Optimal Incentives for Income-Generation in Universities: The Rule of Thumb for the Compton Tax*

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Abstract

In this paper we propose a novel framework to model one of the key links between universities and industry – the undertaking of applied research. We postulate that a basic objective of universities is to undertake fundamental research and that they receive public funding to do so. Nevertheless, faced with tight budget constraints, universities may have incentives to allow their staff to devote some of their time to income-generating activities such as applied research or consultancy. This opens up two channels by which universities can ease their budget constraint:

(i) by allowing academics to supplement their income, universities may be able to hold down academic salaries;

(ii) universities can effectively 'tax' the income that academics raise through applied research or consultancy – for example, through the imposition of 'overhead charges'.

By easing their budget constraint, universities may be able to take on sufficient extra staff to more than offset the time that existing staff are spending on non-fundamental research and thus increase the amount of fundamental research that they can achieve with a given public budget. We develop a model of this link between universities and firms and use it to determine the optimal 'tax' that universities should impose on applied-research income. The Compton Tax, used at MIT in the 1930's, is an early example of the use of this instrument.

Key Words: University-Firm Links, Income-Generation, Fundamental and Applied Research, Compton Tax.

JEL Classification: H29, L31, O31.

1. **Introduction**

Karl Compton (1887-1954) was President of MIT between 1930 and 1948. While today it might seem odd for a university President to discourage links between the academic and business worlds, Compton was concerned that consultancy was damaging MIT's reputation and research and sought to regulate the amount of consulting that MIT academics did by taxing consultancy income. He chose a rate of 50%. What we show in this paper is that a tax of this form can in fact optimize the knowledge base and we propose a simple rule of thumb to help fix the rate at which it should be set. Was Compton right to choose 50%? While we cannot answer a definite "yes" or "no", we do identify the factors that should enter into the determination of its size.

This general debate – pure versus applied research- while not new, is particularly current because, in the recently emerging economics of science, a central issue of concern is the nature of the link between the science base in universities and its application in industry. The importance of this link for the 'new economy' is widely recognized by both academics and policy makers. For example, the existence of geographically mediated spillovers from university research to commercial innovation has been explored in a series of econometric studies following the initial work of Jaffe (1989), whilst the policy importance is nowhere more apparent than in two key policy documents.[1] The first of these was the World Bank's *1998 World Development Report*, which took knowledge as its theme, and the second was the UK Government's White Paper *Our Competitive Future: Building the Knowledge Driven Economy*.

There are a number of dimensions to the link between the science performed in universities and the application of science in industry. In this paper we choose to focus on just one – the incentives for universities to encourage academics to engage in income-generating activities such as applied research and/or consultancy. In the UK there has

[1] See Barba Navaretti et al. (1998) for a survey of recent academic work in this area.
[3] During the 1920s, MIT's large corporate backers became alarmed that MIT would end up a second-rate university because all the professors did was consult. Compton's solution was to tax consultancy earnings at 50% and use the proceeds to fund research leave for non-consulting professors (Geiger (1986) and Maurer (2002)). We thank Steve Maurer for bringing this to our notice.

been encouragement both from government and from the funding councils to establish and develop this type of university-industry link. Moreover, with many western governments now operating much tighter fiscal policies, cash-constrained universities in the UK and elsewhere are themselves realizing the need to promote income-generating activities if they are to fulfill their mission of generating the maximum amount of fundamental research with limited public funds.

However, to the best of our knowledge, there has been no formal economic analysis of the problem of determining the extent to which universities should engage in such activities and the methods they should employ to do so. It is clear that there are a number of important features of the problem that make such an analysis non-trivial.

(a) The time dedicated by academics to income-generating activities comes at the expense of time that could be spent fulfilling one of the primary objectives of universities – the creation and dissemination of fundamental knowledge. This can only be justified if the promotion of this activity eases university budget constraints to a sufficient extent that they can hire enough extra academics to more than replace the fundamental research time that is now being devoted to other activities.

(b) There are two ways in which the promotion of income-generating activities such as applied research and/or consultancy might enable universities to ease their budget constraint. The first is that universities may be able to effectively 'tax' the applied/consultancy income earned by academics – for example by the use of 'overhead charges' or, the Compton tax. [3] The second more indirect route is that, by allowing academics to supplement their income, universities may be able to hold down academic pay, and so hire more academics with any given budget. Other things being equal, the first of these points to a high implicit tax rate, the second to a low one.

(c) Given the public good nature of knowledge, an expansion in fundamental research by universities will also raise the productivity of applied research in the private sector, and so the alternative income that academics could earn in the private sector.

These last two points indicate the importance to universities of being aware of the opportunities for scientists outside the university sector when designing their policies.

In this paper we present a simple model of university-firm linkages through the market for applied research and use it to determine the optimal 'tax' that universities should impose on the income that academics earn from applied research. However, before proceeding there are four issues we would like to clarify.

1. There is a clear distinction between the issues being addressed in this paper and those concerning the "commercialization" of public research. The latter concerns the terms and conditions under which academics make the results of their own fundamental research available to others – especially those in the private sector.[4] There are certainly important issues to be addressed here, and the topic has been the subject of much recent research, particularly in relation to the pharmaceutical industry - see for example Cockburn, Henderson and Stern (1999), Darby and Zucker (1999), Hall, Link and Scott (2000), Jensen and Thursby (2001) and Siegel, Waldman and Link (2003). However, since we wish to focus on other issues of university funding, in this paper we will assume that fundamental research is made available under the normal scientific conditions ('open science'). That is, it is made available free of charge in academic journals as soon as possible, with academic rewards being conditioned on being first to publish. By applied research or consultancy we mean the bringing to bear of the discoveries of fundamental research on applied problems arising in industry. Academics undertaking such work can draw on not just their own fundamental research, but on the entire body of published fundamental research. In order to pursue their

[4] In the U.S. this relates to the effects of the Bayh-Dole Act of 1980. The act changed the legislation on property rights as they apply to publicly funded science. Academic scientists can now themselves patent discoveries made from publicly funded research

own basic research, university scientists have had to master a wide body of fundamental research that gives them the capability to undertake applied research.

2. In what follows we shall treat all fundamental research as being performed in universities. We recognize that some fundamental research is, in practice, also undertaken by firms. There are various reasons why this should happen. Firms may feel it is necessary to fill gaps in the universities' research portfolios. They may do it to acquire patents and earn financial rewards from these (e.g. genome research). They may do it to ensure that they have the necessary understanding to effectively absorb the results of university research (Cohen and Levinthal (1989)). However, since the focus of research in this paper is on incentives within universities and on overcoming tight financial constraints, the assumption that firms do no fundamental research is not critical.

3. Since a focus of this paper is the generation of research, we will ignore teaching as an activity and proceed on the assumption that the sole objective of universities is to maximize the amount of basic/fundamental research they can achieve with given resources.[5]

4. Finally, we suppose that all transfers of resources from firms to universities are in the form of income. In practice of course, firms sometimes transfer real resources – for example by giving academics access to expensive equipment that is not available in university laboratories. However this is a detail that does not affect the substance of the issues we shall explore.

Our principal result is a proposition whose implication is that, when universities are thinking about the overhead levy to charge on external earnings of academics, the ratio of

[5] Jensen and Thursby (2002) examine the effect of patent revenue for applied research on university and faculty incentives, as well as the resulting quality of education for the case of a single university. De Fraja and Iossa (2001) and Del Rey (2001) analyse the case where there is competition to attract students amongst universities that devote funds to teaching and research. De Fraja and Iossa (2001) focus on students' choice of which university to attend depending on mobility costs while Del Rey (2001) derives conditions for different types of research-teaching combinations to occur.

the university salary to the employee's outside option provides a simple rule of thumb for the charge. Though the result is established in the context of a specific model, simulation suggests that there exists a reasonable range of parameter values for which the result is a satisfactory approximation and so a useful rule of thumb as well.

The paper is organized as follows. In section 2 we outline and discuss the model. Then, in section 3 we analyze the model using general functional forms while in section 4 we take this analysis further by studying an illustrative example that gives rise to the proposition and the rule of thumb. Finally, in section 5 we offer some concluding comments.

2 The Model

To capture the university-firm linkages through the market for applied research we distinguish between three types of agent: firms, scientists and universities. We start by describing the role played by firms.

Firms

In the interest of survival, firms are dependent on continual improvements in the quality of their products or processes. These improvements come from applied research. Suppose that **all** of the various types of fundamental research that are undertaken by universities can be applied to helping firms generate higher profits through improved products or processes of production. Thus applied research is to be understood very widely to encompass improved management/legal services as well as the more usual applications of engineering, physical or bio-chemical sciences. Applied research should be understood to encompass a wide range of activities embracing genuine research through to what might more usually be classified as consultancy.

As mentioned in the Introduction, although firms recognise the link between applied and fundamental research, given the generic nature of the latter, they do not themselves undertake any fundamental research. We suppose that firms do not do applied research in-house, but obtain it from independent applied researchers/consultants working in the private sector or in universities, and that the market for applied research is perfectly competitive. Let q be the value of quality improvement, and p the price of applied research/consulting services. In what follows we treat q as exogenous, while p is definitely an endogenous variable that is influenced by the behaviour of universities.

We denote the demand for applied research/consulting services by any firm by the function $Z^d(q/p)$. If there are F firms, F = 1,...F, then the total demand for applied research/consultancy is $FZ^d(q/p)$. Having described the role of firms we proceed to discuss the role of scientists.

Scientists

There is a supply of individuals who have been trained as scientists. These individuals can enter three occupations: (i) university scientists, (ii) private-sector scientists or (iii) an outside option.[6]

(i) <u>University Scientists</u>.

University scientists undertake fundamental research, but may choose to devote some time undertaking applied research. In order to undertake fundamental research, they have to spend a fixed amount of time each period in keeping up with the latest scientific developments[7]. This is independent of the size of the existing knowledge base. We normalize the units of time so that university scientists have one unit of time available to devote either on pure research, r, or applied research, $a = 1 - r$. Further, the knowledge produced by all scientists is K. We then introduce the following assumption.

ASSUMPTION 1 (*Generation of fundamental knowledge). Given K, an amount of time r*, $0 \le r \le 1$, *devoted on fundamental research generates fundamental knowledge according to the production function* $\phi(r, K)$, *with the following properties:*

[6] While the general model can be solved, in principle, with just the two "inside"options, the intuition behind our result, and its interpretation, is enriched by adding the outside option.

[7] See Cohen and Levinthal (1989) for the idea that, in order to undertake research, it is necessary to invest resources in developing the capacity to absorb other people's ideas.

(i) $\phi(r,0) \equiv 0; \quad \phi(0,K) \equiv 0$;

(ii) $\forall K > 0, \quad \phi_r(0,K) = \infty; \quad \phi_{rr}(r,K) < 0; \quad \phi_r(r,K) \to 0 \quad \text{as} \quad r \to \infty,$

(iii) $\forall r > 0, \quad \phi_K(r,0) = \infty; \quad \phi_{KK} < 0; \quad \phi_K(r,K) \to 0 \quad \text{as} \quad K \to \infty.$

Note that $\phi(r,K)$ is a conventional production function. These standard properties imply that there is value in building the knowledge base. At very low levels of knowledge, time spent in fundamental research will be highly productive. However as the knowledge base grows, there are diminishing returns for any given time spent in the activity.

To simplify the analysis we take it that universities find it difficult to monitor accurately either the inputs or outputs of individual scientific research, and so scientists are paid a fixed salary, $w > 0$, that is independent of the amount of time actually devoted to fundamental research.[8]

University scientists can also undertake work on applied research/consultancy. The effort devoted to absorbing the fundamental research of others before undertaking their own fundamental research is sufficient to enable university scientists to undertake applied research. Further, the productivity of an applied researcher depends on the amount of fundamental research that is available to be applied. We then introduce our second assumption describing the generation of applied knowledge.

ASSUMPTION 2 (*Generation of applied knowledge). Given K, an amount of time a,* $0 \le a \le 1$, *devoted on applied research generates applied knowledge according to the production function* $\psi(a,K)$, *with the following properties:*

(i) $\psi(a,0) \equiv 0; \quad \psi(0,K) \equiv 0,$

(ii) $\forall K > 0, \quad \psi_a(0,K) = \infty; \quad \psi_{aa}(a,K) < 0; \quad \psi_a(a,K) \to 0 \text{ as } a \to \infty;$

(iii) $\forall a > 0, \quad \psi_K(a,0) = \infty; \quad \psi_{KK}(a,K) < 0; \quad \psi_K(a,K) \to 0 \text{ as } K \to \infty.$

[8] This is not to deny that universities cannot monitor research at all. Indeed, universities do monitor research behaviour imperfectly, by using information such as journal publications, working papers, conference attendance (i.e. research output) and refereeing activity, editorships, research grants and so on (i.e. research input).

Note that $\psi(a,K)$ too is a conventional production function with standard properties. In addition, if a university scientist spends an amount of time a, $0 \le a \le 1$, on applied research, then the amount of income earned in this activity is $p\psi(a,K)$. Although universities cannot observe the effort that goes into applied research, they can observe the income, and, through a variety of devices, can effectively 'tax' this income. Let t, $0 \le t \le 1$, be this tax rate, and let $\tilde{p} = p(1-t)$ be the net price that university scientists earn from undertaking applied research.

To avoid the possible problem of shirking by university scientists, we posit that all scientists are motivated to engage in fundamental research.[9] This arises from the importance of priority and the culture of 'open' science. From the work of Merton (1957),[10] there are convincing arguments that the goal of scientists is to establish priority of discovery by being the first to communicate a new result and that an important part of the reward structure in science comes from being first.[11] Moreover, those who have studied the behavior of scientists point to the importance of the satisfaction of solving a puzzle – e.g. Hagstrom (1965) and Hull (1988). Taken together, these considerations suggest that time spent on fundamental research will be an important argument in the utility function of scientists.[12] We thus introduce the following assumption.

ASSUMPTION 3 (*Scientists' Utility*). *A scientist's utility function* $u(y,r)$, *where y is income and r is time devoted to fundamental research, has the following properties:*

(i) $\forall y \ge 0, r \ge 0$, $u_y(y,r) > 0$ *and* $u_r(y,r) > 0$*;*

[9] See Stephan (1996), p. 1207.
[10] See also Dasgupta and David (1987).
[11] Paula Stephan (1996) identifies three parts to the reward structure: eponymy, prizes and publication. The importance of reputation should not be underestimated as it provides a non-market method of correcting the market failures associated with knowledge. Arrow (1987) also notes that "The incentive compatibility literature needs to learn the lesson of the priority system; rewards to overcome shirking and the free-rider problem need not be monetary in nature". As Stephan notes (p. 1206), "A reward system based on reputation also provides a mechanism for capturing the externalities associated with discovery. The more a scientist's work is used, the larger is the scientist's reputation and the larger are the financial rewards. It is not only that the reward structure of science provides a means for capturing externalities. The public nature of knowledge encourages use by others, which in turn enhances the reputation of the researcher."
[12] For some evidence on this, see Levin and Stephan (1991).

(ii) *u(y,r) is strictly quasi-concave;*

(iii) $u_r(y,r) \to \infty$ *as* $r \to 0$ *;*

(iv) *both 'goods' are normal.*

This utility function satisfies all the usual properties. Thus choosing a university career will give a scientist a level of utility

$$v^u(w,\tilde{p},K) = \max_{0 \le r \le 1} \; u[w + \tilde{p}\,\psi(1-r,K),r]. \tag{1}$$

It is clear that $v^u(.)$ is a strictly increasing function of all three arguments. Let

$$R(w,\tilde{p},K) = \underset{0 \le r \le 1}{\operatorname{argmax}} \; u[w + \tilde{p}\,\psi(1-r,K),r] \tag{2}$$

be the solution to the above optimization problem.

From Assumption 3 it follows that university scientists will always devote some time to fundamental research – i.e. $R(w,\tilde{p},K) > 0$. Furthermore, from Assumption 2 it follows that as long as there is some reward to applied research then scientists will spend some time doing it. Formally, if $\tilde{p} > 0$ - i.e. if $t < 1$ - then $R(w,\tilde{p},K) < 1$. However if $t = 1$ and there are no rewards to applied research then university scientists will devote all their time to fundamental research – i.e. $R(w,0,K) = 1$.

Conventional demand theory tells us that *R*(.) is an increasing function of *w*. However, for standard reasons, at this level of generality it is impossible to predict the effects of either $\tilde{p}$ or *K* on the amount of fundamental research that is undertaken.

(ii) Private Sector Scientists

The second career that a scientist can pursue is to work as an applied researcher/consultant in the private sector. In this occupation there is no opportunity for fundamental research. Nevertheless, applied scientists have to draw on the fundamental research base in order to do their applied work, and so have to spend some time mastering/absorbing this fundamental research. Suppose that it takes less time to absorb the fundamental research when it is being used solely for applied work, so that a total amount of time $1+\gamma$, $\gamma > 0$, is available for undertaking applied research.

Consequently the income of a private sector scientist will be $p\psi(1+\gamma, K)$, and the associated level of utility if a scientist pursues this career will be

$$v^p(p,K) \equiv u[p\psi(1+\gamma, K), 0]. \tag{3}$$

(iii) <u>The Outside Option</u>

Finally a scientist can pursue a career totally unrelated to science (e.g., management). Income in this career is $\tilde{w} > 0$, which is exogenous. The utility that a scientist will obtain by pursuing this career is

$$v^m(\tilde{w}) = u(\tilde{w}, 0). \tag{4}$$

Having described the options available to a scientist, we then consider universities.

Universities

As pointed out above, the fundamental issue on which we wish to focus concerns the incentives that universities give their scientists to devote some of the time that they could have spent on fundamental research to undertaking income-generating applied research for private firms. As mentioned in the Introduction, the objective of universities is to maximize the amount of basic/fundamental research they can achieve with given resources.

We take it that this fundamental research is conducted across a wide spectrum of different disciplines ranging from medicine and biochemistry to law and social sciences. As already said, we are abstracting from issues to do with the commercialization of fundamental research, and we take it for granted that, given its special characteristics[13], fundamental research is organized in what is now recognized as the conventional scientific fashion[14]. Thus scientific output is produced and instantly disseminated free of charge under a priority system. We shall ignore the problems arising from having teams from different universities competing for the same prize, and treat all universities as operating as a single integrated university sector.

Suppose then that income to fund the production of fundamental knowledge comes from two sources:
(a) a public fund, $B > 0$, which is exogenous and
(b) the revenue retained by universities from the income generated by university scientists through the applied research they carry out for industry.

If university scientists each devote an amount of time r, $0 \leq r \leq 1$, on fundamental research, and if the university sector taxes the income that scientists earn from applied research, then, if the university sector employs n scientists at a wage w, these have to satisfy the university budget constraint:

$$wn \leq B + ntp\psi(1-r, K), \tag{5}$$

while the amount of fundamental knowledge produced by the university sector, K, has to satisfy the condition

$$K = n\phi(r, K). \tag{6}$$

Equation (6) is an equilibrium relationship. It implies that the stock of knowledge is a common pool and that a key feature of each generation of academics is that it embodies the stock of knowledge. Every scientist contributes to the knowledge base and all their research paths are complementary. The equation also makes clear that everyone can

[13] These include the generic nature of research, the high risk attached to it, and the strong complementarities between the outputs of individual researchers.
[14] See for example Merton (1957) and Dasgupta and David (1987).

benefit from drawing on this knowledge in doing their own research, and that all information is fully shared. This modeling seeks to capture scientific endeavor in a straightforward way. Isaac Newton may have said "If I have seen far, it is by standing on the shoulders of giants"; however such giants are worth nothing without the current generation of academics to interpret and carry forward that knowledge.

This completes the description of the general model. In the next section we show how the model can be solved to determine the amount of fundamental knowledge that will be produced for any given 'tax' rate t, and hence the value of the tax that will maximize knowledge for any given budget.

3 Analysis of the Model

As the most interesting case is where scientists are active in all three occupations we concentrate on this case for the rest of the paper. If scientists are indifferent between a university career and the outside option then we must have

$$v^u\left[w, p(1-t), K\right] = u(\tilde{w}, 0), \tag{7}$$

while if scientists are indifferent between a career as private sector scientists and the outside option then we must have

$$v^p \equiv u\left[p\psi(1+\gamma, K), 0\right] = u(\tilde{w}, 0),$$

or,

$$p\psi(1+\gamma, K) = \tilde{w}. \tag{8}$$

Substituting (2) into the budget constraint, (5), and the knowledge equation, (6), we obtain:

$$wn = B + tnp\psi[1 - R(w, p(1-t), K), K] \tag{9}$$

and

$$K = n\phi[R(w, p(1-t), K), K]\,. \qquad (10)$$

Equations (7) – (10) constitute a system of four equations in the four endogenous variables of interest: the scientist's salary/wage, *w*, the total number of scientists employed by the university sector, *n*, the price of applied research, *p*, and the amount of fundamental knowledge, *K*. While simple, the model captures all the considerations raised in the Introduction:

- the complex tensions that universities face in deciding whether allowing academics to undertake applied research will increase the amount of fundamental research that they carry out;
- the two different routes by which universities can ease the budget constraint by giving academics opportunities to engage in income-generating activities;
- the need to pay attention to the various career options that academics face outside the university sector;
- the fact that fundamental knowledge affects the productivity of private sector researchers.

By solving these four equations we can determine how the amount of fundamental knowledge depends on the 'tax' rate, and hence calculate the optimal 'tax' rate – that which maximises fundamental knowledge, *K*. However, in order to illustrate the workings of the model, we shall consider an illustrative example that uses specific functional forms that satisfy the assumptions we made in Section 2.

4 An Illustrative Example

Suppose that the production and utility functions take the following specific functional forms:

$$\phi(r,K) = r^{\alpha}K^{1-\alpha}, \quad 0<\alpha<1$$

$$\psi(a,K) = a^{\beta}K^{1-\beta}, 0<\beta<1$$

$$u(y,r) = y + \delta r^{\beta}, \delta > 0.$$

These production functions are just conventional Cobb-Douglas production functions. We do not impose the requirement that the production functions for fundamental and applied research should be the same, and so allow for the possibility that the coefficients α and β could be different. We take no view on their relative magnitude.[15] The parameters α and β measure the intrinsic productivity of skilled labour in fundamental and applied research respectively. The utility function is quasi-linear in income. This brings an important simplification since it means that there will be no income effects in the individual scientist's supply function for fundamental research. The parameter δ is crucial since it measures the importance of fundamental research to scientists. However, δ cannot be too large, otherwise we will obtain solutions in which scientists end up working in universities for nothing. So we certainly need to bound δ by the condition that $\delta < \tilde{w}$.[16] The major simplifying restriction that we make is that the power coefficient on *r* in the utility function is the same as that which appears in the production function for applied research, β. However, this restriction has a plausible interpretation: notice that the higher productivity in applied research (β) the smaller is the contribution of the relevant term to a scientist's utility, *ceteris paribus*. This reinforces the point made above on a scientist's preference towards fundamental research and puzzle-solving (e.g., Hagstrom (1965) and Stephan (1996)). In addition, this simplification makes the analysis of the model tractable.

[15] Empirical work indicates that in some cases, notably in biotechnology, entrepreneurial faculty has higher productivity than non-entrepreneurial faculty (Louis *et al.* (2001)) and that involvement in applied research increases productivity in basic research (Zucker and Darby (1996) and Siegel *et al.* (2003)).

[16] Later we will have to put further restrictions on the magnitude of δ.

Given these functional forms, the individual supply of fundamental research takes the form[17]:

$$R(\tilde{p},K)=\frac{\delta^{\frac{1}{1-\beta}}}{\delta^{\frac{1}{1-\beta}}+K\tilde{p}^{\frac{1}{1-\beta}}}, \tag{11}$$

while the associated indirect utility function becomes[18]

$$v^{u}(w,\tilde{p},K)=w+\left[\tilde{p}^{\frac{1}{1-\beta}}K+\delta^{\frac{1}{1-\beta}}\right]^{1-\beta}.$$

Consequently equation (7), determining a scientist's indifference between a university career and the outside option, becomes:

$$w=\tilde{w}-\left[\tilde{p}^{\frac{1}{1-\beta}}K+\delta^{\frac{1}{1-\beta}}\right]^{1-\beta}. \tag{12}$$

Next, eliminating n from equations (9) and (10) and then substituting (11) into the resulting expression we obtain: [19]

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[17] Maximizing $U=w+\tilde{p}(1-r)^{\beta}K^{1-\beta}+\delta r^{\beta}$ with respect to r yields the following first-order condition: $\left(\frac{r}{1-r}\right)^{1-\beta}=\frac{\delta}{\tilde{p}\ K^{1-\beta}}$. This can then be solved for r, which is expression (11).

[18] We have $1-R=\left(\frac{K\tilde{p}^{\frac{1}{1-\beta}}}{\delta^{\frac{1}{1-\beta}}+K\tilde{p}^{\frac{1}{1-\beta}}}\right)$, and so can construct

$$v^{u}=w+\frac{\tilde{p}^{\frac{1}{1-\beta}}K}{\left(\delta^{\frac{1}{1-\beta}}+K\tilde{p}^{\frac{1}{1-\beta}}\right)^{\beta}}+\frac{\delta^{\frac{\beta}{1-\beta}}}{\left(\delta^{\frac{1}{1-\beta}}+K\tilde{p}^{\frac{1}{1-\beta}}\right)^{\beta}}.$$ This simplifies to the expression shown.

$$K = B^{\frac{1}{\alpha}}\left[\tilde{w} - \frac{\delta^{\frac{1}{1-\beta}} + \frac{\tilde{p}^{\frac{1}{1-\beta}}K}{1-t}}{\left[\delta^{\frac{1}{1-\beta}} + \tilde{p}^{\frac{1}{1-\beta}}K\right]^{\beta}}\right]^{-\frac{1}{\alpha}} \frac{\delta^{\frac{1}{1-\beta}}}{\delta^{\frac{1}{1-\beta}} + \tilde{p}^{\frac{1}{1-\beta}}K}. \qquad (13)$$

From (8), i.e. the indifference relation between a career as a private sector scientist and the outside option, we have:

$$\tilde{p}^{\frac{1}{1-\beta}}K = \sigma\left[\tilde{w}(1-t)\right]^{\frac{1}{1-\beta}}, \qquad (14)$$

where $\sigma \equiv \left[(1+\gamma)^{\frac{\beta}{1-\beta}}\right]^{-1}$ and $\sigma < 1$.[20]

It is clear that the parameter σ is a measure of the time available for doing research in universities compared to the time available in the private sector, given the additional time that is required to keep up with the subject if scientists are to undertake fundamental research. We could think intuitively of σ as being a measure of the costs of engaging in fundamental research. It reflects the ability of purely applied researchers to specialise in a well-defined area and thus another way of thinking of σ is as 'specialisation effect'.

Substituting (14) into (13) we obtain the equilibrium value of fundamental knowledge,

$$K^{e} = B^{\frac{1}{\alpha}}\left[\tilde{w} - \frac{\delta^{\frac{1}{1-\beta}} + \sigma\tilde{w}^{\frac{1}{1-\beta}}(1-t)^{\frac{\beta}{1-\beta}}}{\left[\delta^{\frac{1}{1-\beta}} + \sigma\left[\tilde{w}(1-t)\right]^{\frac{1}{1-\beta}}\right]^{\beta}}\right]^{-\frac{1}{\alpha}} \frac{\delta^{\frac{1}{1-\beta}}}{\left[\delta^{\frac{1}{1-\beta}} + \sigma\left[\tilde{w}(1-t)\right]^{\frac{1}{1-\beta}}\right]}. \qquad (15)$$

[19] See Appendix for details of the derivation.

[20] As discussed later, σ may be thought of as a "specialization effect".

It is then straightforward to check that when $t = 0$, the second term on the right-hand side of expression (15) becomes $\tilde{w} - \left[\delta^{\frac{1}{1-\beta}} + \sigma \tilde{w}^{\frac{1}{1-\beta}}\right]^{1-\beta}$, which is positive as long as $\delta < (1-\sigma)^{1-\beta}\tilde{w}$ or $\left(\frac{\delta}{\tilde{w}}\right)^{\frac{1}{1-\beta}} < 1-\sigma$. Letting $\xi \equiv \left(\frac{\delta}{\tilde{w}}\right)^{\frac{1}{1-\beta}} < 1$, the latter inequality can be written as $\xi + \sigma < 1$. From now on we will assume that this restriction on ξ and σ holds. The parameter ξ however is another measure of the intrinsic attractiveness of doing fundamental research (or what we could call scientific drive).

Then (15) defines an equilibrium value of knowledge, *K*, for all values of the 'tax' rate, $t \in [0, 1]$. We can re-write (15) as

$$K^e = \frac{M}{J(t;\alpha,\beta,\sigma,\xi)} \tag{16}$$

where *M* is a constant and

$$J(t;\alpha,\beta,\sigma,\xi) \equiv \left[1 - \frac{\xi + \sigma(1-t)^{\frac{\beta}{1-\beta}}}{\left[\xi + \sigma(1-t)^{\frac{1}{1-\beta}}\right]^{\beta}}\right]^{\frac{1}{\alpha}} \left[\xi + \sigma(1-t)^{\frac{1}{1-\beta}}\right]. \tag{17}$$

Note that *J* (.) is a function of *t*, but with four parameters α, β, σ and ξ.

The equation that gives the equilibrium salary/wage for a university-based scientist (equation (12)) can be re-written using (14) and the definition of ξ as

$$w = \tilde{w} - \left[\sigma[\tilde{w}(1-t)]^{\frac{1}{1-\beta}} + \delta^{\frac{1}{1-\beta}}\right]^{1-\beta} = \tilde{w} - \left[\sigma[\tilde{w}(1-t)]^{\frac{1}{1-\beta}} + \xi\tilde{w}^{\frac{1}{1-\beta}}\right]^{1-\beta} .(12')$$

Re-arranging this yields an expression for, ω, the ratio of the equilibrium salary of university scientists to the salary they would earn in 'management':

$$\omega(t) = \frac{w}{\tilde{w}} = 1 - \left[\sigma(1-t)^{\frac{1}{1-\beta}} + \xi\right]^{1-\beta} . \tag{18}$$

This is a strictly increasing function of t. Obviously the more heavily universities 'tax' outside income, the higher are the salaries they have to pay in order to attract scientists to work in the university sector. This corresponds to point (b) in the Introduction.

We shall now show that in the special case of equal productivity in fundamental and applied research, i.e. when $\alpha = \beta$, this ratio is equal to the optimal tax rate.

Define L and N as follows:

$$L \equiv \xi + \sigma(1-t)^{\frac{1}{1-\beta}}; \;\; N \equiv \xi + \sigma(1-t)^{\frac{\beta}{1-\beta}}.$$

Thus, from (17) we can write $J(t;\beta,\sigma,\xi) = \left(1 - \frac{N}{L^\beta}\right)^{\frac{1}{\beta}} L$ and this can be simplified to $J(\cdot) = \left(L^\beta - N\right)^{\frac{1}{\beta}}$. We seek the t that minimizes this expression. Taking the first-order condition results in

$$\frac{dJ}{dt} = 0 = \frac{1}{\beta}\left(L^\beta - N\right)^{\frac{1-\beta}{\beta}} \frac{d}{dt}\left(L^\beta - N\right).$$

However since $L^{\beta} - N > 0$, it is sufficient that $\frac{d}{dt}\left(L^{\beta} - N\right) = 0$ or, $\frac{d}{dt}L^{\beta} = \frac{dN}{dt}$. Writing this condition out yields

$$-\left(\frac{\sigma\beta}{1-\beta}\right)L^{\beta-1}(1-t)^{\frac{\beta}{1-\beta}} = -\left(\frac{\sigma\beta}{1-\beta}\right)(1-t)^{\frac{\beta}{1-\beta}-1}.$$

Canceling terms and re-arranging, we obtain an explicit expression for the optimal tax rate,

$$\hat{t} = 1 - L^{1-\beta} = \frac{w(\hat{t})}{\tilde{w}} = \omega(\hat{t}). \tag{19}$$

Furthermore, from (19) and the definition of L, it also follows that an alternative way to express the optimal tax rate is

$$\hat{t} = 1 - \left(\frac{\xi}{1-\sigma}\right)^{1-\beta}. \tag{19'}$$

Thus we have established the following:

PROPOSITION 1. *If the production functions for basic and applied science and the utility function for scientists take the following forms*:

$$\begin{aligned}
&\phi(r,K) = r^{\beta}K^{1-\beta}, \quad 0 < \beta < 1; \\
&\psi(a,K) = a^{\beta}K^{1-\beta}, \quad 0 < \beta < 1; \\
&u(y,r) = y + \delta r^{\beta}, \quad \delta > 0,
\end{aligned}$$

then the optimal tax rate equals the ratio of the university salary to the salary in the outside option:

$$\hat{t} = \frac{w(\hat{t})}{\tilde{w}} = \omega(\hat{t}) \text{ and } \hat{t} = 1 - \left(\frac{\xi}{1-\sigma}\right)^{1-\beta}.$$

Proposition 1 implies a simple 'rule-of-thumb', i.e. to use the ratio of academic salaries (exclusive of income-generating money) to the salaries scientists could command in non-science jobs in the private sector as a first-order guide to the appropriate 'tax' rate on income-generating activities.[21] The optimal tax rate for the range of feasible values of σ and ξ is shown in Table 1.

Table 1: Optimal Tax Rate (α=0.5; β= 0.5)

	ξ										
σ	0.01	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	.899	.775	.682	.551	.450	.364	.289	.222	.159	.101	.047
0.05	.897	.771	.676	.541	.438	.351	.275	.205	.142	.082	.027
0.1	.895	.764	.667	.529	.423	.333	.255	.184	.118	.057	-
0.2	.888	.750	.646	.500	.388	.293	.209	.134	.065	-	-
0.3	.880	.733	.622	.465	.345	.244	.155	.074	-	-	-
0.4	.870	.711	.592	.423	.293	.184	.087	-	-	-	-
0.5	.859	.684	.553	.368	.225	.106	-	-	-	-	-
0.6	.842	.646	.500	.293	.134	-	-	-	-	-	-
0.7	.817	.592	.423	.184	-	-	-	-	-	-	-
0.8	.776	.500	.293	-	-	-	-	-	-	-	-
0.9	.684	.293	-	-	-	-	-	-	-	-	-

As the above expression and Table 1 make clear, the optimal tax rate is a strictly decreasing convex function of σ and ξ. Recall that σ is a measure of the costs of doing fundamental research (a sort of 'specialisation effect') as opposed to ξ which measures its attractiveness. Obviously as ξ increases, the more attractive do academics find fundamental research, and so a lower 'tax' rate is required in order to induce them to engage in applied research. This result is consistent with the theoretical findings of

[21] As mentioned earlier, the actual tax rate that Karl Compton imposed on MIT faculty in the 1930s was 50%. According to our analysis this value would correspond to σ = 0.2 and ξ = 0.2, σ = 0.8 and ξ = 0.05, or σ = 0.6 and ξ = 0.1, i.e., relatively low attractiveness of fundamental research. In this case, the tax was used as a stick as the view was that faculty members were over-engaging in consultancy (Geiger, 1986). In one UK university with which we are familiar, the tax rate is 25%. On comparing his academic salary with what someone of his age and experience could get as a senior manager in the private sector, the first author (who has an academic management role) noted that the ratio was in fact about 25%!

Holmström and Milgrom (1991) on multi-task principal-agent problems.[22] As σ increases, the costs of engaging in fundamental research (in terms of the time foregone in mastering the knowledge base) fall and this increases the sensitivity of university academics to salary differentials, and so calls for a lower 'tax' rate.

Next, using the definitions for σ and ξ and expression (14), we can re-write the expression giving the optimal amount of time/effort spent on fundamental research *R*, i.e. expression (11), as

$$R(t;\beta,\sigma,\xi) = \frac{\xi}{\xi + \sigma(1-t)^{\frac{1}{1-\beta}}} \tag{20}$$

Substituting (19') into (20) yields the following proposition:

PROPOSITION 2. *If the production functions for basic and applied science and the utility function for scientists take the following forms*:

$$\begin{aligned}
&\phi(r,K) = r^{\beta}K^{1-\beta}, \quad 0<\beta<1; \\
&\psi(a,K) = a^{\beta}K^{1-\beta}, \quad 0<\beta<1; \\
&u(y,r) = y + \delta r^{\beta}, \quad \delta > 0,
\end{aligned}$$

the optimal level of scientific effort is independent of the level of scientific drive, and depends only on the size of the specialisation effect:

$$R(\hat{t};\beta,\sigma,\xi) = 1-\sigma .$$

For the case of unequal productivity in fundamental and applied research, $\alpha \neq \beta$, it is not possible to obtain closed-form solutions. When this is so it is no longer the case that $\omega(\hat{t}) = \hat{t}$, and indeed $\hat{t} \gtrless \omega(\hat{t})$ as $\alpha \gtrless \beta$. We have, in fact, explored the sensitivity of the

[22]There is also evidence of this from empirical work. Stern (1999) has shown the presence of a wage/right-

result in Proposition 1 to deviations of α from β. There is a systematic pattern that is revealed in the numbers that appear in Tables A1-A4 in the Appendix. These show the ratio between the tax rate and the wage ratio and are a representative subset of the simulations we have done.[23] The degree of approximation is indicated by how close the number reported in the table is to unity. As one can see, the "fit" is at its best when σ is at its highest and ξ is at its lowest. In such cases, even when the divergence between α and β is large, the rule of thumb is a remarkably good approximation: within 10% of the true value. For small deviations between α and β, the approximation is particularly good. Thus we are led to conclude that the result in Proposition 1 is indeed a useful rule of thumb.

5. Concluding Remarks

The development and strengthening of the links between universities and industry is currently a topic of major interest to both governments and the public bodies that fund the university systems in the UK and the rest of Europe as well as in the United States. It is important that a framework is developed that will allow this link to be carefully modeled and systematically analyzed.

In this paper we have provided a framework in which to think about the complex issues facing universities in deciding what incentives to give academics to pursue income-generating activities. In particular, we have tried to capture the various ways in which the promotion of income-generating activities can ease university budget constraints, and the need for universities to recognize the range of outside opportunities that academics face.

Within our framework we have shown that there are four factors that affect the optimal incentives for income-generation: the productivity of researchers in both fundamental and applied research, the intrinsic desirability of fundamental research to academics, and

to-publish trade-off in US biotechnology.

[23] A much more complete set of simulation tables can be obtained from the first-named author on request.

the relative amounts of time that need to be spent in keeping up with one's subject in both fundamental and applied research. While the importance of the first three factors may be thought to be relatively 'obvious', the role of the fourth almost certainly is not.

Given the difficulty of measuring these factors, one very simple 'rule-of-thumb' that emerges from our analysis is to use the ratio of academic salaries (exclusive of income-generating money) to the salaries academic scientists could command in non-science jobs in the private sector as a first-order guide to the appropriate 'tax' rate on income-generating activities.

Throughout the paper we have restricted attention to the case where applied science projects that generate income have no direct effect on the creation and dissemination of fundamental knowledge. Further research should extend the model to the more general case where there is an additional effect from applied research to fundamental research, i.e. when fundamental breakthroughs in science originate in scientists' work on applied problems in industry. This is one of the questions on our current research agenda. For example, in our model the objective of universities is to maximize basic knowledge. Clearly, another important objective in practice is the provision of education and the model should be extended to consider that. This, of course, raises issues concerning the quality of the education provided to students when faculty become more heavily involved in income-generating activities and applied research. Jensen and Thursby (2002) take a first step in this direction. Among the other issues in our agenda for future research are to consider the implications for our analysis if not all research is "open science", to relax the assumption that fundamental research is carried out only in universities, to introduce dynamics into the knowledge generation process and to relax of the assumption that the market for scientists is perfectly competitive.

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Appendix

Derivation of expressions (13) and (16)

From expressions (9) and (10), we obtain

$$K = \left(\frac{B}{w - tp\psi} \right) \phi = \left(\frac{B}{w - tp(1-R)^{\beta} K^{1-\beta}} \right) R^{\alpha} K^{1-\alpha} \Rightarrow K = \left(\frac{B}{w - tp(1-R)^{\beta} K^{1-\beta}} \right)^{\frac{1}{\alpha}} R.$$

Using (11) yields

$$(1-R) = \frac{K\widetilde{p}^{\frac{1}{1-\beta}}}{\delta^{\frac{1}{1-\beta}} + K\widetilde{p}^{\frac{1}{1-\beta}}} \Rightarrow (1-R)^{\beta} K^{1-\beta} = \frac{K^{\beta} \widetilde{p}^{\frac{\beta}{1-\beta}} K^{1-\beta}}{\left(\delta^{\frac{1}{1-\beta}} + K\widetilde{p}^{\frac{1}{1-\beta}} \right)^{\beta}} = \frac{K\widetilde{p}^{\frac{\beta}{1-\beta}}}{\left(\delta^{\frac{1}{1-\beta}} + K\widetilde{p}^{\frac{1}{1-\beta}} \right)^{\beta}}.$$

Now, $tp = \frac{t}{1-t} \widetilde{p}$, and using (12) we can write the denominator of the expression for K as

$$\widetilde{w} - \frac{\delta^{\frac{1}{1-\beta}} + K\widetilde{p}^{\frac{1}{1-\beta}}}{\left(\delta^{\frac{1}{1-\beta}} + K\widetilde{p}^{\frac{1}{1-\beta}} \right)^{\beta}} - \frac{\frac{t}{1-t} K\widetilde{p}^{1+\frac{\beta}{1-\beta}}}{\left(\delta^{\frac{1}{1-\beta}} + K\widetilde{p}^{\frac{1}{1-\beta}} \right)^{\beta}} = \widetilde{w} - \frac{\delta^{\frac{1}{1-\beta}} + \frac{K\widetilde{p}^{\frac{1}{1-\beta}} - tK\widetilde{p}^{\frac{1}{1-\beta}} + tK\widetilde{p}^{\frac{1}{1-\beta}}}{1-t}}{\left(\delta^{\frac{1}{1-\beta}} + K\widetilde{p}^{\frac{1}{1-\beta}} \right)^{\beta}}$$

$$= \widetilde{w} - \frac{\delta^{\frac{1}{1-\beta}} + \frac{K\widetilde{p}^{\frac{1}{1-\beta}}}{1-t}}{\left(\delta^{\frac{1}{1-\beta}} + K\widetilde{p}^{\frac{1}{1-\beta}} \right)^{\beta}}.$$

Putting all this together with the expression for the equilibrium R yields equation (13).

Next notice that we can use the definition of ξ to write

$$\delta^{\frac{1}{1-\beta}} + \sigma[\widetilde{w}(1-t)]^{\frac{1}{1-\beta}} = \left[\widetilde{w}^{\frac{1}{1-\beta}}\left(\left(\frac{\delta}{\widetilde{w}}\right)^{\frac{1}{1-\beta}} + \sigma(1-t)^{\frac{1}{1-\beta}}\right)\right] = \widetilde{w}^{\frac{1}{1-\beta}}\left(\xi + \sigma(1-t)^{\frac{1}{1-\beta}}\right).$$

Using this allows us to re-write (15) as

$$K^e = \frac{B^{\frac{1}{\alpha}}}{\left[\widetilde{w} - \dfrac{\widetilde{w}^{\frac{1}{1-\beta}}\left(\xi + \sigma(1-t)^{\frac{1}{1-\beta}}\right)}{\widetilde{w}^{\frac{\beta}{1-\beta}}\left(\xi + \sigma(1-t)^{\frac{1}{1-\beta}}\right)^{\beta}}\right]^{\frac{1}{\alpha}}} \frac{\delta^{\frac{1}{1-\beta}}}{\widetilde{w}^{\frac{1}{1-\beta}}\left(\xi + \sigma(1-t)^{\frac{1}{1-\beta}}\right)}$$

$$= \frac{B^{\frac{1}{\alpha}}\delta^{\frac{1}{1-\beta}}}{\widetilde{w}^{1+\frac{1}{1-\beta}}} \Bigg/ \left[1 - \frac{\left(\xi + \sigma(1-t)^{\frac{\beta}{1-\beta}}\right)}{\left(\xi + \sigma(1-t)^{\frac{1}{1-\beta}}\right)^{\beta}}\right]^{\frac{1}{\alpha}} \left[\xi + \sigma(1-t)^{\frac{1}{1-\beta}}\right] = \frac{M(B,\delta,\widetilde{w})}{J(t;\alpha,\beta,\sigma,\xi)}.$$

which is equation (16) in the text.

Table A1: Tax-Wage Ratio (α=0.5; β= 0.6)

		ξ										
		0.01	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
σ	0.01	.970	.944	.927	.906	.891	.879	.869	.861	.853	.846	.840
	0.05	.971	.945	.929	.908	.893	.882	.873	.864	.856	.849	.843
	0.1	.972	.947	.932	.911	.897	.886	.876	.868	.860	.854	-
	0.2	.974	.952	.937	.918	.904	.893	.884	.876	.869	-	-
	0.3	.977	.956	.942	.924	.910	.901	.892	.884	-	-	-
	0.4	.979	.960	.947	.931	.919	.909	.900	-	-	-	-
	0.5	.982	.965	.953	.938	.926	.917	-	-	-	-	-
	0.6	.984	.969	.959	.945	.934	-	-	-	-	-	-
	0.7	.987	.974	.965	.953	-	-	-	-	-	-	-
	0.8	.990	.979	.972	-	-	-	-	-	-	-	-
	0.9	.993	.986	-	-	-	-	-	-	-	-	-

Table A2: Tax-Wage Ratio (α=0.4; β= 0.6)

		ξ										
		0.01	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
σ	0.01	.927	.870	.835	.793	.766	.744	.727	.712	.699	.688	.677
	0.05	.931	.875	.841	.799	.772	.750	.733	.718	.705	.693	.683
	0.1	.935	.881	.848	.807	.779	.757	.740	.725	.712	.700	-
	0.2	.942	.893	.861	.821	.794	.772	.755	.739	.726	-	-
	0.3	.949	.903	.873	.835	.808	.787	.769	.754	-	-	-
	0.4	.955	.913	.885	.849	.823	.802	.784	-	-	-	-
	0.5	.961	.923	.897	.863	.837	.817	-	-	-	-	-
	0.6	.966	.933	.909	.877	.853	-	-	-	-	-	-
	0.7	.972	.943	.922	.892	-	-	-	-	-	-	-
	0.8	.978	.954	.936	-	-	-	-	-	-	-	-
	0.9	.985	.967	-	-	-	-	-	-	-	-	-

Table A3: Tax-Wage Ratio (α= 0.3; β= 0.7)

						ξ					
		0.01	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	0.01	.757	.654	.604	.553	.522	.500	.488	.469	.457	.447	0
	0.05	.782	.676	.623	.568	.534	.511	.493	.478	.465	.455	0
	0.1	.805	.699	.644	.585	.550	.525	.505	.489	.476	.467	-
	0.2	.838	.737	.681	.618	.579	.551	.529	.511	.498	-	-
	0.3	.862	.768	.712	.648	.606	.576	.553	.533	-	-	-
σ	0.4	.882	.796	.741	.676	.633	.601	.576	-	-	-	-
	0.5	.900	.821	.769	.703	.660	.627	-	-	-	-	-
	0.6	.916	.845	.795	.731	.686	-	-	-	-	-	-
	0.7	.932	.868	.822	.759	-	-	-	-	-	-	-
	0.8	.948	.893	.85	-	-	-	-	-	-	-	-
	0.9	.965	.921	-	-	-	-	-	-	-	-	-

Table A4: Tax-Wage Ratio (α= 0.6; β= 0.4)

						ξ					
		0.01	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	0.01	1.021	1.057	1.090	1.144	1.193	1.238	1.281	1.326	1.372	1.409	1.446
	0.05	1.021	1.057	1.090	1.142	1.190	1.231	1.277	1.318	1.356	1.4	1.424
	0.1	1.020	1.056	1.088	1.140	1.186	1.227	1.271	1.309	1.351	1.394	-
	0.2	1.020	1.054	1.086	1.135	1.178	1.217	1.255	1.292	1.321	-	-
	0.3	1.019	1.054	1.082	1.129	1.168	1.205	1.241	1.27	-	-	-
σ	0.4	1.018	1.051	1.078	1.122	1.158	1.195	1.223	-	-	-	-
	0.5	1.018	1.048	1.073	1.114	1.146	1.178	-	-	-	-	-
	0.6	1.017	1.044	1.069	1.102	1.132	-	-	-	-	-	-
	0.7	1.015	1.040	1.061	1.088	-	-	-	-	-	-	-
	0.8	1.013	1.034	1.05	-	-	-	-	-	-	-	-
	0.9	1.011	1.023	-	-	-	-	-	-	-	-	-