Vertical Foreclosure versus Downstream Competition with Capital Precommitment – Appendix

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In our *IJIO* paper, we claim that $K^*(n) < K^m$ holds under mild regularity conditions. That is, the monopolist underinvests not only relative to the efficient capital input but also relative to the capital level chosen in the integrated monopoly solution. Here is the proof.

Assumptions. We additionally assume that $\widetilde{p}(X)$ and $X = F(Z, K)$ are three times continuously differentiable, and that long run marginal costs are not decreasing too quickly in X, i.e., $dc(X, K^e(X))/\,dX > dp(X)X/\,dX$.

Proposition. *If $c_{XXK} \leq 0$ and $c_{XXX} \leq c_{XX}/\,X$ and $p_{XXX} \geq p_{XX}/\,X$ holds, the level of capital is lower than under vertical integration, i.e., $K^*(n) < K^m$ for all $n \geq 2$.*

Proof. Define $X^e(K) := K^{e^{-1}}(K)$ and $c_K(X^e(K), K) = 0$. Then, equation

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(9) can be written as

$$\int_{X^e(K)}^{X^m(K)} c_{XK}(X,K)dx = \frac{\partial X^*(K,n)}{\partial K}\left[p_X(X)X + p - c_X\right]\Big|_{X=X^*(K,n)} - \int_{X^m(K)}^{X^*(K,n)} c_{XK}(X,K)dX. \tag{1}$$

Hence, $c_{XK} < 0$ implies $X^e(K^*) < X^m(K^*)$ iff the RHS of (1) is negative. Differentiating (7) and using symmetry we get (with n as a continuous variable)

$$\frac{\partial X^*(K,n)}{\partial n} = \frac{p_X X^*}{n(p_{XX}X^* - nc_{XX} + (n+1)p_X)} > 0, \tag{2}$$

where $\partial X^*/\partial n > 0$ follows from the second order conditions for x^* and from $p_X < 0$. Using (2) and (7), the RHS of (1) is negative iff

$$c_{XK}(X^*,K)n(n-1)\frac{\partial X^*(K,n)}{\partial n} < \int_{X^m(K)}^{X^*(K,n)} c_{XK}(X,K)dX. \tag{3}$$

With $c_{XXK} \leq 0$ the lower bound of the RHS of (3) is given by

$$c_{XK}(X^*,K)[X^*(K,n) - X^m(K)]. \tag{4}$$

Therefore, (3) is satisfied if

$$n(n-1)\frac{\partial X^*(K,n)}{\partial n} > X^*(K,n) - X^m(K). \tag{5}$$

for all $n > 1$. Since $n = 1$ implies equality in (5), a sufficient condition for (5) is

$$\frac{\partial}{\partial n}\left[n(n-1)\frac{\partial X^*(K,n)}{\partial n}\right] - \frac{\partial X^*(K,n)}{\partial n} > 0. \tag{6}$$

Evaluating $\partial^2 X^*(K,n)/\partial n^2$ shows that (6) is equivalent to

$$n\frac{\partial X^*(K,n)}{\partial n}\left[nc_{XXX} - p_{XXX}X^*(n,K) - p_{XX}(n+2)\right] + 2p_{XX}X^*(n,K) + 2p_X < 0. \tag{7}$$

Substituting (2) simple algebra shows that $c_{XXK} \leq 0$, $c_{XXX} \leq c_{XX}/X$ and $p_{XXX} \geq p_{XX}/X$imply that (7) holds. ■