Supplementary calculations for

International Mergers and Trade Liberalisation: Implications for Unionised Labour

by

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1 Derivation of the optimal wage levels reported in Section 3

Union utility is given by

$$U = w_1 (x_{11} + x_{12} + x_{13}) \tag{1}$$

To capture all possible union strategies, we have to consider three different market structures, with three possible trade regimes within each market structure.

1.1 The decentralised structure (M_0)

Setting $\overline{w} = 0$, the firms solve the following problems in the Cournot game:

$$\max_{x_{11},x_{12},x_{13}} \pi_1 = (p_1 - w_1) x_{11} + (p_2 - w_1 - t_{12}) x_{12} + (p_3 - w_1 - t_{13}) x_{13} \tag{2}$$

$$\max_{x_{21},x_{22},x_{23}} \pi_2 = (p_1 - t_{12}) x_{21} + p_2 x_{22} + (p_3 - t_{23}) x_{23} \tag{3}$$

$$\max_{x_{31},x_{32},x_{33}} \pi_3 = (p_1 - t_{13}) x_{31} + (p_2 - t_{23}) x_{32} + p_3 x_{33} \tag{4}$$

where

$$p_1 = 1 - x_{11} - x_{21} - x_{31} \tag{5}$$

$$p_2 = 1 - x_{12} - x_{22} - x_{32} \tag{6}$$

$$p_3 = 1 - x_{13} - x_{23} - x_{33} \tag{7}$$

The first-order conditions are given by

$$x_{11} = \frac{1}{2} (1 - x_{21} - x_{31} - w_1) \tag{8}$$

$$x_{12} = \frac{1}{2} (1 - x_{22} - x_{32} - w_1 - t_{12}) \tag{9}$$

$$x_{13} = \frac{1}{2} (1 - x_{23} - x_{33} - w_1 - t_{13}) \tag{10}$$

$$x_{21} = \frac{1}{2}(1 - x_{11} - x_{31} - t_{12}) \quad (11)$$

$$x_{22} = \frac{1}{2}(1 - x_{12} - x_{32}) \quad (12)$$

$$x_{23} = \frac{1}{2}(1 - x_{13} - x_{33} - t_{23}) \quad (13)$$

$$x_{31} = \frac{1}{2}(1 - x_{11} - x_{21} - t_{13}) \quad (14)$$

$$x_{32} = \frac{1}{2}(1 - x_{12} - x_{22} - t_{23}) \quad (15)$$

$$x_{33} = \frac{1}{2}(1 - x_{13} - x_{23}) \quad (16)$$

1.1.1 Regime A

This regime is characterised by intra-industry trade between all markets. Simultaneously solving the system of first-order conditions, (8)-(16), yields the following equilibrium values for output:

$$x_{11} = \frac{1}{4}(1 - 3w_1 + t_{13} + t_{12}) \quad (17)$$

$$x_{12} = \frac{1}{4}(1 - 3w_1 + t_{23} - 3t_{12}) \quad (18)$$

$$x_{13} = \frac{1}{4}(1 - 3w_1 + t_{23} - 3t_{13}) \quad (19)$$

$$x_{21} = \frac{1}{4}(1 + w_1 + t_{13} - 3t_{12}) \quad (20)$$

$$x_{22} = \frac{1}{4}(1 + w_1 + t_{12} + t_{23}) \quad (21)$$

$$x_{23} = \frac{1}{4}(1 + w_1 + t_{13} - 3t_{23}) \quad (22)$$

$$x_{31} = \frac{1}{4}(1 + w_1 + t_{12} - 3t_{13}) \quad (23)$$

$$x_{32} = \frac{1}{4}(1 + w_1 + t_{12} - 3t_{23}) \quad (24)$$

$$x_{33} = \frac{1}{4}(1 + w_1 + t_{23} + t_{13}) \quad (25)$$

The optimal wage in Regime A is found by inserting (17), (18) and (19) into (1), and maximising with respect to w_1, which yields

$$w_1^A = \frac{1}{6} + \frac{1}{9}(t_{23} - t_{13} - t_{12}) \quad (26)$$

1.1.2 Regime B

Assume that $t_{12} < t_{13}$. It may then be optimal for the union to set a wage sufficiently high to prohibit exports to country 3. Solving (8)-(16) with $x_{13} = 0$ yields

$$x_{11} = \frac{1}{4}(1 - 3w_1 + t_{12} + t_{13}) \tag{27}$$

$$x_{12} = \frac{1}{4}(1 - 3w_1 - 3t_{12} + t_{23}) \tag{28}$$

$$x_{21} = \frac{1}{4}(1 + w_1 + t_{13} - 3t_{12}) \tag{29}$$

$$x_{22} = \frac{1}{4}(1 + w_1 + t_{12} + t_{23}) \tag{30}$$

$$x_{23} = \frac{1}{3}(1 - 2t_{23}) \tag{31}$$

$$x_{31} = \frac{1}{4}(1 + w_1 + t_{12} - 3t_{13}) \tag{32}$$

$$x_{32} = \frac{1}{4}(1 + w_1 + t_{12} - 3t_{23}) \tag{33}$$

$$x_{33} = \frac{1}{3}(1 + t_{23}) \tag{34}$$

Inserting (27) and (28) into (1), with $x_{13} = 0$, and maximising with respect to w_1, yields the optimal wage in Regime B:

$$w_1^B = \frac{1}{6} + \frac{1}{12}\left(t_{23} + \frac{1}{12}t_{13} - 2t_{12}\right) \tag{35}$$

1.1.3 Regime C

In this regime the unionised firm serves its home market only. Solving (8)-(16) with $x_{12} = x_{13} = 0$ yields

$$x_{11} = \frac{1}{4}(1 - 3w_1 + t_{12} + t_{13}) \tag{36}$$

$$x_{21} = \frac{1}{4}(1 + w_1 + t_{13} - 3t_{12}) \tag{37}$$

$$x_{22} = \frac{1}{3}(1 + t_{23}) \tag{38}$$

$$x_{23} = \frac{1}{3}(1 - 2t_{23}) \tag{39}$$

$$x_{31} = \frac{1}{4}(1 + w_1 + t_{12} - 3t_{13}) \tag{40}$$

$$x_{32} = \frac{1}{3}(1 - 2t_{23}) \tag{41}$$

$$x_{33} = \frac{1}{3}(1 + t_{23}) \tag{42}$$

Inserting (36) into (1) and maximising with respect to w_1 yields the optimal wage in Regime C:

$$w_1^C = \frac{1}{6}(1 + t_{12} + t_{13}) \tag{43}$$

1.2 A merger between the non-unionised firms (M_{2+3})

Assume, without loss of generality, that $t_{12} < t_{13}$. In this case the merged firm will serve market 1 from production at the plant located in country 2. Denoting the merged entity by subscript m, the profit functions are now given by

$$\pi_1 = (p_1 - w_1)\,x_{11} + (p_3 - w_1 - t_{12})\,x_{12} + (p_3 - w_1 - t_{13})\,x_{13} \tag{44}$$

$$\pi_m = (p_1 - t_{12})\,x_{m1} + p_2 x_{m2} + p_3 x_{m3} \tag{45}$$

where

$$p_1 = 1 - x_{11} - x_{m1} \tag{46}$$

$$p_2 = 1 - x_{12} - x_{m2} \tag{47}$$

$$p_3 = 1 - x_{13} - x_{m3} \tag{48}$$

The first-order conditions for the profit-maximising quantities are given by

$$x_{11} = \frac{1}{2}(1 - x_{m1} - w_1) \tag{49}$$

$$x_{12} = \frac{1}{2}(1 - x_{m2} - w_1 - t_{12}) \tag{50}$$

$$x_{13} = \frac{1}{2}(1 - x_{m3} - w_1 - t_{13}) \tag{51}$$

$$x_{m1} = \frac{1}{2}(1 - x_{11} - t_{12}) \tag{52}$$

$$x_{m2} = \frac{1}{2}(1 - x_{12}) \tag{53}$$

$$x_{m3} = \frac{1}{2}(1 - x_{13}) \tag{54}$$

In this market structure, the different trade regimes are defined (and denoted) identically to the trade regimes in M_0.

1.2.1 Regime A

Simultaneously solving (49)-(54) yields

$$x_{11} = \frac{1}{3}\left(1 - 2w_1 + t_{12}\right) \tag{55}$$

$$x_{12} = \frac{1}{3}\left(1 - 2w_1 - 2t_{12}\right) \tag{56}$$

$$x_{13} = \frac{1}{3}\left(1 - 2w_1 - 2t_{13}\right) \tag{57}$$

$$x_{m1} = \frac{1}{3}\left(1 + w_1 - 2t_{12}\right) \tag{58}$$

$$x_{m2} = \frac{1}{3}\left(1 + w_1 + t_{12}\right) \tag{59}$$

$$x_{m3} = \frac{1}{3}\left(1 + w_1 + t_{13}\right) \tag{60}$$

Inserting (55), (56) and (57) into (1), and maximising with respect to w_1, yields the optimal wage level in Regime A:

$$w_1^A = \frac{1}{4} - \frac{1}{12}\left(t_{12} + 2t_{13}\right) \tag{61}$$

1.2.2 Regime B

Simultaneously solving (49)-(54) with $x_{13} = 0$ yields

$$x_{11} = \frac{1}{3}\left(1 - 2w_1 + t_{12}\right) \tag{62}$$

$$x_{12} = \frac{1}{3}\left(1 - 2w_1 - 2t_{12}\right) \tag{63}$$

$$x_{m1} = \frac{1}{3}\left(1 + w_1 - 2t_{12}\right) \tag{64}$$

$$x_{m2} = \frac{1}{3}\left(1 + w_1 + t_{12}\right) \tag{65}$$

$$x_{m3} = \frac{1}{2} \tag{66}$$

Inserting (62) and (63) into (1), and maximising with respect to w_1, yields the optimal wage in Regime B:

$$w_1^C = \frac{1}{4} - \frac{1}{8}t_{12} \tag{67}$$

1.2.3 Regime C

Simultaneously solving (49)-(54) with $x_{12} = x_{13} = 0$ yields

$$x_{11} = \frac{1}{3}(1 - 2w_1 + t_{12}) \tag{68}$$

$$x_{m1} = \frac{1}{3}(1 + w_1 - 2t_{12}) \tag{69}$$

$$x_{m2} = x_{m3} = \frac{1}{2} \tag{70}$$

Inserting (68) into (1) and maximising with respect to w_1 yields the optimal wage level in Regime C:

$$w_1^C = \frac{1}{4} + \frac{1}{4}t_{12} \tag{71}$$

1.3 A merger involving the unionised firm

Consider, without loss of generality, a merger between firm 1 and 2. There are now three different trade regimes to consider.

1.3.1 Regime I

In this regime the unionised plant serves market 1 only. The profit functions for the merged firm and the outsider (owner 3), respectively, are

$$\pi_m = (p_1 - w_1)\,x_{m1} + p_2 x_{m2} + (p_3 - t_{23})\,x_{m3} \tag{72}$$

$$\pi_3 = (p_1 - t_{13})\,x_{31} + (p_2 - t_{23})\,x_{32} + p_3 x_{33} \tag{73}$$

where

$$p_1 = 1 - x_{m1} - x_{31} \tag{74}$$

$$p_2 = 1 - x_{m2} - x_{32} \tag{75}$$

$$p_3 = 1 - x_{m3} - x_{33} \tag{76}$$

The first-order conditions for profit-maximising output levels are

$$x_{m1} = \frac{1}{2}(1 - x_{31} - w_1) \tag{77}$$

$$x_{m2} = \frac{1}{2}(1 - x_{32}) \tag{78}$$

$$x_{m3} = \frac{1}{2}(1 - x_{33} - t_{23}) \tag{79}$$

$$x_{31} = \frac{1}{2}(1 - x_{m1} - t_{13}) \tag{80}$$

$$x_{32} = \frac{1}{2}(1 - x_{m2} - t_{23}) \tag{81}$$

$$x_{33} = \frac{1}{2}\left(1 - x_{m3}\right) \tag{82}$$

By simultaneously solving (77)-(82) we obtain

$$x_{m1} = \frac{1}{3}\left(1 - 2w_1 + t_{13}\right) \tag{83}$$

$$x_{m2} = \frac{1}{3}\left(1 + t_{23}\right) \tag{84}$$

$$x_{m3} = \frac{1}{3}\left(1 - 2t_{23}\right) \tag{85}$$

$$x_{31} = \frac{1}{3}\left(1 + w_1 - 2t_{13}\right) \tag{86}$$

$$x_{32} = \frac{1}{3}\left(1 - 2t_{23}\right) \tag{87}$$

$$x_{33} = \frac{1}{3}\left(1 + t_{23}\right) \tag{88}$$

The optimal *unconstrained* wage is given by

$$w_1^I = \arg\max\left\{U = w_1 x_{m1}\right\} \tag{89}$$

By using (83), we obtain

$$w_1^I = \frac{1}{4}\left(1 + t_{13}\right) \tag{90}$$

The optimal wage is constrained by $w_1^I \leq t_{12}$.

1.3.2 Regime II

This trade regime is characterised by the merged firm using the unionised plant to serve markets 1 and 3, which implies $t_{13} < t_{23}$. In this case the merged firm solves

$$\max_{x_{m1},x_{m2},x_{m3}} \pi_m = \left(p_1 - w_1\right) x_{m1} + p_2 x_{m2} + \left(p_3 - w_1 - t_{13}\right) x_{m3} \tag{91}$$

where prices are given by (74)-(76). First-order conditions for optimal quantities set by the merged firm are given by

$$x_{m1} = \frac{1}{2}\left(1 - x_{31} - w_1\right) \tag{92}$$

$$x_{m2} = \frac{1}{2}\left(1 - x_{32}\right) \tag{93}$$

$$x_{m3} = \frac{1}{2}\left(1 - x_{33} - w_1 - t_{13}\right) \tag{94}$$

Simultaneously solving (80)-(82) and (92)-(94) yields

$$x_{m1} = \frac{1}{3}\left(1 - 2w_1 + t_{13}\right) \tag{95}$$

$$x_{m2} = \frac{1}{3}(1 + t_{23}) \tag{96}$$

$$x_{m3} = \frac{1}{3}(1 - 2w_1 - 2t_{13}) \tag{97}$$

$$x_{31} = \frac{1}{3}(1 + w_1 - 2t_{13}) \tag{98}$$

$$x_{32} = \frac{1}{3}(1 - 2t_{23}) \tag{99}$$

$$x_{33} = \frac{1}{3}(1 + w_1 + t_{13}) \tag{100}$$

The optimal *unconstrained* wage in Regime II is given by

$$w_1^{II} = \arg\max \{U = w_1 (x_{m1} + x_{m3})\} \tag{101}$$

By using (95) and (97), we obtain

$$w_1^{II} = \frac{1}{4} - \frac{1}{8} t_{13} \tag{102}$$

In order to induce Regime II as an equilibrium outcome in terms of trade flows, two constraints on the union's wage setting are necessary:

$$w_1^{II} \leq t_{12} \tag{103}$$

and

$$w_1^{II} \leq t_{23} - t_{13} \tag{104}$$

1.3.3 Regime III

The final trade regime in this market structure is characterised by the merged firm using the unionised plant for export production to market 3 only. In this case the merged firm solves

$$\max_{x_{m1}, x_{m2}, x_{m3}} \pi_m = (p_1 - t_{12}) x_{m1} + p_2 x_{m2} + (p_3 - w_1 - t_{13}) x_{m3} \tag{105}$$

where the prices are given by (74)-(76). The first-order conditions for (105) are given by

$$x_{m1} = \frac{1}{2}(1 - x_{31} - t_{12}) \tag{106}$$

$$x_{m2} = \frac{1}{2}(1 - x_{32}) \tag{107}$$

$$x_{m3} = \frac{1}{2}(1 - x_{33} - w_1 - t_{13}) \tag{108}$$

Simultaneously solving (80)-(82) and (106)-(108) yields

$$x_{m1} = \frac{1}{3}(1 - 2t_{12} + t_{13}) \tag{109}$$

$$x_{m2} = \frac{1}{3}(1 + t_{23}) \tag{110}$$

$$x_{m3} = \frac{1}{3}(1 - 2w_1 - 2t_{13}) \tag{111}$$

$$x_{31} = \frac{1}{3}(1 + t_{12} - 2t_{13}) \tag{112}$$

$$x_{32} = \frac{1}{3}(1 - 2t_{23}) \tag{113}$$

$$x_{33} = \frac{1}{3}(1 + w_1 + t_{13}) \tag{114}$$

The optimal unconstrained wage level in Regime III is given by

$$w_1^{III} = \arg\max \{U = w_1 x_{m3}\} \tag{115}$$

By using (111), we obtain

$$w_1^{III} = \frac{1}{4} - \frac{1}{2}t_{13} \tag{116}$$

The relevant constraint in this regime is

$$w_1^{III} \leq t_{23} - t_{13} \tag{117}$$

2 Symmetric trade costs

If $t_{12} = t_{13} = t_{23}$ we see that the constraints (104) and (117) imply a wage below the reservation level in Regimes II and III. Consequently, Regime I is the only relevant trade regime in M_{1+2} (and M_{1+3}) when the trade cost structure is symmetric. Furthermore, since $t_{12} = t_{13}$, there is no wage level for which Regime B could emerge as an equilibrium outcome in M_0 or M_{2+3}.

The equilibrium values of profits and union utility reported in Appendix A.2 in the paper are easily calculated by inserting the equilibrium values of wages and output into the relevant profit and utility functions, and imposing symmetry with respect to trade costs.