\left(1 + \frac{1}{\Theta^*}\right)$) *or* ($\Theta^* < -1$ *and* $s_i^* < \widetilde{s}$).

As first noted by Seade (1985), corollary 3.1 shows that the average impact (AI) depends on the concavity of the inverse demand function (around the equilibrium). If $P(.)$ is concave ($\Theta^* > 0$), production is shifted from inefficient to efficient firms (that is from small to large market share firms). Therefore if firm i is efficient enough, its profit might increase despite the increase of its marginal cost. On the contrary, if $P(.)$ is convex ($\Theta^* < 0$), production is shifted from efficient firms to inefficient firms. Therefore, firm i's profit can increase only if it is relatively inefficient, that is, if firm i's market share is low enough. One must be careful with these interpretations because $\widetilde{s}$ can be negative or greater than one for some parameter values, hence the restrictions given in corollary 3.1 on both n and Θ^*.

Corollary 3.2. (Bergstrom and Varian (1985)) *If* $\overline{\gamma} = 0$ *only the heterogeneity effect is present and* $\frac{\partial \pi_i^*}{\partial w} > 0 \Leftrightarrow \gamma_i < 0$.

Corollary 3.2 shows that a mean-preserving shock benefits only to firms whose marginal cost is reduced. Corollary 3.3 extends this result to any shock which, on average, penalizes firms:

Corollary 3.3. *If* $\overline{\gamma} \geq 0$, $\gamma_i < 0$ *implies* $\frac{\partial \pi_i^*}{\partial w} > 0$.

Proof. If $s_i^* > 2/(n+1)$, then $\frac{n\overline{\gamma}}{(n+1+\Theta^*)}\left[\Theta^* s_i^* - \frac{2}{n}\left(1+\Theta^*\right)\right]$ is increasing with Θ^*. Therefore, as $\Theta^* > -2$

$$\frac{n\overline{\gamma}}{(n+1+\Theta^*)}\left[\Theta^* s_i^* - \frac{2}{n}\left(1+\Theta^*\right)\right] > -2\overline{\gamma}\frac{ns_i^* - 1}{n-1} > -2\overline{\gamma}$$

which proves that $\gamma_i < 0$ implies $\frac{n\overline{\gamma}}{(n+1+\Theta^*)}\left[\Theta^* s_i^* - \frac{2}{n}\left(1+\Theta^*\right)\right] + 2\left(\overline{\gamma} - \gamma_i\right) > 0$.

If $s_i^* < 2/(n+1)$, then $\frac{n\overline{\gamma}}{(n+1+\Theta^*)}\left[\Theta^* s_i^* - \frac{2}{n}\left(1+\Theta^*\right)\right]$ is decreasing with Θ^*. Therefore, as $\Theta^* < +\infty$

$$\frac{n\overline{\gamma}}{(n+1+\Theta^*)}\left[\Theta^* s_i^* - \frac{2}{n}\left(1+\Theta^*\right)\right] > \overline{\gamma}(ns_i^* - 2) > -2\overline{\gamma}$$

which proves the result. $\square$

It means that if, on average, marginal costs increase after the shock, then firms whose marginal cost decreases benefit from it. The results provided in corollary 3.2 and 3.3 are fairly intuitive: given that on average the shock increases marginal costs ($\overline{\gamma} \geq 0$) firms which enjoy a good shock ($\gamma_i < 0$) increase their profits. Less obvious results have been exhibited in the literature, that is, cases where $\gamma_i > 0$ and still $\frac{\partial \pi_i^*}{\partial w} > 0$.

The following corollaries detail some special cases and relate our result to previous contributions. Proposition 2 allows a complete analysis of the linear case as shown in corollary 3.4.

Corollary 3.4. (Linear case) *If* $\Theta^* = 0$, *then* $\frac{\partial \pi_i^*}{\partial w} > 0 \Leftrightarrow \frac{n}{n+1}\overline{\gamma} > \gamma_i$.

In particular, in Zhao (2001), $\gamma_i = 1$ for all $i \in T$ and $\gamma_j = 0$ for all $j \notin T$, with $\#T = k$, which implies that $\frac{\partial \pi_i^*}{\partial w} < 0$. When demand is linear, corollary 3.4 shows that firm i's profit increases iff its sensitivity to the shock is lower than a fraction $\frac{n}{n+1}$ of the average sensitivity $\overline{\gamma}$. This result can be extended (using proposition 2) to the general case as follows:

Corollary 3.5. *Let assume that* $\overline{\gamma} > 0$. *If* $\frac{n}{n+1}\overline{\gamma} > \gamma_i$, *then* $\left(s_i^* > \frac{2}{n}\frac{\gamma_i}{\overline{\gamma}} \text{ and } \Theta^* \geq 0\right)$ *or* $\left(s_i^* < \frac{2}{n}\frac{\gamma_i}{\overline{\gamma}} \text{ and } \Theta^* \leq 0\right)$ *implies* $\frac{\partial \pi_i^*}{\partial w} > 0$.

Yet, note that corollary 3.5 only provides sufficient conditions.

Corollary 3.6 analyses the case of a shock which hits uniformly a subgroup of the firms and does not affect the others. This case generalizes Zhao (2001) (as demand is no longer linear), Lahiri and Ono (1988) (as the subgroup is no longer a singleton) and Linnemer (2003) (where the subgroup contains $n-1$ firms). The limit case where the subgroup includes all firms is the subject of corollary 3.1.

Corollary 3.6. (Subgroup shock) *If* $\gamma_i = 1$ *for all* $i \in T$ *and* $\gamma_j = 0$ *for all* $j \notin T$, *with* $\#T = k \leq n-1$, *then on the one hand* $\frac{\partial \pi_j^*}{\partial w} > 0$ *and on the other* $\frac{\partial \pi_i^*}{\partial w} > 0 \Leftrightarrow n \geq 3$, $k \geq 3$, $\Theta^* > \frac{2(n+1-k)}{k-2}$, *and* $s_i^* > \frac{2}{k}\left(1 + \frac{(n+1)-k}{\Theta^*}\right)$.

In particular, in Lahiri and Ono (1988), $k = 1$ and therefore $\frac{\partial \pi_i^*}{\partial w} < 0$. In Linnemer (2003), the above formulae apply for $n \geq 4$ as $k = n-1$. The case $k = n$ which is presented in corollary 3.1 is the only one where (for some i) $\frac{\partial \pi_i^*}{\partial w}$ can be positive when $\Theta^* < 0$.

4 Impact of the shock on aggregate profit and on welfare

Do firms that benefit from an increase of w gain more than what is lost by other firms? The study of aggregate profit is an important step towards the analysis of total surplus. As consumers's surplus decreases after a shock, a necessary condition for total surplus to increase is that aggregate surplus rises. It is also interesting in itself as it provides conditions under which firms, as a whole, would like a more hostile environment. For example, higher taxes or input prices.

Before stating our results, note that after the shock, firms spend (or gain) the amount $w \sum \gamma_i q_i^* (w)$. In this paper (following Lahiri and Ono (1988) and Zhao (2001)) it is assumed that the amount $w \sum \gamma_i q_i^* (w)$ is not collected by any member of the society and therefore that it does not enter in the social surplus. For instance (assuming that in the short run firms cannot substitute one input by another), if w is a price increase of an input produced by foreign firms, their profits do not modify national welfare.[8]

To answer these questions, proposition 3 shows the effect of w on both aggregate profit and welfare.

Proposition 3. *Let* $H^* = \sum (s_i^*)^2$ *is the ex ante Herfindahl index, and let* $\operatorname{cov}(\gamma^*, s^*) = \frac{1}{n} \sum (\gamma_i - \overline{\gamma})(s_i^* - 1/n)$ *denote the covariance between the shock sensitivities and the market shares.*

The aggregate profit increases with w if and only if

$$\frac{\overline{\gamma}}{2(n+1+\Theta^*)} \left[\Theta^* H^* - \frac{2}{n}(1+\Theta^*) \right] > \operatorname{cov}(\gamma^*, s^*) \ , \tag{8}$$

an exogenous shock increases total welfare if, and only if,

$$\frac{\overline{\gamma}}{2(n+1+\Theta^*)} \left(\Theta^* H^* - \frac{2}{n}(1+\Theta^*) - 1 \right) > \operatorname{cov}(\gamma^*, s^*) \ . \tag{9}$$

Proof. Using $\sum s_i^* = 1$ and equation 7, it follows that

$$\frac{\partial \Pi^*}{\partial w} = \sum \frac{\partial \pi_i^*}{\partial w} = Q^* \left[2 \sum (\overline{\gamma} - \gamma_i) s_i^* + \frac{n \overline{\gamma}}{n+1+\Theta^*} \left(\Theta^* H^* - \frac{2}{n}(1+\Theta^*) \right) \right] \tag{10}$$

[8] In Linnemer (2003), $w \sum \gamma_i q_i^* (w)$ is part of the social welfare.

By definition:

$$\frac{\partial W^*}{\partial w} = \frac{\partial S^*}{\partial w} + \frac{\partial \Pi^*}{\partial w}$$

using (4) and (10), it follows that

$$\frac{\partial W^*}{\partial w} = Q^* \left[2 \sum (\overline{\gamma} - \gamma_i) s_i^* + \frac{n\overline{\gamma}}{n+1+\Theta^*} \left(\Theta^* H^* - \frac{2}{n}(1+\Theta^*) - 1 \right) \right] \tag{11}$$

The results follow from $\text{cov}(\gamma^*, s^*) = \frac{1}{n} \sum (\gamma_i - \overline{\gamma}) (s_i^* - 1/n) = \frac{1}{n} \sum (\gamma_i - \overline{\gamma}) s_i^*$. $\square$

Conditions 8 and 9 have been written as condition 5 in terms of, first, an average effect and, second, an heterogeneity effect (through the covariance term).

The sign of the average impact (AI) depends on the sign of $\left(\Theta^* H^* - \frac{2}{n}(1+\Theta^*) - 1\right)$, that is, on the magnitude of the Herfindahl index. This effect only depends on the market structure before the shock and on the curvature of the inverse demand function around equilibrium.

The second effect is due to the heterogeneity of the shock. The sign of this heterogeneity effect (HE) is the opposite of the sign of the covariance[9] between γ and s^*. A positive correlation between s^* and γ is not in favour of a welfare increase. Corollaries 4.1 and 4.2 study each effect separately. Corollary 4.1 begins with the heterogeneity effect.

Corollary 4.1. (Bergstrom and Varian (1985)) *If $\overline{\gamma} = 0$ only the heterogeneity effect is at play and $\frac{\partial W^*}{\partial w} = \frac{\partial \Pi^*}{\partial w} > 0 \Leftrightarrow \text{cov}(\gamma, s^*) < 0$.*

That is, aggregate profit and welfare increase when the diffusion of the shock is negatively correlated with the market shares (a large firm is less likely to be affected by the shock). When $\overline{\gamma} = 0$, $\text{Sign}(\partial H^*/\partial w) = \text{Sign}(\partial V/\partial w) = -\text{Sign}(\text{cov}(\gamma, s^*))$. That is, for a mean-preserving

[9] It is also the sign of $\partial V/\partial w$ where $V(.)$ is the variance of the marginal cost distribution. That is, HE is linked to the evolution of the market structure. Formally, $V(w) = 1/n \sum (\overline{c} - c_i + w(\gamma - \gamma_i))^2$, therefore, (at $w = 0$) $\partial V/\partial w = (-2P'(Q^*)/n) \sum (\overline{\gamma} - \gamma_i) s_i^* = (-2P'(Q^*)) (-\text{cov}(\gamma_i, s_i^*))$.

shock, a marginal shock increases welfare if and only if the Herfindahl index increases. It is important to note, however, that when $\overline{\gamma} \neq 0$, $\operatorname{cov}(\gamma, s^*)$ and $\partial H^*/\partial w$ can be of opposite signs.[10] Consequently, results have to be interpreted in terms of the covariance between γ and s^* or equivalently in terms of the variation of the variance of the marginal costs but not in terms of the variation of the Herfindahl index.

Corrolary 4.2 extends the work of Kimmel and Seade as they did study the effect of a common shock on individual profit but neither on welfare nor on aggregate profit.

Corollary 4.2. (Seade (1985) and Kimmel (1992)) *If for all i, $\gamma_i = \overline{\gamma}$ (or more generally if $\operatorname{cov}(\gamma, s^*) = 0$) and $\overline{\gamma} > 0$ only the average effect is present and*

$\frac{\partial \Pi^*}{\partial w} > 0 \Leftrightarrow \Theta^* H^* > \frac{2}{n}(1+\Theta^*)$, *that is,* $n \geq 3$, $\Theta^* > \frac{2}{n-2}$ *and* $H^* > \frac{2}{n}\left(1 + \frac{1}{\Theta^*}\right)$

$\frac{\partial W^*}{\partial w} > 0 \Leftrightarrow \Theta^* H^* > 1 + \frac{2}{n}(1+\Theta^*)$, *that is,* $n \geq 3$, $\Theta^* > \frac{n+2}{n-2}$ *and* $H^* > \frac{1}{\Theta^*} + \frac{2}{n}\left(1 + \frac{1}{\Theta^*}\right)$

Intuitively, a common shock (or a shock the diffusion of which is uncorrelated with the distribution of the market shares) that on average penalized firms ($\overline{\gamma} > 0$) should decrease aggregate profit and therefore welfare. Corrolary 4.2 shows, this always happens for a duopoly ($n = 2$). For larger oligopoly, $n \geq 3$, the aggregate profit decreases when the inverse demand function is convex or not too concave, that is, when $\Theta^* \leq 2/(n-2)$. When, however, the inverse demand function is concave enough $\Theta^* > 2/(n-2)$ the common shock reallocates production from inefficient to efficient firms. Therefore, aggregate profit increases if and only if the market is concentrated enough ($H^* > \frac{2}{n}\left(1 + \frac{1}{\Theta^*}\right)$), which means that some firms are significantly more productive than some others.[11] When the inverse demand function is even

[10] More precisely, $n\partial V/\partial w = \left[P'(Q^*)\, Q^*\right]^2 \partial H^*/\partial w + 2P'(Q^*)(1+\Theta^*)\, n\overline{\gamma}\,(H^* - 1/n)/(n+1+\Theta^*)$. For instance, if $\Theta^* > -1$, $\partial V/\partial w > 0$ implies $\partial H^*/\partial w > 0$ but the reverse is not true.

[11] By allowing Θ^* to be lower than -2, Kimmel exhibited a positive effect on Π^* of a common shock (in fact

more concave around the equilibrium quantity, $\Theta^* > (n+2)/(n-2)$, this positive effect on aggregate profit is large enough to compensate the consumers' surplus loss.

As any shock is a combination of an average shock and a mean-preserving one, the effect on welfare (or aggregate profit) of a general shock can be understood by combining the intuitions provided by corollaries 4.1 and 4.2. When both effects go in the same direction the combination of the two is clear-cut.

First, if the covariance effect and the average impact both deteriorate welfare, then the combination of the two necessarily reduces welfare. This happens when the shock is positively correlated with the market shares and the inverse demand function is either not concave enough or sufficiently concave but the market is not concentrated enough.

Next, if both the covariance effect and the average impact improve welfare, then their combination also increases total surplus. For this situation to happen the shock has to be negatively correlated with the market shares and conjointly the inverse demand should be concave enough while the market should be sufficiently concentrated.

In the other cases (one positive and one negative effects) the impact of the shock on welfare is ambiguous. Corollaries 4.3 and 4.4 show that, in general, each effect can dominate the other. To clear up this ambiguity we, next, put more structure on the form of shock.

Corollary 4.3. (Symmetric case) *Let $\mathcal{N}$ denote the set of all neighborhoods of the symmetric market share list ($s_1^* = s_2^* = \ldots = s_n^* = 1/n$). Whatever γ such that $\overline{\gamma} > 0$, there is $\mathcal{N}_0(\overline{\gamma}) \in \mathcal{N}$ such that for any $s^* \in \mathcal{N}_0(\overline{\gamma})$, $\frac{\partial \Pi^*}{\partial w} < 0$ and $\frac{\partial W^*}{\partial w} < 0$.*

Proof. When $s^* = (1/n, \ldots, 1/n)$, $\operatorname{cov}(\gamma, s^*) = 0$, and $H^* = 1/n$. Therefore condition (8) simplifies in: $\Theta^* + 2 < 0$ which implies $\frac{\partial \Pi^*}{\partial w} < 0$. Similarly, condition (9) becomes: $n + \Theta^* + 2 <$

a case where for all i $\frac{\partial \pi_i^*}{\partial w} > 0$). In this case, however, second order conditions are not automatically verified.

0 which means that $\frac{\partial W^*}{\partial w} < 0$. The results follow by continuity. $\square$

In the neighborhood of the symmetric market share distribution, the covariance between any γ and s^* is almost zero, while H^* is almost equal to $1/n$. Now, given that H^* is small, the average effect is strictly negative. The heterogeneity effect could be positive but for s^* close enough to the symmetric distribution it is arbitrarily small. Therefore the potential beneficial effect of production redistribution cannot compensate a strictly negative average impact.

In a Cournot oligopoly with asymmetric firms, the larger the number of (active) firms, the more similar they have to be. Therefore for a large number of Cournot oligopolists, a shock is less likely to improve welfare.

Yet, corollary 4.4 shows that as soon as one firm is slightly more efficient than one another, a shock can be constructed which increases welfare.

Corollary 4.4. (Welfare improving shocks) *For any $s^* \neq (1/n, \ldots, 1/n)$, there is γ, $\overline{\gamma} > 0$ such that $\frac{\partial W^*}{\partial w} > 0$ and $\frac{\partial \Pi^*}{\partial w} > 0$.*

Proof. As $s^* \neq (1/n, \ldots, 1/n)$ there are i and j such that $s_i^* > s_j^*$. Let $A > 0$ and γ be such that $\gamma_i = -A$, $\gamma_j = A + 1$ and $\gamma_k = 0$ for $k \neq i, j$. Then, $\overline{\gamma} = 1/n > 0$. It is readily confirmed that $\operatorname{cov}(\gamma, s^*) = \frac{1}{n} \sum \gamma_k s_k^* - 1/n^2 = -A \left(s_i^* - s_j^* \right) / n + s_j^* - 1/n^2$. As $s_i^* > s_j^*$, $\operatorname{cov}(\gamma, s^*)$ can be as negative as one wishes, which means that for A large enough, $\frac{\partial W^*}{\partial w} > 0$ and therefore $\frac{\partial \Pi^*}{\partial w} > 0$. $\square$

Before turning to more specific shocks, the linear demand case is a special case of interest. Indeed, if $\Theta^* = 0$, then the average effect is strictly negative and its value is independent of the market structure (H^*). Therefore, for a shock to increase welfare it has to induce a large enough heterogeneity effect.

Corollary 4.5. (Linear case) *If* $\Theta^* = 0$*, then*

$$\frac{\partial \Pi^*}{\partial w} > 0 \Leftrightarrow \operatorname{cov}(\gamma, s^*) < \frac{-\overline{\gamma}}{n(n+1)} \;\; and \;\; \frac{\partial W^*}{\partial w} > 0 \Leftrightarrow \operatorname{cov}(\gamma, s^*) < \frac{-(n+2)\overline{\gamma}}{2n(n+1)}$$

Zhao (2001) emphasized that, in the linear case and for a subgroup common shock, $\frac{\partial \Pi^*}{\partial w} > 0$ or $\frac{\partial W^*}{\partial w} > 0$ iff the total market share of the firms affected by the shock is low enough (see corollary 4.6). The generalization of this result to any shock is that the weighted market share sum $\sum \gamma_i s_i^*$ is low enough. It is, however, better understood in terms of $\operatorname{cov}(\gamma, s^*)$ which has to be negative enough. That is, the shock should be harsh enough on small firms an soft enough large ones.

One can extend corollary 4.5 to strictly concave inverse demand functions to obtain a sufficient condition for welfare to increase: when $\Theta^* > 0$, $\operatorname{cov}(\gamma, s^*) < \frac{-(n+2)\overline{\gamma}}{2n(n+1)}$ implies $\frac{\partial W^*}{\partial w} > 0$. Indeed, welfare is always more likely to increase when demand is strictly concave rather than linear because concavity implies a beneficial reallocation of the production.

We now illustrate how proposition 3 applies when the structure of γ is known better. First, corollary 4.6 focuses on the case of a shock which hits uniformly a subgroup of the firms. This particular case has been widely considered in the literature (in fact besides mean-preserving shocks all previously studied shocks are of this kind).

Corollary 4.6. (Subgroup common shock) *If* $\gamma_i = 1$ *for all* $i \in T$ *and* $\gamma_j = 0$ *for all* $j \notin T$*, with* $\#T = k$*, then*

$\frac{\partial \Pi^*}{\partial w} > 0 \Leftrightarrow \frac{1}{k} \sum_{i \in T} s_i^* < \frac{1}{n} + \frac{1}{2(n+1+\Theta^*)} \left[\Theta^* H^* - \frac{2}{n}(1+\Theta^*)\right]$

$\frac{\partial W^*}{\partial w} > 0 \Leftrightarrow \frac{1}{k} \sum_{i \in T} s_i^* < \frac{1}{n} + \frac{1}{2(n+1+\Theta^*)} \left[\Theta^* H^* - \frac{2}{n}(1+\Theta^*) - 1\right]$

Lahiri and Ono (1988) restrict themselves to the case $\Theta^* > -1$ and show that $s_i^* \to 0$ implies an increase of the aggregate profit. Their result extends (as shown by corollary 4.6) to a subgroup common shock when $\frac{1}{k}\sum_{i\in T} s_i^* \to 0$. Note that it is not necessarily true for $-2 < \Theta^* < -1$.

In Zhao (2001), $\Theta^* = 0$ and the threshold values simplify further: $\frac{\partial \Pi^*}{\partial w} > 0 \Leftrightarrow \frac{1}{k}\sum_{i\in T} s_i^* < 1/(n+1)$ and $\frac{\partial W^*}{\partial w} > 0 \Leftrightarrow \frac{1}{k}\sum_{i\in T} s_i^* < 1/\left[2(n+1)\right]$. When demand is not linear, however, these kinds of threshold results are to be used carefully. Indeed, in the conditions given in corollary 4.6, $\frac{1}{k}\sum_{i\in T} s_i^*$ is on the left hand side but also in the right hand side through the H^* term.

Corollary 4.7 characterizes the impact of a shock which hits negatively (but uniformly) a subgroup of firms and positively (but uniformly) the subgroup of the other firms. Given this particular structure of γ, the covariance between γ and s^* simplifies. This case could happen, for example, when firms are located in two separate labor pools: an important worker migration from one location to the other would depress labor cost in one and increase it in the other.

Corollary 4.7. (Anti-shocks) *If $\gamma_i = 1$ for all $i \in T$ and $\gamma_j = -1$ for all $j \notin T$, with $\#T = k$ and $k > n/2$, then*

$$\frac{\partial \Pi^*}{\partial w} > 0 \Leftrightarrow \frac{1}{k}\sum_{i\in T} s_i^* < \frac{1}{n} + \frac{2k-n}{4k(n+1+\Theta^*)}\left[\Theta^* H^* - \frac{2}{n}(1+\Theta^*)\right]$$

$$\frac{\partial W^*}{\partial w} > 0 \Leftrightarrow \frac{1}{k}\sum_{i\in T} s_i^* < \frac{1}{n} + \frac{2k-n}{4k(n+1+\Theta^*)}\left[\Theta^* H^* - \frac{2}{n}(1+\Theta^*) - 1\right]$$

The comparison between the threshold values involved in a subgroup common shock and an anti-shock reflect the potentially counter intuitive effect of a shock. Indeed, one would except that for a given s^* and T, if a subgroup common shock increases welfare, then an anti-shock (based on the same T) would also increase welfare. After all, firms not affected by the subgroup common shock, enjoy a reduction of their marginal cost in the anti-shock. In fact this intuition is correct when a total common shock reduces welfare (which is always true for a duopoly, for example) while the reverse is true when a total common shock improves welfare.

The intuition is that when γ_j changes from 0 to -1 the effect on welfare can be either positive or negative depending first on the market structure (Θ^* and H^*), second on the size of firm j (s_j^*). The threshold values given in corollary 4.7 have to take into account the worse scenario. For example, when a firm j is very inefficient while demand is very concave from a welfare perspective $\gamma_j = 0$ is better than $\gamma_j = -1$, hence a lower threshold.

Finally, corollary 4.8 applies our general framework to a comparison between unit and ad valorem taxes a subject analyzed for both Cournot and price competition in Anderson, de Palma, and Kreider (2001a,b).

Corollary 4.8. (Anderson, de Palma, and Kreider (2001b)) *From the same market structure, a marginal ad valorem tax is strictly better than a marginal unit tax. The introduction of marginal unit tax increases welfare if a marginal ad valorem tax rises welfare.*

Proof. Under a unit tax w, the first order condition of the maximization of firm i's profit is: $q_i P' + P = c_i + w$. Therefore, the introduction of a marginal unit tax is equivalent to a shock $\gamma_U = (1, \ldots, 1)$. The introduction of an ad valorem tax, τ such that $1 - \tau = \frac{1}{1+w}$ (that is $\tau \simeq w$), on the other hand, leads to the following f.o.c.: $q_i P' + P = \frac{c_i}{1-\tau} = c_i + c_i w$. Therefore, the introduction of a marginal unit tax is equivalent to a shock $\gamma_A = \left(\frac{c_1}{\overline{c}}, \ldots, \frac{c_n}{\overline{c}}\right)$ (where the normalization by $\overline{c}$ is done without loss of generality to insure that $\overline{\gamma_U} = \overline{\gamma_A} = 1$).

The average impact term in condition 9 of proposition 3 is independent of γ and therefore takes the same value for both γ_U and γ_A. The heterogeneity effect depends, however, on γ but $\text{cov}(\gamma_U, s^*) = 0$ while $\text{cov}(\gamma_A, s^*) < 0$. Now, equation 11 in the proof of proposition 3 shows that $\frac{\partial W^*}{\partial w}\big|_{\gamma=\gamma_U} < \frac{\partial W^*}{\partial w}\big|_{\gamma=\gamma_A}$ which proves the result. $\square$

The fact that a marginal ad valorem tax is more efficient than a marginal unit tax is similar to the findings of Anderson, de Palma, and Kreider (2001b) (see their proposition 2 page 238).

5 Conclusion

In this note, we show how the impact of idiosyncratic shocks on a Cournot oligopoly is better understood when decomposed into two terms: an average shock which hits all firms similarly plus an individual shock which on average does not affect firms. We specify under which conditions the individual profit, the aggregate profit, and social welfare increase or decrease after a shock. In doing so we built a bridge between two strands of literature (the one on mean-preserving shocks and the other on subgroup common shocks).

The mechanism at work in this paper does not necessarily depend on the specific assumption of Cournot competition. Under oligopoly price competition production is not efficiently shared between firms. Therefore a shock on marginal cost could also (relatively) reduce the production of the less efficient firms and (relatively) increase the one of the more productive ones, leading to a welfare increase.

Acknowledgements

We are very grateful to Rabah Amir, Jean Gabszewicz, and Michael Visser for helpful comments. The quality of the paper has been improved throughout the refereeing process and we are very thankful to both anonymous referees as well as to the editor Simon Anderson. All remaining errors are our own.

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