Economics 380
Homework #3
Solutions

1. Banks of transfer and interbank clearance

This problem proceeds through the steps in the example of banks of transfer and interbank clearance from the lecture notes. The tables used in the notes to show endowments, allocations, trades, and merchants' balances at the banks are all provided in the problem. You need to fill in all blank cells of the tables. Table 1 shows an example of market prices, merchants' endowments, and merchants' allocations after trading is finished.

	Commodities				
	Dyes	Linen	Spices	Wool	
p	8	7	8	9	Value
E1	—	—	—	10	90
F2	—	10	—	—	70
I1	10	—	—	—	80
I2	—	—	10	—	80

	Commodities				
	Dyes	Linen	Spices	Wool	
p	8	7	8	9	Cost
E1	5	—	5	—	80
F2	5	—	5	—	80
I1	—	10	—	—	70
I2	—	—	—	10	90

Table 1: Commodity endowments and their values (left); commodity allocations and their costs (right)

(a) (2 points) Using the prices in the row labeled p, fill in the value of each merchant's endowment on the left side of table 1 and fill in the cost of each merchant's allocation on the right side of table 1.

Buy.	Sell.	Comm.	Units	Price	Cost
E1	I1	Dyes	5	8	£40
E1	I2	Spices	5	8	£40
I1	F2	Linen	10	7	£70

Buy.	Sell.	Comm.	Units	Price	Cost
F2	I1	Dyes	5	8	£40
F2	I2	Spices	5	8	£40
I2	E1	Wool	10	9	£90

Table 2: Transactions cleared at bank B1 (left) and at bank B2 (right)

(b) (5 points) Table 2 shows the buyer for each of six trades that lead from the endowments to the allocations in table 1. Complete the table by filling in the cells that are blank.

Merch.	Credit	Debit	Bal. (A)	Bal. (B)
E1	£0	£80	−£80	£10
I1	£40	£70	−£30	£10

Merch.	Credit	Debit	Bal. (A)	Bal. (B)
F2	£0	£80	−£80	−£10
I2	£40	£90	−£50	−£10

Table 3: Merchants' accounts at bank B1 (left) and at bank B2 (right)

(c) (5 points) Table 3 shows a ledger for the account of each merchant. Assume that merchants E1 and I1 have accounts at bank B1; assume that merchants F2 and I2 have accounts at bank B2. Show all the debits for each merchant and show the credits that result from trades with merchants who have an account at the same bank. Show the balance for these trades in the column labeled "Bal. (A)."

(d) (5 points) There are some trades that clear across banks. After these trades are cleared across banks, some merchants will have an additional credit. In the column labeled "Bal. (B)," show the balance of each merchant after these interbank trades are credited appropriately.

Solution

Merchants E1 and I1 each have a credit due to them from bank B2. The credit due to E1 is for £90 (which results from his sale to I2). The credit due to I1 is for £40.

Merchants F2 and I2 each have a credit due to them from bank B1. The credit due to F2 is for £70 (which results from his sale to I1). The credit due to I2 is for £40.

(e) (5 points) What is the amount of money that needs to be transferred between the two banks? The transfer payment that one of the banks makes is a debit of the bank. Show that it has an equal offsetting credit. The transfer payment received by the other bank is a credit of the bank. Show that it has an equal offsetting debit.

Solution

Bank B1 transfers £40 to bank B2 on behalf of merchant I2, and also transfers £70 to bank B2 on behalf of merchant F2. Bank B2 transfers £130 to bank B1 on behalf of merchants I1 (£40) and E1 (£90). The net balance owed by bank B2 to B1 is £20.

The debit of £20 that bank B2 has to bank B1 if offset by two credits. These are the £10 owed to bank B2 by merchant F2 and the £10 owed to bank B2 by merchant I2. These two credits balance the £20 debit that B2 has to B1.

The credit that bank B1 has from bank B2 is offset by the two debits that the bank has for £10 each. Bank B1 owes merchant E1 £10 and also owes merchant I1 £10.

2. Problem 5.1 from the lecture notes (4 points) The demand for agent j is given in equation (5.7). Use that equation, evaluated at the market equilibrium interest rate for this example, to find the demand for loans or supply of loans by each agent. (If the demand for loans is negative for an agent, that means that that agent will supply loans at the market equilibrium interest rate.) How much will each agent consume in the first period? How much will each consume in the second period? Which agent consumes more in total across the two periods? What is the cause of the difference in total consumption?

Solution

The demand for loans by agent 1 is

$$z_1^1(i) = \frac{1}{2}\left(\frac{100}{1+i} - 100\right).$$

At the equilibrium interest rate $i^* = 0.4$, her excess demand is

$$z_1^1(0.4) = \frac{1}{2}\left(\frac{100}{1.4} - 100\right) \doteq -14.29.$$

Since her demand for loans is negative, that means that she will lend this amount at the equilibrium interest rate. Her consumption level in period 1 is $100 - 14.29 = 85.71$.

The demand for loans by agent 2 is

$$z_1^2(i) = \frac{2}{3}\left(\frac{100}{1+i} - 50\right).$$

At the equilibrium interest rate $i^* = 0.4$, his excess demand is

$$z_1^2(0.4) = \frac{2}{3}\left(\frac{100}{1.4} - 50\right) \doteq 14.29.$$

Since his demand for loans is positive, that means that he will borrow this amount at the equilibrium interest rate. His consumption level in period 1 is $100 + 14.29 = 114.29$.

The loan size is 14.29, and the interest rate is 40%, so the amount repaid on the loan is the principal (14.29) plus the interest rate times the principal ($14.29 \cdot 0.4$). This is $14.29 + 0.4 \cdot 14.29 = 20$. The lender receives this amount from the borrower, so consumption levels in period 2 are adjusted by the amount of this payment. Both have the endowment 100 in period 2, so agent 1 consumes 120 and agent 2 consumes 80.

Total consumption by agent 1 is $85.71 + 120 = 205.71$. Total consumption by agent 2 is $114.29 + 80 = 194.29$. The interest on the loan is $0.4 \cdot 14.29 \doteq 5.71$. This is what agent 1 earned by lending, and what agent 2 paid for borrowing.

3. Market interest rates Suppose that each of two agents in an exchange economy have the demand function

$$z_1^j(i) = \frac{1}{\beta_j + 1}\left(\frac{\omega_2}{1+i} - \beta_j\,\omega_1\right) \tag{1}$$

for loans. Assume that agent 1 has the discount factor $\beta_1 = 10/11$ and agent 2 has the discount factor $\beta_2 = 10/12$.

(a) (1 point) Suppose that agent 1 has the endowment $(\omega_1, \omega_2) = (300, 0)$ and agent 2 has the endowment $(\omega_1, \omega_2) = (0, 300)$. Write down the demand for each agent. (To do this, use equation (1) to find the demand function $z_1^1(i)$ for agent 1 and the demand function $z_1^2(i)$ for agent 2. Write down the market excess demand for loans.

Solution

The excess demand for X_1 by agent 1 is

$$\begin{aligned} z_1^1(i) &= \frac{1}{\beta_1 + 1}\left(\frac{\omega_2}{1+i} - \beta_1\,\omega_1\right) \\ &= \frac{1}{10/11 + 1}\left(0 - 10/11 \cdot 300\right) \\ &= -\frac{11}{21}\cdot\frac{10}{11}\cdot 300. \\ &= -\frac{10}{21}\cdot 300. \end{aligned}$$

The excess demand for X_1 by agent 2 is

$$\begin{aligned} z_1^2(i) &= \frac{1}{\beta_2 + 1}\left(\frac{\omega_2}{1+i} - \beta_2\,\omega_1\right) \\ &= \frac{1}{10/12 + 1}\left(\frac{300}{1+i} - 0\right) \\ &= \frac{12}{22}\cdot\frac{300}{1+i}. \end{aligned}$$

Market excess demand is

$$\begin{aligned} z_1(i) &= z_1^1(i) + z_1^2(i) \\ &= -\frac{10}{21}\cdot 300 + \frac{6}{11}\cdot\frac{300}{1+i}. \end{aligned}$$

(b) (1 point) Equate the market demand to the market supply to determine the market equilibrium interest rate i^*.

Solution

Since excess demand of each agent is demand minus endowment, market excess demand is equal to demand minus supply, therefore demand equals supply when excess demand is zero. So

$$\frac{6}{11}\cdot\frac{300}{1+i} = \frac{10}{21}\cdot 300.$$

After canceling terms that appear on both sides of this equation and simplifying fractions, the equation is equivalent to

$$\frac{6}{11} \cdot \frac{1}{1+i} = \frac{10}{21}.$$

When this is solved for i, the result is $i^* = 8/55$, which is approximately 14.5%.

4. Yield of a treasury bill (1 point) If the price of a simple bond that pays \$100 after a one year maturity is \$96, what is the yield (or the interest rate i) on the bond? (Think of this as a bond with a single coupon payment $C = \$100$ in one year.)

Solution

The yield on the bill is determined from the equation $p\,(1+i) = \$100$. Since $p = \$96$, this can be written as $1 + i = \$100/\$96$. So $i = 1/24 \doteq 0.0417$ or 4.17%.

5. Problem 5.2 in the lecture notes (2 points) What was the range for the interest rate on the Monte Vecchio between 1320 and 1350 when prices varied between 80 and 102?

Solution

The price of a perpetual bond that pays an annual coupon of C at interest rate i is

$$PV = C/i.$$

When the coupon payment on the Monte Vecchio was 5, and its price was 102, the interest rate is the solution to the equation $102 = 5/i$, so $i \doteq 4.90\%$.

When the coupon payment on the Monte Vecchio was 5, and its price was 80, the interest rate is the solution to the equation $80 = 5/i$, so $i = 6.25\%$.

6. Finite term bonds and perpetual bonds Suppose that a bond makes two coupon payments. Each payment is C. The first payment is made one year after the issue date of the bond and the second payment is made two years after the issue date. Use the convention that the bond is issued at year 0, the first coupon payment is made after one year (which we call year 1), and the second payment is made after two years (which we call year 2).

(a) (1 point) Find the present value (at year 0) of the bond. Your present value formula should be a function of the interest rate i and the coupon payment C. (Hint: Find the present value of the first coupon and the second coupon separately and add them together to get $PV = PV_1 + PV_2$.)

Solution

The present value at year 0 of the first bond payment, which is made after 1 year, is

$$PV_1 = \frac{C}{1+i}.$$

The present value at year 0 of the second bond payment, which is made after 2 years, is

$$PV_2 = \frac{C}{(1+i)^2}.$$

The present value of the bond is therefore

$$\begin{aligned} PV &= PV_1 + PV_2 \\ &= \frac{C}{1+i} + \frac{C}{(1+i)^2}. \end{aligned}$$

(b) (1 point) In the lecture and the lecture notes we found that the present value (also at year 0) of a perpetual bond with payments C every year forever is $PV = C/i$ where i is the interest rate or yield on the bond. Imagine that the perpetual bond B is separated into two parts. One part B_b includes the first two coupon payments; the other part B_e includes all of the coupon payments after the first two coupon payments. (The subscripts are b for 'beginning' and e for 'end'.) The present value of this perpetual bond at year 0 is $PV = PV_b + PV_e$. We also know that at year 2 the present value of B_e is C/i. What is the present value PV_e of B_e at year 0?

Solution

At year 2 the bond B_e has value C/i. The present value in year 0 of an asset that is worth C/i in two years is

$$\frac{C/i}{(1+i)^2}.$$

(c) (1 point) The bond in part (a) is equivalent to acquiring a perpetual bond today and selling it two years from today. In other words, the bond in part (a) is equivalent to the bond B_b in part (b). Find the present value PV_b of B_b as the difference $PV_b = PV - PV_e$.

Solution

The present value of the bond B_b in year 0 is

$$\begin{aligned} PV_b &= PV - PV_e \\ &= C/i - \frac{C/i}{(1+i)^2}. \\ &= \frac{C}{i}\left(1 - \frac{1}{(1+i)^2}\right). \end{aligned}$$

(d) (1 point) Evaluate your formula from part (a) when the coupon payment is $C = 100$ and the interest rate is 25%. (This interest rate is unnaturally high, but it makes the equation work out simply.)

Solution

The present value of the bond as determined in part (a) is

$$\begin{aligned} PV &= PV_1 + PV_2 \\ &= \frac{C}{1+i} + \frac{C}{(1+i)^2}. \end{aligned}$$

When this is evaluated at the interest rate $i = 25\%$ and the coupon payments $C = 100$, the present value is

$$\begin{aligned} PV &= \frac{100}{1+0.25} + \frac{100}{(1+0.25)^2} \\ &= \frac{100}{1.25} + \frac{100}{(1.25)^2} \\ &= 100\,\frac{4}{5} + 100\,\frac{16}{25} \\ &= 80 + 64 \\ &= 144 \end{aligned}$$

(e) (1 point) Evaluate your formula from part (c) when the coupon payment is $C = 100$ and the interest rate is 25%.

Solution

The present value of the bond as determined in part (c) is

$$\begin{aligned} PV_b &= PV - PV_e \\ &= \frac{C}{i}\left(1 - \frac{1}{(1+i)^2}\right). \end{aligned}$$

When this is evaluated at the interest rate $i = 25\%$ and the coupon payments $C = 100$, the present value is

$$\begin{aligned} PV_b &= \frac{100}{0.25}\left(1 - \frac{1}{(1+0.25)^2}\right) \\ &= 400\left(1 - \frac{1}{(1.25)^2}\right) \\ &= 400\left(1 - \frac{16}{25}\right) \\ &= 400\,\frac{9}{25} \\ &= 9 \cdot 16 \\ &= 144. \end{aligned}$$

7. Problem 6.1 in the lecture notes (1 point) If Bill B is payable after three months, what is the interest rate earned by Marsden on the bill? (Recall that there were 20 shillings per £.)

Solution

Bill B is issued on the basis of the loan in Bill A. Bill A earns 4 s. interest, which is £0.2. Since the loan amount is £10, this is a return of 2% in three months or about 8% per year. (It's actually a return of $(1 + 0.02)^4 - 1 = 0.0824322$ or almost 8.25% on an annual basis, with quarterly compounding.)