

Effects of Information Revelation Policies under Cost Uncertainty

The paper presents insights regarding the key learning-related factors a buyer in a procurement context should consider when choosing the information policy, i.e., the extent to which information about bids are revealed in an auction. We offer the insights by analyzing the following two first-price sealed bid policies in a private-value sequential auction with no winner drop-outs: (i) IIS: where only the winner’s bid is revealed, and (ii) CIS: where all bids are revealed. Our analysis identifies two important learning effects – the extraction and the deception effects – as having significant welfare implications. While both these effects arise because of a bidder’s desire to gain informational advantage relative to its competitors, their manifestations are different. The extraction effect occurs because of a bidder’s incentive to learn about its competitors, and the deception effect is a consequence of the incentive to prevent an opponent from gaining the information. Both effects lead to higher bid prices, and either may be dominant from a procurer surplus standpoint. Because of the deception effect, social welfare can decrease even when the number of suppliers increases, a result which is counterintuitive. The paper also discusses how insights regarding the learning effects might apply to other policies.

[Keywords: Information Revelation, Electronic Markets, Economics of Information Systems, Perfect Bayesian Nash Equilibrium, Auctions]

1. Introduction and Problem Motivation

The research problem in this paper is motivated from conversations with Freemarkets (now Ariba), a firm that executes electronic reverse auctions (or *market-sessions*) at the request of the procurers. Each procurer initiating a market-session has to choose an information policy, which determines the nature of information—bids submitted, rank of the bidder, etc.—that is revealed to suppliers at the beginning, during, and/or at the end of a market-session. In Freemarkets, a procurer can choose from among a wide range of information policies to accept sealed bids with minimal feedback to the suppliers; provide rank information as feedback; or conduct an open-cry auction. In fact, different information policies have been adopted in various electronic (Jap, 2002, 2003) and traditional procurement markets (McAfee and McMillan, 1988, Hausch, 1986). Even with electronic procurement auction systems installed at individual firms, the information policy is an important design element (for example, see Elmaghraby and Keskinocak, 2000). Our interest in this paper is to provide insights guiding the choice of information policy in a procurement auction.

In procurement scenarios, the nature of competition is known to be similar across market-sessions with the same set of suppliers repeatedly competing (Milgrom and Weber, 1982, as well as anecdotal evidence from Freemarkets). With the repeated competition, depending on the infor-

mation revealed, suppliers alter their behaviors in order to learn about the private information of their opponents such as costs. Thus, the information policy choice affects the market outcomes. It can be extrapolated from other contexts that market outcomes, including procurer surplus, do not always vary monotonically with the information revealed. However, that extrapolation in itself is insufficient to guide the choice of information policy. Moreover, to the best of our knowledge, no generic analysis of information policies applicable to this setting is available. Our paper provides insights regarding some of the key bidding related factors which a buyer should consider when choosing an information policy.

We study a first-price sealed bid private value sequential auction with no winner drop-out under the following two policies:

- Complete Information Policy (CIS): All bids are revealed at the end of the market-session. This policy is similar to the one adopted for municipal construction contracts (Hausch, 1986).
- Incomplete Information Policy (IIS): Only the winner's bid is revealed at the end of the market-session. Some government procurement auctions are required by statutes to adhere to this policy (McAfee and McMillan, 1988).

CIS and IIS are two of the many policies available to a procurer arriving at Freemarkets (Arora et al., 2007). Not only are they commonly adopted in electronic and traditional markets but they have also been analyzed in prior works (although in different contexts). For example, Arora et al. (2007), and Greenwald et al. (2009) compare CIS and IIS; Ortega-Reichert (1968) and Hausch (1986) investigate CIS.

Our analysis of the two policies identifies two learning related effects – the extraction and the deception effects – as among the key factors having significant welfare implications. While both effects are consequences of a bidder's desire to gain an informational advantage relative to its competitors, their manifestations are quite different. The extraction effect, which occurs under IIS, is a consequence of a bidder's interest in learning about his opponent; but the deception effect, which is observed under CIS, is an outcome of a bidder's interest to prevent an opponent from learning about the bidder. Each of these effects leads to a higher bid price and is significant enough to dominate the other from a supplier profit standpoint. In our setting, loosely speaking, the supplier profit comparisons are the opposite of the procurer surplus comparisons. Thus, our analysis also

provides insights for a procurer choosing an information policy. It is not just the managerial insights that are interesting but also the manner in which we demonstrate them. Corresponding to the condition when both the effects occur, neither CIS nor IIS has a generic closed form expression for the equilibrium strategies; yet, we prove the comparison results.

We also evaluate the social welfare generated under the policies. When analyzing its sensitivity to the exogenous parameters, we find that the learning effects lead to a counterintuitive observation. One would expect that, as the number of competitors increases, the market becomes more competitive and, hence, more efficient. However, we find that the social welfare can also decrease with an increase in the number of competitors. This result has implications for a social planner.

The research problem investigated in our paper overlaps many domains, which appeal to the information systems (IS) research audience. First, several papers by IS researchers (e.g., Koppius, 2002, Arora et al., 2007, Mithas and Jones, 2007, Adomavicius et al., 2008, Greenwald et al., 2009) have analyzed the impact of the information revealed in procurement contexts (the focus has been different). Our paper contributes to that growing body of knowledge. Second, we investigate the dynamics arising because of varying information levels. Specifically, our paper focuses on the learning related dynamics. The topic of informational dynamics and information transparencies is also of interest to the IS community (e.g., Granados et al., 2005, Goes et al., 2009). Third, our paper offers insights regarding electronic markets, which is also a research theme of interest to many in the IS area (e.g., Yoo et al., 2007, Kumar et al., 2007, Adomavicius et al., 2009).

The rest of the paper is organized as follows. In Section 2, we review the related literature. Following that, we present our model in Section 3 and solve for equilibria in Section 4. The policies are compared in Section 5. Section 6 presents some generalization results. Finally, we discuss in Section 7 and conclude in Section 8.

2. Literature Review

In this section, we survey the literature in the following streams of research: literature studying the impact of information on auction markets, sequential auctions in economics, and information policies in procurement contexts.¹

¹Note also that there are similarities with the market microstructure literature in Finance. However, there are institutional differences between the two settings. Typically, financial exchanges are double-sided auctions whereas electronic reverse-markets are single-sided auctions.

2.1 Impact of Information in Auctions

Several studies have considered the information impact on auction performances. In order to be brief, we only survey a few of the key ones in this section. Wilson (1977) is one of the first papers to study the information impact in auctions. He considers a single period auction where the bidders are unaware of their valuation. He shows that when bidders base their bids only on the private sample information they get, the maximum bid is almost surely equal to the true value. Winkler and Brooks (1980) investigated the impact of uncertainty in a common value auction. They demonstrate how the valuation dependence is related to “winner’s curse.” The seminal work from Milgrom and Weber (1982) also studies the information impact in one of the subsections. In that, they recommend that the auctioneer reveal all bidder related valuation to maximize surplus based on their analysis of a single-period auction setting where the bidder valuations are affiliated. This result is often extended to our setup to claim that, since bids are surrogate for valuation information, it is optimal for the procurer to reveal all the information. As we show later, such a result is not applicable to our context.

2.2 Sequential Auctions Literature

There is a large body of literature on sequential auctions. Within that, there are two main streams. In one, the winning bidder drops out of the future periods while, in the other, the winner continues to participate. The reverse auction contexts we study, because they involve repeated competition among the same set of sellers (Milgrom and Weber, 1982, as well as anecdotal evidence from Freemarkets), relate to the latter stream. However, as Klemperer (2004) notes, much of the literature has focused on the former. Krishna (2009, Page 221) observes the main problem affecting analysis of the “no drop out” scenario is that “even if bidders are symmetric ex ante, multiunit demands [or simply, the no drop out scenario] introduce asymmetries in later auctions.” Thus, as Laffont (1997) concludes, “the theory is more complex.” It is also because of the differences between the two sequential auction streams, one cannot extrapolate results from one to the other. For example, the revenue equivalence between the first- and the second-price auctions holds when the winner drops out (Krishna, 2009) but not with “no drop out” (Hausch, 1988).

Among the articles which model the winner not dropping out, Ortega-Reichert (1968) and Hausch (1986) have considered problems related to ours. Ortega-Reichert (1968) is the first study

to focus on sequential auctions with no drop-outs. He analyzes a first-price sealed bid common-value auction with two people and two market-sessions (each of which corresponds to a period), under CIS-like policy. He discovers is that there exists a symmetric pure strategy equilibrium which reveals the bidder's information in the first period. Hausch (1986) builds on the setting and compares CIS against a simultaneous auction scenario. He shows that, under different conditions, either policy may be better from a bidder standpoint. Our private-value auction framework is different from the common-value auction studied in both papers. We also study IIS.

More recently, Tu (2005) in his thesis also considers the same problem even though the results are different. Note that his work at best can be considered to be parallel to ours (an earlier version of our paper can be easily shown to precede Tu, 2005). He models a two-player game and characterizes the equilibrium by making a critical assumption. Specifically, he explicitly assumes that pooling of bids is not permitted (i.e., players do not fake their types), an assumption which we believe is unrealistic in this context. With such an assumption, he shows that CIS always generates higher procurer surplus than IIS. However, we believe that one of the key reasons our results are different is because we do not make the same assumption. As mentioned before, we demonstrate that IIS can generate a higher procurer surplus. We also demonstrate the robustness of our result for any arbitrary number of suppliers, and not just for two. Extending the proofs beyond two suppliers involves additional complications.

There is also some overlap with the literature on mergers. Thomas (1996), which is also a working paper, analyzes how the vertical mergers may be different to horizontal mergers. With a two-player game, where the player type of either high or low is equally likely, he concludes that the merged entities with complete information flow between them is better for the procurer than un-merged ones. Contrary to his conclusion, our analysis shows that a policy which does not completely reveal information may be dominant. Moreover, the stochastic dominance results and the social welfare results we later prove in our paper are not even present there. Furthermore, we demonstrate our results for any arbitrary number of suppliers.

2.3 Information Policies in Procurement Settings

Our paper best fits this stream of research. As mentioned earlier, there is an increasing interest in the IS community to study information policies. The most closely related work is Arora et al.

(2007), which compares CIS and IIS in a setting with homogeneous bidders facing uncertainty about the number of their competitors. Thus, theirs is a common-value setup. They prove that the procurer surplus varies non-monotonically with the information revealed and recommend CIS from a procurer standpoint. First, note that ours is a private value setup. Second, one of their key assumptions – which is the non-zero probability of a bidder being a monopolist in the market – does not always hold in real life. This assumption is violated when requirements regarding the minimum number of suppliers is imposed (e.g., OMB, 1993, United-Nations, 2007). Third, our results are different since, in our setting, the procurer surplus under IIS can also be higher.

Another related paper is Greenwald et al. (2009) and they primarily develop a computational test-bed to enable comparisons of policies by building on a previous version of our paper. Their paper also demonstrates the usefulness of the test-bed by comparing CIS and IIS policies in an analytically intractable setup where both types of uncertainties – uncertainty about the number of competitors as well as about their costs – are jointly present. The development of their test-bed treats bidder behavior as a black box, and considers the procurer surplus as their only metric of analysis. In fact, no equilibrium bid distribution is specified in their paper. Bid distributions are relevant only to demonstrate the performance of the proposed algorithm. Even to do that, they primarily consider a single period game. However, for such a game, the deception and extraction effects never arise. In short, their paper neither establishes the presence of the learning effects nor demonstrates the dominance of the effects. To the contrary, our paper identifies the learning effects and also studies their welfare implications.

Koppius (2002) has experimentally analyzed the impact of revelation policies on bidder behavior in a multidimensional procurement auction. He infers that, when the suppliers have the least level of uncertainty about the weights a procurer places on the multiple dimensions, the supplier profits are maximized. Our focus is quite different in that we do not analyze the impact of information processing capacities of the suppliers but consider the strategic behavior that suppliers exhibit with the nature of the information revealed.

Mithas and Jones (2007) presents an empirical analysis based on data from a real-life electronic marketplace. The paper does not specifically consider the policies that we consider. Their analysis shows that there is no monotonicity in the procurer surplus with the information revealed to the bidders. The analysis is, however, limited in its ability to explicate the strategies adopted by the

suppliers. This is so because they do not have the data to track how suppliers bid across market-sessions. The focus of our work is quite different from theirs in that we are not only comparing the procurer surplus but also developing an understanding of the bidding behavior in response to information policies. Moreover, the methodology adopted is different.

In a recent study, Adomavicius et al. (2008) use a laboratory experiment to study the role of information in a combinatorial procurement auction setting, where each market-session involves multiple rounds. They study three policies which differ in the information revealed: (1) the provisional allocation (e.g., the current rank) is revealed; (2) task-related information (e.g., the price or level of quality needed to win the auction) specific to each bidder is provided; and (3) a menu of prices along with the possible ranks and the profits generated, again specific to the bidder is provided. Based on the experimental data, they infer that the procurer generates the maximum surplus with the second policy. The paper also empirically investigates the changes in bidder behaviors observed with the varying levels of feedback.

Paper	Setting	Relationship between Procurer Surplus and the Information Revealed
Hausch (1986)	Common Value Setting	CIS or simultaneous auction; it varies depending on the priors.
Arora et al. (2007)	Common Value Setting but with numbers uncertainty	CIS preferred from a procurer standpoint.
Thomas (1996)	Private Value Setting with 2 bidders and equally likely cost types	CIS preferred from a procurer standpoint.
Tu (2005)	Private Value Setting with 2 bidders, and no pooling strategy allowed	CIS preferred from a procurer standpoint.

Table 1: Summary of Prior Analytical Works on Information Policies.

Table 1 provides a short comparison of the settings and results in prior works.

3. Model

Our model is closely related to those in Hausch (1986) and Arora et al. (2007). We treat the procurement context as an independent private value auction. Such a treatment is generally ac-

cepted (e.g., Maskin and Riley, 2000b, Bajari, 2001, Brosig and Reiß, 2007, Kostamis et al., 2008).² Our model employs a first-price sealed bid auction although we also briefly discuss the second-price mechanism in Section 7. Because we focus on the learning effects across market-sessions, we consider two sequentially occurring market-sessions or periods, each initiated by a different procurer. We assume that, in each period, the suppliers submit bids without any knowledge of their opponents’ bids for that period. Thus, ours is a simultaneous move game in each period.

We assume that there are $n + 1$ suppliers in the marketplace, where n is common knowledge. (Table 6 in Appendix A shows the notations used in the paper.) Hausch (1986) considers $n = 1$, i.e., a two player game. Each supplier can be one of two types depending on its marginal cost of production: a low- or a high-cost type. We assume that the marginal cost is c_l for a low-cost type while it is c_h for a high-cost type, with $c_l < c_h$.³ (In Section 6.1, we also discuss the result when the model involves three cost types.) Each supplier is aware of his own type while uncertainty persists for him regarding the knowledge of his opponents’ types. Let the probability of a supplier being a low-cost type be θ , which is common knowledge.⁴ Once a supplier’s cost is determined from the random draw, we assume that it is retained the same across both periods.⁵ Thus, the only difference between the two periods is the information set.

Bids submitted by the suppliers are often multi-dimensional in reality. The procurer may

²“Each potential contractor has essentially the same information about the nature of the project but a different opportunity cost of completing it.” (Maskin and Riley, 2000b)

³As we show later, a signaling game arises in our context. The analysis of signaling games with non-binary types has been known to quickly run into technical complexities. For this reason, models of job markets, which involve signaling games, are typically solved with two types of players (See, for example, Spence, 1973, Daughety and Reinganum, 1994). In games involving incomplete information, Laffont and Martimort (2002) argue that the intuitions for binary types carry over to continuous types as well and that the continuous type model provides very few additional insights. In that regard, consider, for example, Milgrom and Roberts (1982), which analyzes a limit pricing game with both binary and continuous types. Even in the context of auctions, Fudenberg and Tirole (1994) has shown similar results to hold.

⁴Note that ours is a two-period incomplete information game, with players unaware of their opponents’ cost types. Harsanyi’s Nobel-prize winning works (Harsanyi, 1967, 1968a,b) show that an incomplete game can be analyzed by considering an imperfect game, with an initial move probabilistically made by Nature and the probabilities being common knowledge. Our modeling of the game is consistent with this well-established technique. We let θ correspond to Nature’s move and assume it to be common knowledge.

In our model, we assume a common priors for the bidders. The assumption “enables one to zero on purely informational issues in analyzing economic (and other interactive) models of uncertainty” (Aumann, 1987). Recall that our paper focuses on the informational dynamics of revelation policies. Therefore, to be able to attribute the procurer surplus differences solely to the revelation policies, we indeed assume common priors.

The assumptions regarding the common prior and common knowledge of the probability are no different from what are commonly made in the auctions literature. Our specification of probabilities θ and $1 - \theta$ over the cost types c_l and c_h is similar to specifying a distribution function over a set of valuation/cost types.

⁵This assumption is reasonable considering the anecdotal evidence from Freemarkets which suggests that often times the same set of suppliers repeatedly compete. The assumption that the supplier type is the same across periods is also made in Hausch (1986).

place weights over the different attributes and use a single metric to compare the bids. In certain markets, for example in the coal-marketplace,⁶ bidders may even be aware of the weights that the procurer places on each attribute. In such cases, a multi-dimensional bid may be collapsed into a single dimensional bid without loss of generality. This logic motivates the assumption of a single dimensional bid in electronic reverse markets in prior works (e.g., Carr, 2003). We also assume that the bid submitted is a single dimensional *price* p , which is required to be non-negative.

Next, we define the sequence of moves in our game. In the first period, all $n + 1$ suppliers submit the bid prices simultaneously. The supplier with the lowest price is picked as the winner and, in case of a tie, it is broken randomly. At the end of the period, information is revealed according to the revelation policy. At this stage, under both CIS and IIS, the winning supplier is awarded the contract, and paid the bid amount; the supplier manufactures the product, and incurs the marginal cost associated with his type. For the convenience of the reader, Table 2 captures the information that is revealed under the different information policies without the details regarding the awarding of the contracts. The second period is repeated in the same manner as the first.

	Complete Information Policy(CIS)	Incomplete Information Policy (IIS)
Beginning of the first period	Uncertain	Uncertain
End of the first period	All bids are revealed	Only winner’s bid revealed
Beginning of the second period	All suppliers aware of all bids	<ul style="list-style-type: none"> – Each loser is aware of the the winner’s bid. – Winner from the first market-session continues to be uncertain.

Table 2: Information Revealed under each Policy.

4. Equilibrium Analysis

Since our multi-period game is a “game with incomplete information”, we use the *perfect Bayesian Nash* solution concept to determine the equilibrium. Such a concept allows us to account for the interdependencies among the first period strategies, the updated belief for the second period, and

⁶Since the thermal equation, which relates the heat generated from coal to factors such as the ash content, and water content, is generally known, bidders in coal markets are also aware of the weights that the procurer places on each attribute.

the second period strategies. The following points are relevant before we solve for the equilibrium:

- We use the backward induction technique by first solving the second period game. Our second period game turns out to be a single period game where the beliefs of the suppliers regarding the others may or may not be asymmetric. Games similar to the ones we encounter in the second period have already been analyzed in Maskin and Riley (1985) and it is well-known they do not always have an equilibrium. To overcome the problem, we employ the same technique as Maskin and Riley (1985). Actually, their Footnote 2 becomes directly applicable to our context: “As our model is formulated, an equilibrium in the sealed-bid auction may not exist. The nonexistence problem, however, is an artifact of our allowing literally a continuum of possible bids. In fact, we can restore existence even with a continuum by allowing the possibility of positive but infinitesimal bids, which we implicitly assume in our analysis.” For the sake of readability, we also make the same implicit assumption in the main manuscript and in the corresponding content in the appendix. The key difference is that, while they employ it for their single period game, we extend it to our two period game.⁷
- A high-cost supplier always bids a price of c_h (equal to his marginal cost) and it is quite easy to demonstrate so.⁸ Compared to it, computing the equilibrium of a low-cost type is fairly involved. So, the rest of the analyses focuses on the low-cost suppliers’ bidding behavior.

⁷We also make available in the supplementary material (Section G) all the results, including the equilibrium analyses of the policies, for a case where the explicit assumption is similar to the implicit one in Maskin and Riley (1985).

⁸One may wonder if the high cost has any incentive to participate if his profit is zero. We can justify this zero profit case in the following manner: First, let the cost types of suppliers actually be from a distribution and the suppliers incur a participation cost. Then, only the types with costs less than the participation cost participate. The last type which participates - referred to here as the margin type - is important to our analysis. The other participating cost types attempt to deceive their opponents by pretending to be of the margin type. In the discretized model, the margin type could be infinitesimally close to the participation cost, which we refer to as simply c_h for the sake of simplicity. In the two-cost model (main model) and the three-cost model extension, we show that the other cost types pretend to be type c_h so as to deceive their opponents, illustrating the importance of the margin type. Second, on a related note, the assumption that the last type generates zero profits is quite common in economics, including in principal-agent models (Salanie, 2000) and in auctions (Krishna, 2009). For example, the classic revenue equivalence result in single period auctions holds only if the last type generates zero profit (Krishna, 2009). Third, there are motives other than profits affecting bidder participation in real life. A procurement manager provided us with the following example. Even if a vendor does not have the distributorship of a manufacturer whose products are being procured, the vendor participates knowing well that he will not win the bid. The vendor’s main intention for participation is to remind the procurer about the vendor for future auctions.

4.1 Single Period Game

In this section, we develop equilibrium results for a game which generalizes most of the single period games that arise in our model context. This helps us avoid repeating the same intuitions. Consider the following game, which will later refer to as the *SPG* (Single Period Game). An instance of the game, for example, occurs in the second periods of IIS and CIS. The players in the game can be categorized into two sets. In the first set, there is only one low-cost supplier, R_1 (the subscript refers to the only player), while the other n suppliers, each referred to as R_o (the subscript refers to the other players), are in the second set. For example, under IIS in the second period, the first period winner corresponds to R_1 while each loser is R_o . The game is such that R_1 believes that each of his n opponents (R_o) is of a low-cost type with a probability of α . Each R_o believes that R_1 is a low-cost type with probability β and that each of the other R_o suppliers is a low-cost type with probability of α . We always find that $\beta \geq \alpha$ and that both β and α are common knowledge.

As mentioned earlier, an auction game similar to SPG has already been analyzed by Maskin and Riley (1985, Section 3). Adhering to their procedure, we solve for the equilibrium, which is unique (see Appendix B). The equilibrium is obtained by solving the following profit function expressions for R_1 and a low-cost R_o from bidding any price p :

$$\Pi_{R_1}(p) = \left(\overbrace{\underbrace{(1-\alpha)}_{\text{That supplier is a high-cost type}} + \underbrace{\alpha(1-F_{R_o}(p))}_{\text{That supplier is a low-cost type and is outbid}}}_{\text{Consider each of the } R_o\text{s.}} \right)^n (p - c_l), \text{ and} \quad (1)$$

$$\Pi_{R_o}(p) = \underbrace{((1-\beta) + \beta(1-F_{R_1}(p)))}_{\text{For } R_1} \underbrace{((1-\alpha) + \alpha(1-F_{R_o}(p)))^{n-1}}_{\text{For each of the other } R_o\text{s}} (p - c_l). \quad (2)$$

Here, $F_{R_1}(p)$ is the cumulative density function (cdf) of the bid distribution for R_1 and $F_{R_o}(p)$ for R_o . There are only two key results from this game we carry along for the rest of the paper: (i) a low-cost type always bids $< c_h$ and never bids like a high-cost type, i.e., it is a *separating equilibrium*, and (ii) $\Pi_{R_1} = \Pi_{R_o} = (1-\alpha)^n(c_h - c_l)$. Some may find the result surprising but it has been observed before for certain special cases of asymmetric auctions, including by Maskin and Riley (1985, Section 3) and Arora et al. (2007, Lemma 2). It can also be seen from the equilibrium calculations that, when $\alpha = \beta$, i.e., bidders are symmetric, their bid distributions are identical.

Note that the values of α and β vary across the policies, and they depend on how, in the first period of each policy, the suppliers bid. Our analysis will take into account these variations to derive the first period strategies.

4.2 Two Period Games

The steps involved in determining the perfect Bayesian Nash equilibrium (Fudenberg and Tirole, 1994) are as follows. Under each policy, we first derive the second period equilibrium when the first period game has a separating equilibrium, i.e., a low-cost type bids $p < c_h$. Following that, we obtain the first-period separating equilibrium and the conditions when such an equilibrium is valid in the first period. Eventually, for those conditions when a separating equilibrium does not exist in the first period, we determine the equilibrium for both periods.

4.2.1 IIS

Under this policy, only the winner's bid is revealed at the end of the first period. Let us represent the revealed bid to be p_w . Suppose a separating equilibrium exists in the first period. If $p_w = c_h$, it indicates that all participating suppliers are of high-cost type. They continue to bid c_h in the second period. Hence, analyzing that case is uninteresting.

If at least one of the suppliers is a low-cost type, then $p_w < c_h$. In that case, let us map the second period game to SPG. Set R_1 to be the winner, and R_o to represent every loser. Because the winner's bid is announced and all the other suppliers observe the winning bid $p_w < c_h$, they learn that the winner is a low-cost type, i.e., $\beta = 1$. However, the first period winner continues to be uncertain about the cost-types of other suppliers. Let the winner's belief about each of the other suppliers being a high-cost type be $x(p_w)$ and let it be updated in a Bayesian manner:

$$x(p_w) = \frac{\overbrace{(1 - \theta)}^{\text{Prob. of a supplier being a high-cost type}}}{\underbrace{1 - \theta + \theta(1 - F_{1,\text{IIS}}(p_w))}_{\text{Prob. of outbidding the supplier with a bid } p_w}}, \quad (3)$$

where $F_{1,\text{IIS}}(p)$ is the cdf of the first-period bid distribution at equilibrium. Hence, the common knowledge of p_w translates into the common knowledge of $x(p_w)$ since bidders know the first period bid distribution. So, $\alpha = 1 - x(p_w)$. With those values for α and β , our SPG analysis shows that the expected profits for any low-cost supplier is $(x(p_w))^n(c_h - c_l)$ in the second period.

Next, we compute the first period equilibrium. If the supplier wins with a bid p_1 in the first period, the conditional and unconditional second period profits are straightforward to compute since $p_w = p_1$. If the supplier loses with a bid p_1 , the expected second period profit has to consider all possible $p_w < p_1$ and the probability of the winning bid being p_w . Thus, the expected profit across both periods from bidding p_1 in the first period is:

$$\begin{aligned} \Pi_{\text{IIS}}(p_1) = & \overbrace{(1 - \theta + \theta(1 - F_{1,\text{IIS}}(p_1)))^n (p_1 - c_l)}^{\text{First period profits from bidding } p_1} + \overbrace{(1 - \theta + \theta(1 - F_{1,\text{IIS}}(p_1)))^n (x(p_1))^n (c_h - c_l)}^{\text{Second period profits when he wins the first period with } p_1} \\ & + \underbrace{\int_{p_l^{\text{IIS}}}^{p_1} (x(p))^n (c_h - c_l) \sum_{k=1}^n \binom{n}{k} (1 - \theta)^{(n-k)} \theta^k \underbrace{k f_{1,\text{IIS}}(p) (1 - F_{1,\text{IIS}}(p))^{k-1}}_{\text{Prob. that one of them wins with } p} dp}_{\substack{\text{Prob. of } k \text{ low-cost rivals} \\ \text{Second period profits when the supplier loses the first period with } p_1}} \end{aligned} \quad (4)$$

where $f_{1,\text{IIS}}(p)$ is the probability density function (pdf) of the first period bid distribution and p_l^{IIS} is the infimum of the first period strategy set.

Using the above expressions, we compute (in Appendix C.1) the first period equilibrium bid distribution, $F_{1,\text{IIS}}(p)$, to be a solution to the following non-linear equation:

$$(1 - \theta)^n (1 - n \log(1 - \theta)) (c_h - c_l) = (1 - \theta + \theta(1 - F_{1,\text{IIS}}(p)))^n (p - c_l) - (1 - \theta)^n n (c_h - c_l) \log(1 - \theta + \theta(1 - F_{1,\text{IIS}}(p))). \quad (5)$$

Although $F_{1,\text{IIS}}(p)$ cannot be expressed in closed-form, the distribution can be computed numerically. However, from Equation 5, one can analytically derive comparative results such as, $\frac{\partial F_{1,\text{IIS}}(p)}{\partial n} \geq 0$, $\frac{\partial F_{1,\text{IIS}}(p)}{\partial \theta} \geq 0$, validating our intuition that, as the intensity of competition increases, bids tend to be lower. Among the sensitivity results, the most interesting one is:

Lemma 1 *Consider two settings which may have a different θ and/or n . If the winning bid is the same in both settings, the winner's second belief that none of the suppliers is a low-cost type across the two settings is the same, and independent of both θ and n .*

Under IIS, bidders do not have an incentive to deviate from the separating equilibrium in the first period. We next present an informal argument to explain why a pooling strategy, which is to bid c_h , is not sustained. Suppose a low-cost supplier deviates in the first period and bids $p = c_h$. He is worse off now than before when none of his opponents is a low-cost type. Even if at least one

low-cost supplier is present, he loses the first period. Since no other bid apart from the winner's is observed anyway, the pooled bid does not serve to alter the second period beliefs of the winner. In that words, he is not better off in the latter case either. Therefore, by pooling, the bidder is actually worse-off. The interesting aspect of our result is that even though suppliers have an opportunity to signal their types differently, they have no incentive to do so. Because suppliers do not signal their types differently, our result under IIS is mathematically similar to that from Arora et al. (2007). Note that suppliers in Arora et al. (2007) do not even have the opportunity to signal their types differently in that context.

As a consequence of the equilibrium, the total expected profits for a low-cost supplier across both the periods in IIS is:

$$\Pi_{\text{IIS}} = (1 - \theta)^n (2 - n \log(1 - \theta))(c_h - c_l). \quad (6)$$

Compared to a single period version of the same, the bid distributions in the first period are skewed to the right. This is because, under IIS, suppliers are willing to lose in the first period in order to gain the informational advantage over the winner. This effect, which we refer to as the *extraction effect*, accounts for the supplier's action to increase his bid in order to extract information or improve his information endowment about his opponents' types for the future periods.

4.2.2 CIS

Under this policy, recall that all bids are revealed. For the equilibrium analysis, we initially consider the case when only a separating equilibrium exists in the first period. The separating equilibrium is valid only under a certain condition. Eventually, we compute the alternate equilibrium.

Separating Equilibrium

Suppose a separating equilibrium exists in the first period. Then, since all bids are revealed under this policy, the second period is a Bertrand game. The expected second period profit for each low-cost supplier is: $\Pi_{2,\text{CIS}}^{\text{sep}} = (1 - \theta)^n (c_h - c_l)$.

We next consider the first period equilibrium. The total expected profits across both periods,

if $F_{1,\text{CIS}}^{\text{sep}}(p_1)$ represents the cdf of the first period bid distribution for $p_1 < c_h$, is:

$$\Pi_{\text{CIS}}^{\text{sep}}(p_1) = \overbrace{\left(1 - \theta + \theta(1 - F_{1,\text{CIS}}^{\text{sep}}(p_1))\right)^n (p_1 - c_l)}^{\text{First period profits from bidding } p_1} + \overbrace{(1 - \theta)^n (c_h - c_l)}^{\text{Second period profits when he is the only low-cost type}}. \quad (7)$$

The second period payoff is a constant independent of p_1 and can be ignored for equilibrium calculations. Without the term, the expected profit expressions are identical to SPG with $\alpha = \beta = \theta$ and hence, the equilibrium bid distributions are also the same.

The total expected profits across both periods in this case is:

$$\Pi_{\text{CIS}}^{\text{sep}} = 2(1 - \theta)^n (c_h - c_l). \quad (8)$$

We verify if the separating equilibrium is always valid in the first period. By comparing the expected profits between the separating equilibrium and the faking strategies, we find in Appendix D.1 that a separating equilibrium in the first period only exists for $\theta \leq \theta_n^{\text{fake}} = \frac{1}{2+n}$. In the same appendix, we show that only a semi-pooling strategy exists for $\theta > \theta_n^{\text{fake}}$. A semi-pooling equilibrium is one where a low-cost supplier mixes with bids $p < c_h$ and also $p = c_h$ (which is to fake).

The existence of a semi-pooling equilibrium implies that a supplier has an incentive to fake in order to inhibit his opponent from gaining information regarding the faking supplier's type. By faking and deceiving his opponent, the supplier gains an opportunity to "low-ball" his opponents in the second period and gain. The resulting profit is higher than if the low-cost supplier reveals his type in the first period (so long as $\theta > \theta_n^{\text{fake}}$). We refer to this faking as the *deception effect*.⁹

Semi-Pooling Equilibrium

In this subsection, we characterize the semi-pooling equilibrium. For a given θ and n , let $\gamma_{\theta,n}$ represent the probability with which suppliers fake, i.e., the probability with which a low-cost supplier bids $p = c_h$, in the first period. Because a pooling equilibrium does not exist, $\gamma_{\theta,n} < 1$. The first period equilibrium computation involves characterizing $\gamma_{\theta,n}$ in addition to characterizing the expression for how a low-cost supplier bids $p < c_h$, the cdf of which we denote by $F_{1,\text{CIS}}^{\text{semi}}(p)$.

The first period equilibrium is driven by the second period equilibria, which are computed as

⁹Hausch (1986) also finds that suppliers have an incentive to deceive in his common-value setup although its characteristic is different.

Case #	Possibilities for the First Period Game	Relationship to SPG	Second Period Profits for a Low-Cost Type
1.	All suppliers bid $p = c_h$. No one bids $p < c_h$.	Symmetric Game; Set $\alpha = 1 - y$ and $\beta = 1 - y$.	$y^n(c_h - c_l)$
2.	Only one of the suppliers bids $p < c_h$. All others bid $p = c_h$. (The low-cost types may even be faking.)	Asymmetric Game; Set $\alpha = 1 - y$ for the winner. $\beta = 1$ for the other low-cost types.	$y^n(c_h - c_l)$
3.	At least two suppliers bid $p < c_h$.	None. In equilibrium, the low-cost suppliers bid $p = c_l$.	0

Table 3: The second period game under CIS.

follows. If a low-cost supplier does not observe any price $p < c_h$ bid in the first period, he updates his second period belief that each of its opponents is a high-cost type to

$$y = \frac{(1 - \theta)}{1 - \theta + \theta \gamma_{\theta,n}}$$

in a Bayesian manner. Instead, if he observes a price $p < c_h$ in the first period, the second period belief about having low-cost supplier is 1.

Depending on whether in the first period all suppliers faked, only one supplier reveals to be a low-cost type, or more than one suppliers revealed their low cost structures, the outcome of the second period game is different. Table 3 shows the three possible games in the second column. Corresponding to each game, the third column matches the beliefs from each of the cases in SPG. In the last column, we show the expected supplier profits for each case. These second period profits are taken into account while characterizing the first period equilibrium.

The total expected profit across both periods is expressed as follows. The profit for supplier i from bidding $p_1 < c_h$:

$$\Pi_{\text{CIS}}^{\text{semi}}(p_1 | p_1 < c_h) = \overbrace{\left(1 - \theta + \theta(1 - F_{1,\text{CIS}}^{\text{semi}}(p_1))\right)^n (p_1 - c_l)}^{\text{First period profits from bidding } p_1 < c_h} + \underbrace{\left(1 - \theta + \theta \gamma_{\theta,n}\right)^n y^n (c_h - c_l)}_{\text{Second period profits: case 2 in Table 3; } i \text{ is the only supplier bidding } p_1 < c_h}. \quad (9)$$

The profit for supplier i from bidding $p_1 = c_h$ is:

$$\begin{aligned}
\Pi_{\text{CIS}}^{\text{semi}}(c_h) &= \overbrace{\left((1 - \theta + \theta\gamma_{\theta,n})^n \frac{(c_h - c_l)}{n+1} + [1 - \theta + \theta\gamma_{\theta,n}]^n y^n(c_h - c_l) \right)}^{\text{All the suppliers bid } p = c_h} + \overbrace{\left(y^n(c_h - c_l) \sum_{k=1}^n \left(\binom{n}{k} (1 - \theta)^{n-k} \theta^k \right) \left(k(1 - \gamma_{\theta,n}) \gamma_{\theta,n}^{k-1} \right) \right)}^{\text{Second period profits: Case 2 in Table 3 when some supplier } j \neq i \text{ is the only supplier bidding } p_1 < c_h} \\
&\quad \text{First period profits} \quad \text{Second period profits: Case 1 in Table 3} \\
&\quad \text{At least one opponent exists} \quad \text{Prob. of having } k \text{ low-cost opponents} \quad \text{Only one of them does not fake}
\end{aligned} \tag{10}$$

Using the above two expressions, we compute the first period equilibrium (see Appendix D.2). The semi-pooling equilibrium includes a bid distribution $F_{1,\text{CIS}}^{\text{semi}}(p)$ according to which a supplier bids a price $p < c_h$ and the probability $\gamma_{\theta,n}$ with which he fakes. The two terms are solutions to the following two equations:

$$(1 - \theta + \theta\gamma_{\theta,n})^n (c_h - c_l) = (1 - \theta + \theta(1 - F_{1,\text{CIS}}^{\text{semi}}(p)))^n (p - c_l), \tag{11}$$

$$(1 - \theta + \theta\gamma_{\theta,n})^{n+1} = (n+1)(1 - \gamma_{\theta,n})(1 - \theta)^n \theta. \tag{12}$$

Based on the equilibrium analysis, the total expected profit for a low-cost supplier under CIS when only a semi-pooling equilibrium exists is:

$$\Pi_{\text{CIS}}^{\text{semi}} = ((1 - \theta + \theta\gamma_{\theta,n})^n + (1 - \theta)^n)(c_h - c_l). \tag{13}$$

Note that, from Equation 12, we cannot solve $\gamma_{\theta,n}$ for an arbitrary n . This is due to the Abel–Ruffini impossibility theorem, which states that no general solution exist *in radicals* for a polynomial equation of order five or more. The limitation perhaps might be the reason why prior papers have only considered $n = 1$ when a quadratic equation can be solved in radicals. Despite the limitation, we compare the policies using the properties of Equation 12 presented next.

In general, the variation of $\gamma_{\theta,n}$ with respect to θ is similar to the one in Figure 1. The function is zero and increasing at θ_n^{fake} (recall that it is the value of θ beyond which the semi-pooling equilibrium is valid); attains a maxima; and again reaches zero at $\theta = 1$. Let γ_n^{max} be the maximum value attained for a given n and $\theta_n^{\text{max}\gamma}$ be the θ at which the maxima is attained.

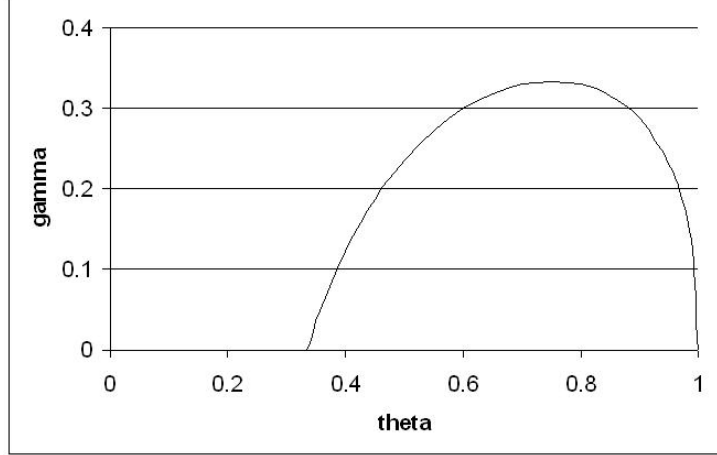


Figure 1: Variation of $\gamma_{\theta,1}$ assuming $c_l = 0$, $c_h = 1$.

Proposition 1 *For any arbitrary n , the following properties of Equation 12 are observed:*

1. $\gamma_{\theta,n} > 0 \forall \theta \in (\theta_n^{fake}, 1)$. There is only one point of inflection, which occurs at $\theta_n^{\max\gamma} = \frac{1}{1 + \frac{n^{1+n}}{n^n + (1+n)^n}}$.
2. $\frac{\partial \theta_n^{fake}}{\partial n} < 0$.
3. For $\theta > \theta_1^{\max\gamma}$, $\gamma_{\theta,1} \geq \gamma_{\theta,n}$ for any $n > 1$.

(See Appendix D.3 for the proof.) We will use these properties when comparing the policies in the following section.

Compared to the single period, the bid distribution is skewed to the right here again. In fact, the bid distributions under CIS can be shown to be stochastically dominant over that for the single period game. The skew is because of the deception effect we identified earlier.

5. Comparisons

In this section, we compare the policies on the procurer surplus, the expected supplier profits, and the social welfare generated. In doing so, we are implicitly comparing the welfare implication of the two learning effects. As we will show soon, the procurer surplus and the supplier profit comparisons are quite related. So, we first deal with them. Following that, we compare the social welfare generated. Note that, independent of the policies, the metrics are identical across the policies for $\theta = 1$ and so, we ignore this uninteresting case in our results.

If SP_ϕ represents the unconditional supplier profits across both periods under policy ϕ , where $\phi \in \{\text{IIS}, \text{CIS}\}$, then:

$$SP_\phi = \theta\Pi_\phi + (1 - \theta)0$$

where Π_ϕ is the expected supplier profits conditional on a supplier being a low cost type and is characterized by Equations 6, 8 and 13 under the respective policies. From the above expression, it is sufficient to compare Π_ϕ for comparing the expected supplier profits.

Based on the expression for the unconditional profits for each bidder, we compute the total expected payment by the procurer across both periods, EP_ϕ . Under the three cases, they are:

$$\begin{aligned} EP_{\text{IIS}} &= 2c_l + (n + 1)\theta\Pi_{\text{IIS}} + 2(1 - \theta)^{n+1}(c_h - c_l), \\ EP_{\text{CIS}}^{\text{sep}} &= 2c_l + (n + 1)\theta\Pi_{\text{CIS}}^{\text{sep}} + 2(1 - \theta)^{n+1}(c_h - c_l), \\ EP_{\text{CIS}}^{\text{semi}} &= 2c_l + (n + 1)\theta\Pi_{\text{CIS}}^{\text{semi}} + (2(1 - \theta)^{n+1} + \Delta)(c_h - c_l), \end{aligned}$$

where $\Delta \geq 0$ is the probability a high-cost type wins in the first period of CIS when at least one low supplier is present. From these expressions, we see that comparing the procurer surplus between CIS-separating equilibrium and IIS is the opposite of comparing their respective expected supplier profits. It should also be obvious from the expressions that, whenever CIS-separating equilibrium generates higher expected supplier profits, the expected payment for the buyer is also higher. Therefore, comparing Π_ϕ is also relevant to the procurer surplus comparisons.

5.1 Comparing Π_ϕ

The result from comparing CIS under the separating equilibrium and IIS is straightforward.

Theorem 1 $\Pi_{\text{IIS}} > \Pi_{\text{CIS}}^{\text{sep}}$ and $EP_{\text{IIS}} > EP_{\text{CIS}}^{\text{sep}}$ for $\theta \in [0, \theta_n^{\text{fake}})$.

(Proofs for all the results in Section 5 are available in Appendix E.) Recall that the extraction effect exists under IIS but no such effect plays a role under CIS when separating equilibrium is valid. As a result, the bids tend to be higher under IIS. Thus, the extraction effect is the primary cause for the theorem. Note that the comparison in the above theorem has been executed for any n but the result under the semi-pooling equilibrium case is not as straightforward.

Var.	Explanation
θ	Probability of a supplier being a low-cost type.
θ_n^{fake}	For a given n , faking occurs in CIS for $\theta > \theta_n^{\text{fake}}$.
$\theta_n^{\text{max}\gamma}$	For a given n , θ when the γ attains the maximum.
θ_n^{equal}	For a given n , the θ value when profits from CIS under the semi-pooling equilibrium case equals that under IIS.

Table 4: Different representations of θ which we employ when comparing the policies.

Let us introduce a variable θ_n^{equal} to represent, for a given n , the value of θ when $\Pi_{\text{CIS}}^{\text{semi}} = \Pi_{\text{IIS}}$. (For the benefit of the reader, Table 4 shows the different representations of θ used during the comparisons.) Let us first execute the comparison for the case when $n = 1$.

Proposition 2 *For $n = 1$, there exists a $\theta_1^{\text{equal}} < 0.87$ such that $\Pi_{\text{CIS}}^{\text{semi}} > \Pi_{\text{IIS}}$ and $EP_{\text{CIS}}^{\text{semi}} > EP_{\text{IIS}}$ for $\theta > \theta_1^{\text{equal}}$ and; $\Pi_{\text{CIS}}^{\text{semi}} < \Pi_{\text{IIS}}$ for $\theta_1^{\text{fake}} < \theta < \theta_1^{\text{equal}}$.*

Note that the proof *analytically* establishes that there can be only one value of θ when $\Pi_{\text{CIS}}^{\text{semi}} = \Pi_{\text{IIS}}$. Numerical calculations are only needed to establish the directionality of the difference.

Combining the proposition with the last theorem, notice that there are three regions depending on the nature of the equilibrium under CIS and the policy comparison outcomes: (i) $\theta \in [0, \theta_1^{\text{fake}})$ where the separating equilibrium exists and IIS generates higher supplier profits; (ii) $\theta \in [\theta_1^{\text{fake}}, \theta_1^{\text{equal}})$ where the semi-pooling equilibrium exists but IIS continues to generate higher supplier profits; and (iii) $\theta \in [\theta_1^{\text{equal}}, 1)$ where the semi-pooling equilibrium exists and CIS generates higher supplier profits. When $n = 1$, we can compute $\theta_1^{\text{fake}} = \frac{1}{3}$ and $\theta_1^{\text{equal}} \approx 0.8688$. As we discuss below, θ_n^{equal} can even be less than 0.5.

Unlike in the previous theorem, here, either policy may be dominant. Hence, recommending one policy over the other is not quite correct. Actually, the choice should depend on the value that the probability of observing a low-cost opponent takes. Notice that the outcome of the comparison is driven by the learning effects we demonstrated earlier. The observation can be explained as follows. When the probability of a low-cost type is high, there is lesser motive for a player to learn about its opponent. However, the incentive to fake is still dominant. In contrast, for lower values of θ , the deception effect is not the dominant enough to decrease the procurer surplus.

The next question that arises is whether our result is robust for any arbitrary n . Such a question is especially valid since even the nature of the equilibrium (separating or semi-pooling)

changes depending on n . Moreover, even the probability of faking for a given θ is not monotonic in n . This leads to additional difficulties in proving the results. The trick to overcoming the difficulty is to focus on the sensitivity of the difference with respect to n by building on the result established in the previous proposition for $n = 1$.

Theorem 2 *Suppose $n > 0$. There always exists a $\theta_n^{equal} < 0.87$ such that $\Pi_{IIS} < \Pi_{CIS}^{semi}$ and $EP_{IIS} < EP_{CIS}^{semi}$ for any $\theta \in [\max\{\theta_1^{\max\gamma}, \theta_n^{equal}\}, 1)$.*

Thus, the results from the previous proposition qualitatively hold even for any n .¹⁰

The above analysis does not show how low θ_n^{equal} can be. It is reasonable to consider that, if the range $(\theta_n^{equal}, 1]$ is quite small, nothing much is lost from a procurer standpoint if we always choose CIS. To assess how small θ_n^{equal} can be, we did some numerical experiments.¹¹ Our experiments demonstrate that θ_n^{equal} decreases with n . We found that θ_n^{equal} is even less than 0.5 when $n > 16$. This means that, even if the probability of a low-cost type is a simple coin toss, the procurer should consider revealing all the bids if the market involves 17 or more suppliers. This is not a large number. In Freemarkets, when a procurer initiates a market-session seeking suppliers to be cast, 30 or more of them participate. This example illustrates the importance of considering either policy. Next, suppose the procurer is unaware of our recommendation and chooses to always adhere to one policy. How serious will the impact be? In some regards, the answer to this question explains the contribution of our paper. Again, through the same numerical experiments, for $n = 1$, we found that the expected payments for the procurer can be as large as 68% if CIS is always chosen and 25% if IIS is always chosen. These numerical analyses show the value of considering the trade-off between the extraction and the deception effects. We are not aware of any prior work that have shown the existence of such a trade-off. These examples also illustrate the problem with concluding that one policy is always better than the other.

Next, we investigate how the two effects are driving the bidding behaviors of the suppliers. The following theorems provide some insights in that regard:

¹⁰Suppose in the original model, we introduce another Bernoulli process for each of the $n + 1$ suppliers to decide on its participation. Even in that case, based on the results already established, we can argue that: (i) Bidder profits under IIS are at least as high as under CIS for $\theta \leq \frac{1}{n+2}$; and (ii) Bidder profits are under CIS are at least as high as under IIS for $\theta \geq 0.87$.

¹¹Note that every aspect of our paper holds if we normalize our cost values and hence we will assume the same for the rest of the discussion in this paragraph.

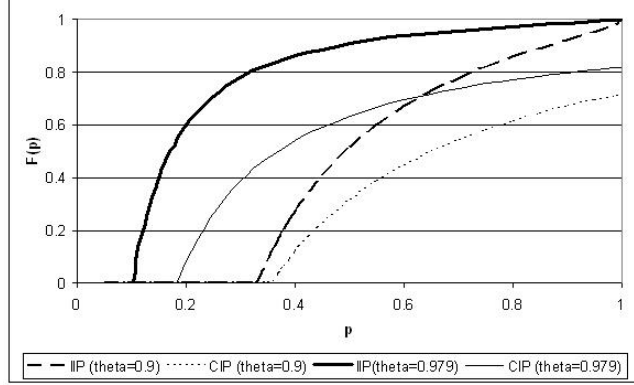


Figure 2: Bid distribution under both IIS and CIS for two different values of $\theta = \{0.9, 0.979\}$, $n = 1$, $c_h = 1$ and $c_l = 0$.

Theorem 3 For $\theta > \theta_n^{equal}$, the first period bid distribution under CIS first order stochastically dominates that under IIS.

Theorem 4 For $\theta < \theta_n^{fake}$, the first period bid distributions under IIS first order stochastically dominates that under CIS.

Figure 2 provides a visual idea how the bid distributions compare under the case corresponding to Theorem 3. In the figure, under both the policies, as θ increases, the bid distributions tend to be lower, consistent with our expectation. If one considers the first period bid distribution under IIS, notice that the mass of the cdf in the first period tends to be on lower bids when $\theta > \theta_n^{equal}$. Thus, the first order stochastic dominance results proved in the theorem can be seen. The stochastic dominance result also affects the second period profit comparisons. For $\theta > \theta_n^{equal}$, because of the stochastic dominance result, the beliefs held by the winner tend to be lower under IIS. This also leads to lower second period profits under IIS than under CIS. Therefore, we observe the expected profit comparison in Theorem 2. A similar explanation is valid for $\theta < \theta_n^{fake}$.

5.2 Comparing the Social Welfare

In this subsection, we are interested in comparing the efficiencies of the policies. Let SW_ϕ represent the social welfare generated under a specific policy $\phi \in \{\text{IIS}, \text{CIS}\}$. It actually represents the total welfare of the procurers and the sellers. Note that the procurer utilities are identical across the policies, prices are simply transfer of rents, and only the costs of the winning-suppliers matter. So, the focus of social welfare comparison is the probability of a high-cost supplier winning. Formally:

Theorem 5 $SW_{IIS} = SW_{CIS}^{sep}$ for $\theta \in [0, \theta_n^{fake}]$. $SW_{IIS} > SW_{CIS}^{semi}$ for $\theta \in (\theta_n^{fake}, 1)$.

It is clear that CIS generates lesser social welfare than IIS. This is so because suppliers in CIS have an incentive to fake. When faking occurs, the probability of a high-cost type winning is non-zero, which in turn leads to a lower social welfare. It is important to distinguish here the impact of the extraction effect as opposed to the faking effect. Note that, while bids are higher under IIS because of the extraction effect, they are never equal to c_h . This is the reasoning behind why CIS is not socially optimal. These insights may be useful to a policy maker. Note that the government, which is typically considered a social welfare maximizer, often conducts auctions which are similar to CIS. One example is the municipal auctions, which we described earlier. Another example is the auctioning of sulfur dioxide emission permits by the environmental protection agency (EPA). Here, the emission rights are packaged into smaller sets and sequentially auctioned using a first-price sealed bid mechanism. At the end of each auction, all of the sealed bids are revealed, consistent with the complete information policy analyzed in our paper (Culligan, 2008). Our results demonstrate that such a complete revelation of bids can lead to decreasing social welfare even if the number of competitors increases.

When studying the sensitivity of the social welfare to different parameters, we observed the following counterintuitive result:

Theorem 6 For every $\theta \in [0, \theta_1^{fake}]$, there exist n_1 and n_2 , such that $\frac{\partial SW_{CIS}}{\partial n} < 0$ for $n_1 \leq n < n_2$.

One would expect that as the number of competitors increases, the efficiency also increases. With respect to that, the above theorem may be surprising. The key point to note to this that the deception effect under CIS blunts the ferocity of competition and enables the suppliers to submit the high bids, resulting in decreased social welfare. From a technical standpoint, the result is a consequence of θ_n^{fake} decreasing with n (also shown as Property 2 in Proposition 1). Fix the number of suppliers to be n . For an arbitrary $\theta' \in (\theta_{n+1}^{fake}, \theta_n^{fake})$, the faking probability is zero. For the same θ' , increase the number of suppliers to $(n + 1)$. Then, the suppliers start to fake. Thus, the increased competition can be seen to decrease social welfare.

6. Generalization

This section demonstrates that the results we have already established hold in more generic settings. In the first subsection, we consider the scenario where there are three cost types. In the second

subsection, we consider a stylized model to study the sensitivity of the faking effect to the direct benefit from faking.

6.1 Three Cost Type

Consider the original model specification in Section 3 with some modifications. Instead of the two cost type model, we assume that each supplier can be one of three types c_l , c_m , or c_h . We will focus on the special case of $n = 1$ (i.e., a two bidder game), and normalized costs. Let c_l type incur $c_l = 0$ cost; c_h incur $c_h = 1$; and c_m type incur $c_m = c$ with $0 < c < 1$. The probability that a supplier's cost type is c_l is $\{l | 0 \leq l \leq 1\}$; c_m is $\{m | 0 < m \leq 1\}$; and c_h is $\{1 - l - m | l + m \leq 1\}$. The mathematical details turn out to be such that the results in the original model can be retrieved by setting $l = 0$. As in the original model, we implicitly assume throughout the discussion that discontinuities exist in the strategy space to avoid the equilibrium non-existence problem. All the newly introduced variables are assumed to be common knowledge. In this game, at equilibrium, a supplier of cost type c_h always bids 1. We are focused on the bidding behavior of the other two types. (See Appendix F for additional details.)

6.1.1 Second Period Game

There are four scenarios in the second period game which are relevant to our analysis. In all the four scenarios, whenever there is uncertainty about the opponent's type, let l' and m' be the beliefs that a supplier has about the opponent being of type c_l and c_m , respectively.

In Scenario 1, suppliers know each other's cost type and the scenario resembles a Bertrand game. In Scenario 2, the uncertainty that each supplier has about his opponent is identical. At equilibrium, a supplier of type c_l and c_m respectively generate profits of $\Pi_{c_l}^{\text{Sc:2}} = c(1 - l') + (1 - l' - m')(1 - c)$ and $\Pi_{c_m}^{\text{Sc:2}} = (1 - l' - m')(1 - c)$. The subscripts indicate the cost type, and the superscript shows the scenario. In Scenario 3, only one supplier has his type revealed to the other and let the revealed type be c_m . At equilibrium, the profits for the different players are: $\Pi_{c_m}^{\text{Sc:3}; R} = (1 - l' - m')(1 - c)$, $\Pi_{c_m}^{\text{Sc:3}; H} = \frac{(1 - l' - m')}{1 - l'}(1 - c)$ and $\Pi_{c_l}^{\text{Sc:3}; H} = c + \frac{(1 - l' - m')}{1 - l'}(1 - c)$. The additional term R or H in the superscript indicates whether the supplier's type is revealed or not. In Scenario 4, only one supplier has his type revealed to the other and let the type revealed be c_l . Two different equilibria are feasible depending on whether or not $(1 - l' - m') > (1 - l')c$. If the condition is valid, the equilibrium profits are as follows: $\Pi_{c_l}^{\text{Sc:4}(1); R} = (1 - l' - m')$, $\Pi_{c_m}^{\text{Sc:4}(1); H} = \frac{(1 - l' - m')}{1 - l'} - c$, and $\Pi_{c_l}^{\text{Sc:4}(1); H} =$

$\frac{(1-l'-m')}{1-l'}$. Otherwise, the following are the equilibrium profits: $\Pi_{c_l}^{\text{Sc:4-(2); R}} = \Pi_{c_l}^{\text{Sc:4-(2); H}} = (1-l')c$, and $\Pi_{c_m}^{\text{Sc:4-(1); H}} = 0$.

6.1.2 First Period Game: IIS

It turns out that only a separating equilibrium exists in the first period here. The profit expressions for the equilibrium computations are:

$$\begin{aligned} \pi_{\text{IIS}, c_m}(p) &= \overbrace{(1-l-m)(p-c + \Pi_{c_m}^{\text{Sc:3; R}})}^{\text{Opponent is type } c_h} + \overbrace{m(1-F_m(p))(p-c + \Pi_{c_m}^{\text{Sc:3; R}})}^{\text{Opponent is type } c_m \text{ but he lost first period}} \\ &\quad + \underbrace{mF_m(p)(0 + \Pi_{c_m}^{\text{Sc:3; H}})}_{\text{Opponent is type } c_m \text{ but he won first period}} + \underbrace{l\Pi_{c_m}^{\text{Sc:4; H}}}_{\text{Opponent is type } c_l} \\ \pi_{\text{IIS}, c_l}(p) &= \overbrace{((1-l)(p + \Pi_{c_l}^{\text{Sc:4; R}}))}^{\text{Opponent is not type } c_l} + \overbrace{l(1-F_l(p))(p + \Pi_{c_l}^{\text{Sc:4; R}})}^{\text{Opponent is type } c_l \text{ but he lost first period}} + \overbrace{lF_l(p)(0 + \Pi_{c_l}^{\text{Sc:4; H}})}^{\text{Opponent is type } c_l \text{ but he won}} \end{aligned}$$

Assuming Bayesian update for the beliefs, the equilibrium for $F_m(p)$ can be computed. The infimum of the strategy set is $p_{l,m}^{\text{IIS}} = c + (1-c)\frac{(1-l-m)}{1-l}(1 - \log(\frac{1-l-m}{1-l}))$. The total profit for type c_m is: $(1-l-m)(1-c)(2 - \log(\frac{1-l-m}{1-l})) + x\Pi_{c_m}^{\text{Sc:4; H}}$. We discuss the expansion on $\Pi_{c_m}^{\text{Sc:4; H}}$ next.

In Scenario 4, $m' = m$ and $l' = \frac{l(1-F_l(q))}{1-lF_l(q)}$. Let $p_{l,l}^{\text{IIS}}$ be such that $F_l(p_{l,l}^{\text{IIS}}) = 0$. Computing the profit expressions for that scenario has to consider different conditions.

1. If $(1-l-m) > (1-l)c$, then $(1-l'-m') > (1-l')c$. So, $\Pi_{c_l}^{\text{Sc:4; R}} = \frac{1-l}{1-lF_l(p)} - m$, $\Pi_{c_l}^{\text{Sc:4; H}} = \frac{1}{F_l(p)} \int_{p_{l,l}^{\text{IIS}}}^p (\frac{1-l-m(1-lF_l(q))}{1-l}) f_l(q) dq$ and $\Pi_{c_m}^{\text{Sc:4; H}} = \frac{1}{F_l(p)} \int_{p_{l,l}^{\text{IIS}}}^p (\frac{1-l-m(1-lF_l(q))}{1-l} - c) f_l(q) dq$.
2. If $(1-c) < m$, then $(1-l'-m') < (1-l')c$. So, $\Pi_{c_l}^{\text{Sc:4; R}} = \frac{(1-l)c}{1-lF_l(p)}$, $\Pi_{c_m}^{\text{Sc:4; H}} = 0$, and $\Pi_{c_l}^{\text{Sc:4; H}} = \frac{c}{F_l(p)} \int_{p_{l,l}^{\text{IIS}}}^p (\frac{1-l}{(1-lF_l(q))}) f_l(q) dq$.
3. Otherwise, two sub-cases arise. Let p_e be such that $F_l(p_e) = \frac{1}{l}(1 - \frac{(1-l)(1-c)}{m})$
 - If $p \leq p_e$, $\Pi_{c_m}^{\text{Sc:4; H}} = 0$, $\Pi_{c_l}^{\text{Sc:4; R}} = \frac{(1-l)c}{1-lF_l(p)}$, and $\Pi_{c_l}^{\text{Sc:4; H}} = \frac{c}{F_l(p)} \int_{p_{l,l}^{\text{IIS}}}^p (\frac{1-l}{(1-lF_l(q))}) f_l(q) dq$.
 - For $p > p_e$, $\Pi_{c_l}^{\text{Sc:4; R}} = \frac{1-l}{1-lF_l(p)} - m$, $\Pi_{c_m}^{\text{Sc:4; H}} = \frac{1}{F_l(p)} \int_{p_e}^p (\frac{1-l-m(1-lF_l(q))}{1-l} - c) f_l(q) dq$, and $\Pi_{c_l}^{\text{Sc:4; H}} = \frac{c}{F_l(p)} \int_{p_{l,l}^{\text{IIS}}}^{p_e} (\frac{1-l}{(1-lF_l(q))}) f_l(q) dq + \frac{1}{F_l(p)} \int_{p_e}^p (\frac{1-l-m(1-lF_l(q))}{1-l}) f_l(q) dq$.

Now, to compute the bid distribution in each case, we invoke the property that the strategies in the mixed equilibrium yield the same profit and that $p_{l,m}^{\text{IIS}}$ is the supremum of the strategy space for type c_l . The explicit expressions for the different cases have not been shown for the sake of

simplicity. We use the expression to compare against the bid distribution of a single period game. The single period game has the same bid distribution as Scenario 2 except for $l' = l$ and $m' = m$. One can show the stochastic dominance of IIS bid distribution in each case, validating the existence of the extraction effect. One can also prove deviations are not more profitable.

6.1.3 First Period Game: CIS Separating Equilibrium

Recall that all bids are revealed under CIS. If the separating equilibrium is valid in the first period, then the second period is a Bertrand game. The expected payoff for both cost types are:

$$\begin{aligned}\pi_{\text{CIS},c_m}^{\text{sep}}(p) &= (1-l-m)(p-c+1-c) + m(1-F_m(p))(p-c) \\ \pi_{\text{CIS},c_l}^{\text{sep}}(p) &= (1-l-m)(p+1) + m(p+c) + l(1-F_l(p))p\end{aligned}$$

The equilibrium bid distributions in this case are similar to Scenario 2 except for $l' = l$ and $m' = m$. The expected profits are $\pi_{\text{CIS},c_l}^{\text{sep}} = 2(1-l-m+mc)$ and $\pi_{\text{CIS},c_m}^{\text{sep}} = 2(1-l-m)(1-c)$.

Next, we show why the separating equilibrium does not always exist. Fixing the opponent's action, a bidder of c_m type generates a profit of $(1-l-m)(\frac{1-c}{2} + (1-c)) + m(1-c) + l(1-c)$, which is greater than $2(1-l-m)(1-c)$ if $3l + 3m > 1$. Similarly, fixing the opponent's action, c_l type fakes as c_h if $3l + (3+4c)m > 1$. Both these conditions are feasible in the valid range of l and m , proving the existence of the faking effect. The exact conditions when a separating equilibrium exists has to consider when one or both opponent types fake, and one can obtain that from the analysis of the semipooling equilibrium case. One can easily rule out a pooling equilibrium.

6.1.4 First Period Game: CIS Semipooling Equilibrium

Suppose γ_l and γ_m are the respective probabilities that each of type c_l and c_m bid like a c_h type. Let $F_{c_l}^{\text{semi}}(p)$ and $F_{c_m}^{\text{semi}}(p)$ be the cdfs of the bid distributions for c_l and c_m types, respectively. It can be shown that the lower support for type c_l is lower than that for type c_m . Assume for now that p_m^{CIS} is the lower end of the bid distribution for type c_m . In this case, the expected profits for the different types and different bids are:

$$\begin{aligned}\Pi_{c_m}^{\text{semi}}(p|p < 1) &= (1-l-m+l\gamma_l+m\gamma_m)(p-c+\Pi_{c_m}^{\text{Sc:3;R}}) + m(1-F_{c_m}^{\text{semi}}(p)-\gamma_m)(p-c) \quad (14) \\ \Pi_{c_m}^{\text{semi}}(p=1) &= (1-l-m+l\gamma_l+m\gamma_m)(\frac{1-c}{2}+\Pi_{c_m}^{\text{Sc:2}}) + m(1-\gamma_m)\Pi_{c_m}^{\text{Sc:3;H}} + l(1-\gamma_l)\Pi_{c_m}^{\text{Sc:1;H}}\end{aligned}$$

$$\begin{aligned} \Pi_{c_l}^{\text{semi}}(p|p < p_m^{\text{CIS}}) &= (1 - l - m + l\gamma_l + m\gamma_m)(p + \Pi_{c_l}^{\text{Sc:4; R}}) + m(1 - F_{c_m}^{\text{semi}}(p) - \gamma_m)(p + c) \\ &\quad + l(1 - F_{c_l}^{\text{semi}}(p) - \gamma_l)p \end{aligned} \quad (16)$$

$$\Pi_{c_l}^{\text{semi}}(p = 1) = (1 - l - m + l\gamma_l + m\gamma_m)\left(\frac{1}{2} + \Pi_{c_l}^{\text{Sc:2}}\right) + m(1 - \gamma_m)\Pi_{c_l}^{\text{Sc:3; H}} + l(1 - \gamma_l)\Pi_{c_l}^{\text{Sc:4; H}} \quad (17)$$

Equations 15 and 17 are the profit expressions when faking. We note that $F_{c_l}^{\text{semi}}(\approx 1) = 1 - \gamma_l$, $F_{c_m}^{\text{semi}}(\approx 1) = 1 - \gamma_m$, $F_{c_l}^{\text{semi}}(p_m^{\text{CIS}}) = 1 - \gamma_l$, and $F_{c_m}^{\text{semi}}(p_m^{\text{CIS}}) = 0$; and that the expected profits are the same across all prices in the strategy space, to compute the expected profits. In this case, the beliefs l' and m' are the beliefs held by the opponent of a non-revealing bidder (the opponent can be non-revealing also). Unlike in the previous policy, the second period beliefs are the same independent of the cost type holding it and it is so because all bids are revealed. Using Bayesian update, $(1 - l' - m') = \frac{(1-l-m)}{1-l(1-\gamma_l)-m(1-\gamma_m)}$ and $(1 - l') = \frac{1-l-m(1-\gamma_m)}{1-l(1-\gamma_l)-m(1-\gamma_m)}$. One has to solve the four equations subject to the constraints that $1 \geq \gamma_l \geq 0$ and $1 \geq \gamma_m \geq 0$ and the updated beliefs should be considered for the appropriate Scenario 4. If a solution does not exist, then the equilibrium is a separating one.

6.1.5 Numerical Comparison

c	l	m	γ_l	γ_m	Π^{CIS}	Π^{IIS}
0.5	0	0.1	0	0	0.09	0.095
		0.5	0	0.236	0.280	0.337
		0.9	0	0.287	0.206	0.193
	0.2	0.2	0	0.211	0.404	0.455
		0.75	0	0.250	0.291	0.216
		0.75	0.2	0.431	0.156	0.516
0.75	0.55	0.25	0.04	0.263	0.451	0.528

Table 5: Numerical Comparison

In this subsection, we show that one effect dominates the other using numerical simulations. Under each policy, we have already characterized the payoff for the bidder conditional on his type. We use that to compare the unconditional payoff for each bidder under either policy. We use Π^{CIS} to represent the unconditional bidder profit under CIS and Π^{IIS} as that under IIS. We continue to impose the restriction that $l + m < 1$. Table 5 shows the results from the numerical comparisons. The first three rows show the comparisons which are similar to that in the original model. Rows four and five show that either effect may dominate for nonzero l . The last two rows show examples

of γ_l being non-zero and how one effect may dominate the other.

6.2 Robustness of the Faking Effect

In the original model, the maximum feasible number of low-cost types is the same as the number of suppliers in the market. When the total numbers of suppliers in the market increases, faking may become less attractive and it could be that the original results may not be sustained. The question we address in this section is: how strong are the results if the direct benefit from faking decreases? Using a stylized model, we show below that our original results qualitatively continue to hold.

Suppose there are n_l low-cost suppliers in the world and all others are of high-cost type. Let θ be the probability with which a low-cost type participates in the auction. Assume that there are M participating suppliers and the value of M is common knowledge. Everything else from the original model is assumed to hold, including the same bidder characteristics across both periods.

If $M \leq n_l$, the original equilibrium results are applicable except that n in all the equations will be replaced with M . The reason for the results being similar is that the question considered by each of the suppliers is also similar in both cases: Which of the M (instead of n in the earlier formulation) suppliers are of low-cost types? Under this case, therefore, the extraction and the deception effects become relevant. Note that the deception effect occurs only when $\theta \geq \frac{1}{2+M}$.

When $M > n_l$, the equilibrium results are slightly different: (i) n in all the other equilibrium equations except Equations 10 and 12 are replaced with n_l . (ii) Equations 10 and 12 are respectively changed to the following:

$$\begin{aligned}
 \Pi_{\text{CIS}}^{\text{semi}}(c_h) &= [1 - \theta + \theta\gamma_{\theta, n_l}]^{n_l} \overbrace{\frac{(c_h - c_l)}{M + 1}}^{\text{Note the change}} + [1 - \theta + \theta\gamma_{\theta, n_l}]^{n_l} y^{n_l}(c_h - c_l) \\
 &\quad + \sum_{k=1}^{n_l} \binom{n_l}{k} (1 - \theta)^{n_l - k} \theta^k \left[k(1 - \gamma_{\theta, n_l}) \gamma_{\theta, n_l}^{k-1} \right] y^{n_l}(c_h - c_l) \\
 (1 - \theta + \theta\gamma_{\theta, n_l})^{n_l + 1} &= n_l(1 - \gamma_{\theta, n_l})(1 - \theta)^{n_l} \theta \underbrace{\frac{(M + 1)}{M}}_{\text{Note the change}}. \tag{18}
 \end{aligned}$$

The question considered is the same as the one in the original model: Which of the n_l suppliers are of low-cost types? However, the difference is that the suppliers have to consider M other suppliers when faking. From these equilibrium expressions, it is obvious that the deception and the extraction effects become relevant even in this case. From Equation 18, we can identify that the

deception effect occurs for $\theta > \frac{M}{n_l(M+1)+M}$, which we refer to as $\theta_{n,M}^{\text{fake}}$. We also introduce another notation $\theta_{n,M}^{\text{equal}}$ to refer to the θ when the supplier profits under CIS and IIS are equal.

Based on the equilibrium analysis, we find the following sensitivity results. First, as M increases, the incentive to fake decreases. Second, as M increases, $\theta_{n,M}^{\text{fake}}$. In fact, $\lim_{M \rightarrow \infty} \frac{M}{n_l(M+1)+M} = \frac{1}{n_l+1}$. This shows that even if the direct benefit from faking is zero, faking occurs for $\theta > \frac{1}{n_l+1}$. Even more surprising is the result that $\theta_{n,\infty}^{\text{equal}} \approx 0.95$. It means that for θ larger than that the supplier profits under CIS are higher than under IIS. Qualitatively, the results we had established in the base model qualitatively hold even under this special case. Combining the results from these two cases, we can establish that, independent of M , the expected bidder profits are higher under IIS for $\theta < \frac{1}{n_l+2}$ and, for $\theta > 0.95$, they are higher under CIS.

7. Discussion

We first discuss the implication of replacing the first price auction in our model setup with a second price auction. With the second price auction, the equilibrium strategies are as follows. Each supplier has an incentive to truthfully bid his valuation independent of the policy and it is procurer surplus maximizing in our context. However, in general, the second price auction is not always procurer surplus maximizing (Maskin and Riley, 2000a). Even if one considers the first price auction in those scenarios where the second price sealed bid is not optimal, the learning/signaling effects that we identify in this paper become relevant. So, our analysis becomes relevant even in such a scenario. Furthermore, the approach of employing a model with a first-price sealed bid so that the analysis can expose the key insights useful to a procurement manager is consistent with other prior works e.g., Arora et al. (2007), Bajari (2001) and Brosig and Reiß (2007). In each of the examples, the second price auction is the procurer surplus maximizing mechanism.

Our consideration of the first price auction is also a matter of relevance. The examples mentioned earlier, the municipal construction auctions as well as the government procurement auctions, are often conducted as first price sealed bid auctions. Even in the electronic marketplaces we interacted with, the second price auction or its equivalent, the open-cry auction, is not commonly adopted in practice. In fact, the second price auction is not available as an information policy to the procurers. We learned from Freemarkets that the open-cry policy is often not adopted in practice either. Similarly, during a discussion with AT Kearney, we learned that the software provided

by AT Kearney’s procurement group (which competes with Freemarkets) does not even enable an open-cry auction. It may be that the procurers do not find the second price auction surplus maximizing, a result which, as we mentioned earlier, has already been shown theoretically. For these reasons, the model we studied is also reasonable.

The main driver behind our results is how the beliefs held by the suppliers (about the presence of each of the others) evolve across the periods. Under IIS, the belief terms change because of the extraction effect while, under CIS, it is due to the faking effect. In IIS, a supplier alters his bid in order to affect his *own* belief and improve his information endowment about the presence of any other low-cost opponent. On other hand, the key reason for a supplier to fake in CIS is to influence his *opponent’s* belief and, specifically, to inhibit the competitor’s information endowment about himself. It should be clear that these two effects have different objectives.

The belief updates occur in our setup only because the supplier cost types are identical in both periods. If the cost types were randomly drawn in each period, there would be no benefit from making the belief changes. Then, independent of the information policy, the equilibrium strategies under the two period game would coincide with those under two single period games. Thus, the procurer surplus generated under the policies policies would be identical, making the policy comparisons uninteresting. In reality, from a supplier’s perspective, the cost-types drawn for every opponent is not independent across the periods. Often, they are consistent across periods. As mentioned earlier, anecdotal evidence from Freemarkets supports this claim (see also Arora et al., 2007). So long as the draws are not completely independent, suppliers have an incentive to alter the beliefs; but the degree to which they are altered depend on the consistency of draws across periods and, of course, the information policy. The suppliers under IIS continue to have an incentive to bid in a manner that creates an information advantage over the winner. Similarly, the suppliers in CIS also continue to have an incentive to fake. So, qualitatively the results should hold.

8. Conclusion

In conclusion, we study an important problem that was motivated by a real-world marketplace. Procurers in third party marketplaces have to choose an information policy for the market-sessions they initiate. Suppliers who face uncertainty regarding their opponents’ costs, respond by altering their bidding behaviors according to the policy. Thus, the choice of the policy affects the procurer

surplus. Given this, our paper focuses on providing managerial insights to procurers in choosing an appropriate policy. Specifically, our objective is to offer insights regarding the key effects affecting revelation policies which a procurer needs to consider when choosing the policy.

We accomplish our objective by analytically comparing the policies referred to as CIS and IIS. Our analysis reveals two important effects, the extraction and the deception effects, a procurer has to consider. The extraction effect impacts the bidding behavior under IIS, and the deception effect under CIS. When we compare the policies and the effects, we use the following metrics: the expected supplier profits, the expected procurer surplus and the social welfare generated. Loosely speaking, in our setup, the procurer surplus comparisons turn out to be the opposite of the supplier profit comparisons. From a procurer standpoint, we show that either effect may dominate the other depending on the probability of observing a low-cost opponent. Specifically, we find that the procurer surplus is higher under IIS when the probability of observing a low-cost opponent is high, and vice-versa. We also demonstrate through analytical models and numerical experiments the importance of understanding the trade-offs between the two effects and when they occur.

From a social welfare standpoint, we find that IIS is better than CIS. This observation is also a consequence of the faking effect. When low-cost suppliers fake, the probability of a high-cost type winning the contract is non-zero. However, under IIS, the probability of a high-cost type winning is zero when at least one low-cost type is present. We also analyzed the sensitivity of the social welfare results. We find it surprising that the social welfare under CIS may decrease as the number of opponents increases.

Let us briefly contrast our results from those in related prior papers. When the uncertainty about the number of competitors is considered, Arora et al. (2007) demonstrates that the IIS always generates higher supplier profits than CIS. As we show in the paper, that result does not hold when the uncertainty about the opponents' cost is considered. Compared to Tu (2005), a work that was simultaneously pursued, our paper highlights how his assumption disallowing pooling equilibrium affects the policy comparison. We also show that, counter to the simple extension of the revelation principle, IIS can generate higher procurer surplus.

In our opinion, in addition to the insights, the methodologies adopted in this paper are also interesting. We determine the perfect Bayesian Nash equilibrium of two policies. In general, both CIS and IIS did not have closed form expressions for the bid distribution. Even the expected supplier

profits under CIS could not be expressed in a closed form for any arbitrary number of players. In this paper, we overcome the issues related to the lack of closed-form expression and still execute the comparisons. Note that prior works, which have dealt with similar issues, have avoided this problem by only considering a two player game.

The insights offered in the paper also appear to shed some light on the implications of policies other than those analyzed here. Recall that, under IIS, suppliers attempt to learn about the nature of the competition i.e., the presence of the other low-cost competitors, by bidding high prices (the extraction effect). When this result was presented to Freemarkets, they mentioned that they had also observed a similar phenomenon when only the bidder rank was provided as feedback. Apparently, in those cases, bidders submitted higher bids initially to learn about the total of competitors, another aspect related to the nature of competition. Perhaps, the extraction effect may also explain the lower procurer surplus observed by Koppius (2002) when rank was provided as feedback. It would be interesting to validate if the extraction effect is relevant to other policies as well.

As regards future research, our paper can be extended in many ways. We have already highlighted some of the questions earlier. Apart from those, one can also consider extending the model to incorporate procurers' ability to learn across auctions as well. It would also be interesting to analyze other policies adopted in the marketplaces. Another interesting way to extend our paper would be to relax the common prior assumption on θ . Such an analysis will shed light on how the policy comparison results alter with different priors.

References

- Adomavicius, G., A. Gupta, and P. Sanyal (2008). Design and Evaluation of Feedback Schemes for Multi-Attribute Procurement Auctions. In *Proceedings of the International Conference on Information Systems*.
- Adomavicius, G., A. Gupta, and D. Zhdanov (2009). Designing Intelligent Software Agents for Auctions with Limited Information Feedback. *Information Systems Research*.
- Arora, A., A. Greenwald, K. N. Kannan, and R. Krishnan (2007). Effects of Information-Revelation Policies Under Market-Structure Uncertainty. *Management Science* 53(8), 1234–1248.
- Aumann, R. (1987). Correlated Equilibrium as an Expression of Bayesian Rationality. *Econometrica* 55, 118.

- Bajari, P. (2001). Comparing Competition and Collusion in Procurement Auctions: A Numerical Approach. *Economic Theory* 18, 187–205.
- Brosig, J. and J. P. Reiß (2007). Entry decisions and bidding behavior in sequential first-price procurement auctions: An experimental study. *Games and Economic Behavior* 58, 50–74.
- Carr, S. (2003). Note on Online Auctions with Costly Bid Evaluations. *Management Science* 49(11), 1521–1528.
- Culligan, K. (2008). SO₂ Allowance in the US. In *2nd Global Carbon Market Forum: Auctioning carbon allowances- towards robust auction design and implementation*.
- Daughety, A. and J. Reinganum (1994). Asymmetric information acquisition and behavior in role choice models: an endogenously generated signaling game. *International Economic Review* 35(4), 795–819.
- Elmaghraby, W. and P. Keskinocak (2000). Technology for Transportation Bidding at the Home Depot. Teaching case available from http://www2.isye.gatech.edu/people/faculty/Pinar_Keskinocak/home-depot-case.zip.
- Fudenberg, D. and J. Tirole (1994). *Game Theory*. Cambridge, MA: MIT Press.
- Goes, P., G. Karuga, and A. Tripathi (2009). Understanding Willingness-to-Pay Formation of Repeat Bidders in Sequential Online Auctions. *Information Systems Research*.
- Granados, N., A. Gupta, and R. J. Kauffman (2005). Transparency Strategy in Internet-Based Selling. In K. Tomak (Ed.), *Advances in the Economics of Information Systems*, pp. 80–112. Harrisburg, PA: Idea Group Publishing.
- Greenwald, A., K. N. Kannan, and R. Krishnan (2009). On evaluating Information Revelation Policies in Procurement Auctions: A Markov Decision Process Approach. *Information Systems Research*. Forthcoming.
- Harsanyi, J. (1967). Games with Incomplete Information Played by Bayesian Agents. *Management Science* 14, 159–182.
- Harsanyi, J. (1968a). Games with Incomplete Information Played by Bayesian Agents. *Management Science* 14, 320–334.
- Harsanyi, J. (1968b). Games with Incomplete Information Played by Bayesian Agents. *Management Science* 14, 486–502.
- Hausch, D. (1986). Multi-object Auctions: Sequential vs Simultaneous sales. *Management Science* 32(12), 1599–1610.
- Hausch, D. (1988). A Model of Sequential Auctions. *Economics Letters* 26, 227–233.

- Jap, S. D. (2002). Online Reverse Auctions: Issues, Themes, and Prospects for the Future. *Journal of the Academy of Marketing Science* 30, 506–25.
- Jap, S. D. (2003, July). An Exploratory Study of the Introduction of Online Reverse Auctions. *Journal of Marketing* 67, 96–107.
- Klemperer, P. (2004). *Auctions: Theory and Practice*. Princeton, New Jersey: Princeton University Press.
- Koppius, O. R. (2002). *Information architecture and electronic market performance*. Ph. D. thesis, Erasmus University.
- Kostamis, D., D. Beil, and I. Duenyas (2008). Total-Cost Procurement Auctions: Should the Buyer Reveal Suppliers' Quality? Forthcoming in *Management Science*.
- Krishna, V. (2009). *Auction Theory*. San Diego, California: Academic Press.
- Kumar, S., M. Dawande, and V. S. Mookerjee (2007). Optimal Scheduling and Placement of Internet Banner Advertisements. *IEEE Transactions on Knowledge Data and Engineering* 19(11), 1571–1584.
- Laffont, J. (1997). Game Theory and Empirical Economics: The Case of Auction Data. *European Economic Review* 41, 1–35.
- Laffont, J. and D. Martimort (2002). *The theory of incentives. The principal agent model*. Princeton, NJ: Princeton University Press.
- Maskin, E. and J. Riley (1985). Auction Theory in Private Values. *American Economic Review* 75(2), 150–155.
- Maskin, E. and J. Riley (2000a). Asymmetric Auctions. *The Review of Economic Studies* 67(3), 413–438.
- Maskin, E. and J. Riley (2000b). Equilibrium in Sealed High Bid Auctions. *The Review of Economic Studies* 67(3), 439–454.
- McAfee, P. and J. McMillan (1988). *Incentives in Government Procurements*. Toronto, Canada: University of Toronto Press.
- Milgrom, P. R. and J. Roberts (1982). Limit pricing and entry under incomplete information: an equilibrium analysis. *Econometrica* 50(2), 443–459.
- Milgrom, P. R. and R. J. Weber (1982). A Theory of Auctions and Competitive Bidding. *Econometrica* 50, 1089–1122.
- Mithas, S. and J. Jones (2007). Do Auction Parameters Affect Buyer Surplus in E-Auctions for Procurement? *Production and Operations Management* 16(4), 455–470. Forthcoming.

- Narasimhan, C. (1988). Competitive Promotional Strategies. *The Journal of Business* 61(4), 427–449.
- OMB (1993). CIRCULAR A-110: Uniform Administrative Requirements for Grants and Agreements with Institutions of Higher Education, Hospitals, and Other Non-Profit Organizations. Revised on 11/19/93 and further amended on 9/30/99.
- Ortega-Reichert, A. (1968). *Models for Competitive Bidding Under Uncertainty*. Ph. D. thesis, Department of Operations Research, Stanford University.
- Salanie, J. (2000). *The Economics of Contracts*. MIT Press.
- Spence, M. (1973). Job Market Signaling. *Quarterly Journal of Economics* 87(3), 355–374.
- Thomas, C. (1996). Market Structure and the Flow of Information in Repeated Auctions. Working paper, Federal Trade Commission.
- Tu, Z. (2005). Why do we use Dutch Auction to Sell Flowers? Information Disclosure in Sequential Auctions. Working Paper, Department of Economics, University of Pittsburgh.
- United-Nations (2007). United Nations Procurement Manual. Revision 4.
- Wilson, R. (1977). A Bidding Model of Perfect Competition. *Review of Economic Studies* 44, 511–518.
- Winkler, R. and D. Brooks (1980). Competitive Bidding with Dependent Value Estimates. *Operations Research* 28, 603–613.
- Yoo, B., V. Choudhary, and T. Mukhopadhyay (2007). Electronic B2B Marketplaces with Different Ownership Structures. *Management Science* 53(6), 952–961.

Appendix

A. Variables Used

See Table 6 for the variables used.

B. Deriving the Equilibrium for the Asymmetric Game

Here, we only provide the proof sketch for deriving the equilibrium because the details are identical to those in Maskin and Riley (1985) and Narasimhan (1988). In this section, when we refer to a proposition, it corresponds to the one in Narasimhan (1988). If a pure equilibrium exists, every supplier can be shown to have an incentive to undercut the other suppliers. Beyond a certain price the supplier does not have an incentive to undercut anymore but switch to a high price, leading

	Primary Usage	Vars	Explanation
Exogenous Parameters		n	<i>Number of bidders in the market</i>
		c_l	<i>Cost incurred by a low type</i>
		c_h	<i>Cost incurred by a high type</i>
		θ	<i>Probability that a bidder is a low cost</i>
Endogenous Parameters	SPG §4.1	α	Probability that R_o believes each of his opponents is a low-cost
		β	Probability that R_1 believes R_o is a low-cost
	IIS §4.2.1	$F_{1,IIS}(p)$	CDF of the first period bid distribution
		x	Probability that the first-period winner believes each of his opponents is a low type
	CIS §4.2.2	$F_{1,CIS}^{sep}(p)$	CDF of the first period bid distribution under separating equilibrium
		$F_{1,CIS}^{semi}(p)$	CDF of the first period bid distribution under semi-pooling equilibrium
		y	Probability that the first-period bidder, who bid $< c_h$, believes each of his opponents is a low type
		$\gamma_{\theta,n}$	Probability that a low cost type fakes
	Comparisons §5	Π_ϕ	Total profits earned by a bidder conditional on being a low type under policy ϕ
		SP_ϕ	Unconditional total profits earned by a bidder under policy ϕ
EP_ϕ		Expected price paid by a bidder under policy ϕ	

Table 6: Variables Used.

to another round of undercutting. Thus, only a mixed strategy equilibrium exists (see Proposition 1 in Narasimhan, 1988). As a result, the profit expressions should involve cdfs and, therefore, we obtain $\Pi_{R_1}(p)$ and $\Pi_{R_o}(p)$ in Equations 1 and 2. The strategy sets are convex (Proposition 2 there) and have no mass points in the interior (Proposition 3). The infimum of the strategy space is identical across both types. The strategy sets are identical; if mass points exist, they are only present for one of the two types and that too at the supremum of the strategy space (Proposition 4). By the results established so far, we can conclude that an equilibrium is feasible only if $F_{R_o}(p)$ does not have a mass point at the supremum of the strategy set, i.e., $F_{R_o}(p) = 1$ at the supremum. If we can determine the supremum of R_o 's equilibrium strategy, we can exploit the definition of the mixed strategy equilibrium and set the profit from the supremum to be equal to $\Pi_{R_1}(p)$. The supremum strategy is the bid infinitesimally less than c_h (In Maskin and Riley (1985), where a forward auction is considered, its equivalent is the infimum, which is explicitly mentioned to occur

at a bid infinitesimally greater than zero). Similar to what Maskin and Riley (1985) have implicitly done, we let the unconditional revenue from the supremum of the strategy to be equal to c_h . The unconditional revenue is so because we assume a continuum of infinitesimally incremental bids in the strategy space. Thus, the profit from the supremum is $(1 - \alpha)^n(c_h - c_l)$. By setting up the profits to be equal, we obtain

$$F_{R_o}(p) = 1 - \frac{(1 - \alpha)}{\alpha} \left(\left(\frac{c_h - c_l}{p - c_l} \right)^{\frac{1}{n}} - 1 \right).$$

The lower support for the bid distribution is obtained by setting $F_{R_o}(p) = 0$, which is $c_l + (1 - \alpha)^n(c_h - c_l)$. From the preliminaries (Proposition 4), the lower support of R_1 's bid distribution is also the same. If it were anything else, the winner can improve his profits by shifting the lower support. So, $F_{R_1}(c_l + (1 - \alpha)^n(c_h - c_l)) = 0$. Substituting for that price, we obtain $\Pi_{R_o}(c_l + (1 - \alpha)^n(c_h - c_l))$. This profit should be the same independent of the price bid according to the definition of a mixed strategy equilibrium. By setting them up to be equal, we obtain:

$$F_{R_1}(p) = 1 - \left(\frac{(1 - \alpha)}{\beta} \left(\frac{c_h - c_l}{p - c_l} \right)^{\frac{1}{n}} - \frac{1 - \beta}{\beta} \right),$$

The cdf is such that a mass point of $\frac{\beta - \alpha}{\beta}$ exists at the supremum.

Substituting these expressions back in the profit expressions, the expected profit for each supplier is equal to $(1 - \alpha)^n(c_h - c_l)$.

Suppose one of the low-cost supplier deviates to bid $p = c_h$ while the other low-cost suppliers bid according to the separating equilibrium we identified above. Let it be one of the R_o . Since R_1 is always a low-cost type, such a deviation for R_o would only result in a zero profit, which is less than $(1 - \alpha)^n(c_h - c_l)$ it secured before the deviation. Let R_1 deviate and bid $p = c_h$. He secures a profit only every R_o is a low cost type. Even then, the unconditional profit is $(1 - \alpha)^n \frac{c_h - c_l}{n+1}$. This profit is less than the profit before deviation. Along similar lines, it can be proved that suppliers have an incentive to deviate from the pooling equilibrium. Thus, only a separating equilibrium exists.

C. IIS Results

C.1 IIS: First Period

Algebraic simplification of Equation 4 yields the right hand side of Equation 5. We next exploit the definition of the mixed strategy equilibrium by computing the payoff for a specific bid in the strategy space and setting it to be the left hand side of Equation 5. The supremum of the strategy space in this game turns out to be the same as that under SPG. Similar to Maskin and Riley (1985) and Section B, we treat the unconditional revenue at the supremum, which is infinitesimally less than c_h , to be equal to c_h (recall that it is a consequence of our implicit assumption regarding the strategy space being the continuum of infinitesimally incremental bids). After substituting the expressions and simplifying the algebra, we obtain the left hand side of Equation 5.

C.2 Proof for Lemma 1

Lemma 1 *Consider two settings which may have a different θ and/or n . If the winning bid is the same in both settings, the winner's second belief that none of the suppliers is a low-cost type across the two settings is the same, and independent of both θ and n .*

Proof. Recall that the winner's second period belief about every supplier being a high cost type is x as defined in Equation 3. From this, the belief that none of the suppliers is a low cost type is $x' = x^n$. With that, let us now rearrange Equation 5:

$$1 - (\log x') = \frac{1}{x'} \frac{p - c_l}{c_h - c_l}.$$

x' is a solution to the above equation. It is clear that x' is independent of θ and n , but is a constant for a given p . ■

D. CIS Results

D.1 CIS: Condition for Separating versus Semi-Pooling Equilibrium

We first prove the restriction on the separating equilibrium. The profit from following the separating equilibrium strategy, as we showed before, is: $\Pi_{\text{CIS}}^{\text{sep}} = 2(1 - \theta)^n(c_h - c_l)$. While all other suppliers are playing that strategy, let us consider the incentive for one of the suppliers to deviate and bid

c_h . The profit from such deviation is:

$$\Pi^{\text{dev}} = \underbrace{(1-\theta)^n}_{\text{All } n \text{ suppliers are high-cost type}} \left(\overbrace{\frac{1}{n+1}(c_h - c_l)}^{\text{First period profit}} + \overbrace{(c_h - c_l)}^{\text{Second period profit: from SPG}} \right) + \underbrace{n\theta(1-\theta)^{n-1}}_{\text{Only one other supplier is a low-cost type}} \left(\underbrace{0}_{\text{First period profit}} + \underbrace{c_h - c_l}_{\text{Second period profit}} \right).$$

The first line in the profit expression corresponds to the scenario when all n suppliers are high-cost types. The supplier under consideration, say i , wins the first period with a probability of $\frac{1}{n+1}$, realizes before the beginning of the second period that none of the other suppliers is a high-cost type, and wins the second period with a bid which is infinitesimally smaller than c_h . The second line corresponds to the case when only one of the other suppliers is a low-cost type, whom we refer to as j . Because j wins the first period, i obtains zero profits in the first period. Since, by assumption, j continues its separating equilibrium strategy in the second period, i can outbid j in the second period with a bid price two infinitesimal increments below c_h and hence, the last term (see the previous sections in the appendix for discussions on infinitesimal bids and the assumptions when considering the unconditional revenue). We obtain θ_n^{fake} by solving for θ when $\Pi^{\text{dev}} > \Pi_{\text{CIS}}^{\text{sep}}$.

We next prove the existence of the semi-pooling equilibrium. The previous part of the proof has already established the incentive for a low-cost supplier to bid $p = c_h$. It is not clear whether or not only a pooling equilibrium exists. Suppose there indeed was a pooling equilibrium where both types of suppliers bid the same price, i.e., $p = c_h$, then the profit from the pooling scenario would be:

$$\Pi^{\text{pool}} = \overbrace{\frac{1}{n+1}(c_h - c_l)}^{\text{First period profit}} + \overbrace{(1-\theta)^n(c_h - c_l)}^{\text{Second period profit from SPG since beliefs do not change}}.$$

While all suppliers are playing the pooling equilibrium, let one of the suppliers deviate to bid in the first period a price infinitesimally smaller than c_h . The expected profit in that case would be:

$$\Pi^{\text{dev}} = \overbrace{(c_h - c_l)}^{\text{First period profit}} + \overbrace{(1-\theta)^n(c_h - c_l)}^{\text{Second period profit: SPG with } R_1 \text{ being the deviating supplier and } R_o \text{ refers to the others}}.$$

Comparing the two profits, it should be clear that the supplier will have an incentive to deviate. Therefore, only a semi-pooling equilibrium exists in our model.

D.2 CIS: Semi-pooling Equilibrium

There are two unknowns, $F_{1,\text{CIS}}^{\text{semi}}(p)$ and $\gamma_{\theta,n}$. To determine them, we need at least two equations. These two equations are obtained by setting three different expressions to be equal. For this, we invoke the definition of mixed strategy equilibrium, in which a supplier's expected profits from all his actions are identical. The first expression is the expected profit for any $p_1 < c_h$, which is Equation 9. The second expression we use is the expected profit faking, which is Equation 10. The third expression is the profit from submitting the maximum feasible bid less than c_h . Consistent with Section B and Maskin and Riley (1985), we consider the point to be a price infinitesimally less than c_h (because of the infinitesimally incremental bid assumption) and its unconditional revenue to be c_h . At that bid, $F_{1,\text{CIS}}^{\text{semi}}(p) = 1 - \gamma_{\theta,n}$. Equation 11 is obtained by setting the first two expressions equal. Equation 12 is obtained by setting the last two expressions equal.

D.3 Proof of Proposition 1

Proposition 1 *For any arbitrary n , the following properties of Equation 12 are observed:*

1. $\gamma_{\theta,n} > 0 \forall \theta \in (\theta_n^{\text{fake}}, 1)$. There is only one point of inflection, which occurs at $\theta_n^{\text{max}\gamma} = \frac{1}{1 + \frac{n^{1+n}}{n^n + (1+n)^n}}$.
2. $\frac{\partial \theta_n^{\text{fake}}}{\partial n} < 0$.
3. For $\theta > \theta_1^{\text{max}\gamma}$, $\gamma_{\theta,1} \geq \gamma_{\theta,n}$ for any $n > 1$.

Proof. The proof for Property 1 is as follows. Note from Equation 12 that $\gamma_{\theta,n} = 0$ both at $\theta = 1$ and again at θ_n^{fake} . Next, we obtain the first order derivative from Equation 12:

$$\frac{\partial \gamma_{\theta,n}}{\partial \theta} = \frac{(1 - \gamma_{\theta,n})(1 - \theta - n\theta\gamma_{\theta,n})}{(1 - \theta)\theta(1 + n\theta(1 - \gamma_{\theta,n}))}$$

When the derivative is computed at θ_n^{fake} , the slope is found to be positive. We will use the same first order derivative expression to show that only one point of inflection occurs for every $\theta \in (\theta_n^{\text{fake}}, 1)$. Since both the numerator and denominator are polynomial expressions, to demonstrate that $\gamma_{\theta,n}$ is continuous in the range of θ of interest to us, consider when the denominator becomes zero. The

denominator is zero for $\gamma_{\theta,n} = 1 + \frac{1}{n\theta} > 1$ or for $\theta = 0$ or for $\theta = 1$. The probability of faking can never be greater than 1. Faking occurs only for $\theta \geq \theta_n^{\text{fake}} > 0$. At $\theta = 1$, the numerator also becomes zero (because of the last term in the numerator). One can apply L'Hospital's rule to determine the slope. We will not concern about the nature of the function at $\theta = 1$ for two reasons. First, we already know that $\gamma_{\theta,n} = 0$ at $\theta = 1$. Second, comparing the policies for $\theta = 1$ is uninteresting. So, we next consider the zeros of the numerator. The value $\gamma_{\theta,n} = 1$ is ruled out by the definition of a semi-pooling equilibrium. The zero in the numerator occurs when $\gamma_{\theta,n}^{\text{max}} = \frac{1-\theta}{n\theta}$. Substituting back in Equation 12, we solve for θ and obtain the expression for $\theta_n^{\text{max}\gamma}$; the corresponding second order derivative is negative.

The proof for Property 2 is as follows. $\frac{\partial \theta_n^{\text{fake}}}{\partial n} = -\frac{1}{(2+n)^2}$.

Now, the proof for Property 3. The solution, $\gamma_{\theta,n}$, is the value of x that makes $(1 - \theta + \theta x)^{n+1}$, referred to as *LHS*, equal to $(1 - \theta)^n \theta (n + 1)(1 - x)$, referred to as *RHS*. Notice that *LHS* is an increasing function of x while *RHS* is a decreasing function of x . The point of intersection of *LHS* and *RHS* yields us the solution $x^* = \gamma_{\theta,n}$. First, note that $LHS < RHS$, at $x = 0$ for $\theta > \theta_1^{\text{max}\gamma}$. It is also obvious that, at $x = 1$, $LHS > RHS$. These two statements mean that a solution exists in the $x \in (0, 1)$ range for the case we consider. If we set $x = y$, for some y , and observe $LHS > RHS$, then it must be that $y > x^*$. We use this logic to prove the property. Specifically, we set $x = \gamma_{\theta,1}$ and show that $LHS > RHS$ for $\theta > \theta_1^{\text{max}\gamma}$ and $n > 1$. Of course, when $x^* = \gamma_{\theta,1}$ and $n = 1$, $LHS = RHS$.

We first prove the property for $n \geq 3$ and then for $n = 2$. When $x = \gamma_{\theta,1}$, $LHS - RHS$ is:

$$(1 - \theta + \theta \gamma_{\theta,1})^{n+1} - (1 - \theta)^n \theta (n + 1)(1 - \gamma_{\theta,1}) = (n + 1)(1 - \theta)^n \left(-2 + \theta + \sqrt{(3 - 4\theta + \theta^2)} \right) + \left(-1 + \theta + \sqrt{(3 - 4\theta + \theta^2)} \right)^{n+1}.$$

Because $\left(-1 + \theta + \sqrt{(3 - 4\theta + \theta^2)} \right) > 0$, $LHS > RHS$ so long as the following is true:

$$\begin{aligned} -(1 + n)(1 - \theta)^n + \left(-1 + \theta + \sqrt{(3 - 4\theta + \theta^2)} \right)^{n+1} &>? 0. \\ L' = \left(\frac{(\sqrt{(3 - \theta)} + \sqrt{1 - \theta})^{n+1}}{n + 1} \right)^{\frac{2}{n-1}} &>? (1 - \theta). \end{aligned}$$

Independent of n , it is observed that $\frac{\partial L'}{\partial \theta} > 0$. If we can establish that $L' = 1 - \theta$ for some specific θ' and n' ; and L' at θ' is always increasing in n ; then we have established that $LHS > RHS$ for all $n \geq n'$ and $\theta \geq \theta'$. Compute L' at $\theta = \theta_1^{\max \gamma}$.

$$L'|_{\theta_1^{\max \gamma}} = \left(\frac{1}{n+1} \right)^{\frac{2}{n-1}}.$$

From this, it can be shown that $\left(\frac{1}{n+1} \right)^{\frac{2}{n-1}} = 1 - \theta_1^{\max \gamma}$ when $n = 3$. It can also be seen that $\frac{\partial}{\partial n} \left(\frac{1}{n+1} \right)^{\frac{2}{n-1}} > 0$. Thus, we have proved that $LHS > RHS$ for $n \geq 3$ and $\theta \geq \theta_1^{\max \gamma}$. For $n = 2$ and $\theta \geq \theta_1^{\max \gamma}$, LHS can be shown to be greater than RHS through direct substitution. ■

E. Proofs for the Comparisons

E.1 Proof of Theorem 1

Theorem 1 $\Pi_{IIS} > \Pi_{CIS}^{sep}$ and $EP_{IIS} > EP_{CIS}^{sep}$ for $\theta \in [0, \theta_n^{fake})$.

Proof. $\Pi_{IIS} > \Pi_{CIS}^{sep}$ can be established through the Taylor series expansion of the logarithmic expression, the result can be easily observed. Given this result, the comparison involving the expected prices are obtained directly by considering the expressions. ■

E.2 Proof of Proposition 2

Proposition 2 For $n = 1$, there exists a $\theta_1^{equal} < 0.87$ such that $\Pi_{CIS}^{semi} > \Pi_{IIS}$ and $EP_{CIS}^{semi} > EP_{IIS}$ for $\theta > \theta_1^{equal}$ and; $\Pi_{CIS}^{semi} < \Pi_{IIS}$ for $\theta_1^{fake} < \theta < \theta_1^{equal}$.

Proof. So long as $(\Pi_{CIS}^{semi} - \Pi_{IIS})/(1 - \theta) > 0$, it implies $(\Pi_{CIS}^{semi} - \Pi_{IIS}) > 0$. The function $(\Pi_{CIS}^{semi} - \Pi_{IIS})/(1 - \theta)$ evaluated at $n = 1$ has only one point of inflection. This implies that there are only two θ values when the function is zero. The first one occurs at some $\theta < 0$ and the other is at $\theta \approx 0.8688$. The directionality of the difference can be verified by substituting the value of θ . From this, it should be evident that $\theta_1^{equal} < 0.87$. Thus, when $n = 1$, we have proved that, for $\theta > \theta_1^{equal}$, $\Pi_{CIS}^{semi} > \Pi_{IIS}$; and that, for $\theta_1^{fake} < \theta < \theta_1^{equal}$, $\Pi_{CIS}^{semi} < \Pi_{IIS}$. The comparisons involving EP_{CIS}^{semi} and EP_{IIS} can be obtained from how the expected payments vary with Π_ϕ . ■

E.3 Proof of Theorem 2

Theorem 2 Suppose $n > 0$. There always exists a $\theta_n^{equal} < 0.87$ such that $\Pi_{IIS} < \Pi_{CIS}^{semi}$ and $EP_{IIS} < EP_{CIS}^{semi}$ for any $\theta \in [\max\{\theta_1^{\max \gamma}, \theta_n^{equal}\}, 1)$.

Proof. There are two parts to prove the supplier profit comparison. We begin with the first part.

For any n :

$$\begin{aligned} \Pi_{\text{CIS}}^{\text{semi}} &= \left(((n+1)(1-\gamma_{\theta,n})(1-\theta)^n \theta)^{\frac{n}{n+1}} + (1-\theta)^n \right) (c_h - c_l) \\ &\stackrel{\text{Property 3 in Proposition 1}}{\geq} \left(((n+1)(1-\gamma_{\theta,1})(1-\theta)^n \theta)^{\frac{n}{n+1}} + (1-\theta)^n \right) (c_h - c_l) = \kappa. \end{aligned}$$

Of course, Property 3 can only be invoked for $\theta > \theta_1^{\max \gamma}$. Thus, κ is an envelope to the expected profit expression for the semi-pooling equilibrium under CIS and that too for a specific range of θ .

For the second part of the proof, we compare κ and Π_{IIS} . Specifically, consider the following difference function:

$$\begin{aligned} \frac{\kappa - \Pi_{\text{IIS}}}{(1-\theta)^n} &= \left(\frac{(n+1)(2-\theta-\sqrt{3-4\theta-\theta^2})}{1-\theta} \right)^{\frac{n}{n+1}} - 1 + n \log(1-\theta) \\ (1+n)^2 \left(\frac{\partial}{\partial n} \frac{\kappa - \Pi_{\text{IIS}}}{(1-\theta)^n} \right) &= (1+n)^2 \log[1-\theta] + A^{\frac{n}{1+n}} (n + \log A) \end{aligned}$$

where $A = \frac{(1+n)(-2+\theta+\sqrt{3-4\theta+\theta^2})}{-1+\theta}$. Rearranging the terms, we can find $(1+n)^2 \left(\frac{\partial}{\partial n} \frac{\kappa - \Pi_{\text{IIS}}}{(1-\theta)^n} \right) > 0$ for any arbitrary n and $\theta > \theta_1^{\max \gamma}$. This states that, suppose the difference between κ and IIS is positive for a given $n = n'$, and $\theta > \theta_1^{\max \gamma}$; it will remain positive for all $n > n'$. Now, we combine the results from Proposition 2 with that from the two parts proved here:

$$\Pi_{\text{CIS}}^{\text{semi}} \stackrel{\text{By Part 1}}{>} \kappa \stackrel{\text{By Part 2}}{>} \Pi_{\text{IIS}} \text{ for } \theta > 0.87.$$

As in the previous theorems, the expected payment comparisons simply follow the expected supplier profit comparisons under the specific cases. ■

E.4 Proof of Theorem 3

Theorem 3 For $\theta > \theta_n^{\text{equal}}$, the first period bid distribution under CIS first order stochastically dominates that under IIS.

Proof. The condition $\theta > \theta_n^{\text{equal}}$, implies that $\Pi_{\text{CIS}}^{\text{semi}} \geq \Pi_{\text{IIS}}$, which is the same as

$$\left[1 - \theta + \theta(1 - F_{1,\text{CIS}}^{\text{semi}}(p)) \right]^n (p - c_l) - \left[1 - \theta + \theta(1 - F_{1,\text{IIS}}(p)) \right]^n (p - c_l) +$$

$$(1 - \theta)^n n \log(1 - \theta + \theta(1 - F_{1,\text{IIS}}(p)))(c_h - c_l) \geq 0. \quad (19)$$

This condition holds even without the last term on the left hand side equation, since that term is a negative one. Then, for any $p > c_l$,

$$\begin{aligned} [1 - \theta F_{1,\text{CIS}}^{\text{semi}}(p)]^n - [1 - \theta F_{1,\text{IIS}}(p)]^n &\geq 0 \\ \theta(F_{1,\text{IIS}}(p) - F_{1,\text{CIS}}^{\text{semi}}(p))(\text{Positive terms.}) &\geq 0 \end{aligned}$$

This implies that $F_{1,\text{IIS}}(p) \geq F_{1,\text{CIS}}^{\text{semi}}(p)$ for any given $p > c_l$. ■

E.5 Proof of Theorem 4

Theorem 4 For $\theta < \theta_n^{\text{fake}}$, the first period bid distributions under IIS first order stochastically dominates that under CIS.

Proof. The condition $\theta < \theta_n^{\text{fake}}$ implies that $\Pi_{\text{CIS}}^{\text{sep}} \leq \Pi_{\text{IIS}}$. From the expected profit expressions, we compute the actual difference. Therefore,

$$\begin{aligned} [1 - \theta + \theta(1 - F_{1,\text{IIS}}(p))]^n (p - c_l) - [1 - \theta + \theta(1 - F_{1,\text{CIS}}^{\text{sep}}(p))]^n (p - c_l) = \\ -(1 - \theta)^n n \log \frac{1 - \theta}{(1 - \theta + \theta(1 - F_{1,\text{IIS}}(p)))} (c_h - c_l). \end{aligned}$$

The right hand side expression is positive. So,

$$\begin{aligned} [1 - \theta + \theta(1 - F_{1,\text{IIS}}(p))]^n (p - c_l) - [1 - \theta + \theta(1 - F_{1,\text{CIS}}^{\text{sep}}(p))]^n (p - c_l) &\geq 0 \\ \theta(F_{1,\text{CIS}}^{\text{sep}}(p) - F_{1,\text{IIS}}(p))(\text{Positive terms.}) &\geq 0 \end{aligned}$$

This implies that $F_{1,\text{IIS}}(p) \leq F_{1,\text{CIS}}^{\text{sep}}(p)$ for any given $p > c_l$. ■

E.6 Proof of Theorem 5

Theorem 5 $SW_{\text{IIS}} = SW_{\text{CIS}}^{\text{sep}}$ for $\theta \in [0, \theta_n^{\text{fake}}]$. $SW_{\text{IIS}} > SW_{\text{CIS}}^{\text{semi}}$ for $\theta \in (\theta_n^{\text{fake}}, 1)$.

Proof. The social welfare comparison is the inverse of the probability of faking or bidding $p = c_h$.

The only scenario when faking occurs is in CIS under the semi-pooling equilibrium. ■

E.7 Proof of Theorem 6

Theorem 6 For every $\theta \in [0, \theta_1^{fake}]$, there exist n_1 and n_2 , such that $\frac{\partial SW_{CIS}}{\partial n} < 0$ for $n_1 \leq n < n_2$.

Proof. The higher the probability of faking, the lower is the social welfare. By Property 2 in Proposition 1, we know that probability of faking can increase with n for a certain range. Corresponding to that range, the social welfare decreases. ■

F. Three Cost Type Model

Consider the model specification in Section 6.1.

Second Period Game

There are four scenarios in the second period game which are relevant to our analysis. In all the four scenarios, whenever there is uncertainty about the opponent's type, let l' and m' be the beliefs that a supplier has about the opponent being of type c_l and c_m , respectively. An interesting property observed in the four scenarios is as follows. Suppose bidder A of a particular type mixes his bids over the strategy space $[p_1, p_2]$ such that bidder B of type α is indifferent among prices. If $\alpha = c_l$, the expected profit for B of type c_m increases with the bid price in the same range. Instead, if $\alpha = c_m$, the profit for B of type c_l decreases in the range. We refer to this property as profit variation property, or simply *PVP*.

Scenario 1: The suppliers know each other's cost type. The equilibrium in this case is straightforward and the scenario resembles a Bertrand game.

Scenario 2: The uncertainty that each supplier has about his opponent is identical. The expected profit for supplier of type α – independent of whether it is c_l or c_m – from bidding a price of p is: $(1 - l'F_{c_l}^I(p) - m'F_{c_m}^I(p))(p - \alpha)$, where $F_{\beta}^I(p)$ is the cumulative density function of bid distribution from the opponent when his type is β . In this case, the equilibrium is such that $F_{c_m}^I(p) = \frac{1}{m'} \left(1 - l' - \frac{(1-l'-m')(1-c)}{p-c}\right)$ for $p > (c + \frac{(1-l'-m')}{1-l'}(1-c))$ and $F_{c_l}^I(p) = \frac{1}{l'} \left(1 - (c + \frac{1-l'-m'}{1-l'}(1-c))\frac{1-l'}{p}\right)$ defined for $c(1-l') + (1-l'-m')(1-c) < p < c + \frac{(1-l'-m')}{1-l'}(1-c)$. Suppliers of type c_l and c_m respectively generate profits of $\Pi_{c_l}^{Sc:2} = c(1-l') + (1-l'-m')(1-c)$ and $\Pi_{c_m}^{Sc:2} = (1-l'-m')(1-c)$. Deviations can be ruled out because of PVP and also because profit for either type from faking as c_h is less than the profit already generated.

Scenario 3: Only one supplier has his type revealed to the other and let the revealed type be

c_m . The revealed supplier faces uncertainty about the other supplier and his profit from bidding a price p is $(1 - l'F_{c_l}^{H;R=c_m}(p) - m'F_{c_m}^{H;R=c_m}(p))(p - c)$, where $F_{\alpha}^{H;R=c_m}(p)$ is the cdf of the bid distribution for the supplier hiding. The profit for the hiding supplier of type α is $(1 - F_{c_m}^R(p))(p - \alpha)$ where $F_{c_m}^R(p)$ is the bid distribution of the revealed supplier. The equilibrium is as follows: the revealed type bids according to the distribution $F_{c_m}^R(p) = \left(1 - \frac{(1-l'-m')(1-c)}{(p-c)(1-l')}\right)$ with a mass point of $1 - \frac{m'}{1-l'}$ at the supremum (infinitesimally close to 1); the hiding supplier of type c_m bids $F_{c_m}^{H;R=c_m}(p) = \frac{1}{m'} \left(1 - l' - \frac{(1-l'-m')(1-c)}{p-c}\right)$ for $p > c + \frac{1-l'-m'}{1-l'}(1-c)$; and the hiding supplier of type c_l submits a pure strategy bid of $c + \frac{1-l'-m'}{1-l'}(1-c)$. The profits for the respective cases are: $\Pi_{c_m}^{\text{Sc:3; R}} = (1-l'-m')(1-c)$, $\Pi_{c_m}^{\text{Sc:3; H}} = \frac{(1-l'-m')}{1-l'}(1-c)$ and $\Pi_{c_l}^{\text{Sc:3; H}} = c + \frac{(1-l'-m')}{1-l'}(1-c)$. Deviations can be ruled out because of PVP. Also, a mixed strategy for c_l type is unsustainable.

Scenario 3a: A variant of Scenario 3 is useful later for considering deviations. Let a hiding supplier, B , be of type c_m or c_h and a revealed supplier, A , be uncertain about it. Suppose the beliefs A has about B are $l' = 0$ and m' , and they be common knowledge. Also, suppose B incorrectly believes that A is of type c_m when the type actually is c_l . In this case, $F_{c_m}^{H;R=c_m}(p)$, as before, is the bid distribution for B when he is type of c_m . Given the strategy by B , A 's strategy (because of PVP) is to bid $c + \frac{1-l'-m'}{1-l'}(1-c)$, generating the same as the profit $\Pi_{c_l}^{\text{Sc:3-a; H}}$. This is the only profit of interest to us.

Scenario 4: Only one supplier has his type revealed to the other and let the type revealed be c_l . The revealed supplier generates a profit of $(1 - l'F_{c_l}^{H;R=c_l}(p) - m'F_{c_m}^{H;R=c_l}(p))p$ from bidding a price p , where $F_{\alpha}^{H;R=c_l}(p)$ is the cdf of the bid distribution for the supplier hiding who is of type α when the revealed type is c_l . The profit for the hiding supplier is $(1 - F_{c_l}^R(p))(p - \alpha)$ where α is the cost of the opponent ($=0$ or c depending on the supplier's type) and $F_{c_l}^R(p)$ is the bid distribution of the revealed supplier. Two different equilibria are feasible depending on whether or not $(1 - l' - m') > (1 - l')c$:

- If the condition is valid, the equilibrium bids are as follows: for the revealed supplier, the cdf is $F_{c_l}^R(p) = \left(1 - \left(\frac{(1-l'-m')}{1-l'} - c\right)\frac{1}{p-c}\right)$ for $p > \frac{1-l'-m'}{1-l'}$ and with a masspoint of $1 - \frac{m'}{(1-c)(1-l')}$ at the supremum (infinitesimally close to 1); the cdf for the hiding supplier, whose type is c_m , is $F_{c_m}^{H;R=c_l}(p) = \frac{1}{m'} \left(1 - l' - \frac{(1-l'-m')}{p}\right)$; and the hiding supplier, whose type is c_l , bids $\frac{(1-l'-m')}{1-l'}$. The profits for the respective cases are $\Pi_{c_l}^{\text{Sc:4-(1); R}} = (1 - l' - m')$, $\Pi_{c_m}^{\text{Sc:4-(1); H}} = \frac{(1-l'-m')}{1-l'} - c$,

and $\Pi_{c_l}^{\text{Sc:4-(1); H}} = \frac{(1-l'-m')}{1-l'}$.

- If the condition is invalid, i.e., $(1-l'-m') < (1-l')c$, the following are the equilibrium strategies: the revealed type bids according to the distribution $F_{c_l}^R(p) = \frac{1}{l'} \left(1 - \frac{(1-l')c}{p}\right)$, whose supremum is c ; the hiding supplier of type c_m bids his cost; and the hiding supplier of type c_l bids in the same manner as revealed supplier. The profits in this case are: $\Pi_{c_l}^{\text{Sc:4-(2); R}} = \Pi_{c_l}^{\text{Sc:4-(2); H}} = (1-l')c$, and $\Pi_{c_m}^{\text{Sc:4-(1); H}} = 0$.

First Period Game: IIS

It turns out that only a separating equilibrium exists in the first period here. We first characterize the equilibrium and later argue that deviations from that equilibrium do not exist.

$$\begin{aligned}
\pi_{\text{IIS}, c_m}(p) &= \overbrace{(1-l-m)(p-c + \Pi_{c_m}^{\text{Sc:3; R}})}^{\text{Opponent is type } c_h} + \overbrace{m(1-F_m(p))(p-c + \Pi_{c_m}^{\text{Sc:3; R}})}^{\text{Opponent is type } c_m \text{ but he lost first period}} \\
&+ \underbrace{mF_m(p)(0 + \Pi_{c_m}^{\text{Sc:3; H}})}_{\text{Opponent is type } c_m \text{ but he won first period}} + \underbrace{l\Pi_{c_m}^{\text{Sc:4; H}}}_{\text{Opponent is type } c_l} \\
&= (1-l-mF_m(p))(p-c + \Pi_{c_m}^{\text{Sc:3; R}}) + mF_m(p)\Pi_{c_m}^{\text{Sc:3; H}} + l\Pi_{c_m}^{\text{Sc:4; H}} \\
\pi_{\text{IIS}, c_l}(p) &= \overbrace{((1-l)(p + \Pi_{c_l}^{\text{Sc:4; R}}))}^{\text{Opponent is not type } c_l} + \overbrace{l(1-F_l(p))(p + \Pi_{c_l}^{\text{Sc:4; R}})}^{\text{Opponent is type } c_l \text{ but he lost first period}} + \overbrace{lF_l(p)(0 + \Pi_{c_l}^{\text{Sc:4; H}})}^{\text{Opponent is type } c_l \text{ but he won}} \quad (20) \\
&= (1-lF_l(p))(p + \Pi_{c_l}^{\text{Sc:4; R}}) + lF_l(p)\Pi_{c_l}^{\text{Sc:4; H}}
\end{aligned}$$

We will continue to assume Bayesian update of beliefs. Consider the first equation first. When Scenario 3 occurs in the second period of IIS, $l' = 0$ and $m' = \frac{m(1-F_m(q))}{1-l-mF_m(q)}$, where q is the winning bid. This implies that $\Pi_{c_m}^{\text{Sc:3; R}} = \frac{1-l-m}{1-l-mF_m(p)}$ since $q = p$ in this case. When computing $\Pi_{c_l}^{\text{Sc:4; H}}$ for supplier A, l' and m' correspond to the beliefs held by the winning opponent B, whose bid now is q . So, if $p_{l,m}^{\text{IIS}}$ is such that $F_m(p_{l,m}^{\text{IIS}}) = 0$, then $\Pi_{c_m}^{\text{Sc:3; H}}$ as a function of a losing bid p submitted by A is: $\frac{1}{F_m(p)} \int_{p_{l,m}^{\text{IIS}}}^p \left(\frac{1-l-m}{1-l-mF_m(q)}\right) f_m(q)(1-c) dq = -(1-c) \frac{(1-l-m)}{yF_m(p)} \log\left(\frac{1-l-mF_m(p)}{1-l}\right)$. Substituting for these expressions:

$$\begin{aligned}
\pi_{\text{IIS}, c_m}(p) &= (1-l-mF_m(p))(p-c) - (1-l-m)(1-c) \log\left(\frac{1-l-mF_m(p)}{1-l}\right) \\
&+ l\Pi_{c_m}^{\text{Sc:4; H}} + (1-l-m)(1-c) \quad (21)
\end{aligned}$$

Given the separating nature of the equilibrium, $\Pi_{c_m}^{\text{Sc:4; H}}$ is independent of p . The last two terms in

Equation 21, which are independent of p , can be ignored while computing the equilibrium. One can easily compute the equilibrium bid distribution by noting that the expected payoff from any price p in the strategy set is the same and that p infinitesimally close to 1 is the supremum of the strategy set. The equilibrium for $F_m(p)$ is thus the solution to the following:

$$(1-l-m)(1-\log(\frac{1-l-m}{1-l})) = (1-l-mF_m(p))\frac{(p-c)}{(1-c)} - (1-l-m)\log(\frac{1-l-mF_m(p)}{1-l}).$$

The infimum of the strategy set is

$$p_{l,m}^{\text{IIS}} = c + (1-c)\frac{(1-l-m)}{1-l}(1-\log(\frac{1-l-m}{1-l})). \quad (22)$$

The total profit for type c_m is: $(1-l-m)(1-c)(2-\log(\frac{1-l-m}{1-l})) + l\Pi_{c_m}^{\text{Sc:4; H}}$. We discuss about the computation of $\Pi_{c_m}^{\text{Sc:4; H}}$ next.

Computing the profit expressions for Scenario 4 occurring in the second period of IIS has to consider different conditions. Note that, here, $m' = m$ and $l' = \frac{l(1-F_l(q))}{1-lF_l(q)}$. Let $p_{l,l}^{\text{IIS}}$ be such that $F_l(p_{l,l}^{\text{IIS}}) = 0$.

1. If $(1-l-m) > (1-l)c$, then $(1-l'-m') > (1-l')c$. So, $\Pi_{c_l}^{\text{Sc:4; R}} = \frac{1-l}{1-lF_l(p)} - m$, $\Pi_{c_l}^{\text{Sc:4; H}} = \frac{1}{F_l(p)} \int_{p_{l,l}^{\text{IIS}}}^p (\frac{1-l-m(1-lF_l(q))}{1-l}) f_l(q) dq$ and $\Pi_{c_m}^{\text{Sc:4; H}} = \frac{1}{F_l(p)} \int_{p_{l,l}^{\text{IIS}}}^p (\frac{1-l-m(1-lF_l(q))}{1-l} - c) f_l(q) dq$.
2. If $(1-c) < y$, then $(1-l'-m') < (1-l')c$. So, $\Pi_{c_l}^{\text{Sc:4; R}} = \frac{(1-l)c}{1-lF_l(p)}$, $\Pi_{c_m}^{\text{Sc:4; H}} = 0$, and $\Pi_{c_l}^{\text{Sc:4; H}} = \frac{c}{F_l(p)} \int_{p_{l,l}^{\text{IIS}}}^p (\frac{1-l}{(1-lF_l(q))}) f_l(q) dq$.
3. Otherwise, two sub-cases arise. Let p_e be such that $F_l(p_e) = \frac{1}{l}(1 - \frac{(1-l)(1-c)}{y})$
 - If $p \leq p_e$, $\Pi_{c_m}^{\text{Sc:4; H}} = 0$, $\Pi_{c_l}^{\text{Sc:4; R}} = \frac{(1-l)c}{1-lF_l(p)}$, and $\Pi_{c_l}^{\text{Sc:4; H}} = \frac{c}{F_l(p)} \int_{p_{l,l}^{\text{IIS}}}^p (\frac{1-l}{(1-lF_l(q))}) f_l(q) dq$.
 - For $p > p_e$, $\Pi_{c_l}^{\text{Sc:4; R}} = \frac{1-l}{1-lF_l(p)} - m$, $\Pi_{c_m}^{\text{Sc:4; H}} = \frac{1}{F_l(p)} \int_{p_e}^p (\frac{1-l-m(1-lF_l(q))}{1-l} - c) f_l(q) dq$, and $\Pi_{c_l}^{\text{Sc:4; H}} = \frac{c}{F_l(p)} \int_{p_{l,l}^{\text{IIS}}}^{p_e} (\frac{1-l}{(1-lF_l(q))}) f_l(q) dq + \frac{1}{F_l(p)} \int_{p_e}^p (\frac{1-l-m(1-lF_l(q))}{1-l}) f_l(q) dq$.

The equilibrium is different in each of the cases. Now, to compute the bid distribution, we invoke the property that the strategies in the mixed equilibrium yield the same profit and that $p_{l,m}^{\text{IIS}}$ is the supremum of the strategy space for type c_l . The explicit expressions for the different cases have not been shown for the sake of simplicity. We use the expression to compare against the bid distribution

of a single period game. The single period game has the same bid distribution as Scenario 2 except for $l' = l$ and $m' = m$. One can show the stochastic dominance of IIS bid distribution in each case, validating the presence of the extraction effect.

We next consider deviations from the equilibrium. The deviations for type c_m are straightforward to discuss since they most closely mirror the discussion in the original model. Type c_m faking as type c_h does not yield any benefit since it does not bias the opponent's belief and only the winner's bid is revealed. When type c_m fakes as type c_l , the opponent bids aggressively, resulting in lower profits for the faking bidder. A similar argument holds for type c_l except that Scenario 3a arises.

First Period Game: CIS Separating Equilibrium

Recall that all bids are revealed under CIS. If the separating equilibrium is valid in the first period, then the second period is a Bertrand game. The expected payoffs from bidding p are:

$$\begin{aligned}
\pi_{\text{CIS},c_m}^{\text{sep}}(p) &= (1-l-m)(p-c+1-c) + m(1-F_m(p))(p-c) \\
&= (1-l-mF_m(p))(p-c) + (1-l-m)(1-c) \\
\pi_{\text{CIS},c_l}^{\text{sep}}(p) &= (1-l-m)(p+1) + m(p+c) + l(1-F_l(p))p \\
&= (1-lF_l(p))p + ((1-l-m) + mc).
\end{aligned}$$

Note that, in each equation, the last term is independent of p and will not affect the equilibrium calculations. Ignoring that term, the expected profit expressions are similar to the expected profit in Scenario 2 of the second period game. Therefore, the expressions for the equilibrium bid distributions are identical to that scenario. The infimum of type c_m 's strategy set is $p_{l,m}^{\text{sep}} = c + \frac{(1-l-m)}{1-l}(1-c)$. The expected profits are $\pi_{\text{CIS},c_l}^{\text{sep}} = 2(1-l-m+mc)$ and $\pi_{\text{CIS},c_m}^{\text{sep}} = 2(1-l-m)(1-c)$. Similar to the original model, the separating equilibrium does not always exist. Given the opponent's action as fixed, a bidder of c_m type generates a profit of $(1-l-m)(\frac{1-c}{2} + (1-c)) + m(1-c) + l(1-c)$, which is greater than $2(1-l-m)(1-c)$ if $3l + 3m > 1$. Along similar lines, fixing the opponent's action, c_l type bidder fakes as c_h if $3l + (3+4c)m > 1$. It is important to note that these conditions are feasible in the space of $0 < l < 1$ and $0 < m < 1-l$ and, hence, the proof demonstrates the existence of the faking effect. The exact conditions when a separating equilibrium exists can be

obtained from the semipooling equilibrium we characterize next. We can easily rule out a pooling equilibrium.

First Period Game: CIS Semipooling Equilibrium

Suppose γ_l and γ_m are the respective probabilities that each of type c_l and c_m bid like a c_h type. Type c_l faking as c_m is not sustained as an equilibrium because of a PVP-lie property. Let $F_{c_l}^{\text{semi}}(p)$ and $F_{c_m}^{\text{semi}}(p)$ be the cdfs of the bid distributions for c_l and c_m types, respectively. It can be shown that the lower support for type c_l is lower than that for type c_m . Assume for now that p_m^{CIS} is the lower end of the bid distribution for type c_m . In this case, the expected profits for the different types and different bids are:

$$\Pi_{c_m}^{\text{semi}}(p|p < 1) = (1 - l - m + l\gamma_l + m\gamma_m)(p - c + \Pi_{c_m}^{\text{Sc:3; R}}) + m(1 - F_{c_m}^{\text{semi}}(p) - \gamma_m)(p - c) \quad (23)$$

$$\Pi_{c_m}^{\text{semi}}(p = 1) = (1 - l - m + l\gamma_l + m\gamma_m)\left(\frac{1 - c}{2} + \Pi_{c_m}^{\text{Sc:2}}\right) + m(1 - \gamma_m)\Pi_{c_m}^{\text{Sc:3; H}} + l(1 - \gamma_l)\Pi_{c_m}^{\text{Sc:4; H}} \quad (24)$$

$$\begin{aligned} \Pi_{c_l}^{\text{semi}}(p|p < p_m^{\text{CIS}}) &= (1 - l - m + l\gamma_l + m\gamma_m)(p + \Pi_{c_l}^{\text{Sc:4; R}}) + m(1 - F_{c_m}^{\text{semi}}(p) - \gamma_m)(p + c) \\ &+ l(1 - F_{c_l}^{\text{semi}}(p) - \gamma_l)p \end{aligned} \quad (25)$$

$$\Pi_{c_l}^{\text{semi}}(p = 1) = (1 - l - m + l\gamma_l + m\gamma_m)\left(\frac{1}{2} + \Pi_{c_l}^{\text{Sc:2}}\right) + m(1 - \gamma_m)\Pi_{c_l}^{\text{Sc:3; H}} + l(1 - \gamma_l)\Pi_{c_l}^{\text{Sc:4; H}} \quad (26)$$

Equations 24 and 26 are the profit expressions when faking. We note that $F_{c_l}^{\text{semi}}(\approx 1) = 1 - \gamma_l$, $F_{c_m}^{\text{semi}}(\approx 1) = 1 - \gamma_m$, $F_{c_l}^{\text{semi}}(p_m^{\text{CIS}}) = 1 - \gamma_l$, and $F_{c_m}^{\text{semi}}(p_m^{\text{CIS}}) = 0$; and that the expected profits are the same across all prices in the strategy space, to compute the expected profits. In this case, the beliefs l' and m' are the beliefs held by the opponent of a non-revealing bidder (the opponent can be non-revealing also). Unlike in the previous policy, the beliefs are the same independent of the cost type holding it and it is so because all bids are revealed. Using Bayesian update, $(1 - l' - m') = \frac{(1-l-m)}{1-l(1-\gamma_l)-m(1-\gamma_m)}$ and $(1 - l') = \frac{1-l-m(1-\gamma_m)}{1-l(1-\gamma_l)-m(1-\gamma_m)}$. One has to solve the four equations subject to the constraints that $1 \geq \gamma_l \geq 0$ and $1 \geq \gamma_m \geq 0$ and the updated beliefs should be considered for the appropriate Scenario 4. If a solution does not exist, then the equilibrium is a separating one.