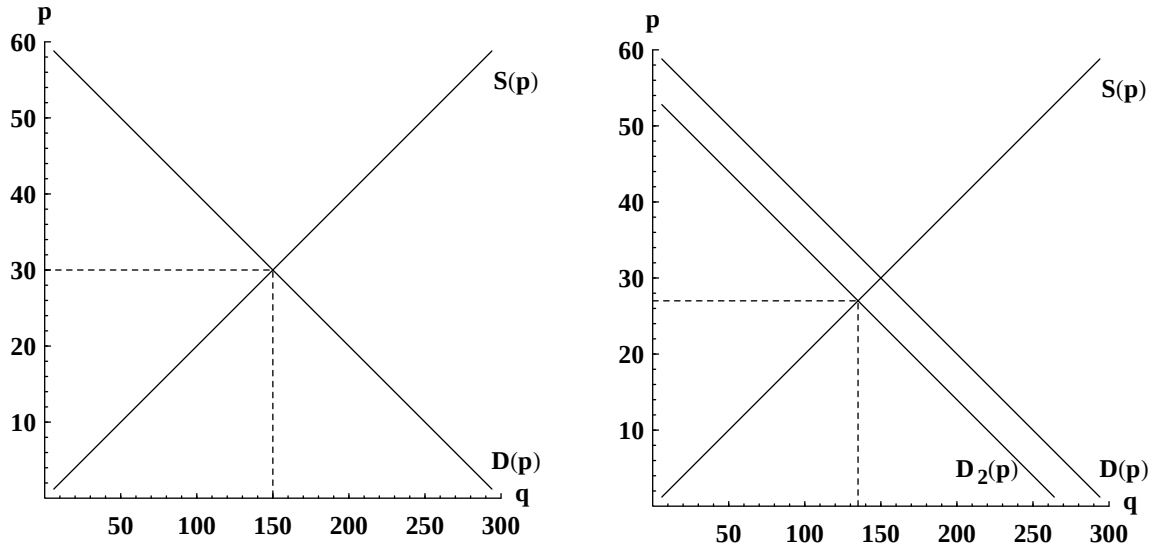
Economics 251
Homework #3 solutions

The first problem in this assignment is based on the lecture notes on long-run equilibrium. The problems progress through steps similar to those taken in the lecture and the lecture notes, but with firm contraction in response to a tax on consumers, rather than firm entry or expansion due to a demand increase. Each part of problem 1 is worth 1 point. Problem 2 is worth 4 points.

1. Suppose that originally the demand in a market is $D_1(p) = 300 - 5\,p$ and the supply in the market is $S_1(p) = 5\,p$. The supply and demand are shown on the left side of the figure below.



Suppose that in this market a sales tax of \$6 per unit is placed on the buyers. Demand of the buyers then shifts to

$$\begin{aligned} D_2(p) &= 300 - 5\,(p+6) \\ &= 270 - 5\,p \end{aligned}$$

as shown on the right side of the figure above.

(a) Assume that the production function is the same one that we worked with in the lecture notes: $q = f(K, L) = \bar{K}^{1/2} L^{1/2}$. Assume that the factor prices are $r = 9$ and $w = 25$. Find the cost function for each of ten firms if their capital stock is fixed in the short-run at $\bar{K} = 25$. Recall that the cost function is

$$\begin{aligned} C(q) &= r\bar{K} + w L(q) \\ &= r\bar{K} + w q^2/\bar{K}. \end{aligned}$$

Solution With the factor prices $r = 9$ and $w = 25$ and capital level $\bar{K} = 25$, the cost function is

$$\begin{aligned} C(q) &= 9 \cdot 25 + 25 q^2/25 \\ &= 225 + q^2. \end{aligned} \tag{1}$$

(b) Find the marginal cost function for the firm, using the equation $MC(q) = \frac{1}{\delta}(C(q) - C(q - \delta))$.

Solution The marginal cost function for each firm is

$$\begin{aligned} MC(q) &= \frac{1}{\delta}(C(q) - C(q-\delta)) \\ &= \frac{1}{\delta}(225 + q^2 - (225 + (q-\delta)^2)) \\ &= \frac{1}{\delta}(q^2 - (q-\delta)^2) \\ &= \frac{1}{\delta}(q^2 - (q^2 - 2\delta q + \delta^2)) \\ &= \frac{1}{\delta}(2\delta q - \delta^2) \\ &= 2q - \delta \\ &\doteq 2q. \end{aligned} \tag{2}$$

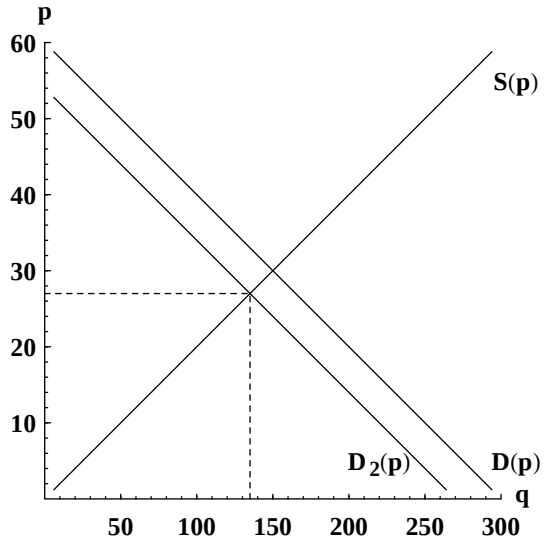
(c) Use the marginal cost function that you've found (dropping the δ from $MC(q)$ if you haven't already done so) and the profit maximization condition $p = MC(q)$ to find the supply function for a firm in this market.

Solution The supply function for a firm in this market is determined by solving the profit maximization condition $p = MC(q)$ for $q = S^i(p)$. In this case the profit maximization condition is $p = 2q$ so $q = S^i(p) = p/2$.

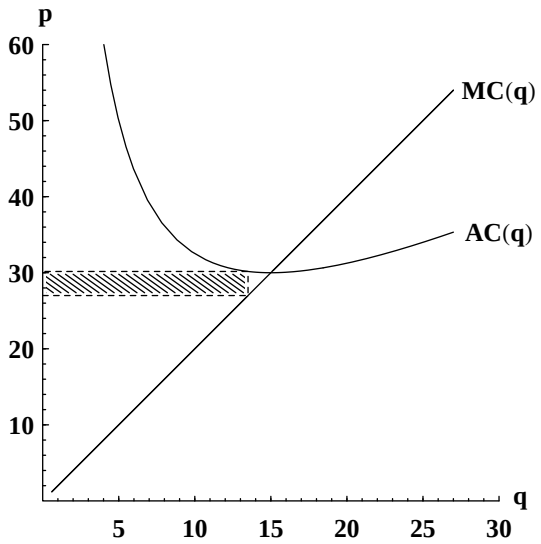
(d) Find the average cost function for the firm, using the equation $AC(q) = C(q)/q$.

Solution The average cost function for the firm, using the definition of the average cost $AC(q) = C(q)/q$ and the cost function from part (a), is $AC(q) = 225/q + q$.

(e) The graph on the right side of the figure below shows the marginal cost and average cost for a typical firm in this market. Check the marginal cost and average cost functions that you've found in parts (b) and (d) by comparing your equations to the functions shown in the graph.



(a) Market conditions



(b) Firm conditions

Solution The table below shows values of the average cost and marginal cost functions at several output levels. These values are consistent with the ones shown in the graph.

q	$AC(q)$	$MC(q)$
5	50.00	10
10	32.50	20
15	30.00	30
20	31.25	40

Table 1: Values of average cost and marginal cost at several output levels.

(f) Calculate the economic profit for each firm in this market at the original equilibrium price p_1^* and at the equilibrium price p_2^* for the market after the demand shifts due to the introduction of the sales tax.

Solution At the original equilibrium price $p_1^* = 30$ each firm produces $q_1^* = S^i(p_1^*) = S(30) = 15$ units. The average cost at output $q_1^* = 15$ is $AC(q_1^*) = AC(15) = 225/15 + 15 = 30$. Since price minus average cost is zero, the economic profit is zero: the original configuration of supply and demand was a long-run equilibrium.

After the tax is imposed, the equilibrium price declines. The market supply is $S(p) = 5\,p$ and market demand fell to $D_2(p) = 270 - 5\,p$ so $p_2^* = 27$. Each firm therefore supplies $q_2^* = S^i(p_2^*) = S(27) = 27/2$

units. The average cost of a firm at this level of output is

$$\begin{aligned} AC(q_2^*) &= AC(27/2) \\ &= \frac{225}{27/2} + 27/2 \\ &= \frac{450}{27} + 27/2 \\ &= \frac{50}{3} + 27/2 \\ &= 181/6. \end{aligned}$$

The economic profit for each firm in the new equilibrium is

$$\begin{aligned} \pi(q) &= (p_2^* - AC(q_2^*))\, q_2^* \\ &= \left(27 - \frac{181}{6}\right) \frac{27}{2} \\ &= \left(\frac{162}{6} - \frac{181}{6}\right) \frac{27}{2} \\ &= -\frac{19}{6}\,\frac{27}{2} \\ &= -\frac{19}{2}\,\frac{9}{2} \\ &= -42.75. \end{aligned}$$

(g) Calculate the accounting profit (or producer's surplus for an individual firm at each of these two equilibrium prices) and subtract the total capital cost ($r\,\bar{K}$) from the accounting profit. How does economic profit compare to accounting profit minus capital cost?

Solution In the original equilibrium, prior to the shift to demand, the market price was $p_1^* = 30$. Output for each firm was $q_1^* = 15$. Since $S^i(p) = p/2$, the supply function intersects the price axis at $p = 0$. So the area of the producer's surplus triangle is $PS^i = \frac{1}{2} \cdot 15 \cdot 30 = 225$. The capital cost for the firm is $r\,\bar{K} = 9 \cdot 25 = 225$, which is equal to the accounting profit. The firm meets its capital cost exactly, and has economic profit zero, because economic profit is accounting profit minus capital cost.

After the demand decrease due to the imposition of the sales tax, the equilibrium price declines to $p_2^* = 27$. Output for each firm declines to $q_2^* = 27/2$. Since $S^i(p) = p/2$, the supply function again intersects the price axis at $p = 0$. So the area of the producer's surplus triangle is $PS^i = \frac{1}{2}\,\frac{27}{2}\,27 = 182.25$. The capital cost is still $r\,\bar{K} = 9 \cdot 25 = 225$. Accounting profit minus capital cost is $182.25 - 225 = -42.75$, which we found is the economic profit after the shift to demand. Since the market price fell below the average cost of production for the firms, they begin to experience an economic loss. Economic profit is accounting profit minus capital cost, which in this case is negative.

(h) Suppose that in response to the economic losses that result from the decrease to demand the firms in the market begin to reduce their capital investment. Suppose that each firm reduces its capital stock to $\bar{K} = 20$. Find the new cost function for the firm.

Solution When the capital stock is $\bar{K} = 20$ the cost function for the firm is

$$\begin{aligned} C(q) &= r\,\bar{K} + w\,q^2/\bar{K} \\ &= 9 \cdot 20 + 25\,q^2/20 \\ &= 180 + 1.25\,q^2. \end{aligned} \tag{3}$$

(i) Find the marginal cost function for the firm. Use the marginal cost function to determine the supply for a firm with its reduced level of capital. Find the new market supply $S_3(p)$ by adding together the supply functions of the individual firms.

Solution The marginal cost function is calculated just like in part (b), but the cost function differs. The marginal cost for each firm is

$$\begin{aligned} MC(q) &= \frac{1}{\delta}\left(C(q) - C(q-\delta)\right) \\ &= \frac{1}{\delta}\left(180 + 1.25\,q^2 - (180 + (1.25\,q - \delta)^2)\right) \\ &= \frac{1}{\delta}\left(1.25\,q^2 - (1.25\,q - \delta)^2\right) \\ &= \frac{1}{\delta}\left(2.5\,\delta\,q - \delta^2\right)) \\ &= 2.5\,q - \delta \\ &\doteq 2.5\,q. \end{aligned}$$

From the profit maximization condition $p = MC(q)$ for a firm in a competitive market, the firm chooses q so that $p = 2.5\,q$ or $q = 0.4\,p$. This is the supply function for the firm: $S^i(p) = 0.4\,p$. There are ten firms in the market, each with an identical supply function, so the market supply after all firms reduce their capital level is $S_3(p) = 10 \cdot 0.4\,p = 4\,p$.

(j) Find the equilibrium price p_3^* in the market with the demand after the tax is imposed and the supply after the firms have reduced their capital in response to the economic losses. What is the output for a typical firm at this new equilibrium price?

Solution The demand is $D_2(p) = 270 - 5\,p$ after the tax is imposed. Once firms have all reduced their capital stock to $\bar{K} = 20$, the new market supply is $S_3(p) = 4\,p$. The new equilibrium price is the solution to $D_2(p) = S_3(p)$, which in this case is the equation $270 - 5\,p = 4\,p$. So $p_3^* = 30$. The contraction of the capital stock of each firm leads to reduced supply and an increase in the market price.

The output of a typical firm at this new equilibrium price is $q_i^* = S^i(p_3^*) = 0.4 \cdot 30 = 12$.

(k) What is the average cost for a typical firm at the level of output that you found in part (j)? Find the difference between the equilibrium price p_3^* and the average cost for the typical firm at this level of output.

Solution The average cost function for the firm, using the definition of the average cost $AC(q) = C(q)/q$ and the new cost function $C(q) = 180 + 1.25\,q^2$, is $AC(q) = 180/q + 1.25\,q$. The average cost at the equilibrium output level $q_3^* = 12$ is $AC(12) = 30$.

(l) Economic profit for each firm is the difference between price and average cost times the number of units sold. Calculate the economic profit.

Solution As in the original equilibrium (before demand decreased), the firm's average cost is equal to the market price, so the economic profit of the firm is zero.

(m) Should firms continue to reduce their capital investment? Briefly explain why or not. (One to two sentences should be enough to explain this.)

Solution Since the firms now meet both their variable cost and their capital cost, they have no further incentive to reduce their capital investment (nor any incentive to increase it): the market has returned to a new long-run equilibrium.

(n) What is the accounting profit for the typical firm in this new equilibrium?

Solution The accounting profit for the typical firm in this new equilibrium is determined from the usual formula:

$$\begin{aligned} PS &= \frac{1}{2}\, q_i^* \,(p^* - s_0) \\ &= \frac{1}{2} \cdot 12 \cdot 30 \\ &= 180. \end{aligned}$$

(o) How does accounting profit for the typical firm compare to the capital costs of the firm? (The capital cost is the rental rate of capital r times the amount $\bar{K}$ of capital utilized, which is $r\,\bar{K} = 9 \cdot 20$ after the capital level of the firms declines.)

Solution The accounting profit is $PS = 180$, which is equal to the capital cost.

(p) How does the difference between accounting profit and capital cost (your answer to part (o)) compare to the economic profit (your answer to part (l))? Give a one or two sentence description of the relationship between these three measures.

Solution As always in this model, the economic profit (zero) is equal to the accounting profit (180) minus the capital cost ($r\,\bar{K} = 9 \cdot 20$).

2. Suppose that a monopolist faces the demand curve $q = D(p) = a - b\,p$.

(a) (1 point) Find the inverse demand function $p = D^{-1}(q)$ for the monopolist and substitute this into the monopolist's revenue function $R(q) = p\,q$ to get the revenue as a function of output.

Solution The inverse demand function is determined by solving the equation $q = a - b\,p$ for p. Add $b\,p$ to both sides of this equation to get $q + b\,p = a$. Then subtract q from both sides of the new equation to get $b\,p = a - q$. Finally, divide both sides of this equation by b to get $p = D^{-1}(q) = (a - q)/b$.

The revenue function is $R(q) = p\,q$. Since the price varies with q, we want to substitute in the inverse demand function for p, because the inverse demand function specifies how the price varies with q. The result is

$$R(q) = D^{-1}(q)\,q = \frac{1}{b}\,(a\,q - q^2).$$

(b) (1 point) Find the level of output q^* that maximizes the monopolist's revenue. Also, substitute the profit maximizing price into the inverse demand equation $p = D^{-1}(q)$ to find the price charged by the monopolist.

Solution One simple way to find the output q^* that maximizes the monopolist's revenue is to complete the square. The revenue function is

$$\begin{aligned} R(q) &= \frac{1}{b}\,(a\,q - q^2) \\ &= -\frac{1}{b}\,q^2 + \frac{a}{b}\,q. \end{aligned}$$

This function has its maximum at $q^* = -\frac{a/b}{-2\cdot 1/b} = a/2$. It is easy to find the price once you know the output. The monopolist charges the price

$$\begin{aligned} p^* &= D^{-1}(q^*) \\ &= \frac{1}{b}\,(a - q^*) \\ &= \frac{1}{b}\,(a - a/2) \\ &= \frac{a}{2\,b}. \end{aligned}$$

(c) (2 points) Calculate the elasticity of demand for the monopolist at the output that maximizes revenue for the monopolist. In the elasticity formula take $p_0 = p^*$ and $q_0 = q^*$. (To get q_1 and p_1 you can take q_1 slightly larger than q_0 and use the inverse demand function to find p_1.)

Solution Starting from the initial price $p_0 = p^* = a/(2\,b)$ and the initial quantity $q_0 = q^* = a/2$, increase the price by some small amount. A price change to $p_1 = p_0 + 1$ would work, but some of the algebra is simpler if

$$\begin{aligned} p_1 &= p_0 + \frac{1}{2\,b} \\ &= \frac{a}{2\,b} + \frac{1}{2\,b} \\ &= \frac{a+1}{2\,b}. \end{aligned}$$

Demand is

$$\begin{aligned} q_1 &= a - b\,p_1 \\ &= a - b\,\frac{a+1}{2\,b} \\ &= a - \frac{a+1}{2} \\ &= (a-1)/2. \end{aligned}$$

Now all of the prices and quantities are calculated that are needed to find the elasticity of demand at the revenue maximizing output. The elasticity of demand is

$$\begin{aligned} \epsilon_d &= \left|\frac{(q_1^d - q_0^d)/q_0^d}{(p_1 - p_0)/p_0}\right| \\ &= \left|\frac{q_1^d - q_0^d}{p_1 - p_0} \cdot \frac{p_0}{q_0^d}\right| \\ &= \left|\frac{(a-1)/2 - a/2}{(a+1)/(2\,b) - a/(2\,b)} \cdot \frac{a/(2\,b)}{a/2}\right| \\ &= \left|\frac{-1/2}{1/(2\,b)} \cdot \frac{a/(2\,b)}{a/2}\right| \\ &= |-1|. \end{aligned}$$