

1. **Values, costs, and equilibrium (7 points)** Suppose that a competitive market has two buyers and two sellers. Suppose both buyers have the same value schedule and both sellers have the same cost schedule, as shown in the table.

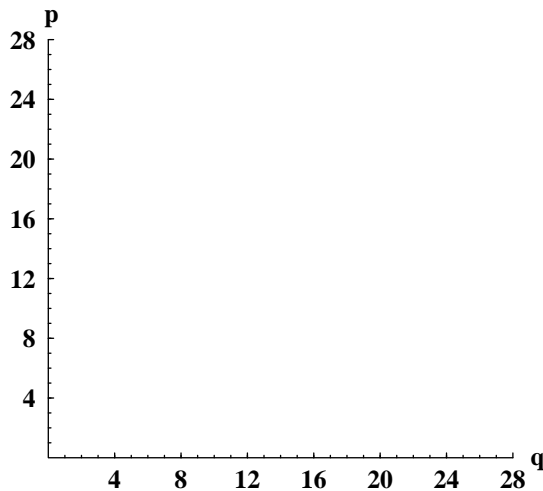
B1: 24, 22, 20, 18, 16, 14, 12

B2: 24, 22, 20, 18, 16, 14, 12

S1: 4, 7, 10, 13, 16, 19, 22

S2: 4, 7, 10, 13, 16, 19, 22

(a) (2 points) Graph the combined list of unit values (for B1 and B2) and graph the combined list of unit costs. Since the graph of the list of values and the graph of demand are the same, label the graph of the list of values as  $D(p)$ . Also label the list of unit costs as  $S(p)$ .



(b) (1 point) Find the equilibrium price and equilibrium quantity of trade. Show the equilibrium price and the equilibrium quantity on the graph.

(c) (1 point) Calculate the consumers' surplus at the equilibrium price. Also, calculate the producers' surplus at the equilibrium price.

(d) (1 point) If a sales tax of \$5 per unit is imposed on the buyers, then the list of buyers' values after the tax is imposed is

B1: 19, 17, 15, 13, 11, 9, 7

B2: 19, 17, 15, 13, 11, 9, 7.

On the figure on page 1, graph the new list of values. Since the graph of the list of values and the graph of demand are the same, label the graph of the list of values after the sales tax as  $D_t(p)$ .

(e) (1 point) What is the new equilibrium price  $p_s^*$  and equilibrium quantity  $q_s^*$ ?

(f) (1 point) What fraction of the sales tax is paid by the buyers after the market reaches its new equilibrium price  $p_s^*$ ?

**2. Linear supply and demand (8 points)** About half of the material that we covered in the first week involved lists of unit values and costs, and about half involved linear supply and demand curves. The purpose of this problem is to describe an explicit connection between these two approaches. This problem involves a sequence of steps that demonstrates that the price, quantity, surplus, and split of tax burden are all similar whether we use lists of unit values and costs or linear approximations to the unit values and costs. The first two steps in the problem are to construct the linear approximation to the demand and the linear approximation to the supply. The remaining steps repeat parts (b) through (f) of problem 1 with linear demand and supply instead of lists of values and costs. Your answers should be the same for some parts and they should differ only a small amount for others, due to the approximation of demand and supply.

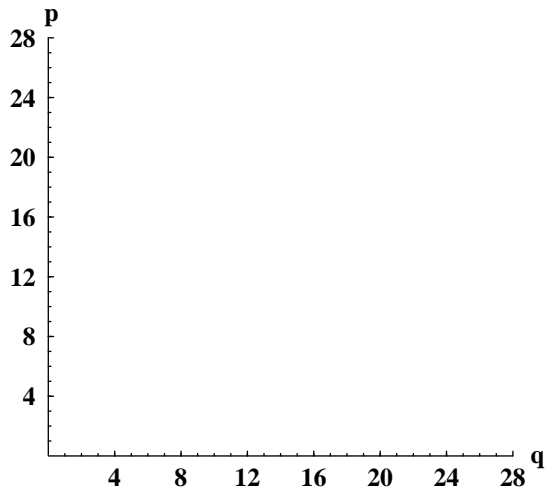
(a) (1 point) One point on the original demand function  $D(p)$  is  $p_0 = 24$  and  $q_0 = 1$ . Choose another price and quantity pair  $(p_1, q_1)$  on the original demand function  $D(p)$ . (Use the midpoint of one of the horizontal segments of the demand function. The values of  $q$  at the other midpoints are  $q = 3, 5, 7, \dots, 13$ . Select one of these other six points to determine the price quantity pair  $(p_1, q_1)$ .)

Use the equations  $q_0 = D(p_0) = a - bp_0$  and  $q_1 = D(p_1) = a - bp_1$  to find the linear demand function that approximates the demand function from the list of unit values. (Since  $p_0 = 24$  and  $q_0 = 1$  the first equation is  $1 = a - b24$ . Write down the other equation using the other price and quantity pair  $(p_1, q_1)$  that you selected from the demand graph. Then subtract one equation from the other to solve for the slope term  $b$ . Once you have  $b$  you can use either equation to find  $a$ . Then write out the demand function  $q = D(p) = a - bp$ , by substituting the values  $a$  and  $b$  that you found into the demand equation.

(b) (1 point) Follow steps similar to those in part (a) to find the linear supply function that approximates the list of costs in problem 1. One point on the original supply function  $S(p)$  is  $p_0 = 4$  and  $q_0 = 1$ . Choose another price and quantity pair  $(p_1, q_1)$  on the original supply function  $S(p)$ .

Use the equations  $q_0 = S(p_0) = c + dp_0$  and  $q_1 = S(p_1) = c + dp_1$  to find the linear supply function that approximates the supply function from the list of unit costs. (Since  $p_0 = 4$  and  $q_0 = 1$  the first equation is  $1 = c + d4$ . Write down the other equation using the other price and quantity pair  $(p_1, q_1)$  that you selected from the supply graph. Then subtract one equation from the other to solve for the slope term  $d$ . Once you have  $d$  you can use either equation to find  $c$ . Then write out the supply function  $q = S(p) = c + dp$ , by substituting the values  $c$  and  $d$  that you found into the supply equation.

(c) (2 points) Graph the linear demand function from part (a) and the linear supply function from part (b) below.



(d) (1 point) Find the equilibrium price and equilibrium quantity of trade. Show the equilibrium price and the equilibrium quantity on the graph.

(e) (1 point) Calculate the consumers' surplus at the equilibrium price. Also, calculate the producers' surplus at the equilibrium price.

(f) (1 point) If a sales tax of \$5 per unit is imposed on the buyers, then the new demand function is  $D_t(p) = D(p + 5)$  where  $D(p)$  is the original demand. On the figure above, graph the new demand and label it  $D_t(p)$ .

(g) (1 point) What is the new equilibrium price  $p_s^*$  and equilibrium quantity  $q_s^*$ ?

(h) (1 point) What fraction of the sales tax is paid by the buyers after the market reaches its new equilibrium price  $p_s^*$ ?

**3. Elasticity (4 points)**

(a) (1 point) Give a one sentence definition of the elasticity of demand.

(b) (1 point) Write down a formula for the elasticity of demand  $\epsilon_d$ .

(c) (1 point) Give a one sentence definition of the elasticity of supply.

(d) (1 point) Write down a formula for the elasticity of supply  $\epsilon_s$ .

**4. Elasticity and tax shares (5 points)** Suppose that the demand in a market is  $D(p) = 25 - p$  and the supply in the market is  $S(p) = (2p - 5)/3$ .

(a) (1 point) Find the elasticity of demand when the initial price  $p_0$  is the market equilibrium price  $p^* = 16$ .

(b) (1 point) Find the elasticity of supply when the initial price  $p_0$  is the market equilibrium price  $p^* = 16$ .

(c) (2 points) Compute the buyers' tax share from the formula  $T_b = \frac{\epsilon_s}{\epsilon_d + \epsilon_s}$ , where  $\epsilon_d$  is your answer to question 4 (a) and  $\epsilon_s$  is your answer to 4 (b).

(d) (1 point) Comment on the relationship between your answer to part (c) and your answers to problems 1 (f) and 2 (h).