Economics 380
Exam #1 solutions
Sunday September 16, 2007

1. An exchange economy This problem consists of six parts. In the first part, you'll find a graphical solution to the demand for agent type A at two different price vectors $p = (1, 1)$ and $p = (1, 2)$. In the second part, you'll repeat that exercise for agent type B. In the third part, you'll describe why one of the price vectors is an equilibrium price vector and why the other is not. In the fourth part, you'll find the demand functions for agent type A and compare the quantities demanded at the two price vectors $p = (1, 1)$ and $p = (1, 2)$ to the graphical demands that you found in the first part. The fifth part repeats that problem for agent type B. The sixth part applies the budget identity to the demand functions that you obtained for agent type B.

(a) (10 points) Agent type A has the utility function $u^A(x_1, x_2) = \min\{2x_1, x_2\}$ and the endowment $\omega^A = (60, 0)$. Show the endowment point on the diagram below. Draw the budget line that corresponds to the price vector $p = (p_1, p_2) = (1, 1)$. (Recall that the budget line passes through the endowment, and use the price of X_2 relative to the price of X_1 to figure out what combinations of X_1 and X_2 are available to the agent at these prices.) Draw the indifference curve $u^A(x_1, x_2) = 40$. What are the optimal consumption levels of X_1 and X_2 for this agent when the prices are $p = (p_1, p_2) = (1, 1)$?

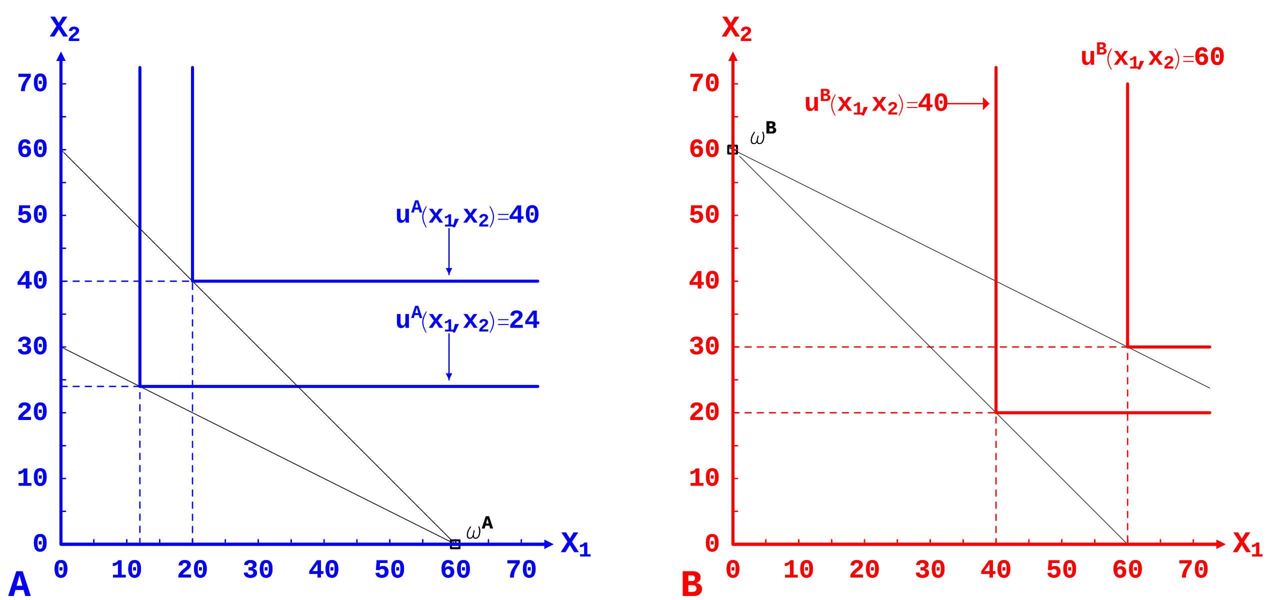

Figure 1: Graphs for agent types A (left) and B (right).

Solution The graph on the left side of figure 1 shows the endowment of a type A agent as a square at $\omega^A = (60, 0)$. The budget set that corresponds to the price vector $p = (p_1, p_2) = (1, 1)$ passes through the endowment and has slope -1, since one unit of X_2 can be acquired if one unit of X_1 is given up. The indifference curve $u^A(x_1, x_2) = 40$ has a corner at $(x_1, x_2) = (20, 40)$. (This is easy to determine. If $u^A(x_1, x_2) = 40$ then there are two possible cases: (1) $2x_1 = 40$ and $x_2 \geq 40$, or (2) $2x_1 \geq 40$ and $x_2 = 40$. These can be rewritten as: $(1)'$ $x_1 = 20$ and $x_2 \geq 40$, or $(2)'$ $x_1 \geq 20$ and $x_2 = 40$. The indifference curve corresponds to the conjunction of these two inequalities.) A type A agent would choose to consume at the corner of this indifference curve where $(x_1, x_2) = (20, 40)$ when the prices are $(p_1, p_2) = (1, 1)$ because no other point has as high a utility level. (This too is easy to show. Suppose that the agent instead decides to consume $(x_1(a), x_2(a)) = (20 + a, 40 - a)$ where a is some number that can be either positive or negative. For every $a \in [-20, 40]$, $(x_1(a), x_2(a))$ lies on the budget set. The utility level at $(x_1(a), x_2(a)) = (20 + a, 40 - a)$ is

$$\begin{aligned} u^A(x_1(a), x_2(a)) &= \min\{2\,x_1(a), x_2(a))\} \\ &= \min\{2\,(20 + a), 40 - a\} \\ &= \min\{40 + 2\,a, 40 - a\}. \end{aligned}$$

If $a > 0$, then the second term is less than 40; if $a < 0$, then the first term is less than 40. If $a = 0$, then both terms are 40, so the utility is maximized at $(x_1, x_2) = (20, 40)$, which is the corner of the indifference curve.)

On the same diagram (on page 1), draw the budget line that corresponds to the price vector $p = (p_1, p_2) = (1, 2)$. Draw the indifference curve $u^A(x_1, x_2) = 24$. What are the optimal consumption levels of X_1 and X_2 for this agent when the prices are $p = (p_1, p_2) = (1, 2)$?

Solution The budget set that corresponds to the price vector $p = (p_1, p_2) = (1, 2)$ passes through the endowment and has slope $-1/2$, since one unit of X_2 can be acquired if two units of X_1 are given up. The indifference curve $u^A(x_1, x_2) = 24$ has a corner at $(x_1, x_2) = (12, 24)$. A type A agent would choose to consume at the corner of this indifference curve when the prices are $(p_1, p_2) = (1, 2)$.

(b) (10 points) Agent type B has the utility function $u^B(x_1, x_2) = \min\{x_1, 2\,x_2\}$ and the endowment $\omega^B = (0, 60)$. Show the endowment point on the diagram below. Draw the budget line that corresponds to the price vector $p = (p_1, p_2) = (1, 1)$. Draw the indifference curve $u^B(x_1, x_2) = 40$. What are the optimal consumption levels of X_1 and X_2 for this agent when the prices are $p = (p_1, p_2) = (1, 1)$?

Solution The budget set that corresponds to the price vector $p = (p_1, p_2) = (1, 1)$ passes through the endowment and has slope -1, since one unit of X_2 can be acquired if one unit of X_1 is given up. The indifference curve $u^B(x_1, x_2) = 40$ has a corner at $(x_1, x_2) = (40, 20)$. A type B agent would choose to consume at the corner of this indifference curve when the prices are $(p_1, p_2) = (1, 1)$.

On the same diagram, draw the budget line that corresponds to the price vector $p = (p_1, p_2) = (1, 2)$. Draw the indifference curve $u^B(x_1, x_2) = 60$. What are the optimal consumption levels of X_1 and X_2 for this agent when the prices are $p = (p_1, p_2) = (1, 2)$?

Solution The budget set that corresponds to the price vector $p = (p_1, p_2) = (1, 2)$ passes through the endowment and has slope $-1/2$, since one unit of X_2 can be acquired if three units of X_1 are given up. The indifference curve $u^B(x_1, x_2) = 60$ has a corner at $(x_1, x_2) = (60, 30)$. A type B agent would choose to consume at the corner of this indifference curve when the prices are $(p_1, p_2) = (1, 2)$.

(c) (5 points) How much of commodity X_2 is wanted when the prices are $p = (p_1, p_2) = (1, 2)$? How much is available? Is $p = (p_1, p_2) = (1, 2)$ an equilibrium price vector?

Solution Agent type A wants 24 units of X_2 when the prices are $p = (p_1, p_2) = (1, 2)$. Agent type B wants 30 units at these prices. The total wanted is 54 units, but the amount of X_2 available is 60 units, since the endowment of commodity X_2 is the 60 units of X_2 that agent B starts out with. The market is not in equilibrium at the prices $p = (p_1, p_2) = (1, 2)$: the price of X_2 needs to fall so that the demand for X_2 will increase from 54 to 60.

How much of commodity X_2 is wanted when the prices are $p = (p_1, p_2) = (1, 1)$? How much is available? Is $p = (p_1, p_2) = (1, 1)$ an equilibrium price vector?

Solution Agent type A wants 40 units of X_2 when the prices are $p = (p_1, p_2) = (1, 1)$. Agent type B wants 20 units at these prices. The total wanted is 60 units, which is equal to the amount of X_2 that is available. The market is in equilibrium at the prices $p = (p_1, p_2) = (1, 1)$.

(d) (5 points) Use the utility function $u^A(x_1, x_2) = \min\{2x_1, x_2\}$ and the budget equation for agent A to find this agent's demand function for commodity X_2. (Remember that an important step is to find the relationship between the consumption levels x_1 and x_2.) After you have the demand for X_2, write down the demand for X_1. How much of X_1 and X_2 does this agent want at the prices $p = (p_1, p_2) = (1, 1)$? How much of X_1 and X_2 does this agent want at the prices $p = (p_1, p_2) = (1, 2)$?

Solution The budget equation for agent type A is

$$p_1 x_1^A + p_2 x_2^A = p_1 \omega_1^A + p_2 \omega_2^A.$$

An agent with the utility function $u^A(x_1, x_2) = \min\{2x_1, x_2\}$ chooses optimally when $2x_1 = x_2$. This relationship can be substituted into the budget equation and solved for x_2 to get the demand function for X_2 by a type A agent:

$$p_1 x_2^A/2 + p_2 x_2^A = p_1 \omega_1^A + p_2 \omega_2^A$$

or

$$x_2^A(p_1, p_2) = 2\,\frac{p_1 \omega_1^A + p_2 \omega_2^A}{p_1 + 2p_2}.$$

The relationship between the optimal consumption of X_1 and X_2 by this agent, $2x_1 = x_2$, can be used once again to get

$$x_1^A(p_1, p_2) = \frac{p_1 \omega_1^A + p_2 \omega_2^A}{p_1 + 2p_2}.$$

When the prices are $p = (p_1, p_2) = (1, 1)$, a type A agent wants

$$\begin{aligned} x_1^A(1,1) &= \frac{1 \cdot 60 + 1 \cdot 0}{1 + 2 \cdot 1} \\ &= 20 \end{aligned}$$

units of X_1 and

$$\begin{aligned} x_2^A(1,1) &= 2\,x_1^A(1,1) \\ &= 40 \end{aligned}$$

units of X_2. These amounts are the same as those shown on the left graph in figure 1 for the prices $p = (p_1, p_2) = (1, 1)$.

When the prices are $p = (p_1, p_2) = (1, 2)$, a type A agent wants

$$\begin{aligned} x_1^A(1,2) &= \frac{1 \cdot 60 + 2 \cdot 0}{1 + 2 \cdot 2} \\ &= 12 \end{aligned}$$

units of X_1 and

$$\begin{aligned} x_2^A(1,2) &= 2\, x_1^A(1,2) \\ &= 24 \end{aligned}$$

units of X_2. These amounts are the same as those shown on the left graph in figure 1 for the prices $p = (p_1, p_2) = (1, 2)$.

(e) (5 points) Use the utility function $u^B(x_1, x_2) = \min\{x_1, 2\,x_2\}$ and the budget equation for agent B to find this agent's demand function for commodity X_2. After you have the demand for X_2, write down the demand for X_1. How much of X_1 and X_2 does this agent want at the prices $p = (p_1, p_2) = (1, 1)$? How much of X_1 and X_2 does this agent want at the prices $p = (p_1, p_2) = (1, 2)$?

Solution The budget equation for agent type B is

$$p_1\, x_1^B + p_2\, x_2^B = p_1\, \omega_1^B + p_2\, \omega_2^B.$$

An agent with the utility function $u^B(x_1, x_2) = \min\{x_1, 2\,x_2\}$ chooses optimally when $x_1 = 2\,x_2$. This relationship can be substituted into the budget equation and solved for x_2 to get the demand function of a type B agent for X_2:

$$p_1\, 2\,x_2^B + p_2\, x_2^B = p_1\, \omega_1^B + p_2\, \omega_2^B$$

or

$$x_2^B(p_1, p_2) = \frac{p_1\, \omega_1^B + p_2\, \omega_2^B}{2\,p_1 + p_2}. \tag{1}$$

The relationship between the optimal consumption of X_1 and X_2 by this agent, $x_1 = 2\,x_2$, can be used once again to get

$$x_1^B(p_1, p_2) = 2\,\frac{p_1\, \omega_1^B + p_2\, \omega_2^B}{2\,p_1 + p_2}. \tag{2}$$

When the prices are $p = (p_1, p_2) = (1, 1)$, a type B agent wants

$$\begin{aligned} x_1^B(1,1) &= 2\,\frac{1 \cdot 0 + 1 \cdot 60}{2 \cdot 1 + 1} \\ &= 40 \end{aligned}$$

units of X_1 and

$$\begin{aligned} x_2^B(1,1) &= x_1^B(1,1)/2 \\ &= 20 \end{aligned}$$

units of X_2. These amounts are the same as those shown on the right graph in figure 1 for the prices $p = (p_1, p_2) = (1, 1)$.

When the prices are $p = (p_1, p_2) = (1, 2)$, a type B agent wants

$$\begin{aligned} x_1^B(1,2) &= 2\,\frac{1 \cdot 0 + 2 \cdot 60}{2 \cdot 1 + 2} \\ &= 60 \end{aligned}$$

units of X_1 and

$$\begin{aligned} x_2^B(1,2) &= x_1^B(1,2)/2 \\ &= 30 \end{aligned}$$

units of X_2. These amounts are the same as those shown on the right graph in figure 1 for the prices $p = (p_1, p_2) = (1, 2)$.

(f) (5 points) Use the demand functions $x_1^B(p_1, p_2)$ and $x_2^B(p_1, p_2)$ for agent B that you found in part (e) to verify the budget identity

$$p_1\, x_1^B(p_1, p_2) + p_2\, x_2^B(p_1, p_2) \equiv p_1\, \omega_1^B + p_2\, \omega_2^B.$$

Solution The budget identity for a type B agent with the demand in equations (2) and (1) is

$$\begin{aligned} p_1\, x_1^B(p_1, p_2) + p_2\, x_2^B(p_1, p_2) &= p_1\, 2\, \frac{p_1\, \omega_1^B + p_2\, \omega_2^B}{2\, p_1 + p_2} + p_2\, \frac{p_1\, \omega_1^B + p_2\, \omega_2^B}{2\, p_1 + p_2} \\ &= (2\, p_1 + p_2)\, \frac{p_1\, \omega_1^B + \ \omega_2^B}{2\, p_1 + p_2} \\ &= p_1\, \omega_1^B + p_2\, \omega_2^B. \end{aligned}$$

2. Barter sequences in an exchange economy Suppose that there are three agents (A, B, and C) in an exchange economy that has three commodities X_1, X_2, and X_3. The left panel of table 1 shows endowments for agents A, B, and C. If the equilibrium prices for the three commodities are $p^* = (p_1^*, p_2^*, p_3^*) = (1, 2, 3)$, then the desired allocations for three agents are shown in the right panel of the table.

	X_1	X_2	X_3
A	16	18	20
B	14	20	22
C	14	24	22

	X_1	X_2	X_3
A	0	14	28
B	12	0	36
C	32	48	0

Table 1: Agents' endowments (left) and desired consumption (right)

(a) (8 points) Verify that for agent A the value of her endowment is equal to the value of her allocation. Do the same for agents B and C.

Solution The budget equation for agent i (for $i = A, B$, or C) is

$$p_1\, x_1^i + p_2\, x_2^i + p_3\, x_3^i = p_1\, \omega_1^i + p_2\, \omega_2^i + p_3\, \omega_3^i.$$

For agent A, the value of the endowment is $p_1\, \omega_1^A + p_2\, \omega_2^A + p_3\, \omega_3^A = 1 \cdot 16 + 2 \cdot 18 + 3 \cdot 20 = 112$.
For agent A the value of the allocation is $p_1\, x_1^A + p_2\, x_2^A + p_3\, x_3^A = 1 \cdot 0 + 2 \cdot 14 + 3 \cdot 28 = 112$.

For agent B, the value of the endowment is $p_1\, \omega_1^A + p_2\, \omega_2^A + p_3\, \omega_3^A = 1 \cdot 14 + 2 \cdot 20 + 3 \cdot 22 = 120$.
For agent B the value of the allocation is $p_1\, x_1^B + p_2\, x_2^B + p_3\, x_3^B = 1 \cdot 12 + 2 \cdot 0 + 3 \cdot 36 = 120$.

For agent C, the value of the endowment is $p_1\, \omega_1^A + p_2\, \omega_2^A + p_3\, \omega_3^A = 1 \cdot 14 + 2 \cdot 24 + 3 \cdot 22 = 128$.
For agent C the value of the allocation is $p_1\, x_1^C + p_2\, x_2^C + p_3\, x_3^C = 1 \cdot 32 + 2 \cdot 48 + 3 \cdot 0 = 128$.

(b) (12 points) Table 2 shows several trades in a sequence that leads from the endowments in the left panel of table 1 to the equilibrium allocations in the right panel of table 1. Fill in the table with two more trades to complete the sequence of barters. Describe each of the two trades that complete the sequence of barters in the space below table 2.

	X_1	X_2	X_3
A	16	18	20
B	14	20	22
C	14	24	22

	X_1	X_2	X_3
$\underline{A}$	4	18	24
B	14	20	22
$\underline{C}$	26	24	18

	X_1	X_2	X_3
A	4	18	24
$\underline{B}$	14	2	34
$\underline{C}$	26	42	6

	X_1	X_2	X_3
$\underline{A}$	0	20	24
$\underline{B}$	18	0	34
C	26	42	6

	X_1	X_2	X_3
$\underline{A}$	0	14	28
B	18	0	34
$\underline{C}$	26	48	2

	X_1	X_2	X_3
A	0	14	28
$\underline{B}$	12	0	36
$\underline{C}$	32	48	0

Table 2: Sequence of interim allocations (starting from the endowment) with barter

Solution One possible fourth trade is between agents A and C. (This is the trade shown in the middle panel of the second row of interim allocations.) In this trade, agent C gives up 4 units of X_3 to agent A and receives 6 units of X_2 in return. The interim allocation of agent C was $(26, 42, 6)$ before this trade and it was $(26, 48, 2)$ after the trade; the interim allocation of agent A was $(0, 20, 24)$ before this trade and it was $(0, 14, 28)$ after the trade. Agent A is done trading at this point.

The last trade is between agents B and C. In this trade, agent C gives up 2 units of X_3 to agent B and receives 6 units of X_1 in return. The interim allocation of agent C was $(26, 48, 2)$ before this trade and the final allocation is $(32, 48, 0)$ after the trade; the interim allocation of agent B was $(18, 0, 34)$ before this trade and the final allocation is $(12, 0, 36)$ after the trade. Agents B and C are also done trading at this point.

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3. Arbitrage with commodity money This problem involves a comparison between two arbitrage situations and also some interpretation of the results.

(a) (7 points) Suppose that the prices in the arbitrage example between the British and the French camps are 10 for tea and 9 for coffee in the British camp. In the French camp prices are 9 for tea and 10 for coffee. If a British officer has 81 units of commodity money, what arbitrage strategy could he follow to yield a profit of 19 units of the commodity money or more?

Solution A British officer could use his 81 units of commodity money to purchase 9 ounces of coffee in the British camp, sell the 9 ounces of coffee in the French camp for 10 each to obtain 90 units of the commodity money. He could use the 90 units to purchase 10 ounces of tea, and sell the tea for 100 units of the commodity money in the British camp. This arbitrage strategy yields a profit of 19 units of the commodity money.

In this simple example, the movement of goods from one market to the other doesn't alter the prices in either market. That is a reasonable assumption only if the total amount of goods shipped is small relative to the sizes of the two markets. When trade volumes are high enough, the decrease in the amount of coffee in the English camp will tend to increase its price, and the increase in the amount of tea in the English camp will decrease its price. These price movements will continue so long as coffee costs less in the English camp than in the French camp and tea costs more in the English camp than in the French camp. (Notice that the argument isn't that the prices of tea and coffee end up equal. Rather, the argument is that their relative prices should be similar in the two camps). As long as the price of coffee is lower in the English camp than in the French camp, English traders have an incentive to move coffee from their camp to the French camp where it commands a high price. The same English traders can also convert their earnings from the sale of coffee in the French camp to tea, which costs more in the English camp than in the French camp. Once the relative prices are driven together in the two camps, there is no more incentive additional trade, so at that point the relative prices should stabilize.

(b) (7 points) Now suppose that everything is the same as in the previous problem, but the price of coffee in the French camp is now 8 instead of 10. (So the prices in the British camp are 10 for tea and 9 for coffee. In the French camp prices are 9 for tea and 8 for coffee.) If a British officer has 81 units of capital, what arbitrage strategy could he follow to yield a profit of 10 units of the commodity money?

Solution In this case, there is no profitable trade that involves the purchase of a commodity in the British camp and its transfer to the French camp. The best strategy must involve transfer of the commodity money to the French camp. If the officer takes 81 units of the commodity money to the French camp, purchases 10 ounces of coffee at a cost of 80, and sells the coffee for 90 units of commodity money per ounce of coffee in the British camp, the yield will be 10 units of commodity money. (He ends up with the 90 units of commodity money from the sale of the coffee, and one unit of commodity money that he brought to the French camp but didn't use.)

(c) (6 points) In part (a) the best arbitrage strategy involves commodities being moved from each camp to the other. In part (b) the best arbitrage strategy involves moving commodity money from one camp to the other. Why is this the case? What characteristic of the prices of the commodities drive the British officers to trade in commodities in part (a) but in currency in part (b)?

Solution In the second example, the price of each commodity in the French camp is lower than the price of that commodity in the British camp, so nothing can be purchased in the British camp and sold for a profit in the French camp. So the British camp exports money and imports commodities.

Situations of this sort are common. For many centuries, silver flowed first from the mines of Germany and Silesia through Italy and the middle east to China, and commodities flowed from China west. Later, after the massive flow of silver began from South America to Europe, a great deal of this silver found its way to China. It is likely that due to small amounts of money and a large amount of produced goods that all prices in China were below those in the west. This would lead to a pattern of trade in which money flows to China and goods flow west.

One interesting feature of this situation is that, in the economy that has the inflow of money and outflow of goods, the money supply increases and the supply of goods decreases. Both of these tend to increase the price level, because there is more money available to purchase fewer goods. (Money is relatively less scarce and goods are relatively more scarce than before the inflow of money and the outflow of goods.) In our simplified version of Fisher's equation of exchange (without the velocity of money) $P = M/Q$. Since M increases and Q decreases, the price level should rise. The opposite effect occurs in the economy that exports money and imports goods. There money becomes relatively more scarce and goods become relatively less scarce, so that the price level falls. In the equation $P = M/Q$, since M falls and Q increases, the price level falls. Eventually the overall price level in the two economies should come together.

My arguments in parts (a) and (c) taken together suggest that in the absence of trade costs, relative price levels should be similar across regional economies, and with a commodity money, the overall price levels should also be similar.

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4. Money supply, output, and inflation (8 points) In the exchange economy from the lecture notes that we used to derive a version of the "equation of exchange," the relationship between the equilibrium price and the endowments of commodity money and commodity is $p^* = \omega_1^B/\omega_2^S$. In this model we interpret ω_1^B as the money supply and we interpret ω_2^S as output.

Suppose that output at year 0 is $\omega_2^S = 15$ and the money supply is $\omega_1^B = 1500$ so that the price level is $p^* = 1500/15 = 100$.

If in year 1 output ω_2^S increases to $\omega_2^S = 18$ and the money supply ω_1^B increases to $\omega_1^B = 1926$, find the new price level. What is the rate of of inflation between years 0 and 1?

Solution The price level in year 1 is $p^* = \omega_1^B/\omega_2^S = 1926/18 = 107$. The rate of inflation is 7%.

Short answer questions

5. Functions of money (6 points) List the three functions of money.

Solution The three functions of money are

1. a medium of exchange,
2. a store of value, and
3. a unit of account.

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6. Number of markets with barter versus commodity money (6 points) Suppose that there are 16 commodities in an exchange economy. How many markets could possibly operate if agents barter with one another? How many markets could there be if agents all coordinate and use commodity 1 as money? What is the ratio of the number of markets open with barter to the number with commodity money?

Solution With sixteen goods, commodity 1 could be traded for any one of 15 other commodities, commodity 2 could be traded against any one of 14 other commodities (since we already counted commodity 2 trading against commodity 1), commodity 3 could be traded against any one of 13 other commodities (since we already counted commodity 3 trading against commodities 1 and 2), and so forth. When these are added up the result is

$$M_B = 15 + 14 + 13 + \cdots + 1 = 120.$$

With a commodity money, say commodity 1, there would be only 15 possible markets.

There would be $120/15 = 8$ times as many possible markets open with barter than with commodity money.