Supplementary material for the paper

**Price and non-price restraints**

*when retailers are vertically differentiated*

by

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This note contains the proof of Proposition 3 and presents the numerical solutions of the model in the imperfect customer restrictions case that are reported in Section 5.2.

**Proof of Proposition 3:** Given a uniform wholesale price, \( w \), and since \( \theta \) is distributed uniformly on the interval \([0, \bar{\theta}]\), the retailers’ profits are:

\[
\pi_H(w) = \left(1 - \frac{\theta_H}{\bar{\theta}}\right)(p_H - c_H - w),
\]  

(B-1)

and

\[
\pi_L(w) = \begin{cases} 
\left(\frac{\theta_H - \theta_L}{\bar{\theta}}\right)(p_L - c_L - w), & p_H > p_L/\gamma, \\
0, & p_H \leq p_L/\gamma.
\end{cases}
\]  

(B-2)

Given \( w \), the two retailers simultaneously choose \( p_H \) and \( p_L \) to maximize their respective profits. Let the Nash equilibrium prices be \( p_H(w) \) and \( p_L(w) \).

Now, suppose that both retailers operate in the market, i.e., \( p_H(w) > p_L(w)/\gamma \). Then, using equation (3), \( p_H(w) \) and \( p_L(w) \) are defined by the following best-response functions:

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To facilitate the analysis, we will characterize the Nash equilibrium in terms of \( \theta_H \) and \( \theta_L \) that are induced by \( p_H \) and \( p_L \) rather than directly by \( p_H \) and \( p_L \). Equation (3) indicates that whenever \( p_H > p_L / \gamma \), then \( p_H = \gamma \theta_L +(1-\gamma) \theta_H \) and \( p_L = \theta_L \gamma \). Substituting these expressions in (B-3) and (B-4) and solving, yields

\[
\theta_H(w) = \frac{(2-\gamma)c_H - c_L + (1-\gamma)(2-\gamma)\bar{\theta} + (1-\gamma)w}{(1-\gamma)(4-\gamma)},
\]

\[
\theta_L(w) = \frac{\gamma c_H + 2c_L + \gamma (1-\gamma)\bar{\theta} + (2+\gamma)w}{\gamma(4-\gamma)}.
\]

Equation (2) implies that both \( Q_H \) and \( Q_L \) are positive (i.e., both retailers are active) only if \( \theta_H(w) > \theta_L(w) \). From (B-5) it follows that \( \theta_H(w) > \theta_L(w) \) only if

\[
w < w^{**} = \frac{\gamma (1-\gamma)\bar{\theta} + \gamma c_H - (2-\gamma)c_L}{2(1-\gamma)}.
\]

M sets the wholesale price, \( w \), to maximize his revenue from wholesale:

\[
\pi(w) = (Q_H(w) + Q_L(w))w,
\]

where \( Q_H(w) \) and \( Q_L(w) \) are given by (2), evaluated at \( \theta_H(w) \) and \( \theta_L(w) \). Substituting from (B-5) into (B-7) and rearranging terms, M’s profit is
Differentiating this expression and evaluating the derivative at \( w = w^{**} \), we obtain:

\[
\pi'(w^{**}) = \frac{(2 + 2\gamma - \gamma^2) c_L - 3\gamma c_H + \gamma (1 - \gamma)^2 \bar{\theta}}{\gamma (4 - \gamma)(1 - \gamma)\bar{\theta}} > \frac{(2 + 2\gamma - \gamma^2) \gamma c_H - 3\gamma c_H + \gamma (1 - \gamma)^2 \bar{\theta}}{\gamma (4 - \gamma)(1 - \gamma)\bar{\theta}}
\]

\[
= \frac{(1 - \gamma)(\bar{\theta} - c_H)}{(4 - \gamma)\bar{\theta}} > 0,
\]

where the first inequality follows because by assumption, \( c_L > \gamma c_H \), and the last inequality follows because by assumption \( \bar{\theta} > c_H \). Noting from (B-8) that \( \pi(w) \) is strictly concave, it follows that it is never optimal to set \( w \leq w^{**} \), so in equilibrium L is effectively foreclosed.

When \( H \) is the sole provider of \( M \)'s product, its profit is given by (B-1) with \( p_H = \theta_H \). The optimal choice of \( H \) is given by

\[
\theta_H^*(w) = \frac{\bar{\theta} + c_H + w}{2}.
\]

Since \( M \) deals only with \( H \), \( M \)'s profit is

\[
\pi(w) = \left(1 - \frac{\theta_H^*(w)}{\bar{\theta}} \right)w = \frac{\left(\bar{\theta} - c_H - w\right)w}{2\bar{\theta}},
\]

This expression is maximized at \( w^* = (\bar{\theta} - c_H)/2 \). The assumptions that \( \gamma < 1 \) and \( c_H < c_L/\gamma \), ensure that \( w^* > w^{**} \). Given \( w^* \), the lowest type that is served is \( \theta_L = \theta_H^*(w^*) = (3\bar{\theta} + c_H)/4 \). Since \( F(\theta) \) is uniform on the interval \([0, \bar{\theta}]\), Lemma 1 shows that under vertical integration, the lowest type that is served is \( \theta^* = (\bar{\theta} + c_H)/2 \) which is above \((3\bar{\theta} + c_H)/4 \) since \( c_H < \bar{\theta} \). Hence \( M \) sells less than in vertical integration case.
Numerical solution of the model under imperfect customer restrictions:
The numerical solution is based on the following assumptions:

(i) The distribution of $\theta$ is uniform on the interval $[0,1]$
(ii) The distribution of $\tilde{\varepsilon}$ is uniform on the interval $[-\varepsilon,\varepsilon]$

Figure 3 presents $M$'s profit, $\pi(z_{CR})$, for $c_L = 1/8$, $c_H = 1/4$, $\gamma = 3/8$, and 4 different values of $\varepsilon$: 0, 0.05, 0.1, and 0.15.\(^1\) When $\varepsilon = 0$, we are back in the perfect CR case, so $z_{CR}^* = \theta_{CR}$. The horizontal line, marked $\pi_H^*$, represents $M$'s profit when $H$ is an exclusive distributor. The figure shows that although $\pi(z_{CR}^*)$ is always above $\pi_H^*$, $\pi(z_{CR}^*)$ decreases with $\varepsilon$. Moreover, $z_{CR}^*$ also decreases with $\varepsilon$. Hence, as the signal $z$ becomes less informative about $\theta$, CR become less profitable and $M$ gives $H$ a larger segment of the market. Consequently, Figure 4 shows that as $\varepsilon$ increases, $H$'s sales increase and $L$'s sales and $M$'s aggregate sales decrease. The figure also compares the sales of $H$, $L$, and $M$ with their corresponding sales when $M$ deals exclusively with $H$ (this case is denoted by *). The figure shows that even when CR are imperfect, $H$ sells less, $L$ sells more, and $M$ sells more than they do in the case where $H$ is an exclusive distributor.

The effect of imperfect CR on consumers and on welfare is shown in Figures 5 and 6. These figures present consumers’ surplus and social welfare as functions of $\varepsilon$ for $c_L = 1/8$, $c_H = 1/4$, $\gamma = 3/8$ and were obtained by raising $\theta$ from 0 to 0.15 in steps of 0.0001 and solving the model numerically each time. The perfect CR case corresponds to $\varepsilon = 0$. In each figure we also show consumers’ surplus and welfare when $M$ deals exclusively with $H$ (again, this case is denoted by *). Figure 5 shows that under imperfect CR, consumers are better off than under perfect CR although they are worse off than in the case where $L$ is foreclosed. Moreover, consumers’ surplus is increasing with $\varepsilon$, so although $M$’s aggregate sales fall with $\varepsilon$, the fact that more consumers are served by $H$ who provides more customer services than $L$, imply that overall consumers become better-off. Figure 6 shows that relative to the case where $H$ is an exclusive distributor, CR is welfare enhancing when $\varepsilon$ is small but welfare decreasing otherwise. This

\(^1\) Qualitatively, the picture does not change when we use other values of $c_L$, $c_H$, $\gamma$, and $\varepsilon$, such that $c_L < c_H < c_L/\gamma < 1$ and $c_L/\gamma < \theta^*$.\[\]
supports the conclusion from Proposition 7 that CR may or may not be socially desirable and therefore should be considered under the rule of reason.
Figure 3: The manufacturer's profit under imperfect CR, as a function of $z_{CR}$

(c_L = 1/8, c_H = ¼, $\gamma$ = 3/8)

Figure 4: $Q_L$, $Q_H$, and the total quantity, $Q$, as functions of $\varepsilon$
Figure 5: Consumers’ surplus as a function of $\varepsilon$

![Figure 5: Consumers’ surplus as a function of $\varepsilon$](image)

Figure 6: Social welfare as a function of $\varepsilon$

![Figure 6: Social welfare as a function of $\varepsilon$](image)