

Economics 380

Homework #4

Solutions

Problem 8.1 (5 points) In an area further east, such as Ohio, a new railroad would serve a larger initial population with a smaller growth rate. Suppose that the fixed cost for this other line is $F = \$1$ million, the interest rate is $i = 0.08$ (8%), the initial population of the area served is 20,000, and the profit per person is \$5 per year. Find the present value of the Ohio railroad as a function of the growth rate. Graph the function for growth rates between 0 and 0.07, as in figure 1.

Solution The present value of the railroad with the higher initial population is

$$\begin{aligned} PV &= 20,000 \cdot \$5 \frac{1.08}{0.08 - g} - \$1,000,000 \\ &= \$100,000 \left(\frac{1.08}{0.08 - g} - 10 \right). \end{aligned} \quad (1)$$

The graph of this function is shown on the right side of figure 1.

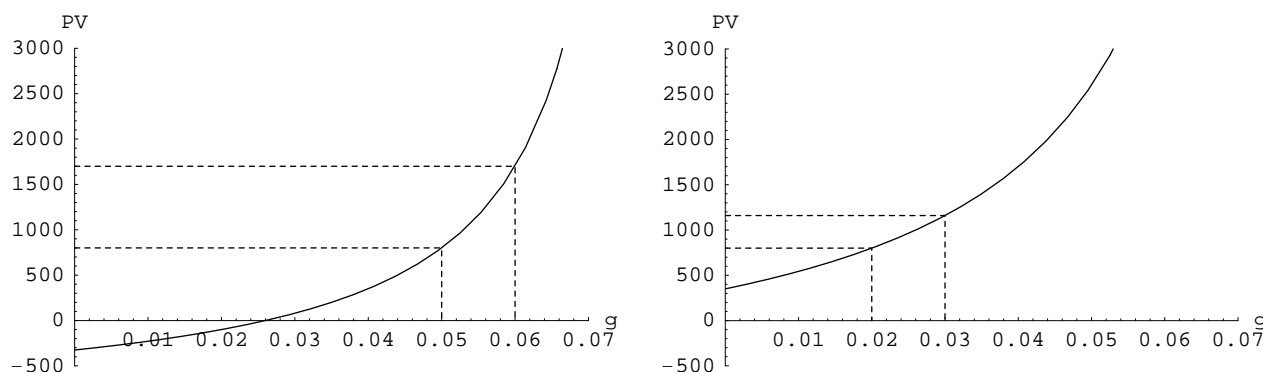


Figure 1: Present values of each railroad (in \$1000) as a function of the growth rate g .

Problem 8.2

- (a) (1 point) Compare the present value of the first example in equation (2) if the growth rate is 6% to the present value of the railroad in problem 8.1 if the growth rate is 3%.

Solution The present value of the western railroad (where the growth is higher) is

$$\begin{aligned} PV &= 10,000 \cdot \$5 \frac{1.08}{0.08 - g} - \$1,000,000 \\ &= \$50,000 \left(\frac{1.08}{0.08 - g} - 20 \right). \end{aligned} \quad (2)$$

When the growth rate is 6% the present value of this railroad is

$$\begin{aligned} PV &= \$50,000 \left(\frac{1.08}{0.08 - 0.06} - 20 \right) \\ &= \$50,000 (54 - 20) \\ &= \$1,700,000. \end{aligned}$$

The present value of the eastern railroad (where the growth rate is lower) is

$$\begin{aligned} PV &= \$100,000 \left(\frac{1.08}{0.08 - 0.03} - 10 \right) \\ &= \$100,000 \cdot 11.6 \\ &= \$1,160,000. \end{aligned}$$

These are shown with dashed lines in the two graphs in figure 1.

- (b) (1 point) How do the present values of the two investments change if growth rates are 1% less in each region than the growth rate that was originally anticipated (i.e., the growth rate is 2% rather than 3% in Ohio and it is 5% rather than 6% in the west)?

Solution If the growth rates are 2% in Ohio and 5% in the west, then calculations similar to those in part (a) result in the present values $PV = \$800,000$ for both railroads.

When the growth rate increases by 1% on both railroads, the present value of the western line (with the higher base growth rate) is more than 100% (from \$800,000 to \$1,700,000), whereas the present value of the eastern line increases by less than 50% (from \$800,000 to \$1,160,000).

- (c) (1 point) What happens to the present value of each investment if growth rates are only 2/3 as big as what was originally anticipated in each region (i.e., the growth rate is 2% rather than 3% in Ohio and it is 4% rather than 6% in the west)?

Solution If the growth rates are 2% in Ohio and 4% in the west, then calculations similar to those in part (a) result in the present values $PV = \$800,000$ for the Ohio railroad and $PV = \$350,000$ for the railroad in the west.

Problem 8.3 Suppose that there are eleven banks in a clearinghouse and one of the banks, Bank A, experiences a bank run. Suppose that Bank A has \$1 million in reserves, and that the amount of specie drawn from Bank A during the run is \$200,000 per day. In the normal course of a business day, each bank receives \$20,000 in checks from each of the other banks. If Bank A receives \$1 million

in clearinghouse loan certificates, those can be used to cover the checks that are drawn on Bank A and presented to Bank A by Bank B, by Bank C, and so forth. Bank B though will send \$20,000 per day to Bank A, because Bank A receives \$20,000 in checks drawn on Bank B each day. Bank A also receives \$20,000 per day from Bank C, Bank D, and so forth.

- (a) (1 point) How long would it be before Bank A is out of reserves without the clearinghouse loan certificate?

Solution Without loan certificates, Bank A will pay its depositors \$200,000 per day. Since the payments for check clearing to and from other banks net out to zero, its reserves will be depleted by \$200,000 per day, so its reserves will be gone in five days.

- (b) (1 point) How many days will it take before the \$1 million credit issued to Bank A is used up? What will the reserves of Bank A be at that point?

Solution Since each of the other ten banks in the clearinghouse present to Bank A for payment \$20,000 of checks drawn on Bank A each day, and since Bank A uses loan certificates to settle those checks, Bank A will use \$200,000 of loan certificates per day. The \$1 million of loan certificates will be gone in five days.

The checks that Bank A presents to each of the other ten banks will be paid with \$20,000 of currency, so the total amount of money that is sent from the other ten bank each day is \$200,000. This balances the amount that is distributed to the depositors, so Bank A is able to preserve its reserves during the five days from the time that it receives the loan certificate until the time that the certificates are used up.

- (c) (2 points) How has the money supply increased as a result of the loan certificates?

Solution A loan certificate authorized by the clearinghouse is an asset of the member bank that receives the certificate from Bank A and it is a liability of Bank A. This is very similar to a bank note issued by Bank A. A bank note issued by Bank A is an asset for the holder of the note and it is a liability of Bank A. Loan certificates circulated just like bank notes, except that loan certificates only circulated among the clearinghouse banks. For example, the loan certificates in figure 2 indicate that they “will be received in payment of balances at the Clearing House for the sum of Ten [Twenty] Thousand Dollars from any member of the Clearing House Association.”

No. 3437 **\$10,000.**

Loan Committee of the New York Clearing House Association.

New York, _____ 1873.

This Certifies that the _____

has deposited with this Committee securities in accordance with the proceedings of a Meeting of the Association, held September 20th, 1873, upon which this Certificate is issued. This Certificate will be received in payment of balances at the Clearing House for the sum of **TEN THOUSAND DOLLARS**, from any Member of the Clearing House Association, until November 1st, 1873.

On the surrender of this Certificate by the depositing Bank above named, the Committee will endorse the amount as a payment on the obligation of said Bank, held by them, and surrender a proportionate share of the collateral securities held therefor. This Certificate may also be redeemed prior to the 1st of November, if desired.

\$10,000.

_____ } Committee.

TEN THOUSAND DOLLARS.

National Monetary Commission

FORM OF CLEARING-HOUSE LOAN CERTIFICATE USED BY NEW YORK CLEARING-HOUSE IN 1873. LATER ISSUES DIFFER FROM THIS ONLY IN THE OMISSION OF THE RESTRICTIVE DATE.

No. _____ **\$20,000**

LOAN COMMITTEE OF THE SAN FRANCISCO CLEARING HOUSE ASSOCIATION

SAN FRANCISCO, _____ 1907

This Certifies that the _____

has deposited with this Committee securities in accordance with the proceedings of a Meeting of the Association held October 28, 1907, upon which this Certificate is issued. This Certificate will be received in payment of balances at the Clearing House for the sum of **Twenty Thousand Dollars** from any member of the Clearing House Association.

On the surrender of this Certificate by the depositing bank above named, the Committee will endorse the amount as a payment on the obligation of said bank held by them, and surrender a proportionate share of the collateral securities held therefor.

\$20,000

_____ } Committee.

Figure 2: Clearinghouse loan certificates

Problem 8.4 According to Calomiris and Schweikart [1991, p. 830], no Alabama bank failed during the banking crisis in 1857, yet Calomiris and Schweikart [1991, p. 818] also indicate that the modal discount on Alabama bank notes in New York City on October 7, 1857 was 10%. If the probability of failure of every bank in Alabama was $Pr[F] = 1$ and if every bank in Alabama had assets equal to

90% of its liabilities, then the expected loss (using the equation from Section 9.4 of the lecture notes) would be

$$\begin{aligned} E[\text{Loss}] &= Pr[F](1 - A/L) \\ &= 1(1 - 0.9) \\ &= 0.1 \end{aligned}$$

or 10%. This suggests a significant overestimation of the probability of failure and the loss per bank at Alabama banks.

Many different combinations of the probability of failure $Pr[F]$ and the ratio of assets to liabilities A/L are consistent with the 10% discount for Alabama bonds. The five questions below are related to this problem.

- (a) (1 point) Suppose that the New York market believed that the probability of failure at the Alabama banks was $Pr[F] = 0.5$. What asset to liability ratio is consistent with the note discount 10% in this case?

Solution The equation $Pr[F](1 - A/L) = 0.1$ with $Pr[F] = 0.5$ is $1 - A/L = 0.2$ so $A/L = 0.8$. Assets would be 80% of liabilities.

- (b) (1 point) Suppose instead that the New York market believed that the probability of failure at the Alabama banks was $Pr[F] = 0.2$. What asset to liability ratio is consistent with the note discount 10% in this case?

Solution The equation $Pr[F](1 - A/L) = 0.1$ with $Pr[F] = 0.2$ is $1 - A/L = 0.5$ so $A/L = 0.5$. Assets would be 50% of liabilities.

- (c) (2 points) Solve the equation $Pr[F](1 - A/L) = 0.1$ for the asset to liability ratio A/L to get a formula that determines all combinations of the asset to liability ratio and the probability of failure that result in a discount of 10%.

Solution The equation $Pr[F](1 - A/L) = 0.1$ is equivalent to $1 - A/L = 0.1/Pr[F]$. Solved for A/L , this is $A/L = 1 - 0.1/Pr[F]$.

- (d) (2 points) Make a graph with the probability of failure on the horizontal axis and the asset to liability ratio (A/L) on the vertical axis. Show all combinations of the asset to liability ratio and

the probability of failure that are consistent with the discount 10%. Do any of these combinations seem like plausible estimates, in light of the fact that no Alabama bank failed?

Solution The graph below shows the combinations of the probability of bank failure $Pr[F]$ and the asset to liability ratio A/L that are consistent with the bank note discount 0.1.

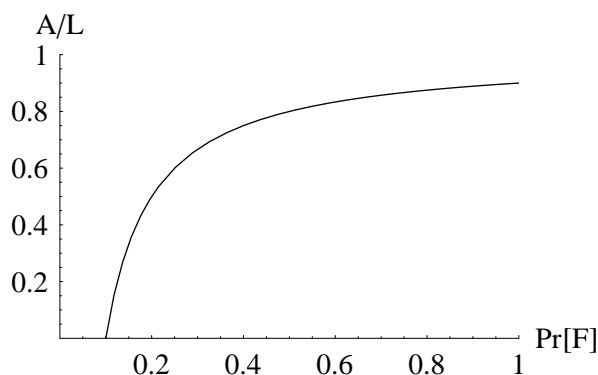


Figure 3: This graph shows all combinations of bank asset to liability ratios and probabilities of bank failure that result in a note discount of 10%.

All of the combinations of $Pr[F]$ and A/L that lead to a discount of 0.1 have either a very high probability of bank failure or a very low asset to liability ratio. Neither of these are consistent with the observation that no bank failed in Alabama during the panic of 1857.

- (e) (2 points) Is there a more plausible explanation than expected loss on the notes for the large discount on Alabama bank notes during the panic?

Solution The discounts on Alabama bills were much larger than what seems reasonable in light of the fact that no Alabama bank failed during the crisis. Many asset prices were falling during the financial crisis, primarily due to the illiquidity of many firms, banks, and individuals. We've seen that even U.S. Treasury bonds, payable in 1861, declined 10%. This is almost surely not the result of a much increased chance of default on these bonds by the U.S. Treasury: rather it is because the potential buyers of these bonds became illiquid.

Deposits and reserves were both flowing out of the banks quickly between late August and the middle of October. Banks were so illiquid that they were forced to sell many of their own liquid assets. Most banks were not in a position to acquire new assets in the form of notes issued even by solvent banks if those banks were far away, because those notes would take weeks to convert back into reserves.

Bank loans also dropped dramatically during this period. Their customers also faced significant liquidity problems as a result of loan curtailments. Non-bank firms were also much less able to acquire new assets due to their liquidity constraints. This general liquidity crisis rendered many firms unable to purchase notes of distant banks, such as those in Alabama or Ohio, even if the notes were excellent values.