Economics 380
Exam #3
Solutions

Problem 1 According to de Roover, "banking originated in exchange and not in credit." Banks of exchange mediate substantial volumes of trade while economizing on monetary resources as they accomplish this task. The real consequence of this trade intermediation is that, by tying up few resources in trade, more resources are available to merchants for other purposes, such as production and acquisition of merchandise.

This problem examines a simple model of trade mediated by exchange banks, as in the example in the lecture notes and the homework problem.

(a) (5 points) Table 1 shows an example of market prices, merchants' endowments, and merchants' allocations after trading is finished. Using the prices in the row labeled p, fill in the value of each merchant's endowment on the left side of table 1 and fill in the cost of each merchant's allocation on the right side of table 1.

	Commodities				
	Dyes	Linen	Spices	Wool	
p	8	7	9	8	Value
E1	—	—	—	10	80
F2	—	10	—	—	70
I1	10	—	—	—	80
I2	—	—	10	—	90

	Commodities				
	Dyes	Linen	Spices	Wool	
p	8	7	9	8	Cost
E1	5	—	5	—	85
F2	5	—	5	—	85
I1	—	10	—	—	70
I2	—	—	—	10	80

Table 1: Commodity endowments and their values (left); commodity allocations and their costs (right)

(b) (5 points) (1 point) Table 2 shows the buyer for each of six trades that lead from the endowments to the allocations in table 1. Complete the table by filling in the cells that are blank.

Buy.	Sell.	Comm.	Units	Price	Cost
E1	I1	Dyes	5	8	£40
E1	I2	Spices	5	9	£45
I1	F2	Linen	10	7	£70

Buy.	Sell.	Comm.	Units	Price	Cost
F2	I1	Dyes	5	8	£40
F2	I2	Spices	5	9	£45
I2	E1	Wool	10	8	£80

Table 2: Transactions cleared at bank B1 (left) and at bank B2 (right)

(c) (5 points) Table 3 shows a ledger for the account of each merchant. Assume that merchants E1 and I1 have accounts at bank B1; assume that merchants F2 and I2 have accounts at bank B2. Show all the debits for each merchant and show the credits that result from trades with merchants who have an account at the same bank. Show the balance for these trades in the column labeled "Bal. (A)."

Mer.	Cr.	Debit	Bal. (A)	Tran.	Bal. (B)
E1	£0	£85	−£85	£80	−£5
I1	£40	£70	−£30	£40	£10

Mer.	Cr.	Debit	Bal. (A)	Tran.	Bal. (B)
F2	£0	£85	−£85	£70	−£15
I2	£45	£80	−£35	£45	£10

Table 3: Merchants' accounts at bank B1 (left) and at bank B2 (right)

(d) (5 points) There are some trades that clear across banks. After these trades are cleared across banks, some merchants will have an additional credit transferred. Show the transferred credit in the column labeled "Tran." In the column labeled "Bal. (B)," show the balance of each merchant after these interbank trades are credited appropriately.

(e) (5 points) What is the amount of money that needs to be transferred between the two banks? The transfer payment that one of the banks makes is a debit of the bank. Show that it has an equal offsetting credit. The transfer payment received by the other bank is a credit of the bank. Show that it has an equal offsetting debit.

Solution

Bank B2 transfers £40 to Bank B1 on behalf of merchant I1, and also transfers £80 to bank B1 on behalf of merchant E1. Bank B1 transfers £115 to Bank B2 on behalf of merchants I2 (£45) and F2 (£70). The net balance owed by B2 to B1 is £5.

The debit of £5 that B2 has to B1 is offset by its net balance due from customers. One customer owes the bank £15 and the other is owed £10. This net credit balances the £5 debit that B2 has to B1.

The credit that bank B1 has from bank B2 is offset by the net debit that the bank has for £5. Bank B1 owes merchant I1 £10 and is owed £5 by merchant E1.

(f) (5 points) What is the total value of goods and services traded in this example? How much money is needed to carry out the exchanges? What fraction of the total value of trade is this?

Solution

The answer depends on assumptions about how balances are cleared. I've provided two possible answers below. In the first, balances across the two banks are cleared with cash, whereas balances of merchants at the banks are retained as either deposits or loans. In the second answer, merchants' balances are also cleared with cash payments to the banker (in the case of merchants with debts) or cash payments from the banker (in the case of merchants with credits).

The total value of the trades is £320. All but £5 of this is transferred on the books of the bankers. At the end, there is £5 in currency that is transferred between banks. This is 5/320 or about 1.5% of the total trade.

If the balances that merchants have with the banks are not carried over as deposits and loans, then those too need to be paid out. In B2, merchant F2 could pay £15 to the banker, who could then give £10 of that to I2 and £5 to B1. Then B1 could take the £5 that he received from B2 and the £5 due to him from E1 to settle his account with I1. In this case, £20 in currency is required to clear the balances, which is 20/320 of the total, or about 6.0%.

In either case, the amount is a small percentage of the total trade.

Problem 2 Commercial banks make resources available for profitable enterprises. In the section on trade, we examined how firms can earn profits from trading. When commercial bankers began to accept deposits, there were three parties who all had to find this practice beneficial: depositors, bankers, and the merchants who borrowed the deposited funds. How was each group affected by this practice?

(a) (5 points) What benefit did depositors obtain? (Assume that they already wanted to save, but that rather than putting their money in a secure place where it remained idle, they instead placed it on deposit.)

Solution

The depositors were able to earn interest on their deposits. Without deposit banks, their only alternative would be to invest directly in commercial ventures, which involves substantial risk. A banker who invests in these ventures though is not affected as much when one of his loans is not repaid, because he has many loans outstanding at one time. Since the banker's income from his loans is relatively stable, he can afford to repay depositors even when some loans aren't repaid. So the depositors can partake in some of the profits of commercial activity with very low risk.

(b) (5 points) What benefit did commercial banks obtain from deposits?

Solution

When banks accept deposits, they have a larger pool of funds to lend to the merchants. The gap between the interest that they charge borrowers and that they pay to depositors adds to their profit.

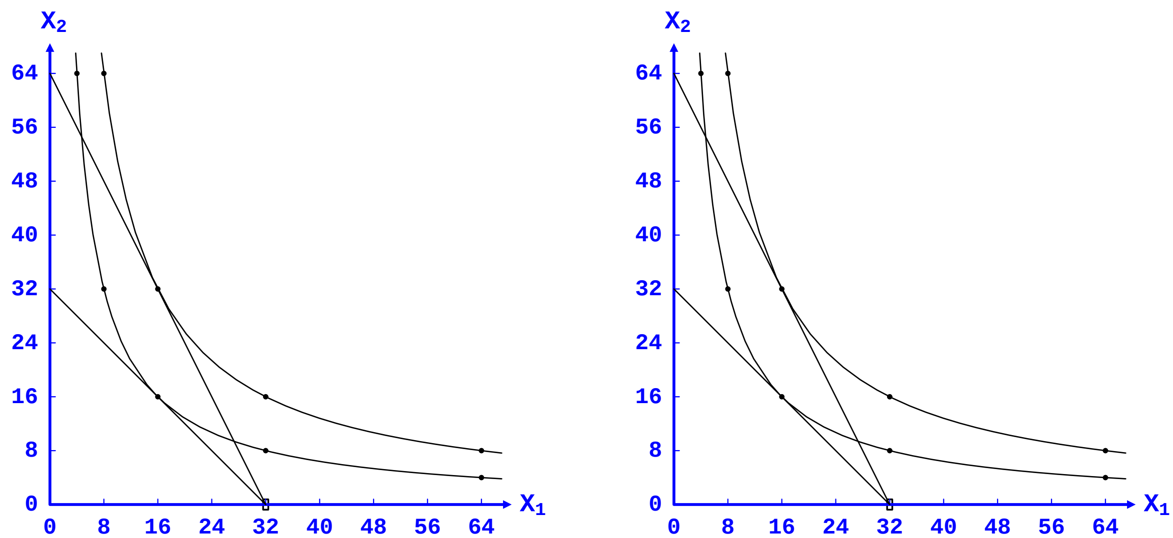
(c) (5 points) How did merchants benefit from this practice?

Solution

With a larger supply of funds for loans, the interest rate falls.

Problem 3 In the lecture notes and homework, we had two models of motives for saving. In one, agents are risk averse and uncertain about future income. In the other, agents have different preferences over present versus future consumption, and a change to the interest rate changes the price of future consumption relative to consumption in the present. These two models can be combined. This problem outlines some steps in the process of combining them.

(a) (5 points) Suppose that one agent has the utility function $u(x_1, x_2) = x_1\, x_2$ over present and future consumption. Draw the indifference curves $u(x_1, x_2) = 256$ and $u(x_1, x_2) = 512$ on the left graph below. (Note: This is a problem about a comparison between the utility functions of two agents. It is not an exchange economy problem.)



(b) (5 points) Suppose that another agent has the utility function $v(x_1, x_2) = \log_2 x_1 + \log_2 x_2$ over present and future consumption. Draw the indifference curves $v(x_1, x_2) = 8$ and $v(x_1, x_2) = 9$ on the right graph above. [Hint: Write down the equation for an indifference curve, such as $v(x_1, x_2) = 8$, and then use the rules for manipulation of logarithms to graph this. The two rules that you need are $\log_2(a) + \log_2(b) = \log_2(a \cdot b)$ and $2^{\log_2(x)} = x$.]

Solution

The first indifference curve, $v(x_1, x_2) = 8$ implies that $\log_2 x_1 + \log_2 x_2 = 8$. This can be written $\log_2(x_1\, x_2) = 8$. Then $2^{\log_2(x_1\, x_2)} = 2^8$ so $x_1\, x_2 = 256$. This is the same as one of the indifference curves in part (a). The second indifference curve is $x_1\, x_2 = 512$, which is the same as the other one in part (a).

(c) (5 points) What amounts of X1 and X2 would each consumer choose if prices were $p_1 = 1$, $p_2 = 1$, and the consumer had an endowment of 32 units of X_1? What amounts would they consume if $p_1 = 1$ but $p_2 = 1/2$? How does the behavior of the two agents in this example compare?

Solution

For the prices $(p_1, p_2) = (1, 1)$, the optimal consumption levels for both consumers are $(x_1^*, x_2^*) = (16, 16)$. For the prices $(p_1, p_2) = (1, 1/2)$, the optimal consumption levels for both consumers are $(x_1^*, x_2^*) = (16, 32)$.

Problem 4 In the previous problem we compared the utility functions $u(x_1, x_2) = x_1 \, x_2$ and $v(x_1, x_2) = \log_2 x_1 + \log_2 x_2$. In fact, when one utility function $v(x_1, x_2) = g(u(x_1, x_2))$, where $g(y)$ is an increasing function, then the two utility functions represent the same preferences. So $u(x_1, x_2) = x_1 \, x_2$ and $v(x_1, x_2) = \ln x_1 + \ln x_2$ are also equivalent, because $v(x_1, x_2) = \ln(u(x_1, x_2))$ and $\ln(y)$ is an increasing function. We can use this equivalence to translate our model of precautionary savings into the exchange model and vice-versa.

(a) (5 points) In the model of precautionary savings, the consumer had the endowment $(\omega_1, \omega_2) = (c, 0)$ with probability π (if unemployed in period 2), and endowment $(\omega_1, \omega_2) = (c, c)$ with probability $1 - \pi$ (if employed in period 2). To simplify the problem, we had interest rate zero. The consumer would choose to save the amount that maximizes the expected utility function

$$\begin{aligned} E[u(s)] &= u(c - s) + \pi \, u(s) + (1 - \pi) \, u(c + s) \\ &= \ln(c - s) + \pi \, \ln(s) + (1 - \pi) \, \ln(c + s). \end{aligned}$$

If we assume also that there is a positive interest rate r, then the amount s saved accumulates to $(1 + r) \, s$ in period 2, so that expected utility is

$$E[u(s)] = \ln(c - s) + \pi \, \ln((1 + r) \, s) + (1 - \pi) \, \ln(c + (1 + r) \, s).$$

Suppose that $\pi = 0$. Make that substitution into the last equation for expected utility. Also, substitute in the usual notation (ω_1, ω_2) for the endowment into the equation. Your last equation is an expression for the expected utility for this person if there is no uncertainty, and the rate of interest is positive.

Solution

If $\pi = 0$ is substituted into the equation for $E[u(s)]$, the equation reduces to

$$E[u(s)] = \ln(c - s) + \ln(c + (1 + r) \, s).$$

If we substitute the usual notation (ω_1, ω_2) for endowments, with $(\omega_1, \omega_2) = (c, c)$, this can be written as

$$E[u(s)] = \ln(\omega_1 - s) + \ln(\omega_2 + (1 + r) \, s).$$

(b) (5 points) Now suppose instead that the person has the utility function $u(x_1, x_2) = x_1\, x_2$, the endowment $(\omega_1, \omega_2) = (c, c)$ with certainty, and the interest rate on loans is r. Rather than work with the utility function $u(x_1, x_2) = x_1\, x_2$ that we are familiar with, we can work with the equivalent utility function $v(x_1, x_2) = \ln x_1 + \ln x_2$. Write down the utility function for this person if she takes s units of X_1 from her endowment and uses it to acquire $(1+r)\, s$ units X_2. (Hint: In this case, her consumption x_1 of the first period commodity X_1 will be $x_1 = \omega_1 - s$. Since her consumption levels x_1 and x_2 depend only on her choice of a savings level s, her utility can be written as a function of s.)

Solution

Since the endowment is specified in advance and the interest rate is determined in the market, the decision maker only chooses the amount to save. Then $x_1 = \omega_1 - s$ and $x_2 = \omega_2 + (1+r)\, s$. So the utility level is

$$u(s) = \ln(\omega_1 - s) + \ln(\omega_2 + (1+r)\, s).$$

The expression from the solution to part (a) is the same as this expression, so we could view the decision maker in the model of precautionary savings as an agent in an exchange economy with uncertainty.

Problem 5 In the multiple deposit creation example in the lecture notes, some new "base money" arrives in Venice. (Now that we know that the bond market attracted gold coins from all over Europe to Venice, we can suppose that wealthy princes, kings, bishops, popes, and pirates – in short, all of the thieves of Europe – brought their money to Venice to invest.) The money is used to buy issues of the Monte from a number of bond holders, who then have the gold coins.

The amount of the base money retained by the sellers of the bonds, the amount of their initial deposits, the amount of these initial deposits held by the banks in reserves, and the amount of the initial deposits lent by the bank at the first stage are all shown in the first row of table 4. The second row shows what happens after the money is received by people who provide goods and services to the borrowers from the first stage, and so forth.

Circulation	Deposits	Reserves	Loans
2,500 *d.*	7,500 *d.*	1,500 *d.*	6,000 *d.*
1,500 *d.*	4,500 *d.*	900 *d.*	3,600 *d.*
900 *d.*	2,700 *d.*	540 *d.*	2,160 *d.*

Table 4: Summary of each stage in the multiple deposit creation example

We worked out the total amount of deposits in the lecture notes with the equation

$$\begin{aligned} D &= 0.75 \cdot 10,000 + (0.75 \cdot 10,000) \cdot 0.8 \cdot 0.75 + (0.75 \cdot 10,000 \cdot 0.8 \cdot 0.75) \cdot 0.8 \cdot 0.75 + \cdots \\ &= 0.75 \cdot 10,000 \cdot (1 + 0.8 \cdot 0.75 + 0.8^2 \cdot 0.75^2 + \cdots \\ &= 0.75 \cdot 10,000 \cdot \frac{1}{1 - 0.8 \cdot 0.75} \\ &= 7,500 \, \frac{1}{0.4} \\ &= 18,750. \end{aligned}$$

In this equation, the fraction f held as currency by the public (that is, outside the banks) is $f = 0.25$, the fraction of deposits held as reserves is $r = 0.2$, and the increase to the monetary base is $B = 10,000$.

(a) (5 points) Suppose that the fractions r of deposits held as reserves, the fraction f of currency held by the public, and the amount of the initial change to the base money supply B are variables instead of fixed. Write down an equation for the amount of deposits in the banks.

Solution

The total amount of deposits in the notes with the equation

$$\begin{aligned} D &= (1-f) \cdot B + (1-f) \cdot B \cdot (1-r) \cdot (1-f) \\ &\quad + (1-f) \cdot B \cdot (1-r) \cdot (1-f) \cdot (1-r) \cdot (1-f) \; + \; \cdots \\ &= (1-f) \cdot B \cdot \Big(1 + (1-r) \cdot (1-f) + (1-r)^2 \cdot (1-f)^2 \; + \; \cdots \Big) \\ &= (1-f) \cdot B \cdot \frac{1}{1 - (1-r) \cdot (1-f)}. \end{aligned}$$

(b) (5 points) How is the money supply affected when the reserve ratio r falls? (The money supply is the amount of "base money," which in this case is the gold coins held by the public plus the amount of deposits.)

Solution

If r declines, then $1-r$ increases, so $(1-r)\,(1-f)$ increases. That implies that $1-(1-r)\,(1-f)$ decreases, so its reciprocal increases. So deposits increase when the reserve ratio falls. The money supply is the sum of base money held by the public and deposits. Since the amount of money held as reserves declines, the amount held by the public increases, so that both the the base money in circulation and deposits increase. Therefore the money supply increases.

Alternatively, it is possible to look at the right-hand side of the first equality in the equation for deposits to see that deposits increase. The first term includes the factor $1-r$, the second term includes the factor $(1-r)^2$, and in general, term n includes the factor $(1-r)^n$. When

r declines, $1 - r$ increases, so $(1 - r)^n$ also increases. That means that every term in the infinite sequence increases, so deposits increase, and the money supply increases also.

(c) (5 points) How is the money supply affected when the fraction f of base money held by the public (outside the banks) falls?

Solution

When the fraction of the base money held by the public decreases, deposits increase. The argument that shows this is similar to the one that shows that deposits increase when the reserve ratio falls.

The argument that shows that deposits increase more than the decline in currency held by the public is more subtle.

If f decreases to $f + \Delta f$ (with $\Delta f < 0$), then the amount of money held by the public falls from $f\,B$ to $(f + \Delta f)\,B$, which is a change of $-\Delta f\,B$.

Deposits change from

$$D = \frac{(1 - f)\,B}{1 - (1 - r)\,(1 - f)}$$

to

$$\begin{aligned} \tilde{D} &= \frac{(1 - (f + \Delta f))\,B}{1 - (1 - r)\,(1 - (f + \Delta f))} \\ &= (1 - f)\,\frac{B}{1 - (1 - r)\,(1 - (f + \Delta f))} - \Delta f\,\frac{B}{1 - (1 - r)\,(1 - (f + \Delta f))}. \end{aligned}$$

The first term in the last line of the this equation is positive. The second term is also positive and it is greater than $\Delta f\,B$, which was the reduction in currency held by the public, so deposits increase by more than the amount that currency held by the public falls. Therefore the money supply increases.

(d) (5 points) How is the money supply affected when the amount of the increase to the monetary base B goes up?

Solution

When the base money increase, the change to the money supply is straightforward. Both deposits and the amount of base money held by the public increases in direct proportion to the increase in the money supply.

Problem 6 (5 points) In the example of the loans that the English merchant Marsden provided, and the bills of exchange that he issued to provide the loans, he drew on his Blackwell Hall factor, so the amount of his loans was constrained by the accumulated balance he had at Blackwell Hall. An example of Marsden's note is shown in figure 1.

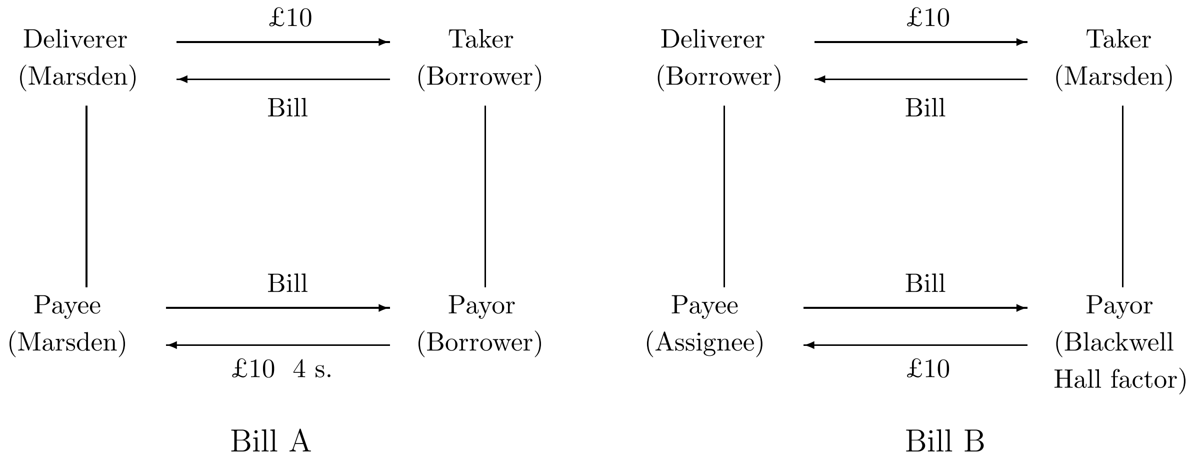


Figure 1: Two bills of exchange that create a circulating medium of exchange

Soon afterward, when the London goldsmith banker Edward Backwell issued his loans and bills, he both issued the bill and acted payor on the bill. In effect, he created money with paper and pen, since he issued the loan (which he collects interest on) and in return gives the borrower a bill of exchange that the borrower can transfer to a third party to make a purchase. The person who acquires the bill from the original borrower can transfer it to another person to make a purchase, and so on, until the bill eventually comes back to Backwell for payment. An example of Backwell's note is shown in figure 2.

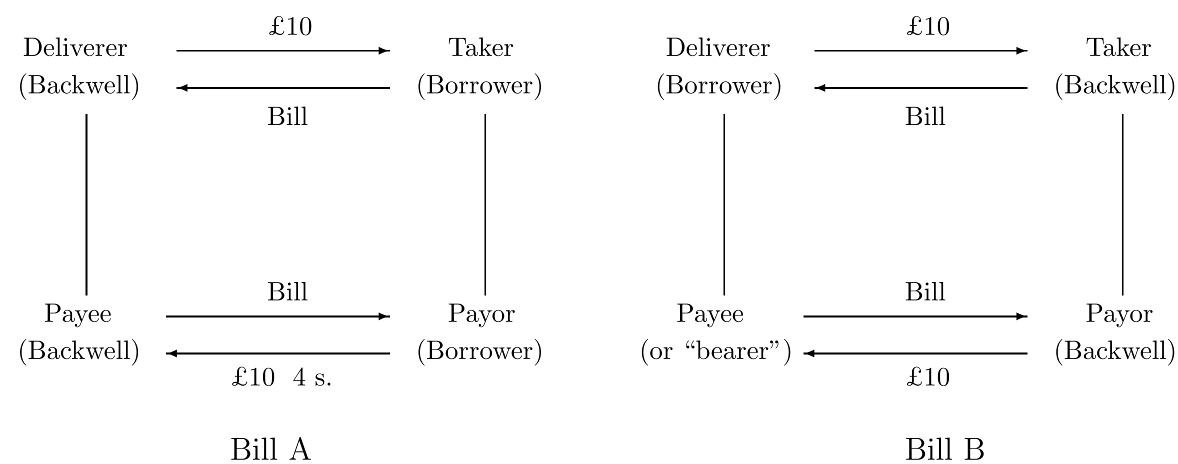


Figure 2: Two bills of exchange create a note payable to order (or to the bearer). Bill A is a promissory note from the borrower to the goldsmith banker Backwell. Bill B is a note that can circulate as money.

The bills issued by Backwell differ only slightly in form from those issued by Marsden, but the economic difference is great. Marsden's notes were backed by his balances with his Blackwell

Hall factor. Backwell's were backed by himself.

What constrains the amount of money created by Backwell in this way? Put slightly differently, he earns interest on each loan, so why would he not issue as many loans as possible?

Solution

The only constraint that Backwell faces is that he needs to be able to introduce the notes into circulation by lending. He needs to find borrowers who are good credit risks to take his notes as a form of loan, since he is ultimately liable to the bearer of the note who presents it to him for repayment. The only way to offset this liability is to have an asset of equal or greater value. In this case, the offsetting asset would be the value of the loan that Backwell has issued.

Physically, there is no constraint on his issue of the notes other than availability of paper, and his ability to write out the notes.

Problem 7 (5 points) In both problems 5 and 6, the money supply is increased. (In problem 5 the money supply is increased by fractional reserve banking and multiple deposit creation. In problem 6, it is increased by creation of a circulating financial instrument.) In the equation of exchange, $PQ = MV$, which we simplified to $P = M/Q$ by assuming that velocity of money is one. In this equation, an increase to the money supply has no effect on output. How can we reconcile the importance of the monetary factors in the economy (everyone pays attention when the Federal Reserve makes its announcements about changes to the prime rate) with the apparent fact that changes to the money supply only affect the price level?

Solution

The short answer to this question is that there must be some connection between the availability of credit and the real sector of the economy. Any answer that points toward a connection between the money supply and the real sector of the economy is worth full credit. This issue though is the most important one that we will consider in the remainder of this course, so a longer answer should be helpful at this point.

In a paper in 1983, Ben Bernanke addressed the problem of how monetary policy might impact the real sector of the economy.

> "There is much support for the monetary view. However, it is not a complete explanation of the link between the financial sector and aggregate output in the 1930's. One problem is that there is no theory of monetary effects on the real economy that can explain protracted nonneutrality. Another is that the reductions of the money supply in this period seems quantitatively insufficient to explain the subsequent falls in output."

Although Bernanke was writing about the depression, the issues he raised affect any economy. The question is, how do the money supply and availability of credit impact the productive sector of the economy? The monetarist view only says that the money supply affects the price level. This seems to hold, but there must be more to the story, otherwise the only effect of a change to the money supply would be a change to the price level. In real terms, that wouldn't change anything. There would be more money and prices would be proportionally higher. That is what Bernanke means when he writes that "there is no theory of monetary effects on the real economy that can explain protracted non-neutrality." (Monetary neutrality is exactly the claim in the "equation of exchange": the price level is equal to the money supply divided by output.) On the surface, it might look like Bernanke is calling the monetarist view into question, but the monetarist program was the point of departure for contemporary monetary policy, going back to the tenure of William McChesney Martin as Fed chairman in 1951 – 1969. Bernanke was among the first to examine (perhaps the first) how changes to monetary policy propagate from the financial system into the real sector of the economy. In the same paper he summarizes his proposal for a mechanism that could have transmitted monetary changes into the real sector of the economy during the great depression.

> "The basic premise is that, because markets for financial claims are incomplete, intermediation between some classes of borrowers and lenders requires nontrivial market-making and information gathering services. The disruptions of 1930 – 33 (as I shall try to show) reduced the effectiveness of the financial sector as a whole in performing these services. As the real costs of intermediation increased, some borrowers (especially households, farmers, and small firms) found credit to be expensive and difficult to obtain. The effects of this credit squeeze on aggregate demand helped convert the severe but not unprecedented downturn of 1929-30 into a protracted depression."

In my view, this is a crucial step in understanding the role of monetary policy. Bernanke examined how a disruption to the monetary system led to problems with credit intermediation (the banking sector), and how the resulting problems with credit intermediation led to problems in the real sector of the economy. The next step is to turn the perspective around and consider how monetary growth and the resulting increased access to credit might foster economic growth. This will be the main topic throughout the remainder of the course, but a short answer to the question is worthwhile.

At any given time, there is some specified amount of goods and services available in the economy. A change to the money supply can change the price level, but the real resources in the economy aren't affected by the change. Suppose that the total money supply increases by a small amount, because the Federal Reserve increases the base money supply. Then the banks, through multiple deposit creation, will have more money available to make loans. Just to set up a contrast with the ideal distribution of the credit, imagine that the money were somehow distributed equally to everyone in proportion to the amount of money that they spend. Then every one has say, 2% more money, and they're all trying to obtain the same goods as before with that money, so prices would go up and nothing would change. That is money neutrality.

Suppose though that the bankers could foresee the future perfectly. A banker could look at every business proposal placed before her and determine instantly what the rate of return would be, and how much the business would grow. The banker would want to provide credit for the proposals that would have the highest profit and growth, because then their own business would grow, and they would never have losses from defaults. The profits of these businesses represent the difference between the value of the inputs that the businesses utilize and the products that they sell, so wealth is increased by the distribution of credit. It is also increased if the businesses grow and add to employment. So credit shifts the claims to the fixed amount of real goods and services available in the economy. If it is provided wisely, then the shift can be toward activities that add to aggregate wealth.

Bernanke, Ben S. (1983). "Nonmonetary Effects of the Financial Crisis in the Propagation of the Great Depression," *American Economic Review*, **73**: 257 – 276.