Endogenous Selection of a Trade Mechanism in a Search-Theoretic Environment*

Gabriele Camera
Purdue University
Gcamera@mgmt.purdue.edu

Alain Delacroix
Purdue University
Delacroixa@mgmt.purdue.edu

October 26, 2001

Abstract

We study the endogenous determination of the trading mechanism in a search economy with a continuum of homogenous sellers and heterogeneous buyers. Prices can be either negotiated or simply posted. We inquire how heterogeneity in buyers' preferences influences the sellers' choice of trading mechanism. This entails a trade-off between how fast the trade is expected to be completed and the realized gain. We consider both bargaining under symmetric and asymmetric information, regarding buyer's preferences, to address the role of commitment to a price mechanism. A *price setting externality* arises because of a strategic complementarity between sellers' choices of a mechanism.

JEL No. C78, D4, D82, D83

Keywords: Search, pricing, strategic complementarities, asymmetric information

*We wish to thank participants at the 2000 SED Meetings, the Fall 2000 NBER - Cleveland Fed Meetings, the 2001 Midwest Macro Meetings, the 2001 Midwest Theory Meetings and Indiana University for their remarks. We also thank Derek Laing, Antonio Merlo and Randall Wright for helpful comments. In particular, we want to express our gratitude to Ricardo Lagos for comments on an earlier version of the paper.

1 Introduction

Casual observation indicates that the price of some goods tends to be negotiated, while for other goods, the price tends to be posted with no recourse to bargaining. For example, it is rather uncommon that the first price requested by the seller of a house is also the final one. In fact, home prices are generally negotiated. Prices of goods purchased at a grocery store, on the other hand, tend to be posted by the seller and are not subject to any haggling. The object of this paper is to endogenize the seller's choice of a trading mechanism, between bargaining and price posting. We are interested, in particular, in determining how heterogeneity in buyers' preferences for the good being sold affects the sellers' decision of which mechanism to select. This is important at the empirical level, but it is also relevant at the theoretical level, since most models, and in particular the search theoretical ones in the tradition of Acemoglu and Shimer (2000), Moen (1997), Mortensen and Pissarides (1994), Pissarides (2000) and Trejos & Wright (1995) exogenously assume that the terms of trade are determined by either bargaining or price posting, generally without justification. Doing so is not without consequence, however, since it not only influences equilibrium allocations, but may also affect existence of equilibria[1].

Some aspects of this problem have been addressed in the literature. The contributions can be broken down into four main areas. First, some authors, Riley and Zeckhauser (1983), Wang (1995) and Arnold and Lippman (1999), have studied the endogenous choice of trading mechanism of a monopolist seller facing a large number of heterogeneous buyers. Specifically, Riley and Zeckhauser find that, if commitment is possible, sellers should use fixed price strategies. Bargaining allows to price discriminate, but it encourages buyers to refuse a high price, in the hope of getting a lower one. Wang focuses on the cost differences between price posting and bargaining. When bargaining, the monopolist seller is able to discriminate among buyers, but has to incur more costs (haggling costs in addition to displaying costs). If the common displaying costs are large enough, the seller always chooses bargaining. Arnold and Lippman show that, when the seller faces a distribution of buyers' bargaining abilities and private valuations, he finds it optimal to bargain,

[1] See the random matching models of Camera and Winkler (2000) and Curtis-Soller and Wright (2000). They show that monetary equilibria do not exist if sellers are allowed to post prices.

when the average bargaining ability is sufficiently low. The second line of related research looks at the issue of commitment in the choice of a mechanism. Bester (1994) considers an environment where a continuum of sellers can commit, at a cost, to a fixed price, or costlessly bargain. He finds that the benefit from commitment depends on the commitment decision of all others. Thus, there is possibility of multiple equilibria. He finds that, if search costs are low enough relative to commitment costs, bargaining is an equilibrium. Adachi (1999) studies a strategic game, where two sellers offer differentiated goods to a large number of heterogeneous buyers. The possibility of costless commitment to price posting alleviates the buyers' fears that they might be exploited once bargaining takes place. Masters and Muthoo (2000) study endogenous price formation in markets characterized by match-specific heterogeneity and allow for renegotiation of prices, if mutually beneficial. In particular, they are able to determine the proportion of trades that occur at the price specified in the ex-ante contract, rather than the renegotiated bargained price. In the third line of research, Bester (1993) studies the interplay between quality uncertainty and pricing mechanism. Buyers have to visit stores to determine quality. When posted, prices are determined before the quality is observed, and because of switching costs to move to another store, there is an incentive for sellers to provide low quality. With bargaining, however, the price is determined after the buyer has learned the good's quality. Fourth, Peters (1991) and McAfee (1993) study the endogenous determination of trade mechanisms in economies where search is directed. In those economies, the mechanism retained by every seller[2] is public information and the latter can use the mechanism to increase the probability that they will be visited by buyers. Peters (1991, 1995) considers ex-ante price offers, and shows that bargaining is not a stable institution, since there is always an incentive for sellers to post ex-ante prices when prices are determined by Nash bargaining. In McAfee (1993), sellers are allowed to choose between a vast array of mechanisms, and there is a unique equilibrium where sellers hold identical auctions and buyers randomize over which auction they participate in.

We complement the existing literature, by studying a search environment which allows for strategic interactions among sellers, as opposed to Riley and Zeckhauser (1983), Wang (1995) and Arnold and Lippman (1999). There is a continuum of homogeneous sellers, who attempt to sell a good

[2] As well as her location.

of known quality to a continuum of heterogeneous buyers, whose private valuations are unknown to the sellers ex-ante. Buyers are of two types. Sellers must choose a trading mechanism between bargaining and posting a price, taking as given the actions of other buyers and sellers. In order to focus entirely on the strategic choices embedded in the model, we do not consider any exogenous cost differential between the two mechanisms, contrary to Wang (1995). Given the assumption of random search, the choice of mechanism does not affect the arrival rate of buyers, only their acceptance probability: upon encountering a seller, buyers can reject the trade and continue their quest for an acceptable one. This set-up is ideal to investigate how the distribution of buyers' private valuations affects this strategic choice. Furthermore, we contrast the case where the seller and the buyer leave the market after completing a transaction, with the case where sellers and buyers never leave the market but resume search after a successful trade. We interpret the former as a market for durable goods, and the latter as a market for non-durable goods. Because bargained prices are determined *after* a match has taken place, we also study the role that availability of information to the seller has on her choice of trading mechanism. First, we consider a model where a seller who has committed to bargaining observes the buyer's type and the price is determined via the axiomatic Nash bargaining solution[3]. We then relax the assumption of commitment to the mechanism and study negotiations as a strategic bargaining game of imperfect information between the buyer and the seller.

Our first contribution is to show that, when choosing a trading mechanism, the seller faces a trade-off between how fast the trade is expected to be completed and the gain from a realized trade. By posting a price, a seller targets one particular category of buyers, extracting their *entire* surplus[4]. On the other hand, negotiating allows sellers to target a larger variety of types. However, if one assumes first that the buyers' type is learned at the start of the bargaining, sellers have to cede some

[3] We recently became aware of a study by Ellingsen and Rosen (1997) that focuses only on the occurrence of a bargaining equilibrium in a labor market setting, under similar commitment to a mechanism. While a general proof of existence and uniqueness is not provided, they discuss conditions under which a bargaining equilibrium may exist (and provide a specific example).

[4] To be precise, sellers extract the *entire* surplus from *one* type of buyers and may extract *some* surplus from buyers with a higher valuation.

surplus to *every one* of their potential trade partners, as specified by the Nash rule. Bargaining is thus preferred when it ensures larger expected gains, relative to those generated by extracting the entire surplus from one particular type of buyers. This occurs if: (i) sellers have a high enough bargaining power; (ii) the disparity in buyers' valuations is neither too large nor too small, given a particular proportion of types; or (iii) buyers' types are not too unevenly distributed, given their relative valuations. As a corollary, heterogeneity among buyers is necessary for bargaining to take place in equilibrium.

If instead, the buyer's type is unknown at the start of the bargaining process, the trade-off faced by the seller is of a different nature (but leads to similar implications). Specifically, in this case, multi-period strategic negotiations allow the seller to discriminate among buyers' types, by offering different prices at various stages of the process. This, however, causes costly delays in settling the transaction. Once again, bargaining takes place when trading with a variety of buyers is better than focusing on one particular type. Thus, price posting is an equilibrium when the disparity in buyers' valuations is extreme, either very high or very low. A consequence of this trade-off is that when sellers meet buyers more frequently than buyers meet sellers (i.e. a sellers' market), a seller never chooses to discriminate between the two types of buyers and hence bargaining cannot be an equilibrium.

Our second contribution is to show that due to strategic complementarities, the choice of a trading mechanism by sellers entails a *price setting externality.* When sellers choose a pricing mechanism, they influence the buyers' value of staying in the market and therefore, other sellers' optimal choice of a trade mechanism. We show that the flows of buyers and sellers in and out of the market matter. When exit from the market occurs after a transaction, a buyer's choice to accept or not a particular mechanism depends on the value of remaining in the market and searching for an alternative trade mechanism. Hence, a seller's decision is influenced by what other sellers in the market are doing, and we have the possibility of multiple equilibria. On the contrary, when agents never leave the market and resume search after completing a trade, the buyers' acceptance decision cannot be influenced by the value of being in the market and therefore the strategic complementarities are absent and multiple equilibria impossible.

In section 2, we define the environment. We first assume that negotiated prices are the result of the Nash bargaining solution. In section 3, we look at the case of one-time purchases, where buyers and sellers leave the market after completing a transaction. In section 4, we look at the case of repeat purchases, where buyers and sellers stay in the market and resume search after completing a trade. We then assume that bargained prices are the outcome of a game of incomplete information and look at that case in section 5. Finally, we conclude in section 6.

2 Environment

Time is discrete and infinite. The economy is comprised of two sets of agents, *sellers* and *buyers*. The sellers have a good from which they derive no utility and intend to sell. The buyers have no inventory and receive utility from consumption of the sellers' good. Buyers are heterogenous in their preferences for the good. A proportion λ of them derives low utility u_L from its consumption; the remaining $1-\lambda$ have a high valuation for the good, $u_H > u_L$. The buyer's type, $i \in \{L, H\}$, is private information ex-ante. Buyers can transfer utility to sellers. The amount of utility that a seller receives from a buyer is denoted by P, which will be referred to as the "price" at which a trade takes place. Hence, if a trade takes place at a price P, the value of the trade is P to the seller and $u_i - P$ to buyer i. Let $\delta = \frac{1}{1+r}$ denote the discount factor for future utility.

At the end of each period t, the seller chooses to either *post a price* or to *negotiate* with buyers at date $t+1$, committing to her choice for the following period. Choosing to post also implies the selection of a price. If the seller commits to negotiating, she acquires the ability to observe the buyer's type upon starting the bargaining process (this assumption is relaxed in section 5). Thus, a seller can refuse to negotiate with a particular type of buyers. This also implies that if negotiation takes place, the price is settled *after* the resolution of uncertainty regarding the buyer's type. This is in contrast with price posting, where the terms of trade are determined *before* the match takes place. The timing of events is the following. At the end of period t, the seller chooses and commits to one trading mechanism for the next period. During period $t+1$, sellers and buyers search for a trading partner. Sellers meet buyers with probability σ, while buyers meet sellers with probability α. Conditional on a meeting, and contingent on the trading mechanism chosen, seller and buyer can both choose to trade or walk away from the match. If they both choose to trade, consumption immediately follows. If no meeting takes place or if one party refuses to trade with the other, the seller chooses a trading mechanism once again, for period $t+2$, when agents have another chance for a trade.

3 One time purchases

In this section, we confine our attention to markets where individuals transact only once. Buyers and sellers leave the economy after a transaction and are replaced by identical agents[5]. This can be viewed as a decentralized market for durable goods: once a purchase has been made, the buyer need not search for another one and the seller has nothing to sell. We study rational expectations symmetric stationary equilibria where identical agents follow identical strategies, taking prices and strategies of others as given. In equilibrium, decisions are individually optimal, given the correctly perceived strategies of others, and the choice of trade mechanism is based on the correct evaluation of the gains from trade in each possible match.

3.1 Strategies

Upon meeting, buyers and sellers decide whether to trade, depending on the trade mechanism adopted by the seller and the price buyers would have to pay to complete the transaction. When selecting a *trading mechanism* a seller chooses to negotiate with probability $\pi \in [0,1]$, and to post a

[5]This implies that the distribution of buyers' types in the searching pool is constant. In principle, the agents' strategies influences the distribution of agents exiting the economy, and therefore the distribution in the searching pool. An alternative would be to endogenize the latter by assuming a constant distribution of buyers coming into the economy and using a steady state condition. This however, may lead to multiple equilibrium distributions, as it generates a "sorting externality". We assume away the sorting externality–not a focus of our study–by considering that every buyer leaving the economy is replaced by a buyer of same type.

price with probability $1-\pi$. Conditional on having met a type i buyer, a seller who is not posting prices must also choose the probabilities $\pi_i \in [0,1]$ to bargain with that type of buyers. Prices resulting from a negotiation with a buyer of type i are denoted by P_i^N, while P^P denotes a posted price. Contingent on a match, the buyer must choose whether to trade or not. If the seller is willing to bargain, a type i buyer accepts to trade with probability $\eta_i \in [0,1]$. If the seller is posting price P^P, a type i buyer makes the purchase with probability $\beta_i\left(P^P\right) \in [0,1]$. We let η_i^*, $\beta_i^*\left(P^P\right)$, π^*, π_i^* denote the expectations about the strategies adopted by others in equilibrium[6].

3.2 Equilibrium Prices

Let V_i denote the value of search to a buyer of type i. V_N and $V_P(P^P)$ denote the value of search to a seller who has chosen, respectively, to negotiate and to post price P^P. Let $V_P = \max\limits_{P^P} V_P(P^P)$. Define the trade surplus to an agent, as the difference between the value of completing a trade and the value of search. Hence, given a transaction price P, the surplus to a type i buyer is $u_i - P - V_i$, and the surplus to a seller is $P - V_N$ (if negotiating) or $P - V_P$ (if price posting).

For convenience, we next describe a feature of price posting that simplifies the derivation of the value functions (jumping ahead of the analysis of equilibrium strategies). Specifically, in equilibrium, the distribution of posted prices will have at most two mass points P_L^P and P_H^P[7].

Lemma 1 *In equilibrium, a posted price can take at most two values, each of which makes one buyer type indifferent between buying or not:* $P^P \in \left\{P_i^P\right\}_{i=L,H}$*, where:*

$$P_i^P = u_i - V_i \tag{1}$$

By posting a price, the seller can extract the entire surplus from one type of buyers only. By leaving a positive surplus to both buyers, the seller does not increase the probability of a trade. She can therefore increase her expected profit by extracting the entire surplus from one type of buyers. Thus, the seller will optimally select P^P from at most two prices, each of which extracts the surplus

[6] Asterisks denote equilibrium values, as opposed to individual choice variables, which do not carry asterisks.

[7] The proofs of all lemmas and propositions are collected in appendix.

from only one type of buyers. Note that it is possible that the seller be indifferent between posting either one of them, because, while choosing the higher price generates a higher revenue from each sale, it also reduces the probability of a sale. Hence, we let γ^* denote the equilibrium proportion of price posting sellers who post price P_H^P, and $1-\gamma^*$ the proportion of those who post P_L^P.

When negotiation takes place, the seller learns the buyer's type at the start of the process. The resulting price P_i^N is assumed to be given by the axiomatic Nash bargaining solution, where the threat points are the respective values of search[8]. The amount of surplus allocated to the seller depends on her relative bargaining power, which we denote by $\theta \in (0,1)$. Since the surplus to the seller and to the buyer are $P_i^N - V_N$ and $u_i - P_i^N - V_i$, respectively, the total surplus to be split is $u_i - V_i - V_N$. The seller obtains a portion θ of that total surplus. Thus, we denote the price generated by bargaining with a buyer of type i as:

$$P_i^N = V_N + \theta \left(u_i - V_i - V_N\right) \tag{2}$$

P_i^N is the sum of the minimum acceptable price for a seller (V_N), plus a proportion θ of the total surplus. We emphasize that, regardless of the exact nature of the bargaining procedure, the main difference between the two candidate trading mechanisms is rooted in the assumption on the observability of types. With price posting, the seller is able to extract the entire trade surplus from (only) one buyer type. By opting to bargain instead, the seller can choose whom to trade with, but must share the surplus with the buyer.

[8] Even though taking the axiomatic approach leaves the nature of the bargaining game unresolved, it is well known that properly defined strategic bargaining games exist that produce the same outcome as particular Nash bargaining problems (Osborne & Rubinstein 1990). Our choice of threat points corresponds to a dynamic alternating offer game, where negotiations during the bargaining game are subject to probabilistic breakdowns, and players get to resume search, in case the negotiations actually broke down.

3.3 Value functions

From the discussion above, it follows that the value functions can be define recursively, and must satisfy a standard set of flow return conditions. Specifically:

$$\begin{aligned} rV_i &= \alpha\pi^*\pi_i^* \max_{\eta_i} \left[\eta_i \left(u_i - P_i^N - V_i\right)\right] + \alpha\left(1-\pi^*\right)\gamma^* \max_{\beta_i\left(P_H^P\right)} \left[\beta_i\left(P_H^P\right)\left(u_i - P_H^P - V_i\right)\right] \\ &\quad + \alpha\left(1-\pi^*\right)\left(1-\gamma^*\right) \max_{\beta_i\left(P_L^P\right)} \left[\beta_i\left(P_L^P\right)\left(u_i - P_L^P - V_i\right)\right] \end{aligned} \tag{3}$$

Equation (3) can be interpreted as follows. A buyer of type i meets a seller with probability α. That seller is willing to negotiate with probability $\pi^* \in [0,1]$, while she posts a price with probability $1-\pi^*$. Conditional on having met a seller who negotiates, there is probability $\pi_i^* \in [0,1]$ that she intends to negotiate with a buyer of type i, in which case the buyer has to decide whether to bargain with her ($\eta_i = 1$), keep searching ($\eta_i = 0$) or randomize over the two options $\eta_i \in (0,1)$. In case a buyer of type i bargains, his trade surplus is $u_i - P_i^N - V_i$. If a buyer meets a seller who is posting a price, there is a probability γ^* $(1-\gamma^*)$ that the price offered takes away the entire surplus from high (low) valuation buyers. In this case, a buyer of type i has to decide whether to go ahead with the purchase ($\beta_i\left(P^P\right) = 1$), keep searching ($\beta_i\left(P^P\right) = 0$) or randomize over the two options ($\beta_i\left(P^P\right) \in (0,1)$). Note that $\beta_i\left(P^P\right)$ is a function of the posted price, while η_i is not a function of the negotiated price. This is because, once the seller is committed to negotiate, the Nash bargaining solution is given so that the price negotiated only depends on the total value to split and the respective threat points.

Similarly, the value of search to a seller who has chosen to negotiate is

$$rV_N = \sigma\lambda\eta_L^* \max_{\pi_L} \left[\pi_L\left(P_L^N - V_N\right)\right] + \sigma\left(1-\lambda\right)\eta_H^* \max_{\pi_H} \left[\pi_H\left(P_H^N - V_N\right)\right] \tag{4}$$

A negotiating seller meets buyers with probability σ. A proportion λ of them have low valuation for the good, while $1-\lambda$ have a high valuation. For a given type k of buyer, the seller expects that, with probability η_k^*, the buyer will accept to negotiate, in which case the seller optimally decides to negotiate with that particular type, if and only if negotiating brings her a higher value than search.

Using lemma 1, let $V_P = \max\left\{V_P(P_L^P), V_P(P_H^P)\right\}$ where

$$rV_P\left(P_i^P\right) = \sigma\left[\lambda\beta_L^*\left(P_i^P\right) + \left(1-\lambda\right)\beta_H^*\left(P_i^P\right)\right]\left[P_i^P - V_P\left(P_i^P\right)\right] \tag{5}$$

A seller who opts to post a price P_i^P meets low valuation buyers with probability $\sigma\lambda$ and expects that, with probability $\beta_L^*\left(P_i^P\right)$, these buyers will accept the posted price P_i^P. The seller meets high valuation buyers with probability $\sigma(1-\lambda)$ and expects that with probability $\beta_H^*\left(P_i^P\right)$, these buyers will accept the posted price.

3.4 Equilibrium Strategies

We discuss the equilibrium strategies of the representative seller and buyer, using (3)-(5). Consider a seller. Recall that the selection of trade mechanism entails choosing a price to charge, contingent on having selected price posting, and the choice of whom to trade with, contingent on having selected to bargain. Recall also that in equilibrium the selection of strategies is based on the correct evaluation of the gains from trade in each possible match. Contingent on having chosen to negotiate, the seller chooses to bargain with a buyer of type i if, by doing so, she receives positive surplus, will randomize if the surplus is zero, and will not trade otherwise:

$$\pi_i \begin{cases} = 1 & \text{if } P_i^N > V_N \\ \in [0,1] & \text{if } P_i^N = V_N \\ = 0 & \text{if } P_i^N < V_N \end{cases} \tag{6}$$

Similarly, the seller strictly prefers to negotiate if the value from doing so is strictly greater than the value of the alternative (posting prices). Hence, in equilibrium, the individually optimal choice of trade mechanism must satisfy:

$$\pi \begin{cases} = 1 & \text{if } V_N > V_P \\ \in [0,1] & \text{if } V_N = V_P \\ = 0 & \text{if } V_N < V_P \end{cases} \tag{7}$$

Recall from lemma 1 that, contingent on having chosen to post prices, the seller selects one of two prices, P_L^P and P_H^P. She optimally chooses to extract the entire surplus from type i buyers, posting price P_i^P, if the value from doing so is the larger. If indifferent, she randomizes between P_L^P

and P_H^P with probability γ. Thus:

$$\gamma \begin{cases} = 1 & \text{if } V_P\left(P_H^P\right) > V_P\left(P_L^P\right) \\ \in [0,1] & \text{if } V_P\left(P_H^P\right) = V_P\left(P_L^P\right) \\ = 0 & \text{if } V_P\left(P_H^P\right) < V_P\left(P_L^P\right) \end{cases} \tag{8}$$

Now consider a buyer of type i. Contingent on having met a seller who is willing to negotiate, type i agrees to bargaining if by doing so he is able to obtain a positive net surplus, randomizes if the surplus is zero, and refuse to trade otherwise:

$$\eta_i \begin{cases} = 1 & \text{if } u_i - P_i^N - V_i > 0 \\ \in [0,1] & \text{if } u_i - P_i^N - V_i = 0 \\ = 0 & \text{if } u_i - P_i^N - V_i < 0 \end{cases} \tag{9}$$

Contingent on a match with a seller posting price P^P, type i buyer chooses his trading strategy $\beta_i\left(P^P\right)$ similarly:

$$\beta_i\left(P^P\right) \begin{cases} = 1 & \text{if } u_i - P^P - V_i > 0 \\ \in [0,1] & \text{if } u_i - P^P - V_i = 0 \\ = 0 & \text{if } u_i - P^P - V_i < 0 \end{cases} \tag{10}$$

Since we are looking for symmetric rational expectations equilibria, the strategy of an individual must be identical to that adopted by all other individuals of identical type. That is, in equilibrium:

$$\pi = \pi^*, \;\; \gamma = \gamma^*, \;\; \pi_i = \pi_i^*, \; \eta_i = \eta_i^*, \;\; \beta_i\left(P^P\right) = \beta_i^*\left(P^P\right) \;\; \forall i \in \{L, H\} \tag{11}$$

Definition *A symmetric stationary equilibrium is a set of value functions* $\left\{V_P\left(P_i^P\right), V_N, V_i\right\}_{\forall i}$, *prices* $\left\{P_i^P, P_i^N\right\}_{\forall i}$, *and strategies* $\{\pi, \gamma, \pi_i, \eta_i, \beta_i(P)\}_{\forall i}$ *such that:*

(i) Each agent maximizes his expected lifetime utility, i.e. the value functions satisfy (3)-(5) and the strategies satisfy (6)-(10) when $\left\{P_i^P, P_i^N\right\}_{\forall i}$ *and* $\{\pi^*, \pi_i^*, \eta_i^*, \gamma^*, \beta_i^*(P)\}_{\forall i}$ *are taken as given,*

(ii) The prices $\{P_i^P, P_i^N\}_{\forall i}$ *satisfy (1)-(2) when* $\{V_P(P_i^P), V_N, V_i\}_{\forall i}$ *are taken as given,*

(iii) Expectations are rational, i.e. (11) is satisfied when $\{V_P(P_i^P), V_N, V_i\}_{\forall i}$ *and* $\{P_i^P, P_i^N\}_{\forall i}$ *are taken as given.*

3.5 Characterization of equilibrium

In this section we investigate which types of equilibria can arise and the sets of parameters that support them, focusing attention to pure strategies[9]. To find conditions for the existence of a particular equilibrium, we conjecture its existence and solve the system of equations (defined in the previous section) for the candidate equilibrium. Once equilibrium prices are determined, this also involves checking for profitable deviations. The calculations are cumbersome and reported in appendix (together with functional forms).

Proposition 1 summarizes the conditions that guarantee existence of each possible pure strategy equilibrium. Due to our interest in studying how the selection of a trading mechanism is affected by the degree of heterogeneity in buyers' preferences, we discuss existence by partitioning the parameter space in regions defined over λ, the proportion of low valuation types and u_L/u_H, their relative valuation. These two variables summarize the information that sellers possess about the pool of buyers. We also investigate how the bargaining power θ affects the sellers' decision to negotiate or post prices. To begin, we show that bargaining will not take place in equilibrium unless there is *some* heterogeneity in buyers' valuations for the good, that is $\lambda \notin \{0, 1\}$.

Lemma 2 *When buyers are homogenous, the only equilibrium is a price posting equilibrium.*

When buyers are homogenous, the sellers know for sure the buyers' type and are able to extract the entire surplus from them. If all sellers are posting a price equal to the buyers' valuation for the good, the buyers have no other option but to buy the good at that price. It cannot be profitable for an individual seller to commit to negotiating with buyers, since she would have to cede some of the

[9] Although the setup developed enables us to look for both pure and mixed strategy equilibria, we restrict our attention to pure strategies. It is shown later that a pure strategy equilibrium always exists. Notice that allowing for mixed strategies would allow us to consider the issue of price dispersion.

surplus, but would not make trade more likely. Thus, given our focus on studying the competition between bargaining and price posting, we assume $0 < \lambda < 1$ in the remainder of the paper.

Lemma 3 *At most three pure-strategy equilibria are possible when buyers are heterogenous:*

(i) a bargaining equilibrium, where all sellers bargain with both types, i.e. $\pi^*=\pi_H^*=\pi_L^*=1$,

(ii) a high price posting equilibrium, where all sellers post P_H^P, *i.e.* $\pi^* = 0$ *and* $\gamma^* = 1$,

(iii) a low price posting equilibrium, where all sellers post P_L^P, *i.e.* $\pi^* = 0$ *and* $\gamma^* = 0$,

denoted as B-, H-, and L-equilibria, respectively.

Equilibria where sellers are only willing to bargain with a single type are not sustainable. Assume that all sellers decide to bargain with only type i buyers. In that case, the other buyers are shut out of the market and the situation is similar to an economy with homogenous buyers. Then, it is always profitable for a seller to deviate by posting a price that extracts the entire surplus from the buyers who remain active in the market. Hence, equilibria where $\pi^* = \pi_H^* = 1, \pi_L^* = 0$ or $\pi^* = \pi_L^* = 1, \pi_H^* = 0$ are not sustainable. However, any parameterization supports at least some of the remaining three pure strategy equilibria.

Proposition 1 *There always exists a pure-strategy equilibrium. In particular,*

(i) No B-equilibrium exists when $\theta \leq \frac{1}{2}$, regardless of other parameter values. *If* $\theta > \frac{1}{2}$, *such an equilibrium exists if and only if* $0 < \widetilde{\lambda}(\theta) < \lambda < 1$ *and* $0 < \underline{u}(\lambda, \theta) < \frac{u_L}{u_H} < \overline{u}(\lambda, \theta) < 1$.

(ii) An H-equilibrium exists if and only if $0 \leq \frac{u_L}{u_H} < \widetilde{u}(\lambda, \theta)$, *given* λ *and* θ

(iii) An L-equilibrium exists if and only if $\widehat{u}(\lambda, \theta) < \frac{u_L}{u_H} \leq 1$, *given* λ *and* θ

(Functional forms for $\widetilde{\lambda}(\theta)$, $\underline{u}(\lambda, \theta)$, $\overline{u}(\lambda, \theta)$, $\widetilde{u}(\lambda, \theta)$ *and* $\widehat{u}(\lambda, \theta)$ *are available in appendix. Notice that* $\widetilde{u}(\lambda, \theta)$ *and* $\widehat{u}(\lambda, \theta)$ *are decreasing in* λ.*)*

In a nutshell, the intuition is that, when bargaining, sellers are willing to cede some of the surplus to the buyer (depending on the bargaining power θ), since bargaining allows them to learn about the buyer's type and therefore to "price discriminate" between types. Hence, bargaining arises as an equilibrium, when (1) the implicit cost sellers have to pay to learn about buyers' preferences is not too high (high θ), and (2) the cost of neglecting one type of customers is high (intermediate values of λ and u_L/u_H). On the other hand, price posting arises when sellers are better off focusing on a specific type of customers, which, of course, depends on their distribution and preferences.

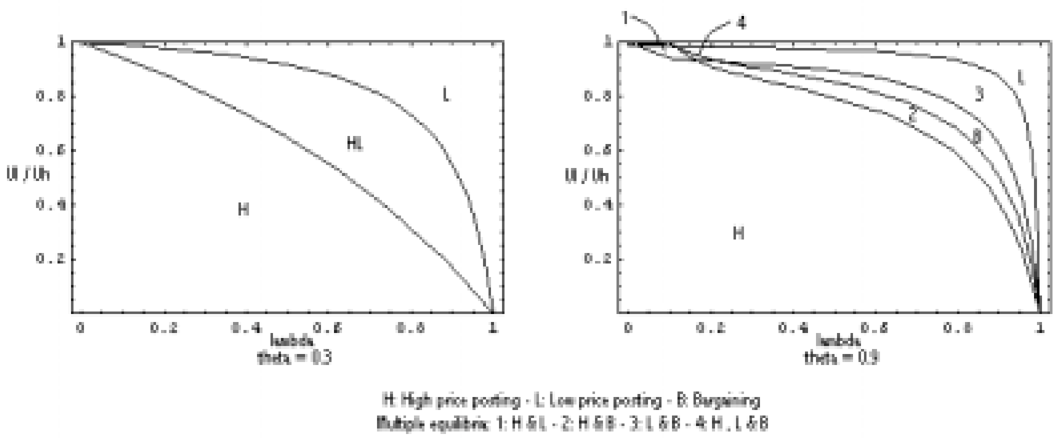

Figure 1: Equilibrium regions: $r = 0.05,\ \alpha = 0.25,\ \sigma = 0.5$

Figure 1 provides a graphical illustration of the equilibrium regions for particular parameter values. It shows that the equilibrium outcome depends on the proportion λ of each type, and their relative valuation (u_L/u_H) in an intuitive manner, as the various equilibria involve a trade-off between how fast the trade is expected to be completed and the gain from a realized trade. With heterogeneous buyers, there is an opportunity cost of trading with the first potential partner, which is the gain from trading with the other type of buyers. Depending on the proportion of each type in the market and the differences in gains from trade, as influenced by the relative valuation u_L/u_H, any one of the three equilibria may arise. All else equal, the larger the proportion λ, the more attractive is targeting the low-valuation buyers; the smaller u_L/u_H, the greater are the incentives to target the high-valuation buyers. For given λ and u_L/u_H, the share of surplus obtainable is also an important parameter, as illustrated by the following table:

	Seller's share of surplus with H	Seller's share of surplus with L
H-equilibrium	1	*No trade*
L-equilibrium	*Some*	1
B-equilibrium	θ	θ

Given λ, there is an H-equilibrium when buyers' valuations differ substantially, i.e. $u_L/u_H < \widetilde{u}(\lambda, \theta)$. If there are buyers who value the good very highly, it is optimal for sellers to post a premium price even though by doing so they decrease their probability of trading during the period. By the same token, the lower bound $\widetilde{u}(\lambda, \theta)$ is decreasing in λ. As λ increases, the expected probability of a transaction drops. Posting high prices remains individually optimal only in a market where buyers have an even more dramatic disparity in their valuations for the good being offered.

Given λ, there is an L-equilibrium when buyers have close valuations for the good, i.e. $u_L/u_H > \widehat{u}(\lambda, \theta)$. When buyers have similar valuations, the seller does not relinquish too much surplus to the high type by posting the lower price, yet she is able to transact with both types of buyers. As the proportion of low types increases, however, $\widehat{u}(\lambda, \theta)$ falls. The less likely it is to meet high valuation buyers, the less the seller foregoes by posting the lower price.

Finally, given λ, a bargaining equilibrium arises for intermediate values of u_L/u_H. A high value of θ, the sellers' bargaining power, is also necessary for such equilibria. From Lemma 3, we know that a seller who chooses to negotiate, always does so with both types of buyers. Hence, by bargaining, a seller maximizes the frequency of trades (as she would in an L-equilibrium). However, she also gives away some of the surplus to both types. In order for her to prefer negotiating to posting a low price, it is necessary that λ and u_L/u_H not be too high. Similarly, when λ and u_L/u_H are not too low, she does not have any incentive to post a high price. In addition, if her bargaining power θ is too low, she has to cede too much surplus in each possible match and the two conditions on λ and u_L/u_H cannot hold simultaneously (in particular, if she has to give more than half of the surplus to both types when negotiating, she will prefer to post one of two prices and receive the entire surplus from just one type). By posting a price, sellers "focus" on a particular segment of the population. When they post a high price, they extract the entire surplus from high types, but do not trade with low types. When they post a low price, they extract the whole surplus from low types, but little surplus from high types. Hence, sellers prefer to negotiate when it is costly to neglect a segment of the population. That is the case when the population is sufficiently heterogenous (λ not too close to 0 or 1) or when type L and H preferences are neither too similar nor too distinct (u_L/u_H not too close to 0 or 1). Of course, if the seller has to cede too much surplus (low θ) when negotiating, she will prefer to focus on one type of buyers.

Figure 1 reveals the possibility of multiple equilibria. The multiplicity is due to strategic complementarities in the choice of trading mechanisms. As opposed to Riley and Zeckhauser (1983), Wang (1995) and Arnold and Lippman (1999), there is a continuum of sellers who, when selecting a mechanism, have to take into account what other sellers are doing. That choice gives rise to a *price setting externality*. When sellers opt for a particular method of determining prices, they affect buyers' search values, and in turn, the value to other sellers of choosing an alternative mechanism. Hence, the mechanism retained by some sellers also influences other sellers' profit per trade. By deciding to negotiate, for example, sellers increase the value of search to buyers, who can expect to extract some of the surplus from trading. In this case, a price poster has to reduce the price she needs to post to make buyers just indifferent between accepting the price and continuing search. Another way to view this is that, if price posters do not adjust their prices, then their probabilities of trading goes to zero.

The multiplicity of equilibria can be intuitively explained as follows. First assume that parameters are such that a high price posting equilibrium exists. Now suppose that all sellers post a low price. By doing so, they increase the value of search V_H to high valuation types, who now experience a strictly positive surplus from trade. A seller considering posting a high price would be forced to decrease her price, thereby reducing her value from posting a high price. It is thus possible that sellers would find it preferable to post P_L^P, given that everybody else posts that price. This may hold, despite the fact that if everybody posted P_H^P, then individual sellers would also prefer to post a high price. Similarly, assume that parameters are such that a price posting equilibrium is sustainable. Now assume that all sellers agree to bargain. Splitting the surplus between buyers and sellers increases the value of search to both buyers. In that case, sellers wishing to post a price have to set a lower price, possibly decreasing their value of posting a price below the value of bargaining.

4 Repeat purchases

In this section, we consider the case where sellers and buyers separate after a transaction, return to the search pool, and independently resume search for a trading partner.

Since agents return to the search pool after a completed exchange, the payoff from a transaction is now equal to the immediate value of consuming the trade, P to the seller and $u_i - P$ to a type i buyer, augmented by the value of search. Thus, the surplus obtained by a type i buyer from a purchase at price P is independent of his value of search, since $V_i + u_i - P - V_i = u_i - P$. Similarly, the surplus to the seller is P. It follows that a trade will take place if and only if $0 < P < u_i$. Hence, the value functions must satisfy:

$$\begin{aligned} rV_i &= \alpha\pi^*\pi_i^* \max_{\eta_i}\left[\eta_i\left(u_i - P_i^N\right)\right] + \alpha\left(1-\pi^*\right)\gamma^* \max_{\beta_i\left(P_H^P\right)}\left[\beta_i\left(P_H^P\right)\left(u_i - P_H^P\right)\right] \\ &\quad + \alpha\left(1-\pi^*\right)\left(1-\gamma^*\right) \max_{\beta_i\left(P_L^P\right)}\left[\beta_i\left(P_L^P\right)\left(u_i - P_L^P\right)\right] \end{aligned} \tag{12}$$

$$rV_N = \sigma\lambda\eta_L^* \max_{\pi_L}\left[\pi_L P_L^N\right] + \sigma\left(1-\lambda\right)\eta_H^* \max_{\pi_H}\left[\pi_H P_H^N\right] \tag{13}$$

$$rV_P\left(P_i^P\right) = \sigma\lambda\beta_L^*\left(P_i^P\right)P_i^P + \sigma\left(1-\lambda\right)\beta_H^*\left(P_i^P\right)P_i^P \tag{14}$$

It is easy to see that, as in section 3, there can be at most two posted prices in equilibrium. These prices are set to extract the entire trade surplus from one type of buyers. Since the buyers' surplus is equal to $u_i - P$, then $P^P \in \left\{P_i^P\right\}_{i=L,H}$ where:

$$P_i^P = u_i \tag{15}$$

The price resulting from a negotiation with a buyer of type i divides the total surplus according to the respective bargaining powers. Hence:

$$P_i^N = \theta u_i \tag{16}$$

The analysis of equilibrium strategies is similar to section 3: π and γ must satisfy (7)-(8), while

$$\pi_i \begin{cases} = 1 & \text{if } P_i^N > 0 \\ \in [0,1] & \text{if } P_i^N = 0 \\ = 0 & \text{if } P_i^N < 0 \end{cases} \tag{17}$$

$$\eta_i \begin{cases} = 1 & \text{if } u_i > P_i^N \\ \in [0,1] & \text{if } u_i = P_i^N \\ = 0 & \text{if } u_i < P_i^N \end{cases} \tag{18}$$

$$\beta_i\left(P^P\right) \begin{cases} = 1 & \text{if } u_i > P^P \\ \in [0,1] & \text{if } u_i = P^P \\ = 0 & \text{if } u_i < P^P \end{cases} \tag{19}$$

Equilibrium can be defined as in section 3: the value functions must satisfy (12)-(14) and the strategies (7)-(8) and (17)-(19); prices are given by (15)-(16); expectations are rational, i.e. (11) is satisfied.

4.1 Characterization of equilibrium

It is possible to determine the equilibrium regions in a similar fashion to the one-time purchase case. There are similarities between the two cases. Lemmas 2 and 3 also hold for repeat purchases. If buyers were homogenous, sellers would be able to extract the entire surplus from trade and would always post prices. For repeat purchases, there are also only three possible pure strategy equilibria. Proposition 1 also has its equivalent in the repeat purchase case (see appendix). That is, a pure strategy equilibrium always exists. Also, for a bargaining equilibrium to exist, the seller's bargaining power θ needs to be high enough (greater than 1/2), λ needs to greater than $\widetilde{\lambda}(\theta)$ and u_L/u_H needs to be in an interval, whose boundaries depend on λ and θ, and which is bounded away from 0 and 1. Likewise, high price posting equilibria, for a given λ and θ, are obtained for any values of u_L/u_H below some threshold, while low price posting equilibria are supported for u_L/u_H above some threshold. For the sake of comparison, the equilibrium regions are depicted in figure 2. However, there is an important difference between the one-time and the repeat purchase case.

Proposition 2 *Equilibria are unique in the case of repeat purchases.*

Uniqueness follows from the absence of a price setting externality. Since the agents resume search after consuming the trade, a necessary and sufficient condition for an exchange to take place is that the instantaneous values of consuming the trade (P to the seller and $u_i - P$ to the buyer) be positive. Therefore, the respective values of search do not enter the agents' trade surpluses and the terms of trade do not depend on the values of search. It follows that the decision of an individual seller cannot be affected by what the other sellers are doing. Hence, there is no strategic complementarities present in this case.

5 Bargaining under asymmetric information

In sections 3 and 4, we assumed that, upon bargaining, the seller learned the buyer's type. That is, ex-ante private information was revealed before negotiations actually took place (but after the seller committed to negotiating). Hence, we assumed commitment to the price mechanism selected. In this section, we instead relax the assumption of commitment and assume that types are not revealed prior to the negotiations. Rather, the seller and the buyer engage in a multi-period asymmetric information bargaining game, where the seller may try to extract some information about the buyer's preferences by observing the buyer's response to previous price offers. Hence, information may get

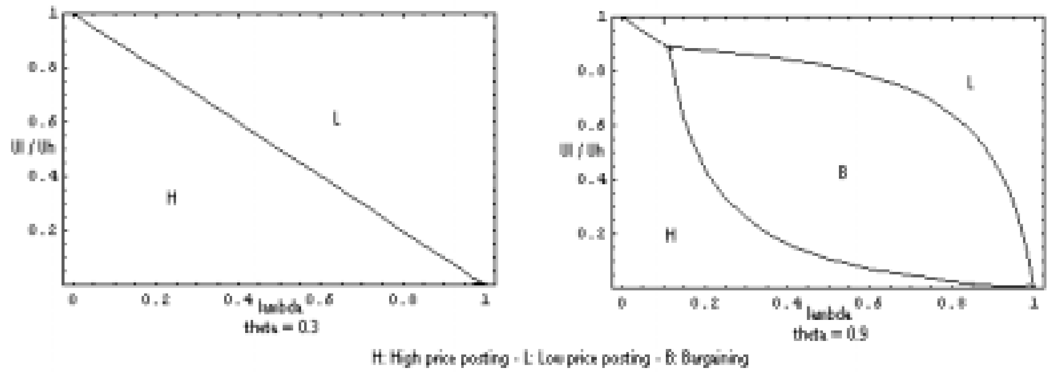

Figure 2: Equilibrium regions: $r = 0.05,\ \alpha = 0.25,\ \sigma = 0.5$

revealed *as part of* the bargaining process, but at a cost to the seller, due to bargaining delays. This is to be contrasted with the previous section, where the seller's ability to price discriminate by means of bargaining, came at the cost of having to cede some of the surplus. In what follows, we limit the discussion to the case of one-time-purchases.

5.1 The bargaining game

The structure of the bargaining game very closely follows Fudenberg and Tirole (1983). This is a simple framework where the seller may use the bargaining process to extract information about the buyer's type. It also has the advantage of providing a unique, parametric solution to the bargaining game. Since there is only one type of sellers, only the buyer's type is private information. The seller has a prior belief that a proportion $\Lambda \in [0, 1]$ of those buyers willing to negotiate with her are of type L. Bargaining takes place in two stages. Conditional on a meeting between two agents willing to negotiate, in the first period (at date t), the seller makers an offer p_1. The buyer may accept or refuse the offer. In case the offer is accepted, the trade is completed and the payoffs are p_1 to the seller and $u_i - p_1$ to a buyer of type i. In case the offer is refused, the players go to a second period (date $t+1$). Then, using any information she can deduce from the buyer's first-stage response, the seller offers price p_2. If the buyer accepts, the payoffs are δp_2 and $\delta(u_i - p_2)$ to the seller and buyer respectively, where $\delta = \frac{1}{1+r}$ (the period's payoffs are discounted back to date t). If the offer is refused by the buyer, the payoffs are δV_N and δV_i (this is because the two players then return to search and have a possibility of a meeting only at date $t+2$). If an agreement is not reached in the first period, both agents must bargain for another period, without the possibility of meeting any one else[10].

The solution concept is the Perfect Bayesian equilibrium[11]. The methodology used and the solution to the bargaining are described in appendix. We show in it that there can only be two possible equilibrium price offers in the second stage of the game: $p_2^i = u_i - V_i,\ i \in \{L, H\}$ that leave type i indifferent between accepting and refusing. Consider the following table:

		undiscounted 2nd stage payoffs to:	
		type L	*type H*
2nd stage prices	$u_L - V_L$	V_L	$u_H - u_L + V_L$
	$u_H - V_H$	V_L	V_H

Although it is necessary to consider all possible prices when looking for equilibrium, one can see that three first-stage prices p_1 are pertinent. These are the prices that make the buyer indifferent between accepting the first-stage price or getting one of the three possible second-stage payoffs. That is $p_1 \in \left\{p_1^L, \widehat{p}, p_1^H\right\}$ where: $u_L - p_1^L = \delta V_L$, $u_H - \widehat{p} = \delta(u_H - u_L + V_L)$ or $u_H - p_1^H = \delta V_H$. Note that

[10]Because negotiations may last two periods, the decision to bargain may affect the type distribution in the searching pool. We abstract away from this point.

[11]It requires that in every period, each type of buyers maximizes his payoff, taking the seller's strategy as given. For a given first-stage price offer p_1, the seller chooses a price $p_2(p_1)$ that maximizes her expected second-stage payoff, taking the buyer's response to first- and second-stage offers as given, and updating her posterior beliefs (on the buyer's type) using Bayes' rule. Finally, the seller chooses a first-stage offer p_1 that maximizes her first-stage expected payoffs, taking buyer's strategies and her second-stage strategy $p_2(p_1)$ as given.

$p_1^L < \widehat{p} < p_1^H$. Offers in between do not affect the probability the offer will be accepted, but reduce the profit to the seller conditional on acceptance. As a result, in the presentation, we can restrict our attention to these three prices.

The solution to the bargaining game depends on the seller's prior beliefs regarding the distribution of buyers willing to negotiate, denoted by $\Lambda \in [0,1]$. If in equilibrium, $\Lambda \in \{0,1\}$, there is no uncertainty regarding the buyer's type. The seller exploits this information by extracting the entire surplus from the buyer. That is, if $\Lambda = 0$, then $p_1 = p_1^H$ and if $\Lambda = 1$, $p_1 = p_1^L$. In equilibrium, these prices are accepted and negotiations terminate after one period. If there is some uncertainty about the buyer's type ($0 < \Lambda < 1$), but the seller gains from trading with only one type, then the seller also extracts the entire surplus from that particular type. If there are gains from trade with both types, the seller may choose a sequence of offers that allows her to extract information about the buyers type. In fact, there are three possible equilibria. In two of these equilibria, the seller offers, in each period, a price that extracts the entire surplus only from one type. In the remaining one, the seller discriminates between the two types. The first stage offer captures type H buyers. If this offer is rejected, the seller lowers the offer to capture what is now known to be type L (even though the seller may be able to discriminate in that way, it is costly to her, because of bargaining delays). This discussion is summarized below:

$$\{p_1, p_2\} = \begin{cases} \left\{p_1^H, p_2^H\right\} & \text{if } \Lambda = 0 \\ \left\{p_1^L, p_2^L\right\} & \text{if } \Lambda = 1 \\ \left\{p_1^L, p_2^L\right\}, \left\{p_1^H, p_2^H\right\}, \left\{\hat{p}, p_2^L\right\} & \text{if } \Lambda \in (0,1) \end{cases}$$

The conditions under which a particular equilibrium arises and the associated payoffs are reported in appendix. Notice that restricting the bargaining game to two periods is not restrictive, since it is enough to allow the seller to discriminate between the two buyers' types.

5.2 Definition of equilibrium

Value functions

The main difference with sections 3 and 4 is that the seller does not have the ability to distinguish between the two types, conditional on having chosen to negotiate. Thus, a particular type cannot be excluded from the negotiations and π_i ($i \in \{L, H\}$) are not decision variables any longer. Denote by $\Pi_s(p_1, p_2)$ ($\Pi_i(p_1, p_2)$) the payoff to the seller (type i buyer) from a negotiation in which equilibrium offers $\{p_1, p_2\}$ are made. Thus, keeping the same notations as previously:

$$\begin{aligned} rV_i &= \alpha\pi^* \max_{\eta_i} \left[\eta_i \left(\Pi_i(p_1, p_2) - V_i\right)\right] + \alpha(1-\pi^*)\gamma^* \max_{\beta_i\left(P_H^P\right)} \left[\beta_i\left(P_H^P\right)\left(u_i - P_H^P - V_i\right)\right] \\ &\quad + \alpha(1-\pi^*)(1-\gamma^*) \max_{\beta_i\left(P_L^P\right)} \left[\beta_i\left(P_L^P\right)\left(u_i - P_L^P - V_i\right)\right] \end{aligned} \tag{20}$$

$$rV_N = \sigma\left(\lambda\eta_L^* + (1-\lambda)\eta_H^*\right)\left[\Pi_s(p_1, p_2) - V_N\right] \tag{21}$$

while $V_P\left(P_i^P\right)$ satisfies (5).

Similarly to section 3, the equilibrium strategies must satisfy (7)-(8), (10) and (22) where:

$$\eta_i \begin{cases} = 1 & \text{if } \Pi_i(p_1, p_2) - V_i > 0 \\ \in [0, 1] & \text{if } \Pi_i(p_1, p_2) - V_i = 0 \\ = 0 & \text{if } \Pi_i(p_1, p_2) - V_i < 0 \end{cases} \tag{22}$$

That is, type i enters the bargaining process if expected payoffs is no less than his value of search.

The seller has to form a prior belief Λ, regarding the distribution of types of buyers willing to negotiate. In equilibrium, this prior belief has to be consistent with the ex-ante distribution of types in the entire economy, as well as the buyers' equilibrium strategies. The proportion of low types among buyers willing to negotiate is given by:

$$\Lambda = \frac{\lambda\eta_L^*}{\lambda\eta_L^* + (1-\lambda)\eta_H^*} \tag{23}$$

Definition of equilibrium *A symmetric stationary equilibrium is a set of value functions, prices, and strategies such that:*

(i) The value functions satisfy (5) and (20)-(21), and the strategies satisfy (7)-(8), (10) and (22),

(ii) Posted prices satisfy (1) and negotiated price offers must be equilibrium outcomes of the bargaining game, as described in appendix,

(iii) The sellers' prior beliefs Λ are consistent with equilibrium strategies, i.e. (23) is satisfied,

(iv) Expectations about the other agents's strategies are rational, i.e. (11) is satisfied.

5.3 Characterization of equilibrium

It is proved in appendix that a pure strategy equilibrium always exists. Again we can find existence conditions, by postulating an equilibrium candidate and checking for possible deviations (the methodology is provided in appendix). Rather than to contrast the equilibrium conditions in this case with the ones previously found, we want instead to emphasize the new trade-off faced by the seller in selecting her price mechanism. Remember that when we assume Nash bargaining, the seller may want to negotiate with buyers, because even though she has to cede a fixed proportion of the surplus in each transaction, it allows her to trade more often. In the present case however, the negotiating seller can always price to extract the entire surplus from the low types in either period of the game. But by offering different prices in each period ($\hat{p}$ and p_2^L), the seller is able to (intertemporally) discriminate between types[12]. This is not perfect discrimination though, since (i) she gets the low type buyers only in the second period and (ii) she cannot extract the entire surplus from high types in the first period, because these buyers have the option to wait for another period and pay the lower price. Hence, the seller prefers bargaining to posting a low price when the extra surplus she can make by targeting the high types in the first period makes up for having to wait another period to get the low types[13]. And she prefers negotiating to posting a high price, when she is better off trading with both types and discriminating (to the extent explained above) rather than trading with only high types and extracting the entire surplus from them. In that case, the surplus lost in trades with high types is made up in trades with low types. Even though the nature of the trade-off is

[12] As explained in appendix, bargaining equilibria where the price sequences offered are $\{p_1^{\mathsf{H}}, p_2^{\mathsf{H}}\}$ or $\{p_1^{\mathsf{L}}, p_2^{\mathsf{L}}\}$, are indistinguishable from H- or L-equilibria. Hence, we refer to B-equilibria as ones where the price sequence is $\{\hat{p}, p_2^{\mathsf{L}}\}$.

[13] Specifically, $(1-\lambda)[\widehat{p} - u_{\mathsf{L}}] > \lambda [u_{\mathsf{L}} - \delta u_{\mathsf{L}}] \Longrightarrow (1-\lambda)[u_{\mathsf{H}} - \delta(u_{\mathsf{H}} - u_{\mathsf{L}}) - u_{\mathsf{L}}] > \lambda [u_{\mathsf{L}} - \delta u_{\mathsf{L}}]$
$\Longrightarrow (1-\lambda) > \frac{\mathsf{u}_{\mathsf{L}}}{\mathsf{u}_{\mathsf{H}}}$.

different than in sections 3 and 4, it leads to the same qualitative result. For a given λ, if u_L is close to u_H, the seller does not want to delay trades (with low types) and hence, does not negotiate. And if u_L/u_H is close to 0, the seller does not mind foregoing trades with low types and is not willing to give up too much surplus to the high types, and thus does not discriminate either, but rather posts a high price. Hence, as illustrated in figure 3, B-equilibria are only possible for "intermediate" values of u_L/u_H where discriminating between types dominates targeting a particular type of buyers, while price posting is an equilibrium when u_L/u_H is either close to 0 or 1.

Notice that, in fact, we dot not even have to require sellers to choose a particular trade mechanism each period. We could instead assume that agents always play the bargaining game described above, once they meet a partner willing to trade. In equilibrium, one of three things would happen: (i) the price targeted at the high type would be offered in both periods and accepted in the first, (ii) the price targeted at the low type would be offered in both periods and accepted in the first, or (iii) two different prices would be quoted in the first and second period, high types accepting to trade in the first period and low types in the second period. It is straightforward to prove that it would result in the exact same equilibrium prices and equilibrium regions than under the setup actually described above. Hence, trade patterns would look identical to an outside observer under the two setups. Sellers would not have to commit to price posting or bargaining, but simply play the bargaining game all the time. Sometimes, it would look like prices are posted (offers do not change and are accepted immediately), and sometimes it would look like a bargaining game (offers changes between periods).

The graphs also reveal two properties of the model. First, when buyers meet sellers infrequently, there is no bargaining in equilibrium. Second, as in section 3, multiple equilibria are a possibility.

Proposition 3 *When $\sigma > \alpha$, there cannot be a bargaining equilibrium.*

Notice that a high value for σ and a low value for α are incompatible with a B-equilibrium. In that case, both types of buyers have infrequent trading opportunities. Sellers considering posting a price targeted at either type i can take advantage of that fact to set a high value for P_i^P. If,

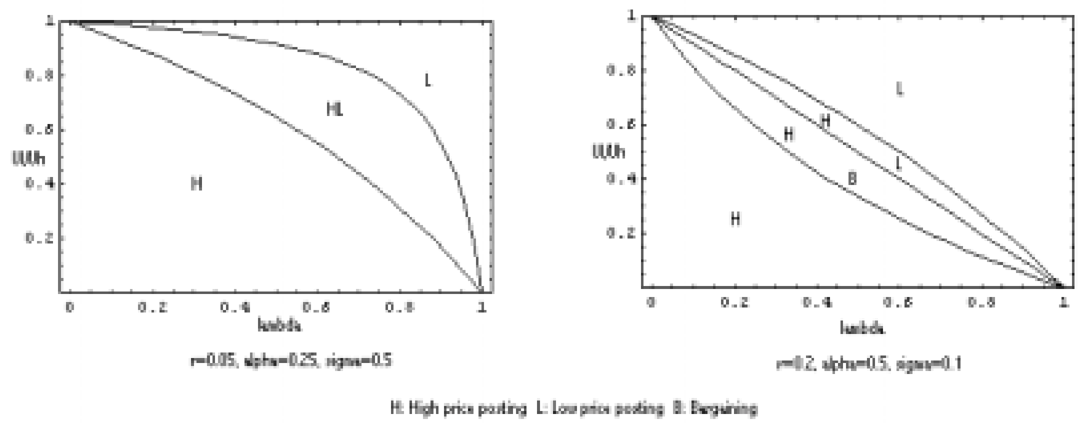

Figure 3: Equilibrium regions - asymmetric bargaining

in addition, sellers themselves have no difficulty meeting buyers, they prefer to target a particular type of buyers, rather than negotiate. Proposition 3 establishes that a necessary condition for a B-equilibrium is that $\sigma \leq \alpha$ or that buyers meet sellers more frequently than sellers meet buyers (a buyers' market).

Finally, this version of the model also allows for the possibility of multiple equilibria. This happens for the same reason as in section 3: choosing a trade mechanism influences the buyers' search values and, hence, other sellers' choice of a price mechanism. Therefore, we also have a *price setting externality* in this version of the model.

6 Conclusion

We studied the endogenous determination of the trading mechanism, between bargaining and price posting, when there is a multitude of both buyers and sellers. By relaxing the assumption of a monopolist seller, we allowed for strategic interactions among sellers. We analyzed how heterogeneity in buyers' preferences for the good being sold influenced the sellers' decision to bargain or to post a price. There are two main results to our study. First, the determination of the pricing mechanism entails a trade-off between how fast the trade is expected to be completed and the gain from a realized trade. When sellers post a price, they focus on a particular type of buyers. When we assume commitment to the pricing mechanism, Nash bargaining allows the seller to trade with a variety of types at the cost of having to cede a fixed portion of the surplus to the buyer.

When instead, we relax the assumption on commitment and consider bargaining under asymmetric information, then negotiating allows the seller to discriminate, to some degree, between buyers by offering different prices at different stages of the negotiations. This, of course, comes at the cost of possible delays in reaching an agreement. Even though the two cases are different, they have similar implications. Sellers choose to negotiate with buyers, when it is not optimal for them to focus on one particular category of buyers.

The second finding of the paper is the presence, in some cases, of a *price setting externality*. When sellers choose a pricing mechanism, they influence the buyers' value of staying in the market and therefore, other sellers' optimal choice of a trade mechanism. We showed that these strategic complementarities are only relevant when agents leave the market after the transaction is completed, but not when agents resume search after completing a trade.

This set-up can be used to address both empirical and theoretical questions. For example, the predictions of this model regarding the types of goods that tend to be bargained over, could be directly tested. Unfortunately, as far as we are aware, there is no empirical literature, beyond casual observation, as to which products are negotiated. From a theoretical point of view, this model also sheds some light on how appropriate some exogenous assumptions about trading mechanisms can be, in models of search or others. One could also use this set-up, allowing for mixed strategies, to study the issue of price dispersion. Finally, this model can be easily extended to address various issues. For example, one could add costs associated with the different mechanisms. It could also be adjusted to allow for auctions as an endogenous trading mechanism, as is done in Julien, Kennes and King (2000), Kultti (1999), Lu and McAfee (1996) and Wang (1993), in different contexts.

Appendix

Proof of lemma 1

Note that a buyer will buy if the price leaves him some positive surplus and that the surplus to the seller (buyer) is increasing (decreasing) in P^P. Suppose the seller posts a price P^P which leaves some types of buyers with strictly positive surplus, and some others possibly with strictly negative surplus, but no type with zero surplus. If P^P is such that all buyers' surpluses are negative, this cannot be an equilibrium. Let i_o denote the type of the buyer who receives the smallest strictly positive surplus from a purchase. Since $u_{i_o} - V_{i_o} > P^P$, a seller can increase P^P by ε, such that $u_{i_o} - \left(P^P + \varepsilon\right) - V_{i_o} > 0$, for any arbitrarily small ε. This increase will give the seller a larger surplus from each sale, while the probability of selling to any buyer of type i_o will not decrease. What about the probability to sell to buyers of type $k \neq i_o$? If $u_{i_o} - V_{i_o} \leq u_k - V_k$, buyers of type k will also buy, and if $u_i - V_i > u_k - V_k$ then the probability to sell to buyer k is also unaffected by the proposed increase in P^P (they never buy anyway). This implies that the optimal price choice for a price-posting seller must be chosen among one of two possible quantities, that is $P^P \in \left\{P_i^P\right\}_{i=L,H}$ where $P_i^P = u_i - V_i$.

Proof of Lemma 2

Assume that buyers are homogeneous. Without loss of generality, suppose that $\lambda = 1$. Also, assume that all buyers decided to bargain. Thus, (3) and (4) imply that $rV = \alpha\left(1-\theta\right)\left(u - V - V_N\right)$ and $rV_N = \sigma\theta\left(u - V - V_N\right)$. An individual seller who would like to deviate and post a price, would post the price $\widetilde{P}^P = u - V$. This would bring her a value of posting a price given by $rV_P(\widetilde{P}^P) = \sigma(u - V - V_P(\widetilde{P}^P))$. Since $\theta < 1$ and $u - V > 0$, it is immediate that $V_P(\widetilde{P}^P) > V_N$.

Proof of Lemma 3

It suffices to show that $\{\pi^* = \pi_H^* = 1, \pi_L^* = 0\}$ and $\{\pi^* = \pi_L^* = 1, \pi_H^* = 0\}$ cannot be sustained as equilibria. We will show that this is true of the first candidate equilibrium. The proof is similar for the other one. Assume that sellers follow the strategy $\{\pi^* = \pi_H^* = 1, \pi_L^* = 0\}$. Solving (3) and (4), it is easy to obtain that $V_L = 0$, $V_H = \frac{\alpha(1-\theta)}{r+\alpha(1-\theta)+\sigma(1-\lambda)\theta} U_H$ and $V_N = \frac{\sigma(1-\lambda)\theta}{r+\alpha(1-\theta)+\sigma(1-\lambda)\theta} U_H$. An individual seller wishing to post a price targeted at the high valuation types would post a price $\widetilde{P}_H^P = u_H - V_H = \frac{r+\sigma(1-\lambda)\theta}{r+\alpha(1-\theta)+\sigma(1-\lambda)\theta} U_H$. Notice that, by trading with the high valuation types at price $\widetilde{P}_H^P$, the seller receives a value $V^P(\widetilde{P}_H^P)$ such that $rV^P(\widetilde{P}_H^P) \geq \sigma(1-\lambda)(\widetilde{P}_H^P - V^P(\widetilde{P}_H^P))$. Hence, $V^P(\widetilde{P}_H^P) \geq \frac{\sigma(1-\lambda)}{r+\sigma(1-\lambda)} \frac{r+\sigma(1-\lambda)\theta}{r+\alpha(1-\theta)+\sigma(1-\lambda)\theta} U_H$. From the equation for V_N, $V^P(\widetilde{P}_H^P) \geq \frac{r+\sigma(1-\lambda)\theta}{[r+\sigma(1-\lambda)]\theta} V_N > V_N$. Hence, the candidate strategy cannot be supported as an equilibrium.

Proof of Proposition 1

We first establish existence conditions for the different types of pure strategy equilibria and use these to derive the existence of at least one such equilibria, for all parameter values. There are three possible pure strategy equilibria ($\{\pi^* = \gamma^* = 0\}$, $\{\pi^* = 0, \gamma^* = 1\}$ and $\{\pi^* = \pi_L^* = \pi_H^* = 1\}$). For each candidate equilibrium, it is possible to solve for the equilibrium value functions and prices. This is not enough to guarantee equilibrium, though. For this to be the case, we need to check that no individual seller has an incentive to deviate by offering another mechanism. Hence, we need to compute the deviating price(s) and the values of offering the other mechanisms given the deviating prices. For example, for the equilibrium where $\pi^* = 1$, $\pi_L^* = 1$, $\pi_H^* = 1$, we compute the equilibrium value functions V_L, V_H and V_N. With these, it is possible to compute the (deviating) prices $\widetilde{P}_L^P$ and $\widetilde{P}_H^P$ that an individual seller considering posting a price, given that all other sellers are bargaining, would pick. One can then check that $V_N > V^P(\widetilde{P}_L^P)$ and $V_N > V^P(\widetilde{P}_H^P)$ (in that case, it is also necessary to verify that $u_i - V_i - V_N > 0$, $i \in \{L, H\}$). Notice that it is enough to check for deviations in pure strategies (and not in mixed ones), since a deviating seller would only want to randomize, if she were indifferent between the possible deviations, which would bring her the same value as any one of the pure strategy deviations already considered. The results are summarized below (more detailed calculations are available upon request):

Case where $\pi^* = 1,\ \pi_L^* = 1,\ \pi_H^* = 1$. One needs $\underline{u}(\lambda,\theta) < \frac{u_L}{u_H} < \overline{u}(\lambda,\theta)$:

$$\begin{aligned}\text{where } \underline{u}(\lambda,\theta) &= 1 + \frac{[r+\alpha(1-\theta)][1-\lambda-\theta]}{\lambda\theta[r+\alpha(1-\theta)+\sigma(1-\lambda)]} \\ \text{and } \overline{u}(\lambda,\theta) &= 1 - \frac{(1-\theta)[r+\alpha(1-\theta)]}{(1-\theta)[r+\alpha(1-\theta)]+[r+\sigma+\alpha(1-\theta)]\theta(1-\lambda)}\end{aligned}$$

Case where $\pi^* = 0,\ \gamma^* = 1$. One needs $\frac{u_L}{u_H} < \widetilde{u}(\lambda,\theta)$:

$$\text{where } \widetilde{u}(\lambda,\theta) = Min\left\{1 - \frac{r\lambda}{r\lambda+(r+\sigma)(1-\lambda)}, 1 + \frac{r(1-\theta-\lambda)}{\lambda\theta[r+\sigma(1-\lambda)]}\right\}$$

Case where $\pi^* = 0,\ \gamma^* = 0$. One needs $\frac{u_L}{u_H} > \widehat{u}(\lambda,\theta)$:

$$\text{where } \widehat{u}(\lambda,\theta) = Max\left\{1 - \frac{(r+\alpha)\lambda}{(r+\alpha)\lambda+(r+\sigma)(1-\lambda)}, 1 - \frac{(r+\alpha)(1-\theta)}{(r+\alpha)(1-\theta)+(r+\sigma)\theta(1-\lambda)}\right\}$$

B-equilibrium: We know that we need $\frac{u_L}{u_H} > \underline{u}(\lambda,\theta)$. It can be readily verified that $\underline{u}(\lambda,\theta) > 0$ and $\underline{u}(\lambda,\theta) < 1$, if $\lambda > 1-\theta$. For a B-equilibrium, we also need $\frac{u_L}{u_H} < \overline{u}(\lambda,\theta)$. It is immediate that $0 < \overline{u}(\lambda,\theta) < 1$. One can verify that $\underline{u}(\lambda,\theta) < \overline{u}(\lambda,\theta) \Longleftrightarrow \lambda > \frac{1}{\theta} - 1 = \widetilde{\lambda}(\theta)$. $\widetilde{\lambda}(\theta)$is positive, however for $\widetilde{\lambda}(\theta) < 1$, and hence to have values of u_L/u_H consistent with bargaining, we need that $\theta > \frac{1}{2}$ (notice that $\lambda > \widetilde{\lambda}(\theta)$ ensure that $\lambda > 1-\theta$).

H-equilibrium: We know that we need $\frac{u_L}{u_H} < \widetilde{u}(\lambda,\theta)$. Since the two functions inside the brackets are decreasing in λ, so is $\widetilde{u}(\lambda,\theta)$.

L-equilibrium: We need $\frac{u_L}{u_H} > \widehat{u}(\lambda,\theta)$. The monotonicity of $\widehat{u}(\lambda,\theta)$ can be proved as above.

Existence of a pure-strategy equilibrium: One can show that $\widetilde{u}(\lambda,\theta) = 1 - \frac{r\lambda}{r\lambda+(r+\sigma)(1-\lambda)} \Longleftrightarrow \lambda < \widetilde{\lambda}(\theta)$ and that $\widehat{u}(\lambda,\theta) = 1 - \frac{(r+\alpha)\lambda}{(r+\alpha)\lambda+(r+\sigma)(1-\lambda)} \Longleftrightarrow \lambda < \widetilde{\lambda}(\theta)$. It can be verified that when $\lambda < \widetilde{\lambda}(\theta)$, $\widehat{u}(\lambda,\theta) < \widetilde{u}(\lambda,\theta)$, which ensures existence of either an H- or an L-equilibrium (or both). When $\lambda > \widetilde{\lambda}(\theta)$, $\widetilde{u}(\lambda,\theta) = 1 + \frac{r(1-\theta-\lambda)}{\lambda\theta[r+\sigma(1-\lambda)]}$ and $\widehat{u}(\lambda,\theta) = 1 - \frac{(r+\alpha)(1-\theta)}{(r+\alpha)(1-\theta)+(r+\sigma)\theta(1-\lambda)}$. If $\widehat{u}(\lambda,\theta) < \widetilde{u}(\lambda,\theta)$, then again we have existence of an H- or an L-equilibrium. If $\lambda > \widetilde{\lambda}(\theta)$, one can easily show that $\underline{u}(\lambda,\theta) < \widetilde{u}(\lambda,\theta)$ and $\widehat{u}(\lambda,\theta) < \overline{u}(\lambda,\theta)$, guaranteeing the existence of a B-equilibrium, in the case where an H- or an L-equilibrium may not exist.

Repeat purchase case: equilibrium conditions, existence of a pure-strategy equilibrium and proof of Proposition 2

Using the same methodology as in the one-time purchase case, we also find that there are only three possible pure strategy equilibria. For a B-equilibrium, we need $\underline{v}(\lambda,\theta) < \frac{u_L}{u_H} < \overline{v}(\lambda,\theta)$. For an H-equilibrium, we need $\frac{u_L}{u_H} < Min\{1-\lambda, \underline{v}(\lambda,\theta)\}$, while for an L-equilibrium, we need $\frac{u_L}{u_H} > Max\{1-\lambda, \overline{v}(\lambda,\theta)\}$, where $\underline{v}(\lambda,\theta) = \frac{(1-\theta)(1-\lambda)}{\lambda\theta}$ and $\overline{v}(\lambda,\theta) = \frac{\theta(1-\lambda)}{1-\lambda\theta}$. One can show that when $\lambda < \widetilde{\lambda}(\theta)$, equilibrium conditions are consistent with either an H- or an L-equilibrium. Similarly, when $\lambda < \widetilde{\lambda}(\theta)$, and the conditions for either an H- or an L-equilibrium are not met, then a B-equilibrium exists. Thus, a pure-strategy equilibrium always exists. Furthermore, by examining the three sets of conditions, it is straightforward to see that multiple equilibria are impossible, for any parameter values.

Solving for the bargaining game under asymmetric information

We remind the reader of notation introduced in section 5.1:

$$p_1^i = u_i - \delta V_i\ ,\ p_2^i = u_i - V_i\ ,\ \widehat{p} = (1-\delta)\,u_H + \delta\,(u_L - V_L)$$

The seller's strategy is a first-period price p_1, and a second-period price $p_2(p_1)$, for each choice of p_1. A buyer of type i accepts p_1 with probability $A_1^i(p_1)$ and p_2 with probability $A_2^i(p_2)$. Given this set-up, we can already discuss the possible price choices in the second stage. As was discussed in lemma 1, it is not optimal for the seller to leave strictly positive surplus to both types of buyers. Thus, the seller takes the entire surplus from one type of buyers and $p_2 \in \left\{p_2^L, p_2^H\right\}$. Hence, consider the probability that the seller takes away the entire surplus from the low type as the following random variable: $\zeta_2(p_1) = \Pr ob\left[p_2(p_1) = p_2^L\right]$.

In the following discussion, we focus on the interesting case where there is a positive surplus in each match, regardless of the worker's type ($Min\{u_H - V_H, u_L - V_L\} > V_N$). Obviously, if $u_L - V_L < u_H - V_H < V_N$, there is never a surplus from trade and thus no trade takes place. Second, if there is positive surplus in only one type of matches, only one type of buyers buys and does so in the first stage. Notice that, in the stage game, the price offer also has to be greater than V_N. Assuming that $Max\{u_L - V_L, u_H - V_H\} = u_H - V_H$, it follows that if $u_L - V_L < V_N < u_H - V_H$, then the second stage price $p_2 = p_2^H$ and the first stage price $p_1 = p_1^H$. In this case, type H accepts the offer in the first period, while type L never accepts. A similar argument holds if $Max\{u_L - V_L, u_H - V_H\} = u_L - V_L$.

To start, notice that if the seller has a degenerate prior $\Lambda \in \{0, 1\}$, the bargaining game has a unique equilibrium characterized as follows. Given the seller believes that the buyer she is negotiating with is of type L ($\Lambda = 1$), there is no informational problem, thus in equilibrium, $p_1 = p_1^L$ and $p_2 = p_2^L$. Similarly, if $\Lambda = 0$, then $p_1 = p_1^H$ and $p_2 = p_2^H$. We can now focus on the case where $0 < \Lambda < 1$. For that, we follow the methodology of Fudenberg and Tirole (1983).

Assume that $u_H - V_H > u_L - V_L$ (this will be confirmed later in equilibrium). Also suppose $u_L - V_L > V_N$ (this is one of the equilibrium conditions to check). Clearly, in equilibrium, $A_2^i(p_2) = 1$, if $p_2 \leq p_2^i$ and $A_2^i(p_2) = 0$ otherwise. Furthermore, since type L is guaranteed δV_L in the second period, he refuses any $p_1 > p_1^L$ and accepts any price less or equal to p_1^L. Therefore, in equilibrium, $A_1^L(p_1) = 1$, if $p_1 \leq p_1^L$ and $A_1^L(p_1) = 0$ otherwise. Hence, the non-trivial strategies are $A_1^H(p_1)$, p_1 and $\zeta_2(p_1)$. First notice that if $p_1 = p_1^L$, then both types of buyers accept the first-stage offer ($p_1^L < \widehat{p}$ implies that type H also accepts that price). Now consider the case where $p_1 > p_1^L$ so that $A_1^L(p_1) = 0$. From Bayes' rule, the probability Λ^{POS} that the buyers's type is L, given that p_1 is refused, is:

$$\begin{aligned}\Lambda^{POS} &= \frac{\Pr ob\,(p_1 \text{ refused} \mid L)\Pr ob\,(L)}{\Pr ob\,(p_1 \text{ refused} \mid L)\Pr ob\,(L) + \Pr ob\,(p_1 \text{ refused} \mid H)\Pr ob\,(H)} \\ &= \frac{\Lambda}{1 - A_1^H(p_1)(1-\Lambda)} \geq \Lambda\end{aligned}$$

First consider the second stage. The expected payoff to the seller from offering p_2^H is:

$$\Lambda^{POS} V_N + \left(1 - \Lambda^{POS}\right) p_2^H = \Lambda V_N + (1-\Lambda)\, p_2^H + \left(\Lambda^{POS} - \Lambda\right)\left(V_N - p_2^H\right) \leq \Lambda V_N + (1-\Lambda)\, p_2^H$$

From this expression, we see that there are two cases to consider.

Case 1: $\Lambda V_N + (1-\Lambda)\, p_2^H < p_2^L$

In the second stage, the seller offers p_2^L given p_1 was refused. Given that $p_2 = p_2^L$, the highest p_1 that can be accepted by type H is $\widehat{p}$ (the seller can post any arbitrarily small amount below $\widehat{p}$). Denote by $\Pi_s(p_1, p_2)$ the equilibrium payoff to the seller from offering p_1 in the first-stage and p_2 in the second period. There can be two equilibria. She always offers p_2^L in the second stage. In the first-stage, she can offer either $p_1 = p_1^L$, which is accepted by both types of buyers, or $p_1 = \widehat{p}$, which is accepted by type H only. In the first stage, her two possible equilibrium payoffs are:

$$\begin{aligned}
\text{When } u_L - V_L &> \Lambda V_N + (1-\Lambda)(u_H - V_H) \\
\Pi_s\left(p_1^L, p_2^L\right) &= p_1^L = u_L - \delta V_L \\
\Pi_s\left(\widehat{p}, p_2^L\right) &= \Lambda \delta p_2^L + (1-\Lambda)\widehat{p} = \delta(u_L - V_L) + (1-\Lambda)(1-\delta)u_H
\end{aligned}$$

The seller chooses the sequence of prices that brings her the higher Π_s. Hence, we have the following result. If $(1-\Lambda)u_H > u_L$, the seller offers $\left\{\widehat{p}, p_2^L\right\}$. Otherwise, she offers $\left\{p_1^L, p_2^L\right\}$.

Case 2: $\Lambda V_N + (1-\Lambda)p_2^H > p_2^L$

Consider type H buyer's decision to accept or reject a first-stage price p_1. Any price $p_1 \leq \widehat{p}$ is accepted by type H, regardless of the second-stage price. Denote by A^H the value of $A_1^H(p_1)$ that makes the seller indifferent between choosing the following two sequences of offers: $\left\{p_1^H, p_2^H\right\}$ and $\left\{p_1^H, p_2^L\right\}$[14]. Now consider $p_1 > \widehat{p}$. One can show that, for a given $A_1^H(p_1)$, the seller receives a higher payoff from playing $\left\{p_1, p_2^H\right\}$ than from playing $\left\{p_1, p_2^L\right\}$, if and only if[15] $A_1^H(p_1) < A^H$. Therefore, if $A_1^H(p_1) > A^H$, the seller plays p_2^L in the second stage, but then type H buyer would

[14] A^{H} satisfies:

$$\Lambda\delta V_{\mathsf{N}} + (1-\Lambda)\left[A^{\mathsf{H}} p_1^{\mathsf{H}} + \left(1 - A^{\mathsf{H}}\right)\delta p_2^{\mathsf{H}}\right] = \Lambda\delta p_2^{\mathsf{L}} + (1-\Lambda)\left[A^{\mathsf{H}} p_1^{\mathsf{H}} + \left(1 - A^{\mathsf{H}}\right)\delta p_2^{\mathsf{L}}\right]$$

Hence:

$$A^{\mathsf{H}} = 1 - \frac{\Lambda}{1-\Lambda}\frac{u_{\mathsf{L}} - V_{\mathsf{L}} - V_{\mathsf{N}}}{u_{\mathsf{H}} - V_{\mathsf{H}} - u_{\mathsf{L}} + V_{\mathsf{L}}}$$

[15] When $p_1 > \widehat{p}$:

$$\begin{aligned}
\Pi_{\mathsf{s}}\left(p_1, p_2^{\mathsf{H}}\right)\Big|_{A_1^{\mathsf{H}}} &= \Lambda\delta V_{\mathsf{N}} + (1-\Lambda)\left[A_1^{\mathsf{H}} p_1 + \left(1 - A_1^{\mathsf{H}}\right)\delta p_2^{\mathsf{H}}\right] \\
\Pi_{\mathsf{s}}\left(p_1, p_2^{\mathsf{L}}\right)\Big|_{A_1^{\mathsf{H}}} &= \Lambda\delta p_2^{\mathsf{L}} + (1-\Lambda)\left[A_1^{\mathsf{H}} p_1 + \left(1 - A_1^{\mathsf{H}}\right)\delta p_2^{\mathsf{L}}\right]
\end{aligned}$$

refuse $p_1 > \widehat{p}$ and $A_1^H(p_1) = 0$, which is a contradiction. And if $A_1^H(p_1) < A^H$, the seller plays p_2^H, and the buyer would prefer to accept p_1, and $A_1^H(p_1) = 1$, another contradiction (except in the limit case where $p_1 = p_1^H$). Hence, for $\widehat{p} < p_1 < p_1^H$, the optimal strategy for type H is to accept the price with probability A^H. Randomization implies that type H buyer is indifferent between accepting and refusing the first-stage offer given the second-stage offer. Recall that, given p_1, the seller offers p_2^L with probability $\zeta_2(p_1)$. Thus $u_H - p_1 = \delta\left[\zeta_2\left(u_H - p_2^L\right) + (1-\zeta_2)\left(u_H - p_2^H\right)\right]$, which implies that:

$$\zeta_2(p_1) = \frac{u_H - p_1 - \delta V_H}{\delta(u_H - V_H - u_L + V_L)}$$

Notice that ζ_2 decreases from 1 to 0, as p_1 increases from $\widehat{p}$ to p_1^H. Finally, $A_1^H\left(p_1^H\right) = A^H$, since the seller can approximate the buyers' payoff arbitrarily closely by offering a price p_1 arbitrarily smaller than p_1^H. Since acceptance probabilities are constant for p_1 between p_1^L and $\widehat{p}$, and between $\widehat{p}$ and p_1^H, the seller's best strategy is to offer the first period price that brings her the highest expected payoff:

$$\begin{aligned}
\text{When } u_L - V_L &< \Lambda V_N + (1-\Lambda)(u_H - V_H) \\
\Pi_s\left(p_1^L, p_2^L\right) &= p_1^L = u_L - \delta V_L \\
\Pi_s\left(\widehat{p}, p_2^L\right) &= \Lambda\delta p_2^L + (1-\Lambda)\widehat{p} = \delta(u_L - V_L) + (1-\Lambda)(1-\delta)u_H \\
\Pi_s\left(p_1^H, p_2^H\right)\big|_{A^{\mathsf{H}}} &= \Lambda\delta V_N + (1-\Lambda)\left[A^H p_1^H + \left(1-A^H\right)\delta p_2^H\right] \\
&= \Lambda\delta V_N + (1-\Lambda)\left[A^H u_H(1-\delta) + \delta(u_H - V_H)\right]
\end{aligned}$$

We summarize the payoffs to the buyers in cases 1 and 2, for each possible equilibrium $\{p_1, p_2\}$:

$$\begin{aligned}
\Pi_L\left(p_1^L, p_2^L\right) &= \delta V_L \text{ and } \Pi_H\left(p_1^L, p_2^L\right) = u_H - u_L + \delta V_L \\
\Pi_L\left(\widehat{p}, p_2^L\right) &= \delta V_L \text{ and } \Pi_H\left(\widehat{p}, p_2^L\right) = \delta(u_H - u_L + V_L) \\
\Pi_L\left(p_1^H, p_2^H\right) &= \delta V_L \text{ and } \Pi_H\left(p_1^H, p_2^H\right) = \delta V_H
\end{aligned}$$

By definition, A^{H} satisfies $\Pi_{\mathsf{s}}\left(p_1^{\mathsf{H}}, p_2^{\mathsf{H}}\right)\big|_{A^{\mathsf{H}}} = \Pi_{\mathsf{s}}\left(p_1^{\mathsf{H}}, p_2^{\mathsf{L}}\right)\big|_{A^{\mathsf{H}}}$. Using this equality, one can show that:

$$\Pi_{\mathsf{s}}\left(p_1, p_2^{\mathsf{H}}\right)\Big|_{A_1^{\mathsf{H}}} - \Pi_{\mathsf{s}}\left(p_1, p_2^{\mathsf{L}}\right)\Big|_{A_1^{\mathsf{H}}} = (1-\Lambda)\,\delta\,(u_{\mathsf{H}} - V_{\mathsf{H}} - u_{\mathsf{L}} + V_{\mathsf{L}})\left(A^{\mathsf{H}} - A_1^{\mathsf{H}}(p_1)\right)$$

Equilibrium conditions under asymmetric bargaining

We need to ensure that we have equilibrium in the bargaining game, whether we consider bargaining as an equilibrium candidate or as a deviation from equilibrium. First consider bargaining as a candidate equilibrium. We know that there are three possible price sequences offered in the game ($\{p_1^L, p_2^L\}$, $\{p_1^H, p_2^H\}$ or $\{\hat{p}, p_2^L\}$). If we posit $\{p_1^L, p_2^L\}$ ($\{p_1^H, p_2^H\}$) as an equilibrium sequence, then the search values to buyers and sellers are identical to the case where low (high) price posting is a candidate equilibrium, and therefore indistinguishable from it. Hence, for the rest of the discussion and graphical analysis, we consider only $\{\hat{p}, p_2^L\}$ as characteristic outcome of the bargaining game. We can now calculate the associated payoffs from the bargaining game to the two types of buyers, Π_L and Π_H, as well as the associated V_L and V_H. At this point, it is possible to verify whether each type of buyers will be willing to enter the game (η_L,η_H) and thus to compute the distribution Λ of buyer types willing to negotiate, as rationally expected by sellers, based on the price sequence[16]. Then, the seller's payoff from the game Π_s and her value of bargaining V_N can be computed[17]. Then, one can check the conditions under which this constitutes an equilibrium of the bargaining game[18]. Since under these conditions, $\{\hat{p}, p_2^L\}$ is the equilibrium outcome of the bargaining game (given V_N, V_L and V_H), buyers rationally expect this sequence given they enter the game. Finally, given this equilibrium of the bargaining game, one needs to verify that deviating by posting a high or a low price are not profitable deviations.

If one considers, for example, high price posting as a candidate equilibrium strategy and bargaining as a possible deviation instead, the method is the same except that $\overline{V}_L$ and $\overline{V}_H$ are derived from

[16]Notice that $\Pi_L(\hat{p}, p_2^L) = \delta V_L$. This implies that $V_L = 0$. We consider that type L still enters the bargaining game, since the seller can always offer $p_2^L + \varepsilon$ in the second period, for any arbitrarily small ε, ensuring that type L enters the negotiations for sure, while replicating the equilibrium payoffs.

[17]Notice that the seller is always willing to enter the game. Remark that when solving the bargaining game, we assumed that $V_N < u_L - V_L < u_H - V_H$, which ensures that all price offers played by the seller makes her better off than her reservation value V_N. If this inequality does not hold, the seller offers either p_1^L or p_2^L, cases that are considered when looking at H- or L- as candidate equilibria.

[18]Under Case 1, that $\Pi_s(\hat{p}, p_2^L) > \Pi_s(p_1^L, p_2^L)$ and under Case 2, that $\Pi_s(\hat{p}, p_2^L) > \Pi_s(p_1^L, p_2^L)$ and $\Pi_s(\hat{p}, p_2^L) > \Pi_s(p_1^H, p_2^H)\big|_{AH}$. Also, one needs to verify that $V_N < u_L - V_L < u_H - V_H$.

the equilibrium candidate strategy. Once conditions for an equilibrium of the bargaining game are established, one can check when such an equilibrium would be a profitable deviation from posting a high price. One also needs to check that posting a low price is not profitable.

Existence of a pure-strategy equilibrium in section 5

The proof is based on existence conditions for the different types of equilibria that can be provided upon request[19]. It can be shown that the set of conditions for existence of an H-equilibrium collapses into the single condition that $u_L/u_H < A(\lambda)$, where $A(\lambda) = 1 - \frac{r\lambda}{r\lambda + (r+\sigma)(1-\lambda)}$. For an L-equilibrium, we need to consider two cases. First, when $\sigma > \alpha$, all existence conditions also collapse into a single condition: $u_L/u_H > B(\lambda)$, where $B(\lambda) = 1 - \frac{(r+\alpha)\lambda}{(r+\alpha)\lambda + (r+\sigma)(1-\lambda)}$. In that case, one can easily verify that there always exists an H- or an L-equilibrium or both, as $A(\lambda) > B(\lambda)$. Looking at the case where $\sigma < \alpha$, there are more than one condition to ensure the existence of an L-equilibrium. It is still necessary that $u_L/u_H > B(\lambda)$ (i.e. posting a high deviating price $\widetilde{P}_H^P$ is not profitable). Notice that when $\sigma < \alpha$, $B(\lambda) < 1 - \lambda$. For bargaining to be a profitable deviation from an L-equilibrium, it must be the case that $u_L/u_H < 1 - \lambda$. Hence, if $u_L/u_H > 1 - \lambda$, neither bargaining nor posting a high price can be a profitable deviation from an L-equilibrium and that entire parameter region supports an L-equilibrium. Notice that $A(\lambda) > 1 - \lambda$, so that the parameter region where high price posting is an equilibrium strictly includes the region where $u_L/u_H < 1 - \lambda$. As a result, existence of either an H- or an L-equilibrium can also be guaranteed, when $\sigma < \alpha$.

Proof of Proposition 3

Assume all sellers bargain. To ensure that posting a price targeted at type H is not a profitable deviation, it is necessary that $u_L/u_H > \frac{r+\sigma+(r+\alpha)\sigma\lambda}{r+\sigma+(\alpha-\sigma)\lambda}(1-\lambda)$ and to ensure that posting a price targeted at type L is not a profitable deviation, it is necessary that $u_L/u_H < (1-\lambda)$. If $\sigma > \alpha$, both conditions cannot hold simultaneously.

[19] This also is true for the proof of proposition 3.

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