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## The sufficiency of the ‘lens condition’ for factor price equalization in the case of two factors

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### Abstract

Factor price equalization (FPE) is a central theme in trade theory, for which [Dixit, A.K., Norman, V., 1980. *Theory of International Trade*, Cambridge University Press, London] establish the necessary and sufficient condition (the Dixit–Norman condition). [Deardorff, A.V. 1994. The possibility of factor price equalization: revisited, *Journal of International Economics* 36, 167–175.] provides a more intuitive condition (the lens condition) and establishes its necessity in general, as well as its sufficiency for the case of two countries. In this paper, I prove that the lens condition is sufficient for FPE in the case of two factors. This theorem has implications for empirical work. © 2001 Elsevier Science B.V. All rights reserved.

*Keywords:* Factor price equalization; Integrated world economy; Intermediate lens; Lens condition

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### 1. Introduction

The concept of factor price equalization (FPE) is central to the theory of international trade. Dixit and Norman (1980) establish that with perfect competition and constant returns to scale, the necessary and sufficient condition for FPE (the Dixit–Norman condition) is that the outputs of the ‘integrated world economy’ (IWE) can be produced using the technology of the IWE, with each

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factor of each country fully employed.<sup>1</sup> To help visualize this somewhat abstract condition, a factor endowment box (FEB)<sup>2</sup> is presented both for the case of two factors, two countries and two goods and that of two factors, two countries and three goods. In the FEB, a polygon (in the case of two goods, a parallelogram) can be drawn, with each side being the factor usage vector of one sector in the IWE. When there are only two countries, the distribution of factor endowments can be represented by a point in the FEB. It is then straightforward that FPE is achieved if and only if the point lies within the polygon. A natural question to ask is, if the numbers of factors, goods and countries are arbitrary, can we still visualize the Dixit–Norman condition in a similar fashion?

Deardorff (1994) explores this idea and formalizes the visualization as the ‘lens condition’; i.e. the lens spanned by the vectors of the countries’ factor endowments (the endowment lens) lies inside the lens spanned by the factor usage vectors of all sectors (the factor usage lens) in the IWE. In addition to being easier to visualize than the Dixit–Norman condition, the lens condition also formalizes the intuition that, to achieve FPE, the variation across countries in relative endowments must be less, in some sense, than the variation across industries in factor intensities. Deardorff proceeds to show that the lens condition is necessary for FPE in general, and in the case of two countries it is also sufficient. He also conjectures that the lens condition is sufficient for an arbitrary number of factors, goods and countries.

In this paper, I show that the conjecture holds in the case of two factors for an arbitrary number of goods and countries. The case of two factors deserves attention because the factor endowment box has probably become the most widely used tool to analyze factor price equalization, and in almost all cases, such a box is drawn in two dimensions. It is therefore important to notice, and comforting to know, that such a box drawn in two dimensions is indeed appropriate for an arbitrary number of goods and countries. What’s more, the theorem proved in this paper has empirical usefulness as well. For instance, some authors have noticed that the Heckscher–Ohlin–Vanek (HOV) theorem performs poorly even among developed countries.<sup>3</sup> Could this be because even developed countries have very different factor endowments so that FPE fails to hold among them? This is an empirical question, and a useful first step towards answering it is to test the lens condition in the case of two factors. Compared with the Dixit–Norman condition, the lens condition is easy to test. While both conditions start from the factor endowment and factor usage data of the IWE, the Dixit–Norman condition requires verifying the existence of an allocation of activities across countries, which is a matrix satisfying several conditions that it might be hard to find even with a computer search. The lens condition, on the other hand, can be directly

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<sup>1</sup>Blackorby et al. (1993) provide alternative necessary and sufficient conditions for factor price equalization.

<sup>2</sup>Fig. 4.4 and 4.6 in Dixit and Norman (1980).

<sup>3</sup>See, for example, Brecher and Choudhri (1988), and Gabaix (1997).

calculated from those same data. Moreover, if we consider only two factors, the lens condition is not only necessary for FPE, but also sufficient, as shown in this paper. Finally, in the special case of two factors, the lens condition is easy to visualize.

Two caveats merit mentioning, though. The first is the omission of other factors of production, such as land. With more than two factors, even if the lens condition is satisfied for every pair of factors, it could still be violated, as shown in Deardorff (1994). The other is simply that we are unable to observe the IWE factor usage lens.<sup>4</sup>

Debaere and Demiroglu (1998) test the lens condition for two factors (capital and labor) and for different groups of countries.<sup>5</sup> The testing is done by using the observed sector factor usage vectors to construct the factor usage lens and comparing it with the corresponding endowment lens. They accept the lens condition for a group of developed countries, and reject the condition for a group consisting of both developed and developing countries. Their work suggests that among developed countries, factor endowments are similar enough for FPE to hold.<sup>6</sup> Therefore, it seems appropriate to conclude that the poor performance of HOV among developed countries<sup>7</sup> is not due to a failure of FPE that is caused by very different factor endowments.

## 2. The setup, and the lemma

Consider a multi-dimensional Heckscher–Ohlin setup. Suppose there are  $f$  factors,  $n$  sectors, and  $m$  countries. Index the factors, sectors and countries by  $j$ ,  $i$  and  $c$ , respectively. Each sector uses some or all of the factors as inputs, and produces one distinct consumption good as the output. For each sector  $i$ , the production technology is identical across countries, and characterized by constant returns to scale. All the domestic and international markets are perfectly competitive. Factors are mobile within a country, but immobile internationally.

If, in the above model, factors are allowed instead to move freely across country

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<sup>4</sup>The second caveat applies to testing the Dixit–Norman condition as well.

<sup>5</sup>They cited Demiroglu, U., 1997. ‘The Lens Condition: The Two Factor Case’, mimeo, University of Michigan and Qi, L., 1998. ‘Deardorff’s Condition for Factor Price Equalization in the Two-factor Case’, mimeo, Kyoto University. Both authors proved the same theorem that I prove here. However, my work is entirely independent, and through private conversation with Ufuk Demiroglu, I learned that both proofs were long and not easily understandable. I also learned that Demiroglu’s mimeo is unfinished.

<sup>6</sup>Arguably, given the caveats presented above and other problems such as transportation cost, the evidence is not overwhelming. However, I believe that their work is a useful first step.

<sup>7</sup>Davis and Weinstein (1998) find good empirical support for HOV among developed countries after considering such extensions as technological differences and non-FPE.

borders, the economy becomes the ‘integrated world economy’ (IWE). As argued by Dixit and Norman (1980), FPE can be achieved without international factor mobility if and only if the world outputs of the IWE can be replicated using the IWE production techniques while fully employing each factor in each country.

Let  $v$  be an  $n \times f$  matrix that has as its rows the factor usage vectors of the sectors of the IWE; i.e.  $v_{ij}$  is the amount of factor  $j$  demanded by sector  $i$  in the IWE. Call  $v$  the ‘IWE factor usage matrix.’ Let  $V$  be an  $m \times f$  matrix that has as its rows the factor endowment vectors of all the countries; i.e.  $V_{cj}$  is country  $c$ ’s endowment of factor  $j$ . Call  $V$  the ‘factor endowment matrix.’ Let  $u_n$  be a  $1 \times n$  row vector of ones, and  $u_m$  a  $1 \times m$  row vector of ones. Since all factors are fully employed in the IWE, the individual sectors’ demands for each factor must add up to the world endowment of that factor. In mathematical notation,  $u_n v = u_m V$  ( $v$  is said to be an ‘equal-sum matrix’ to  $V$ ).

One way of stating the Dixit–Norman condition formally is to define an operator ‘–FPE–’ as follows:

**Definition.** ((–FPE–))  $v$ –FPE– $V$  if and only if there exists an  $m \times n$  matrix,  $\lambda$ , (referred to as the ‘FPE matrix’) such that:

$$\lambda v = V \tag{1}$$

$$\forall c = 1, 2 \dots m \quad \text{and} \quad i = 1, 2 \dots n, \lambda_{ci} \in [0, 1] \tag{2}$$

$$u_m \lambda = u_n \tag{3}$$

The FPE matrix  $\lambda$  tells us how, to achieve FPE, the production of the world outputs of the IWE should be distributed across the  $m$  countries. Namely,  $\lambda_{ci}$  is the share of the IWE output of good  $i$  that country  $c$  has to produce using the IWE production technique. Condition (1) then says that after such a distribution, each factor in each country has to be fully employed. Condition (2) says that all the shares have to be between 0 and 1. Condition (3) says that for a given good  $i$ , the shares of all the countries have to add up to 1.

It turns out that the operator ‘–FPE–’ has the following important property:

**Lemma.** Let the  $n \times f$  matrix  $v$ ,  $q \times f$  matrix  $M$ , and  $m \times f$  matrix  $V$  be equal sum matrices such that  $v$ –FPE– $M$  and  $M$ –FPE– $V$ . Then  $v$ –FPE– $V$ ; i.e. the operator ‘–FPE–’ is transitive.

**Proof.** By definition of –FPE–, there exist a  $q \times n$  FPE matrix  $\lambda_1$  and an  $m \times q$  FPE matrix  $\lambda_2$  such that

$$\lambda_1 v = M$$

$$\lambda_2 M = V$$

$$u_q \lambda_1 = u_n, \quad u_m \lambda_2 = u_q \tag{4}$$

The first two equations yield:

$$\lambda_2 \lambda_1 v = V \tag{5}$$

Let  $\lambda_3 = \lambda_2 \lambda_1$ . It can be shown that

$$\lambda_3 \text{ is an FPE matrix} \tag{6}$$

(i) By definition of  $\lambda_3$

$$\lambda_{3ci} = \sum_p \lambda_{2cp} \lambda_{1pi} \quad \forall c = 1, 2, \dots, m, \quad i = 1, 2, \dots, n.$$

For each  $i$ , the sum of  $\lambda_{1pi}$  over  $p$  is 1 since  $\lambda_1$  is an FPE matrix. Therefore,  $\lambda_{3ci}$  is the weighted average of  $\lambda_{2cp}$ , all of which lie between 0 and 1. Thus, each element of the matrix  $\lambda_3$  lies between 0 and 1.

$$(ii) \text{ By (4), } u_m \lambda_3 = u_m \lambda_2 \lambda_1 = u_q \lambda_1 = u_n$$

Together with (5), (i) and (ii) establish claim (6). Because  $v$  and  $V$  are equal sum matrices,  $v$ -FPE- $V$ . QED

Next, I need to define ‘lens’, and formalize the ‘lens condition’. The definitions below follow Deardorff (1994).

Definition. ((lens)) Let  $a$  be an  $n \times f$  matrix. The lens spanned by the row vectors of  $a$  is defined to be:

$$L(a) = \{x \in R^f: x = b a \text{ for some } 1 \times n \text{ vector } b \text{ such that } b_k \in [0,1] \forall k = 1,2, \dots, n\}$$

When there are only two factors, the row vectors of  $a$  are two-dimensional, and  $L(a)$  can be represented by a polygon in the factor endowment box. Each side of the polygon corresponds to one row vector of  $a$ , and the polygon is symmetric about the diagonal of the box.<sup>8</sup>

Definition. ((lens condition)) Let  $v$  be an  $n \times f$  IWE factor usage matrix and  $V$  an  $m \times f$  factor endowment matrix; let  $v$  and  $V$  be equal sum matrices. Matrices  $v$  and  $V$  satisfy the lens condition if and only if  $L(V) \subseteq L(v)$ .

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<sup>8</sup>See, for example, Fig. 2 of Deardorff (1994).

### 3. The theorem, and the intuition

**Theorem.** *Let  $v$  be an  $n \times 2$  IWE factor usage matrix and  $V$  an  $m \times 2$  factor endowment matrix; let  $v$  and  $V$  be equal sum matrices. If  $L(V) \subseteq L(v)$ , then  $v - \text{FPE} - V$ .*

It may not come as a surprise that the lens condition is sufficient in the case of two factors. However, the Theorem is not trivial to prove, and the difficulty lies in having more than two countries.<sup>9</sup> When there are only two countries, no matter how many goods and factors we have, the lens condition ensures that it is possible to assign a fraction of the production of each sector to one country so that the country's factors are fully employed (this I would call an assignment plan). The fact that world factor supplies equal world factor demands then ensures that assigning the rest of world production to the other country also constitutes an assignment plan. Therefore, we have FPE. This, of course, is just what Deardorff (1994) has shown. When there are more than two countries, however, the above argument does not go through. Suppose there are three countries. To use the above argument, we need to make assignment plans for the first two countries. In general, neither plan is unique,<sup>10</sup> and their aggregate might not be feasible. For instance, the first plan may assign 50% of sector 1 production to country 1, while the second assigns 70% of sector 1 production to country 1.

To help illustrate the gist of the proof, I consider a case with three countries and four sectors (i.e.  $m = 3$  and  $n = 4$ ) in the remainder of this section. The exercise also reveals the economic intuition behind the proof.

Call the two factors capital and labor, and suppose the world endowment of capital and labor are  $V_{wk}$  and  $V_{wl}$ , respectively. A  $2 \times 2$  box can be drawn with width  $V_{wk}$  and length  $V_{wl}$ , as in Fig. 1a. In the graph,  $Ov_1, v_1v_2, v_2v_3$  and  $v_3O^*$  are the row vectors of the  $4 \times 2$  IWE factor usage matrix  $v$ , and represent good 1 through good 4, respectively. The polygon  $Ov_1v_2v_3O^*$  is the upper half of the lens  $L(v)$ .<sup>11</sup> Similarly,  $OV_1, V_1V_2$  and  $V_2O^*$  are the row vectors of the  $3 \times 2$  factor endowment matrix  $V$ , and represent country 1 through country 3, respectively. The polygon  $OV_1V_2O^*$  is the upper half of the lens  $L(V)$ . (See Fig. 1a.)

Imagine that, instead of having the original three countries ( $OV_1, V_1V_2$  and  $V_2O^*$ ), we have four,  $OA_1, A_1v_2, v_2v_3$ , and  $v_3O^*$ . It is straightforward to achieve FPE for these four countries: country  $OA_1$  produces  $Ov_1$  and  $v_1A_1$ , and countries  $A_1v_2, v_2v_3$ , and  $v_3O^*$  produce  $A_1v_2, v_2v_3$ , and  $v_3O^*$ , respectively. On the other

<sup>9</sup>The difficulty also lies in having more than two goods. However, viewing the problem from the country perspective enables us to get a better understanding of the proof presented below.

<sup>10</sup>If the factor usage vectors are linearly independent, then the assignment plans are unique. In this case, the lens condition is also sufficient. See Proposition 2 of Demiroglu and Yun (1999).

<sup>11</sup>Because the lens  $L(v)$  is symmetric about the diagonal of the box, I focus on what happens above the diagonal.

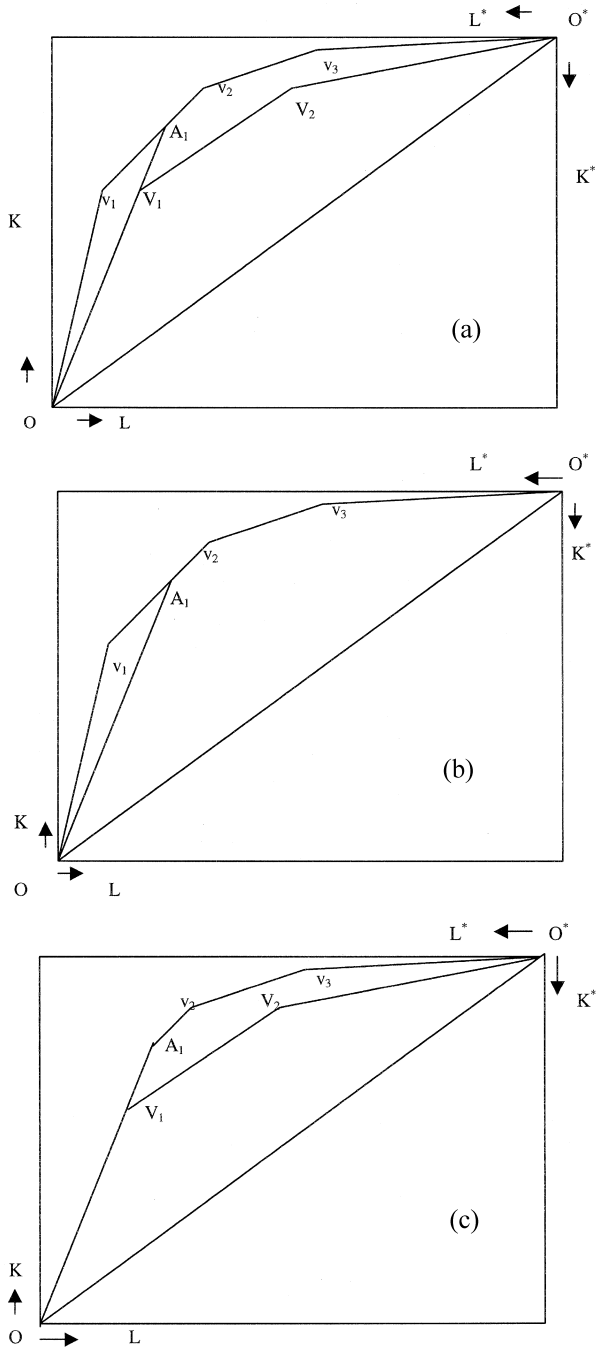


Fig. 1.

hand, the four new countries form a bigger lens than the original three countries, and so in a sense, have more dissimilar factor endowments. Intuition suggests that the more dissimilar factor endowments countries have, the more difficult, in some sense, it is to achieve FPE; as FPE is achievable for the four new countries, it should also be achievable for the original three countries. In fact, this is the intuition behind the Lemma about transitivity in Section 2.

In the following, I present the proof in this special case in a more rigorous manner.

The logic of the proof is as follows. First, I construct an intermediate lens  $L(M)$  such that  $L(V) \subseteq L(M) \subseteq L(v)$ . Second, I show that  $v$ -FPE- $M$ . Third, I use a proof-by-induction argument to show that  $M$ -FPE- $V$ . Finally, I apply the Lemma to show that  $v$ -FPE- $V$ .

Extend  $OV_1$  so that it intersects the polygon  $Ov_1v_2v_3O^*$  at  $A_1$ . The polygon  $OA_1v_2v_3O^*$  is the upper half of a new lens spanned by  $OA_1$ ,  $A_1v_2$ ,  $v_2v_3$  and  $v_3O^*$ . Denote this lens by  $L(M)$ . Clearly  $L(M)$  lies within  $L(v)$  but contains  $L(V)$ . Also, make the standard proof-by-induction assumption that the Theorem holds for the case of  $m=2$ . Call this Assumption (X).

First of all, imagine a new world with the same sectors but four fictional countries, as shown in Fig. 1b. In this world, the IWE factor usage matrix is still  $v$ , but the factor endowment matrix is  $M$ . Namely, the factor endowments of countries 1', 2', 3' and 4' are  $OA_1$ ,  $A_1v_2$ ,  $v_2v_3$  and  $v_3O^*$ , respectively. It is straightforward to achieve FPE in this world. Country 1' produces good 1 and a share  $(v_1A_1/v_1v_2)$  of the IWE output of good 2. Country 2' produces the remaining share  $(A_1v_2/v_1v_2)$  of the IWE output of good 2. Country 3' produces good 3 and country 4' produces good 4<sup>12</sup>. Therefore,  $v$ -FPE- $M$ .

Second, consider another world shown in Fig. 1c, where there are four fictional sectors and the same three original countries. The factor endowment matrix here is still  $V$ , but the IWE factor usage matrix is  $M$ . Namely, the factor usage vectors of sectors 1', 2', 3' and 4' are  $OA_1$ ,  $A_1v_2$ ,  $v_2v_3$  and  $v_3O^*$ , respectively<sup>13</sup>. To achieve FPE in this world, I could assign country 1, the most capital-abundant country, to

<sup>12</sup>The corresponding FPE matrix is

$$\lambda = \begin{pmatrix} 1 & \frac{v_1A_1}{v_1v_2} & 0 & 0 \\ 0 & \frac{A_1v_2}{v_1v_2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Of course, this is only one of the many possible allocations.

<sup>13</sup>Notice that the row vectors of  $M$  play the role of fictional countries in proving  $v$ -FPE- $M$  and now they play the role of fictional sectors.



produce nothing but good 1', the most capital-intensive good. By the construction of this new world, the factor usage vector of good 1' is proportional to, and at least as large as, the factor endowment vector of country 1, and therefore the factors of country 1 are fully employed. Now all I need to do is to assign the production of the remaining IWE outputs (whose factor usage vectors are  $V_1A_1$ ,  $A_1v_2$ ,  $v_2v_3$  and  $v_3O^*$ ) to the remaining two countries ( $V_1V_2$  and  $V_2O^*$ ), and I know I can do that by Assumption (X). Therefore,  $M$ -FPE- $V$ .

Finally, with  $v$ -FPE- $M$  and  $M$ -FPE- $V$ , the Lemma can be applied to show that  $v$ -FPE- $V$ .

Having presented the proof in this special case, let me proceed to articulate why this is a good route to take when the difficulty of the problem lies in having more than two countries. Briefly speaking, it is because the construction of the intermediate lens reduces the number of countries by 1, enabling us to use the proof-by-induction argument. Before the construction, there are three countries, and the endowment box is  $OO^*$ . After the construction, we need to show that: 1. When viewed as an endowment lens, the intermediate lens can achieve FPE with the original factor usage lens. 2. When viewed as a factor usage lens, the intermediate lens can achieve FPE with the original endowment lens. The first claim can be proved easily by looking at Fig. 1a; to prove the second claim, it is sufficient to show that FPE holds in the world characterized by the endowment box  $V_1O^*$ , the factor usage lens  $V_1A_1v_2v_3O^*$  and the endowment lens  $V_1V_2O^*$ . In this world there are only two countries ( $V_1V_2$  and  $V_2O^*$ ) rather than three, and so FPE holds by the proof-by-induction argument. With both statements proved, the Lemma about transitivity can then be applied.

#### 4. The proof

The proof is closely related to Fig. 2. Denote the row vectors of the  $n \times 2$  IWE factor usage matrix  $v$  by  $Ov_1, v_1v_2, \dots, v_{n-1}O^*$ , as in Fig. 2, and similarly those of the  $m \times 2$  factor endowment matrix  $V$  by  $OV_1, V_1V_2, \dots, V_{m-1}O^*$ .

The proof is by induction.

**(A). The Theorem holds for all  $n$  and  $m = 1$ .** When there is only one country, the country is the world itself, therefore the Theorem holds trivially.

**(B). Assume that the Theorem holds for all  $n$  and  $m - 1$ .** If it can be shown that the Theorem holds for all  $n$  and  $m$ , I am home.

As in Fig. 2, extend  $OV_1$  to intersect the polygon  $Ov_1v_2 \dots v_{n-1}O^*$  at  $A_1$ . Without loss of generality, assume that point  $A_1$  is on  $v_{k-1}v_k$ . Let  $\alpha = v_{k-1}A_1 / v_{k-1}v_k$  ( $\alpha \in [0, 1)$ ). It is straightforward to check that the  $(n - k + 2) \times 2$  matrix with row vectors  $OA_1, A_1v_k, v_kv_{k+1}, \dots, v_{n-1}O^*$  (denote the matrix by  $M$ ) is an 'equal-sum' matrix to both  $v$  and  $V$ .

**(C)  $v$ -FPE- $M$ .**

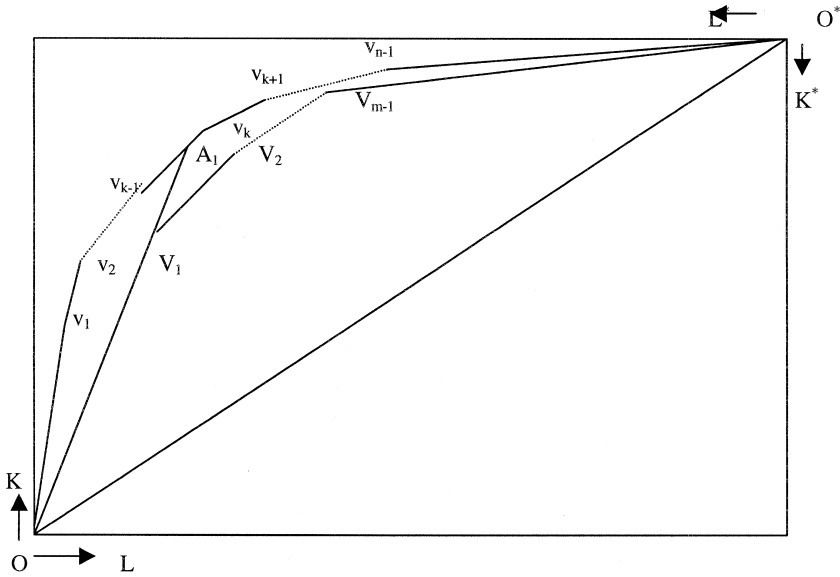


Fig. 2.

**Proof.** By examining Fig. 2, the following  $(n - k + 2) \times n$  matrix can be constructed:

$$\lambda = \begin{pmatrix} \underbrace{1 \dots 1}_{k-1} & \alpha & 0 & \dots & 0 & 0 \\ \underbrace{0 \dots 0}_{k-1} & 1-\alpha & 0 & \dots & 0 & 0 \\ \underbrace{0 \dots 0}_{k-1} & 0 & 1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & 0 & 1 \\ \underbrace{0 \dots 0}_{k-1} & 0 & 0 & \dots & 0 & 1 \end{pmatrix} \left. \begin{matrix} \right\} 2 \\ \left. \right\} n-k \end{matrix} \right.$$

Noting that  $A_1 v_k = (1 - \alpha) v_k - v_{k-1}$ , it is easy to check that  $\lambda$  is the FPE matrix needed so that  $v$ -FPE- $M$ ; namely,  $\lambda v = M$ ,  $u_{n-k+2} \lambda = u_n$ , and  $\lambda_{ci} \in [0,1] \forall c, i$ . QED

**(D) M-FPE-V**

**Proof.** Let  $\beta = OV_1/OA_1$ ,  $\beta \in (0,1]$ . Then  $V_1 A_1 = (1 - \beta) OA_1$ .

Consider the following  $(n - k + 2) \times 2$  matrix  $M_1$  and  $(m - 1) \times 2$  matrix  $W$ :

$$M_1 = \begin{pmatrix} V_1 A_1 \\ A_1 v_k \\ \vdots \\ v_{n-1} O^* \end{pmatrix}, \quad \text{and} \quad W = \begin{pmatrix} V_1 V_2 \\ V_2 V_3 \\ \vdots \\ v_{m-1} O^* \end{pmatrix}.$$

It is easy to check that they are ‘equal-sum’ matrices, and that  $L(W) \subseteq L(M_1)$ . Therefore, by (B), there exists an  $(m - 1) \times (n - k + 2)$  FPE matrix  $\lambda'$  such that  $M_1$ -FPE- $W$ .

Partition  $\lambda'$  as:

$$\lambda' = \left( \underbrace{\lambda_1}_1 \quad \underbrace{\lambda_{-1}}_{n-k+1} \right)$$

Since  $\lambda'$  is an FPE matrix, we have:

$$\begin{aligned} u_{m-1} \lambda_1 &= 1 \\ u_{m-1} \lambda_{-1} &= u_{n-k+1} \end{aligned} \tag{7}$$

Now I want to show that the following  $m \times (n - k + 2)$  matrix is the FPE matrix needed so that  $M$ -FPE- $V$ :

$$\begin{aligned} \lambda'' &= \begin{pmatrix} \beta & 0 \\ \underbrace{(1 - \beta)\lambda_1}_1 & \underbrace{\lambda_{-1}}_{n-k+1} \end{pmatrix} \\ \lambda'' M &= \begin{pmatrix} \beta & 0 \\ \underbrace{(1 - \beta)\lambda_1}_1 & \underbrace{\lambda_{-1}}_{n-k+1} \end{pmatrix} M = \begin{pmatrix} \beta O A_1 \\ \lambda' M_1 \end{pmatrix} = \begin{pmatrix} O V_1 \\ W \end{pmatrix} = V, \end{aligned} \tag{I}$$

where the second equality uses the fact that  $(1 - \beta)O A_1 = V_1 A_1$ , and the third equality follows from the definition of  $\beta$  and that  $\lambda'$  is an FPE matrix.

$$\begin{aligned} u_m \lambda'' &= (1 \quad u_{m-1}) \begin{pmatrix} \beta & 0 \\ \underbrace{(1 - \beta)\lambda_1}_1 & \underbrace{\lambda_{-1}}_{n-k+1} \end{pmatrix} \\ &= (\beta + (1 - \beta)u_{m-1} \lambda_1 \quad u_{m-1} \lambda_{-1}) = (1 \quad u_{n-k+1}) = u_{n-k+2} \end{aligned} \tag{II}$$

where the third equality uses Eq. (7).

$$\text{It is straightforward to check that each element of } \lambda'' \text{ is in } [0,1]. \tag{III}$$

By (I) (II) and (III), I have shown that  $\lambda''$  is the FPE matrix needed so that  $M$ -FPE- $V$ . QED

**(E)  $v$ -FPE- $V$**

**Proof.** The lemma, together with (C) and (D), implies that  $v$ -FPE- $V$ . QED

## 5. Conclusion

This paper proves the sufficiency of the lens condition for FPE in the case of two factors. Together with the work of Deardorff (1994) and Demiroglu and Yun (1999), this result helps form a clear picture: the lens condition is sufficient for factor price equalization in the case of two countries,<sup>14</sup> two factors, or two goods<sup>15</sup>. When there are more than two factors, Demiroglu and Yun (1999) show that the lens condition may be insufficient for FPE by providing two counter examples, both involving three factors.

Moreover, this result justifies the use of the two-dimensional factor endowment box to analyze factor price equalization for an arbitrary number of countries and goods. In addition, this result has empirical values in the context of testing the lens condition in the case of two factors. One such example is Debaere and Demiroglu (1998), who assess the similarity of country endowment proportions using capital and labor.

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<sup>14</sup>The sufficiency in the case of two countries is proved in Deardorff (1994).

<sup>15</sup>The sufficiency in the case of two goods is proved in Demiroglu and Yun (1999).