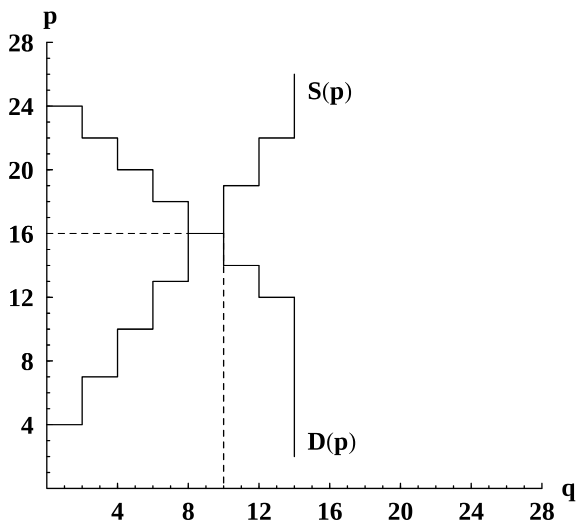
Economics 251 HW #1
Answer key

1. Values, costs, and equilibrium (7 points) Suppose that a competitive market has two buyers and two sellers. Suppose both buyers have the same value schedule and both sellers have the same cost schedule, as shown in the table.

B1: 24, 22, 20, 18, 16, 14, 12
B2: 24, 22, 20, 18, 16, 14, 12
S1: 4, 7, 10, 13, 16, 19, 22
S2: 4, 7, 10, 13, 16, 19, 22

(a) (2 points) Graph the combined list of unit values (for B1 and B2) and graph the combined list of unit costs. Since the graph of the list of values and the graph of demand are the same, label the graph of the list of values as $D(p)$. Also label the list of unit costs as $S(p)$.



(b) (1 point) Find the equilibrium price and equilibrium quantity of trade. Show the equilibrium price and the equilibrium quantity on the graph.

Solution At the competitive equilibrium, all units with values greater than or equal to the cost will be traded. From the graph, it is apparent that the equilibrium price is $p^* = 16$ and there will be between 8 and 10 units traded at that price.

(c) (1 point) Calculate the consumers' surplus at the equilibrium price. Also, calculate the producers' surplus at the equilibrium price.

Solution The consumers' surplus is determined by adding up the difference between the unit value and the equilibrium price for each unit sold. So

$$\begin{aligned} CS &= 2(24-16) + 2(22-16) + 2(20-16) + 2(18-16) + 2(16-16) \\ &= 16 + 12 + 8 + 4 + 0 \\ &= 40. \end{aligned} \quad (1)$$

Solution The producers' surplus is found by adding up the difference between the equilibrium price and the unit cost for each unit sold. So

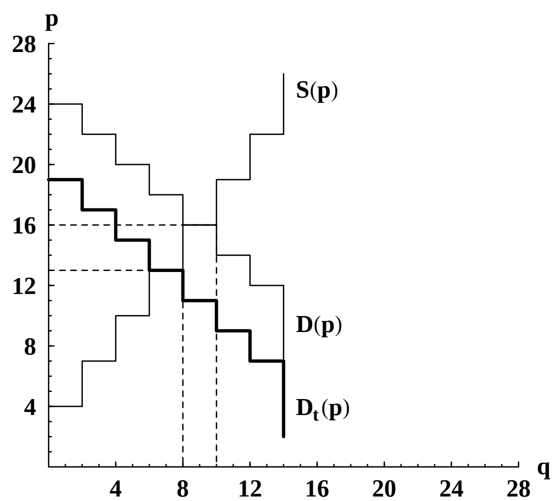
$$\begin{aligned} PS &= 2(16-4)+2(16-7)+2(16-10)+2(16-13)+2(16-16) \\ &= 24+18+12+6+0 \\ &= 60. \end{aligned} \quad (2)$$

(d) (1 point) If a sales tax of \$5 per unit is imposed on the buyers, then the list of buyers' values after the tax is imposed is

B1: 19, 17, 15, 13, 11, 9, 7

B2: 19, 17, 15, 13, 11, 9, 7.

On the figure on page 1, graph the new list of values. Since the graph of the list of values and the graph of demand are the same, label the graph of the list of values after the sales tax as $D_t(p)$.



(e) (1 point) What is the new equilibrium price p_s^* and equilibrium quantity q_s^*?

Solution The graph shows that the equilibrium price is $p^* = 13$ and there will be 8 units traded at that price.

(f) (1 point) What fraction of the sales tax is paid by the buyers after the market reaches its new equilibrium price p_s^*?

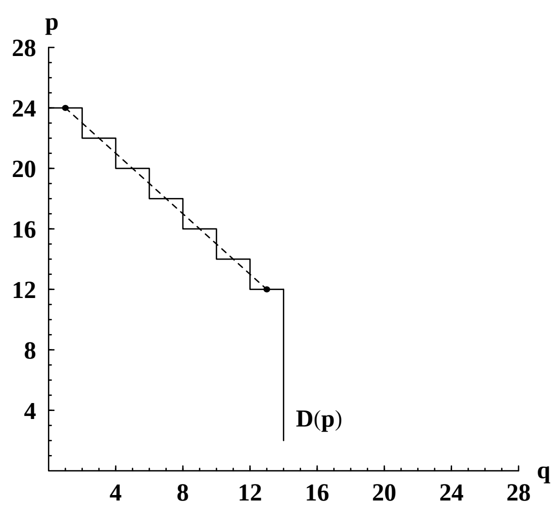
Solution The buyers pay \$5 per unit in tax, but the price that the buyers pay dropped from $p^* = 16$ to $p^* = 13$, so the buyers' net price increase is the amount of the tax minus the \$3 price reduction. Buyers therefore pay 2/5 of the tax.

2. Linear supply and demand (9 points) About half of the material that we covered in the first week involved lists of unit values and costs, and about half involved linear supply and demand curves. The purpose of this problem is to describe an explicit connection between these two approaches. This problem involves a sequence of steps that demonstrates that the price, quantity, surplus, and split of tax burden are all similar whether we use lists of unit values and costs or linear approximations to the unit values and costs. The first two steps in the problem are to construct the linear approximation to the demand and the linear approximation to the supply. The remaining steps repeat parts (b) through (f) of problem 1 with linear demand and supply instead of lists of values and costs. Your answers should be the same for some parts and they should differ only a small amount for others, due to the approximation of demand and supply.

(a) (1 point) One point on the original demand function $D(p)$ is $p_0 = 24$ and $q_0 = 1$. Choose another price and quantity pair (p_1, q_1) on the original demand function $D(p)$. (Use the midpoint of one of the horizontal segments of the demand function. The values of q at the other midpoints are $q = 3, 5, 7, \ldots, 13$. Select one of these other six points to determine the price quantity pair (p_1, q_1).)

Use the equations $q_0 = D(p_0) = a - b\,p_0$ and $q_1 = D(p_1) = a - b\,p_1$ to find the linear demand function that approximates the demand function from the list of unit values. (Since $p_0 = 24$ and $q_0 = 1$ the first equation is $1 = a - b\,24$. Write down the other equation using the other price and quantity pair (p_1, q_1) that you selected from the demand graph. Then subtract one equation from the other to solve for the slope term b. Once you have b you can use either equation to find a. Then write out the demand function $q = D(p) = a - b\,p$, by substituting the values a and b that you found into the demand equation.

Solution The equation from the first point is $1 = a - b\,24$. When the point $q_1 = 13$ is chosen, the price on the demand graph is $p_1 = 12$. These two points and the line between them are shown in the figure below.



From the demand equation $q_1 = D(p_1) = a - b\,p_1$ we get the equation $13 = a - b\,12$ at this point. Subtract the first equation from this equation:

$$\begin{array}{rcl} 13 & = & a - b\,12 \\ 1 & = & a - b\,24 \\ \hline 12 & = & b\,12 \end{array}$$

The coefficient b is therefore equal to 1. Then substitute $b = 1$ into the first equation $1 = a - 1 \cdot 24$ to get $a = 25$. The demand function is therefore $D(p) = 25 - p$.

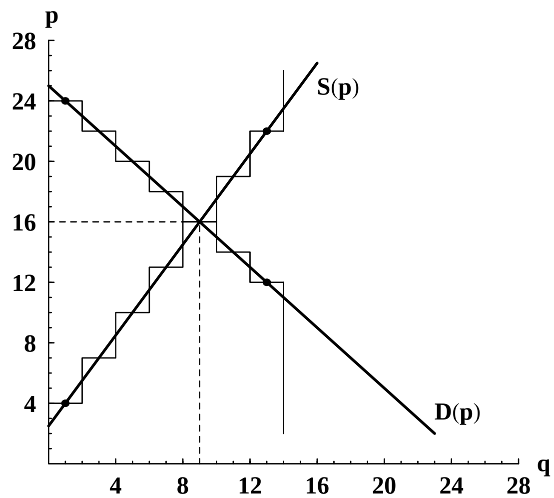
(b) (1 point) Follow steps similar to those in part (a) to find the linear supply function that approximates the list of costs in problem 1. One point on the original supply function $S(p)$ is $p_0 = 4$ and $q_0 = 1$. Choose another price and quantity pair (p_1, q_1) on the original supply function $S(p)$.

Use the equations $q_0 = S(p_0) = c + d\,p_0$ and $q_1 = S(p_1) = c + d\,p_1$ to find the linear supply function that approximates the supply function from the list of unit costs. (Since $p_0 = 4$ and $q_0 = 1$ the first equation is $1 = c + d\,4$. Write down the other equation using the other price and quantity pair (p_1, q_1) that you selected from the supply graph. Then subtract one equation from the other to solve for the slope term d. Once you have d you can use either equation to find c. Then write out the supply function $q = S(p) = c + d\,p$, by substituting the values c and d that you found into the supply equation.

Solution The equation from the first point is $1 = c + d\,4$. When the point $q_1 = 13$ is chosen, the price on the demand graph is $p_1 = 22$. The second equation for this choice of q_1 is $13 = c + d\,22$. The first equation can be subtracted from the second equation to get $12 = d\,18$. So the slope coefficient is $d = 2/3$. Substitute this into the first equation to get $1 = c + \frac{2}{3}4$. Then $c = -\frac{5}{3}$ and the supply equation is

$$\begin{aligned} S(p) &= -\tfrac{5}{3} + \tfrac{2}{3}\,p \\ &= \tfrac{1}{3}\,(2\,p - 5). \end{aligned}$$

(c) (2 points) Graph the linear demand function from part (a) and the linear supply function from part (b) below.



(d) (1 point) Find the equilibrium price and equilibrium quantity of trade. Show the equilibrium price and the equilibrium quantity on the graph.

Solution The demand equation is $D(p) = 25 - p$. Supply is $S(p) = \frac{1}{3}(2p - 5)$. The equilibrium price is the one where demand equals supply:

$$\begin{aligned} D(p) &= S(p) \\ 25 - p &= \tfrac{1}{3}(2p - 5) \\ 75 - 3p &= 2p - 5 \\ 80 &= 5p. \end{aligned}$$

So $p^* = 16$. When $p^* = 16$ the equilibrium quantity is $q^* = D(p^*) = 25 - 16 = 9$.

(e) (1 point) Calculate the consumers' surplus at the equilibrium price. Also, calculate the producers' surplus at the equilibrium price.

Solution The consumers' surplus is

$$\begin{aligned} CS &= \tfrac{1}{2} q^* (d_0 - p^*) \\ &= \tfrac{1}{2} 9 (25 - 16) \\ &= 40.5. \end{aligned}$$

The producers' surplus is

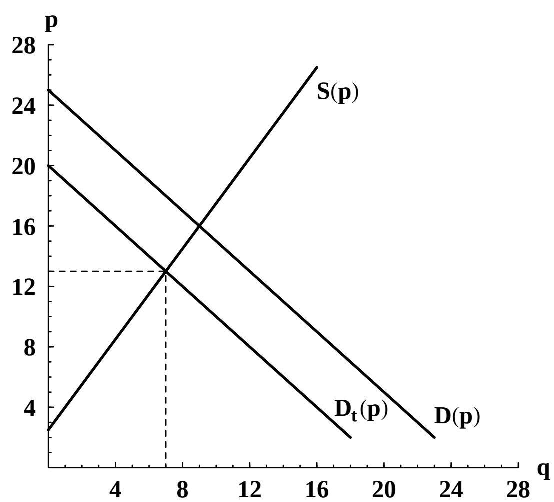
$$\begin{aligned} PS &= \tfrac{1}{2} q^* (p^* - s_0) \\ &= \tfrac{1}{2} 9 (16 - 2.5) \\ &= \tfrac{9}{2} \tfrac{27}{2} \\ &= 60.75. \end{aligned}$$

(f) (1 point) If a sales tax of \$5 per unit is imposed on the buyers, then the new demand function is $D_t(p) = D(p+5)$ where $D(p)$ is the original demand. On the figure above, graph the new demand and label it $D_t(p)$.

Solution The new demand is

$$\begin{aligned} D_t(p) &= D(p+5) \\ &= 25 - (p+5) \\ &= 20 - p. \end{aligned}$$

Both demand curves and the supply curve are shown in the figure below.



(g) (1 point) What is the new equilibrium price p_s^* and equilibrium quantity q_s^*?

Solution The new demand equation is $D_t(p) = 20 - p$. Supply is $S(p) = \frac{1}{3}(2p - 5)$. At the equilibrium

$$\begin{aligned} D_t(p) &= S(p) \\ 20 - p &= \tfrac{1}{3}(2p - 5) \\ 60 - 3p &= 2p - 5 \\ 65 &= 5p. \end{aligned}$$

So $p^* = 13$. When $p^* = 13$ the equilibrium quantity is $q^* = D_t(p^*) = 20 - 13 = 7$.

(h) (1 point) What fraction of the sales tax is paid by the buyers after the market reaches its new equilibrium price p_s^*?

Solution The buyers pay \$5 per unit in tax, but the price that the buyers pay dropped from $p^* = 16$ to $p^* = 13$, so the buyers net price increase is the amount of the tax minus the \$3 price reduction. Buyers therefore pay 2/5 of the tax.

3. Elasticity (4 points)

(a) (1 point) Give a one sentence definition of the elasticity of demand.

Solution The definition of elasticity of demand is "the percentage change in the quantity demanded divided by the percentage change in the price."

(b) (1 point) Write down a formula for the elasticity of demand ϵ_d.

Solution The elasticity of demand is

$$\begin{aligned} \epsilon_d &= \left| \frac{\% \, \Delta \, \text{demand}}{\% \, \Delta \, \text{price}} \right| \\ &= \left| \frac{(q_1^d - q_0^d)/q_0^d}{(p_1 - p_0)/p_0} \right|. \end{aligned}$$

(c) (1 point) Give a one sentence definition of the elasticity of supply.

Solution The definition of elasticity of supply is "the percentage change in the quantity supplied divided by the percentage change in the price."

(d) (1 point) Write down a formula for the elasticity of supply ϵ_s.

Solution The elasticity of supply is

$$\begin{aligned} \epsilon_s &= \left| \frac{\% \, \Delta \, \text{supply}}{\% \, \Delta \, \text{price}} \right| \\ &= \left| \frac{(q_1^s - q_0^s)/q_0^s}{(p_1 - p_0)/p_0} \right|. \end{aligned}$$

4. Elasticity and tax shares (5 points) Suppose that the demand in a market is $D(p) = 25 - p$ and the supply in the market is $S(p) = (2p - 5)/3$.

(a) (1 point) Find the elasticity of demand when the initial price p_0 is the market equilibrium price $p^* = 16$.

Solution At the initial price $p_0 = 16$ the quantity demanded is $q_0 = 9$. At a new price $p_1 = 17$ the quantity demanded is $q_1 = 8$. The elasticity of demand is

$$\begin{aligned} \epsilon_d &= \left| \frac{(q_1^d - q_0^d)/q_0^d}{(p_1 - p_0)/p_0} \right| \\ &= \left| \frac{(8-9)/9}{(17-16)/16} \right| \\ &= \left| \frac{-1/9}{1/16} \right| \\ &= \frac{16}{9}. \end{aligned}$$

(b) (1 point) Find the elasticity of supply when the initial price p_0 is the market equilibrium price $p^* = 16$.

Solution At the initial price $p_0 = 16$ the quantity supplied is $q_0 = 9$. At a new price $p_1 = 17$ the quantity supplied is $q_1 = 29/3$. The elasticity of supply is

$$\begin{aligned}\epsilon_s &= \left|\frac{(q_1^s - q_0^s)/q_0^s}{(p_1 - p_0)/p_0}\right| \\ &= \left|\frac{(29/3 - 9)/9}{(17 - 16)/16}\right| \\ &= \left|\frac{2/27}{1/16}\right| \\ &= 32/27.\end{aligned}$$

(c) (2 points) Compute the buyers' tax share from the formula $T_b = \frac{\epsilon_s}{\epsilon_d + \epsilon_s}$, where ϵ_d is your answer to question 4 (a) and ϵ_s is your answer to 4 (b).

Solution The buyers' tax share is

$$\begin{aligned}T_s &= \frac{\epsilon_s}{\epsilon_d + \epsilon_s} \\ &= \frac{32/27}{16/9 + 32/27} \\ &= \frac{32}{48 + 32} \\ &= 2/5.\end{aligned}$$

(d) (1 point) Comment on the relationship between your answer to part (c) and your answers to problems 1 (f) and 2 (h).

Solution The buyers' share of the tax is the same in problems 1 (f), 2 (h), and 4 (c). Problem 2 (h) is similar to problem 1 (f) because in parts (a) and (b) of problem 2 we found the linear demand and linear supply that approximate the value and cost schedules from problem 1. The linear approximations from problem 2 are $D(p) = 25 - p$ and $S(p) = (2p - 5)/3$, which are the supply and demand functions given in problem 4.

The discussion of elasticity and the split of the tax burden on page 26 and the top of page 27 demonstrates that whenever the supply and demand are linear, tax shares are determined by the formulas $T_b = \epsilon_s/(\epsilon_d + \epsilon_s)$ and $T_s = \epsilon_d/(\epsilon_d + \epsilon_s)$, so the answer to problem 2 (h) must be the same as the answer to problem 4 (c).