Divisibility and Tie Breaking Rules in Experimental Duopoly Markets

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June 2005

Abstract

This study investigates experimentally pricing behavior of sellers in duopoly markets with posted prices and market power. The two treatment variables are given by divisibility of the price space and tie breaking rules. A change in divisibility is modeled by making the sellers’ price space finer or coarser. The second treatment variable deals with the rule under which demanded units are allocated between sellers in case of a price tie. We find that lower divisibility tends to generate lower prices. Furthermore, posted prices and the incidence of perfect collusion are significantly higher under the sharing tie breaking rule than under the random (coin-toss) one.

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1 Introduction

The goal of this study is to shed some light on factors that affect pricing behavior and collusion incentives in market experiments. Previous experimental research suggests that the type of trading institution employed can prominently affect pricing behavior (e.g., Plott and Smith [16]). Furthermore, a well established experimental research program studies pricing behavior within the context of a particular market institution (see, for instance, Davis, Holt and Villamil [6], Friedman and Hoggatt [10], Alger [1]), and seems to suggest that environmental details play an important role. This paper falls in this second strand of literature.

We examine the effect of divisibility and tie breaking rules on prices in simple posted offer duopoly markets with simulated buyer behavior.

We model divisibility by making the sellers’ price space coarser (Less Divisible) or finer (Continuum). Nash equilibrium predictions imply that mean prices should be higher when the price space is less divisible. The reason why we decided to focus on this treatment variable is that there are several environments where divisibility might affect pricing behavior. An example might be provided by currency redenomination since it might cause a change in money divisibility. For instance, most of the countries that adopted the Euro currency experienced a decrease in divisibility and a price increase that has involved mostly services and some small-ticket items during the changeover.

Furthermore, notice that a decrease in divisibility makes price ties more relevant. This leads us to pay careful attention to the rule used to allocate units in case of equal prices, and to adopt a second treatment variable. The second treatment variable, namely the tie breaking rule, deals with the rule under which demanded units are allocated between sellers in case of equal prices. We explore two possible such rules. Under the random tie breaking rule (referred to as R) ties are broken randomly, i.e., the simulated buyer picks randomly which seller to approach first. On the other hand, the buyer equalizes purchases among the

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2The term divisibility refers to the fact that money can be broken in smaller or larger units. In Italy, for instance, the divisibility of currency has decreased approximately by a factor of 2. The smallest unit is now 1 euro cent which is worth approximately 20 liras. The factor of 2 is coming from the fact that the smallest unit circulating before the introduction of the Euro was given by 10 liras.

3Note that Eurispes used different methods and bundles than Istat to calculate inflation. According to Istat the increase in grocery prices has been 3.8% (between november 2001 and november 2002). The same calculation employed by Eurispes reports an increase in prices equal to 13%, four times larger than the one officially declared by Istat.
tied sellers under the sharing tie breaking rule (referred to as S). Under the assumption of risk neutrality and the Nash equilibrium concept, the tie braking rule should not affect pricing behavior.

The emphasis on tie breaking rules is not minor. Indeed, random and sharing tie breaking rules are used by government agencies in procurement auctions environments and might have an impact on collusion (see McAfee and McMillan [13] and Davis and Wilson [7]). For instance, McAfee and McMillan [13] observe that government agencies often employ a random tie breaking rule to deter collusive behavior.

The paper focuses on some issues such as: Is divisibility going to affect median prices? Is a sharing tie breaking rule going to facilitate tacit perfect collusion?

We answer these (and other) questions in a very simple environment. In particular, we concentrate our attention on duopoly markets and we keep pairs fixed since this makes pricing decisions easier for subjects (who have to worry only about one other player’s strategy). Furthermore, since the buyer is simulated, the focus is clearly on the sellers’ side of the market. We think it is reasonable to take such a duopolistic setting as a starting point.

We find that tie breaking rules have a significant effect on prices. In particular, a sharing tie breaking rule generates higher prices and facilitates perfect collusion. As far as divisibility is concerned, median prices are not higher under a less divisible space. In fact, there is some evidence that lower divisibility generates lower median prices.

The layout of the paper is as follows. Section 2 provides a brief literature review. Section 3 contains a discussion of the equilibrium predictions. Then, in Section 4, we describe the experimental design and procedures. The experimental results are presented in Section 5. Finally, in Section 6, we offer some concluding remarks.

2 Literature Review

Numerous experiments have been conducted to shed light on factors that affect pricing behavior. One of the most significant factors is the type of trading institution employed (see Plott and Smith [16]). A well established experimental research program studies pricing behavior within specific market institutions. For instance, prior posted offer experiments focused on factors such as the number of

\footnote{In the stage game.}
sellers (see Davis, Holt and Villamil [6]), the amount of information provided to
sellers (e.g., Kruse, Rassenti, Reynolds and Smith [4]), the role of subject expe-
rience (Friedman and Hoggatt [10], Alger [1], Benson and Faminow [2]), mergers
(e.g., Davis and Holt [5]), and so on.5

Some of these studies examine duopoly markets as well (e.g., Davis, Holt and
Villamil [6], Friedman and Hoggatt [10], Alger [1], Benson and Faminow [2]).

To the best of our knowledge, the effect of divisibility and tie breaking rules
has not been previously studied in posted-offer markets. The effect of tie breaking
rules has been investigated by Davis and Wilson (see [7]) in a variant of posted-
offer trading rules appropriate to a procurement auction environment. In this
experiment, every market lasts for forty trading periods and consists of four
sellers. One of their findings is that a change in the tie-breaking rule does not
affect behavior.

On the other hand, as far as divisibility is concerned, lower divisibility gen-
erates a simplification of the price decision space. The latter has been examined
in contestable markets by Brown-Kruse [3]. In this study, sellers’ offers are re-
stricted to multiples of 0.25 in one of the treatments. Under this restriction,
the strategy space shrinks from 250 choices to 10 choices. In particular, in this
work, a less complex specification of the choice space allows sellers to identify
alternative strategies of tacit collusion, even if the impact on mean prices is not
significant. They conjecture that one of the reasons why the results are not sig-
nificant might rely in the specification of the design generating losses for sellers
in case of matched prices.6 Note that in our environment, on the other hand,
sellers do not incur any loss in case of a price tie.

Thus, the contribution of our paper, relative to the existing literature, is to
investigate the effect of a simplification of the choice space of sellers and the
effect of tie breaking rules on market power (in particular on the distribution of
prices) and collusive behavior, in duopoly posted offer markets.

5 According to Davis et al.[6], static market power leads to price increases in posted offer
markets with three sellers, while the effect is not clearcut for duopolies. Both across sessions
(see [2]) and within session (see [1]) subject experience seems to increase the likelihood of
collusive outcomes.

6 “Under the decreasing average costs of a natural monopoly, if sellers match prices and share
the market, this can result in substantial losses when prices are near the competitive range”
([3], p.144).
3 Equilibrium Distributions

The model consists of a finitely repeated game. The stage game is described as follows. There are two identical firms producing a homogeneous good whose demand curve is

\[ q(p) = \begin{cases} 
0 & \text{if } p > 550 \\
6 & \text{if } 0 \leq p \leq 550.
\end{cases} \]

Each firm has a capacity constraint of 4. The cost of production is 41 per unit and production is to-order as sellers incur costs only if a unit is sold. Firms simultaneously choose prices and quantities. That is, each seller has a two dimensional strategy space \( B = (p, q) \in P \times Q \), where \( Q = \{0, 1, 2, 3, 4\} \). We will consider two games whose distinction trait is the price space, namely, either \( P = P_C = [0, 10000] \) or \( P = P_{LD} = \{0, 50, \ldots, 10000\} \).

Note that it is a dominant strategy for every firm to post prices exceeding 41 and offer all the units available for sale. Thus, we can treat this game as if the strategy space is in fact one dimensional and firms simultaneously choose only prices. If the prices are unequal, the low-price firm sells 4 units and the high-price firm sells the remaining 2 units. If the firm choose the same price, i.e., in case of a price tie, either they share the market equally, or the seller to be approached first is chosen randomly. Thus, under the assumption that the firms are risk neutral, the payoff functions are given by the following expression for \( i = 1, 2 \):

\[ u_i(p_1, p_2) = \begin{cases} 
(p_i - 41) \times 4 & \text{if } p_i < p_j, p_i \leq 550 \\
(p_i - 41) \times 3 & \text{if } p_i = p_j, p_i \leq 550 \\
(p_i - 41) \times 2 & \text{if } p_i > p_j, p_i \leq 550 \\
0 & \text{if } p_i > 550
\end{cases} \]

In the next sections, we will characterize the Nash Equilibrium for this game for both \( P_C = [0, 10000] \) and \( P_{LD} = \{0, 50, \ldots, 10000\} \). Before doing that, let us highlight some of the common features of the equilibrium analysis.

Note that in both cases the competitive equilibrium \((p = 41 \text{ and quantity demanded } = 6)\) is not a Nash equilibrium, since either of the sellers can profit from a unilateral deviation, i.e., there is static market power. Specifically, we will see that because of the market power the noncooperative equilibrium for the market game leads to prices that exceed the competitive level (e.g., see [11]).
Obviously, no pure strategy Nash Equilibrium exists in this game and nonco-operative firms randomize to avoid being slightly undercut in the cycle. There is a unique symmetric mixed strategy equilibrium in both cases that requires mixing over the range of the Edgeworth cycle. Notice that in our case, the support of the distribution coincides with the range of the Edgeworth cycle. (See [12].)

Furthermore, since the stage game has a unique equilibrium, by backward induction the unique subgame perfect equilibrium of the repeated game is to play the static equilibrium in every subgame.

3.1 Equilibrium in the continuum case

In this section, we let $P_C = [0, 10000]$. By the discussion at the end of the previous section, it is easy to see that the game does not have a pure-strategy Nash Equilibrium. The noncooperative equilibrium involves randomization over the range of prices called “Edgeworth cycle.” The support of the noncooperative equilibrium distribution is determined as follows. The upper end of the range of the Edgeworth cycle is 550. At a price of 550, the residual demand is of two units, so that either seller can secure himself a profit of $2[550 - 41] = 1018$. If a seller chooses a price of 550, the other seller’s best response is to post a price just below it and sell four units. Best responses consist in undercutting until a price of 295.5 is reached. In fact, selling four units at a price below 295.5 is not as profitable as selling two units at a price of 550 ($4[295.5 - 41] = 1018$).

Let $G(p)$ denote the probability that the price $p$ posted by seller $i$ is the highest price posted in the market for a period. Since any two sellers in a market have identical profit functions, we will focus on a symmetric mixed equilibrium, so that $G(p)$ can be considered as the common price distribution. Furthermore, note in this game there is a unique equilibrium in mixed strategies and that the probability of a price tie is zero (for a proof see [15] and [8]). If a price $p$ is the highest price, the seller who posted it will sell two units and will earn $H(p) = 2[p - 41]$. On the other hand, if $p$ is the lowest price, the seller will sell four units and his earnings are $L(p) = 4[p - 41]$. Hence, the expected profit for a seller is given by

$$G(p)H(p) + (1 - G(p))L(p).$$

Next, observe that sellers must be indifferent over all prices in the support of the

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\(^7\)Each period each firm posts a price slightly below the prices posted in the previous period. Eventually prices decrease to a level where a firm’s best response is to charge the monopoly price for the residual demand. The other firms follow and a new price cycle begins.
distribution, and they can secure themselves a profit of 1018 by choosing a price of 550. By substituting for the expression $H(p)$ and $L(p)$, we obtain

$$G(p)2[p - 41] + (1 - G(p))4[p - 41] = 1018.$$  

Solving for $G(p)$ we have the equilibrium cumulative distribution function of prices

$$G(p) = \begin{cases} 0 & \text{if } 0 < p \leq 295.5 \\ \frac{501 - 2p}{41 - p} & \text{if } 295.5 \leq p \leq 550, \end{cases}$$

which is depicted in the following figure, for $295.5 \leq p \leq 550$.

![Figure 1](image)

**Figure 1.** Cumulative distribution in the continuum case.

### 3.2 Equilibrium in the discrete case

Now, let $P_{LD} = \{0, 50, ..., 10000\}$, i.e., sellers are allowed to post only prices that are multiples of 50. We now deal with a finite game, and as such, we know that there exists an equilibrium. With the help of some specialized software (see either [14] or [17]) we are able to find the equilibrium in mixed strategies. Furthermore, the equilibrium is unique and it is symmetric. The equilibrium distribution assigns zero probability to all prices below 300, and it is described in the table below.
<table>
<thead>
<tr>
<th>price</th>
<th>probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 − 300</td>
<td>0</td>
</tr>
<tr>
<td>350</td>
<td>0.38</td>
</tr>
<tr>
<td>400</td>
<td>0.12</td>
</tr>
<tr>
<td>450</td>
<td>0.26</td>
</tr>
<tr>
<td>500</td>
<td>0.04</td>
</tr>
<tr>
<td>550</td>
<td>0.20</td>
</tr>
</tbody>
</table>

TABLE I. Equilibrium distribution in the discrete case.

3.3 Summary of Equilibrium Predictions

Under risk neutrality, the choice of the tie breaking rule does not affect Nash equilibrium predictions. Consequently, based on the previous two sections, we can summarize the equilibrium predictions in Table II. For completeness, we include also the predictions from competitive equilibria and perfectly collusive behavior.8

<table>
<thead>
<tr>
<th></th>
<th>Mean Price</th>
<th>Median Price</th>
<th>St. Dev.</th>
<th>Profit/Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Competitive Equ.</td>
<td>41</td>
<td>41</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Perfectly Collusive</td>
<td>550</td>
<td>550</td>
<td>0</td>
<td>1527</td>
</tr>
<tr>
<td>Nash Equ. (Continuum)</td>
<td>393.81</td>
<td>380.33</td>
<td>71.16</td>
<td>1018</td>
</tr>
<tr>
<td>Nash Equ. (LD)</td>
<td>427.67</td>
<td>400</td>
<td>75.44</td>
<td>1118.69</td>
</tr>
</tbody>
</table>

TABLE II. Theoretical predictions.

From a qualitative standpoint the static Nash equilibrium predicts the following differences under the two divisibility regimes:

Hypothesis 1. Mean and median prices are higher under the Less Divisible regime than under the Continuum one.

Hypothesis 2. Price dispersion is higher under LD than under C.

8Perfectly collusive profits are the per seller profits associated with the limit price. Note that we include also the predictions arising from perfectly collusive behavior even if it is not an equilibrium in our game (we deal with a finite horizon game). Nonetheless, in the infinitely repeated counterpart, in order to sustain perfect collusion as an equilibrium in the continuum and less divisible case the probability of continuation should exceed 0.49 and 0.43, respectively.
Hypothesis 3. Equilibrium profit is higher under LD than under C.
A change in the tie-breaking rule does not affect the equilibrium predictions under risk neutrality.

4 Experimental Design and Procedures

Every market consisted of sixty trading periods during which pairs are fixed. Supply and demand arrays for each market are shown in Figure 2 (in the appendix). Note that the two sellers in every market were symmetric. They had identical costs and identical capacity constraints, i.e., they both were endowed with four units at a cost of 41 each.

All markets were conducted under posted-offer rules with a simulated, fully revealing buyer. In every period, sellers simultaneously made price/quantity decisions in each market. Every seller was allowed to post only one price at which he was willing to sell the posted units. After all prices were posted, the simulated buyer purchased up to six units in each market at prices up to 550 (and no units at a price exceeding 550). The buyer made all profitable purchases, buying first from the seller with the lowest posted price, then from the other seller.

Sellers were fully informed about other seller’s cost, about the preferences and shopping behavior of the simulated buyer, and about the matching protocol.

Production was to-order as sellers incurred costs only if a unit was sold. Thus, the payoff for every seller was given by

\[ \text{Payoff} = [(\text{selling price} \times \text{number of units sold}) - (\text{production cost of units sold})]. \]

We adopted two treatment variables.

The first treatment variable dealt with the divisibility of sellers’ strategy space (modeled by making each seller’s strategy space coarser or finer).

In particular, in the Continuum treatment sellers were allowed to post prices that were numbers up to three decimal places, while in the Less Divisible treatment, sellers were allowed to post only prices that were multiples of 50. That is, in the Continuum treatment (C thereafter) \( p \in P_C = [0, 0.001, \ldots, 10000] \) and \( q \in Q = \{0, 1, 2, 3, 4\} \). On the other hand, in the Less Divisible treatment (LD), \( p \in P_{LD} = [0, 50, \ldots, 10000] \) and \( q \in Q = \{0, 1, 2, 3, 4\} \).

The second treatment variable changed the rules according to which the simulated buyer made purchases in the event of a price tie. In case of identical prices, in design S (S for sharing) the buyer equalized purchases among the tied
sellers, while in design R (R for random), the buyer chose randomly which seller
to approach first.\(^9\)

We conducted fifty-six homogeneous-product duopoly markets, run in ten
sessions with 8, 10, 12, 14 or 16 participants (see Table III).

The experiments are divided into four cells based on the two treatment con-
ditions.

<table>
<thead>
<tr>
<th></th>
<th>Continuum (C)</th>
<th>Less Divisible (LD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sharing (S)</td>
<td>7 markets (1 session)</td>
<td>16 markets (3 sessions)</td>
</tr>
<tr>
<td>Random (R)</td>
<td>16 markets (3 sessions)</td>
<td>17 markets (3 sessions)</td>
</tr>
</tbody>
</table>

Table III. Matrix of treatments and sessions summary.

Note that the matrix in Table III also displays the number of sessions run
under each treatment. All the sessions took place at the Vernon Smith Ex-
perimental Economics Laboratory at Purdue University. The experiment was
programmed and conducted with the software z-Tree (See [9]). Subjects for the
experiments were recruited from undergraduate students at Purdue University.
Subjects were inexperienced, where by experience we mean previous participa-
tion in one or more of the posted offer experiments.\(^{10}\) Instructions were read
aloud to participants as they followed along in their own copies.\(^{11}\) Subjects were
given explicit information regarding the purchasing decision of the simulated
buyer as well as the fact that sellers were identical. Furthermore, they were told
that they were paired with the same person throughout the whole experiment,
that the experiment would consist of 60 periods\(^{12}\) and would run for up to one

\(^9\)Note that from the theoretical viewpoint, this treatment variable does not affect (in case
of risk neutrality of the sellers) the outcome. Under risk aversion and discrete strategy space, on
the other hand, this might affect behavior. Indeed, note that if two sellers price tie between 50
and 550 and post four units each, then under S they will sell three units each, while under R,
one of them (randomly selected) will sell four, and the other only two.

\(^{10}\)On the other hand, if by experience we mean previous participation to any type of economics
experiment, our subjects were experienced. As a matter of fact, two sessions were run with
subjects who had never participated before to any type of experiment. The results from these
sessions appear significantly different from the ones obtained from the others, suggesting the
introduction of a third (experience) treatment variable, that goes beyond the scope of this study.

\(^{11}\)The instruction for the treatments Sharing Less Divisible, Random Less Divisible and Ran-
don Continuum are contained in the appendix. The instructions for the treatment Sharing
Continuum is obtained as an obvious modification of the ones included.

\(^{12}\)That is, the stopping rule of 60 periods was publicly announced.
hour and a half. A typical experiment lasted about an hour. Earnings ranged between $11 and $22 per subject. Average earnings were $15.95.

5 Experimental Results

Before providing a detailed discussion of our results, let us point out that in our experiment markets consisted of fixed pairs, so that we can treat markets as statistically independent observations. For the sake of completeness, we tested our hypotheses by using the price medians as well as the mean transaction prices and the mean posted prices.\textsuperscript{13}

In this section, we will adopt the following notation:

\begin{itemize}
  \item \textit{rc} = random tie breaking rule and continuum price space
  \item \textit{rld} = random tie breaking rule and less divisible price space
  \item \textit{sc} = sharing tie breaking rule and continuum price space
  \item \textit{sld} = sharing tie breaking rule and less divisible price space.
\end{itemize}

The most important conclusions of our analysis are derived from mean and median transaction prices, from a regression analysis and from a probit model.

When the markets in each treatment cells are pooled, the median prices for periods 11-60\textsuperscript{14} are plotted in Figure 3 (in the appendix). Figure 4 (in the appendix) plots average prices pooled by treatment and every 10 periods.

The data are characterized by the following features:

(i) In all treatments, prices do not follow a specific trend, but they exhibit an unstable pattern.

(ii) The quantitative theoretical predictions are rejected (including the competitive one) by the data. Median prices are higher than predicted by the noncooperative Nash Equilibrium\textsuperscript{15} and lower than monopolistic ones.

(iii) The treatment \textit{sld} is characterized by the highest incidence of median prices equal to 550.

The next sections contain a more detailed analysis of the data.

\textsuperscript{13}Note that the noncooperative equilibrium predictions for the one-shot game regard posted prices.
\textsuperscript{14}We chose to focus on this range to account for some initial noise and end-game effects.
\textsuperscript{15}The Kolmogorov-Smirnov test leads to reject the equality of distributions of theoretical and observed prices at any level of significance (and both for the continuum and less divisible cases).
5.1 Tie Breaking Rules

Does a change in the tie breaking rule affect pricing behavior? Recall that under the assumption of risk-neutrality, the tie breaking rule (since we are also dealing with duopolies) should not affect prices. On the other hand, our data analysis seems to support the opposite. More specifically, we focus on the effect of this treatment variable on perfect tacit collusion, as well as on posted prices.

In order to study the effect of the tie breaking rules on perfect tacit collusion, we will focus on the data collected under the less divisible (LD) treatment (since few observations were collected under the continuum with sharing tie breaking rule treatment).\textsuperscript{16}

5.1.1 Perfect tacit collusion

Tie breaking rules affect the rates of perfect tacit collusion. In particular, perfect tacit collusion occurs more frequently under the sharing tie breaking rule (see Figure 6 in the appendix). When the sharing rule is employed, 7 markets (out of 16 markets) converge to perfect collusion for 27, 21, 49, 54, 40, 21 and 41 consecutive periods respectively.\textsuperscript{17} In five of them perfect collusion was broken in the last period, and in two of them in the last two periods. On the other hand, when the random rule is employed, perfect collusion is not sustained by any of the 17 markets.\textsuperscript{18} A Mann-Whitney two-tailed test carried on longest ties confirms that this difference is significant (p-value= 0.0627).

We also estimate the likelihood of perfect collusion (i.e., price ties at 550) in the last thirty periods by employing probit models. Probit analysis was conducted using the following model:

\[ Ties_{30t} = \beta_0 + \beta_1 sld_{30} + \varepsilon_t, \]

\textsuperscript{16}It must be mentioned that, even though the sample size is small for the sc treatment, a change in the divisibility of the price space from less divisible to continuum seems not affect our results on the effect of tie breaking rule on perfect collusion. In particular, 2 out of 7 markets converged to perfect collusion in the sc treatment, while none of the markets did in the rc treatment. More specifically, our conjecture is that, the sharing tie breaking rule facilitates perfect collusion, no matter what the degree of divisibility of the price space is. Perfect collusion might be slightly more fragile under a Continuum price space (e.g., in one of the markets that converged to perfect collusion, the latter was broken and resumed with a certain regularity), but still survives (see Figure 6 in the appendix).

\textsuperscript{17}This provides us with the highest number of consecutive ties. The total number of ties for these markets were 28, 44, 49, 54, 43, 43, 41, respectively.

\textsuperscript{18}Three markets tied for 15, 27 and 12 periods, but not consecutively. In particular, the longest sequences consisted of 6, 16 and 4 ties respectively.
as well as a random effects probit model

\[ Ties_{30, it} = \beta_0 + \beta_1 \text{sl}_i + \epsilon_{it}. \]

where \( i \) refers to markets.

We focused on the effect of the tie breaking rules on the probability of perfect collusion in the last thirty periods. The dependent variable \( Ties_{30} \) is a binary variable which is equal to 1 in case of a tie at 550 and 0 otherwise, \( sl_{d30} \) is a dummy variable whose value is 1 if the market is under the treatment sharing and less divisible.

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>Regression Coefficients</th>
<th>Regression Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Probit (robust)</td>
<td>RE probit</td>
</tr>
<tr>
<td>( Ties_{30} )</td>
<td>( \beta_0 )</td>
<td>( \beta_0 )</td>
</tr>
<tr>
<td></td>
<td>( \beta_1(sld) )</td>
<td>( \beta_1(sld) )</td>
</tr>
<tr>
<td></td>
<td>-1.32</td>
<td>-1.32</td>
</tr>
<tr>
<td></td>
<td>1.13***</td>
<td>1.14***</td>
</tr>
<tr>
<td></td>
<td>(-6.02 )</td>
<td>(-17.13)</td>
</tr>
<tr>
<td></td>
<td>(3.11 )</td>
<td>(11.85 )</td>
</tr>
</tbody>
</table>

Table IV. Probit Results.

Probit (robust) is robust to assumptions about within-market correlation.

***Statistically significant at 1% level.

Note that the coefficient of \( sld (\beta_1) \) is positive and significant under both probit specifications. This implies that the sharing tie breaking rule is an important determinant of perfect collusion, and that it increases the likelihood of observing collusion in the last thirty periods.

A simple calculation also shows that the probability of observing a tie at 550 (i.e., perfect collusion) is approximately 33% higher under the sharing rule treatment, relative to the random rule treatment (keeping divisibility low).

**Conclusion:** Tacit perfect collusion is significantly higher in the Sharing Less Divisible \( (sl_{d}) \) treatment than in the Random Less Divisible \( (rl_{d}) \) one.

### 5.1.2 Posted Prices

The empirical distributions of posted prices are different under the two tie breaking rules (see Figure 5, in the appendix). Note that the frequency of prices equal to 550 is 43.7 under the sharing tie breaking rule, and 26.08 under the random one (given a less divisible price space).
We investigate the effects of tie breaking rules on posted prices also by using an analysis of variance (ANOVA) procedure and a subject-specific random effects (RE) regression model.

The ANOVA statistics are calculated by running an OLS regression with price as the dependent variable and dummy variables as independent variables. The regression was run for all the price data except for period 1 and the last two periods. We estimated the following models

\[ p_t = \alpha_0 + \alpha_1 (1/t) + \alpha_2 rc + \alpha_3 sld + \alpha_4 sc + \varepsilon_t. \]

\[ p_t = \beta_0 + \beta_1 (1/t) + \beta_2 rc + \beta_3 sld + \beta_4 rld + \varepsilon_t. \]

The variable \((1/t)\) is an explanatory variable equal to the inverse of the trading period. We employed four dummy variables, one, \(rc\), is equal to 1 if the price is observed under the treatment with random tie breaking rule and continuum choice space, while the other, \(sld\), is 1 if the price is observed under the treatment with sharing tie breaking rule and less divisible choice space. Similarly, the dummy variable \(sc\) equals 1 if the posted price was observed in the treatment with sharing tie breaking rule and continuum choice space.\(^{19}\)

We estimated the same model under the random effects framework,

\[ p_{it} = \alpha_0 + \alpha_1 (1/t) + \alpha_2 rc_i + \alpha_3 sld_i + \alpha_4 sc_i + \nu_i + \varepsilon_{it}. \]

\[ p_{it} = \beta_0 + \beta_1 (1/t) + \beta_2 rc_i + \beta_3 sld_i + \beta_4 rld_i + \nu_i + \varepsilon_{it}. \]

where \(i\) refers to individual sellers.

The next table contains the results for price regressions from both approaches. The \(t\)-test statistics (for the null of no difference from zero) are printed below the coefficient estimates.

\(^{19}\)The dummy variable \(rld\) is the omitted category in the first equation, and \(sc\) is the omitted category in the second one.
### Regression Coefficients

**ANOVA**

<table>
<thead>
<tr>
<th></th>
<th>(\alpha_0)</th>
<th>(\alpha_1(1/t))</th>
<th>(\alpha_2(rc))</th>
<th>(\alpha_3(sld))</th>
<th>(\alpha_4(sc))</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>465.85**</td>
<td>-284.92**</td>
<td>14.70**</td>
<td>21.75**</td>
<td>-2.18</td>
</tr>
<tr>
<td></td>
<td>(190.50)</td>
<td>(-19.11)</td>
<td>(4.55)</td>
<td>(6.73)</td>
<td>(-0.52)</td>
</tr>
</tbody>
</table>

**Statistically significant at 5% level. Number of Obs.=6384, \(R^2 = 0.062\).**

### Random Effects

**Regression Coefficients (individual sellers)**

<table>
<thead>
<tr>
<th></th>
<th>(\beta_0)</th>
<th>(\beta_1(1/t))</th>
<th>(\beta_2(rc))</th>
<th>(\beta_3(sld))</th>
<th>(\beta_4(rld))</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>-506.69**</td>
<td>-284.92**</td>
<td>16.89**</td>
<td>23.94**</td>
<td>2.18</td>
</tr>
<tr>
<td></td>
<td>(-62.37)</td>
<td>(-19.11)</td>
<td>(4.02)</td>
<td>(5.69)</td>
<td>(0.52)</td>
</tr>
</tbody>
</table>

**Statistically significant at 5% level. Number of Obs.=6384, \(R^2 = 0.062\).**

### Table V. Anova Regression Results.

### Random Effects

**Regression Coefficients (individual sellers)**

<table>
<thead>
<tr>
<th></th>
<th>(\beta_0)</th>
<th>(\beta_1(1/t))</th>
<th>(\beta_2(rc))</th>
<th>(\beta_3(sld))</th>
<th>(\beta_4(rld))</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>463.84***</td>
<td>-284.92***</td>
<td>16.89</td>
<td>23.94</td>
<td>2.18</td>
</tr>
<tr>
<td></td>
<td>(13.57)</td>
<td>(-21.91)</td>
<td>(1.04)</td>
<td>(1.47)</td>
<td>(0.14)</td>
</tr>
</tbody>
</table>

**Statistically significant at 1% level. *Statistically significant at 10% level.**

Number of Obs.=6384, \(R^2 = 0.071\)

### Table VI. Random Effects Regression Results.
The negative and significant coefficient on $t$ implies an upward trend in prices for the early periods. The positive and significant coefficient\(^{20}\) on $sl_d$ in the regression analysis (1) means that sellers with “sharing” tie breaking rule posted higher prices than the ones posted by sellers with the random one (given the less divisible choice space).\(^{21}\)

**Conclusion:** Under a low divisibility regime, sellers post significantly higher prices under the sharing tie breaking rule than under the random one.

### 5.2 Divisibility

Following the Nash Equilibrium prediction (See Table II) we have three hypotheses, that we test next.

**Hypothesis 1:** Mean and median prices are higher under the Less Divisible regime than under the Continuum one.

To test this conjecture we employed data collected under the random tie breaking rule (since the sample size is small under the treatment $sc$). This conjecture is not supported by the data. In fact, there is some evidence of the opposite effect. In particular, the positive coefficient on $rc$ in regression (1) suggests that sellers with “continuum” choice space posted higher prices than with less divisible choice space, given a random tie breaking rule. Moreover, median market prices are significantly higher under $rc$ than under $rld$ (according to Kolmogorov-Smirnov test, $p$-value=0.052\(^{22}\)).

**Hypothesis 2:** Price dispersion is higher under LD than under C.

The data collected under the random tie breaking rule does not support this conjecture. In particular, the standard deviation of prices for markets under $rld$ is significantly smaller than for markets under $rc$ (Mann-Whitney, $p$-value=0.07).

**Hypothesis 3:** Equilibrium profit is higher under LD than under C.

This conjecture is not supported by the data as well. In particular, there is evidence that individual sellers’ profits are significantly higher under $rc$ than under $rld$ (Kolmogorov-Smirnov, $p$-value= 0.07).

**Conclusion:** None of the theoretical predictions regarding divisibility is sup-

\(^{20}\)A regression estimation producing consistent standard errors even if residuals across groups are not identically distributed does not change the results.

\(^{21}\)Average (both contract and posted) market prices are marginally significantly higher under $sl_d$ than under $rld$ according to Kolmogorov-Smirnov test ($p = 0.098$).

\(^{22}\)The p-value of 0.052 refers to the combined test. This estimate for small samples is too conservative. A less conservative approximation is 0.028.
ported by the data. For instance, mean and median prices are not higher under the Less Divisible regime than under the Continuum one. Similarly, for price dispersion and profits.

6 Conclusions

We examine the effect of a change in divisibility and tie breaking rules on prices in simple experimental duopoly markets with posted prices and simulated buyer behavior.

We model a change in divisibility by making the sellers’ price space finer or coarser. We explore two possible tie breaking rules, sharing and random.

Theoretically, in our duopoly model prices should be affected by divisibility but not by the tie breaking rule. However, experimentally we find that a change in the tie breaking rule significantly affects pricing behavior. Moreover, there is evidence that in our design divisibility has an opposite effect than predicted by the Nash equilibrium.

In particular, nonparametric tests and probit estimation procedures show that the probability of perfect collusion is higher under the sharing tie breaking rule than under the random (coin-toss) one. Also, using random effects regression analysis, we find that posted prices are significantly higher under the sharing tie breaking rule and lower divisibility does not generate higher prices.

We studied divisibility and tie breaking rules in a very simple environment. For instance, note that we focused only one side of the market, i.e., the sellers’ pricing behavior. It would be interesting to complement this study by employing an experimental design which allows to test also for buyers’ responses. Furthermore, a change in the market structure (e.g., in the number of sellers, or in the cost and demand conditions) might generate different results. Another interesting point to investigate is the effect of tie breaking rules on prices. One possible explanation for this effect might be a failure of risk neutrality. These issues are left for further research.
References


