Econ 635 Fall 2009: Problem Set 1

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Due date: November 11, 2009

Instruction: Answer 2 out of 3 questions below.

1. (20 points: Filtering techniques) Download the Dataset 1 from the course website. It contains quarterly series from the national income accounts of Australia and the U.S. from 1959:3 to 2008:4, and also their nominal exchange rate. (See the "Readme" sheet for the description of data.) Note that you must turn in your m-files with your answers.

(a) (5 points) Filter the trade-balance-to-GDP ratio, and the logarithm of output, consumption, inflation and real exchange rate (RER) series, with the Baxter-King (BK) filter, to keep the periodic components lasting from 6 to 32 quarters. Set the lead/lag number to 12. Which variable is the most volatile? Which variable is the most persistent? Which variables are procyclical? Which variables are countercyclical? Which variable has the highest cross-country correlation?

(b) (5 points) Use the nominal exchange rate series to split the sample into a fixed exchange rate period and a flexible exchange rate period. Redo the exercise in Part (a) for each sub-period. How does exchange rate system influence the business cycles properties of these variables?

(c) (5 points) Experiment the BK filter with the logarithm of RER with the following lead/lag numbers: 10, 11, 13 and 14. Discuss how they change the business cycle properties of RER, comparing to Part (a).

(d) (5 points) Filter the logarithm of RER with the 1st-difference filter and the HP filter with $\lambda = 1600$. Compare the results with the properties of RER in Part (a).

2. (20 points: Current account in a two-period model) Consider a two-period small-open economy model taking the world interest rate as given. There is a representative consumer who lives for 2 periods. The consumer faces the world real interest
rate $r$. Assume the following period utility function.

$$u(C_t) = -\gamma \exp(-C_t/\gamma)$$

$\gamma > 0$ The representative individual maximizes $U(C_1) + \beta U(C_2)$. The output of consumption good $Y_t$ is produced from the capital stock $K_t$ with the production function $Y_t = A_t K_t^\alpha$. The initial capital stock is given by $K_1$. The capital accumulation process is as:

$$K_2 = K_1 + I_1.$$  

A unit of the capital good is produced from $n$ units of the consumption good. The reverse transformation from a unit of capital good to $n$ units of the consumption good is also possible. For simplicity, assume that $\beta = 1/1 + r$.

(a) (5 points) Define the consumer’s utility maximization problem. Then derive the optimal choice of the investment in the 1st period.

(b) (5 points) Suppose the efficiency in the capital-good sector increases in the 1st period. How does this change affect current account? Explain analytically and diagrammatically.

(c) (5 points) Suppose the efficiency in the capital-good sector is anticipated to rise in the 2nd period. How does this change affect current account? Explain analytically and diagrammatically.

(d) (5 points) Suppose the efficiency in the capital-good sector increases permanently (i.e. in both periods). How does this change affect current account? Explain analytically and diagrammatically.

3. (20 points: Current account in a multi-period model) Assume perfect foresight and consider a small-open economy in which $\beta = 1/1 + r$ and government consumption is zero. The period utility function is given by:

$$U(C_t) = \log(C_t).$$

The investment $I_t$ is subject to the installation cost given by:

$$\Phi(I_t, K_t) = \frac{\chi I_t^2}{2K_t}.$$  

The production technology is given by:

$$F(K_t) = A_t K_t^\alpha.$$  

The dynamics of capital accumulation is subject to a depreciation rate $\delta$, $K_{t+1} = (1 - \delta)K_t + I_t$. Assume that $A_t = 1$ in the long run.
(a) (5 points) Derive the shadow price of capital, or Tobin’s $q$, in the long run. How does it depend on the adjustment cost parameter $\chi$?

(b) (5 points) Derive the dynamics of capital stock and Tobin’s $q$.

(c) (5 points) Linearize the dynamic equations in Part (b) around the steady state, and illustrate the dynamics using the 2-dimension plane $(K_t, q_t)$.

(d) (5 points) Explain the effect of an unexpected permanent increase in productivity on the correlation of saving and investment.