

Economics 251

Homework #2 solutions

These exercises demonstrate the ideas from the utility theory lecture on May 22 and May 23.

1. Suppose that a consumer has an income of $M = 120$, and that several indifference curves from her preferences over two commodities are shown in the top figure on page 3. Assume also that the price of commodity Y is fixed at $p_y = 4$.

Follow the instructions in steps (a) and (b) to fill in the left table below. Use the information in the table to answer question (c). Then follow the instructions in part (d) to fill in the table on the right.

p_x	$D_x(p_x)$	$D_y(p_x)$
2	40	10
4	15	15
6	8	18
8	5	20

p_x	$D_x(p_x)$	$D_y(p_x)$
2	40	10
4	15	15
6	8	18
8	5	20

- (a) (1 point) Plot the budget line that corresponds to a price of X of $p_x = 2$ and determine the quantity of X consumed as well as the quantity of Y consumed. Enter the amount of X chosen for price $p_x = 2$ in the column labeled $D_x(p_x)$ in the row labeled 2. Enter the amount of Y chosen for price $p_x = 2$ in the column labeled $D_y(p_x)$ in the row labeled 2. Also, plot the price of X and the corresponding quantity of X consumed in the lower graph on page 3.

Solution The budget line that corresponds to $p_x = 2$ is the one that intersects the X axis at $x = 60$. This budget line is tangent to an indifference at point A, where the consumption of X is $D_x(p_x) = 40$ and the consumption of Y is $D_y(p_x) = 10$.

- (b) (1 point) Repeat part (a) for a price of X of $p_x = 4$. Then repeat two more times with prices $p_x = 6$, and $p_x = 8$.

Solution The other three budget lines for $p_x = 4$, $p_x = 6$, and $p_x = 8$, intersect the X axis at $x = 30$, $x = 20$, and $x = 15$. Consumption levels of X for these three budgets are $D_x(4) = 15$, $D_x(6) = 8$, and $D_x(8) = 5$.

- (c) (1 point) Are commodities X and Y complements or substitutes? Explain your answer.

Solution Commodities X and Y are substitutes. An increase to the price of X leads to an increase in consumption of Y , so the consumer substitutes Y for X as the price of X increases.

(d) (1 point) The demand function for this consumer for good X is

$$D_x(p_x) = \frac{480}{p_x \cdot (p_x + 4)}.$$

The demand for good Y as a function of the price of X for this consumer is

$$D_y(p_x) = \frac{120 \cdot p_x}{4 \cdot (p_x + 4)}.$$

By entering the four different prices for X ($p_x = 2$, $p_x = 4$, $p_x = 6$, and $p_x = 8$) into these demand functions, fill in the right hand table on the previous page. Show your calculation for each of the eight cells in the table. Compare the results of the two tables.

Solution For the first row, with $p_x = 2$, the demand for X is $D_x(2) = \frac{480}{2 \cdot (2+4)} = 40$ and the demand for Y when the price of X is $p_x = 2$ is $D_y(2) = \frac{120 \cdot 2}{4 \cdot (2+4)} = 10$.

In the second row, with $p_x = 4$, the calculations are $D_x(4) = \frac{480}{4 \cdot (4+4)} = 15$ and $D_y(4) = \frac{120 \cdot 4}{4 \cdot (4+4)} = 15$.

With $p_x = 6$, the calculations are $D_x(6) = \frac{480}{6 \cdot (6+4)} = 8$ and $D_y(6) = \frac{120 \cdot 6}{4 \cdot (6+4)} = 18$.

In the last row, with $p_x = 8$, the calculations are $D_x(8) = \frac{480}{8 \cdot (8+4)} = 5$ and $D_y(8) = \frac{120 \cdot 8}{4 \cdot (8+4)} = 20$.

(e) (1 point) Verify that at the chosen points, the budget equation is satisfied. That is, verify that $p_x x + p_y y = M$ when $x = D_x(p_x)$ and $y = D_y(p_x)$.

Solution When the price of X is $p_x = 2$, the demand for X is $x = D_x(2) = 40$ and the demand for Y is $y = D_y(2) = 10$. The cost of this level of consumption is

$$p_x x + p_y y = 2 \cdot 40 + 4 \cdot 10 = 120.$$

When the price of X is $p_x = 4$, the demand for X is $x = D_x(4) = 15$ and the demand for Y is $y = D_y(4) = 15$. The cost of this level of consumption is

$$4 \cdot 15 + 4 \cdot 15 = 120.$$

When the price of X is $p_x = 6$, the demand for X is $x = D_x(6) = 8$ and the demand for Y is $y = D_y(6) = 18$. The cost of this level of consumption is

$$6 \cdot 8 + 4 \cdot 18 = 120.$$

When the price of X is $p_x = 8$, the demand for X is $x = D_x(8) = 5$ and the demand for Y is $y = D_y(8) = 20$. The cost of this level of consumption is

$$8 \cdot 5 + 4 \cdot 20 = 120.$$

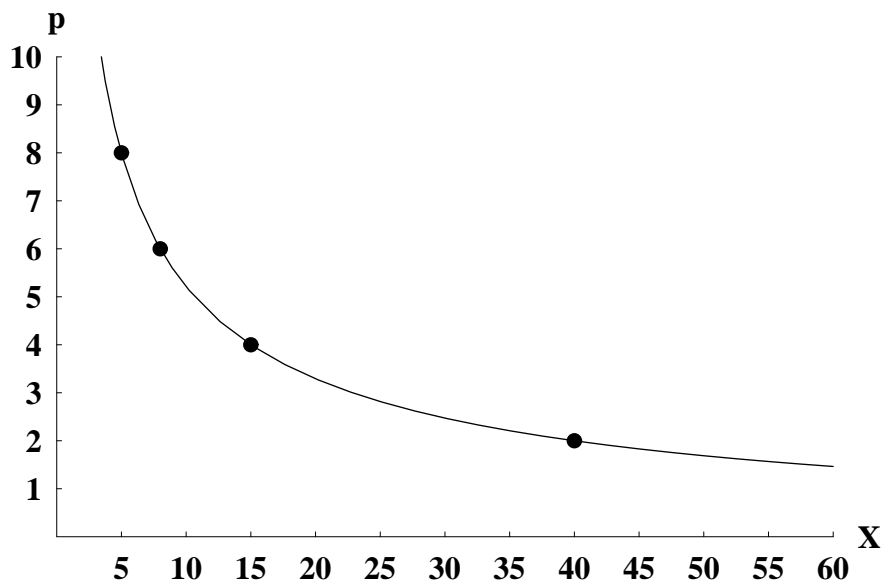
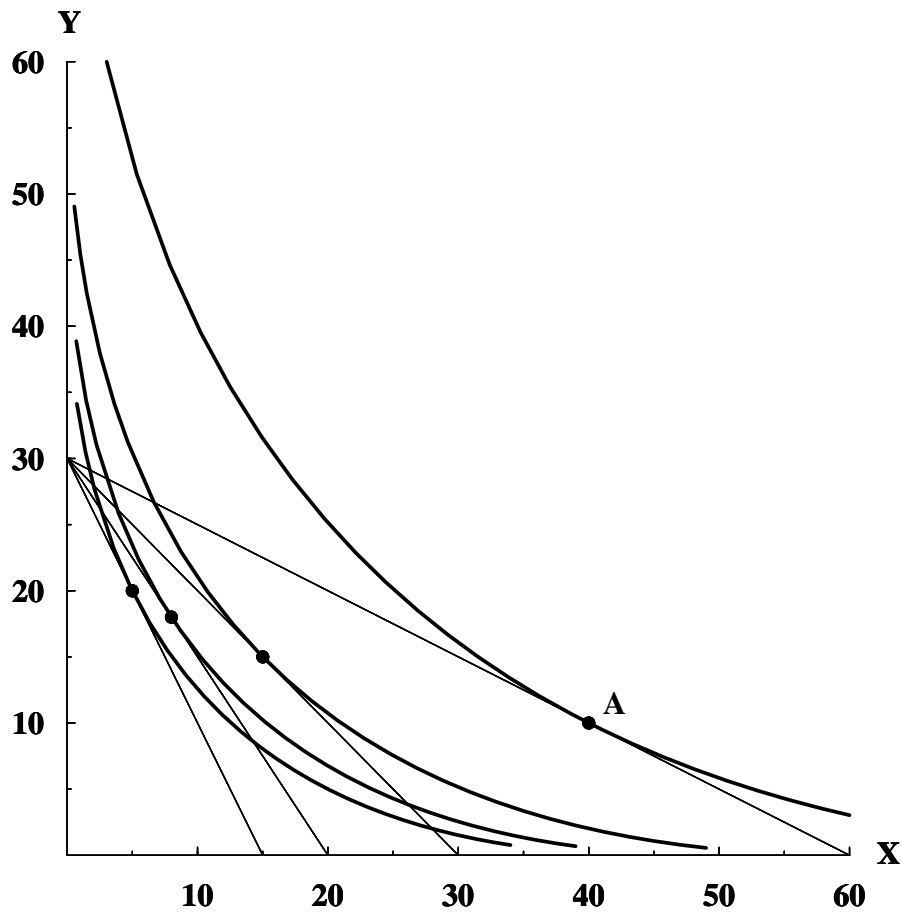


Figure 1: The top graph shows the four budget sets for the prices and incomes given, and the tangent indifference curve for each of these budget sets. For each of these four prices of X , the price and the amount of X chosen are shown on the bottom graph.

Problem 2 is the same as problem 1, except that the indifference curves in this problem come from a different utility function.

2. Suppose that a consumer has an income of $M = 120$, and that several indifference curves from his preferences over two commodities are shown in the top figure on page 6. Assume also that the price of commodity Y is fixed at $p_y = 4$.

Follow the instructions in steps (a) and (b) to fill in the left table below. Use the information in the table to answer question (c). Then follow the instructions in part (d) to fill in the table on the right.

p_x	$D_x(p_x)$	$D_y(p_x)$
2	25	18
4	15	15
6	11	13.5
8	9	12.5

p_x	$D_x(p_x)$	$D_y(p_x)$
2	24.85	17.57
4	15	15
6	11.01	13.48
8	8.79	12.43

- (a) (1 point) Plot the budget line that corresponds to $p_x = 2$ and determine the quantity of X consumed as well as the quantity of Y consumed. Enter your answers in the table. Also, plot the price of X and the corresponding quantity of X consumed in the lower graph on page 6.

Solution As in problem 1, the budget line that corresponds to $p_x = 2$ is the one that intersects the X axis at $x = 60$. This budget line is tangent to an indifference at the point $(x, y) \doteq (25, 18)$, where the consumption of X is $D_x(p_x) \doteq 25$ and the consumption of Y is $D_y(p_x) \doteq 18$.

- (b) (1 point) Repeat part (a) for a price of X of $p_x = 4$. Then repeat two more times with prices $p_x = 6$, and $p_x = 8$.

Solution The other three budget lines for $p_x = 4$, $p_x = 6$, and $p_x = 8$, intersect the X axis at $x = 30$, $x = 20$, and $x = 15$, just as in problem 1. Consumption levels of X for these three budgets are approximately $D_x(4) = 15$, $D_x(6) = 11$, and $D_x(8) = 9$.

- (c) (1 point) Are commodities X and Y complements or substitutes? Explain your answer.

Solution Commodities X and Y are complements. An increase to the price of X leads to a decrease in consumption of Y . The consumer decreases consumption of Y almost as much as she decreases consumption of X as the price of X increases.

(d) (1 point) The demand function for this consumer for good X is

$$D_x(p_x) = \frac{120}{p_x + 2p_x^{0.5}}.$$

The demand for good Y as a function of the price of X for this consumer is

$$D_y(p_x) = \frac{120}{4 + 2p_x^{0.5}}.$$

By entering the four different prices for X ($p_x = 2$, $p_x = 4$, $p_x = 6$, and $p_x = 8$) into these demand functions, fill in the right hand table on the previous page. Show your calculation for each of the eight cells in the table. Compare the results of the two tables.

Solution For the first row, with $p_x = 2$, the demand for X is $D_x(2) = 120/(2 + 2 \cdot 2^{0.5}) = 24.85$ and the demand for Y when the price of X is $p_x = 2$ is $D_y(2) = 120/(4 + 2 \cdot 2^{0.5}) = 17.57$.

In the second row, with $p_x = 4$, the demand for X is $D_x(4) = 120/(4 + 2 \cdot 4^{0.5}) = 15$ and the demand for Y is $D_y(4) = 120/(4 + 2 \cdot 4^{0.5}) = 15$.

In the third row, with $p_x = 6$, the demand for X is $D_x(6) = 120/(6 + 2 \cdot 6^{0.5}) = 11.01$ and the demand for Y is $D_y(6) = 120/(4 + 2 \cdot 6^{0.5}) = 13.48$.

In the last row, with $p_x = 8$, the demand for X is $D_x(8) = 120/(8 + 2 \cdot 8^{0.5}) = 8.79$ and the demand for Y is $D_y(8) = 120/(4 + 2 \cdot 8^{0.5}) = 12.43$.

(e) (1 point) Verify that at the chosen points, the budget equation is satisfied. That is, verify that $p_x x + p_y y = M$ when $x = D_x(p_x)$ and $y = D_y(p_x)$.

Solution When the price of X is $p_x = 2$, the demand for X is $x = D_x(2) = 24.85$ and the demand for Y is $y = D_y(2) = 17.57$. The cost of this level of consumption is

$$p_x x + p_y y = 2 \cdot 24.85 + 4 \cdot 17.57 \doteq 120.$$

When the price of X is $p_x = 4$, the demand for X is $x = D_x(4) = 15$ and the demand for Y is $y = D_y(4) = 15$. The cost of this level of consumption is

$$4 \cdot 15 + 4 \cdot 15 = 120.$$

When the price of X is $p_x = 6$, the demand for X is $x = D_x(6) \doteq 11.01$ and the demand for Y is $y = D_y(6) \doteq 13.48$. The cost of this level of consumption is

$$6 \cdot 11.01 + 4 \cdot 13.48 \doteq 120.$$

When the price of X is $p_x = 8$, the demand for X is $x = D_x(8) = 8.79$ and the demand for Y is $y = D_y(8) = 12.43$. The cost of this level of consumption is

$$8 \cdot 8.79 + 4 \cdot 12.43 \doteq 120.$$

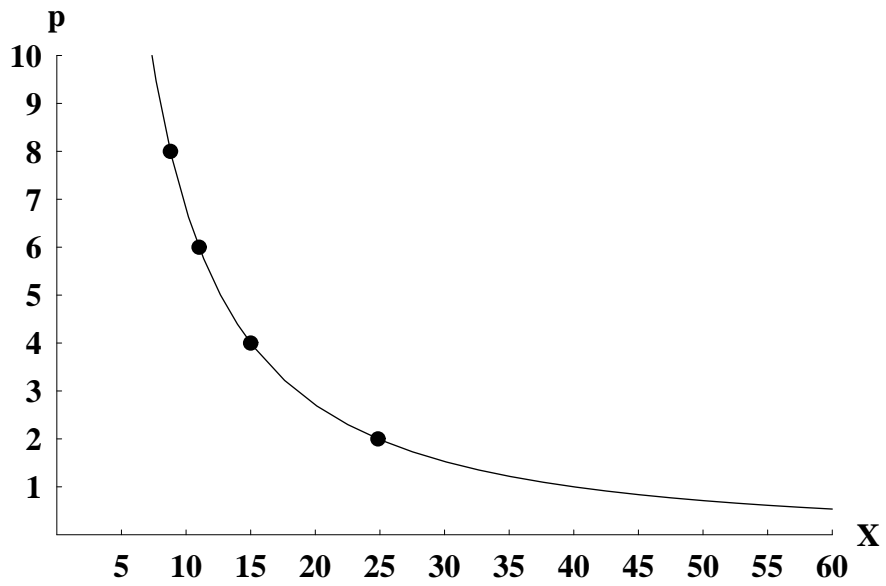
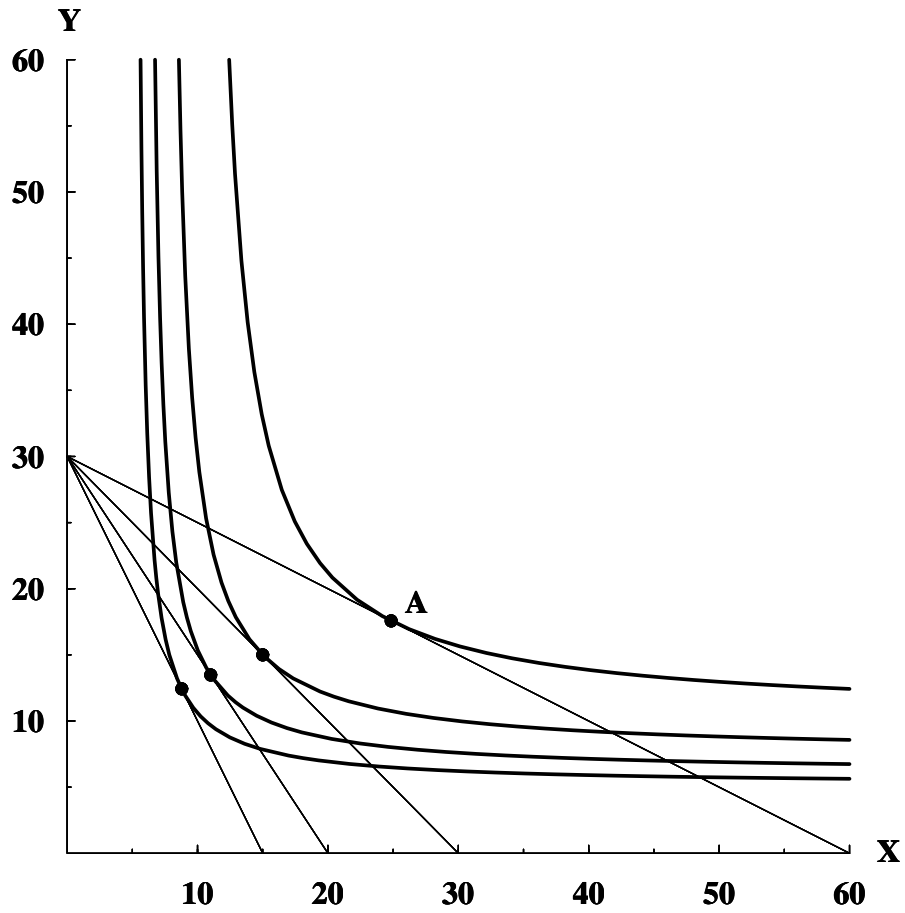


Figure 2: The top graph shows the four budget sets for the prices and incomes given, and the tangent indifference curve for each of these budget sets. For each of these four prices of X , the price and the amount of X chosen are shown on the bottom graph.

3. Suppose that a consumer has an income of $M = 120$, and that several indifference curves from his preferences over two commodities are shown in the top figure on the next page. Assume also that the price of commodity Y is fixed at $p_y = 4$.

Follow the instructions in steps (a) and (b) to fill in the left table below. Use the information in the table to answer question (c). Then follow the instructions in part (d) to fill in the table on the right.

p_x	$D_x(p_x)$	$D_y(p_x)$
2	30	15
4	15	15
6	10	15
8	7.5	15

p_x	$D_x(p_x)$	$D_y(p_x)$
2	30	15
4	15	15
6	10	15
8	7.5	15

- (a) (1 point) Plot the budget line that corresponds to a price of X of $p_x = 2$ and determine the quantity of X consumed as well as the quantity of Y consumed. Enter your answers in the table. Also, plot the price of X and the corresponding quantity of X consumed in the lower graph on the next page.

Solution The budget lines are the same as in problems 1 and 2.

- (b) (1 point) Repeat part (a) for a price of X of $p_x = 4$. Then repeat two more times with prices $p_x = 6$, and $p_x = 8$.
- (c) (1 point) Are commodities X and Y complements or substitutes? Explain your answer.

Solution For this consumer, the demand for commodity Y is unaffected by the price of commodity X , so Y is neither a substitute for nor a complement to commodity X .

- (d) (1 point) The demand function for this consumer for good X is

$$D_x(p_x) = \frac{60}{p_x}.$$

The demand for good Y as a function of the price of X for this consumer is

$$D_y(p_x) = 15.$$

Use these demand functions to fill in the right hand table above.

- (e) (1 point) Verify that at the chosen points, the budget equation is satisfied. That is, verify that $p_x x + p_y y = M$ when $x = D_x(p_x)$ and $y = D_y(p_x)$.

Solution The expenditure on commodity Y for this consumer is $4 \cdot 15 = 60$ at all four prices of X . Expenditure on X is $2 \cdot 30 = 60$, $4 \cdot 15 = 60$, $6 \cdot 10 = 60$, and $8 \cdot 7.5 = 60$ at the four different prices. The total expenditure on X and Y is 120 at all four prices of X .

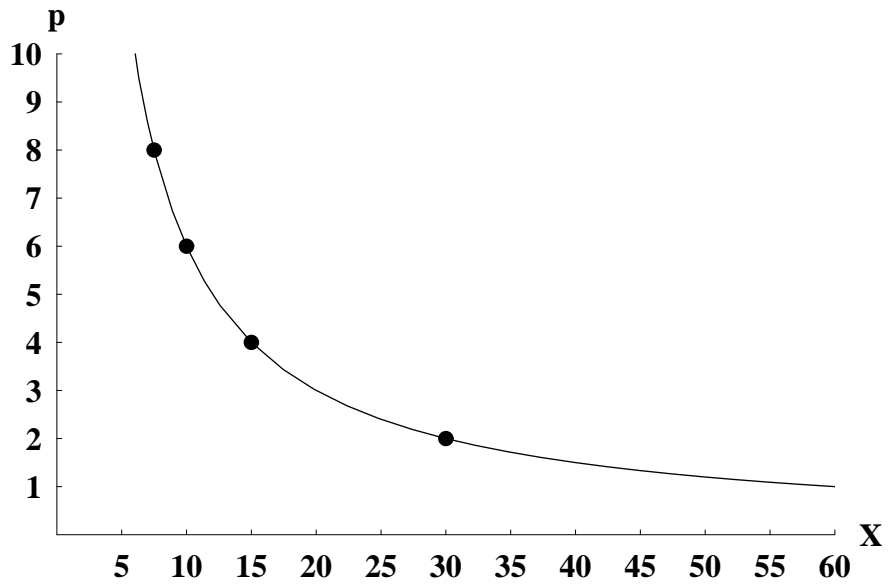
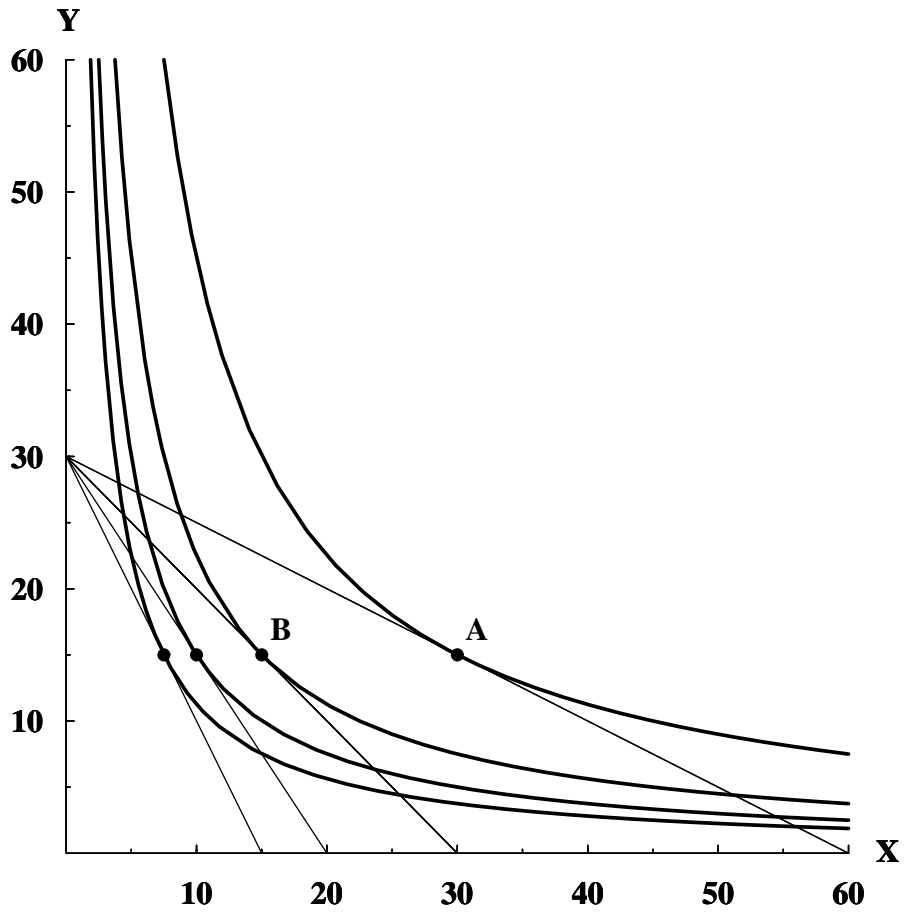


Figure 3: The top graph shows the four budget sets for the prices and incomes given, and the tangent indifference curve for each of these budget sets. For each of these four prices of X , the price and the amount of X chosen are shown on the bottom graph.

4. Suppose that a consumer has an income of $M = 120$, and that several indifference curves from her preferences over two commodities are shown in the top figure on page 12. Assume also that the price of commodity Y is fixed at $p_y = 4$.

Follow the instructions in steps (a) and (b) to fill in the left table below. Use the information in the table to answer question (c). Then follow the instructions in part (d) to fill in the table on the right.

p_x	$D_x(p_x)$	$D_y(p_x)$
2	35	12.5
4	25	5
6	15	7.5
8	5	20

p_x	$D_x(p_x)$	$D_y(p_x)$
2	35	12.5
4	25	5
6	15	7.5
8	5	20

- (a) (1 point) Plot the budget line that corresponds to a price of X of $p_x = 2$ and determine the quantity of X consumed as well as the quantity of Y consumed. (Since these indifference curves are so flat, you'll need to draw the budget lines carefully to get find the tangency.) Enter your answers in the table. Also, plot the price of X and the corresponding quantity of X consumed in the lower graph on the next page.

Solution The budget lines are the same as in problems 1 and 2.

- (b) (1 point) Repeat part (a) for a price of X of $p_x = 4$. Then repeat two more times with prices $p_x = 6$, and $p_x = 8$.
- (c) (1 point) Are commodities X and Y complements or substitutes? Explain your answer.

Solution For this consumer, the demand for commodity Y initially falls when the price of X increases from $p_x = 2$ to $p_x = 4$. For this price range commodities X and Y are complements. For prices between $p_x = 4$ and $p_x = 8$, consumption of Y increases as the price of commodity X increases. In this higher price range, commodities X and Y are substitutes.

- (d) (1 point) The demand function for this consumer for good X is $D_x(p_x) = 45 - 5p_x$. The demand for good Y as a function of the price of X for this consumer is $D_y(p_x) = (120 - 45p_x + 5p_x^2)/4$. Use these demand functions to fill in the right hand table above.

- (e) (1 point) Verify that at the chosen points, the budget equation is satisfied. That is, verify that $p_x x + p_y y = M$ when $x = D_x(p_x)$ and $y = D_y(p_x)$.

At $p_x = 2$, expenditure is $2 \cdot 35 + 4 \cdot 12.5 = 120$.

At $p_x = 4$, expenditure is $4 \cdot 25 + 4 \cdot 5 = 120$.

At $p_x = 6$, expenditure is $6 \cdot 15 + 4 \cdot 7.5 = 120$.

At $p_x = 8$, expenditure is $8 \cdot 5 + 4 \cdot 20 = 120$.

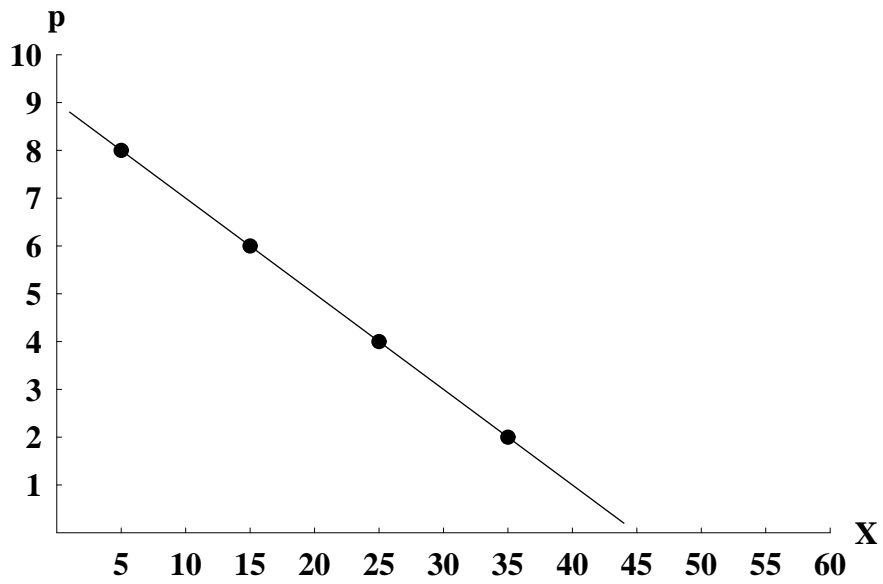
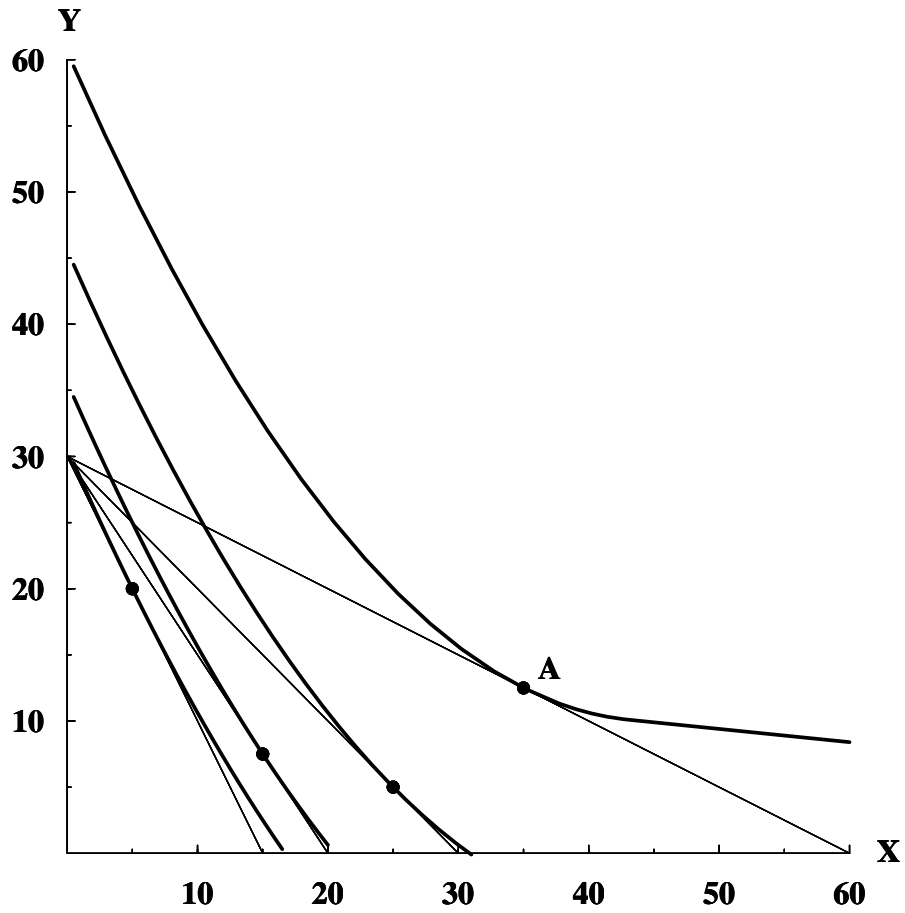


Figure 4: The top graph shows the four budget sets for the prices and incomes given, and the tangent indifference curve for each of these budget sets. For each of these four prices of X , the price and the amount of X chosen are shown on the bottom graph.