Analyzing competition among sellers in a B2B exchange*

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Abstract

Business-to-business exchanges are expected to bring several advantages to the participants of the exchange. One of the principal advantages of such exchanges lies in their promise of lowering prices for buyers due to increased competition among the sellers for the buyers’ business. This paper models the competition between similar sellers vying for the same business by bidding on price. It captures some of the unique features of such a competition that has been made possible by the existence of an online marketplace. We consider the case where the capacities of the sellers are such that they individually cannot cater to the entire demand, but their cumulative capacity is more than the required demand. This type of price competition is distinct from the standard Bertrand price competition model, where sellers compete on prices and end up supplying at their marginal cost. It is also distinct from a capacity constrained competition that has been extensively analyzed in existing literature. We also take into account the nature of the demand for the commodities in such marketplaces, which is distinct from consumer markets. We find that there exists a mixed strategy equilibrium in prices among the sellers, and specify the nature of this equilibrium. The model is then extended to an $n$-seller case, and the results are seen to be consistent with the expectations of lower average prices with increasing $n$. A generalized model with only $m$ of the $n$ bidding sellers being required to cater to the demand is then considered. The results show that it is crucial for the sellers to be informed about the total quantity that has already been bid for before deciding whether or not to participate in the auction, and the availability of this information is crucial for the “fairness” of the exchange. In the final part of the paper, we analyze the effect of cost variations between sellers (effectively modeling competitors facing same cost distributions rather than same costs), and show that the nature of the equilibrium is different from that with fixed costs. In the process, we come up with an alternate explanation of price randomization that is often seen in such marketplaces.
Introduction

“...the new entity [Covisint, the online B2B exchange formed by the Big Three automakers] would command $240 billion in purchasing power through 90,000 member companies worldwide, generate an estimated $3 billion a year in short-term revenue and...eventually lop off $3,000 from the production cost of your average 21st century car.”

- “Some Assembly Required”, Business 2.0, February 2001

While the media hype around the business-to-business (or B2B) exchanges has died down considerably in recent times, industry observers and analysts continue to feel that they will deliver a sizeable portion of their promises within a few years. For example, Roland Berger, one of the principal consultants in the automotive industry, recently concluded that while a cost savings in the excess of $3,000 might not be possible in the foreseeable future, significant cost reductions are certainly in the cards for the North American automobile industry:

“Our estimates of roughly $1,150-$1,200 in savings per vehicle for the North American industry (after five years of B2B implementation throughout the value chain) compares with estimates of several thousand dollars per vehicle in many other industry forecasts... (this is not to say that they are not meaningful – automakers celebrate when they are bale to pull $10 out of the cost of a vehicle, much less $1,000).”

[Emphasis added]

- Roland Berger Automotive e-Commerce report with Deutsche Bank, June 2000

Estimates of the size of the market are still very large. Even after several downward revisions of forecast estimates, the online research and consulting firm Jupiter Media Metrix predicts that $5.4 trillion in goods and services transacted online among businesses by 2006. Forrester Research indicates that in Q3 2001, 49% of organizations that buy more than $1 billion per year reported using an online auction, with most of them increasing their usage of these venues. Signaling these trends, FreeMarkets, a leading e-sourcing software and services provider, recently updated its Q4 2001 revenue estimates from earlier analyst estimates in a market that has otherwise been witness to profit and revenue warnings [Sanders, Hurd and Temkin, 2001]. In the first six months of 2001, the founders of Covisint had channeled $33 billion, or 13% of their annual purchase transactions, through the exchange [Helper and MacDuffie, 2001].
The industry literature speaks of several advantages of B2B transactions, like expanding market reach, lowering buyers’ costs, identifying best practices, etc. [Kerrigan, et al, 2001]. Our focus in this paper centers on the potential advantage of lower prices for buyers that might result from the increased competition between sellers. As a pertinent example, Ford announced in July 2001 that it had saved $70 million through Covisint in terms of reduced paperwork and lower supplier prices, which is more than its initial investment in the exchange [Helper and MacDuffie, 2001].¹ The Roland Berger research estimates that about 33% of the cost savings in North American auto industry will come from purchase-related savings.

Suppliers however have been increasingly discomfited with the process of reverse auction in these exchanges. OEM buyers might argue that the price competition induces efficient competition that was lacking before, but the suppliers fear that this competition might benefit only the buyers, resulting in a disproportionate sharing of the surplus [Jap, 2001].

[The buyer] talks about the relationship being a partnership and this [the auction] really takes that away. There is not a partnership there at all. What they do is take your existing business that you have worked very hard to achieve and maintain. You work with them to give them cost reductions over the years and they send it out across the board for a competitive bid. I just do not think that is fair.

- Supplier comment [Jap, 2001]

As Dai and Kauffman [2000] have noted, sellers have reduced information about buyers in open exchanges, which might make the effects of the competition stacked heavily against the sellers. However, given the monopsony power of a limited number of large buyers, sellers have limited options. A recent Forrester Research survey found that 57% of auto suppliers polled said they respond to online RFQs currently, and 70% said they will do so by 2003 [Sanders, Hurd and Temkin, 2001].

We analyze the case of competition between similar suppliers (we use the terms suppliers and sellers interchangeably throughout the paper) of a single raw material or component

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¹ It has been estimated that in terms of reduction of paperwork alone, B2B exchanges can bring down costs per purchase order from $75-$150 to $10-$30 [Helper and MacDuffie, 2001].
vying for business from a handful of buyers (or a single large buyer) within a B2B exchange framework. We concentrate on the mechanism of a one-sided, sealed bid reverse auction. We assume that there is no combined capacity constraint as such: the sellers were supplying to the entire demand before the birth of the exchange, and continue to do so after it comes into play. However, it is conceivable that the firms individually cannot supply to the entire market (in fact, we show that it is to their benefit not to have too much capacity, when there is – as per the assumptions of our model – industry overcapacity in the first place). This means that while there is a competition between the firms to be the low-price bidder, it is not as extreme as a Bertrand game that results in prices equal to marginal cost. However, there remains an incentive to be the low-price bidder and have the “first invitation” to supply a requirement.

There are several objectives of the paper. We explore the nature of the pricing strategy that the suppliers employ in such dynamic environments in a game-theoretic framework. A related issue is to consider the conditions under which this competition would be debilitating for the suppliers, as many of them fear it would be. We also quantify the cost savings that is induced by the competition between the sellers (remembering that these are theoretical values, the actual savings would be lower due to transaction costs).

Prior experience with bidding in industrial markets suggests that managers resort to price randomizations, and indeed our analysis shows that there exists a support of prices where the sellers randomize their bids. The exact nature of the equilibrium (whether it is pure or mixed) depends on the characterization of the cost function of the sellers. From our characterization of equilibrium, we obtain an explicit expression for the average price paid by the buyers, and see that for some “typical” values of the parameters within the model, the exchange promotes competition to lower prices significantly even in a two-seller model, with the effects more pronounced with increasing number of sellers. Our results also point to some crucial requirements for the exchange to be seen as promoting “fair” competition, as opposed to extracting the entire surplus from the sellers, which has been one of the main reasons for the reluctance of the sellers’ to join various exchanges. The final part of the paper considers the effect of variable costs among competing sellers,
and comes up with a practical explanation as to why managers might resort to price randomizations while bidding in these environments.

**Evolution of B2B marketplaces**

Sawhney and Kaplan [2000] differentiate between exchanges (they use the term ‘hubs’) that employ a variety of market-making mechanisms:

- A listing or catalog model that creates value by aggregating buyers and sellers.
- An auction model that creates value by spatial matching of buyers and sellers (our paper models one variant of this type of exchange).
- An exchange model that creates value by matching temporary demand and supply.
- A barter model that creates value between two parties that possess reciprocal assets.

B2B marketplaces differ in some significant ways from their predecessors: the interorganizational systems (or IOS), like EDI (Electronic Data Interchange) systems and extranets, that large businesses used to communicate electronically with both their vendors and customers. Given the large cost of using such systems [Waltner, 1997], the use and benefits of IOS was limited to only the largest of organizations and their partners.

While the positive effects of these electronic markets have been well documented (see for example, [Malone, Yates and Benjamin, 1987], [Bakos, 1991], and [Kauffman, McAndrews and Wang, 2000]), there is also the negative externalities effect on the supplier’s adoption of IOS [Riggins, Kriebel and Mukhopadhyay, 1994], and as a result, buyers have often induced suppliers to join EDI systems [Barua and Lee, 1997]. Buyers often give suppliers incentives to make relation-specific investments and not be too concerned about increased competition [Bakos and Brynjolfsson, 1993]. Also, by electronically sharing data, IOS link buyers and suppliers closely, thus encouraging the buyer to develop closer ties with a small number of suppliers [Clemons, Row and Reddi, 1994].
The decreasing cost of hardware and software (a Internet-enabled personal computer and
a web browser is often all that is required to access an exchange) and the extensive use of
open standards in developing the electronic exchanges of today have ensured that the
benefits of B2B marketplaces are experienced by a large number of both buyers and
sellers. However, the relative magnitude of the benefits enjoyed by the buyers and sellers
has been markedly different. Dai and Kauffman [2000] argue that in contrast to closed
extranets, which bring about several advantages to the limited circle of suppliers, today’s
electronic markets are open networks where buyers have increased access to product and
price information, while sellers on the other hand have reduced level of information
sharing about buyers. As an example, suppliers may obtain ordering information, but not
inventory information. Buyers however have everything to gain: lower costs due to
automating the procurement process, reverse auctions, interoperability among users,
collaborative planning and collaborative design [Helper and MacDuffie, 2001]. Thus,
while the benefits of the buyers are readily apparent, sellers have to analyze the costs and
benefits of joining the exchange. (Indeed, as Baumgartner, Kajuter and Van [2001] point
out, sellers have been extremely reluctant to join these exchanges in several instances,
fearing increased competition.) Dai and Kauffman model the environment where there is
one large buyer and two suppliers. The suppliers are similar, realizing the same costs and
benefits. By assigning the various costs and benefits of the electronic network and the
exchange for both the buyer and the sellers, they develop several simple what-if scenarios
that determine whether a supplier prefers an extranet or an electronic network. Our paper
continues to investigate this model, and considers the effects when the two sellers (and
later generalizing the model to n sellers) have decided to participate in the exchange, and
are competing for any business within that framework.

Model environment and assumptions

As Dai and Kauffman [2000] point out, unlike closed extranets, open electronic markets
expose sellers to increased competition. It is this competition that we model in this paper.
In the process, we look at a competitive game that is unique in several respects.
To illustrate this, let us consider a hypothetical example: the Big Three automakers that have joined forces for making the auto exchange Covisint (http://www.covisint.com/) are looking for similar-sized tires for competing models of cars. The number of tire manufacturers are limited, and usually each of the three automakers have their own set of preferred suppliers, since the cost of analyzing quotations from the entire universe of suppliers (dedicated account management teams for buyers, sales force for sellers, cost of sending individual RFPs to the entire universe of sellers, etc.) outweigh the benefits. The demand for tires can be considered inelastic below a certain reservation price (while the demand for the final product, i.e. the car, is certainly elastic, it is inconceivable that the demand for the tires, which constitute a very small fraction of the value of the car, is elastic also. It is more likely that the buyers have a reservation price for such products – set by industry norms or internal costing data, for example – and would buy from their established list of sellers as long as the price is less than this reservation price).\(^2\) To put it in another way, the demand for the end product might go up or down because of its price, but the demand for the raw materials that are used to make the product will not show such a behavior – more likely, the manufacturer of the product would revise its reservation price for the raw material. Note that this feature is a crucial distinction between B2B and B2C (or business to consumer) transactions. Individual consumers buy end products, whose demand does vary with their prices.

Since the set of suppliers are limited, all of whom are reputed in the market, the buyers would not mind getting their orders fulfilled by any one or several of these suppliers. With the advent of Covisint, the buyers put their requirements on the exchange only once, instead of contacting the sellers individually. The sellers look at the entire requirement, and then decide to quote their prices. The lowest priced seller is first invited to supply the

\(^2\) We contacted Roland Berger to confirm this assumption. We found out that auto manufacturers indeed have “target cost” structures for components, which is equivalent to the reservation price in our nomenclature. See also footnote 3.
required quantity, and if there is any residual demand, it passes on to the seller with the second lowest price, and so on till the demand is completely satisfied.\(^3\)

As mentioned before, we consider the mechanism of a one-sided, sealed bid auction mechanism: the buyer post a request for purchase (RFP) and invites specific suppliers to view and respond to the RFP. The suppliers are also given a date (usually several weeks in the future) by which they are expected to place their bids.

From the modeling point of view, it is immaterial whether the sellers respond to an aggregate demand of several buyers or one single demand from a buyer\(^4\) – what is important to note is that the entire requirement is auctioned to the sellers, and for any unfulfilled demand, a lower priced bidder is invited before a higher priced bidder to satisfy it.

It is readily apparent that with unlimited capacity, the sellers respond with a Bertrand competition in prices with the seller or sellers with the lowest marginal cost outbidding the others. This is not to the advantage of the sellers. Kreps and Scheinkman [1983] (and several variants of the original model, such as [Allen, et al 1996])) show that if sellers could limit capacity, then a quantity precommitment and Bertrand competition yield Cournot outcomes that have equilibrium prices above marginal cost.

At the other end of the spectrum, if the total capacity of the sellers is so limited as to be less than the total demand, it is easy to see that the sellers can sell their entire capacities at the buyer’s reservation price.

Our model is different from either of these cases. It is realistic to think of sellers having limited capacities so that any one seller cannot meet market demand. However in our

\(^3\) A confirmation of this mechanism was obtained from IndiaMarkets.com, India’s largest B2B portal. The software used at IndiaMarkets.com is licensed from one of the pioneer B2B portals, Ariba. Figure 3 (in the Appendix) shows an actual RFP at this site, as presented to suppliers. Identities have been concealed for confidentiality reasons.

\(^4\) The assumption in the case of multiple buyers, of course, is that all of them have the same reservation price. If that is not so, we would have to consider the contract between the buyers that specifies the way they would divide a seller’s output between themselves.
setting we add the conditions that the aggregate output of the sellers exceeds total quantity demanded and that a firm sells all it can produce only if it is the low price seller. That is, the lowest priced seller sells to capacity, but a higher priced seller only sells to a residual demand. Sellers therefore are pulled by two opposing “forces” – on one hand, higher prices fetch higher margins, but on the other, higher prices bring about increased chances of being underbid by competition.

We first consider a simplified version of this above scenario. We assume that there are two buyers, B1 and B2, and two sellers, S1 and S2, with B1 buying initially from S1, and B2 buying from S2. The two sellers are similar, with same (constant) marginal costs $c$ and capacity $K$. With the advent of the exchange, each buyer puts its RFP on the exchange, and the two sellers compete for being the low bidder. The low bidder can sell to capacity, but the higher priced bidder cannot. Thus, there is an incentive for a seller not to bid very high, but at the same time, not bid so low as to “leave money on the table”.

**The model**

We start off with the analysis of a two-seller model (i.e. two sellers selling individually to two buyers), the mechanics of which allow us to generalize the model later to $n$ sellers. Before the advent of the exchange, due to the lack of competition, the sellers could afford to sell the required quantities to the buyers at their reservation price, which we assume to be the same for both buyers at $r$. With the advent of the exchange, the buyers put forward their requirements to the exchange, and the sellers can then bid for the total requirement from both buyers.

We consider the case when both sellers have equal capacities $K$ that is more than the respective individual requirements of the buyers, but is less than the total requirement of both buyers $Q$ (i.e. $(2K - Q) > 0$). In such a setting, the lower priced seller is invited first to sell the required quantity, and after he has supplied his total capacity $K$, the other seller
can then sell the residual demand \((Q - K)\). Both sellers have a common fixed marginal cost of production, \(c\).

A summary of all the variables used appears in Table 1 in the Appendix.

**Proposition 1**: There can be no pure strategy in prices.

**Proof**: Let there be a price \(\hat{p}\) such that \((\hat{p} - c)K = (r - c)(Q - K)\). Since \(2K - Q > 0\), \(K > (Q - K)\), which implies that \(\hat{p} < r\). At these prices, both sellers make the same profits. The low priced seller has a lower margin, but sells more; the high price seller meets residual demand at a price equal to the buyers’ reservation value. In this setting, the higher priced seller is content to sell to the residual demand at the reservation price when the lower priced seller sets his own price at \(\hat{p}\). However, when one seller sets its price at \(r\), the best response for the other seller is to price his product at \((r - \varepsilon)\) which prompts the first seller to undercut the second by an infinitesimal amount and so on, resulting in the prices spiraling down till one of the sellers reaches the price \(\hat{p}\), at which point the other seller responds with a price of \(r\), and the cycle starts once again. Thus, there can be no pure strategy equilibrium in prices (and there is no incentive for any seller to undercut below the price \(\hat{p}\)). (In fact, the existence of mixed strategy equilibrium in a game with discontinuous payoffs has been proved by [Dasgupta and Maskin, 1986] – our paper finds out the specific nature of the solution under the proposed set of demand and cost assumptions)

In a mixed strategy equilibrium, each seller chooses a price according to a probability density function \(f(p)\). \(f(p)\) thus effectively defines the strategy of the sellers: when the requirements are put forth by the exchange, the sellers respond with a price \(p\) according to its density function \(f(p)\). In its choice of pricing strategy, each seller takes the other seller’s pricing strategy and the demand behavior of the buyers as given. If its price turns out to be the lower of the two prices, it sells to capacity \(K\) and has profits \(\Pi, (p)\). The other seller then sells the residual demand \((Q-K)\) and has profits \(\Pi, (p)\). We analyze the
case of a symmetric equilibrium, when both sellers choose the same pricing strategy (which is reasonable, since both the sellers are similar).

**Proposition 2**: $f(p) = 0$ for $p > r$ or $p \leq c$.

**Proof**: Above the reservation price, nothing will be bought. At a price equal to marginal cost, it is better to be undersold and sell to the residual demand and make positive profits. €

**Proposition 3**: There is no equilibrium where both sellers charge the same price.

**Proof**: Similar to that of Proposition 1. The best response of each seller is to undercut the other by $\varepsilon$, so charging the same price cannot be a Nash equilibrium. €

We can therefore concentrate on establishing the nature of a price randomizing solution.

**Proposition 4**: There are no point masses in the equilibrium pricing strategies.

**Proof**: If the price $p$ were charged with some positive probability, there would be a positive probability of a tie at $p$. By Proposition 3, that is not possible. €

Since there are no point masses in the equilibrium density function, we can concentrate on a cumulative distribution function that will be continuous on some range $(p_1, p_2)$, such that $c < p_1 < p_2 \leq r$. Let $F(p)$ be the cumulative distribution function for $f(p)$; from Proposition 4, we can state that $f(p) = F'(p)$ almost everywhere. In other words, $(p_1, p_2)$ represents the range of the support of the mixed strategy in prices that the two players engage in. In this respect, the model has some similarities to [Varian, 1980].

**Proposition 5**: $p_2 = r$.

**Proof**: If a seller is to sell to the residual demand (since at price $p_2$ it will be definitely undersold – there cannot be any ties), it is best to sell it off at the highest possible price that he can get, which is $r$. €
We now proceed to expected profit function of a seller. When a store charges a price \( \hat{p} \), two events are possible. It may be that \( \hat{p} \) is the smallest price charged, an event which we term as a *success*, in which case, the seller sells to capacity with profit \( \Pi_s(\hat{p}) \). This happens when the other store charges a price more than \( \hat{p} \), an event happens that with probability \( 1 - F(\hat{p}) \). On the other hand, the seller might be undersold (we call that event a *failure*), and in that case, the profit it makes is \( \Pi_f(\hat{p}) \), an event that occurs with probability \( F(\hat{p}) \). (Proposition 3 shows that we can leave out the possibility of a tie.)

Hence the expected profit of either seller at any price \( \hat{p} \) is

\[
\Pi_s(\hat{p})(1 - F(\hat{p})) + \Pi_f(\hat{p})F(\hat{p})
\]

where

\[
\Pi_s(\hat{p}) = \hat{p}K - cK = (\hat{p} - c)K
\]

and

\[
\Pi_f(\hat{p}) = \hat{p}(Q - K) - c(Q - K) = (\hat{p} - c)(Q - K)
\]

We next prove that the expected profit at any price is constant. Let the strategy space of seller \( i \ (i = 1, 2) \) be denoted by \( S_i \). Consider a sequence of finite approximations \( S^n_i \) of

\[
\times_i S^n_i
\]

has a mixed strategy equilibrium. The expected profit at any price in the discrete strategy space has to be equal, for otherwise a player would place higher probabilities for those prices that yield higher payoffs. By the law of weak convergence, there is a subsequence of Nash-equilibrium mixed-strategy profiles, which without loss of generality can be taken to the above sequence itself, which converges to the mixed strategy \( f(p) \) on \( S_i \). Since the expected payoff (i.e. the profit) is equal for any price in the discrete mixed-strategy profiles, the same holds true when the sequence converges onto \( f(p) \). Thus, we must have for all prices \( p \)

\[
\Pi_s(p)(1 - F(p)) + \Pi_f(p)F(p) = \Pi
\]
where $\Pi$ is the value of the expected profit at any price.

Re-arranging (4), we get

$$F(p) = \frac{\Pi_s - \Pi}{\Pi_s - \Pi_f}$$  

At $p = r$, the seller in question sells to the residual demand, with (expected) profit

$$\Pi_f(r) = (r - c)(Q - K)$$

By the reasoning above, this should be equal to $\Pi$.

Similarly, at $p = p_1$, the seller sells to capacity, with (expected) profit

$$\Pi_s(p_1) = (p_1 - c)K = \Pi$$, as before.

Using (2), (3) and (5), we get the expression for $F(p)$:

$$F(p) = \frac{(p - c)K - (r - c)(Q - K)}{(p - c)(2K - Q)}$$,

which is of the form

$$F(p) = A - \frac{B}{(p - c)}$$,

where

$$A = \frac{K}{(2K - Q)} \text{ and } B = \frac{(r - c)(Q - K)}{(2K - Q)}$$ are constants.

We must have $F(p_1) = 0$ (and $F(r) = 1$), which gives (from (8) above)

$$p_1 = \frac{(r - c)(Q - K)}{K} + c$$
Note that this expression can also be obtained from the fact that $\Pi_i(p_i) = \Pi_f(r) = \Pi$, as well as from the proof of Proposition 1. We note that since $r > c$ and $Q > K > 0$, $p_i > c$, i.e., the sellers always charge higher than their marginal cost.

Note that at the limit when $2K \to Q$ (i.e., the two sellers can just clear the market), $p_i \to r$, i.e., the density function tends to a point mass at $p = r$. This is expected since the two firms need not compete to quote the lower price, and can afford to price their quantities at the market-clearing price, which is the reservation price $r$.

The following proposition defines the nature of $F(p)$.

**Proposition 6:** There is no interval $(p', p'')$ such that $f(p) = 0$ $\forall p \in (p', p'')$.

**Proof:** If not, let $p' < \hat{p} < p''$. Now $\hat{p}$ succeeds in being the lowest price in exactly the same circumstances that $p'$ succeeds in being the lowest price: when the other price is higher than $p''$. Similarly, $\hat{p}$ fails to be the lowest price when some store charges a price less than $p'$, in which case $p'$ also fails to be the lowest price. But in either case, since $\hat{p} > p'$, charging $\hat{p}$ will result in larger profits than charging $p'$.

Thus, $F(p)$ is strictly increasing in its support.

We now have a complete characterization of the equilibrium price density function: $f(p) > 0$ for all $p$ in $(p_1, r)$ and $f(p) = F'(p)$, where $F(p)$ is given by (8) above. We thus get

$$f(p) = F'(p) = \frac{(r-c)(Q-K)}{(2K-Q)(p-c)^2}$$

From (11) we get
(13) \[ f(p_1) = \frac{(r-c)(Q-K)}{(2K-Q)[(r-c)(Q-K)-cK]} \]

and

(14) \[ f(r) = \frac{(Q-K)}{(2K-Q)(r-c)} \]

Equations (12), (13) and (14) characterize the entire equilibrium price distribution function.

Figure 1 shows that approximate shape of the graph of \( F(p) \):

![Graph of F(p)](image)

Figure 1: Graph of \( F(p) \)

Figure 2 shows the approximate nature of the graph of \( f(p) \):

![Graph of f(p)](image)
Thus, the sellers randomize between prices \((p_1, r)\) with monotonously decreasing probability densities as prices increase.

The reason for this behavior of \(f(p)\) can be understood by looking at the nature of the profit function in Equation (4). We re-write the profit function of the seller as follows:

\[
\Pi = (p - c)K + (1 - F(p))[(p - c)(Q - 2K)]
\]  

(15)

Differentiating both sides of Equation (16) with respect to \(p\) gives us:

\[
(Q - 2K)[- (p - c)f(p) + (1 - F(p))] = 0
\]

or

\[
[- (p - c)f(p) + (1 - F(p))] = 0
\]

(16a)

The first term in Equation (16a) can be interpreted as the effect on profit because of the margin, while the second term is the effect due to the probability of success (i.e., of being the lower-priced seller and sell to capacity) at any price. These two opposing “forces” balance each other at any equilibrium (which is why their sum equals zero at any equilibrium). As \(p\) increases, the profit margin goes up, while the probability of success goes down (as \((1 - F(p))\) goes down). To counter this effect, \(f(p)\) has to decrease with increasing prices.
Another way to describe the nature of \( f(p) \) would be as follows. Differentiating (4) with respect to \( p \) on both sides, we get on re-arranging:

\[
(17) \quad f(p)[\Pi_s(p) - \Pi_f(p)] + F(p)[\Pi_s(p) - \Pi_f(p)]' = K
\]

The left hand side of Equation (17) can be seen to be the derivative of \( F(p)[\Pi_s(p) - \Pi_f(p)] \), which can be termed as the expected “regret” of being the higher priced seller at any price \( p \). Since this expected regret increases linearly with \( p \) (the derivative is a constant), sellers will tend to stack up their equilibrium densities at lower prices, explaining the monotonously decreasing curve of \( f(p) \).

It will be of interest to find out the average price paid by the buyers under the equilibrium price density function. The average price is simply

\[
(18) \quad \bar{p} = \int_{\Pi_s}^{\Pi_f} pf(p)dp
\]

Integrating by substitution gives us

\[
(19) \quad \bar{p} = \frac{(r-c)(Q-K)}{(2K-Q)} \left[ \ln \left( \frac{r-c}{p_1-c} \right) + c \left( \frac{1}{p_1-c} - \frac{1}{r-c} \right) \right]
\]

The expression for \( \bar{p} \) is complex and does not allow for a ready comparison with \( r \), which is the price paid before the advent of the exchange. However we can compare the values by taking some reasonable values of the parameters. We illustrate the results with a numerical example.

Let each buyer require a quantity 1 of the commodity, and have a reservation price equal to 1. Each seller can supply a maximum quantity of 1.5. They also have a marginal cost of 0.2. That is, \( Q = 2, K = 1.5, r = 1, c = 0.2 \).

Inserting these values in equations (11) and (17) gives us \( p_1 = 0.467 \) and \( \bar{p} = 0.64 \).
Previously, the sellers used to sell at the reservation price of $r = 1$, and due to the competition induced by the exchange, the average price has fallen by 36%. Thus, if the cost of putting up the RFQ at the exchange (which includes the cost of setting up the RFQ and the fees of the exchange) is less than this amount, it makes sense for the buyers to join the exchange.

It must be noted that the average drop in prices as found from the above analysis is probably somewhat higher than what could be expected in real life. It is conceivable that large buyers, even in the absence of competition among sellers, would manage to force their preferred vendors to charge less than their reservation price $r$, by introducing the realistic threat of taking their business elsewhere otherwise. For an industry with a limited number of buyers, this is a credible threat indeed. However, it is instructive to note the effect of the exchange, whereby a substantial price reduction can be brought about by just the effect of increased competition.

Let us denote the lower quoted price to be $p_{\min}$. Then the density function of $p_{\min}$ is given by

\begin{equation}
    f(p_{\min}) = (1 - F(p)) f(p)
\end{equation}

Thus the average of the lower quoted price, $\bar{p}_{\min}$, is given by

\begin{equation}
    \bar{p}_{\min} = \int_{p_1}^{r} p f(p_{\min}) dp
\end{equation}

This gives us the value of $\bar{p}_{\min}$ as

\begin{equation}
    \bar{p}_{\min} = \frac{c(r-c)(Q-K)^2}{2K-Q} \ln \left( \frac{r-c}{p_1-c} \right) + \frac{(r-c)(Q-K)}{2K-Q} (r-p_1)
\end{equation}

\begin{equation}
    - \frac{(r-c)(Q-K)}{2K-Q} \left[ 1 - \frac{cK}{(2K-Q)} - \frac{1}{p_1-c} \right]
\end{equation}

\begin{equation}
    - \frac{c(r-c)(Q-K)}{2(2K-Q)} \left( \frac{1}{(r-c)^2} - \frac{1}{(p_1-c)^2} \right)
\end{equation}
For the numerical values we assumed to calculate $p_1$ and $\bar{p}$, the value of $\bar{p}_{\text{min}}$ turns out to be 0.55.

Similarly, let us denote the higher quoted price to be $p_{\text{max}}$. Then the density function of $p_{\text{max}}$ is given by

$$f(p_{\text{max}}) = F(p)f(p)$$

Then the average of the lower quoted price, $\bar{p}_{\text{max}}$, is given by

$$\bar{p}_{\text{max}} = \int_{p_1}^{r} pf(p_{\text{max}})dp$$

This gives us the value of $\bar{p}_{\text{max}}$ as

$$\bar{p}_{\text{max}} = \frac{c}{2} \left[ \frac{(r-c)(Q-K)}{(2K-Q)} \right]^2 \left( \frac{1}{(r-c)^2} - \frac{1}{(p_1-c)^2} \right)$$

$$+ \frac{(r-c)(Q-K)}{(2K-Q)} \left[ \frac{(r-c)(Q-K) - cK}{(2K-Q)} \right] \left( \frac{1}{r-c} - \frac{1}{p_1-c} \right)$$

$$+ \frac{K(r-c)(Q-K)}{(2K-Q)} \ln \left( \frac{r-c}{p_1-c} \right)$$

Again, for our numerical values above, we compute $\bar{p}_{\text{max}} = 0.76$. Thus, even the highest price charged is discounted by 24% as compared to prices before the advent of the exchange.

Finally, Table 2 in the Appendix presents a summary of comparative statics computations that are of interest. The signs of the behavior of the various parameters are as expected.

**Extending the model to a $n$-seller case**
A similar argument can be followed to find the equilibrium density function in the case when there are $n$ sellers. The expressions for the various functions will be more complicated than the two-seller model, but we can get enough insight to comment on the nature of the competition when there are $n$ players.

We first consider the simpler case when all the $n$ sellers are required to supply the entire demand, but however their total capacity exceeds that demand, i.e. $(n-1)$ sellers supply to capacity, and the last seller supplies to the residual demand ($(nK-Q) > 0$). The demand is fulfilled the same way, with a lower priced seller getting preference over a higher priced seller to supply any residual demand. A seller fails if its price turns out to be the highest price among all the sellers (an event that happens with a probability of $(F(p))^{n-1}$) and succeeds otherwise (an event that occurs with a probability of $(1-(F(p))^{n-1})$). The profits during these events are signified by $\Pi_s(p)$ and $\Pi_f(p)$ respectively, as before, and the profit at any price on the support is $\Pi$. Using similar arguments as before, we get

$$\Pi_f(F(p))^{n-1} + \Pi_s(1-(F(p))^{n-1}) = \Pi$$

which gives

$$F_n(p) = \left( \frac{\Pi_s - \Pi}{\Pi_s - \Pi_f} \right)^{\frac{1}{n-1}},$$

where we introduce the subscript $n$ in the last step to differentiate between the current expression and (5) above.

The expression for $\Pi_s(p)$ remains unchanged, while the expression for $\Pi_f(p)$ changes to

$$\Pi_f(p) = (p-c)(Q-(n-1)K)$$

For $n=2$, the expression in Equation (27) reduces to (5), as expected. It can also be easily checked that the support of the strategy remains unchanged, i.e., $p_1$ is given by (11) as before and $p_2 = r$. 

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Note that in Equation (25), the expression within the brackets is less than 1 (since \( \Pi_s > \Pi > \Pi_f \)), and therefore taking the \((n-1)^{th}\) root of the expression makes it larger than \( \left( \frac{\Pi_s - \Pi}{\Pi_s - \Pi_f} \right) \), which is the expression for \( F(p) \). That is, we have

\[
F_n(p) > F(p) \quad \text{for all } p, \text{ and } n > 2
\]

and the difference increases as \( n \) increases.

The expression for \( f_n(p) \) can be found to be

\[
f_n(p) = \frac{1}{n-1} \left( \frac{(p-c)K-(r-c)(Q-K)}{(p-c)(nK-Q)} \right)^{2-\frac{n-1}{n}} \left( \frac{(r-c)((Q-K)}{(nK-Q)(p-c)^2} \right)
\]

There does not seem any easy way to find the expression for the average price from the above expression, but we can gain sufficient insights about its behavior as \( n \) increases. Note that as \( n \) goes up, the difference \( (F_n(p) - F(p)) \) also increases, which implies that as \( n \) goes up, \( f_n(p) = F'(p) \) is stacked increasingly towards lower prices. Therefore, the average price \( \bar{p}_n = \int_{p_1}^{r} pf_n(p) dp \) would decrease with increasing \( n \). This is exactly what is to be expected, since increased competition should lower prices.

**Generalized model**

We now extend our model to a more generalized framework – that of \( n \) identical suppliers with capacity \( K \) each, each competing to get a part of the total requirement \( Q \); however, only \( m \) suppliers are needed to supply the entire quantity, with \( (m-1) \) suppliers supplying to capacity, and one supplier supplying the residual, i.e.,

\[
(m-1)K < Q < mK
\]
The remaining \((n - m)\) suppliers do not supply anything, where \((n - m) > 1\), to distinguish this scenario with the previous ones.

As in the case for two sellers, we can show that

**Proposition 9:** If the suppliers supply above cost, there can be no pure strategy in prices.

**Proof:** If there is, then the resulting Nash equilibrium should result in the same payoff for all players, as the players are equivalent. If that is not the case, a player choosing a price (strategy) that gives him a lower payoff is better off choosing a price that gives a higher payoff, and with similar players that employ similar strategies with similar payoffs, that is not possible. To get the same payoff, all players must choose the same price in a pure strategy, but that clearly is not a best response, since the best response by a player to a price \(p\) chosen by the other \((n - 1)\) players is to select a price \((p - \varepsilon)\).

We can also show using similar reasoning that there cannot be an equilibrium with a price tie above cost, and therefore there are no point masses in the equilibrium pricing strategies.

Therefore, if there exists a support for prices above marginal cost, there can be only a mixed strategy equilibria in that support. Let the highest price in that support be \(p'\). Since only a fraction of the suppliers supply can cater to the demand, at the price \(p'\), the supplier does not sell anything, and therefore his profits are zero. Further, as \(p'\) lies on the support, the expected profit at that price should equal the expected profit at any other price, i.e., the expected profit at any price should be zero. Therefore, the seller does not any incentive to charge \(p'\), and since \(p'\) can in fact be any price above marginal cost, it is apparent that the seller does not have any incentive to charge any price that is above the marginal cost \(c\). Thus, the support of the strategy collapses to a point \(p = c\): all suppliers supply at cost.
In fact, the intuition of this result can be gleaned from the expression for $F(p)$ in equation (8). If $K \rightarrow Q$, we have two sellers each of who is able to supply the entire demand. Then $F(p) \rightarrow 1$ for any $p > c$, and since no supplier is willing to supply below cost (i.e., $F(p) = 0$ for $p < c$), we have a degenerate probability density function $f(p)$ with point mass at $p = c$.

Thus, the very existence of a situation where only a fraction of the sellers can sell (as opposed to the earlier cases where the seller with the highest price is at least able to sell the residual) is enough to result in a Bertrand competition. In other words, with $m = 1$, we have a mixed strategy with prices above marginal cost. But increasing $m$ to 2 is enough to reduce the competition to Bertrand.

The implications of the result are clear. It is in the sellers’ interest to enter into the auction as long as the total capacity of the bidding sellers is such that at the very least the highest-priced bidder can sell to a residual demand. However, if there is the slightest overcapacity, such that there is one extra seller than required to supply to the required demand, the sellers are reduced to selling at marginal cost. On the other hand, it is in the interest of the buyers to introduce as much competition as possible, so as to drive down the price (since the average price falls with increasing $n$, and is reduced to marginal cost for any $m > 1$.

It is very important in the sellers’ interest therefore that the reverse auction mechanism lets each prospective bidder know at the very least the total quantity being bid for so far, so that he can decide whether it makes sense for him to bid or not. This is a crucial design requirement, as sellers have been reluctant to the myriad exchanges that have cropped up for every industry, fearing (and as our analysis shows, rightfully so) that it will lead to the buyers extracting the entire surplus from the sellers in a transaction. The exchanges should promote competition, but suitable information sharing will ensure that the competition results in an equitable division of the surplus.
Modeling variable costs

The fact that all sellers have the same marginal cost is very much an artificial constraint that allows us to model the competition. One option was to consider differing marginal costs, but the analysis becomes very complicated beyond two players.\(^5\) The more interesting, and realistic, option is to consider a situation where the players operate under similar industry environments, and therefore have similar but not the same costs (it can be argued that similar-sized competitors making the same product have similar cost structures).

A related issue is to explore the true nature of the price randomizing strategy of the sellers. Our solution in the preceding sections assumes that the sellers choose randomly between prices in the support of their mixed strategy (if a mixed strategy exists) that is determined by external conditions, i.e. their competition. However, firms might have their own internal reasons for choosing between high and low prices – when cost of raw materials is lower, sellers can bid lower than when cost of production is higher, for example. Modeling variable costs lets us explore this issue. We therefore consider the case when different sellers might have different marginal costs of production depending on the states of nature.

We assume two suppliers having similar cost structures, but instead of assuming that their costs remain constant, we consider the case where the costs vary according to some probability distribution. In such a situation, a seller will have incomplete information about the strategy of the other sellers. Thus, at any point of time, a supplier can assume that its competing suppliers have the same distribution in costs, but are not aware of the exact cost that these suppliers might be facing at a particular point in time. We assume that the two suppliers have capacities of \(K\) each, and are facing costs that are uniformly

\(^5\) The symmetric equilibrium result of the two-player case allows us to see the intuition behind the equilibrium, however: the lower priced seller can always force the other player to lose. In fact, he can do better – he can randomize his prices between a price that is determined by the marginal cost of the higher priced seller and the reservation price \(r\), such that the other player is pegged at selling the residual at \(r\). For a proof of the nature of the equilibrium, see Appendix A4.
distributed in the interval \((c_1, c_2)\), and consider the expected profits of either supplier, \(E(\Pi)\) by charging price \(p\):

\[
E(\Pi) = (p - c)[K(1 - F(p)) + (Q - K)F(p)]
\]

As the seller wants to maximize his profits, we get the resulting first order condition:

\[
(p - c)[K(1 - f(p)) + (Q - K)f(p)] + [K(1 - F(p)) + (Q - K)F(p)] = 0
\]

Thus, the strategy of a seller is as follows: For any cost \(c\) that he draws from his cost distribution, he chooses a price \(p\) which is a function of that cost, i.e. \(p = g(c)\), such that the density function of that price \(F(p)\) (which in turn is determined by the density function of \(c\), and the nature of the transformation \(p = g(c)\)) solves for the relationship specified by (33).

Since we are assuming that the cost has a uniform distribution, what we need therefore is a relationship \(p = g(c)\) (that in turn specifies \(F(p)\)), which solves for the equation in (33).

It is readily apparent that it is difficult to solve for \(g(c)\) and \(F(p)\) simultaneously so as to satisfy (33). We therefore solve the problem in a somewhat roundabout fashion – we start off by assuming that the resulting distribution for price is also uniform, and thus determine \(g(c)\). This determines \(c = g^{-1}(p)\), and we then check whether this relationship between \(p\) and \(c\) results in a uniform distribution for \(c\). If this is indeed the case, we can then go back and validate our \emph{a priori} assumption of \(f(p)\) being uniform and using the known parameters \(c_1\) and \(c_2\), we can determine its parameters.

Thus, let \(p\) be uniform in the interval \((a,b)\), where \(a\) and \(b\) are the parameters to be determined. Utilizing this information in (33) and rearranging, we get
\begin{equation}
(p - a - 4)K + 2Q = c[(b - a - 2)K + Q] + [aQ - (a + b)K],
\end{equation}

which shows that \( p = g(c) \) is a linear function of \( c \). It is also readily apparent that \( c = g^{-1}(p) \) is also linear, and therefore if \( f(p) \) is uniform, so is \( h(c) \), the distribution of \( c \). Since \( h(c) \) is actually uniform, our assumption of \( f(p) \) being uniform is therefore validated. What remain to be determined are the parameters \( a \) and \( b \). These can be determined from the fact that \( a = g(c_1) \) and \( b = g(c_2) \), and also \( F(a) = 0 \) and \( F(b) = 1 \). Using these boundary conditions in (33) gives us two simultaneous equations with the two unknowns \( a \) and \( b \) (they can also be equivalently obtained from (34)).

Thus, if the suppliers face identical uniform cost distributions within the range \((c_1, c_2)\), their best response is a pure strategy in price that is given by (34) (as opposed to a mixed strategy solution in the earlier case). This does not prevent a randomization in prices, whose distribution is uniform in the interval \((a, b)\). However, unlike the previous case with fixed costs, this price randomization is driven not by a mixed strategy, but by the fact that the pure strategy chosen in a particular instant is driven by the underlying cost, which itself is a random variable. This result seems to be a more “practical” explanation of the observed price randomizations – it is more realistic to think of managers in real life reacting to the underlying cost structure in their pricing strategy, than resorting to game theoretic results.

While our strategy for determining the price distribution cannot be applied for more general cost structures (it will be difficult to find a \((g(c), F(p))\) combination for any generalized density function), we do gain a valuable insight into the nature of the equilibrium.

**Managerial insights**

The inherent attraction of B2B marketplaces lies in their promise of bringing efficiency to the supply chain. Buyers can get better prices from suppliers simply from the effects of
increased liquidity and therefore the increased competition among sellers. In our introduction we quoted the comments of a seller who argued that the exchanges hamper any long-term partnership, and it is now easy to see why – since sellers randomize their pricing, they are never sure if they are going to win.

The increased liquidity and transparency in the marketplace also means that industry overcapacities will be severely penalized. Previously, each buyer would conceivably have a limited set of suppliers for supplying a particular raw material or component. Thus, we can think of scenarios where there is overcapacity as a whole in the industry for that component, but since only a limited number of sellers competed for any business, they could extract economic profits from a transaction. But with an open exchange, the number of competitors for any RFP is greatly increased. As our analysis shows, the presence of just a single “superfluous” supplier reduces the competition to Bertrand. Thus, component manufacturers can expect to be more stringent in dealing with any excess capacities, which in turn means that any demand fluctuations for the end product would swiftly be felt all through the supply chain. Managers will have greater responsibility in forecasting industry downturns or upswings, and scalability (in either direction) in manufacturing capabilities would be a major competitive edge. This will ring even more true as competitive pressures of the online marketplaces are expected to drive commoditizing of the raw materials.

In order to survive such potentially fatal price reductions, suppliers will therefore be needed to innovate and implement new features in their products. However, much of the impetus of doing so will have to be self-motivated, since OEMs might not be willing to try out higher-priced components that raise the cost of the end product. This will possibly mean that suppliers will have to proactively consider the end consumer, in order to educate them of their products’ innovations in order to create a ‘pull’ for these products (Forrester Research provides an example: To get consumers to buy its active head-restraint system through GM's online configurator, Autoliv, a Tier 1 supplier, must reach consumers and educate them on the system’s benefits – something Autoliv never had to do when it was just selling to the OEMs [Dixon Bunger, et al]).
Future directions of research

Some future directions of research would include considering asymmetric costs and capacities. Generalizations to $n$ sellers would be difficult in such cases, but it would be interesting to consider the equilibria where some sellers have inherent cost or capacity advantages (Footnote 5 and Appendix A4 refer to the intuition behind the equilibrium in a game between two players).

We are currently trying to implement a B2B reverse auction in a synthetic environment, where semi-intelligent sellers learn over time to converge to an optimum strategy. Some questions that we hope to answer include: can such agents learn over time to converge to an optimal game-theoretic equilibrium? If yes, how fast do they converge to those equilibria? Our findings might have useful implications in the implementation of automated mechanisms by sellers in transactions of this kind.

Conclusion

We have shown the effect of increased competition on prices within a B2B marketplace. We analyzed the equilibrium for a two-seller case, and then extended our insights from there to an $n$-seller case. The generalized model showed the effects of overcapacity, and how the presence of even a single superfluous bidder can reduce the competition to Bertrand. A final section analyzes the effect of cost variations, and provides an alternate explanation into the nature of the price randomizations that sellers participate in.

The model simplifies real-life conditions, since sellers cannot be expected to have similar costs and capacities in real life. A future direction of research can be to include these changes in the analysis. However, the simplified model does bring out the essential characteristics of such a competition. Industrial marketing experience suggests that
sellers would resort to price randomizations, which is what our analysis shows. Further, as expected, the equilibrium density function of prices $f(p)$ decreases monotonously with price. The model captures some of the unique features that we can expect to see in an exchange (limited number of sellers with huge buying power, modified capacity constraints, inelastic demand function below reservation price), and shows how competition in those environments can bring down the overall price paid by the buyers.

We are currently researching whether synthetic agents can learn from experience to implement our results. Our findings will form the basis of our future research.
### A1 - Figure 3: A B2B RFQ at IndiaMarkets.com, India’s largest B2B portal

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**Total Package Start Bid Price:** 66,863,721.0000

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**Comments:**

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<td>$F(p)$</td>
<td>Sellers’ cumulative density function of prices</td>
</tr>
<tr>
<td>$\Pi_s(p)$</td>
<td>Profit on selling to capacity</td>
</tr>
<tr>
<td>$\Pi_f(p)$</td>
<td>Profit on being the highest priced seller who sells something</td>
</tr>
<tr>
<td>$\Pi$</td>
<td>Expected profit at any price level $p$</td>
</tr>
<tr>
<td>$\bar{\Pi}$</td>
<td>Average profit paid by the buyers</td>
</tr>
<tr>
<td>$\bar{\Pi}_{\text{min}}$</td>
<td>Expected minimum price</td>
</tr>
<tr>
<td>$\bar{\Pi}_{\text{max}}$</td>
<td>Expected maximum price</td>
</tr>
<tr>
<td>$f_s(p)$</td>
<td>Density function of prices when there are $n$ sellers</td>
</tr>
<tr>
<td>$F_s(p)$</td>
<td>Cumulative density function of prices when there are $n$ sellers</td>
</tr>
<tr>
<td>$\bar{\Pi}_n$</td>
<td>Average price paid by the buyers when there are $n$ sellers</td>
</tr>
<tr>
<td>$E(\Pi)$</td>
<td>Expected profits with variable costs</td>
</tr>
<tr>
<td>$p = g(c)$</td>
<td>Price $p$ as a function of cost $c$</td>
</tr>
<tr>
<td>$(c_1, c_2)$</td>
<td>Range of uniformly distributed cost $c$</td>
</tr>
<tr>
<td>$(a, b)$</td>
<td>Range of uniformly distributed price $p$ corresponding to cost distribution above</td>
</tr>
</tbody>
</table>

**A2 - Table 1**: Explanation of variables used in the model

<table>
<thead>
<tr>
<th></th>
<th>$R$</th>
<th>$c$</th>
<th>$Q$</th>
<th>$K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_1$</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>$\bar{\Pi}$</td>
<td>+</td>
<td>?</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>$\bar{\Pi}_{\text{min}}$</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>$\bar{\Pi}_{\text{max}}$</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

**A3 - Table 2**: Summary of comparative statics
Nature of the asymmetric equilibrium with two players: If $c_L$ and $c_H$ are the marginal costs of production of the two suppliers, where $c_L < c_H$, it is easy to see from prior analysis that neither supplier has any incentive to set his price below $p^*_H$, where

$$p^*_H = \frac{(r - c_H)(Q - K)}{K} + c_H.$$  

(A1)

Note that the seller with the inherent cost advantage can always win by pricing his bid at $p^*_H$, which invokes the best response from the other seller to price his bid always at $r$. However, pricing his bid at $p^*_H$ is not the best response of the lower priced seller (note also that a pure strategy response by both sellers is not possible). In fact, he can do better: he can randomize his bids in the interval $(p^*_H, r)$ in such a fashion so that the best response of the higher priced seller is still the pure strategy of $p = r$. In other words, his best response to the pure strategy of the higher-priced seller (who, since he has no other option but to lose, set his price to the maximum to extract the highest possible surplus) is to randomize his prices in a manner that maximizes his expected profit, and still keep his competition pegged at $r$. Mathematically the problem is as follows:

$$\text{(A2) } \text{Find } F(p) \text{ that maximizes } \int_{p^*_H}^{r} K(p - c_L) f(p)$$

(where $f(p) = F'(p)$)

subject to the condition:

$$\text{(A2a) } (p - c_H)[K(1 - F(p)) + (Q - K)F(p)] < (Q - K)(r - c_H),$$

with $F(c_H) = 0$ and $F(r) = 1$.

$F(p)$ and $r$ establish the Nash equilibrium of the game.

A4 – Nature of asymmetric equilibrium with two players
References


