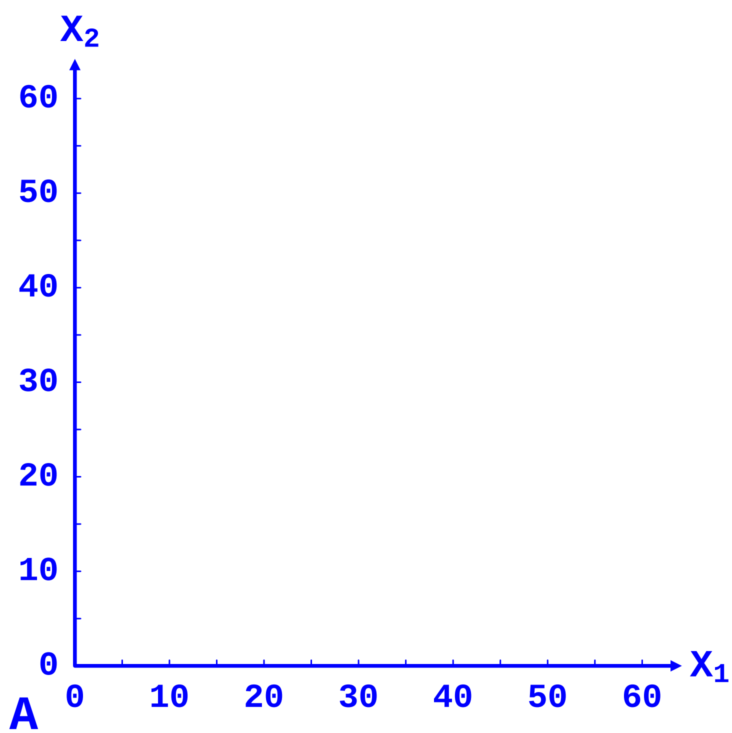
Economics 380 Name: ____________________
Homework #1
Due: Tuesday September 11, 2007 Division: ___

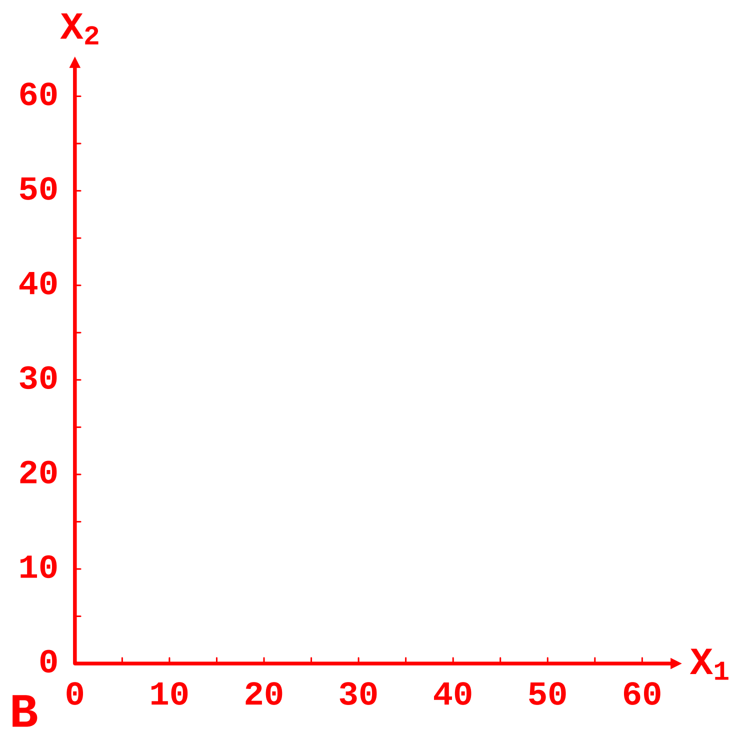
1. An exchange economy This problem consists of six parts. In the first part, you'll find a graphical solution to the demand for agent type A at two different price vectors $p = (1,1)$ and $p = (1,3)$. In the second part, you'll repeat that exercise for agent type B. In the third part, you'll describe why one of the price vectors is an equilibrium price vector and why the other is not. In the fourth part, you'll find the demand functions for agent type A and compare the quantities demanded at the two price vectors $p = (1,1)$ and $p = (1,3)$ to the graphical demands that you found in the first part. The fifth part repeats that problem for agent type B. The sixth part applies the budget identity to the demand functions that you obtained for agent type B.

(a) (2 points) Agent type A has the utility function $u^A(x_1, x_2) = \min\{3\,x_1, x_2\}$ and the endowment $\omega^A = (30, 30)$. Show the endowment point on the diagram below. Draw the budget line that corresponds to the price vector $p = (p_1, p_2) = (1,1)$. (Recall that the budget line passes through the endowment, and use the price of X_2 relative to the price of X_1 to figure out what combinations of X_1 and X_2 are available to the agent at these prices.) Draw the indifference curve $u^A(x_1, x_2) = 45$. What are the optimal consumption levels of X_1 and X_2 for this agent when the prices are $p = (p_1, p_2) = (1,1)$?



On the same diagram (on page 1), draw the budget line that corresponds to the price vector $p = (p_1, p_2) = (1, 3)$. Draw the indifference curve $u^A(x_1, x_2) = 36$. What are the optimal consumption levels of X_1 and X_2 for this agent when the prices are $p = (p_1, p_2) = (1, 3)$?

(b) (2 points) Agent type B has the utility function $u^B(x_1, x_2) = \min\{x_1, 2x_2\}$ and the endowment $\omega^B = (30, 30)$. Show the endowment point on the diagram below. Draw the budget line that corresponds to the price vector $p = (p_1, p_2) = (1, 1)$. Draw the indifference curve $u^B(x_1, x_2) = 40$. What are the optimal consumption levels of X_1 and X_2 for this agent when the prices are $p = (p_1, p_2) = (1, 1)$?



On the same diagram, draw the budget line that corresponds to the price vector $p = (p_1, p_2) = (1, 3)$. Draw the indifference curve $u^B(x_1, x_2) = 48$. What are the optimal consumption levels of X_1 and X_2 for this agent when the prices are $p = (p_1, p_2) = (1, 3)$?

(c) (1 point) How much of commodity X_2 is wanted when the prices are $p = (p_1, p_2) = (1, 1)$? How much is available? Is $p = (p_1, p_2) = (1, 1)$ an equilibrium price vector?

How much of commodity X_2 is wanted when the prices are $p = (p_1, p_2) = (1, 3)$? How much is available? Is $p = (p_1, p_2) = (1, 3)$ an equilibrium price vector?

(d) (1 point) Use the utility function $u^A(x_1, x_2) = \min\{3x_1, x_2\}$ and the budget equation for agent A to find this agent's demand function for commodity X_2. (Remember that an important step is to find the relationship between the consumption levels x_1 and x_2.) After you have the demand for X_2, write down the demand for X_1. How much of X_1 and X_2 does this agent want at the prices $p = (p_1, p_2) = (1, 1)$? How much of X_1 and X_2 does this agent want at the prices $p = (p_1, p_2) = (1, 3)$?

(e) (1 point) Use the utility function $u^B(x_1, x_2) = \min\{x_1, 2\,x_2\}$ and the budget equation for agent B to find this agent's demand function for commodity X_2. After you have the demand for X_2, write down the demand for X_1. How much of X_1 and X_2 does this agent want at the prices $p = (p_1, p_2) = (1, 1)$? How much of X_1 and X_2 does this agent want at the prices $p = (p_1, p_2) = (1, 3)$?

(f) (1 point) Use the demand functions $x_1^B(p_1, p_2)$ and $x_1^B(p_1, p_2)$ for agent B that you found in part (e) to verify the budget identity

$$p_1\, x_1^B(p_1, p_2) + p_2\, x_2^B(p_1, p_2) \equiv p_1\, \omega_1^B + p_2\, \omega_2^B\,.$$

2. Barter sequences in an exchange economy Suppose that there are three agents (A, B, and C) in an exchange economy that has three commodities X_1, X_2, and X_3. The left panel of table 1 shows endowments for agents A, B, and C. If the equilibrium prices for the three commodities are $p^* = (p_1^*, p_2^*, p_3^*) = (1, 2, 3)$, then the desired allocations for three agents are shown in the right panel of the table.

	X_1	X_2	X_3
A	14	20	22
B	14	20	22
C	14	20	22

	X_1	X_2	X_3
A	0	15	30
B	12	0	36
C	30	45	0

Table 1: Agents' endowments (left) and desired consumption (right)

(a) (2 points) Verify that the value of each agent's endowment equals the value of that agent's allocation.

(b) (2 points) Table shows several trades in a sequence that leads from the endowments in the left panel of table 1 to the equilibrium allocations in the right panel of table 1. Fill in the table with two more trades to complete the sequence of barters. Describe each of the two trades that complete the sequence of barters in the space below table 2.

	X_1	X_2	X_3
$\underline{A}$	2	20	26
B	14	20	22
$\underline{C}$	26	20	18

	X_1	X_2	X_3
A	2	20	26
$\underline{B}$	14	2	34
$\underline{C}$	26	38	6

	X_1	X_2	X_3
$\underline{A}$	0	21	26
$\underline{B}$	16	1	34
C	26	38	6

	X_1	X_2	X_3
A	0	21	26
$\underline{B}$	18	0	34
$\underline{C}$	24	39	6

	X_1	X_2	X_3
A			
B			
C			

	X_1	X_2	X_3
A			
B			
C			

Table 2: Sequence of interim allocations with barter

3. Arbitrage with commodity money This problem involves a comparison between two arbitrage situations and also some interpretation of the results.

(a) (2 points) Suppose that the prices in the arbitrage example between the British and the French camps are 10 for tea and 9 for coffee in the British camp. In the French camp prices are 9 for tea and 10 for coffee. If a British officer has 81 units of commodity money, find an arbitrage strategy that the officer could follow that would yield a profit of 19 units of the commodity money.

(b) (2 points) Now suppose that everything is the same as in the previous problem, but the price of coffee in the French camp is now 8 instead of 10. (So the prices in the British camp are 10 for tea and 9 for coffee. In the French camp prices are 9 for tea and 8 for coffee.) If a British officer has 81 units of capital, what arbitrage strategy could he follow to yield a profit of 9 units of the commodity money or more?

(c) (1 point) In part (a) the best arbitrage strategy involves commodities being moved from each camp to the other. In part (b) the best arbitrage strategy involves moving commodity money from one camp to the other. Why is this the case? What characteristic of the prices of the commodities drive the British officers to trade in commodities in part (a) but in currency in part (b)?

4. Money supply, output, and inflation (1 point) In the exchange economy from the lecture notes that we used to derive a version of the "equation of exchange," the relationship between the equilibrium price and the endowments of commodity money and commodity is $p^* = \omega_1^B/\omega_2^S$. In this model we interpret ω_1^B as the money supply and we interpret ω_2^S as output.

Suppose that output at year 0 is $\omega_2^S = 15$ and the money supply is $\omega_1^B = 1500$ so that the price level is $p^* = 1500/15 = 100$.

If in year 1 output ω_2^S increases to $\omega_2^S = 18$ and the money supply ω_1^B increases to $\omega_1^B = 1926$, find the new price level. What is the rate of of inflation between years 0 and 1?

Short answer questions

5. Functions of money (1 point) List the three functions of money.

6. Characteristics of circulating money (1 point) List three characteristics that a commodity will typically have if it is to overcome trade frictions and serve effectively as money.