Joint Transportation-and-Inventory Problems in Supply Chains: A Review

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This article surveys published research on a class of supply-chain management problems we call Joint Transportation-and-Inventory Problems (JTIPs). These problems are characterized by the presence of both transportation and inventory considerations, either as policy-variables or constraints. We define the general JTIP and classify 49 contemporary JTIP papers (i.e., mostly published since 1990). We also suggest problems that deserve further research and possible ways to solve them.

The set of supply-chain management problems surveyed and classified in this article are characterized by the presence of two management concerns: transportation policy and inventory policy. "*Transportation*" involves activities related to the physical movement of goods between different geographic points. "*Inventory*" is concerned with characteristics of the goods being transported, such as demand, required service level, replenishment policies, etc.. Henceforth, these problems are labeled "Joint Transportation-and-Inventory Problems" (JTIPs).

In order to identify contemporary research, we searched the following journals between 1990 and 2003: *Management Science, Operations Research, Transportation Science, Transportation Research, European Journal of Operations Research, Journal of Business Logistics, Annals of Operations Research,* etc.. We also did a citation search of the articles cited by these published papers. Some JTIP working papers are also included.

Our interest in developing this survey stemmed from our interest in working on what we initially thought was "only" a supply-chain (i.e., inventory) management model. Once we became aware of what we now call the JTIP literature, we were surprised that there was no corresponding survey. So, providing a survey for others was our first motive. We also want to attract the interest of other researchers, first, because of the interesting technical challenges JTIPs pose and; second, because of the potential for successful models to provide significant savings in real-world applications.

Since the early 1980's, joint transportation-inventory policies have been successfully implemented in many industries. Bell, et al., (1983) described an on-line, computerized routing-and-scheduling optimizer, developed for a manufacturer of liquid oxygen and nitrogen. Their project involved scheduling a fleet of vehicles that make bulk deliveries stored at a central depot to a set of customer locations. Their optimizer reportedly reduced vehicle-operating costs 6-10%. Golden, et al. (1984) used a simulation model to investigate the interaction between transportation and inventory decisions for a large energy-products company. They reported that the system, which jointly managed transportation and inventory, produced a 8.4% improvement in gallons/hour delivered, reduced stockouts by 50%, and total cost by 23%.

Theoretical studies have also demonstrated that significant savings can be realized when transportation and inventory concerns are considered jointly. Federgruen and Zipkin (1984) incorporated inventory costs into a single-depot vehicle-routing model, and compared the solutions for their JTIP with those of the pure vehicle-routing problem. Their results showed that about 6-7% savings in operating costs can be achieved by using the joint approach. Dror and Ball (1987) considered a JTIP for distributing heating oil to customers with the objective of minimizing the annual delivery and shortage costs. Computational results showed that their approach provided a more than 50% increase in performance (measured in units/hour delivered) over the "manual" rules in use at the time of the study, and a more than 25% increase in performance over another existing system.

Finally, JTIPs pose both interesting and challenging technical problems. From a transportation modeling perspective, for example, JTIPs imbed inventory considerations into the traditional vehicle-routing problem, which is already known to be NP-hard. And, from an inventory-modeling perspective, JTIPs add delivery leadtimes as decision variables to multiple-location inventory models, which are already known to be very difficult to optimize even with fixed leadtimes.

What follows is organized as follows. In Section 2, we describe JTIPs and formulate the corresponding general optimization problem. Section 3 provides a classified bibliography. More detailed summaries on JTIP papers are given in Section 4. Section 5 takes a look into the future by describing several problems for further research.

1. The JTIPs

JTIPs involve managing the activities of supplying (one or more) products from (one or more) geographically-dispersed origins, henceforth called depots, to (one or more) geographically-dispersed destinations, henceforth called retailers/customers (we will use "retailer" and "customer" interchangeably), with a (limited or unlimited) fleet of M capacitated or uncapacitated vehicles during some (finite or infinite) planning horizon of length H. All inventory enters the system through the depot(s). The point-to-point travel distances or times are typically fixed and known. Each retailer experiences deterministic or stochastic external demand on its inventory. The depot may hold inventory or not. Decision-making may be centralized or decentralized.

By definition, JTIPs involve two sets of management concerns: those related to transportation policy and those related to inventory policy. *Transportation policy* includes, but is not limited to, the assignment of vehicles to routes and/or customers, vehicle-capacity constraints, the sequencing of customers on routes, and customer delivery time-windows. *Inventory policy* includes, but is not limited to, determining system (i.e., depot) replenishments, rules for filling customer orders/demands, and allocating vehicle inventory among customers. As we shall see, both of these policies are represented in a given model either as decision variables or as constraints. See Section 4 for more details.

Let *I* be a vector specifying the inventory policy/ies under consideration, and let *T* be a vector specifying the transportation policy/ies under consideration. Let $C_t(I,T)$ represent the cost in period *t, t = 1,…,H*, associated with any given joint policy: $\{I, T\}_{t=1,\ldots,H}$. Then, the general JTIP can be formulated as:

$$\text{JTIP:} \quad \text{Minimize}_{\text{w.r.t. } I,\, T} \; \{\textstyle\sum_{t=1,\ldots,H} C_t(I,T)\} \tag{1}$$

Subject to

$$I \in \Omega \tag{2}$$

$$T \in \pi \tag{3}$$

The objective is to minimize the total (expected) cost (or average (expected) cost/time, in some cases) over the planning horizon *H*. In some cases, a profit or cash flow maximizing objective replaces (1). Inventory policy, *I*, is constrained to be chosen from some given set Ω and transportation policy, *T*, is constrained to be chosen from some given set π. In addition, in multi-period JTIPs, there are typically constraints (e.g., inventory balance constraints, vehicle in-route constraints) that link the result(s) of one period's policies to that of others.

Finally, some JTIPs either consider a single, given, transportation policy, in which case $C_t(I,T)$ in (1) is replaced by $C_t(I|T)$; or a single, given inventory policy, in which case $C_t(I,T)$ in (1) is replaced by $C_t(T|I)$.

2. Literature Classification

Table 2 provides our classification of the JTIP literature, listed in alphabetical order by author(s). Table 1 lists the categories and the abbreviations used in Table 2. Complete citations are in the references. In what follows, we describe the categories and their sub categories.

Table 1: Categories and Abbreviations

<table>
<tr><td rowspan="8">Subject</td><td colspan="2">IRP – Inventory-Routing Problem</td></tr>
<tr><td colspan="2">SIRP – Strategic Inventory-Routing Problem</td></tr>
<tr><td colspan="2">IRPSF – Inventory-Routing Problem with Satellite Facilities</td></tr>
<tr><td colspan="2">VRP – Vehicle-Routing Problem</td></tr>
<tr><td colspan="2">SRP – Ship-Routing Problem</td></tr>
<tr><td colspan="2">IAP – Inventory-Allocation Problem</td></tr>
<tr><td colspan="2">DDP – Delivery-Dispatching Problem</td></tr>
<tr><td colspan="2">O – Other</td></tr>
<tr><td>Horizon</td><td colspan="2">F – Finite
I – Infinite</td></tr>
<tr><td>Periodic/Continuous Review</td><td colspan="2">P – Periodic-Review system
C – Continuous-Review system</td></tr>
<tr><td rowspan="2">Vehicle</td><td>Number</td><td>L – Limited
U – Unlimited</td></tr>
<tr><td>Capacity</td><td>C – Capacitated
U – Uncapacitated</td></tr>
<tr><td>Product</td><td colspan="2">S – Single Product
M – Multiple Product</td></tr>
<tr><td>Orders/Demands</td><td colspan="2">D – Deterministic
S – Stochastic</td></tr>
<tr><td>Assignment*</td><td colspan="2">Ret. Sel. – Retailer Selection
Ret. Grou. – Retailer Grouping
Veh. Assign. – Vehicle Assignment</td></tr>
<tr><td>Routing Policy</td><td colspan="2">S – Static
D – Dynamic</td></tr>
<tr><td>Allocation Policy</td><td colspan="2">S – Static
D – Dynamic</td></tr>
<tr><td>Policy Variable/s</td><td colspan="2">Joint – Joint transportation-inventory policy
Transpo. – Transportation policy
Inv. – Inventory policy</td></tr>
</table>

*The three types listed in this category are not mutually exclusive.

Subject: Subject is divided into seven categories: The **Inventory-Routing Problem (IRP)** and the **IRP with Satellite Facilities (IRPSF)** deal with tactical JTIP activities (e.g., routing an existing fleet of vehicles to visit the retailer; deciding the optimal timing and delivery quantity for every visit to each retailer). The **SIRP**, on the other hand, is motivated by the long lead times (say months or even years) between the signing of purchase or lease agreements and the availability of vehicle(s) for delivery operations. The **SIRP** deals with resource-planning decisions where, for example, the objective is to minimize the size (or cost) of the vehicle fleet required to transport product(s). The **Vehicle-Routing Problem (VRP)**, strictly speaking, is not a JTIP, but is a topic with a long history that, in a sense, sets the stage for JTIPs. We provide a brief overview in Section 4. The **Ship-Routing Problem (SRP)**, which can be viewed as a multi-depot JTIP, involves the design of a set of routes for a fleet of heterogeneous ships servicing a set of production and consumption harbors for a single product (note that no depot is involved). The quantities loaded and discharged are determined by the production and consumption rates of the harbors, possible stock levels, and the ship visiting the harbor. The **Inventory-Allocation Problem (IAP)** assumes that the routes traveled by the vehicles are predetermined and deals only with how to allocate the inventory to the retailers. The **Delivery-Dispatching Problem (DDP)** involves assigning vehicles to "itineraries", i.e., sets of predetermined customers with fixed delivery quantities, at the minimum cost.

***Horizon**:* The planning horizon in the JTIP can be either **Finite** or **Infinite**. Most JTIP models involve only a single planning period.

***Review**:* **Periodic-Review** models divide time into one or more discrete time periods. Correspondingly, information is provided and decisions are made and implemented periodically. **Continuous-Review** models represent information, decision-making and implementation events in continuous time.

***Vehicle**:* The vehicles used to transport product(s) can be **Capacitated,** in the amount of the commodity/ies they can transport at any given time, or **Uncapacitated**, and identical (i.e., same capacity and same operating cost) or not. The number of vehicles can be **Limited** or **Unlimited**.

***Product**:* Although most of the JTIP literature considers the distribution of only a **Single Product**, some models incorporate **Multiple Products**.

***Orders/Demands**:* In some JTIPs, the retailers are viewed as the end customers with either **Deterministic** (i.e., fixed and known) or **Stochastic** (i.e., random variables with know probability distributions) orders. In other JTIPs, the retailers serve customers whose demands are deterministic or stochastic. In the latter case, the JTIP incorporates the inventory-replenishment policy of the retailer. Demand distributions can be either i.i.d. across the retailers or retailer specific. Similarly, when multiple products are considered, their order/demand distributions can be identical or product specific.

Assignment: **Retailer Selection** involves determining the retailers to be visited each time the vehicle(s) leave the depot. **Retailer Grouping** involves assigning retailers to sets, sometimes called regions, which are then managed independently. **Vehicle Assignment** means assigning the retailers to vehicles, vehicles to routes, or limited vehicle space to different products.

Routing Policy: Two types of routing policies (i.e., the sequencing of visits to the retailers on each route) are modeled in literature. **Static Routing** means that a route is always the same whenever the same set of retailers is visited by a single vehicle. Direct delivery (i.e., each retailer is visited independently by dedicated route) is a special case of static routing. **Dynamic Routing** permits the route to the same set of retailers to change over time, depending on the status of the system.

Allocation Policy: Some JTIPs incorporate the allocation of vehicle inventory, either because total retailer orders exceed the amount available on the vehicle or because the inventory policy is constrained to have the vehicle return to the depot empty. Under **Static Allocation** the amount of vehicle inventory to be allocated to the retailers is determined simultaneously, usually when the vehicle(s) leave the depot, and fixed thereafter. Under **Dynamic Allocation** these quantities are determined sequentially, based on the status of the system at the time of delivery to each retailer.

Policy Variable: **Joint** means that the model is attempting to jointly optimize inventory and transportation policy, as in (1) – (3) above. **Transportation Policy** means that the model determines transportation policy for a given inventory policy. **Inventory Policy** means that the model determines inventory policy for a given transportation policy.

Table 2: A Classification of Contemporary JTIP Literature

No.	Author	Year	Subject	Horizon	Review	Vehicle No.	Vehicle Capacity	Product	Orders/Demands	Assignment	Routing Policy	Allocation Policy	Policy Variable	Objective	Solution & Method
1	Adelman	2001	Other	I	P	U	C	M	S, Product specific	Retailer selection	N.A.	Static allocation	Joint	Min cost	Markov-decision process, linear programming, heuristic
2	Anily	1994	Other	I	C	U	C/U (idenical)	S	D (constant), Retailer specific	Retailer grouping	Static routing	Static allocation	Joint	Min cost	Heuristic and lower bound
3	Anily and Federgruen	1990	IRP	I	P	U	C (identical)	S	D (constant), Retailer specific	Retailer grouping	Static routing	Static allocation	Joint	Min cost	Heuristics, lower and upper bounds
4	Anily and Federgruen	1993	IRP	I	P	U	C (identical)	S	D (constant), Retailer specific	Retailer grouping	Static routing	Static allocation	Joint	Min cost	Heuristics, lower and upper bounds
5	Bard, et al.	1998	IRPSF	F	P	L	C (identical)	S	S, Retailer specific	Retailer selection	Dynamic routing	N.A.	Transpo.	Min cost	Heuristics
6	Barnes-Schuster and Bassok	1997	IRP	I	P	U	C (identical)	S	S, Retailer specific	N.A.	Direct delivery	Static allocation	N.A.	Evaluate strategy effectiveness	Lower bound
7	Bassok and Ernst	1995	IAP	I	P	L	C	M	S, Retailer specific /Product specific	Vehicle assignment	Static routing	Dynamic allocation	Inv.	Max profit	Dynamic programming
8	Bell, et al.	1983	IRP	F	P	L	C	S	D	Vehicle assignment	Dynamic routing	Static allocation	Joint	Max profit	Integer programming
9	Berman and Larson	2001	IAP	F	P	Single	C	S	S, Retailer specific	N.A.	Static routing	Dynamic allocation	Inv.	Min cost	Stochastic dynamic programming
10	Bertazzi, et al.	2000	Other	I	P	U	C (non-identical)	M	D, Product specific	Vehicle assignment	Direct delivery	Static allocation	Inv.	Min cost	Heuristic
11	Bertazzi, et al.	2002	IRP	F	P	Single	C	M	D, Retailer specific	Retailer selection	Dynamic routing	N.A.	Transpo.	Min cost	Heuristic
12	Blumenfeld, et al.	1985	Other	F	C	U	C	S	D	Vehicle assignment	Static routing.	Static allocation	Joint	Min cost	Analytical results and heuristic
13	Burns, et al.	1985	Other	F	C	U	C	S	D (constant), same across retailers	Retailer grouping	N.A.	N.A.	Transpo.	Min cost	Theoretical analysis
14	Campbell, et al.	1997	IRP	F/I	C/P	L	C (identical)	S	D/S, Retailer specific	Retailer selection	Static routing	Static allocation	Joint	Min cost	Integer Programming, stochastic Programming, heuristics
15	Cetinkaya and Lee	2000	Other	I	P	Single	C	S	S, i.i.d. across retailers	N.A.	N.A.	Static allocation	Joint	Min cost	Renewal-theoretic model
16	Chan and Simchi-Levi	1998	Other	I	C	U	C (non-identical)	S	D (constant), Retailer specific	Retailer grouping	Static routing	Static allocation	Joint	Min cost	Bounds and heuristic
17	Chan, et al.	1998	IRP	I	P	U	C	S	D, Retailer specific	Retailer grouping	Static routing	Static allocation	Joint	Min cost	Analysis and heuristic

Table 2: A Classification of Contemporary JTIP Literature – Continued

No.	Author	Year	Subject	Horizon	Review	Vehicle No.	Vehicle Capacity	Product	Orders/Demands	Assignment	Routing Policy	Allocation Policy	Policy Variable	Objective	Solution & Method
18	Chien, et al.	1989	IRP	F	P	L	C (non-identical)	S	D, Retailer specific	Vehicle assignment	Dynamic routing	Static allocation	Joint	Max profit	Mixed integer programming, heuristic, and upper bound
19	Christiansen	1999	SRP	F	N.A.	L	C (non-identical)	S	D, Harbor specific	Vehicle assignment	Dynamic routing	Static allocation	Joint	Min cost	Decomposition, integer programming
20	Christiansen and Nygreen	1998 a	SRP	F	N.A.	L	C (non-identical)	S	D, Harbor specific	Vehicle assignment	Dynamic routing	Static allocation	Joint	Min cost	Mixed integer programming
21	Christiansen and Nygreen	1998 b	SRP	F	N.A.	L	C (non-identical)	S	D, Harbor specific	Vehicle assignment	Dynamic routing	Static allocation	Joint	Min cost	Decomposition, dynamic programming
22	Dror and Ball	1987	IRP	F	P	L	C	S	S, Retailer specific	Ret. selec./ Veh. Assign.	Static routing	N.A.	Transpo.	Min cost	Analytical results and heuristic
23	Dror and Trudeau	1996	IRP	F	C	U	C	S	D/S, Single retailer	N.A.	Direct delivery	Static/ Dynamic allocation	Inv.	Optimize cash flow	Analytical research
24	Dror, et al.	1989	VRP	F	P	L	C (identical)	S	S, Retailer specific	N.A.	Static routing	Dynamic allocation	Transpo.	Min distance traveled by the vehicle.	Stochastic Programming, Markov-decision process
25	Dror, et al.	1985	IRP	F	P	L	C	S	S, i.i.d. across retailers	Retailer selection	Static routing	Static allocation	Transpo.	Min cost	Integer programming, decomposition
26	Federgruen and Zipkin	1984	IRP	F	P	L	C (non-identical)	S	S, Retailer specific	Vehicle assignment	Dynamic routing	Static allocation	Joint	Min cost	Nonlinear integer programming
27	Fisher, et al.	1982	VRP	I	P	L	C (non-identical)	S	D	Vehicle assignment	Static routing	Static allocation	Transpo.	Min cost	Integer programming
28	Fumero and Vercellis	1999	Other	F	P	L	C	M	D, Retailer specific /Product specific	N.A.	Dynamic routing	Static allocation	Joint	Min cost	Mixed integer programming, heuristic, and lower bound
29	Gallego and Simchi-levi	1990	Other	I	C	U	C (identical)	S	D (constant), Retailer specific	N.A.	Direct delivery	N.A.	N.A.	Min cost	Heuristic and lower bound
30	Gaur and Fisher	2002	IRP	I	P	U	C	S	D, Retailer specific	Retailer grouping	Static routing	Static allocation	Joint	Min cost	Nonlinear integer programming, heuristic algorithm
31	Golden, et al.	1984	IRP	I	P	L	U	S	S, Retailer specific	Ret. selec./ Veh. Assign.	Dynamic routing	Static allocation	Joint	Min cost	Heuristic, decomposition, and simulation
32	Herer and Levy	1997	IRP	F	P	U	C	S	S, Retailer specific	Retailer selection	Dynamic routing	Static allocation	Transpo.	Min cost	Heuristic
33	Herer and Roundy	1997	Other	I	P	Single	U	S	D (constant), Retailer specific	Retailer selection	Static routing	Static allocation	Joint	Min cost	Heuristics and dynamic programming

Table 2: A Classification of Contemporary JTIP Literature – Continued

No.	Author	Year	Subject	Horizon	Review	Vehicle No.	Vehicle Capacity	Product	Orders/Demands	Assignment	Routing Policy	Allocation Policy	Policy Variable	Objective	Solution & Method
34	Jaillet, et al.	2002	IRPSF	F/I	P	L	C	S	S, Single retailer	N.A.	N.A.	Dynamic allocation	Transpo.	Min cost	Analytical research
35	Jones and Qian	1997	Other	I	C	U	C (identical)	S	D (constant), Retailer specific	N.A.	Direct delivery	N.A.	N.A.	Min cost	Analytical research
36	Kleywegt, et al.	2002 a	IRP	I	P	L	C	S	S, Retailer specific	N.A.	Direct delivery	Static allocation	Joint	Max profit	Markov-decision process and heuristic
37	Kleywegt, et al.	2002 b	IRP	I	P	L	C (identical)	S	S, Retailer specific	Retailer selection	Dynamic routing	Static allocation	Joint	Max profit	Markov-decision process and heuristic
38	Kumar, et al.	1995	IAP	I	P	Single	U	S	S, Retailer specific	N.A.	Static routing	Static/ Dynamic allocation	Inv.	Min cost	Optimal
39	Larson	1988	SIRP	I	P	L	C (identical)	S	S, Retailer specific	N.A.	N.A.	N.A.	N.A.	Min cost	Heuristic
40	Minkoff	1993	DDP	I	P	U	U	S	S, Retailer specific	Vehicle assignment	Static routing	Static allocation	N.A.	Min cost	Markov-decision process and heuristic
41	Park, et al.	2002	IRP	I	P	Single	U	S	S, i.i.d. across retailers	N.A.	Dynamic routing	Dynamic allocation	Joint	Min cost	Heuristic and analytical results.
42	Qu, et al.	1999	IRP	I	P	Single	U	M	S, Product specific	Retailer selection	Dynamic routing	Static allocation	Joint	Min cost	Heuristic and lower bound
43	Reiman, et al.	1999	IAP	I	P	Single	C	S	S, Retailer specific	N.A.	Direct delivery/ Static routing	Static/ Dynamic allocation	Inv.	Min cost	Queuing control theory and heavy traffic analysis
44	Savelsbergh and Goetschalckx	1995	VRP	F	P	L	C (identical)	S	S, Retailer specific	Vehicle assignment	Static routing	Dynamic allocation	Transpo.	Min cost	Heuristic
45	Shen, et al.	2003	Other	F	N.A.	N.A.	N.A.	S	S, Retailer specific	N.A.	N.A.	N.A.	N.A.	Min cost	Nonlinear integer programming
46	Trudeau and Dror	1992	IRP	F	P	L	C	S	S, Retailer specific	Retailer selection	Dynamic routing	Dynamic allocation	Transpo.	Min cost	Heuristic
47	Tyworth	1992	Other	N.A.	P	Single	N.A.	S	S, Single retailer	N.A.	Direct delivery	N.A.	Inv.	Min cost	Framework
48	Viswanathan and Mathur	1997	IRP	I	P	U	U/C (identical)	M	D, Retailer specific /Product specific	Vehicle assignment	N.A.	Static allocation	Joint	Min cost	Heuristic
49	Webb and Larson	1995	SIRP	I	P	L	C (identical)	S	S, i.i.d across periods, but retailer specific.	N.A.	N.A.	N.A.	N.A.	Min fleet size	Heuristic

3. Review of Problems and Models

As background, we will first briefly describe the Vehicle-Routing Problem (VRP). Interested readers should refer to the classification of Bodin and Golden (1981) or, more recently, Anily and Bramel (1999) for an examination of the VRP in the context of supply-chain management.

In its simplest form, the VRP is a single-period problem in which a fleet of M (≥ 1) identical vehicles make deliveries from a single depot to a set of N (≥ 1) retailers. Retailer orders are assumed to be known. The objective is to assign retailers to vehicles and; then, for each vehicle, determine a route originating from and terminating at the depot, such that (i) each retailer is visited exactly once; (ii) each retailer's order is filled; (iii) vehicle-operating constraints (e.g., vehicle capacity constraints) are satisfied; and (iv) total transportation distance (or cost) is minimized. The VRP ignores inventory-related decisions and their associated costs. Delivery quantities are constrained to exactly equal the corresponding retailer orders. Within the context of the JTIP, given any set of delivery quantities for each retailer, VRP procedures could be applied to assign vehicles to retailers and routes to vehicles.

By definition, JTIPs consider both transportation policy and inventory policy. However, *how* these policies are considered varies considerably, depending on how the specific JTIP is formulated. Problem (1)-(3) in section 2 is the most general JTIP. This form of JTIP considers transportation and inventory as *joint policy variables.* Section 4.3 will review this literature. Some JTIPs consider transportation policy under a single, given inventory policy. Section 4.1 reviews this literature. Other JTIPs consider the inventory policy under a single, given transportation policy. They will be reviewed in section 4.2. Finally, a few JTIPs do not fall into any of these categories. They will be reviewed in Section 4.4.

3.1 Transportation Policy under a Given Inventory Policy

The models reviewed in this section consider only a single, given inventory policy, *I*, in solving (1)-(3), in which case $C_t(I,T)$ in (1) is replaced by $C_t(T|I)$. For example, Dror, et al. (1985, 1989), Herer and Levy (1997), Bard, et al. (1998), and Jaillet, et al. (2002) all assume that the retailers follow (s_i, S_i) policies; i.e., the vehicle fills up retailer *i* to full capacity, S_i, whenever it makes delivery to that retailer. These models first identify the retailers to be visited (by reviewing the retailers' inventory position, etc.) and; then, solve (1) by combining these retailers into one or more routes, and assigning a vehicle to each route. Some models employ static routing, others employ dynamic routing. Some of the models in this group assume deterministic demand (Section 4.1.1); others, stochastic demand (Section 4.1.2). Eleven of the 49 papers reviewed are in this group.

3.1.1 Models with Deterministic Demand

JTIPs in this subgroup assume demand is known (either stationary or non-stationary), and generally assume that this demand must be satisfied (i.e., no backorders or lost sales). Typically, the depot holds no inventory.

Bertazzi, et al. (2002) consider a periodic-review, multi-product JTIP with a given inventory policy. The inventory policy, *I*, is restricted to a type such that each retailer has a given a minimum and a maximum level of the inventory for each product; and each retailer must be visited before any of its inventories reach the minimum level. Further, every time a retailer is visited, the quantities delivered are such that the prespecified maximum levels are reached for each product. The problem is to determine for each discrete time period the retailers to be visited and the route of the vehicle. Their heuristic first determines a feasible set of delivery time periods for each retailer to be visited and; then, given these, retailers are inserted in a route. The goal is to minimize the sum of transportation and inventory costs.

Other models in this subgroup are Fisher, et al. (1982) and Burns, et al. (1985).

3.1.2 Models with Stochastic Demand

In this subgroup, customer demands are retailer specific and stochastic; i.e., are represented by a set of random variables with known probability distributions. Since demand is uncertain, these models all address the possibility of customer demand exceeding supply (e.g., expediting, lost sales).

Dror, et al. (1985) consider a stochastic IRP over an annual horizon, in which *I* is a given (s, S) policy. In their model, the retailers' daily demands are assumed to be drawings from i.i.d. normal distributions with known parameters. Their problem is formulated as a two-stage integer program that, first, determines the delivery time for each retailer and; then, schedules customer delivery by route, vehicle, and day of the week.

In a companion paper, Dror and Ball (1987) reduce the annual problem to a shorter planning horizon (*m*-day) problem. The key in doing so is to define short-term costs that reflect the long-term costs. This problem is modeled as a mixed-integer program, in which the effects of current decisions on later periods are accounted for by penalty or incentive factors. In their problem, *I* is restricted to the inventory policy that maintain a specified minimum inventory level at each retailer.

Trudeau and Dror (1992) address a similar stochastic IRP, and employ a retailer-selection submodel, in which the retailers are divided into two sets: retailers that *must* be replenished during the short planning period and retailers that *might* be replenished because of special consideration. They formulate the problem as a mixed-integer program, and develop heuristics for solving (1) – (3).

Bard, et al. (1998) and Jaillet, et al. (2002) examine an IRP with Satellite Facilities (IRPSF), and both models assume that the retailers follow (s, S) policy. In the single-product, periodic model considered by Bard, et al., both the depot and the satellite facilities hold an unlimited supply of the product, and vehicles can reload at any of these locations. However, all vehicle routes must originate from and terminate at the depot. They solve this IRPSF using a two-week rolling-horizon heuristic. More specifically, for a given two-week horizon, they first use a retailer selection, i.e., identify the retailers to be visited. However, only retailers scheduled for the first week are routed. The two-week planning horizon is then rolled forward by a week and the process is repeated. Jaillet, et al. (2002) also reduce the IRPSF from the annual horizon to a biweekly rolling planning-horizon problem, and provide the main justifications behind the retailer selection as well as the justifications behind the derivations of the incremental costs as used in the two-week rolling horizon heuristic of Bard, et al. (1998).

Savelsbergh and Goetschalckx (1995) examine the question of whether static routing is a viable alternative to dynamic routing. They consider managing a single-depot, multi-retailer, stochastic distribution system over a finite horizon under periodic review. The depot serves the retailers with a set of vehicles having identical capacities. The retailer inventory policy, *I,* is assumed to be a base-stock policy. The objective is to minimize the total transportation cost plus the "recourse" cost (a cost associated with service failure). Their numerical studies show that the cost increase caused by static routing was small (less than 10%) compared to dynamic routing. Hence, taking into consideration the possible advantages of static routing policy (e.g., increased performance by the drivers because they become more familiar with static routes over time, increased performance at the facility, decreased management costs), Savelsbergh and Goetschalckx argue that static routing might be an attractive alternative to dynamic routing.

Other stochastic JTIP papers in this subgroup are Dror, et al. (1989) and Herer and Levy (1997).

3.2 Inventory Policy under a Given Transportation Policy

The models reviewed in this section consider only a single, given transportation policy, *T*, in solving (1)-(3), in which case $C_t(I,T)$ in (1) is replaced by $C_t(I|T)$. In determining inventory policy, both static allocation and dynamic allocation policies have been modeled. Note that by postponing allocation, dynamic allocation provides risk-pooling of the inventory on the vehicle(s). Everything else being equal, this improves system performance. Seven of the 49 papers reviewed are in this group. Some of the models in this group assume deterministic demand (Section 4.2.1); others, stochastic demand (Section 4.2.2).

3.2.1 Models with Deterministic Demand

Bertazzi, et al. (2000) consider a multi-product, one-to-one (i.e., single depot and single retailer) distribution system. The given transportation policy is direct delivery under a given, discrete set of shipping frequencies. The objective is to decide how much of each product to ship at each frequency such that $C(I|T)$ is minimized. The solution methodology is branch-and-bound; the authors provide dominance rules to improve performance. Bertazzi, et al. (2000) is the only paper in this deterministic-demand subgroup.

3.2.2 Models with Stochastic Demand

Kumar, et al. (1995) and Reiman, et al. (1999) both consider a single-product JTIP in which a single vehicle (capacitated in Reiman, et al., but uncapacitated in Kumar, et al.) travels along a predetermined static route, allocating its entire inventory to the retailers before returning empty (i.e., the depot holds no inventory). Both static allocation and dynamic allocation are considered. The objective in Kumar, et al. is to minimize the expected inventory (i.e., holding and backordering) cost at the retailers, they define the "risk-pooling incentive" provided by dynamic allocation (i.e., the reduction in system demand variance) and estimate the cost savings of using dynamic allocation versus static allocation using simulation. The objective in Reiman, et al. is to minimize the long-run inventory and transportation cost. Kumar, et al. use dynamic programming, while Reiman, et al. use queueing control theory and heavy-traffic analysis. Although these authors model the problem in much different ways, they both show, under appropriate assumptions, that the optimal system replenishment policy is a base-stock policy. Also, they both conclude that dynamic allocation significantly outperforms static allocation.

Berman and Larson (2001) also consider a stochastic single-depot, single-product, multi-retailer JTIP with static routing and dynamic allocation. The given transportation policy is that a capacitated vehicle leaves the depot filled to capacity and visits all the customers along a predetermined route. The $C_t(I|T)$ considered by Berman and Larson consists of four parts: costs of earliness, lateness, product shortfall, and returning to the depot nonempty. Specifically, each customer has a given "re-service" point, which corresponds to each retailer's desired inventory level just before a delivery is to be made. The system incurs an earliness (lateness) cost if the vehicle makes its delivery when inventory is above (below) this level. A product shortfall cost is incurred if the vehicle leaves the customer's tank less than full. The chosen inventory policy, I, is assumed to be of the following form: While visiting customer j, the vehicle either fills up customer j to capacity or allocates an amount, determined by dynamic programming, whichever is less. They show how to use stochastic dynamic programming to solve the problem.

Bassok and Ernst (1995) is the only model in this subgroup to consider multiple products. Similar to the 3 models above, the transportation policy being considered is that a capacitated vehicle loaded with multiple products visits all the customers along a predetermined sequence. Instead of minimizing $C_t(I|T)$ in (1), however, they formulate a profit maximization problem, decomposing it into two subproblems: a Product Allocation Problem (PAP) and a Space Allocation Problem (SAP). The PAP determines the quantity of each product to allocate to each retailer and the SAP determines the allocation of the limited vehicle space to different products (i.e., system replenishment). Their algorithm starts by solving the PAP with the objective of identifying the minimum quantity of each product i to have on board when the vehicle gets to retailer j, using standard dynamic programming. In solving the SAP, the solution obtained in the PAP is used, i.e., they assume that all the products are allocated to the retailers optimally. Since the SAP is concave in the spaces allocated to each product, given the constraints defined by the vehicle capacity, their method determines the space for each product while maximizing the potential profit.

Finally, Dror and Trudeau (1996) consider both deterministic and stochastic demand. They examine the distribution of a single product over an annual horizon from the perspective of the net present-value of cash flow. Transportation policy, T, is restricted to direct delivery. Their analysis is based on the single-customer case. Their numerical study demonstrates that it would be advantageous for the decision maker to set deliveries for a large percentage of the retailers based on the present value of cash flow. In particular, given stochastic customer demands, deliveries based on the cash-flow consideration will tend to reduce the number of stockouts as compared to those based on the operational efficiency consideration (i.e., to maximize the average number of units delivered per hour).

3.3 Joint Transportation-Inventory Policy

The models reviewed in this section consider transportation and inventory as joint policy variables. Various methodologies have been used on the general JTIP, among them integer programming, stochastic programming, and Markov-decision analysis. For example, among the mathematical-programming models, Bell, et al. (1983), Chien, et al. (1989), and Gaur and Fisher (2002) formulate the JTIP as a mixed-integer program; Campbell, et al. (1997) propose two approaches: integer programming for a deterministic JTIP and dynamic programming for a stochastic JTIP. However, regardless of the method proposed, *optimal* joint transportation-inventory policies are very difficult to find. Hence, heuristics are proposed in all these models. Twenty-four of the 49 papers reviewed are in this group. Some of the models in this group assume deterministic demand (Section 4.3.1); others, stochastic demand (Section 4.3.2).

3.3.1 Models with Deterministic Demand

The most popular methodology used in this subgroup of JTIPs is partitioning. Under a "Fixed Partitioning" (FP) policy, sets of retailers (not necessarily disjoint), often called regions, are created that, together, include all the retailers. Once the routes and the assignment of the vehicles to the routes are determined, each region is then served separately and independently from all other regions. If a retailer belongs to more than one region, then some fraction of its demand is satisfied by each. Typically, each time one of the retailers in a given region is visited, all other retailers in that region are also visited. The following provides an overview of these FP models.

In the single-product, single depot, multi-retailer deterministic model studied by Anily and Federgruen (1990, 1993) and Anily (1994), *T* is restricted to the FP policies proposed by Anily and Federgruen (1986). The objective is to determine a long-term joint transportation-inventory policy that enables all retailers to meet their demands while minimizing system-wide long-run average transportation and inventory costs. One assumption in all three models is that the demand rate μ_i faced by retailer *i* is a multiple of some common quantity μ. A *demand point* is defined as a point facing a demand rate of μ. Hence, a retailer with demand of $K\mu$ is treated as *K* separate demand points, and the partition is over these demand points.

Anily and Federgruen (1993) extends the analysis in Anily and Federgruen (1990) to the case in which the depot can hold inventory. A Combined Routing and Replenishment Strategies Algorithm (CRRSA*), similar to the CRRSA in Anily and Federgruen (1990), is proposed. CRRSA* differs from CRRSA in that the interval between two visits to a region is rounded to a power-of-two series of some prespecified interval. Anily (1994) considers the case where holding costs are retailer dependent, and develops a regional partitioning heuristic, which is asymptotically optimal in the set of FP policies. Anily shows that the optimal solution can be bounded from below by a special partitioning problem with closed-form solution.

Chan, et al. (1998) characterize the asymptotic effectiveness of the class of FP policies and the class of so-called Zero-Inventory Ordering (ZIO) policies, under which a retailer is replenished if and only if its inventory is zero. Their analysis is motivated by the observation that the class of FP policies is a subset of the class of ZIO policies. Worst-case studies as well as probabilistic bounds under a variety of probabilistic assumptions are provided.

Anily and Federgruen (1993) and Herer and Roundy (1997) examine a model in which inventory can be held at the depot. Both papers also adopt a power-of-two heuristic in determining the retailers' reorder intervals. Anily and Federgruen show that the gap between the costs of the power-of-two heuristic and a lower bound for the minimum cost is at most 6% for a sufficiently large numbers of retailers (i.e., $N \to \infty$), and that this gap is typically small even for problems with a moderate number of retailers. Herer and Roundy propose heuristics for finding the power-of-two reorder intervals and present a dynamic-programming algorithm to compute the optimal power-of-two reorder intervals for single-depot multi-retailer supply chains with arbitrary monotone nonnegative order costs.

Gaur and Fisher (2002) examine a periodic-review model of a supermarket chain using FP. Their objective is to determine a weekly delivery schedule that specifies the times when each store should be replenished and the routes for the capacitated vehicles that visit these stores at a minimum transportation cost. The original problem is decomposed into a set-partitioning problem on the stores and a shortest-path problem for each set. They show that the optimal FP policy has at most two deliveries per route, and is polynomially solvable using a generalized minimum weight-matching approach. The implementation at a supermarket chain is described. First-year distribution-cost savings of about 4% are reported.

Viswanathan and Mathur (1997) examine a multi-product model. They propose a power-of-two heuristic algorithm, which generates stationary joint transportation-inventory policies for the cases with capacitated vehicle(s) or not. In their M-product, N-retailer problem, Viswanathan and Mathur define an item as a product at a specific retailer, so there are NM items in total. By determining the reorder interval and quantity for each item, the original problem is transformed into a single-product, MN-retailer problem.

Finally, Chan and Simchi-Levi (1998) consider a multi-depot model. In their three-level supply chain, a fixed number of depots order from a single outside supplier, and supply several geographically-dispersed retailers. The objective is to develop a joint transportation-inventory policy to minimize long-run average transportation and inventory costs. They first develop a lower bound on $C(I,T)$, the long-run average total cost, then, propose a Zero-Inventory Ordering policy and characterize the effectiveness of this policy relative to all other feasible policies. They show that, in a policy that minimizes $C(I,T)$, each depot receives fully-loaded vehicles from the outside supplier but never holds inventory; i.e., each depot serves only as a coordinator of the frequency, time and sizes of deliveries to the retailers.

Other papers in this subgroup include Bell, et al. (1983), Blumenfeld, et al. (1985), Chien, et al. (1989), Christiansen (1999), Christiansen and Nygreen (1998a,b), and Fumero and Vercellis (1999).

3.3.2 Models with Stochastic Demand

Federgruen and Zipkin (1984) published the first JTIP model in this subgroup. They solve a single-day problem and show how some well-known interchange heuristics for the deterministic VRP can be modified to handle the stochastic JTIP. In their model, the quantity of product to be delivered to each retailer is determined on the basis of the level of its inventory. Then, the retailers are assigned to the vehicles and the routes are determined. They model the stochastic JTIP as a nonlinear integer program. The key idea behind their solution methodology is to decompose the original problem into an inventory-allocation problem (using static allocation) and a VRP for each vehicle. Their algorithm constructs an initial feasible solution and iteratively improves it by exchanging customers between routes.

In Golden, et al. (1984), Ω is restricted to the set of inventory policies that maintain an "adequate" level of inventory for all customers, and π is restricted to dynamic-routing policies. Retailer selection is determined by computing the "urgency" (i.e., the ratio of tank size to tank inventory) of each customer, excluding customers whose "urgency" is below a threshold. Their heuristic can be summarized as follows: Initially, a time limit for the total travel time, say T_{MAX}, is set to the number of vehicles multiplied by the length of a day. A large route to all the retailers being visited is constructed iteratively. Customers are added, one at a time, according to the highest ratio of urgency to extra time required to visit this customer. Customers are added until T_{MAX} is reached or there are no more customers left. The final route is partitioned into a set of feasible subroutes by requiring that each customer be filled to capacity whenever it is visited. If this turns out to be impossible, the heuristic can be re-run with a smaller value for T_{MAX}.

Kleywegt, et al. (2002a) formulate an IRP with direct deliveries as a Markov-decision process and propose a dynamic-programming approach. The original problem is decomposed into individual retailer subproblems. Kleywegt, et al. (2002b) extend both the formulation and the approach to handle multiple deliveries per trip. The retailers are grouped into regions to form subproblems. Kleywegt, et al. (2002b) derive a result similar to Gaur and Fisher (2002): if each subset has at most two retailers, then, it can be solved in polynomial time, by solving a maximum-weight perfect matching problem.

Adelman (2001) considers a multi-item inventory-control problem with joint replenishment costs. In his model, a dispatcher periodically monitors inventories for a set of items, where an item may represent a product, a location, or a product-location pair depending on the business setting (note: when the item represents location, it is an JTIP). Objective (1) is decomposed into a collection of functions separated by item. His method first decides which retailers to visit, then, partition these retailers into disjoint subsets. Static allocation is used. Adelman also formulates the problem as a Markov-decision process and studies a price-directed control policy. Rather than considering a myopic policy that minimizes only the costs related to the current replenishment, Adelman approximates the future using dual prices from linear-programming relaxations. Numerical studies show that the price-directed policy performs better than the myopic policy.

Park, et al. (2002) extend the single-product, single-vehicle, single-depot, *N*-retailer stochastic-demand JTIP model considered by Kumar, et al. (1995). In considering dynamic allocation of vehicle inventory, Kumar, et al. assume a given static route and focus attention on optimal allocation and system-replenishment policies, while Park, et al. consider dynamic routing and dynamic allocation. In particular, for a "symmetric" system (in which all retailers are equidistant from the depot and one another), Park, et al. show that a least-inventory-first transportation policy is optimal (i.e., visit next the retailer with the smallest net inventory). They determine dynamic allocation and system inventory-replenishments that minimize the system-wide expected costs between successive depot replenishments.

Other JTIP models in this subgroup are Qu, et al. (1999) and Cetinkaya and Lee (2000).

3.4 Others

Some JTIP models do not fall into any of the categories above, among them the Strategic Inventory Routing Problem (SIRP), the problems considering the effectiveness of direct delivery, the delivery-dispatching problem considered by Minkoff (1993), and the location-inventory problem considered by Shen, et al. (2003). Seven of the 49 papers reviewed are in this group.

The label "Strategic Inventory-Routing Problem" (SIRP), which deals with resource planning decisions; i.e., minimizing the size (or cost) of the vehicle fleet required, was first introduced by Webb and Larson (1995), although the idea had been proposed earlier by Larson (1988). Both papers consider the SIRP in a stochastic setting. Larson (1988) offers a two-step heuristic procedure to solve the SIRP: (i) define an approximate deterministic SIRP, then, (ii) solve this problem to find the fleet size. Webb and Larson (1995) address the case in which the set of retailers being visited on each route can change over time (i.e., dynamic vehicle assignment). To determine the fleet size, *period* (defined as the number of routes taken by the vehicles between two consecutive deliveries to retailer *i*) and *phase* (i.e., the number of routes between the beginning of some predetermined routes and the first route visiting retailer *i*) are introduced as additional decision variables to generalize the approach used in Larson (1988) for the SIRP. Their computational results show that the period-phase approach is cost saving (reflected in average vehicle requirement) in most of their test problems.

Gallego and Simchi-Levi (1990), Jones and Qian (1997), Barnes-Schuster and Bassok (1997) develop models to consider the effectiveness of direct delivery (i.e., the ratio of the long-run average cost of direct delivery to a lower bound on the long-run average cost over all possible policies).

Both Gallego & Simchi-Levi (1990) and Jones & Qian (1997) assume that each retailer faces a constant, retailer-specific, daily demand. The depot holds no inventory. Holding cost and fixed ordering cost are charged only at the retailers. Gallego and Simchi-Levi evaluate the long-run effectiveness of direct delivery and study conditions under which direct delivery is an efficient policy. Lower and upper bounds on $C(I,T)$ are derived. They show that direct delivery is within 6% of optimality under certain restricted parameter settings (i.e., when the economic lot size of each retailer is at least 71% of the vehicle capacity). This result provides useful guidelines for determining when to consider more complex transportation policies. Jones and Qian (1997) extend Gallego and Simchi-Levi by considering a setup cost consisting of two parts, the fixed cost associated with each vehicle trip, which accounts for the driver expense, maintenance cost, etc.; and the fixed cost for each vehicle stop, which accounts for unloading and storing costs at the retailer. (Such cost assumptions have also been used by Anily and Federgruen, 1990 and Burns, et al., 1985.) Jones and Qian show that the fully-loaded direct-delivery policy is optimal among all possible transportation-inventory policies if the vehicle capacity is less than the minimum retailer economic lot size.

Barnes-Schuster and Bassok (1997) examine when it will be effective for the depot to use direct delivery as its transportation policy and a myopic base-stock policy (rounded to full vehicle loads) as its inventory policy. They provide a lower bound on the expected long-run average cost as a sum of the expected inventory holding cost and the expected transportation cost. They conclude that from a practical point of view, in situations where demand distributions are normal or approximately normal, and vehicle capacities are close to the mean demand, then, very good results can be expected from direct delivery.

To summarize, Gallego and Simchi-Levi (1990), Barnes-Schuster and Bassok (1997), Jones and Qian (1997) present very similar results on the effectiveness of direct delivery: *when the vehicle capacity is either close to the mean of customer demand or small enough (i.e., less than the minimum retailer economic lot size), direct delivery would be a simple, but powerful, transportation policy to follow.*

Minkoff (1993) considers a Delivery Dispatching Problem (DDP) with a given set of itineraries, which are characterized by a list of customers to be visited and the quantity to be delivered to each customer. The objective is to dispatch the fleet of vehicles to replenish inventory at these retailers while minimizing the long-run average transportation and inventory costs. They model the problem as a Markov-decision process, and give decomposition heuristic with performance bounds. Inspired by Minkoff, we suggest further improvements on the JTIP might be obtained by incorporating the DDP as a subproblem of the general JTIP, or including the costs related to vehicle rental, dispatching, etc. while solving (1) - (3).

Shen, et al. (2003) consider a joint location-inventory problem. They observe that risk-pooling benefits (i.e., cost savings) might be achieved by allowing some retailers to serve as Distribution Centers (DCs). A DC receives shipments from the depot and distributes directly to the retailers within its jurisdiction. Given a set of retailers, their problem is to determine how many DCs to locate, and where to locate them. They also determine the level of safety stock to maintain at the DCs and the retailers to minimize total location, shipment, and inventory costs, while ensuring a specified level of service. The problem is formulated as two different integer-programming models: a location-allocation risk-pooling model and a set-covering model for determining the partition. Their computational results suggest that as the nonlinear safety stock costs increase relative to the other costs (or equivalently as the specified service level increases), the problem becomes harder to solve. Also, as these costs increase, the number of DCs located decreases.

4. Directions for Future Research

In this section we suggest several possible directions for future research.

Multi-Product JTIP. Most JTIP literature is concerned with the distribution of a single product. Only 5 of the 49 papers surveyed here consider multi-product case: Bassok and Ernst (1995), Viswanathan and Mathur (1997), Fumero and Vercellis (1999), and Bertazzi et al (2000, 2002). Of these 5 papers, Bassok and Ernst (1995) consider stochastic multi-product demand; the other 4, deterministic demand. On the other hand, many real-world distribution scenarios involve multiple products. The fundamental difficulty posed by multiple products, of course, involves the allocation of vehicle capacity to different products.

We suggest two heuristic approaches in solving (1) – (3) for multiple products-: (i) load each vehicle with exactly one product, thus, transforming the *M*-product JTIP into *M* single-product JTIPs (ii) decompose the original problem into three subproblems, a product-allocation problem (to determine the delivery quantities for each product to each retailer), a vehicle-routing problem (to determine the routes and assign the vehicles to different routes), and a capacity-allocation problem (to load different products on to the vehicles).

An associated problem, even given a limited number of uncapacitated vehicles, involves the choice of routes. For example, the inventory policy for one product might favor one particular transportation policy while the inventory policy for another product on the same vehicle might favor a different transportation policy.

Split-Delivery. As noted in Dror and Trudeau (1989), split-delivery (i.e., at least one retailer is assigned to two or more vehicles with each vehicle delivering a portion of that retailer's order or demand) can save money. They study a relaxed version of the VRP in which a delivery to a retailer can be split among any number of vehicles. Their numerical studies demonstrate the potential for cost savings (e.g., in terms of distance traveled and the number of vehicles deployed) through split-deliveries. Under the assumption that demand at each retailer is deterministic and an integer multiple of some common factor, as in Anily and Federgruen (1990), etc., a fixed partitioning approach is appropriate. In Fumero and Vercellis (1999), different vehicles are permitted to deliver to the same customer in the same time period.

Split-delivery brings significant additional complexity when demand is stochastic. Under a fixed-partitioning strategy, when a single retailer is represented by several demand points, capturing shortages at that retailers by summing shortages at its demand points seems inappropriate. Further, if a given retailer is restocked on several routes, whether under static or dynamic allocation, the allocation problem now spans multiple routes. Specifically, an allocation on one route, even to a retailer that is only on that one route, will affect the allocations on another route, if these two routes share at least one retailer.

However, due to the potential cost savings, split-delivery is still an area that deserves some consideration. We think that the JTIP with split-deliveries can be approached from two perspectives: *statically* and *dynamically*. Under a *static* delivery-splitting policy, deliveries are split when the vehicle(s) leaves the depot, and fixed thereafter. Under a *dynamic* delivery-splitting policy, whether the delivery to some retailer *i* should be split is not determined until some vehicle arrives at that retailer. Researchers might also focus on evaluating the "goodness" of split-delivery (i.e., how much cost could be saved), determining the necessary conditions to assure that split-delivery would be efficient and cost effective, and developing good splitting heuristics.

Multi-Depot JTIP. Few models address the case in which there are multiple places where the vehicle(s) can reload and/or originate from/terminate at. Bard, et al. (1998) and Jaillet, et al. (2002) consider the IRP with single depot and satellite facilities. In their models, vehicle(s) must originate from and terminate at the depot, but can reload at any of the satellite facilities while visiting the retailers. In the three-level distribution system (vendor-depot-retailer) considered by

Chan and Simchi-Levi (1998), multiple depots are allowed and vehicles can originate from and terminate at any of these depots. In many real-world distribution systems, multiple depots (or distribution centers) are allowed, which brings more flexibility and convenience.

We see two ways in solving problem (1)- (3) with multiple depots: a *fixed-destination policy* and a *flexible-destination policy*. Under the *fixed-destination policy*, a vehicle must depart from and terminate at the same depot. By incorporating the FP policy, the retailers can be divided into regions, each assigned to one depot; thus, the original problem is decomposed into several independent single-depot, multi-retailer JTIPs. Under the *flexible-destination policy*, a vehicle departs from some depot and can return to any other depots. We also want to mention that by combining with the multi-product concern, another possible area for future research would be to develop the necessary steps for a joint approach to an extended (i.e., multi-product, multi-depot) JTIP.

System Replenishment. By system replenishment, we mean the replenishment policy for the depot (i.e., how frequently the depot should order; and what order quantity should be employed). Seven of the 49 papers address system replenishment: Burns, et al. (1985), Bassok and Ernst (1995), Kumar, et al. (1995), Herer and Roundy (1997), Reiman, et al. (1999), Cetinkaya and Lee (2000), and Berman and Larson (2001). Kumar, et al. (1995) show that under both static- and dynamic- allocation policies, the optimal myopic system replenishment policy is a base-stock policy. Bassok and Ernst (1995) examine system replenishment by solving the problem of loading a capacitated vehicle with multiple products. Most of the JTIP literature ignores system replenishment by assuming that the depot orders with an outside supplier with unlimited supply and instant delivery. However, when the depot holds inventory or leadtimes to the depot are involved (as considered by Herer and Roundy, 1997), then, when and how much to order for the depot must be taken into consideration.

5. Conclusion

This article surveyed a class of problems characterized by the simultaneous presence of transportation and inventory concerns within the framework of a distribution system. The purpose was to provide some links and relationships within the contemporary JTIP literature. We first formulated the general optimization problem in the JTIPs and; then, proposed a detailed classification on the JTIPs. Papers were grouped and reviewed according to the three classes of decision variables, i.e., transportation policy with given inventory policy, inventory policy with given transportation policy, and joint transportation-inventory policy. Relevant JTIP models were compared and summarized. Based on the classification and outline of the contemporary JTIP literature, problems deserving future research effort as well as possible ways to consider them were presented.

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