

Quantification of Qualitative Survey Data and Tests of Consistent Expectations: A New Likelihood Approach*

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Abstract

In this paper, we develop a likelihood approach for quantification of qualitative survey data on expectations and perceptions and we propose a new test for expectation consistency (unbiasedness). Our quantification scheme differs from existing methods primarily by using prior information (perhaps derived from economic theory or well established empirical relations) on the underlying process driving the variable of interest. To investigate the properties of our novel quantification scheme and to analyze the size and power properties of the new expectation consistency test, we perform Monte Carlo simulation studies. Overall, the simulation results are very encouraging and show that efficiency gains from including prior information can be substantial relative to existing quantification schemes. Finally, we provide an empirical illustration. We argue that a nonlinear regime switching model, historically, provides a more adequate characterization of prices changes in the British manufacturing industry relative to a wide range of alternative linear representations. We then show how to use this information to quantify the qualitative survey data on price change expectations and perceptions from the CBI survey in the U.K. manufacturing industry. Our findings support

*Comments and suggestions from John Carlson and an anonymous referee are gratefully acknowledged. The notation used follows the "new" standard proposed by Abadir and Magnus (2002). The software developed for this paper is available from the authors upon request.

the existence of so-called ex post bias, i.e., that rationally formed forecasts might appear biased when compared to ex post realizations. As a result, tests of expectation consistency by comparing forecasts with realizations become invalid. The new test we propose, however, is not affected by the existence of ex post bias and by applying this statistic we cannot reject the expectation consistency hypothesis.

1 Introduction

The analysis on how economic agents form their expectations on economic variables has been treated as one crucial issue in explaining many important economic phenomena. Although there seems to be no consensus among economists as to whether the true expectation formations are rational, adaptive or mixed, the rational expectations hypothesis has been more and more accepted by macroeconomists and is playing a central role in contemporary macroeconomic models. Both new classical economists and later new Keynesian proponents take the rational expectations hypothesis as one of the key assumptions in their respective modeling frameworks. While the rational expectations hypothesis has been applied to numerous economic models, its validity is nevertheless very difficult to verify empirically because tests on the hypothesis and further analysis on the expectation formation process require direct observations on such expectations, which are rarely available.

Many empirical researchers in the past have tried to construct proxy measures for such subjective expectations. Such proxies are usually susceptible to the underlying economic theories, and the empirical tests of any expectations formation theories are in fact joint tests of the economic theory under consideration as well as the validity of the proxies constructed. Subsequently, rejection of the tests might result from the inappropriate choices of proxies, rather than from the falsehood of the hypotheses. Obviously, this potentially possibility of Type I errors is the weakness of the methodology. In addition, procedures using proxies typically suffer from generated regressors problems and measurement errors, which can have unfortunate effects if not properly accounted for.¹ In short, testing hypothesis of expectation formation using proxies is not without drawbacks.

¹On this argument, see Pagan (1984).

Alternative approaches adopted by many researchers to circumvent these problems have been to obtain direct measures of expectations from expectation survey data. With the availability of survey evidence on economic agents' expectations, it is possible to conduct direct tests on rational expectations hypothesis without being concerned with the problems mentioned above. However, in most situations information on expectations obtained from surveys is qualitative instead of quantitative. Survey respondents are only questioned about the expected directions of, say, changes of the variable in question, e.g., whether the variable of interest is expected to "go up", "stay the same", or "go down". This is mainly due to the fact that surveys that require respondents to give point forecasts for the variables in question are more likely to be susceptible to sampling and measurement errors than the surveys that only question about the expected directions of changes, see, e.g., Katona (1975) and Pesaran (1987). Often qualitative variables on expectations are of limited interest since they are difficult to interpret and methods of quantification are therefore needed. While there is a vast literature on this topic, no consensus has emerged among researchers on how to quantify the expectation survey data. The two prevalent pioneering quantification approaches are the subjective probability approach by Theil (1952) and Carlson and Parkin (1975) and the regression model approach by Pesaran (1984, 1987).

Being purely statistical methods, the above mentioned quantification schemes work well in converting qualitative expectations survey data into quantitative time series of expectations. However, these quantification approaches do not attempt to include any prior information about the underlying process driving the variable of interest. Such prior information is typically available from economic theory or from empirical (stylized) facts. We argue that using prior information, say by including a behavioral model of the variable of interest, could lead to significant efficiency gains and, if so, such information should be included in the quantification scheme. One of the aims of this paper is therefore to develop a quantification approach that may take such information into account. In particular, we develop a general likelihood based modeling framework for quantification of qualitative survey data on expectations and perceptions, and propose a likelihood ratio test for the weak form of the rational expectations hypothesis – the consistency of expectations.

The main differences between our approach and previous ones lie in at least

three aspects: First, our approach is likelihood based and estimation of unknown parameters or other population magnitudes of interest therefore not restricted to any of the distributional assumptions previous approaches rely critically upon. In particular, our approach facilitates the use of a very general class of mixture distributions that seems very useful in many situations as we will illustrate later. Because of its generality, the likelihood approach can easily be applied to quantify a wide range of economic variables like inflation rates, unemployment rates, stock prices and foreign exchange rates, etc. Secondly, unlike previous quantification approaches, the new approach is structural, in the sense that the underlying process of the variable of interest is modelled explicitly. The imposed economic structure is a crucial feature in the new test of consistency of expectations. We argue that the (weak) rational expectations hypothesis can only be tested under an explicitly specified economic modeling framework. Without a proper modeling framework to describe the process driving the variable of interest, the validity of inferences on expectations from purely statistical methods might not always be guaranteed, an implication that has been largely ignored in the literature. Thirdly, previous approaches test expectation consistency hypothesis by comparing expectations of the variable of interest with its ex post realizations. As pointed out by Evans and Wachtel (1993) and Dahl and Hansen (2001) this will, due to ex post bias, lead to too many false rejections of the expectation consistency hypothesis in situations where a structural break or a recurrent regime shift has occurred. By contrast, we model the variable of interest taking into account the possible of regime shifts, and the expectation consistency hypothesis is tested by comparing quantified expectations of the variable of interest from the survey data set with the mathematical expectation derived from the postulated behavioral/structural economic model. Therefore, the new test will not suffer from ex post bias.

Monte Carlo simulation results show that the likelihood ratio test of expectation consistency delivers encouraging size and power properties. Moreover, our approach is more efficient in terms of minimizing the mean squared error (MSE) relative to the quantification scheme proposed by Pesaran (1984, 1987).

Finally, an empirical illustration of the likelihood approach is provided by demonstrating how to quantify qualitative survey data on CBI industrial Trends Survey data on the British manufacturing industry. This data set was originally

used by Pesaran (1984, 1987). Statistical evidence is presented, showing that the actual price change rates can be characterized well by a two-state Markov regime-switching model. The empirical results support the expectation consistency hypothesis and also help explain the phenomenon of ex post bias in inflation forecasts, see, e.g., Evans and Wachtel (1993) and Dahl and Hansen (2001).

The paper is organized as follows: In Section 2 the general setup of our quantification scheme is introduced and discussed under alternative survey sampling designs. In Section 3 the size and power properties of the proposed likelihood ratio test of expectation consistency are examined and the relative performance of the likelihood based quantification scheme is analyzed. In Section 4 an empirical illustration is provided using the CBI industrial Trends Survey data on the British manufacturing industry. Finally, Section 5 contains concluding remarks.

2 A likelihood approach for quantification of qualitative survey data

Let $y_{it+1|t} \in \mathbb{N}$, for $t = 1, 2, \dots, T, i = 1, 2, \dots, N$ be a panel survey data set (with T time periods and N respondents) on expectations with respect to $y_{t+1} \in \mathbb{R}$. Assume the survey consists of $j = 1, 2, \dots, J$ discrete response categories. If respondent i anticipates y_{t+1} to fall within an interval $[\beta_{j-1}; \beta_j)$, and we let $G(y_{t+1}|\mathbf{I}_{it})$ define respondent i 's subjective probability function, the survey data sampling scheme can be described as $y_{it+1|t} = \sum_{j=1}^J j \mathbb{1}_{(\beta_{j-1} \leq y_{it+1|t}^* < \beta_j)}$ where $y_{it+1|t}^* = \mathbb{E}(y_{t+1}|\mathbf{I}_{it}) = \int y_{t+1} dG(y_{t+1}|\mathbf{I}_{it}) \in \mathbb{R}$ is respondent i 's latent subjective expectation on the random variable y_{t+1} .² Notice, that expectations are formed conditional on the information set \mathbf{I}_{it} , which includes all the relevant information that respondent i has collected up to time t . When respondents form expectations by correctly using all the information they possess we can write $y_{it+1|t}^* = \mathbb{E}(y_{t+1}|\mathbf{I}_t) + \sigma_\eta \eta_{it+1}$, where $\mathbf{I}_t = \cup_{i=1}^N \mathbf{I}_{it}$ and η_{it+1} is a zero mean idiosyncratic error term with probability function $\Phi(x) = \Pr(\eta_{it+1} \leq x)$. In order to compute $\mathbb{E}(y_{t+1}|\mathbf{I}_t)$ we need to define $G(y_{t+1}|\mathbf{I}_t)$. We suggest a simple regression model approach for this purpose and think of the y_{t+1} as being generated according to $y_t = g(\mathbf{I}_{t-1}; \boldsymbol{\theta}) + \sigma_{ut} u_t$, where $g(\cdot)$ is a general nonlinear measurable function. Let u_t have a stationary distribution with probability function

²Following standard notation, $j_{(\cdot)}$ is an indicator function taking the value j or zero.

$U(x) = \Pr(u_t \leq x)$. Under the hypothesis that $(\delta_1, \delta_2)' = (0, 1)'$ the model can be written as

$$y_t = g(\mathbf{\Gamma}_{t-1}; \boldsymbol{\theta}) + \sigma_{ut}u_t, \quad (1)$$

$$y_{it+1|t}^* = \delta_1 + \delta_2 \mathbb{E}(y_{t+1} | \mathbf{\Gamma}_t) + \sigma_\eta \eta_{it+1}, \quad (2)$$

$$y_{it+1|t} = \sum_{j=1}^J \mathbb{I}_{(\beta_{j-1} \leq y_{it+1|t}^* < \beta_j)}, \quad (3)$$

where $\boldsymbol{\psi} = (\boldsymbol{\theta}', \sigma_{ut}, \delta_1, \delta_2, \sigma_\eta, \beta_1, \dots, \beta_{J-1})'$ is the collection of unknown parameters.³

We suggest to choose $g(\cdot)$ and $\mathbf{\Gamma}_{t-1}$ based on prior information obtain from economic theory. To give a simple example, consider a situation where qualitative data on exchange rate expectations is available. Classical economic theory suggests that the actual exchange rate (measured as the domestic price of foreign currency) would obey the purchasing power property (PPP) and accordingly a natural choice would be $\mathbf{\Gamma}_{t-1} = (p_{t-1}, p_{t-1}^*)'$ and $g(\mathbf{\Gamma}_{t-1}; \boldsymbol{\theta}) = \theta_1(p_{t-1} - p_{t-1}^*)$ where p_{t-1} and p_{t-1}^* denote the domestic and foreign prices in logarithms respectively. In more "restricted" situations, where such information is unavailable we recommend to employ simple dynamic empirical representations. In the simplest AR(1) case one would set $\mathbf{\Gamma}_{t-1} = y_{t-1}$ and $g(\mathbf{\Gamma}_{t-1}; \boldsymbol{\theta}) = \theta_1 y_{t-1}$. In Section 3 and 4 we consider examples of more evolved empirical models where the dynamics governing the variable of interest are allowed to change between regimes. This can be modelled by assuming that, y_t has a mixture of, say, normal distributions. This "flexible" distributional assumption seems relevant when modelling a wide range of "fat tailed" nominal or financial variables such as prices, interest rates, exchange rates, etc. In all situations, however, it is crucial to evaluate the adequacy of the choices made, which we also emphasize in the empirical illustration in Section 4. Misspecification of $g(\cdot)$ and $\mathbf{\Gamma}_{t-1}$ will deteriorate the quality of the estimated quantities and lead too to many rejections of the expectation consistency hypothesis. Finally, it should be noted that economic theory rarely gives any guidance on the short run dynamics which also is the case in the PPP example above. To obtain a well specified representation of equation (1) one would therefore typically be forced to augment $\mathbf{\Gamma}_{t-1}$ and $g(\mathbf{\Gamma}_{t-1}; \boldsymbol{\theta})$ with additional lags of domestic and foreign price levels.

³It is implicitly assumed that $(\beta_0, \beta_J)' = (-\infty, \infty)'$.

To motivate a test for rationality of expectations based on the model given by (1)-(3) we use Muth's (1961) central idea of the rational expectations hypothesis which states that individuals form expectations of a variable rationally only if they are making efficient use of all information pertinent to the determination of that variable. This definition implies that if agents form expectations rationally then the difference between the individuals subjective expectations and the objective/mathematical expectation should be random and equal to zero on average, hence be systematically unbiased. A test for unbiasedness or consistent expectations is often considered as the weak test of rationality, see, e.g., Rich (1990). Focusing on the weak form of rationality, the null hypothesis of rational expectations can be formalized as

$$H_0 : \delta_1 = 0 \wedge \delta_2 = 1. \quad (4)$$

in the model (1)-(3). That is, under the null hypothesis, the conditional mean of respondents' aggregate subjective probability function equals the conditional mathematical mean of the objective probability function.

2.1 Estimation based on individual responses on expectations

In order to quantify the survey expectations and to test hypothesis on expectation formation estimates of the unknown population parameters in ψ must be obtained. Once $\hat{\psi}$ is obtained the qualitative expectations from the survey data set can be quantified simply as

$$\hat{y}_{it+1|t}^* = \hat{\delta}_1 + \hat{\delta}_2 g(\mathbf{I}_t; \hat{\theta}).$$

In what follows we propose a likelihood based approach for estimating $\hat{\psi}$ and to test H_0 given by (4) using the classical likelihood ratio test. Let the joint conditional probability density of the observed sequence $\{(y_t, y_{it+1|t})'\}_{t=1}^T$ for respondent i be given by

$$\prod_{t=1}^T f(y_t, y_{it+1|t} | \mathbf{I}_t; \psi),$$

where $\{\mathbf{I}_t\}_{t=1}^T$ typically is a vector sequence of weakly exogenous variables. Consequently, the log likelihood function defined over all N respondents is simply

$$\ell(\psi) = \sum_{i=1}^N \sum_{t=1}^T \log f(y_t, y_{it+1|t} | \mathbf{I}_t; \psi). \quad (5)$$

Using Bayes rule we can write (5) more conveniently as

$$\ell(\boldsymbol{\psi}) = \sum_{i=1}^N \sum_{t=1}^T (\log f_1(y_{it+1}|t|\mathbf{I}_t; \boldsymbol{\psi}_1) + \log f_2(y_t|\mathbf{I}_{t-1}; \boldsymbol{\psi}_2)), \quad (6)$$

where $f_1(\cdot|\cdot)$ is the conditional density function of $y_{it+1}|t$ given \mathbf{I}_t , $f_2(\cdot|\cdot)$ is the conditional density function of y_t given \mathbf{I}_{t-1} , and $\boldsymbol{\psi} = (\boldsymbol{\psi}'_1, \boldsymbol{\psi}'_2)'$. Using (1)-(3) and by defining $\Phi_{jt+1} = \Pr(\eta_{it+1} < \frac{\beta_j - \delta_1 - \delta_2 E(y_{t+1}|\mathbf{I}_t)}{\sigma_\eta})$ we can write

$$\log f_1(y_{it+1}|t|\mathbf{I}_t; \boldsymbol{\psi}_1) = \sum_{j=1}^J 1_{(y_{it+1}|t=j)} \log(\Phi_{jt+1} - \Phi_{j-1t+1}).$$

The shape of $\log f_2(y_t|\mathbf{I}_{t-1}; \boldsymbol{\psi}_2)$ will depend on the probability function $U(x)$. In the case of normality

$$\log f_2(y_t|\mathbf{I}_{t-1}; \boldsymbol{\psi}_2) = -\frac{1}{2} \log(2\pi) - \frac{1}{2} \log(\sigma_{ut}) - \frac{1}{2} \frac{(y_t - g(\mathbf{I}_{t-1}; \boldsymbol{\theta}))^2}{\sigma_{ut}^2}.$$

For estimation and testing purposes, however, one cannot use (6) directly since $\boldsymbol{\psi}$ is not uniquely identified.⁴ In order to proceed we therefore need to impose a set of identifying assumptions. One possibility is the following scheme:

Identification Scheme 1 *If β_j for all $j = 1, 2, \dots, J - 1$, are known to the respondents, all the unknown parameters of the model given by equation (1)-(3) are uniquely identified.*⁵

The identification scheme is reasonable to impose whenever the categories in which respondents place their answers are pre-specified numerically by the survey organizers. Survey data sets with such pre-specified threshold parameters include the well-known Survey of Consumers Data on price expectations collected by the Survey Research Center (SRC) at the University of Michigan and Livingston Survey collected for the Philadelphia Inquirer.

Often, however, β_j for $j = 1, 2, \dots, J - 1$ are not pre-specified by the surveyors. In this case, it is not reasonable to assume that respondents know the values of

⁴The lack of identification can be seen by noticing that it is only the ratio $\frac{\beta_j}{\sigma_\eta}$ that enters the log likelihood function. Hence β_j and σ_η can be varied proportionally without affecting the log likelihood function.

⁵This result is well known from the discrete choice literature, see, e.g., Amemiya (1985) Ch. 9.

threshold parameters, and consequently identification scheme 1 cannot be used. In the next section we will introduce a second identification scheme that does not rely on this assumption.

2.2 Estimation based on individual responses on expectations and perceptions

Surveys are often organized such that qualitative data on expectations as well as perceptions are collected by the surveyors. That is, in addition to being questioned about future expectations on the variable of interest, respondents are asked about their perceptions on the variable of interest. Observations on perceptions can help to identify the parameters of the model (1)-(3) even when the threshold values are not pre-specified by the surveyors. This idea was originally coined by Pesaran (1984, 1987), who in a different setting derives the parameters used to quantify survey data expectations from the historical relationship between perceptions and actual realizations.

In particular, if $y_{it|t} \in \mathbb{N}$ denotes the categorical observations on respondent i 's perceptions, then by the same line of arguments as used above, our model (1)-(3) can be augmented with the following two equations

$$y_{it|t}^* = \mu_1 + \mu_2 \mathbb{E}(y_t | \mathbf{I}_t) + \sigma_\omega \omega_{it}, \quad (7)$$

$$y_{it|t} = \sum_{j=1}^J \mathbb{1}_{(\alpha_{j-1} \leq y_{it|t}^* < \alpha_j)}, \quad (8)$$

where $y_{it|t}^* \in \mathbb{R}$ is respondent i 's latent subjective perception w.r.t. the random variable y_t , and ω_{it} is an idiosyncratic error term with probability function $\Psi(x) = \Pr(\omega_{it} < x)$. We will use that $\mathbb{E}(y_t | \mathbf{I}_t) = y_t$, since it must be reasonable to assume that $y_t \in \mathbf{I}_t$. The threshold parameters for the perceptions are given by α_j , for $j = 1, 2, \dots, J$. Augmented by observations on perceptions and given the assumptions associated with (7) and (8) the log likelihood becomes

$$\ell(\psi) = \sum_{i=1}^N \sum_{t=1}^T (\log f_1(y_{it+1|t} | \mathbf{I}_t; \psi_1) + \log f_2(y_t | \mathbf{I}_{t-1}; \psi_2) + \log f_3(y_{it|t} | \mathbf{I}_t; \psi_3)), \quad (9)$$

where the additional term is given as

$$\log f_3(y_{it}|t|\mathbf{I}_t; \boldsymbol{\psi}_3) = \sum_{j=1}^J 1_{(y_{it}=j)} \log(\Psi_{jt} - \Psi_{j-1t}),$$

and $\Psi_{jt} = \Psi\left(\frac{\alpha_j - \mu_1 - \mu_2 y_t}{\sigma_\omega}\right)$. It is easy to verify that the unrestricted perception augmented model cannot be identified. We propose to obtain identification by imposing the following scheme:

Identification Scheme 2 *All the unknown parameters of the model given by (1), (2), (3), (7) and (8) will be uniquely identified when the following restrictions are imposed:*

$$(\mu_1, \mu_2, \alpha_1, \alpha_2, \dots, \alpha_{J-1})' = (0, 1, \beta_1, \beta_2, \dots, \beta_{J-1})'$$

Accordingly, identification can be obtained, assuming that respondents are at least capable of forming unbiased perceptions and that respondents use the same set of threshold values when placing their answers in the J expectation and perception categories.

2.3 Estimation based on proportion categories data

The log likelihood functions given above are derived under the assumption that the outcome of each of the individual decisions is observed for every time period. Typically, however, observations on individual expectations are unavailable and only the proportions of respondents replying in each of the categories are reported. Consequently, the log likelihood function have to be modified accordingly by re-defining it over the observed proportions, given as $\hat{p}_{jt}^e = \frac{1}{N} \sum_{i=1}^N 1_{(y_{it+1}=j)}$ for expectations and $\hat{p}_{jt}^p = \frac{1}{N} \sum_{i=1}^N 1_{(y_{it}=j)}$ for perceptions. In that case (6) and (9) becomes

$$\ell_I(\boldsymbol{\psi}) = N \sum_{t=1}^T \sum_{j=1}^J \hat{p}_{jt}^e \log(\Phi_{jt+1} - \Phi_{j-1t+1}) + N \sum_{t=1}^T \log f_2(y_t|\mathbf{I}_{t-1}; \boldsymbol{\psi}_2),$$

and

$$\begin{aligned} \ell_{II}(\boldsymbol{\psi}) &= N \sum_{t=1}^T \sum_{j=1}^J \left(\hat{p}_{jt}^e \log(\Phi_{jt+1} - \Phi_{j-1t+1}) + \hat{p}_{jt}^p \log(\Psi_{jt} - \Psi_{j-1t}) \right) + \\ &N \sum_{t=1}^T \log f_2(y_t|\mathbf{I}_{t-1}; \boldsymbol{\psi}_2), \end{aligned}$$

respectively. It is important to notice that the i.i.d. assumptions on the error terms ω_{it} and η_{it+1} can be relaxed when observations on individual data are available. When only observations on proportion data are available, such degree of generality cannot be obtained, because information on the possible correlation between ω_{it} and η_{it+1} will not be available in this situation. However, under the weak rationality hypothesis it does not seem unreasonable to assume that perception and expectation errors on average are uncorrelated.

3 Monte Carlo experiments

The purpose of this section is two-fold. First, a likelihood ratio test statistic for testing expectation consistency hypothesis given by (4) is derived and some limited evidence on the size and power properties of the statistic is provided using Monte Carlo simulations. Secondly, we compare the efficiency of the likelihood based quantification scheme with Pesaran's (1984, 1987) quantification approach under alternative data generating mechanisms. This section will focus entirely on the situation where only proportion categories data are available from the survey.

3.1 Size and power properties of the expectation consistency test

Let $\hat{\psi}_I = \arg \max_{\psi} \ell_I(\psi)$ and $\hat{\psi}_{II} = \arg \max_{\psi} \ell_{II}(\psi)$ be the unconstrained maximum likelihood estimates obtained under identification scheme 1 and 2 respectively. Similarly, let $\hat{\psi}_I^C$ and $\hat{\psi}_{II}^C$ be the constrained maximum likelihood estimates when the restrictions given by (4) are imposed. The likelihood ratio test for consistent expectations under the two identification schemes are then given as $LR_s = 2 \left(\ell_s(\hat{\psi}_s) - \ell_s(\hat{\psi}_s^C) \right) \stackrel{a}{\sim} \chi^2(2)$ for $s = I, II$.

To investigate the finite sample properties of LR_s we simulate proportion categories data from the model given by (1), (2), (3), (7) and (8) for alternative values of δ_2 ranging from 0.1 to 1.8.⁶ It is assumed that $N = 1000$, $J = 4$, $(\delta_1, \mu_1, \mu_2)' = (0, 0, 1)'$, $(\sigma_{ut}, \sigma_{\eta}, \sigma_{\omega})' = (1, 1, 1)$ and the threshold values chosen are $(\alpha_1, \alpha_2, \alpha_3) = (\beta_1, \beta_2, \beta_3) = (-0.5, 0.5, 1.5)$. The choice of numerical values for the threshold parameters is based on the hypothesis stated by Carlson and Parkin (1975) that there is a range about zero which the respondents cannot

⁶The same type of analysis was performed w.r.t. δ_1 , but the results did not differ and therefore not reported. The results are available from the authors upon request.

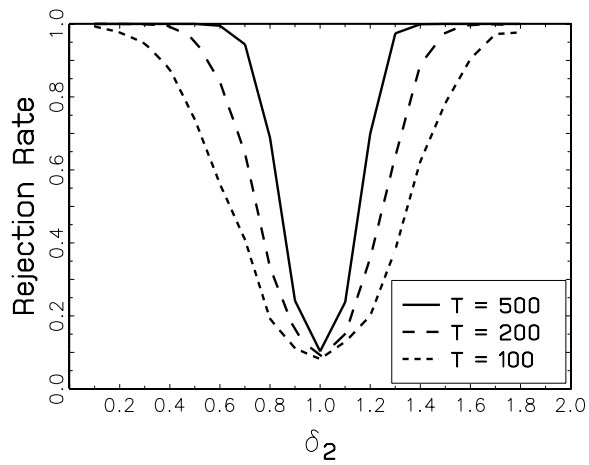


Figure 1: Power of LR_I , at a 5% nominal significance level for alternative sample sizes. The number of Monte Carlo replications equals 1000.

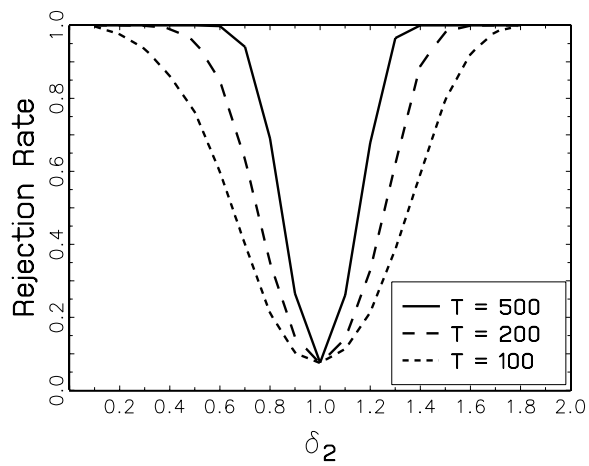


Figure 2: Power of LR_{II} , at a 5% nominal significance level for alternative sample sizes. The number of Monte Carlo replications equals 1000.

distinguish from zero. The magnitudes of the threshold parameters are similar to those used in the SRC survey. For simplicity, the variable of interest y_t is simulated using a Gaussian first order autoregressive process, i.e. $g(\mathbf{\Gamma}_{t-1}; \boldsymbol{\theta}) = 0.5y_{t-1}$ and $u_t \sim N(0, 1)$. The results on the size and power of LR_s on a 5% nominal significance level based on 1000 Monte Carlo replications are summarized in Figure 1 and 2.

When $\delta_2 = 1$, the simulated size of LR_I and LR_{II} are between 5% – 10% for all the sample sizes considered, implying that strictly speaking the size of the tests seems a little too high. We do not, however, consider the size distortion as being significant. In small sample sizes one could use a simple bootstrap version of the tests, suggested by Dahl (2003), to eliminate the risk of possible size distortion. When δ_2 starts deviating from one the power starts approaching one rapidly as expected. There is no strong evidence, that LR_I is more powerful than LR_{II} , though there are fewer estimated parameters involved. In conclusion, both test statistics seem to perform satisfactorily.

3.2 Relative efficiency of likelihood based quantification

In order to analyze the relative performance of the likelihood based quantification scheme a numerical comparison to the quantification scheme suggested by Pesaran (1984,1987) is carried out. Pesaran’s regression model approach uses observations on survey expectations as well as perceptions, so in this comparison focus is on the likelihood approach under Identification Scheme 2.

We consider two data generating mechanisms (DGP’s) that only differ w.r.t. the dynamic process driving the variable of interest. DGP 1 is as defined in the previous section with the exceptions that $J = 3$, $(\alpha_1, \alpha_2) = (\beta_1, \beta_2) = (-0.5, 0.5)$, and that δ_2 is fixed at $\delta_2 = 1$. Under DGP 2 we let y_t be a mixture of normals, by characterizing it as following a two-state Markov regime-switching process. In particular, we assume

$$\begin{aligned} g(\mathbf{\Gamma}_{t-1}; \boldsymbol{\theta}) &= (1 - s_t^*)(c_0 + \phi_0 y_{t-1}) + s_t^*(c_1 + \phi_1 y_{t-1}) \\ \sigma_{ut} u_t &= (1 - s_t^*)\sigma_0 v_{0t} + s_t^*\sigma_1 v_{1t} \end{aligned}$$

where $(c_0, c_1, \phi_0, \phi_1)' = (-1, 2, 0.7, 0.3)'$, $(v_{0t}, v_{1t})' \sim N(\mathbf{0}_2, \mathbf{I}_{2 \times 2})$ and s_t^* is an unobservable random state variable ($s_t^* \notin \mathbf{\Gamma}_{t-1}$) that can take the values zero

Table 1: Comparison of accuracy of Pesaran’s regression model approach with likelihood based quantification. Monte Carlo replications used equals 1000.

Sample Size	DGP 1 (AR(1))		DGP 2 (MS-AR(1))	
	100	500	100	500
MSE: True	1.00	1.00	1.00	1.00
MSE: Pesaran’s approach	1.26	1.38	3.69	3.66
MSE: Likelihood approach	1.00	0.99	1.00	1.00

(regime 1) and one (regime 2) only. We assume that the transition between the two regimes are governed by a Markov chain with transition probability matrix

$$\mathbf{P} = \begin{bmatrix} \Pr(s_t^* = 0 | s_{t-1}^* = 0) & \Pr(s_t^* = 0 | s_{t-1}^* = 1) \\ \Pr(s_t^* = 1 | s_{t-1}^* = 0) & \Pr(s_t^* = 1 | s_{t-1}^* = 1) \end{bmatrix} = \begin{bmatrix} \pi_0 & 1 - \pi_1 \\ 1 - \pi_0 & \pi_1 \end{bmatrix}$$

In the simulations $(\pi_0, \pi_1)' = (0.9, 0.8)'$ is used.⁷ We measure the accuracy of the two alternative quantification schemes using the mean squared difference between the quantified/estimated expectations and the true expectations. The results based on 1000 Monte Carlo replications are reported in Table 1.

When comparing the results based on Pesaran’s approach with those from the likelihood approach the improvement in accuracy by using information on the DGP explicitly seem rather striking. This seems to be particular true when possibly recurrent regime shifts are present in the process driving the variable. Since, as already mentioned, a number of studies have found such evidence in macroeconomic time series on inflation our limited simulation study strongly suggests that the likelihood approach seems to be an important new tool for analysts who wish to quantify qualitative survey data on price change expectations.

4 Empirical illustration

In this section, the CBI industrial Trends Survey data on the British manufacturing industry are used to re-examine the performance of the likelihood approach for quantification and testing the expectation consistency hypothesis. The CBI

⁷For specification of $\log f_2(y_t | \mathbf{I}_{t-1}; \psi_2)$ in the Markov regime switching case, see, e.g., Hamilton (1994), Ch. 22.

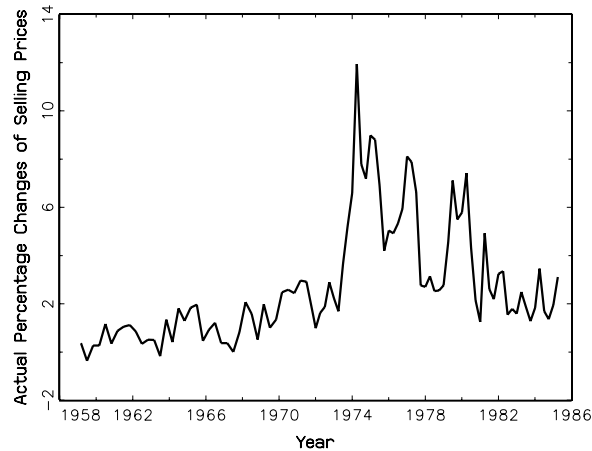


Figure 3: Actual rates of selling prices changes in the British manufacturing industry from 1959q1 - 1985q2.

survey consists of data on qualitative trends for both expectations and perceptions over the past and next four months on average change rates of selling prices of British manufacturing and dates back to June 1958. This data as well as the time series data of the actual rates of price changes in the British manufacturing industry are available from Pesaran (1987) for the period 1959q1 to 1985q2. The data set enables quantification of expected price change rates in British manufacturing industry based on identification scheme 2 of the likelihood based approach and facilitates easy comparison to Pesaran’s regression model approach as discussed in Pesaran (1987).⁸ As shown in Figure 3, the actual price change rates in the British manufacturing industry may have experienced a structural break with the mean and variance of price change rates increasing substantially after 1974. The simulation results reported in the previous section suggest that the quality of the quantified expectation can be improved if we account for the possibility of such regime shifts.

For the likelihood based quantification scheme, we begin by modeling the actual price change rates based on the well-known Box-Jenkins modeling philosophy that simpler models provide more robust forecasts. As mentioned in Hamilton

⁸The detailed results of Pesaran’s regression method applied to this data set are reported in Pesaran (1987).

Table 2: Determining the best linear univariate representation of the price change rate in the British manufacturing industry for the period 1959q1 - 1985q2. Model selection based on alternative information criteria.

Models	Model Selection Criteria		
	AICC	BIC	FPE
AR(1)	323.565	324.635	1.832
AR(2)	325.671	329.587	1.871
AR(3)	325.586	331.814	1.864
AR(4)	327.734	335.88	1.903
MA(1)	370.862	371.634	.
MA(2)	350.815	354.247	.
MA(3)	345.599	351.233	.
MA(4)	335.827	343.471	.
ARMA(1,1)	325.636	329.568	.
ARMA(1,2)	325.993	332.241	.
ARMA(2,1)	327.747	333.967	.
ARMA(2,2)	328.232	336.39	.

(1994), although complicated models can track the data very well over the historical period for which parameters are estimated, they often perform poorly when used for out-of-sample forecasting. Accordingly, to characterize the actual price change rates, we concentrate on low order ARMA models, rather than complicated models with many exogenous variables. In terms of model selection, we first attempt to characterize the actual price change rates with linear representation by selecting the best linear model to fit the actual price change rates data among a pool of candidate ARMA models. The model selection criteria used are the AICC, which is a bias-corrected version of the AIC suggested by Hurvich and Tsai (1989), BIC and the FPE (Final Prediction Error criterion) developed by Akaike (1969). The results reported in Table 2 indicate that the best linear univariate model for the price change rates seems to be an AR(1) representation.

The graphs of sample autocorrelation function (ACF) and partial autocorrelation function (PACF) of AR(1) model shown in Figure 4 further justify the choice of AR(1) model. In particular, the fact that all of the PACF values beyond one lag fall within the bounds, defined as $\pm 1.96/\sqrt{T}$, supports the choice of

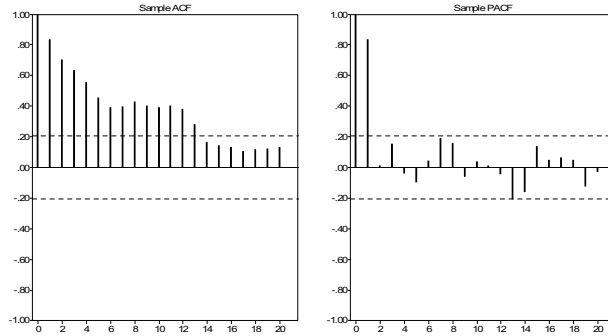


Figure 4: Sample ACF (left) and PACF associated with selling price changes in the British manufacturing industry from 1959q1 - 1985q2.

the AR(1) model for describing the actual price change rates data. In addition, the AR(1) model passes the diagnosis tests of randomness on residuals such as Portmanteau test, McLeod-Li test, implying white noise residuals.

With rather strong indications of possible regime shifts by inspection of Figure 3, a careful study of the stability of the linear models is required. Several tests are conducted to test the stability of the AR(1) model as well as the other candidate ARMA models. Based on the one-step-up Chow test for structural break within the sample period, depicted in Figure 5, the null hypothesis of stability of the AR(1) model is rejected at one percent level. Moreover, none of the other candidate linear univariate models passes the Chow test at this level. We also test structural stability using CUSUM test proposed by Brown, Durbin, and Evans (1975). Figure 5 (lower panel) shows that most CUSUM statistics stay outside the upper confidence bound at 5 percent level, implying the strong instability of the AR(1) model.

Andrews, Lee, and Ploberger (1996) suggest three optimal change point tests for detecting the presence of non-stable coefficients in a normal linear-regression model with unknown breakpoints. The three test statistics, denoted as Sup LM, Exp LM and Ave LM, are reported in Table 3. All three tests reject the hypothesis of stability of all parameters in the AR(1) model. In conclusion, all testing evidence rejects the stability of AR(1) modeling of actual price change rates and it seems fairly safe to reject the adequacy of the linear AR(1) model. To model the apparent shift in the parameters in the linear model a non-linear Markov regime-switching model seems a natural choice. It is, however, crucial to

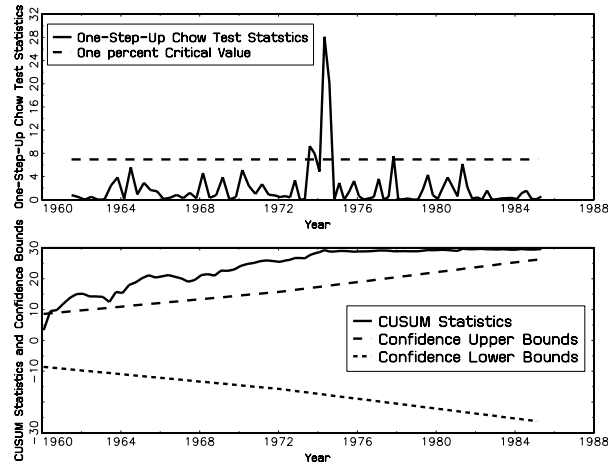


Figure 5: One-step-up Chow test (top) and CUSUM test for stability of the AR(1) model of price changes in the British manufacturing industry from 1959q1 - 1985q2.

Table 3: Andrews-Lee-Ploberger test for instability of the AR(1) model of price change rates in the British manufacturing industry for the period 1959q1 - 1985q2.

		Critical values			
	Statistic	10%	5%	1%	P-value
Sup LM	11.89	10.01	11.79	15.51	0.0436
Exp LM	3.07	2.59	3.22	4.76	0.0574
Ave LM	4.18	3.75	4.61	6.73	0.0699

test whether the Markov regime-switching specification is preferred over the linear model in a statistical sense. For that purpose the specification test proposed by Hansen (1992, 1996) is employed, and the results are summarized in Table 4. In Table 4, LR^* denotes the standardized likelihood ratio test statistics and M denotes the chosen bandwidth, see, Hansen (1996) for more details. Independent of the choice of bandwidth, the associated p-values are all very small, suggesting a rejection of the null of the AR(1) model as an adequate description of the actual price change rates in favor of the two-state Markov regime-switching model.

To determine the order of the MS-AR(p) representation, AIC, HQ (Hannan-Quinn Criterion) and BIC are used. All three model selection criteria give support

Table 4: Testing the MS-AR(1) specification of price change rates in the British manufacturing industry against the AR(1) alternative using Hansen's (1992,1996) standardized likelihood ratio test. The sample period is 1959q1 - 1985q2.

Statistics	P-value				
	M=0	M=1	M=2	M=3	M=4
LR*	5.615	0.0000	0.0000	0.0000	0.0000

Table 5: Determining the best MS-AR(p) representation of price change rates in the British manufacturing industry for the period 1959q1 - 1985q2. Model selection based on alternative information criteria.

Model	Model selection criteria			
	AIC	HQ	BIC	logL
MS-AR(1)	321.61	330.46	343.54	-139.94
MS-AR(2)	322.50	333.63	350.09	-136.74
MS-AR(3)	323.58	337.03	356.92	-133.61
MS-AR(4)	328.41	344.19	367.56	-132.14

to the MS-AR(1) process, which also passes the diagnosis test of randomness of residuals, see Table 5 and 6. Consequently, we will take this representation as the underlying process driving price change rates in the likelihood based quantification scheme.

The main estimation results of our quantification approach with actual price change rates described as a two-state Markov regime-switching model are reported in Table 7. Figure 6 depicts the actual and the quantified expected price change rates. The probabilities for being in the high inflation regime ($s_t^* = 1$) are plotted in the lower part of Figure 6. It is not surprising that the mean

Table 6: Diagnostic tests of the residuals of MS-AR(1) model of inflation in the British manufacturing industry for the period 1959q1 - 1985q2.

Diagnostic Test	Statistics	P-value
Portmanteau (lag=7)	2.9378	0.8166
Normality	2.7014	0.2591
Heteroskedasticity	0.4477	0.7994

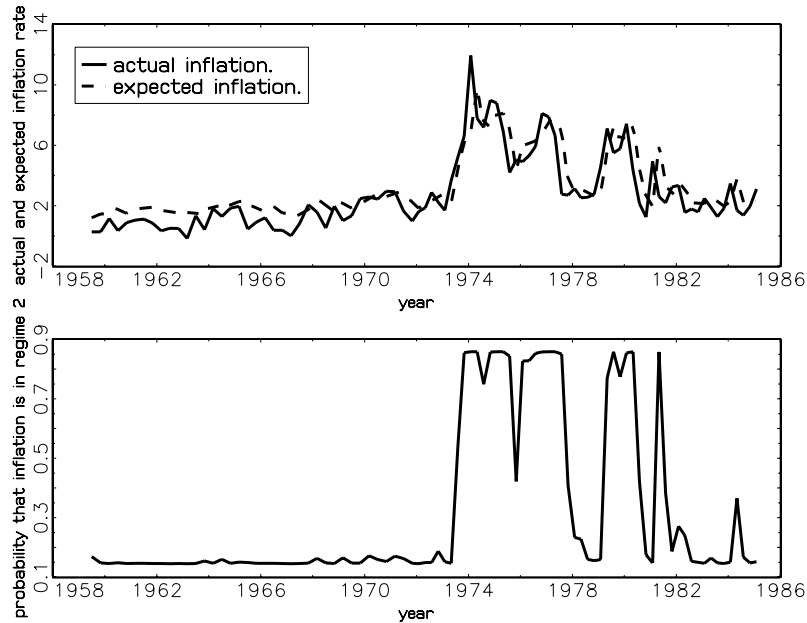


Figure 6: Actual (solid line) and quantified (dashed line) price change expectations in the British manufacturing industry from 1959q1 - 1985q2 (top) and the estimated probability of observing regime 2 ($s_t^* = 1$).

and variance differ substantially in two regimes. In the low-inflation regime, the unconditional mean of the price change rates is equal to 1.5, whereas the unconditional mean equals 6.8 in the high-inflation regime. The estimates of the variance in the two states confirm the conjecture of a positive relationship between the level of inflation and its volatility. That is, higher inflation is associated with higher inflation variance. More importantly, the LR test statistic fails to reject the null hypothesis of expectations consistency, implying that agents make unbiased expectations. This empirical result supports the rational expectations hypothesis. An interesting finding is that the estimates of threshold parameters reveal asymmetry in respondents' decisions on price changes. While a 4.5 percent increase in price makes respondents believe prices are going up, respondents will not believe prices are going down until they observe more than 5.5 percent drop in actual prices. This evidence suggests that economic agents are more sensitive to price increases relative to price decreases.

From Figure 6 (top panel) it appears that respondents tend to over-predict

Table 7: Quantification by the likelihood approach. Estimation results based on a MS-AR(1) representation of price change rates in the British manufacturing industry for the period 1959q1 - 1985q2.

Parameters	Regime 1 ($s_t^* = 0$)	
	Estimates	Std. Errors
c_0	0.761	0.157
ϕ_0	0.493	0.074
σ_0	0.739	0.066
	Regime 2 ($s_t^* = 1$)	
	Estimates	Std. Errors
c_1	3.396	1.248
ϕ_1	0.500	0.174
σ_1	1.633	0.292
$\pi_0 = \Pr(s_t^* = 0 \mid s_{t-1}^* = 0)$	0.953	0.028
$\pi_1 = \Pr(s_t^* = 1 \mid s_{t-1}^* = 1)$	0.858	0.078
	Quantification	
	Estimates	Std. Errors
β_1	-5.533	2.555
β_2	4.499	0.910
σ_2	5.564	1.691
δ_2	0.384	1.46
δ_3	1.043	0.464
σ_3	5.069	1.668
Log-likelihood		-205.566
Constrained Log-likelihood		-205.709
LR test statistic		0.286

price change rates when they are relatively low, and under-predict when the actual price change rates are relatively high. These seemingly biased inflation expectations, however, may not serve as the evidence against the rational expectations hypothesis. As discussed earlier, the LR test cannot reject the expectations consistency and since it is based on comparing two sets of forecasts, ex post bias does not affect the outcome of the test. If one, however, were tempted to test (4) based on a regression of variable of interest on the quantified expectations derived from Pesaran's approach or the likelihood approach, the null hypothesis of weak rationality would be rejected.⁹ Our empirical results support the conclusion of Evans and Wachtel (1993) who claim that forecasters are acting rationally, but face a complicated forecasting problem that makes systematic forecast errors unavoidable. If the price change rates are in one regime but the forecasters perceive that there is possibility that price change rates switch to the other regime, then their expectations will be systematically biased if the switch does not occur.

5 Conclusion

In this paper, a new likelihood approach for quantification of qualitative survey data are introduced and as a by-product a likelihood ratio test for consistent expectation are proposed. Evidences from limited simulation studies and from empirical observation analysis demonstrate that the likelihood approach promises to deliver a more efficient quantification scheme whenever prior information on the underlying process driving the variable of interest is available. When actual price change rates are described by a two-state Markov regime-switching model, the likelihood ratio test conducted on CBI survey data supports hypothesis of consistent expectations. In fact, our empirical conclusions indicates that economic agents behave rationally in making their price change rate forecasts, though they make unavoidable systematic forecast errors due to the possibility of switches of price change regimes.

⁹Testing (4) based on the regression $y_{t+1} = \delta_1 + \delta_2 \hat{q}_{t+1} + \epsilon_t$, using a F-test, where y_{t+1} is actual price change rates and \hat{q}_{t+1} is the quantified expectations gave $F(2, 89) = 6.3256$ [p -value = 0.0027] for the likelihood approach and $F(2, 89) = 4.6285$ [p -value = 0.0122] when \hat{q}_{t+1} was obtained by Pesaran's approach. In this simple illustrations it is ignored that \hat{q}_{t+1} depends on estimated parameters which probably would have lead to a (very moderate) increase of the p-values.

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