

A test for unbiased expectations based on qualitative survey data

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September 12, 2003

Abstract

This paper develops a unified econometric approach for testing unbiasedness of expectations based on direct observations on expectations obtained from qualitative survey data. An ordered multinomial probability model is employed to link unobserved expectations to observations on actual realizations and qualitative expectations. Under alternative survey data sampling schemes I show how to identify the probability model and estimate the parameters asymptotically efficient using an extended version of Berkson's Minimum Chi-square method, see, e.g., Amemiya (1976, JASA) and Gouriéroux (2000, *Econometrics of Qualitative Dependent Variables*, Cambridge University Press). The approach is simple and requires only familiarity with least squares regression techniques. A Wald test for unbiased expectation formation is derived and based on a small simulation study it demonstrates good finite sample properties. Finally, an application to the SRC consumer survey at the University of Michigan and to the British CBI Industrial Trends Surveys used by Pesaran (1987, *The Limits to Rational Expectations*, Basil Blackwell) is provided.

*All the computer codes developed for this paper is available upon request. The scientific notation used follows the standard proposed by Abadir and Magnus (2002).

1 Introduction

It is by now well established that expectations plays a central role in many different macroeconomic contents. While there is a vast literature on this topic, no consensus has emerged among empiricists on how to measure these subjective magnitudes. Empirical researchers in macroeconomics have almost always constructed simple proxies for expectations by applying autoregressive procedures. In such cases empirical tests of any expectation formation theory will be a joint tests of the theory under consideration and the validity of the proxy used. Furthermore, such procedures typically suffer from generated regressor problems and measurement errors, which can have severe adverse effects on the outcome if not properly accounted for. An alternative approach adopted has been to use direct measures on expectations from survey time series. There are now a number of survey time series and they provide information on expectations of several macroeconomic variables, including inflation rates, interest rates and unemployment rates. One advance using direct information on expectations is that conclusions concerning the expectation formation process become invariant to the choice of underlying behavioral model. However, in many situations the survey data set only provides ordinal measures of expectations in the form of categorical data, and some kind of quantification is needed in order to make comparisons to actual realizations. Leading quantification schemes are the subjective probability approach of Theil (1952) and Carlson and Parkin (1975) (TCP) and the regression model approach of Pesaran (1984,1987). Unfortunately, all existing quantification schemes re-introduce the generated regressor problem, which complicates tests of theories of expectation formation based on direct measures of expectations. Another complication typically overlooked is that the sampling interval of the expectations and the expectation horizon of the agents are not equal introducing serial correlation in the expectation errors.

This paper develops a unified econometric approach for testing the unbiasedness of expectations from qualitative survey data series that does not suffer from problem discussed above mainly because quantification and testing is done simultaneously. The approach is probabilistic sharing many characteristics with the TCP approach, however, the statistical inference is based upon an extended version of Berkson's Minimum Chi-square method, see e.g. Amemiya (1976,1985) and Gouriéroux (2000). The approach is simple and requires only familiarity with

least squares regression techniques.

2 Modelling qualitative survey data on expectations

Consider a survey data set containing observations on $j = 1, 2, \dots, J$ discrete outcomes of individual respondents expectations $y_{it+1|t} \in \mathbb{N}$ with respect to $y_{t+1} \in \mathbb{R}$, formed in period t . Let there be $i = 1, 2, \dots, N$ respondents. If respondent i anticipates y_{t+1} to fall within an interval $[\beta_{j-1}; \beta_j]$ then $y_{it+1|t} = j$ is observed. Assuming J categories, the survey data sampling scheme can be described according to

$$y_{it+1|t} = \sum_{j=1}^J j * 1_{(\beta_{j-1} \leq y_{it+1|t}^* < \beta_j)}. \quad (1)$$

where $y_{it+1|t}^* = E(y_{t+1} | \mathbf{I}_{it}) \in \mathbb{R}$, denotes the respondents quantitative expectations on the random variable y at time $t + 1$ formed in period t . It is assumed that the respondents quantitative expectations are unobservable. Suppose that $\mathbf{I}_{it} \subseteq \mathbf{I}_t$, that is, that respondent i possess only a proper subset of all information in period t . In this case expectations will not be informational efficient as defined by Fama (1970,1976). However, rational processing of information does not require full informational efficiency. If the respondents form expectations by correctly using the information they possess then their expectations are said to be rational. The respondents expectation formation can in this situation be formalized as

$$y_{it+1|t}^* = \delta_0 + \delta_1 E(y_{t+1} | \mathbf{I}_t) + \sigma \epsilon_{it+1}. \quad (2)$$

where unbiased information processing (or weak rationality) is attained for the parameter configuration $\{\delta_0, \delta_1\} = \{0, 1\}$. In general, the term $E(y_{t+1} | \mathbf{I}_t)$ could be any forecasting function, but for expository purposes and to avoid complicating the computations I continue by using $E(y_{t+1} | \mathbf{I}_t) = y_{t+1}$, and defer a discussions of the more general case to latter parts of the paper. Finally, let ϵ_{it+1} be an i.i.d. zero mean error term with probability function $F(x) = \Pr(\epsilon_{it+1} \leq x)$. One simplifying assumption of the model given by (1) and (2) relative to the TCP approach is that $\sigma_t = \sigma$ for all t . This implies, that despite having different information sets, all respondents on average predicts y_{t+1} equally well over time. Admittedly this assumption appears rather restrictive and proper testing should

be performed in empirical applications to check the assumption. This, however, will be straightforward within our purely parametric approach.

Based on (1) and (2) the probability of observing $y_{it+1|t} = j$, defined as $p_{jt} \equiv \Pr(y_{it+1|t} = j)$, will be given by $p_{jt} = F(\frac{\beta_j - \delta_0 - \delta_1 y_{t+1}}{\sigma}) - F(\frac{\beta_{j-1} - \delta_0 - \delta_1 y_{t+1}}{\sigma})$ or equivalently

$$\sum_{l=1}^j p_{lt} = F\left(\frac{\beta_j - \delta_0 - \delta_1 y_{t+1}}{\sigma}\right). \quad (3)$$

Equations (1)–(2) constitutes a classical example of a multinomial probability model. As it will be shown in the next section the probability model can be used to infer about the unobservable random variable $y_{it+1|t}^*$ based on realizations of $\{y_{it+1|t}, y_{t+1}\}$ and assumptions regarding their joint distribution function. As of particular interest, I employ the model to derive a test of the null hypothesis of weakly rational information processing given as

$$H_0 : \delta_0 = 0 \wedge \delta_1 = 1. \quad (4)$$

This hypothesis will also be referred to as the unbiasedness hypothesis, since it implies that the conditional mean of respondents subjective probability function equals the conditional mean of the objective probability function.

2.1 Modelling proportion categories data

The multinomial probability model given by (1)–(2) is derived under the assumption that the outcome of each of the N individual decisions are observed for every time period. Typically, however, observations on respondents individual perceptions or expectations are unavailable and only the proportion of respondents replying in each of the J categories is reported. Consequently, the model has to be redefined using the observed proportions: $\hat{p}_{jt} = \frac{1}{N} n_{jt}$, where $n_{jt} = \sum_{i=1}^N 1_{(y_{it+1|t}=j)}$ is the number of respondents in category j . If the probability function $F(\cdot)$ is a one-to-one mapping, equation (3) can be inverted as

$$F^{-1}\left(\sum_{j=1}^j p_{jt}\right) = \frac{\beta_j - \delta_0 - \delta_1 y_{t+1}}{\sigma}, \quad (5)$$

where F^{-1} is the inverse function of F . Using a Taylor approximations for expanding $F^{-1}(\sum_{i=1}^j \hat{p}_{it})$ about $\sum_{i=1}^j p_{it}$ a system of equations given as

$$F^{-1}\left(\sum_{l=1}^j \hat{p}_{lt}\right) = \frac{(\beta_j - \delta_0)}{\sigma} - \frac{\delta_1}{\sigma} y_{t+1} + v_{jt} + w_{jt}, \quad (6)$$

is obtained for $j = 1, 2, \dots, J - 1$, where $v_{jt} = f(F^{-1}(\sum_{l=1}^j p_{lt}))^{-1}(\sum_{l=1}^j \hat{p}_{lt} - \sum_{l=1}^j p_{lt})$, $w_{jt} = (f(F^{-1}(\sum_{l=1}^j p_{lt}^*))^{-1} - f(F^{-1}(\sum_{l=1}^j p_{lt}))^{-1})(\sum_{l=1}^j \hat{p}_{lt} - \sum_{l=1}^j p_{lt})$, $f(x) = \partial F(x)/\partial x$ and $\sum_{l=1}^j p_{lt}^*$ lies between $\sum_{l=1}^j \hat{p}_{lt}$ and $\sum_{l=1}^j p_{lt}$. Because w_{jt} is $O(n_{jt})$, the term can be ignored asymptotically (see, e.g., Amemiya (1985), Ch. 9). Notice, that v_{jt} will be heteroskedastic and dependent across the $J - 1$ equations. In particular, if $\mathbf{v}_t = (v_{1t}, v_{2t}, \dots, v_{J-1t})'$, and $\mathbf{p}_t = \left(\sum_{l=1}^1 p_{lt}, \sum_{l=1}^2 p_{lt}, \dots, \sum_{l=1}^{J-1} p_{lt}\right)'$, then

$$E(\mathbf{v}_t \mathbf{v}_k') = \begin{cases} \boldsymbol{\Omega}(\mathbf{p}_t) & t = k \\ \mathbf{O}_{J-1} & t \neq k \end{cases}, \quad (7)$$

for $\boldsymbol{\Omega}(\mathbf{p}_t) = \frac{1}{\mathbf{f}(\mathbf{F}^{-1}(\mathbf{p}_t))} [\mathbf{p}_t \odot \mathbf{I}_{J-1} - \mathbf{p}_t \mathbf{p}_t'] \frac{1}{\mathbf{f}(\mathbf{F}^{-1}(\mathbf{p}_t))}'$, where $\mathbf{f}(\mathbf{F}^{-1}(\mathbf{p}_t))$ is a $(J-1) \times (J-1)$ lower triangular matrix with typical element $(\mathbf{f})_{ij} = f(\mathbf{F}^{-1}((\mathbf{p}_t)_i)) 1_{(i \leq j)}$ and $\mathbf{F}^{-1}(\mathbf{p}_t) = (\mathbf{F}^{-1}((\mathbf{p}_t)_1), \dots, \mathbf{F}^{-1}((\mathbf{p}_t)_{J-1}))'$. Having specified the stochastic properties of the system given by (6), we can turn to estimation and testing.

3 Estimation and testing strategy

For estimation purposes, notice that the system of equations given by (6) can be written more compactly as

$$\mathbf{F}^{-1}(\hat{\mathbf{p}}_t) = \mathbf{X}_t \mathbf{S} \boldsymbol{\theta} + \mathbf{v}_t, \quad (8)$$

such that $\boldsymbol{\theta} = \left(\frac{\beta_1 - \delta_0}{\sigma}, \frac{\delta_1}{\sigma}, \frac{\beta_2 - \delta_0}{\sigma}, \dots, \frac{\beta_{J-1} - \delta_0}{\sigma}\right)'$, $\mathbf{X}_t = \mathbf{I}_{J-1} \otimes (1, y_{t+1})$ and the $2(J-1) \times J$ matrix \mathbf{S} is restricting the coefficient to y_{t+1} to be identical across all $J-1$ equations in the system given by (6).¹ By defining $\mathbf{A}(\hat{\mathbf{p}}_t) \mathbf{A}(\hat{\mathbf{p}}_t)' = \boldsymbol{\Omega}(\hat{\mathbf{p}}_t)$,

¹In the case $J=3$, $\mathbf{S} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$, and when $J=4$, $\mathbf{S} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 \end{bmatrix}$, etc.

the estimator of θ can be found as a solution to the following moment conditions

$$\frac{1}{T} \sum_{t=1}^T \mathbf{m}(\hat{\mathbf{p}}_t, \mathbf{X}_t \mathbf{S}; \hat{\theta}) = \mathbf{0}_J, \quad (9)$$

where $\mathbf{m}(\hat{\mathbf{p}}_t, \mathbf{X}_t \mathbf{S}; \theta) = (\mathbf{A}(\hat{\mathbf{p}}_t)^{-1} \mathbf{X}_t \mathbf{S})' (\mathbf{A}(\hat{\mathbf{p}}_t)^{-1} \mathbf{F}^{-1}(\hat{\mathbf{p}}_t) - \mathbf{A}(\hat{\mathbf{p}}_t)^{-1} \mathbf{X}_t \mathbf{S} \theta)$. The solution to (9) is the GLS estimator obtained by applying least squares to (8) after pre-multiplying by $\mathbf{A}(\hat{\mathbf{p}}_t)^{-1}$. Finally, the variance covariance matrix of $\hat{\theta}$ is obtained as $\text{var}(\hat{\theta}; \hat{\mathbf{p}}_t, \mathbf{X}_t \mathbf{S}) = \left[\sum_{t=1}^T \mathbf{S}' \mathbf{X}_t' \boldsymbol{\Omega}(\hat{\mathbf{p}}_t)^{-1} \mathbf{X}_t \mathbf{S} \right]^{-1}$.

Unfortunately, it is not possible to test H_0 given by (4) in a classical sense based on (8), since δ_0 and δ_1 are not identified. In what follows I will discuss two alternative approaches, that will enable estimation and testing, by imposing different identifying restrictions on the probability model. The identifying assumptions will depend critically on the nature of the survey data sampling scheme.

3.1 Test for unbiasedness based on known threshold values

It is easy to verify, that if the threshold parameters are known to the respondents, the parameters of probability model given by (1)–(2) are uniquely identified. This suggests that estimation and testing is possible if the qualitative data on expectations are sampled under pre-specified/known β_j for $j = 1, 2, \dots, J$ where $J > 2$. Examples of surveys that are sampled with known threshold values include the well known data set on price expectations collected by the Survey Research Center (SRC) at the University of Michigan.

Notice, that when β is known only 3 unknown parameters, given by δ_0, δ_1 , and σ , need to be estimated in the system of equations given by (8). Imposing the proper cross equation restrictions implied by the identifying assumption that β is known, yields

$$\mathbf{F}^{-1}(\hat{\mathbf{p}}_t) = \mathbf{X}_t \mathbf{S} \mathbf{R}(\beta) \theta^* + \mathbf{v}_t, \quad (10)$$

where

$$\mathbf{R}(\beta) = \begin{bmatrix} 1 & 0 & 0 & \frac{\beta_3 - \beta_2}{\beta_1 - \beta_2} & \frac{\beta_4 - \beta_2}{\beta_1 - \beta_2} & \cdot & \frac{\beta_{J-1} - \beta_2}{\beta_1 - \beta_2} \\ 0 & 1 & 0 & 0 & 0 & \cdot & 0 \\ 0 & 0 & 1 & \frac{\beta_1 - \beta_3}{\beta_1 - \beta_2} & \frac{\beta_1 - \beta_4}{\beta_1 - \beta_2} & \cdot & \frac{\beta_1 - \beta_{J-1}}{\beta_1 - \beta_2} \end{bmatrix}',$$

and $\boldsymbol{\theta}^* = \left(\frac{\beta_1 - \delta_0}{\sigma}, \frac{\delta_1}{\sigma}, \frac{\beta_2 - \delta_0}{\sigma} \right)'$. Let $\widehat{\boldsymbol{\theta}}^* = \left(\widehat{\theta}_1^*, \widehat{\theta}_2^*, \widehat{\theta}_3^* \right)'$ be the solution to (9), where $\mathbf{X}_t \mathbf{S}$ is replaced by $\mathbf{X}_t \mathbf{S} \mathbf{R}(\boldsymbol{\beta})$. By defining $\left(\boldsymbol{\delta}(\widehat{\boldsymbol{\theta}}^*), \sigma(\widehat{\boldsymbol{\theta}}^*) \right)' \equiv \left(\widehat{\delta}_0, \widehat{\delta}_1, \widehat{\sigma} \right)'$, the point estimates can be found as

$$\widehat{\delta}_0 = \beta_1 - \widehat{\theta}_1^* \left(\frac{\beta_1 - \beta_2}{\widehat{\theta}_1^* - \widehat{\theta}_3^*} \right), \quad (11)$$

$$\widehat{\delta}_1 = \widehat{\theta}_2^* \left(\frac{\beta_1 - \beta_2}{\widehat{\theta}_1^* - \widehat{\theta}_3^*} \right), \quad (12)$$

$$\widehat{\sigma} = \frac{\beta_1 - \beta_2}{\widehat{\theta}_1^* - \widehat{\theta}_3^*}. \quad (13)$$

The Wald test corresponding to the null hypothesis in (4) is then

$$W_I = \begin{bmatrix} \widehat{\delta}_0 \\ \widehat{\delta}_1 - 1 \end{bmatrix}' \text{var}(\boldsymbol{\delta}(\widehat{\boldsymbol{\theta}}^*); \widehat{\mathbf{p}}_t, \mathbf{X}_t \mathbf{S} \mathbf{R}(\boldsymbol{\beta}))^{-1} \begin{bmatrix} \widehat{\delta}_0 \\ \widehat{\delta}_1 - 1 \end{bmatrix} \stackrel{a}{\sim} \chi^2(2),$$

where $\text{var}(\boldsymbol{\delta}(\widehat{\boldsymbol{\theta}}^*); \widehat{\mathbf{p}}_t, \mathbf{X}_t \mathbf{S} \mathbf{R}(\boldsymbol{\beta})) = \left[\frac{\partial \widehat{\boldsymbol{\delta}}(\widehat{\boldsymbol{\theta}}^*)}{\partial \widehat{\boldsymbol{\theta}}^*} \right] \text{var}(\widehat{\boldsymbol{\theta}}^*; \widehat{\mathbf{p}}_t, \mathbf{X}_t \mathbf{S} \mathbf{R}(\boldsymbol{\beta})) \left[\frac{\partial \widehat{\boldsymbol{\delta}}(\widehat{\boldsymbol{\theta}}^*)}{\partial \widehat{\boldsymbol{\theta}}^*} \right]'$ and

$$\left[\frac{\partial \widehat{\boldsymbol{\delta}}(\widehat{\boldsymbol{\theta}}^*)}{\partial \widehat{\boldsymbol{\theta}}^*} \right] = \begin{bmatrix} \widehat{\theta}_3 \frac{(\beta_1 - \beta_2)}{(\widehat{\theta}_1 - \widehat{\theta}_3)^2} & 0 & -\widehat{\theta}_1 \frac{(\beta_1 - \beta_2)}{(\widehat{\theta}_1 - \widehat{\theta}_3)^2} \\ -\widehat{\theta}_2 \frac{(\beta_1 - \beta_2)}{(\widehat{\theta}_1 - \widehat{\theta}_3)^2} & \frac{(\beta_1 - \beta_2)}{(\widehat{\theta}_1 - \widehat{\theta}_3)} & \widehat{\theta}_2 \frac{(\beta_1 - \beta_2)}{(\widehat{\theta}_1 - \widehat{\theta}_3)^2} \end{bmatrix}.$$

3.2 Test for unbiasedness based on perceptions augmented surveys

Often proportion categories data on expectations as well as perceptions are collected by the surveyor. This richer data set can be used for identification in a manner very similar to Pesaran's (1984,1987) regression approach. In particular, if $y_{it|t} \in \mathbb{N}$ denotes categorical observations on individual perceptions, then by using the same line of arguments as in the derivation of (1) and (2), the probability model can be augmented with the following equations

$$y_{it|t}^* = \phi_0 + \phi_1 E(y_t | \mathbf{I}_t) + \sigma_\eta \eta_{it}, \quad (14)$$

$$y_{it|t} = \sum_{j=1}^J j * 1_{(\tilde{\beta}_{j-1} \leq y_{it|t}^* < \tilde{\beta}_j)}, \quad (15)$$

where η_{it} is an i.i.d. error term with probability function $G_\eta(x) = \Pr(\eta_{it} < x)$, and where it is assumed that $E(y_t | \mathbf{I}_t) = y_t$. For simplicity, let $G_\eta(x) = F(x)$.

Identification of the model given by the equations (1), (2), (14) and (15) can be obtained by assuming that perceptions are formed unbiased, such that $(\phi_0, \phi_1)' = (0, 1)'$ and that respondent's threshold values are identical for perceptions and expectations, i.e., $\tilde{\beta}_j = \beta_j$, for all $j = 1, 2, \dots, J - 1$. Under these identifying assumptions the unbiasedness hypothesis can be tested using a two-step estimation procedure to be described in the following section. However, before turning to estimation procedure notice that the perception augmented probability model can be viewed as a bivariate probability model and in the general case we could be interested in the modelling the joint probability function given by $\Pr(\epsilon_{it+1} \leq y, \eta_{it} < x,) = F(y, x, \rho)$ where ρ denotes the correlation between η_{it} and ϵ_{it+1} . Under the unbiasedness hypothesis, however, it is reasonable to assume that $\rho = 0$ implying that perception and expectation errors on average are uncorrelated and I will continue under this maintained hypothesis, implying that $\Pr(\epsilon_{it+1} \leq y, \eta_{it} < x,) = F(y) * F(x)$. Allowing $\rho \neq 0$ under the alternative hypothesis would be an interesting extension and could potentially improve upon the power properties of the Wald test statistics, however, such degree of generality is currently outside the scope of the present paper.

3.2.1 A two-step estimation procedure

Assuming that the survey data includes observations on expectation and perception proportions, denoted $\hat{\mathbf{p}}_t$ and $\tilde{\mathbf{p}}_t$ respectively, the unknown parameters of the model can be estimated by the following two-step procedure: 1. In the first step obtain an estimate of $\boldsymbol{\gamma} = \left(\frac{\beta_1}{\sigma_\eta}, \frac{1}{\sigma_\eta}, \frac{\beta_2}{\sigma_\eta}, \dots, \frac{\beta_{J-1}}{\sigma_\eta} \right)'$ as a solution to the moment conditions $\frac{1}{T} \sum_{t=1}^T \hat{\mathbf{m}}_t = \mathbf{0}_J$ where $\hat{\mathbf{m}}_t = \mathbf{m}(\hat{\mathbf{p}}_t, \mathbf{X}_{t-1} \mathbf{S}; \hat{\boldsymbol{\gamma}})$. 2. In the second step continue as described in Section 3.1 now using the estimated threshold parameters obtained in the first step. Let the estimate of $\boldsymbol{\theta}^*$ be a solution to $\frac{1}{T} \sum_{t=1}^T \hat{\mathbf{g}}_t = \mathbf{0}_3$ for $\hat{\mathbf{g}}_t \equiv \mathbf{m}(\hat{\mathbf{p}}_t, \mathbf{X}_t \mathbf{S} \mathbf{R}(\hat{\boldsymbol{\beta}}); \hat{\boldsymbol{\theta}}^*)$. Following Newey (1984), the variance covariance matrix of the estimated parameters $\hat{\boldsymbol{\Psi}} = [\hat{\boldsymbol{\theta}}^*, \hat{\boldsymbol{\Gamma}}']'$ for $\hat{\boldsymbol{\Gamma}} = \left(\hat{\boldsymbol{\beta}}', \hat{\sigma}_\eta \right)'$ under the two-step estimation procedure can then be obtained as

$$\hat{\mathbf{V}}_{\boldsymbol{\Psi}} = \frac{1}{T} \begin{bmatrix} \hat{\mathbf{G}}_{\boldsymbol{\theta}^*}^{-1} & -\hat{\mathbf{G}}_{\boldsymbol{\theta}^*}^{-1} \hat{\mathbf{G}}_{\boldsymbol{\Gamma}} \hat{\mathbf{M}}^{-1} \\ 0 & \hat{\mathbf{M}}^{-1} \end{bmatrix} \begin{bmatrix} \frac{1}{T} \sum_t \hat{\mathbf{g}}_t \hat{\mathbf{g}}_t' & \frac{1}{T} \sum_t \hat{\mathbf{g}}_t \hat{\mathbf{m}}_t' \\ \frac{1}{T} \sum_t \hat{\mathbf{m}}_t \hat{\mathbf{g}}_t' & \frac{1}{T} \sum_t \hat{\mathbf{m}}_t \hat{\mathbf{m}}_t' \end{bmatrix} \times \begin{bmatrix} \hat{\mathbf{G}}_{\boldsymbol{\theta}^*}^{-1} & -\hat{\mathbf{G}}_{\boldsymbol{\theta}^*}^{-1} \hat{\mathbf{G}}_{\boldsymbol{\Gamma}} \hat{\mathbf{M}}^{-1} \\ 0 & \hat{\mathbf{M}}^{-1} \end{bmatrix}', \quad (16)$$

where

$$\begin{aligned}
\widehat{\mathbf{G}}_{\theta^*} &= \frac{1}{T} \sum_t^T \nabla_{\theta^*} \widehat{\mathbf{g}}_t \\
&= \frac{1}{T} \sum_t^T \mathbf{R}(\widehat{\boldsymbol{\beta}})' \mathbf{S}' \mathbf{X}'_t \boldsymbol{\Omega}(\widehat{\boldsymbol{\rho}}_t)^{-1} \mathbf{X}_t \mathbf{S} \mathbf{R}(\widehat{\boldsymbol{\beta}}), \\
\widehat{\mathbf{G}}_{\Gamma} &= \frac{1}{T} \sum_t^T \nabla_{\Gamma} \widehat{\mathbf{g}}_t \\
&= \frac{1}{T} \sum_t^T \left[\left(\mathbf{F}^{-1}(\widehat{\boldsymbol{\rho}}_t) - \mathbf{X}_t \mathbf{S} \mathbf{R}(\widehat{\boldsymbol{\beta}}) \widehat{\boldsymbol{\theta}} \right)' \boldsymbol{\Omega}(\widehat{\boldsymbol{\rho}}_t)^{-1} \mathbf{X}_t \mathbf{S} \otimes \mathbf{I} \right] \left[\frac{\partial \text{vec}(\mathbf{R}(\boldsymbol{\beta}))}{\partial \boldsymbol{\Gamma}'} \right]_{\boldsymbol{\Gamma}=\widehat{\boldsymbol{\Gamma}}} \\
&\quad + \frac{1}{T} \sum_t^T \left[\widehat{\boldsymbol{\theta}}' \otimes \mathbf{R}(\widehat{\boldsymbol{\beta}})' \mathbf{S}' \mathbf{X}'_t \boldsymbol{\Omega}(\widehat{\boldsymbol{\rho}}_t)^{-1} \mathbf{X}_t \mathbf{S} \right] \left[\frac{\partial \text{vec}(\mathbf{R}(\boldsymbol{\beta})')}{\partial \boldsymbol{\Gamma}'} \right]_{\boldsymbol{\Gamma}=\widehat{\boldsymbol{\Gamma}}}, \\
\widehat{\mathbf{M}} &= \frac{1}{T} \sum_t^T \nabla_{\Gamma} \widehat{\mathbf{m}}_t \\
&= \frac{1}{T} \sum_t^T \left[\mathbf{S}' \mathbf{X}'_{t-1} \boldsymbol{\Omega}(\widehat{\boldsymbol{\rho}}_t)^{-1} \mathbf{X}_{t-1} \mathbf{S} \right] \left[\frac{\partial \widehat{\boldsymbol{\gamma}}}{\partial \boldsymbol{\Gamma}'} \right]_{\boldsymbol{\Gamma}=\widehat{\boldsymbol{\Gamma}}},
\end{aligned}$$

and

$$\left[\frac{\partial \widehat{\boldsymbol{\gamma}}}{\partial \boldsymbol{\Gamma}'} \right]_{\boldsymbol{\Gamma}=\widehat{\boldsymbol{\Gamma}}} = \frac{1}{\widehat{\sigma}_\eta} \left[\begin{pmatrix} \mathbf{I}_{J-1} \\ \mathbf{0}'_{J-1} \end{pmatrix} ; -\widehat{\boldsymbol{\gamma}} \right].$$

Having obtained the second-step estimates of δ_0 and δ_1 from (11) and (12) and the estimated variance covariance matrix $\widehat{\mathbf{V}}_\Psi$, the Wald test can be computed as

$$W_{II} = T \begin{bmatrix} \widehat{\delta}_0 \\ \widehat{\delta}_1 - 1 \end{bmatrix}' \left(\left[\frac{\partial \boldsymbol{\delta}}{\partial \boldsymbol{\Gamma}'} \right]_{\boldsymbol{\Gamma}=\widehat{\boldsymbol{\Gamma}}} \widehat{\mathbf{V}}_\Psi \left[\frac{\partial \boldsymbol{\delta}}{\partial \boldsymbol{\Gamma}'} \right]'_{\boldsymbol{\Gamma}=\widehat{\boldsymbol{\Gamma}}} \right)^{-1} \begin{bmatrix} \widehat{\delta}_0 \\ \widehat{\delta}_1 - 1 \end{bmatrix},$$

where

$$\left[\frac{\partial \boldsymbol{\delta}}{\partial \boldsymbol{\Gamma}'} \right] = \left[\left[\begin{array}{ccccc} \theta_3^* \frac{(\beta_1 - \beta_3)}{(\theta_1^* - \theta_3^*)^2} & 0 & -\theta_1^* \frac{(\beta_1 - \beta_3)}{(\theta_1^* - \theta_3^*)^2} & 1 - \frac{\theta_1^*}{(\theta_1^* - \theta_3^*)} & \frac{\theta_1^*}{(\theta_1^* - \theta_3^*)} \\ -\theta_2^* \frac{(\beta_1 - \beta_3)}{(\theta_1^* - \theta_3^*)^2} & \frac{(\beta_1 - \beta_3)}{(\theta_1^* - \theta_3^*)} & \theta_2^* \frac{(\beta_1 - \beta_3)}{(\theta_1^* - \theta_3^*)^2} & \frac{\theta_2^*}{(\theta_1^* - \theta_3^*)} & \frac{-\theta_2^*}{(\theta_1^* - \theta_3^*)} \end{array} \right] ; \mathbf{0}_{2 \times J-2} \right].$$

W_{II} is distributed as W_I asymptotically.² The derivation of the W_I and W_{II} statistics, and the variance matrix of $\widehat{\boldsymbol{\delta}}$ in particular, is based upon the so-called

²Regarding the computation of $\partial \text{vec}(\mathbf{R}(\boldsymbol{\beta})') / \partial \boldsymbol{\Gamma}'$: For example, when $J=3$

$$\left[\frac{\partial \text{vec}(\mathbf{R}(\boldsymbol{\beta})')}{\partial \boldsymbol{\Gamma}'} \right] = \mathbf{0}_{9 \times J},$$

delta method, involving first order expansions of the variance matrices of $\widehat{\boldsymbol{\gamma}}$ and $\widehat{\boldsymbol{\theta}}^*$ respectively. The finite sample properties of the tests can therefore be expected to work most favorable in situations where approximate linearity hold for all likely values of the estimated parameters. In the next section we investigate the finite sample properties of the tests somewhat further by a Monte Carlo simulation study. I propose a simple bootstrap procedure to obtain small sample refinements of the asymptotic distributions of W_I and W_{II} with very rewarding results.

4 Simulation study of size and power properties

To provide a simple illustration the finite sample properties of W_I and W_{II} data is generated from the model given by (1), (2), (14) and (15) for different values of $\boldsymbol{\delta}$ assuming $\boldsymbol{\beta} = (-0.5, 0.5, 1.5)'$, $\sigma = 1$, $\sigma_\eta = 1$, and that y_t follows a stationary first order autoregressive process with i.i.d. $N(0, 1)$ innovations.³ I will focus on the small sample properties of the tests by setting $T = 50$. To obtain better small sample properties of the pivotal test statistics W_I and W_{II} - and to follow the recommendations of Horowitz (2001) - bootstrap p-values will be used. For the W_{II} test statistic such p-values can be computed in the following steps: 1. Estimate the unrestricted model, obtain $\widehat{\boldsymbol{\beta}}$, $\widehat{\sigma}$ and $\widehat{\sigma}_\eta$ and compute $W_{II} = W_{II}(\widehat{\boldsymbol{p}}_t, \widetilde{\boldsymbol{p}}_t, \mathbf{X}_t)$. 2. Resample ϵ_{it+1}° and η_{it}° from the distributions $N(0, \widehat{\sigma}^2)$ and $N(0, \widehat{\sigma}_\eta^2)$ and construct $\widehat{\boldsymbol{p}}_t^\circ$ and $\widetilde{\boldsymbol{p}}_t^\circ$, with typical element $\widehat{p}_{jt}^\circ = \frac{1}{N} \sum_{i=1}^N 1_{(y_{it+1}^\circ=j)}$ and $\widetilde{p}_{jt}^\circ = \frac{1}{N} \sum_{i=1}^N 1_{(y_{it}^\circ=j)}$, under the null hypothesis that $\boldsymbol{\delta} = (0, 1)'$ using (1),

while for $J = 4$

$$\left[\frac{\partial \text{vec}(\mathbf{R}(\boldsymbol{\beta})')}{\partial \boldsymbol{\Gamma}'} \right] = \left[\mathbf{O}'_{9 \times J} : \begin{bmatrix} -\frac{(\beta_3 - \beta_1)}{(\beta_1 - \beta_2)^2} & \frac{(\beta_3 - \beta_1)}{(\beta_1 - \beta_2)^2} & \frac{1}{(\beta_1 - \beta_2)} & 0 \\ 0 & 0 & 0 & 0 \\ \frac{(\beta_3 - \beta_2)}{(\beta_1 - \beta_2)^2} & \frac{(\beta_1 - \beta_3)}{(\beta_1 - \beta_2)^2} & \frac{-1}{(\beta_1 - \beta_2)} & 0 \end{bmatrix} \right]',$$

e.t.c.

³This parameter configuration was chosen to avoid "zero" response frequencies in each category leading to numerical instability of the estimation procedure. A wide range of alternative parameter configurations was also analyzed, but the the size and power properties did not change significantly and the results therefore left out. These results, however, are available upon request.

Table 1: Size and power properties of W_I and W_{II} . Bootstrapped rejection frequencies ($\times 100$) at a 10% significance level using 1000 Monte Carlo replications. Sample size equals 50 observations. Bootstrap resamples equals 200.

		$\delta_1(\delta_0 = 0)$								
		0.7	0.8	0.9	0.95	1	1.05	1.1	1.2	1.3
W_I	100	100	100	100	88	12	95	100	100	100
W_{II}	100	98	98	84	46	9	36	74	88	98

		$\delta_0(\delta_1 = 1)$						
		-0.2	-0.1	-0.05	0	0.05	0.1	0.2
W_I		100	100	97	12	98	100	100
W_{II}		100	99	78	9	88	96	100

(2), (14), (15), and $\beta = \hat{\beta}$.⁴ 3. Compute $W_{II,1}^\diamond = W_{II}(\hat{\mathbf{p}}_t^\diamond, \tilde{\mathbf{p}}_t^\diamond, \mathbf{X}_t)$. 4. Repeat steps 2 and 3 up to $R - 1$ times and obtain $\{W_{II,r}^\diamond\}_{r=2}^R$. 5. The bootstrap p-value is equal to $\frac{1}{R} \sum_{r=1}^R 1_{(W_{II,r}^\diamond > W_{II})}$. The procedure is similar for W_I except that only $\hat{\sigma}$ needs to be obtained in step 1 as β is known. Furthermore, since W_I does not depend on $\tilde{\mathbf{p}}_t$ only ϵ_{it+1}^\diamond has to be resampled in step 2. For each Monte Carlo replication, the bootstrap p-value of step 5 is computed, and the bootstrap rejection frequency is the percentage number of replications in which the bootstrap p-value is less than, say, 10%.

In Table 1 the results from the simulation study is reported. The rejection frequencies of the W_I and W_{II} tests based on the bootstrap p-values when $\delta = (0, 1)$ is 12% and 9%. Hence the actual size of the tests are very close to the 10% nominal significance level as expected. Another important characteristic is that the power of the W_I statistic seems to be uniformly higher than the power of the W_{II} . Since there is more uncertainty involved when β is treated as unknown, as it is the case when computing the W_{II} statistic, the resulting loss in power should be expected. Overall, the finite sample power and size properties of the tests using the bootstrap p-values are very encouraging.

⁴It is crucial that N is large, in order to get precise simulated values of $\hat{\mathbf{p}}_{jt}^\diamond$ and $\tilde{\mathbf{p}}_{jt}^\diamond$. I used $N = 500$. In applications one should use the same N as in the survey data sampling scheme.

Table 2: Test for unbiased expectations using Identification Scheme I based on the SRC data 1978M1 - 2000M12. Numbers in paranthesis are asymptotic standard errors. For the W_I statistic bootstrapped p-values are reported using 1000 resamples. MSE equals the mean value of the squared prediction errors over the estimation period.

	Student		Logistic		Uniform		Normal	
δ_0	-0.425	(0.613)	-2.575	(0.813)	1.826	(0.208)	-0.016	(0.513)
δ_1	0.781	(0.082)	1.086	(0.110)	0.474	(0.028)	0.725	(0.069)
σ	7.902		8.322		3.812		7.475	
W_I	0.000		0.000		0.000		0.000	
MSE	2.339		4.885		2.619		2.150	

5 Empirical illustrations

To illustrate the potential usefulness of the proposed econometric framework we first reexamines price expectations data from the SRC consumer survey and apply the techniques described in Section 3.1 to test this survey series for rationality in terms of unbiasedness. Secondly, a two-step procedure, outlined in Section 3.2, is employed to test the unbiasedness hypothesis based on Pesaran's (1987) perceptions augmented survey data on average selling prices of firms in the British manufacturing sector.

5.1 The SRC expected price change data

The SRC consumer data comes from a survey of randomly selected U.S. households and includes survey respondents expectations on several economic variables. One of the questions asked to the survey respondents concerns their expectations of the rate of price change over the next year. The respondents can answer in the following nine categories: a. Same or down, b. Up by 1-2%, c. Up by 3-4%, d. Up by 5%, e. Up by 6-9%, f. Up by 10-14%, g. Up by more that 15%, h. Don't know (DK) how much up , i. DK. Starting in 1978 the series is published as proportion categories data and is available on a monthly basis. For simplicity I discard the categories h. and i. and adjust the remaining proportions accordingly. In order to avoid zero-valued proportions the categories f. and g. are added together. The

known threshold parameters are then given as $\beta = (0.5, 2.5, 4.5, 5.5, 9.5)'$.⁵

In Table 1 the estimation and test results based on the SRC data on direct expectations are reported for different choices of probability functions $F(\cdot)$. It is noticeable that for the Student and Normal distributions it is not possible to reject the hypothesis that $\delta_0 = 0$. However, for all probability functions, except the logistic, $\hat{\delta}_1$ is significantly below one. Based on the W_I statistic and the bootstrap p-values (as well as the asymptotic p-values which is not reported) the joint null hypothesis $H_0 : \delta_0 = 0 \wedge \delta_1 = 0$ is therefore rejected strongly.

In Figure 1 the quantified inflation expectations one year ahead from all four models are compared to the realized inflation rate. The graph confirms that the strong rejection of unbiasedness hypothesis is a result of respondents under-predicting the actual inflation rate one year ahead. Finally, it is important to notice that the rejection of the unbiasedness hypothesis is closely tied to the use of y_{t+1} in equation (2). Including more realistic forecasting functions may give rise to different and perhaps more insightful results. This will be an important venue for future research.

5.2 Pesaran's (1987) perceived and expected price change data

Pesaran (1987) uses the CBI Industrial Trends Survey to demonstrate his so-called regression approach for quantification of qualitative survey data.⁶ As in the two-step estimation procedure, Pesaran's quantification approach uses proportion categories observations on perceptions as well as on expectations.

The actual inflation rate used in Pesaran's model is depicted in Figure 2. One very characteristic feature is the apparent shift in the inflation rate process after 1974, where the mean and the variance seems to have increased substantially. Unfortunately, a regime shift in the inflation processes, that is not perfectly observable, introduces ex post bias as shown by Evans and Wachtel (1993) and Dahl and Hansen (2001) and makes inference based on a comparison of ex ante forecast from survey series and actual realizations based on ex post realizations ambiguous.

⁵Alternative threshold values consistent with the specified categories was tried, without changing the results.

⁶For a more detailed description of the CBI survey data and the regression model approach, see, e.g., Pesaran (1987).

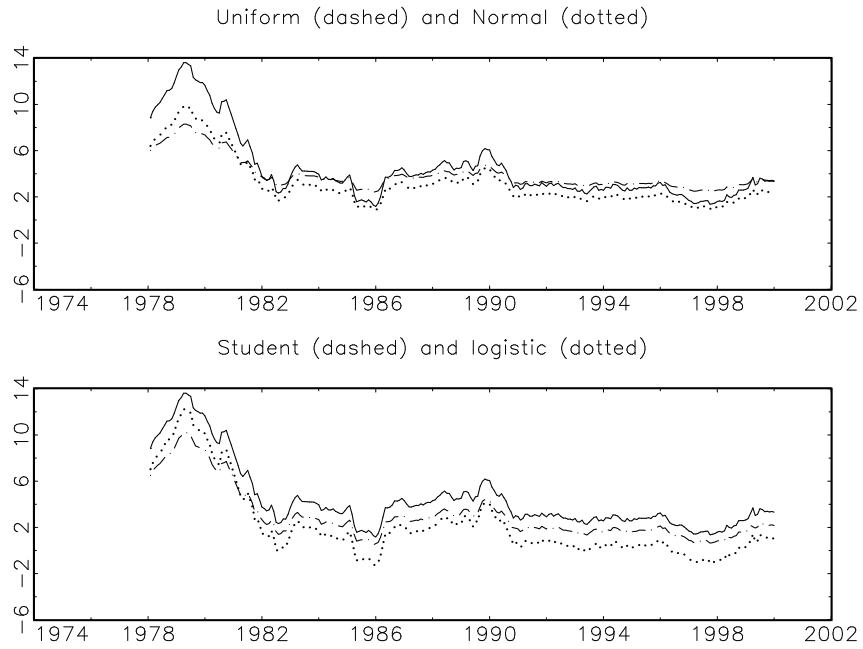


Figure 1: Quantification of inflation expectations using the SRC data for the period 1978M1-2000M12. The solid line is the actual inflation rate.

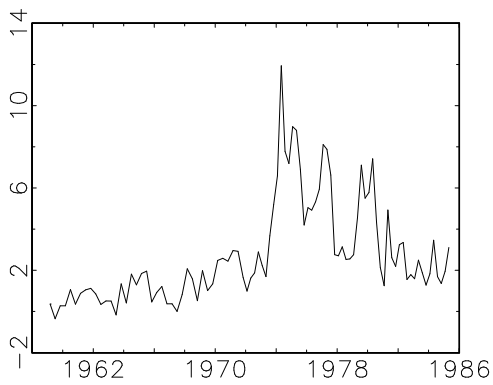


Figure 2: Actual rates of inflation in British manufacturing

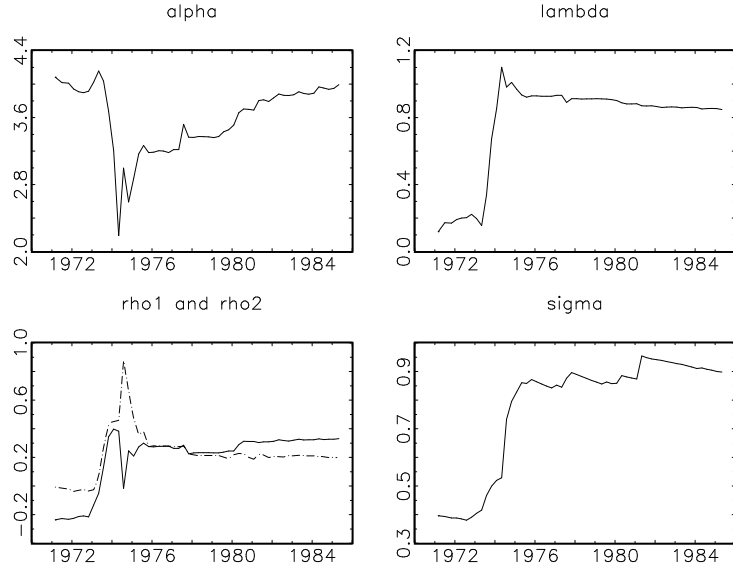


Figure 3: Parameter stability of Pesaran's (1987) preferred model.

As I will argue next, the consequences of the regime shift in terms of non-constant parameter estimates, seems to be present in Pesaran's preferred model represented as

$$\begin{aligned} (1 - \hat{\lambda}\tilde{p}_{1t})y_t &= \hat{\alpha}\tilde{p}_{1t} + \hat{u}_t, \\ \hat{u}_t &= \hat{\rho}_1\hat{u}_{t-1} + \hat{\rho}_2\hat{u}_{t-2} + \hat{\sigma}\omega_t, \end{aligned}$$

where y_t is the actual rate of inflation and \tilde{p}_{1t} is the proportion of firms who reported a rise in prices. By estimating the model recursively one can get a good sense of the stability of the parameters. This exercise reveals that all of the estimated parameters change dramatically after 1974 as shown in Figure 3. The instability of the model is also confirmed by the $ExpLm_{c=\infty} = 8.8910$ test for unknown breakpoint, suggested by Andrews and Ploberger (1994) and Nyblom's (1989) stability test that equals 1.52. The 5% critical values of the two statistics are 4.88 and 1.50 respectively. Consequently, both test statistics reject parameter stability associated with Pesaran's regression model. To avoid the unpleasant consequences of possible regime shifts in the inflation process and to keep the analysis simple for illustrational purposes only the period 1959-1974 is considered when testing the unbiasedness hypothesis based on the W_{II} test.

Table 3: Test for unbiased expectations using Identification Scheme II based on the Pesaran's (1987) data for the sub-period 1959-1974. Numbers in paranthesis are asymptotic standard errors. For the W_{II} statistic bootstrapped p-values are reported using 1000 resamples. MSE equals the mean value of the squared prediction errors over the estimation period.

	Student		Logistic		Uniform		Normal	
β_1	-0.923	(1.667)	-0.952	(1.061)	-2.702	(1.126)	-1.019	(1.492)
β_2	2.086	(0.465)	2.210	(0.268)	3.215	(0.198)	2.055	(0.396)
σ_η	3.038		2.062		2.698		3.257	
δ_0	-0.352	(0.845)	-0.592	(1.182)	0.192	(0.212)	-0.246	(0.682)
δ_1	1.251	(0.589)	1.395	(0.714)	0.905	(0.175)	1.244	(0.635)
σ	3.931		2.977		2.523		4.289	
W_{II}	0.951		0.907		0.336		0.966	
MSE	0.048		0.126		0.012		0.047	

The estimation and test results for the various probability functions are presented in Table 3. The estimated values for threshold parameters are almost identical for the Student, the Logistic and the Normal distributions and the estimates seem plausible from an economic perspective: If respondents expect prices to fall more than 1 percent they report that prices fall, and similarly, if respondents expect prices to increase by more than 2 percent over the next time period they report that prices will go up. This indicates that the so-called difference limen defined as the interval between the estimated threshold values, appears to be asymmetric, something which has often been ignored in the literature. The estimated threshold parameters based on the Uniform distribution are somewhat higher numerically, but still supporting the view that the difference limen might be asymmetric around zero.

Based on estimated threshold parameters the second step estimates of δ becomes close to $(0, 1)'$ for all of the probability models in Table 3. The W_{II} test statistic clearly supports this view, and based on the bootstrap p-values it is not possible to reject the unbiasedness hypothesis for any of the models in Table 3.

In Figure 4 the quantified expectations are drawn along with actual inflation

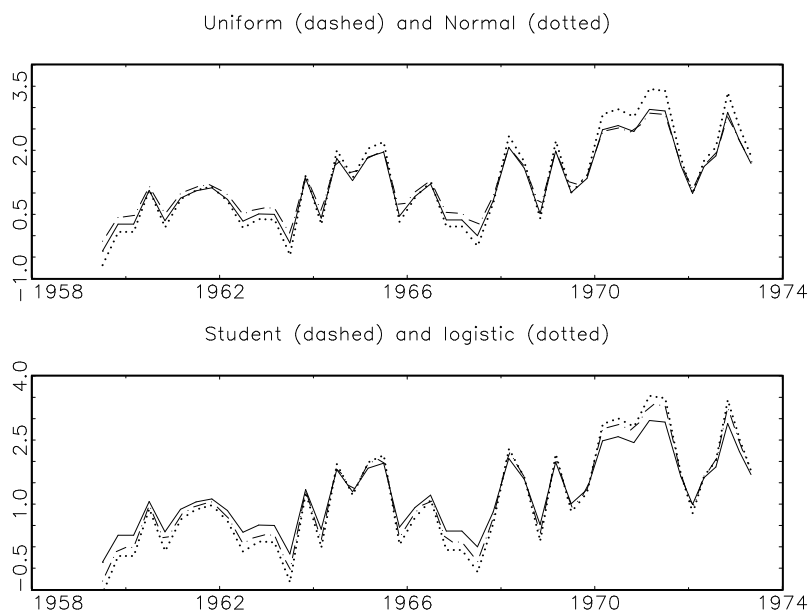


Figure 4: Quantification of inflation expectations using Pesaran’s (1987) perceptions augmented survey data for the sub-period 1959-1974. The solid line is the actual inflation rate.

rates. The inflation expectation measure derived from the Uniform distribution seems to follow the actual rate of inflation closest, and also has the lowest MSE as can be seen from Table 3. However, broadly viewed all the measures of quantified expectations seems to perform quite well, indicating some degree of robustness to the choice of probability function.

6 Conclusion

A new and relative simple parametric approach for testing the unbiasedness hypothesis based on direct observations on expectations obtained from qualitative surveys has been proposed. The finite sample properties of the proposed tests are analysed in a simulation study and the results seems promising. The approach also facilitates quantification of qualitative expectation measures. The approach will work for $J \geq 3$ proportion categories and for any choice of invertible probability functions. The potential strenght of the new approach is finally illustrated

by two empirical applications.

One interesting extension outside the scope of the present paper is to consider the effects of including general forecasting functions in (2), that may depend on estimated parameters. This might be a potential tool for testing the unbiasedness hypothesis in the presence of regime shift. By including, say, a real time forecasting function, the test for unbiasedness will not rely on a direct comparison of ex ante forecast and ex post realizations and the possible adverse effects of ex post bias on the size of existing test will not be present.

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