Economics 380
Homework #1 solutions
Tuesday September 12, 2007

1. An exchange economy This problem consists of six parts. In the first part, you'll find a graphical solution to the demand for agent type A at two different price vectors $p = (1, 1)$ and $p = (1, 3)$. In the second part, you'll repeat that exercise for agent type B. In the third part, you'll describe why one of the price vectors is an equilibrium price vector and why the other is not. In the fourth part, you'll find the demand functions for agent type A and compare the quantities demanded at the two price vectors $p = (1, 1)$ and $p = (1, 3)$ to the graphical demands that you found in the first part. The fifth part repeats that problem for agent type B. The sixth part applies the budget identity to the demand functions that you obtained for agent type B.

(a) (2 points) Agent type A has the utility function $u^A(x_1, x_2) = \min\{3x_1, x_2\}$ and the endowment $\omega^A = (30, 30)$. Show the endowment point on the diagram below. Draw the budget line that corresponds to the price vector $p = (p_1, p_2) = (1, 1)$. (Recall that the budget line passes through the endowment, and use the price of X_2 relative to the price of X_1 to figure out what combinations of X_1 and X_2 are available to the agent at these prices.) Draw the indifference curve $u^A(x_1, x_2) = 45$. What are the optimal consumption levels of X_1 and X_2 for this agent when the prices are $p = (p_1, p_2) = (1, 1)$?

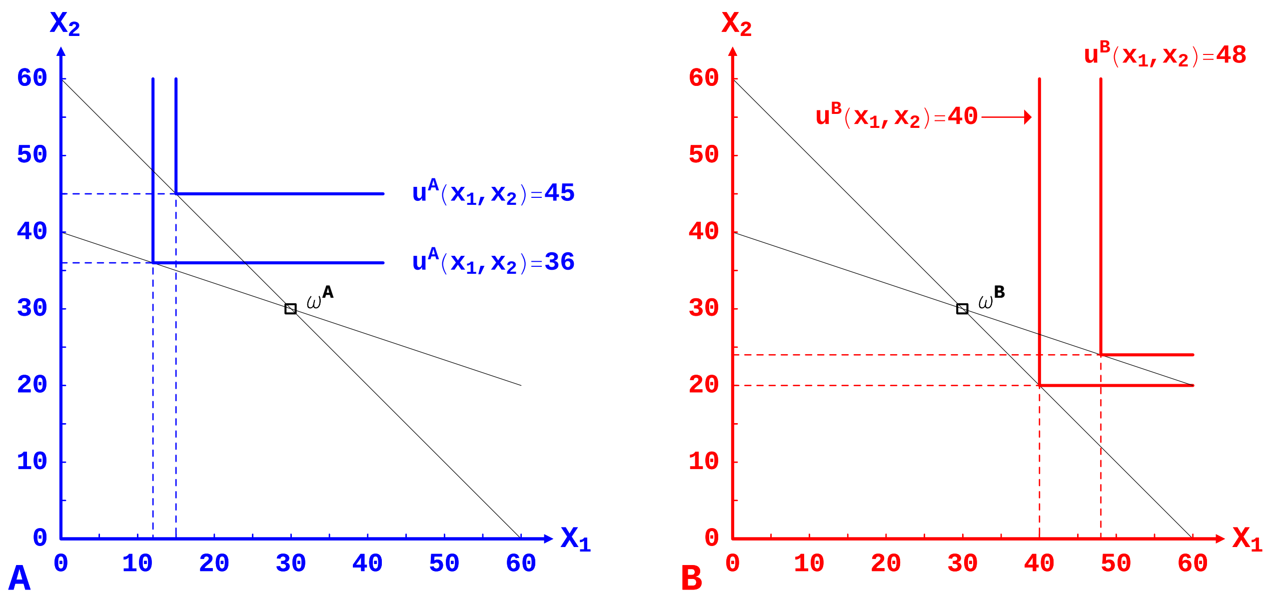

Figure 1: Graphs for agent types A (left) and B (right).

Solution The graph on the left side of figure 1 shows the endowment of a type A agent as a square near the center. The budget set that corresponds to the price vector $p = (p_1, p_2) = (1, 1)$ passes through the endowment and has slope -1, since one unit of X_2 can be acquired if one unit of X_1 is given up. The indifference curve $u^A(x_1, x_2) = 45$ has a corner at $(x_1, x_2) = (15, 45)$. (This is easy to determine. If $u^A(x_1, x_2) = 45$ then there are two possible cases: (1) $3x_1 = 45$ and $x_2 \geq 45$, or (2) $3x_1 \geq 45$ and $x_2 = 45$. These can be rewritten as: $(1)'$ $x_1 = 15$ and $x_2 \geq 45$, or $(2)'$ $x_1 \geq 15$ and

$x_2 = 45$. The indifference curve corresponds to the conjunction of these two inequalities.) A type A agent would choose to consume at the corner of this indifference curve where $(x_1, x_2) = (15, 45)$ when the prices are $(p_1, p_2) = (1, 1)$ because no other point has as high a utility level. (This too is easy to show. Suppose that the agent instead decides to consume $(x_1(a), x_2(a)) = (15 + a, 45 - a)$ where a is some number that can be either positive or negative. For every $a \in [-15, 45]$, $(x_1(a), x_2(a))$ lies on the budget set. The utility level at $(x_1(a), x_2(a)) = (15 + a, 45 - a)$ is

$$\begin{aligned} u^A(x_1(a), x_2(a)) &= \min\{3\,x_1(a), x_2(a))\} \\ &= \min\{3\,(15 + a), 45 - a\} \\ &= \min\{45 + 3\,a, 45 - a\}. \end{aligned}$$

If $a > 0$, then the second term is less than 45; if $a < 0$, then the first term is less than 45. If $a = 0$, then both terms are 45, so the utility is maximized at $(x_1, x_2) = (15, 45)$, which is the corner of the indifference curve.)

On the same diagram (on page 1), draw the budget line that corresponds to the price vector $p = (p_1, p_2) = (1, 3)$. Draw the indifference curve $u^A(x_1, x_2) = 36$. What are the optimal consumption levels of X_1 and X_2 for this agent when the prices are $p = (p_1, p_2) = (1, 3)$?

Solution The budget set that corresponds to the price vector $p = (p_1, p_2) = (1, 3)$ passes through the endowment and has slope $-1/3$, since one unit of X_2 can be acquired if three units of X_1 are given up. The indifference curve $u^A(x_1, x_2) = 36$ has a corner at $(x_1, x_2) = (12, 36)$. A type A agent would choose to consume at the corner of this indifference curve when the prices are $(p_1, p_2) = (1, 3)$.

(b) (2 points) Agent type B has the utility function $u^B(x_1, x_2) = \min\{x_1, 2\,x_2\}$ and the endowment $\omega^B = (30, 30)$. Show the endowment point on the diagram below. Draw the budget line that corresponds to the price vector $p = (p_1, p_2) = (1, 1)$. Draw the indifference curve $u^B(x_1, x_2) = 40$. What are the optimal consumption levels of X_1 and X_2 for this agent when the prices are $p = (p_1, p_2) = (1, 1)$?

Solution The budget set that corresponds to the price vector $p = (p_1, p_2) = (1, 1)$ passes through the endowment and has slope -1, since one unit of X_2 can be acquired if one unit of X_1 is given up. The indifference curve $u^B(x_1, x_2) = 40$ has a corner at $(x_1, x_2) = (40, 20)$. A type B agent would choose to consume at the corner of this indifference curve when the prices are $(p_1, p_2) = (1, 1)$.

On the same diagram, draw the budget line that corresponds to the price vector $p = (p_1, p_2) = (1, 3)$. Draw the indifference curve $u^B(x_1, x_2) = 48$. What are the optimal consumption levels of X_1 and X_2 for this agent when the prices are $p = (p_1, p_2) = (1, 3)$?

Solution The budget set that corresponds to the price vector $p = (p_1, p_2) = (1, 3)$ passes through the endowment and has slope $-1/3$, since one unit of X_2 can be acquired if three units of X_1 are given up. The indifference curve $u^B(x_1, x_2) = 48$ has a corner at $(x_1, x_2) = (48, 24)$. A type B agent would choose to consume at the corner of this indifference curve when the prices are $(p_1, p_2) = (1, 3)$.

(c) (1 point) How much of commodity X_2 is wanted when the prices are $p = (p_1, p_2) = (1, 1)$? How much is available? Is $p = (p_1, p_2) = (1, 1)$ an equilibrium price vector?

Solution Agent type A wants 45 units of X_2 when the prices are $p = (p_1, p_2) = (1, 1)$. Agent type B wants 20 units at these prices. The total wanted is 65 units, but the amount of X_2 available is only 60 units, since the endowment of each type includes 30 units of X_2. The market is not in equilibrium at the prices $p = (p_1, p_2) = (1, 1)$: the price of X_2 needs to rise so that the demand for X_2 will drop from 65 to 60.

How much of commodity X_2 is wanted when the prices are $p = (p_1, p_2) = (1, 3)$? How much is available? Is $p = (p_1, p_2) = (1, 3)$ an equilibrium price vector?

Solution Agent type A wants 36 units of X_2 when the prices are $p = (p_1, p_2) = (1, 3)$. Agent type B wants 24 units at these prices. The total wanted is 60 units, which is equal to the amount of X_2 that is available. The market is in equilibrium at the prices $p = (p_1, p_2) = (1, 1)$.

(d) (1 point) Use the utility function $u^A(x_1, x_2) = \min\{3\,x_1, x_2\}$ and the budget equation for agent A to find this agent's demand function for commodity X_2. (Remember that an important step is to find the relationship between the consumption levels x_1 and x_2.) After you have the demand for X_2, write down the demand for X_1. How much of X_1 and X_2 does this agent want at the prices $p = (p_1, p_2) = (1, 1)$? How much of X_1 and X_2 does this agent want at the prices $p = (p_1, p_2) = (1, 3)$?

Solution The budget equation for agent type A is

$$p_1\, x_1^A + p_2\, x_2^A = p_1\, \omega_1^A + p_2\, \omega_2^A.$$

An agent with the utility function $u^A(x_1, x_2) = \min\{3\,x_1, x_2\}$ chooses optimally when $3\,x_1 = x_2$. This relationship can be substituted into the budget equation and solved for x_2 to get the demand function for X_2 by a type A agent:

$$p_1\, x_2^A/3 + p_2\, x_2^A = p_1\, \omega_1^A + p_2\, \omega_2^A$$

or

$$x_2^A(p_1, p_2) = 3\,\frac{p_1\, \omega_1^A + p_2\, \omega_2^A}{p_1 + 3\,p_2}.$$

The relationship between the optimal consumption of X_1 and X_2 by this agent, $3\,x_1 = x_2$, can be used once again to get

$$x_1^A(p_1, p_2) = \frac{p_1\, \omega_1^A + p_2\, \omega_2^A}{p_1 + 3\,p_2}.$$

When the prices are $p = (p_1, p_2) = (1, 1)$, a type A agent wants

$$\begin{aligned} x_1^A(1,1) &= \frac{1 \cdot 30 + 1 \cdot 30}{1 + 3 \cdot 1} \\ &= 15 \end{aligned}$$

units of X_1 and

$$\begin{aligned} x_2^A(1,1) &= 3\,x_1^A(1,1) \\ &= 45 \end{aligned}$$

units of X_2. These amounts are the same as those shown on the left graph in figure 1 for the prices $p = (p_1, p_2) = (1, 1)$.

When the prices are $p = (p_1, p_2) = (1, 3)$, a type A agent wants

$$\begin{aligned} x_1^A(1,3) &= \frac{1 \cdot 30 + 3 \cdot 30}{1 + 3 \cdot 3} \\ &= 12 \end{aligned}$$

units of X_1 and

$$\begin{aligned} x_2^A(1,1) &= 3\, x_1^A(1,3) \\ &= 36 \end{aligned}$$

units of X_2. These amounts are the same as those shown on the left graph in figure 1 for the prices $p = (p_1, p_2) = (1, 3)$.

(e) (1 point) Use the utility function $u^B(x_1, x_2) = \min\{x_1, 2\,x_2\}$ and the budget equation for agent B to find this agent's demand function for commodity X_2. After you have the demand for X_2, write down the demand for X_1. How much of X_1 and X_2 does this agent want at the prices $p = (p_1, p_2) = (1, 1)$? How much of X_1 and X_2 does this agent want at the prices $p = (p_1, p_2) = (1, 3)$?

Solution The budget equation for agent type B is

$$p_1\, x_1^B + p_2\, x_2^B = p_1\, \omega_1^B + p_2\, \omega_2^B.$$

An agent with the utility function $u^B(x_1, x_2) = \min\{x_1, 2\,x_2\}$ chooses optimally when $x_1 = 2\,x_2$. This relationship can be substituted into the budget equation and solved for x_2 to get the demand function of a type B agent for X_2:

$$p_1\, 2\, x_2^B + p_2\, x_2^B = p_1\, \omega_1^B + p_2\, \omega_2^B$$

or

$$x_2^B(p_1, p_2) = \frac{p_1\, \omega_1^B + p_2\, \omega_2^B}{2\, p_1 + p_2}. \tag{1}$$

The relationship between the optimal consumption of X_1 and X_2 by this agent, $x_1 = 2\,x_2$, can be used once again to get

$$x_1^B(p_1, p_2) = 2\, \frac{p_1\, \omega_1^B + p_2\, \omega_2^B}{2\, p_1 + p_2}. \tag{2}$$

When the prices are $p = (p_1, p_2) = (1, 1)$, a type B agent wants

$$\begin{aligned} x_1^B(1,1) &= 2\, \frac{1 \cdot 30 + 1 \cdot 30}{2 \cdot 1 + 1} \\ &= 40 \end{aligned}$$

units of X_1 and

$$\begin{aligned} x_2^B(1,1) &= x_1^B(1,1)/2 \\ &= 20 \end{aligned}$$

units of X_2. These amounts are the same as those shown on the right graph in figure 1 for the prices $p = (p_1, p_2) = (1, 1)$.

When the prices are $p = (p_1, p_2) = (1, 3)$, a type B agent wants

$$\begin{aligned} x_1^B(1,3) &= 2\,\frac{1 \cdot 30 + 3 \cdot 30}{2 \cdot 1 + 3} \\ &= 48 \end{aligned}$$

units of X_1 and

$$\begin{aligned} x_2^B(1,1) &= x_1^B(1,3)/2 \\ &= 24 \end{aligned}$$

units of X_2. These amounts are the same as those shown on the right graph in figure 1 for the prices $p = (p_1, p_2) = (1, 3)$.

(f) (1 point) Use the demand functions $x_1^B(p_1, p_2)$ and $x_2^B(p_1, p_2)$ for agent B that you found in part (e) to verify the budget identity

$$p_1\, x_1^B(p_1, p_2) + p_2\, x_2^B(p_1, p_2) \equiv p_1\, \omega_1^B + p_2\, \omega_2^B.$$

Solution The budget identity for a type B agent with the demand in equations (2) and (1) is

$$\begin{aligned} p_1\, x_1^B(p_1, p_2) + p_2\, x_2^B(p_1, p_2) &= p_1\, 2\,\frac{p_1\, \omega_1^B + p_2\, \omega_2^B}{2\, p_1 + p_2} + p_2\, \frac{p_1\, \omega_1^B + p_2\, \omega_2^B}{2\, p_1 + p_2} \\ &= (2\, p_1 + p_2)\, \frac{p_1\, \omega_1^B + \omega_2^B}{2\, p_1 + p_2} \\ &= p_1\, \omega_1^B + p_2\, \omega_2^B. \end{aligned}$$

2. Barter sequences in an exchange economy Suppose that there are three agents (A, B, and C) in an exchange economy that has three commodities X_1, X_2, and X_3. The left panel of table 1 shows endowments for agents A, B, and C. If the equilibrium prices for the three commodities are $p^* = (p_1^*, p_2^*, p_3^*) = (1, 2, 3)$, then the desired allocations for three agents are shown in the right panel of the table.

	X_1	X_2	X_3
A	14	20	22
B	14	20	22
C	14	20	22

	X_1	X_2	X_3
A	0	15	30
B	12	0	36
C	30	45	0

Table 1: Agents' endowments (left) and desired consumption (right)

(a) (2 points) Verify that the value of the endowments is equal to the value of the allocations.

Solution The budget equation for one of these agents is

$$p_1\, x_1^i + p_2\, x_2^i + p_3\, x_3^i = p_1\, \omega_1^i + p_2\, \omega_2^i + p_3\, \omega_3^i.$$

For all three agents, the value of the endowment is $p_1\, \omega_1^i + p_2\, \omega_2^i + p_3\, \omega_3^i = 1 \cdot 14 + 2 \cdot 20 + 3 \cdot 22 = 120$.
For agent type A the value of the allocation is $p_1\, x_1^A + p_2\, x_2^A + p_3\, x_3^A = 1 \cdot 0 + 2 \cdot 15 + 3 \cdot 30 = 120$.
For agent type B the value of the allocation is $p_1\, x_1^B + p_2\, x_2^B + p_3\, x_3^B = 1 \cdot 12 + 2 \cdot 0 + 3 \cdot 36 = 120$.
For agent type C the value of the allocation is $p_1\, x_1^C + p_2\, x_2^C + p_3\, x_3^C = 1 \cdot 30 + 2 \cdot 45 + 3 \cdot 0 = 120$.

(b) (2 points) Table 2 shows several trades in a sequence that leads from the endowments in the left panel of table 1 to the equilibrium allocations in the right panel of table 1. Fill in the table with two more trades to complete the sequence of barters. Describe each of the two trades that complete the sequence of barters in the space below table 2.

	X_1	X_2	X_3
A	2	20	26
B	14	20	22
C	26	20	18

	X_1	X_2	X_3
A	2	20	26
B	14	2	34
C	26	38	6

	X_1	X_2	X_3
A	0	21	26
B	16	1	34
C	26	38	6

	X_1	X_2	X_3
A	0	21	26
B	18	0	34
C	24	39	6

	X_1	X_2	X_3
A	0	21	26
B	12	0	36
C	30	39	4

	X_1	X_2	X_3
A	0	15	30
B	12	0	36
C	30	45	0

Table 2: Sequence of interim allocations with barter

Solution One possible fifth trade is between agents B and C. (This is the trade shown in the middle panel of the second row of interim allocations.) In this trade, agent C gives up 2 units of X_3 to agent B and receives 6 units of X_1 in return. The interim allocation of agent C was $(24, 39, 6)$ before this trade and it was $(30, 39, 4)$ after the trade; the interim allocation of agent B was $(18, 0, 34)$ before this trade and it was $(12, 0, 36)$ after the trade. Agent B is done trading at this point.

The last trade is between agents A and C. In this trade, agent C gives up 4 units of X_3 to agent A and receives 6 units of X_2 in return. The interim allocation of agent C was $(30, 39, 4)$ before this trade and the final allocation is $(30, 45, 0)$ after the trade; the interim allocation of agent A was $(0, 21, 26)$ before this trade and the final allocation is $(0, 15, 30)$ after the trade. Agents A and C are also done trading at this point.

3. Arbitrage with commodity money This problem involves a comparison between two arbitrage situations and also some interpretation of the results.

(a) (2 points) Suppose that the prices in the arbitrage example between the British and the French camps are 10 for tea and 9 for coffee in the British camp. In the French camp prices are 9 for tea and 10 for coffee. If a British officer has 81 units of commodity money, what arbitrage strategy could he follow to yield a profit of 19 units of the commodity money or more?

Solution A British officer could use his 81 units of commodity money to purchase 9 ounces of coffee in the British camp, sell the 9 ounces of coffee in the French camp for 10 each to obtain 90 units of the commodity money. He could use the 90 units to purchase 10 ounces of tea, and sell the tea for 100 units of the commodity money in the British camp. This arbitrage strategy yields a profit of 19 units of the commodity money.

(b) (2 points) Now suppose that everything is the same as in the previous problem, but the price of coffee in the French camp is now 8 instead of 10. (So the prices in the British camp are 10 for tea and 9 for coffee. In the French camp prices are 9 for tea and 8 for coffee.) If a British officer has 81 units of the commodity money, what arbitrage strategy could he follow to yield a profit of 9 units of the commodity money or more?

Solution In this case, there is no profitable trade that involves the purchase of a commodity in the British camp and its transfer to the French camp. The best strategy must involve transfer of the commodity money to the French camp. If the officer takes 81 units of the commodity money to the French camp, purchases 9 ounces of tea, and sells the tea for 10 units of commodity money per ounce of tea in the British camp, the yield will be 9 units of commodity money. (Alternatively, he could purchase 10 ounces of coffee for 8 each, sell them for 9 each, and earn a profit of 10.)

(c) (1 point) In part (a) the best arbitrage strategy involves commodities being moved from each camp to the other. In part (b) the best arbitrage strategy involves moving commodity money from one camp to the other. Why is this the case? What characteristic of the prices of the commodities drive the British officers to trade in commodities in part (a) but in currency in part (b)?

Solution In the second example, the price of each commodity in the French camp is lower than the price of that commodity in the British camp, so nothing can be purchased in the British camp and sold for a profit in the French camp. So the British camp exports money and imports commodities.

Situations of this sort are common. For many centuries, silver flowed first from the mines of Germany and Silesia through Italy and the middle east to China, and commodities flowed from China west. Later, after the massive flow of silver began from South America to Europe, a great deal of this silver found its way to China. It is likely that due to small amounts of money and a large amount of produced goods that all prices in China were below those in the west. This would lead to a pattern of trade in which money flows to China and goods flow west.

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4. Money supply, output, and inflation (1 point) In the exchange economy from the lecture notes that we used to derive a version of the "equation of exchange," the relationship between the equilibrium price and the endowments of commodity money and commodity is $p^* = \omega_1^B/\omega_2^S$. In this model we interpret ω_1^B as the money supply and we interpret ω_2^S as output.

Suppose that output at year 0 is $\omega_2^S = 15$ and the money supply is $\omega_1^B = 1500$ so that the price level is $p^* = 1500/15 = 100$.

If in year 1 output ω_2^S increases to $\omega_2^S = 18$ and the money supply ω_1^B increases to $\omega_1^B = 1926$, find the new price level. What is the rate of of inflation between years 0 and 1?

Solution The price level in year 1 is $p^* = \omega_1^B/\omega_2^S = 1926/18 = 107$. The rate of inflation is 7%.

Short answer questions

5. Functions of money (1 point) List the three functions of money.

Solution The three functions of money are

1. a medium of exchange,
2. a store of value, and
3. a unit of account.

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6. Characteristics of circulating money (1 point) List three characteristics that a commodity will typically have if it is to overcome trade frictions and serve effectively as money.

Solution Four properties of circulating money were listed in the notes, and a fifth was discussed in lecture. Any three of the properties can be listed for this question. Money should

1. have low storage cost,
2. have low depreciation,
3. be easy to transport,
4. be easily authenticated, and
5. be readily divisible.