

A Theory of Financial Innovation and Monetary Substitution with an Application to a Century of Data on the M1-Income Ratio

(Preliminary)

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March, 2009

Abstract

I study a DSGE model incorporating a monetary transactions technology in which a representative household invests time to improve its skill (human capital) in making transactions. The model is designed to reconcile salient short- and long-run features of the relationship between money, income, and the opportunity cost of holding money. An econometric reduced form, derived analytically, is revealing for the nature of pathologies perceived by the empirical money demand literature; particularly interesting is the possibility of a moving average root close to the unit circle. The impact elasticity of the M1-income ratio to an innovation to the opportunity cost is about -0.1 , whereas the long-run response to a unit permanent shift is -1 . In comparison to less parsimonious models, parametric restrictions are not rejected, and out-of-sample forecasts are improved. The theory also yields implications for the welfare cost of inflation vastly different from those of standard shopping time models.

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*Thanks are due particularly to Joe Haslag, Kim Huynh, and Mohitosh Kejriwal for thoughtful discussions; and to participants in seminars at Purdue University and the 2009 Midwest Macroeconomics Meetings.

1 Introduction

A large volume of research supports the view that accounting for the effects of financial innovation is important in understanding the relationship among real money, income, and measures of the money opportunity cost. These effects have been recognized as leading candidates in explaining the instabilities perceived in empirical money demand functions since the 1970s. While empirical researchers have demonstrated the explanatory power of a number of proxies for financial innovation, economic theory motivating this research remains relatively inchoate.^{1,2} In this paper, I propose a simple theory of financial innovation in general equilibrium amenable to empirical implementation. Having done so, I derive analytically a reduced form representation closely related to those studied in the conventional “money demand” literature. This exercise admits a reinterpretation of the shortcomings of these models as perceived by Goldfeld (1976) and many others. Finally, I estimate the model using a time series of annual observations of the M1-income ratio and the opportunity cost for the years 1900-2007; and I demonstrate the superior performance of the model from the point of view of relevant criteria.

The following constraints are posed for the construction of a consistent theory of the phenomena of interest. First, the theoretical model should adopt a microfoundational motivation for the necessity of use of money to purchase goods. Second, in line with conventional DSGE modeling norms, it is desirable that these empirical phenomena be consistent with a stationary equilibrium of the model.

With respect to the empirical implications, the view is that two stylized facts (described below) have been troublesome to the construction of (or conspicuously ignored by) models hewing to these constraints. Thus, the ultimate goal is to reconcile theory of the use of money with these features of the data.

The first stylized fact concerns the different long- and short-run elasticities of aggregate M1 to the opportunity cost in the data. Lucas (2000, p.250) summarizes the observation and the conundrum appropriately:

The interest elasticity needed to fit the long-term trends... is much too high to permit a good fit [of a simple parsimonious model] on a year-to-year basis. Of course, it is precisely this difficulty that has motivated much of the money demand research of the last 30 years, and has led to distributed lag formulations of money demand that attempt to reconcile the evidence at different frequencies. In my opinion, this reconciliation has yet to be achieved...

¹Data proxies include time trends, interest rate ratchet variables, and wire transfers volume.

²Ireland’s (1995) paper is significant in the theoretical literature.

The second observation is the long-run decline of the M1-income ratio (apparent in the top panel of Figure 1 below). While address of the first fact has been central to the theoretical motivation for the project from the outset, the second fact arises as an incidental nuisance encountered in the process. Nevertheless, it is a profound feature of the data, and I have accepted the position that one cannot convincingly reconcile the other features of the data without an explanation for it.

The theoretical novelty of the model is to allow that there is an investment aspect of effort spent economizing on the use of money, in the sense that one's habits and organizational skill help to determine the efficiency of time spent shopping. To the extent that the cost of time and money are at a premium, for example, one may make an extra effort to familiarize oneself with all of one's bank's branch locations, so that a stop at the bank can be made more conveniently in the course of other daily activities. Other examples of the types of investment that I have in mind are time spent learning about the services offered at an ATM, memorizing one's debit card password, or making a habit of placing the checkbook in a convenient place and renewing supplies of checks in a timely manner. In each of these examples, acquired organizational skill affects the efficiency of transactions; but it is the enhancement and maintenance of these skills, rather than employment of them, that is costly.

With an eye to the declining use of M1 in the data, the transactions technology is also amended to allow for substitution from M1 to other forms of money as financial institutions make such alternatives more attractive. Thus, households choose the optimal mix of monies considering the difference of the opportunity costs, and the relative efficiency of making transactions with the two forms of money.

The rest of the paper is structured as follows. In the next section, I explore the empirical motivation for the role of financial innovation and substitution effects. In the third section, I explain the details of the model, the equilibrium concept, and provide a characterization of equilibrium that I will use to study the model in the subsequent sections. Next, I provide a detailed analysis of the mechanisms of the model, showing that the implications for the welfare cost of inflation are quite different from those arising from the standard shopping time setup. The fourth section also derives the reduced form econometric representation mentioned above. The fifth section reports the results of the estimation exercise, compares the model's empirical properties to some focal alternatives, and discusses some economic implications. The final section of the paper concludes.

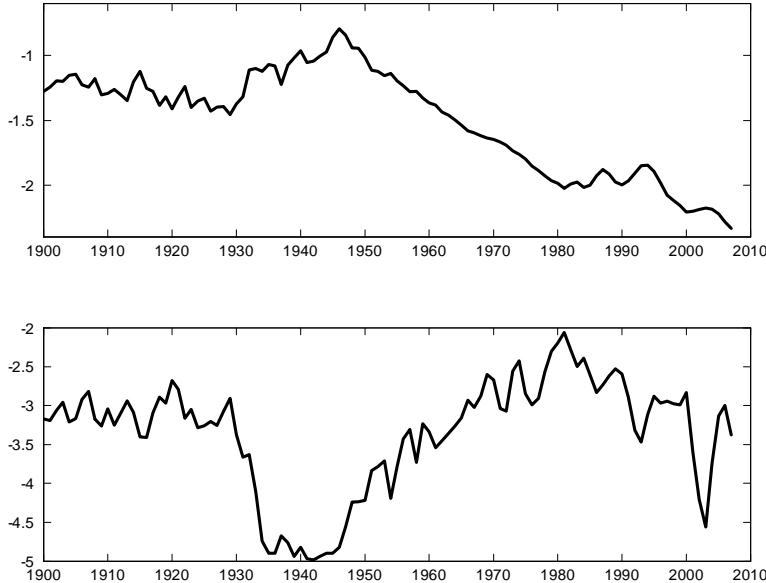


Figure 1: The top panel shows the log of the ratio of M1 to Net National Product for 1900-2007; the bottom panel shows the log of the opportunity cost construct $i_t / (1 + i_t)$, where i_t is the commercial paper rate, over the same period.

2 Empirical Motivation

Following in the footsteps of a vast volume of research, this paper seeks to shed light on the nature of the relationship between the M1-income ratio, shown (in logarithm) in the top panel of Figure 1, and a measure of the opportunity cost of holding M1 money shown in the bottom panel of the same figure. The purpose of this section is to discuss some rudimentary evidence for the approach to be followed, and to relate this approach to the theoretical literature. With respect to the data, the view espoused here is that one or more of the exploratory regressions to be discussed may represent a valid reduced-form representation in the sense that estimated coefficients are consistent for functions of structural parameters. A maintained hypothesis is that the (detrended) M1-income ratio and the nominal interest rate are each stationary ($I(0)$) variables.³ Similarly, I will not attempt to rationalize here the presence of lagged variables on the right-hand side of the regressions; a precise interpretation

³These are not uncontroversial positions; see Stock and Watson (1993) for an informed discussion of related issues.

will be offered in the sections to follow.⁴

Table 1 presents estimates and diagnostic statistics for regressions of the general form

$$my_t = a + \sum_{j=1}^2 b_j my_{t-j} + \sum_{j=0}^2 c_j r_{t-j} + dt + f\bar{r}_t + e_t, \quad (1)$$

where my_t is the logarithm of the ratio of M1 to net national product, r_t is the log-opportunity cost of holding M1, and \bar{r}_t is an interest rate ratchet variable defined as the maximum of the log-opportunity cost up to date t .⁵ The four specifications represented in columns (1)-(4) differ only with respect to restrictions placed upon the coefficients d and f on the trend and ratchet variables. Two panels in the table present results for the full sample of annual data from 1900-2007, and for the subsample from 1900-1973.⁶

The time trend, as in the regressions of columns (2) and (4) of each of these tables, and the interest rate ratchet, as in columns (3) and (4), have been interpreted in the empirical literature as proxies for effects of advancement of the technology for making transactions. The deterministic trend is clearly most easily interpretable as the manifestation of steady *exogenous* progress of such technology.⁷ On the other hand, explanatory power of a ratchet variable has been interpreted as capturing the *endogenous* investment aspect of financial innovation. For example, as warranted by extant opportunity costs, economic agents may be imagined to invest in technologies that reduce their reliance on M1 in making transactions. To the extent that such activity has persistent effects as investment, the effect will not be fully reversed once the opportunity cost has subsided.⁸

⁴One explanation for the presence of one lag of the endogenous variable, invoked since Goldfeld's (1973) seminal article, comes from "partial adjustment" of the endogenous variable to some desired level that is a function of the other explanatory variables. The presence of lags of other variables has been motivated by dependence of real money upon expected or "permanent" measures (Goldfeld (1973, p.600)), as the result of projecting future variables whose expectations affect households' choices on available information (Cuthbertson and Taylor (1987), e.g.), or as general manifestations of more complex dynamic mechanisms (Gordon (1984), Griliches (1967), and Goldfeld and Sichel (1987)).

⁵The data is described more fully in Subsection 5.2 below.

⁶Comparison across these subsamples is intended to shed light on the stability of the regressions after 1973, a subject of much controversy in the empirical money demand literature.

⁷Lieberman (1977, 1979) first articulated this interpretation of the trend in the money demand regression, and stressed its empirical importance.

⁸Enzler, Johnson, and Paulus (1976), Goldfeld (1976), Lieberman (1979), and others used the interest rate ratchet proxy to control for endogenous financial innovation. Goldfeld cites Quick and Paulus (undated) as the first study to do so. Ireland (1995) includes a useful review of this and related work.

	(1)	(2)	(3)	(4)
	1900-2007			
Trend Coef	–	-0.0012 (0.0004)	–	-0.0012 (0.0004)
Ratchet Coef	–	–	-0.072 (0.047)	0.0061 (0.055)
R^2	0.980	0.982	0.981	0.982
DW	1.96	1.96	1.96	1.96
SSE	0.310	0.310	0.331	0.310
	1900-1973			
Trend	–	-0.0016 (0.0005)	–	-0.0012 (0.0006)
Ratchet	–	–	-0.267 (0.104)	-0.097 (0.129)
R^2	0.917	0.929	0.925	0.930
DW	2.05	2.05	2.01	2.04
SSE	0.224	0.224	0.238	0.222
Chow F stat	0.570	0.734	0.751	0.745
p-value (df_1, df_2)	0.038 (34, 66)	0.164 (34, 65)	0.183 (34, 65)	0.177 (34, 64)
Simulation MSE				
1-year	0.007	0.009	0.006	0.006
5-year	0.071	0.067	0.054	0.037
10-year	0.180	0.106	0.088	0.054
20-year	0.390	0.186	0.036	0.089

Table 1: Ordinary least squares estimates of single equation money-income ratio regressions. Each column reports results from OLS regressions of the M1-income ratio on various collections of regressors. Each regression includes a constant; two lags of the dependent variable; and contemporaneous, once- and twice-lagged opportunity cost; coefficients of these regressors are not reported. Regressions in columns 2 and 4 include also a time trend; and the regressions in columns 3 and 4 include an interest rate ratchet variable. Coefficients on the trend and the ratchet are reported along with standard errors (in parentheses). Rows under the heading “Forecast MSE” report mean-square errors for various forecast horizons from dynamic simulations for the years 1974-2007 using coefficients estimated using 1900-1973 data.

Looking across these results, it is apparent that the stability of regression coefficients across subsamples is rejected (by the Chow test in column (1)), unless there is some control for a permanent shift of the M1-income ratio. Perceiving a long-run downward trend in the series plotted in the top panel of Figure 1 and the lack of any such trend in the opportunity cost series in the bottom panel provides some intuition for this finding. That is, the M1-income ratio exhibits a long-run shift that cannot be explained by movement in the opportunity cost series.

The four rows under the label “Simulation MSE” in Table 1 report the mean-square error from “dynamic simulations” of the type used in the literature following Goldfeld (1973) to assess the empirical performance of such models.⁹ Surveying these results, it will be noted that inclusion of either shift proxy, the trend or the ratchet, markedly improves the quality of simulations at all horizons. The best simulation performance is turned in by those specifications that include the ratchet variable, though a performance ordering of the specifications in columns (3) and (4) cannot be established unambiguously. On the other hand, the coefficient on the ratchet variable seems to be significant only in the pre-1974 subsample and only for the specification that does not allow for the time trend (i.e., column (3) of the bottom panel). In contrast, the trend coefficient is significant at conventional levels of inference in each regression in which it has been included, and the point estimate is remarkably stable across the subsamples; this conclusion obtains independently of the inclusion of the ratchet in the regression. Thus, while inclusion of the ratchet variable improves forecast performance, it is unclear that its effect is precisely identified by the data. This is suggestive that some degree of persistent financial innovation follows (endogenously) in response to opportunity cost fluctuations, but it seems that the relevant reduced-form of that process is not captured parsimoniously by inclusion of the opportunity cost ratchet.¹⁰

With respect to the trend, an additional subtlety deserves emphasis. In particular, notice that it is not merely the post-1973 period where financial innovation has affected the money demand relationship. That is, technological advance of the

⁹More precisely, the simulations are constructed recursively for each of the years after 1973 as predictions of the endogenous variable obtained by replacing the exogenous regressors by their realized values, replacing lagged endogenous variables on the right-hand side by value simulated for the previous year, and the recursions are initiated by taking “forecasts” to be consistent with realizations of the endogenous variable for earlier years. For example, the first simulation delivers forecasts $\widetilde{m}y_{1974,t}$ of $\widetilde{m}y_t$ for $t = 1974, \dots, 2007$, where $\widetilde{m}y_{1974,t}$ is obtained from (1) by replacing right-hand side variables my_{t-1} and my_{t-2} by $\widetilde{m}y_{1974,t-1}$ and $\widetilde{m}y_{1974,t-2}$, respectively, and $\widetilde{m}y_{1974,t}$ is taken to agree with my_t for $t < 1974$.

¹⁰These conclusions are consistent with the opinions of Lieberman (1979), who states that “the time trend emerged as the [more] effective of the technological change proxies” (p. 327).

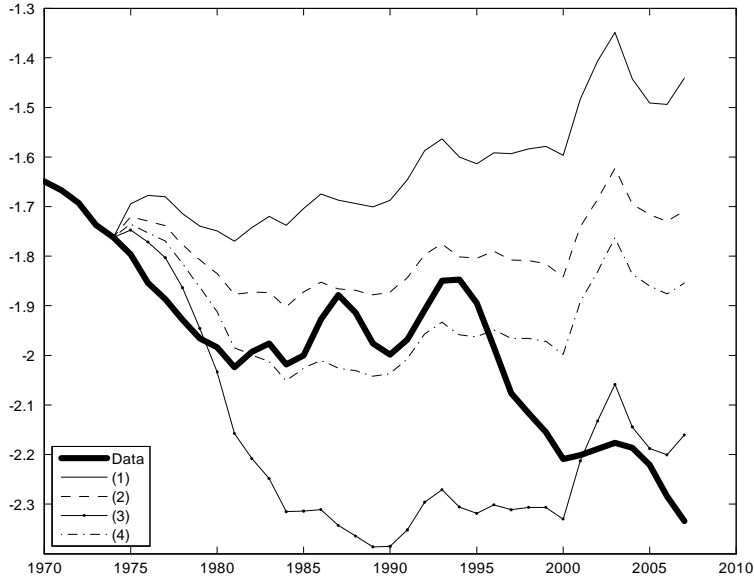


Figure 2: Simulations of M1-income ratio after 1973 from various statistical models.

transactions technology seems to have been preceding long before the 1970s.

For interest here, and for future reference, Figure 2 plots the data and the simulations obtained using each of the regressions imposing unit income elasticity using pre-1974 data.

A hypothesis to be maintained in the rest of the paper is that the effect captured by the time trend is a manifestation of effects of substitution of a broader form of money for M1 as technology for transacting with such an alternative improves, or as the spread between the opportunity costs (*vis a vis* non-monetary assets) of holding the two monies widens. A battery of regressions constructed by augmenting those in Table 1 to include the logarithm of the ratio of the non-M1 component of M2 to income on the right-hand side was also investigated; the time series of this variable is shown in Figure 3. Comparing this series to that of the M1-income ratio in the top panel of Figure 3, it seems plausible a priori that this substitution effect might be identifiable by the data. The regressions, which have not been reported here, do not show strong evidence in favor of a linear specification, however.

To close this section, I summarize the results, and offer some opinion about the implications for economic modelling. First, long-run stability of the relationship between the M1-income ratio and the opportunity cost of holding M1 is empirically plausible only if there is some allowance for permanent shift of the transactions

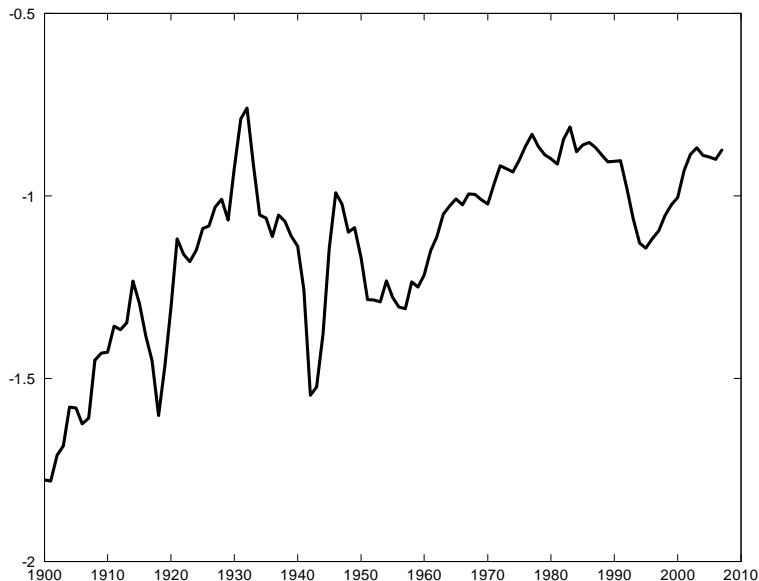


Figure 3: Log of the ratio of non-M1 M2 to Net National Product, 1900-2007.

technology. Second, the relevant shift seems to have an identifiable deterministic component. Finally, the shift seems also to have a component that responds to extant opportunity costs, but (perhaps not surprisingly) its precise nature seems complex.

These findings place important restrictions on the nature of models of narrow money that seek to be empirical relevant, while adhering to conventional paradigms of DSGE modelling. The benchmark models of Cooley and Hansen (1989, 1991, 1995), for example, while employing reference to M1, imply *level*-stationarity of the money-income ratio in the face of real growth. To my knowledge, Ireland (1995) and Uribe (1997) are the only models in the literature allowing simultaneously for balanced growth and a long-run shift of the transactions technology. Neither model allows for continuous financial innovation, or progress in the absence of rising opportunity costs, however; and these perfect foresight models are not adapted to formal econometric estimation.

3 A Model of Financial Innovation and Monetary Substitution

Motivated by the findings of the previous section, this section describes a theory of the response of the ratio of narrow money holdings to income to persistent opportunity cost shocks, and long- and short-run substitution incentives in equilibrium.

3.1 Preferences

A representative consumer has preferences over consumption C_t and leisure L_t as described by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln C_t + \psi_0 \frac{L_t^{1-\psi_1}}{1-\psi_1} \right], \quad (2)$$

where $\beta \in (0, 1)$, and $\psi_0, \psi_1 > 0$. The consumer is assumed to be endowed with a single unit of time which must be allocated in non-negative fractions to work (N_t), shopping (S_t), and skill accumulation (X_t), while leaving leisure non-negative:

$$L_t = 1 - N_t - S_t - X_t \geq 0; \quad N_t, S_t, X_t \geq 0.$$

The nature of these activities will be described as we proceed.

3.2 Production Technology

Output is produced as

$$Y_t = W_t N_t, \quad (3)$$

where the productivity of labor at time t , W_t , is an exogenous stochastic process. Output is used only for consumption, so that the aggregate resource constraint is

$$C_t = Y_t. \quad (4)$$

3.3 Markets and Government Policy

At the beginning of a period, the household has nominal resources in the forms of current income, money and bonds (plus interest) held from the previous period, and a transfer from the government. These resources are used to purchase consumption,

and accumulate money and nominal bonds to be held over into the following period. The representative consumer's budget constraint is

$$P_t C_t + M_t^1 + M_t^2 + B_t \leq P_t W_t N_t + M_{t-1}^1 + (1 + i_{t-1}^m) M_{t-1}^2 + (1 + i_{t-1}) B_{t-1} + T_t. \quad (5)$$

Here, M_t^1 is non-interest-bearing (“narrow”) money, M_t^2 is interest-bearing (“broad”) money, B_t is nominal bonds, T_t is a nominal transfer from the government, P_t is the price level, i_{t-1}^m is the interest earned by broad money held at the end of time $t - 1$, and i_{t-1} is the interest earned by bonds held at the end of period $t - 1$.¹¹

It is assumed that the government controls the nominal supply of each type of money, and that money and interest obligations are serviced from lump-sum tax revenues (negative transfers). The implied budget constraint for the government is

$$0 = i_{t-1}^m M_{t-1}^2 - (M_t^1 - M_{t-1}^1) - (M_t^2 - M_{t-1}^2) + T_t. \quad (6)$$

3.4 Transactions Technology

To capture the role of money in the economy, I extend the rationale described by McCallum and Goodfriend (1987). I assume that the technology for making transactions in the economy requires a combination of real monies M_t^i/P_t , shopping time, and transactions skill Z_{t-1} acquired from previous periods satisfying

$$Z_{t-1}^{1-\nu} S_t^\nu \geq \left\{ \frac{C_t}{\left[\left(\frac{M_t^1}{P_t} \right)^\alpha + \kappa_t \left(\frac{M_t^2}{P_t} \right)^\alpha \right]^{\frac{1}{\alpha}}} \right\}^\eta, \quad (7)$$

where $\nu, \eta > 0$, $\alpha \leq 1$, and $\kappa_t > 0$ for all t .¹² Here κ_t is an exogenous parameter governing the efficiency of use of broad money relative to that of narrow money in making transactions at a point in time. Transactions skill, which is a form of human capital, evolves as a function of the amount of time allotted to its improvement;

¹¹This institutional framework supporting interest-earning money in a shopping time model extends the work of Teles and Zhou (2005).

¹²The elasticity of substitution among narrow and broad money is $1/(1 - \alpha)$. The assumption that $\alpha \leq 1$ implies that these assets are complements (or perfectly substitutable) in making transactions. $\alpha = 1$ is the case of perfect substitutes. The specification becomes Cobb-Douglas in the limit as $\alpha \rightarrow 0$ (substitution elasticity of 1). The case of perfect complements obtains as $\alpha \rightarrow -\infty$. In general, lower α pertains to a higher degree of complementarity

specifically, I assume that

$$Z_t = (1 - \delta + \phi X_t) Z_{t-1}, \quad (8)$$

where $\delta \in (0, 1)$ and $\phi > 0$. Note that the household effectively controls the growth rate of Z_t directly by its choice of the time investment X_t . This assumption is analogous to Lucas's (1988b) "Uzawa-Rosen formulation" of the mechanism by which productive human capital is accumulated.

3.5 Equilibrium

An equilibrium is a collection of stochastic processes

$$\{C_t, N_t, S_t, X_t, M_t^1, M_t^2, B_t, Z_t, Y_t, T_t, P_t, W_t, i_t, i_t^m, \kappa_t\}_{t=0}^\infty,$$

with $(M_{-1}^1, M_{-1}^2, B_{-1}, Z_{-1}, i_{-1}, i_{-1}^m)$ taken as given, such that

1. $\{C_t, N_t, S_t, X_t, M_t^1, M_t^2, B_t, Z_t\}_{t=0}^\infty$ maximize (2) subject to (5), (7), and (8) taking as given $(M_{-1}^1, M_{-1}^2, B_{-1}, Z_{-1}, i_{-1}, i_{-1}^m)$ and stochastic processes $\{T_t, P_t, W_t, i_t, i_t^m, \kappa_t\}_{t=0}^\infty$.
2. The goods market clears, so that (4) holds.
3. The government's budget constraint (6) is satisfied.
4. The private bond market clears, so that $B_{t-1} = 0$ for all t .

3.6 An Assumption about the Nature of the Substitution Incentive

Using first order conditions for the choice of B_t , N_t , M_t^1 , and M_t^2 , one may derive the implication that

$$\frac{M_t^1}{P_t C_t} \left[1 + \kappa_t^{\frac{1}{1-\alpha}} \left(\frac{r_t}{r_t^m} \right)^{\frac{\alpha}{1-\alpha}} \right] = \left(\frac{1}{\lambda_t N_t} - 1 \right) r_t^{-1} \quad (9)$$

and

$$\frac{M_t^2}{M_t^1} = \left(\frac{\kappa_t r_t}{r_t^m} \right)^{\frac{1}{1-\alpha}}, \quad (10)$$

where $\beta^t \lambda_t / W_t$ is a Lagrange multiplier on the household's budget constraint (5) and

$$r_t \equiv \frac{i_t}{1 + i_t} \text{ and } r_t^m \equiv \frac{i_t - i_t^m}{1 + i_t}.$$

The variable r_t (r_t^m) will be interpreted as the interest opportunity cost of holding narrow (broad) money rather than bonds; and the variable

$$q_t \equiv \left[1 + \kappa_t^{\frac{1}{1-\alpha}} \left(\frac{r_t}{r_t^m} \right)^{\frac{\alpha}{1-\alpha}} \right]^{\frac{1}{\alpha}} \quad (11)$$

may be interpreted as a sufficient summary of the variables that determine the relative allocation of purchasing power among the two forms of money. In the rest of the paper, it will be assumed that q_t evolves according to

$$q_t = \bar{\gamma}_q^t u_t, \quad (12)$$

where $\bar{\gamma}_q$ is a positive constant and u_t is a stationary stochastic process.

The phenomenon captured by q_t is the incentive to substitute among narrow and broad monies for use in conducting transactions. The structure suggests two factors that may influence this incentive. The first is the opportunity cost ratio r_t/r_t^m , and the second is the efficiency parameter κ_t . Ceteris paribus, an increase of either of these quantities induces a reduction of $M_t^1/(P_t C_t)$ and a rise in M_t^2/M_t^1 . Thus q_t may be interpreted to proxy for empirical phenomena associated with financial innovation at the level of institutions not modelled in this paper.

In the empirically relevant case, a positive trend of the substitution term, $\bar{\gamma}_q > 1$, allows the model to explain the secular decline of narrow money measures in long time series. This may be interpreted as representing an evolutionary shift within financial institutions that is increasingly favorable to the use of broad money.

3.7 A Characterization of Equilibrium

It is shown in the Technical Appendix to the paper that the elements of equilibrium of interest¹³ may be completely characterized, given $(M_{-1}^1, Z_{-1}, i_{-1})$, by stochastic processes

$$\{\lambda_t, N_t, m_t, \gamma_{z,t}, z_t, r_t, u_t\}_{t=0}^{\infty}$$

¹³Specifying additionally the realization of the productivity process $\{W_t\}_{t=0}^{\infty}$ induces the consumption process through (3) and (4).

satisfying

$$0 = -\lambda_t + \psi_0 \left[1 - \frac{\nu}{\eta\lambda_t} - \left(1 - \frac{\nu}{\eta} \right) N_t - \frac{1}{\phi} (\gamma_{z,t} - 1 + \delta) \right]^{-\psi_1} \quad (13)$$

$$0 = \frac{\nu}{\eta} \left(\frac{1}{\lambda_t} - N_t \right) - m_t^{-\frac{\eta}{\nu}} z_{t-1}^{-\frac{1-\nu}{\nu}} u_t^{-\frac{\eta}{\nu}} \quad (14)$$

$$0 = -m_t + r_t^{-1} \left(\frac{1}{\lambda_t N_t} - 1 \right) u_t^{-\alpha} \quad (15)$$

$$0 = -\lambda_t \gamma_{z,t} + \beta E_t \left\{ \lambda_{t+1} \left[\gamma_{z,t+1} - \phi \left(\frac{1-\nu}{\eta} \right) N_{t+1} \right] + \phi \left(\frac{1-\nu}{\eta} \right) \right\} \quad (16)$$

$$0 = -\gamma_{z,t} + \left(\frac{z_t}{z_{t-1}} \right) \bar{\gamma}_q^{-\frac{(1-\alpha)\eta}{1-\nu}}. \quad (17)$$

To derive this characterization one adopts the following interpretations:

$$\begin{aligned} m_t &\equiv [M_t / (P_t C_t)] \bar{\gamma}_q^{\alpha t} \\ z_{t-1} &\equiv Z_{t-1} \bar{\gamma}_q^{\frac{(1-\alpha)\eta}{1-\nu} t} \\ \gamma_{z,t} &\equiv Z_t / Z_{t-1}. \end{aligned}$$

Thus, m_t is the (real) narrow money-income ratio with adjustment for the trend induced by the substitution effect, and z_{t-1} is the beginning of the period transactions skill with a similar adjustment. $\gamma_{z,t}$ is the growth rate of transactions skill. λ_t is the value of the Lagrange multiplier for the budget constraint in the household's problem weighted by the factor W_t/β^t .

This representation of an equilibrium will turn out to be convenient under the particular assumptions we will adopt for the driving processes of the model. More precisely, given stationary stochastic processes for the opportunity cost r_t and the stochastic component u_t of the substitution term, the vector formed by collecting these variables will be seen to obtain a stationary distribution. Properties of this stationary distribution are the central objects of study in this paper.

4 Qualitative Analysis of Equilibrium

The first subsection of this section describes a balanced growth path of the model under the assumption that the model's driving processes evolve deterministically. This balanced growth path is characterized as a steady-state of the transformed

system presented in Subsection 3.7. Subsequently, properties of an approximating dynamic system are studied analytically in some detail.

4.1 Properties of a Balanced Growth Path

In this subsection, let us assume that $r_t = \bar{r}$, and $u_t = \bar{u}$ for all t . This analysis will be revealing of properties that will be interpreted as “long-run” phenomena, as discussed below. Additionally, this “non-stochastic steady-state” will describe a deterministic path about which a more general model may be linearized in order to study the model’s stochastic dynamics.

Given the assumptions described in the previous paragraph, I will characterize an equilibrium in which N_t , S_t , and X_t are each constant over time; and let us write \bar{N} , \bar{S} , and \bar{X} for the respective constant values. In this case, it is immediate from (8) that $\gamma_{z,t}$ is constant along such a path, with

$$\gamma_{z,t} = \bar{\gamma}_z = (1 - \delta + \phi\bar{X}).$$

Next, (13) shows that λ_t must be constant, as well; and I write $\bar{\lambda}$ for it’s value in the steady-state. Now (15) shows that m_t is constant, and so (14) implies that z_t is constant; and I write \bar{m} and \bar{z} (respectively) for their values.

We have thus shown that the dynamics of the transformed system are consistent with a steady-state equilibrium when the opportunity cost is held constant, and the substitution term q_t grows deterministically at a constant rate. The values of the constants $(\bar{\lambda}, \bar{N}, \bar{m}, \bar{\gamma}_z, \bar{z})$ may now be obtained by solving the system of five equations (13)-(17) under the steady-state assumption.

From (17), the results above show that

$$\bar{\gamma}_z = \bar{\gamma}_q^{-\frac{(1-\alpha)\eta}{1-\nu}}.$$

Now using this result and (16), we can derive

$$(1 - \beta) \gamma_z = \frac{\beta\phi(1 - \nu)}{\eta} \left(\frac{1}{\bar{\lambda}} - \bar{N} \right). \quad (18)$$

Then considering additionally (13), we have that

$$\bar{\lambda} = \psi_0 \left[1 - \frac{\nu}{\eta\bar{\lambda}} - \left(1 - \frac{\nu}{\eta} \right) \bar{N} - \frac{1}{\phi} (\bar{\gamma}_z - 1 + \delta) \right]^{-\psi_1}.$$

An interesting “long-run” property of the model is now apparent. Except for the rate of exogenous financial innovation embodied in $\bar{\gamma}_q$, steady-state values of the variables on that describe real quantities are determined independently of those related to making transactions. To see this, note that, given values of exogenous variables $\bar{\gamma}_q$ and \bar{u} , the last three equations may be solved for $\bar{\lambda}$ and \bar{N} , so that output and the household’s consumption may be evaluated. Moreover, from (8), (7), and (14), the steady-state values of X_t and S_t may be evaluated as

$$\bar{X} = \frac{\bar{\gamma}_z - 1 + \delta}{\phi}$$

and

$$\bar{S} = \frac{\nu}{\eta} \left(\frac{1}{\bar{\lambda}} - \bar{N} \right), \quad (19)$$

respectively; so that the household’s leisure time in the steady-state, say $\bar{L} \equiv 1 - \bar{N} - \bar{S} - \bar{X}$, is also independent of \bar{r} , \bar{m} , and \bar{z} .

This feature of the model marks an important contrast with standard formulations of the shopping-time model.¹⁴ In the present model, for example, to the extent that the effect of inflation in the economy is solely that of increasing the interest opportunity cost of holding money, the long-run welfare cost of an increase of the steady-state inflation rate is zero. The intuition for this result is that, in a steady state with higher opportunity cost, the household optimally achieves and sustains a higher time path of transactions skill Z_{t-1} . By doing so, the household may achieve the same level of transactions (*ceteris paribus*) with the same effort using less real money. The key assumption delivering this result is that investment of time in augmenting transactions skill determines the growth rate of that skill, rather than the size of a fixed increment to it.

The steady-state values of \bar{m} and \bar{z} may now be obtained by plugging into the steady-state representations of (14) and (15) the values $\bar{\lambda}$ and \bar{N} and solving; this gives

$$\bar{m} = \left(\frac{1}{\bar{\lambda}\bar{N}} - 1 \right) \bar{r}^{-1} \bar{u}^{-\alpha}$$

and

$$\bar{z} = \left(\frac{\eta}{\nu} \right)^{\frac{\nu}{1-\nu}} \left(\frac{1}{\bar{\lambda}\bar{N}} - 1 \right)^{-\frac{\eta+\nu}{1-\nu}} \bar{N}^{-\frac{\nu}{1-\nu}} (\bar{r})^{\frac{\eta}{1-\nu}} \bar{u}^{\frac{(1-\alpha)\eta}{1-\nu}}.$$

Thus, consistent with the intuitive arguments above, the model implies that the

¹⁴See Lucas (2000), e.g.

long-run elasticity of the money-income ratio with respect to the opportunity cost is equal to one.¹⁵

4.2 Stochastic Dynamics

4.2.1 Exogenous Processes

I now make precise the nature of the processes that will be assumed to drive the model in the remainder of the paper. I assume that

$$\hat{r}_t \equiv \log r_t - \log \bar{r} = \rho_1 \hat{r}_{t-1} + \rho_2 \hat{r}_{t-1} + \varepsilon_t^r, \quad (20)$$

and

$$\hat{u}_t \equiv \log u_t - \log \bar{u} = \chi \hat{u}_{t-1} + \varepsilon_t^u, \quad (21)$$

where \bar{r} , and \bar{u} are positive constants, and $\varepsilon_t \equiv (\varepsilon_t^r, \varepsilon_t^u)'$ is an i.i.d. innovations process distributed as $N(0, \Sigma)$.

4.2.2 Approximation by a Linear System

I will write variables with carets (“hats”) to denote log-deviations from elements of the vector

$$(\lambda_t, N_t, m_t, \gamma_{z,t}, z_t)$$

from elements of the point defined by the non-stochastic steady-state characterized in Subsection 4.1; for example, I write

$$\hat{\lambda}_t \equiv \log \lambda_t - \log \bar{\lambda}$$

and

$$\hat{N}_t \equiv \log N_t - \log \bar{N}.$$

Then the system of interest may be approximated by log-linearizing (13)-(17):

¹⁵It is certain that this result would change if the mode of accumulation of transactions skill were changed to reflect diminishing returns to time input. It is conjectured, however, that the long-run elasticity would remain higher than the short-run elasticity under conventional alternative assumptions. Similarly, implications for the steady-state welfare cost of inflation would change, but distinction with respect to standard shopping time setups would likely remain.

$$0 = - \left[1 + \frac{\psi_0 \psi_1 (\nu/\eta)}{\bar{L} \bar{\lambda}} \right] \hat{\lambda}_t + \frac{\psi_0 \psi_1 (1 - \nu/\eta) \bar{N}}{\bar{L}} \hat{N}_t + \frac{\psi_0 \psi_1 \bar{\gamma}_z}{\bar{L} \phi} \hat{\gamma}_{z,t} \quad (22)$$

$$0 = -\theta \hat{\lambda}_t - (\theta - 1) \hat{N}_t + (\eta/\nu) \hat{m}_t + [(1 - \nu) / \nu] \hat{z}_{t-1} + (\eta/\nu) \hat{u}_t \quad (23)$$

$$0 = -\hat{m}_t - \hat{r}_t - \theta \hat{\lambda}_t - \theta \hat{N}_t - \alpha \hat{u}_t \quad (24)$$

$$0 = -\hat{\lambda}_t - \hat{\gamma}_{z,t} + \beta E_t \left[\left(1 - \frac{\phi (1 - \nu) \bar{N}}{\eta \bar{\gamma}_z} \right) \hat{\lambda}_{t+1} + \hat{\gamma}_{z,t+1} - \frac{\phi (1 - \nu) \bar{N}}{\eta \bar{\gamma}_z} \hat{N}_{t+1} \right] \quad (25)$$

$$0 = -\hat{\gamma}_{z,t} + \hat{z}_t - \hat{z}_{t-1} \quad (26)$$

where

$$\bar{L} \equiv 1 - \bar{N} - \bar{S} - \frac{1}{\phi} (\bar{\gamma}_z - 1 + \delta)$$

and

$$\theta \equiv \frac{1}{1 - \bar{\lambda} \bar{N}}. \quad (27)$$

4.2.3 Solving the Model

In the rest of this section, I offer an exposition of a special case analytically.¹⁶ Doing so will facilitate greatly understanding the mechanism by which persistence arises in the M1-income ratio endogenously. In fact, it will be seen in the next section that the restrictions imposed in this section are not rejected by the data that I will use.

I assume that $\psi_1 = 0$. In this case, it can be seen from (13) that $\lambda_t = \bar{\lambda} = \psi_0$ and $\hat{\lambda}_t = 0$ for all t . Under these conditions, the equations characterizing the (linearized) system of interest are

$$0 = -(\theta - 1) \hat{N}_t + (\eta/\nu) \hat{m}_t + [(1 - \nu) / \nu] \hat{z}_{t-1} + (\eta/\nu) \hat{u}_t \quad (28)$$

$$0 = -\hat{m}_t - \hat{r}_t - \theta \hat{N}_t - \alpha \hat{u}_t \quad (29)$$

$$0 = -\hat{\gamma}_{z,t} + \beta E_t \left[\hat{\gamma}_{z,t+1} - \frac{\phi (1 - \nu) \bar{N}}{\eta \bar{\gamma}_z} \hat{N}_{t+1} \right] \quad (30)$$

$$0 = -\hat{\gamma}_{z,t-1} + \hat{z}_{t-1} - \hat{z}_{t-2} \quad (31)$$

I begin by solving for a single-equation dynamic system describing the evolution

¹⁶The general case can be analyzed similarly, but the exposition more cumbersome. The interested reader is directed to the Technical Appendix for the details.

of the endogenous state variable \hat{z}_{t-1} . The first two equations, (28) and (29), may be solved to yield

$$\hat{N}_t = \left(\frac{1 - \bar{\lambda}\bar{N}}{\eta/\nu + \bar{\lambda}\bar{N}} \right) \left(\frac{1 - \nu}{\nu} \right) \hat{z}_{t-1} - \left(\frac{1 - \bar{\lambda}\bar{N}}{\eta/\nu + \bar{\lambda}\bar{N}} \right) \frac{\eta}{\nu} \hat{r}_t + \left(\frac{1 - \bar{\lambda}\bar{N}}{\eta/\nu + \bar{\lambda}\bar{N}} \right) \frac{(1 - \alpha)\eta}{\nu} \hat{u}_t$$

and

$$\hat{m}_t = -\frac{(1 - \nu)/\nu}{\eta/\nu + \bar{\lambda}\bar{N}} \hat{z}_{t-1} - \frac{\bar{\lambda}\bar{N}}{\eta/\nu + \bar{\lambda}\bar{N}} \hat{r}_t - \frac{\bar{\lambda}\bar{N}\alpha + \eta/\nu}{\eta/\nu + \bar{\lambda}\bar{N}} \hat{u}_t. \quad (32)$$

Using these expressions and (31), the variables \hat{N}_t , \hat{m}_t , and $\hat{\gamma}_{z,t}$ can be eliminated from (30) to derive the second-order difference equation

$$0 = \hat{z}_{t-1} - \left[1 + \beta + \beta \frac{\omega\phi(1 - \nu)^2 \bar{N}}{\eta\nu\bar{\gamma}_z} \right] \hat{z}_t \quad (33)$$

$$+ \beta E_t \left\{ \hat{z}_{t+1} + \frac{\omega\phi(1 - \nu) \bar{N}}{\nu\bar{\gamma}_z} \hat{r}_{t+1} - \frac{\omega\phi(1 - \nu) \bar{N} (1 - \alpha)}{\nu\bar{\gamma}_z} \hat{u}_{t+1} \right\},$$

where

$$\omega \equiv \frac{1 - \bar{\lambda}\bar{N}}{\eta/\nu + \bar{\lambda}\bar{N}}.$$

Applying the technique of Sargent (1987) (see the Technical Appendix of the paper for details), the lag operator polynomial on z_{t-1} can be factored to write the difference equation as

$$\beta (L^{-1} - \xi) \left(L^{-1} - \frac{1}{\beta\xi} \right) E_t z_{t-1} = \beta E_t \left\{ -\frac{\omega\phi(1 - \nu) \bar{N}}{\nu\bar{\gamma}_z} \hat{r}_{t+1} + \frac{\omega\phi(1 - \nu) \bar{N} (1 - \alpha)}{\nu\bar{\gamma}_z} \hat{u}_{t+1} \right\},$$

where

$$\xi = \frac{1}{2} \left(\frac{1 + \beta + \beta\omega\phi(1 - \nu)^2 \bar{N} / (\eta\nu\bar{\gamma}_z)}{\beta} \right) \quad (34)$$

$$- \sqrt{\frac{1}{4} \left(\frac{1 + \beta + \beta\omega\phi(1 - \nu)^2 \bar{N} / (\eta\nu\bar{\gamma}_z)}{\beta} \right)^2 - \frac{1}{\beta}},$$

and L is the lag operator. Moreover, it is straightforward to show that $\xi \in (0, 1)$.¹⁷

¹⁷See Sargent (1987, pps. 201-202).

Solving the “unstable root” forward, we have

$$\begin{aligned} (L^{-1} - \xi) \hat{z}_{t-1} &= \left(L^{-1} - \frac{1}{\beta\xi} \right)^{-1} E_t \left\{ -\frac{\omega\phi(1-\nu)\bar{N}}{\nu\bar{\gamma}_z} \hat{r}_{t+1} + \frac{\omega\phi(1-\nu)\bar{N}(1-\alpha)}{\nu\bar{\gamma}_z} \hat{u}_{t+1} \right\} \\ &= \beta\xi (1 - \beta\xi L^{-1})^{-1} E_t \left\{ \frac{\omega\phi(1-\nu)\bar{N}}{\nu\bar{\gamma}_z} \hat{r}_{t+1} - \frac{\omega\phi(1-\nu)\bar{N}(1-\alpha)}{\nu\bar{\gamma}_z} \hat{u}_{t+1} \right\} \end{aligned}$$

or

$$\hat{z}_t = \xi \hat{z}_{t-1} + \sum_{j=1}^{\infty} (\beta\xi)^j E_t \left\{ \frac{\omega\phi(1-\nu)\bar{N}}{\nu\bar{\gamma}_z} \hat{r}_{t+j} - \frac{\omega\phi(1-\nu)\bar{N}(1-\alpha)}{\nu\bar{\gamma}_z} \hat{u}_{t+j} \right\}.$$

Next, using our assumptions about the nature of exogenous stochastic processes r_t and u_t , it may be seen that¹⁸

$$\sum_{j=1}^{\infty} (\beta\xi)^j E_t \hat{r}_{t+j} = \frac{\rho_1\beta\xi + \rho_2(\beta\xi)^2}{1 - \rho_1\beta\xi - \rho_2(\beta\xi)^2} \hat{r}_t + \frac{\beta\xi\rho_2}{1 - \rho_1\beta\xi - \rho_2(\beta\xi)^2} \hat{r}_{t-1}$$

and

$$\sum_{j=1}^{\infty} (\beta\xi)^j E_t \hat{u}_{t+j} = \frac{\chi\beta\xi}{1 - \chi\beta\xi} \hat{u}_t.$$

Thus, the solution is

$$\begin{aligned} \hat{z}_t &= \xi \hat{z}_{t-1} + \left[\frac{\omega\phi(1-\nu)\bar{N}}{\nu\bar{\gamma}_z} \right] \frac{\rho_1\beta\xi + \rho_2(\beta\xi)^2}{1 - \rho_1\beta\xi - \rho_2(\beta\xi)^2} \hat{r}_t \\ &\quad + \left[\frac{\omega\phi(1-\nu)\bar{N}}{\nu\bar{\gamma}_z} \right] \frac{\beta\xi\rho_2}{1 - \rho_1\beta\xi - \rho_2(\beta\xi)^2} \hat{r}_{t-1} \\ &\quad - \left[\frac{\omega\phi(1-\nu)\bar{N}(1-\alpha)}{\nu\bar{\gamma}_z} \right] \frac{\chi\beta\xi}{1 - \chi\beta\xi} \hat{u}_t. \end{aligned} \tag{35}$$

¹⁸To derive these results, one may apply formula (91) on page 304 of Sargent (1987). See the Technical Appendix to the paper.

4.3 Understanding Reduced-Form Money Demand Equations

Plugging the solution (35) for the endogenous state variable z_t (lagged one period) into (32) and simplifying, we have

$$\hat{m}_t = \xi \hat{m}_{t-1} + \zeta_1 \hat{r}_t + \zeta_2 \hat{r}_{t-1} + \zeta_3 \hat{r}_{t-2} + \zeta_4 \hat{u}_t + \zeta_5 \hat{u}_{t-1} \quad (36)$$

with

$$\begin{aligned} \zeta_1 &\equiv -\frac{\bar{\lambda}\bar{N}}{\eta/\nu + \bar{\lambda}\bar{N}} \\ \zeta_2 &\equiv \frac{\bar{\lambda}\bar{N}\xi}{\eta/\nu + \bar{\lambda}\bar{N}} - \left[\frac{(1 - \bar{\lambda}\bar{N}) \phi (1 - \nu)^2 \bar{N}}{(\eta/\nu + \bar{\lambda}\bar{N})^2 \nu^2 \bar{\gamma}_z} \right] \frac{\rho_1 \beta \xi + \rho_2 (\beta \xi)^2}{1 - \rho_1 \beta \xi - \rho_2 (\beta \xi)^2} \\ \zeta_3 &\equiv - \left[\frac{(1 - \bar{\lambda}\bar{N}) \phi (1 - \nu)^2 \bar{N}}{(\eta/\nu + \bar{\lambda}\bar{N})^2 \nu^2 \bar{\gamma}_z} \right] \frac{\beta \xi \rho_2}{1 - \rho_1 \beta \xi - \rho_2 (\beta \xi)^2} \\ \zeta_4 &\equiv -\frac{\bar{\lambda}\bar{N}\alpha + \eta/\nu}{\eta/\nu + \bar{\lambda}\bar{N}} \\ \zeta_5 &\equiv \left(\frac{\bar{\lambda}\bar{N}\alpha + \eta/\nu}{\eta/\nu + \bar{\lambda}\bar{N}} \right) \xi + \left[\frac{(1 - \bar{\lambda}\bar{N}) \phi (1 - \nu)^2 \bar{N} (1 - \alpha)}{(\eta/\nu + \bar{\lambda}\bar{N})^2 \nu^2 \bar{\gamma}_z} \right] \frac{\chi \beta \xi}{1 - \chi \beta \xi}. \end{aligned}$$

In much of the literature, and in the analysis to follow, it is of interest to understand the relationship between the money-income ratio and the opportunity cost of holding money. If we assume that innovations to \hat{r}_t and \hat{u}_t are contemporaneously and serially uncorrelated, then we may use the Wold moving average representation $(1 - \chi L)^{-1} \varepsilon_t^u$ to eliminate \hat{u}_t from (36) to derive

$$\begin{aligned} \hat{m}_t &= (\xi + \chi) \hat{m}_{t-1} - \xi \chi \hat{m}_{t-2} + \zeta_1 \hat{r}_t + (\zeta_2 - \chi \zeta_1) \hat{r}_{t-1} + (\zeta_3 - \chi \zeta_2) \hat{r}_{t-2} - \chi \zeta_3 \hat{r}_{t-3} \\ &\quad + \zeta_4 \varepsilon_t^u + \zeta_5 \varepsilon_{t-1}^u. \end{aligned} \quad (37)$$

This equation may be used to understand estimates (and perceived pathologies) of conventional “money demand equations”.

First, the model suggests that the elasticity of real money with respect to income is one; but the model shows how long-run “institutional” trends may induce households to substitute broad money for narrow money. These features, along with the specific assumption about the nature of the substitution effect in (12), are implicit in the formulation (37); in particular, they are embodied in the definition of the

variable \hat{m}_t in Subsection 3.7.

Second, the presence of lags of the money income ratio and of the opportunity cost are induced by the nature of financial innovation in the model in two ways. The role of optimal investment in accumulation of skill, and the role of skill in making transactions, induce dependence upon expectations of future transactions needs. These expectations are formed optimally using current and lagged information. The other channel by which financial innovation affects the reduced form is through the substitution term. Since the stochastic component \hat{u}_t of this process is unobserved in this set-up, its persistence ($\chi \neq 0$) induces another layer of lagged terms through its projection on observables.

Third, the inability to observe the persistent stochastic component \hat{u}_t of the substitution term also introduces the moving average term in the disturbance of the reduced form (37). Qualitatively, this finding is consistent with the fact that residuals from empirical money demand regressions frequently display autocorrelation (c.f., Goodfriend (1985)).

5 Quantitative Results

5.1 Empirical Strategy

Assuming that the disturbances of the model follow Gaussian distributions, it becomes convenient to work within a likelihood framework. I use measures of the logarithm of the money-income ratio, say \tilde{m}_t , and the logarithm \tilde{r}_t of the opportunity cost as described below. Then \hat{m}_t is equal to \tilde{m}_t minus a constant and a linear trend to be estimated; and \hat{r}_t is equal to \tilde{r}_t minus a constant to be estimated.

To estimate the general model, I embed the solution methodology of Uhlig (1999) into the construction of a state-space model with vector of observables $(\tilde{m}_t, \tilde{r}_t)'$; the details of the procedure are described in the Technical Appendix to the paper. The Kalman filter is then applied to the state-space model to factorize the unconditional likelihood of the data.¹⁹ The value of the likelihood can then be evaluated for a given parameterization. Estimates of model parameters are obtained by maximizing the likelihood through a numerical search algorithm.

An important element of the approach that I take is evaluation and maximization of the *exact likelihood* of each model and specification.²⁰ This facilitates comparison

¹⁹A given parameterization of the model induces an unconditional distribution of model state variables; this distribution is used to initialize the Kalman filter. (See, for example, Chapter 13 of Hamilton (1994).)

²⁰With respect to this detail of my analysis, I have found useful the E⁴ toolbox of Matlab

of the theoretical model to more ad hoc statistical formulations within the likelihood framework.²¹

I impose the following a priori: $\beta = 0.96$, $\bar{N} = 0.3$, $\bar{S} = 0.003$. This value of the discount factor is relatively uncontroversial for one-year intervals. The values of \bar{N} and \bar{S} are taken from the calibration of Gavin, Kydland, and Pakko (2007).

Given the values of these calibrated parameters and values for ν and η , we can calculate values for $\bar{\lambda}$ and the ratio $\bar{\gamma}_z/\phi$ from equations (18) and (19). Since the values of parameters of the ratio $\bar{\gamma}_z/\phi$ affect the linearized dynamics only in this fashion, it is sufficient for the purposes of this study to identify their ratio. Similarly, the parameters of the construct $\Psi \equiv \psi_0\psi_1/\bar{L}$ are not separately identified by the objects of study (c.f., equation (22)); thus, Ψ is estimated directly.

While it may be more conventional to remove the mean from \tilde{r}_t and the mean and trend from \tilde{m}_t a priori in order to construct the sample analogues of the variables in the model, I do not follow this approach. Rather, I estimate these objects simultaneously with other parameters of the model. Especially with respect to the trend in the money-income ratio, the structural interpretation that I ascribe to these objects suggests that more formal treatment is desirable. Letting μ_r and μ_m be the theoretical means of r_t and m_t as constructed in the model, and defining $\tilde{\gamma}_q \equiv \alpha \log \gamma_q$ to be the logarithm of the trend in \tilde{m}_t , I append these three parameters to the vector to be estimated.

Finally, the indeterministic components \hat{r}_t and \hat{u}_t of the exogenous processes are characterized by three autoregressive coefficients ρ_1 , ρ_2 , and χ ; and three parameters of the 2×2 covariance matrix Σ .

In summary, there are thirteen parameters,

$$\alpha, \nu, \eta, \Psi, \mu_r, \mu_m, \tilde{\gamma}_q, \rho_1, \rho_2, \chi, \sigma_r, \sigma_u, \rho_{ru},$$

to be estimated. Here, σ_r , σ_u , and ρ_{ru} are the standard errors and the correlation coefficient induced by the covariance matrix Σ .

I estimate the model on two samples of data, a subsample of observations from 1900-1973 and the full sample for 1900-2007, and assess the empirical success of the model based on the following strategy. First, parameter stability can be formally assessed by testing the adequacy of parameters estimated for the full sample within

routines especially useful. The toolbox is available at <http://www.ucm.es/info/icae/e4/>, and is well-documented in Terceiro et al. (2000).

²¹An incidental benefit accrues when the nesting of the theoretical model within the statistical framework can be derived analytically. In the present case, for example, the likelihood of the theoretical model induced by a direct state-space formulation can be verified by evaluating the (exact) likelihood of the reduced form vector ARMA formulation.

the subsample. Second, consistent with the analysis in Subsection 4.2.3, the model induces a restricted VARMAX(3,1) representation of the data;²² thus, the restrictions implied by the model can be tested by comparison to estimates obtained from a less restricted VARMAX(3,1) implementations.²³ Finally, I use estimates obtained from the subsample to compare the quality of out-of-sample forecasts and simulations obtained from the theoretical model to these alternatives.

5.2 Data

The money-income ratio \tilde{m}_t in the data is the logarithm of the ratio of M1 to net national product. For \tilde{r}_t I use the logarithm of the construct $i_t/(1+i_t)$ as implied by the theory, where i_t is taken to be the commercial paper rate.

The data is observed at annual frequency for the years 1900-2007, and has been compiled from several different sources. The data on M1, real Net National Product, and the NNP deflator for 1900-1989 is that used by Stock and Watson (1993), made available through Mark Watson’s webpage. The continuations of these series for the 1990-2007 comes from the Federal Reserve Economic Database (FRED), published online by the Federal Reserve Bank of St. Louis. The splicing of the constructed series is accomplished after renormalizing the NNP deflator from the latter source to be consistent with the normalization in Stock and Watson’s data.

To construct the commercial paper rate series, Stock and Watson’s (1993) measure for 1900-1989 is spliced onto the 3-month commercial paper rate series for 1990-1997, and the 3-month financial commercial paper rate series for 1998-2007, both from FRED.

5.3 Estimates of the Parameters of the Theoretical Model

Table 2 reports the results of estimation of several specifications of the theoretical model for the full dataset and for the 1900-1973 sub-sample. In each case, the likelihood is maximized by choosing $\Psi = 0$, a value on the boundary of the parameter space.²⁴ Thus, I have imposed this value in each treatment below without reporting estimates for this parameter.

²²This result is shown for the general case in the Technical Appendix to the paper.

²³The exogenous variables in the formulation, the “X” in “VARMAX”, are the constant and the trend. Consistent with the approach to estimation of the theoretical model, these elements are always estimated simultaneously with other parameters rather than removed a priori.

²⁴Recall that this restriction induces the simplified version of the model discussed in the previous section.

	1900-2007		1900-1973	
	(1a)	(1b)	(2a)	(2b)
α	-1.308 (0.944)	-1.327 (0.984)	-4.263 (2.02)	-3.380 (1.21)
η	3.291 (6.13)	3.522 (0.863)	2.22 (1.71)	2.69 (0.661)
ν	0.0659 (0.558)	0.413 (0.121)	0.233 (0.479)	0.487 (0.164)
$\tilde{\gamma}_q$	-0.0104 (0.0012)	-0.0106 (0.0013)	-0.0095 (0.0037)	-0.0107 (0.0027)
ρ_1	1.148 (0.0946)	1.138 (0.0910)	1.090 (0.143)	1.041 (0.140)
ρ_2	-0.233 (0.0918)	-0.221 (0.0879)	-0.207 (0.191)	-0.130 (0.159)
χ	0.879 (0.0537)	0.862 (0.061)	0.522 (0.118)	0.522 (0.113)
σ_r	0.254 (0.0174)	0.255 (0.0174)	0.242 (0.024)	0.238 (0.020)
σ_u	0.0591 (0.0062)	0.0686 (0.010)	0.0992 (0.083)	0.146 (0.076)
ρ_{ru}	0.386 (0.437)	[0]	0.308 (0.476)	[0]
μ_m	-0.995 (0.125)	-0.990 (0.131)	-1.008 (0.107)	-1.001 (0.143)
μ_r	-3.201 (0.263)	-3.185 (0.268)	-3.295 (0.344)	-3.200 (0.368)
LL	155.66	155.25	111.94	111.46

Table 2: Model Estimates. Approximate standard errors are reported in parentheses.

Columns labeled “*a*” and “*b*” in Table 2 are distinguished by imposition of the constraint $\rho_{ru} = 0$, implying independence of the shocks to the opportunity cost and the substitution mechanism, in the latter. This constraint is not rejected for either subsample,²⁵ and I will adopt the parameterizations in columns *b* as benchmarks for evaluation of the theory in what follows.

Imposition of the independence constraint improves the identification of the other parameters of the model dramatically. Possibly excepting α , the parameters of the transactions technology (c.f., (7)) are fairly precisely identified. It seems possible that better identification of α might be obtained by utilizing its functional incorporation in the term $\tilde{\gamma}_q \equiv \alpha \log \gamma_q$, but it was deemed important to identify the latter directly given the important role of the trend of the money income ratio in explaining the empirical phenomena of interest. It will be noted that $\tilde{\gamma}_q < 0$, as suggested by the estimates in Table 2, requires $\alpha \neq 0$. On the other hand, estimation of the benchmark model with the additional constraint that $\alpha = 1$ cannot be rejected by a likelihood ratio test.²⁶ Thus, it may be misleading to conclude that $\alpha < 0$, as one might in looking at a one-tailed t-test of the hypothesis for the results in columns (2*a*) and (2*b*).

An interesting finding from the benchmark parameterizations (again, columns *b* in the table) is that η is significantly greater than one, suggesting that there are decreasing returns to time spent shopping and skill in making transactions (c.f., equation (7)).²⁷ The parameterization emerging from column (1*b*), for example,

²⁵A likelihood ratio test may be conducted by comparing twice the difference of the maximized log-likelihood statistics to the $\chi^2(1)$ distribution. The statistics for the full sample and the subsample are

$$2(155.66 - 155.25) = 0.82$$

and

$$2(111.94 - 111.46) = 0.96,$$

respectively, and the 5% critical value for the one-sided test is 3.84.

²⁶The maximized likelihood for the full sample becomes 154.25, and that for the subsample becomes 110.68, so that relevant test statistics, distributed $\chi^2(1)$ are

$$2(155.25 - 154.25) = 2.0$$

and

$$2(111.46 - 110.68) = 1.56,$$

respectively.

²⁷This result stems from the standard one-tailed t-tests suggested by the data in the table, and by likelihood ratio tests for the restriction. (The maximized likelihoods obtained after imposing the constraint $\eta = 1$ on the benchmark model are 145.22 and 100.05 for the full dataset and the subsample, respectively.)

implies an elasticity of money velocity with respect to shopping time of $\nu/\eta = 0.117$. This is at odds with some standard calibrations of the shopping technology that imply that this elasticity is equal to one.²⁸

Estimates of the time trend of the M1-income ratio are remarkably consistent across these subsamples, and precisely estimated in each case. The value of $\tilde{\gamma}_q$ near -0.01 suggests a long-run decline of the ratio of about 1% per year. According to the theory, this stems from a continuous secular trend of substitution from M1 to broader monetary instruments. An important conclusion of this paper is that this trend has been long-running and stable; and the effects of financial innovation on the use of money should not be characterized as having been affected only in the decades after 1965.

Estimates of the autoregressive parameters of the opportunity cost process are roughly consistent with estimates that may be obtained from a univariate estimation of this process; in particular, the parameters support the hypothesis of stationarity.

A test of parameter stability across two subsamples can be constructed by comparing the maximized likelihood of the 1900-1973 subsample to the subsample likelihood evaluated at the estimates obtained from the full sample. For the parameterization in column (1a), the likelihood of the subsample is 105.32; the likelihood ratio test statistic is thus 13.24, which is comfortably below the 5% critical value for the $\chi^2(12)$ distribution of 21.0. For the parameterization in column (1b), the likelihood of the subsample becomes 104.64, and the implied test statistic value of 14.60 does not call for rejection according to the $\chi^2(11)$ critical value of 19.7.

5.4 Assessing the Theoretical Restrictions

In this subsection, I compare the estimated theoretical model to those of less constrained VARMAX(3,1) models that may be represented in the general form

$$\Phi_0 \begin{pmatrix} \tilde{m}_t - \mu_m - \tau t \\ \tilde{r}_t - \mu_r \end{pmatrix} = \sum_{j=1}^3 \Phi_j \begin{pmatrix} \tilde{m}_{t-j} - \mu_m - \tau(t-j) \\ \tilde{r}_{t-j} - \mu_r \end{pmatrix} + \begin{pmatrix} e_t^m \\ e_t^r \end{pmatrix} + \Theta \begin{pmatrix} e_{t-1}^m \\ e_{t-1}^r \end{pmatrix}, \quad (38)$$

where Φ_0 is a matrix with ones on the diagonal,

$$\begin{pmatrix} e_t^m \\ e_t^r \end{pmatrix} \sim N(0, \Sigma_e),$$

²⁸Examples include Gavin, Kydland, and Pakko (2007) and other projects involving these authors.

and Σ_e is a diagonal matrix. It is important to note that, consistent with the results of Section 4, the model implies a reduced form that is nested within each of the statistical formulations to be considered.

Table 3 reports the results of estimation of the structural VARMAX model (in columns (1*b*) and (2*b*)) defined by the exclusion restrictions

$$\Phi_{0,21} = \Phi_{3,11} = \Phi_{3,21} = \Phi_{3,22} = 0$$

and

$$\Theta_{12} = \Theta_{21} = \Theta_{22} = 0,$$

along with the values of the coefficients implied by the benchmark parameterization of the theoretical model (in column (1*a*) and (2*a*)).²⁹ It will be noted that the estimates of most of the coefficients are numerically and statistically quite close, and a likelihood ratio test of the theoretical restrictions cannot reject the model within either subsample.³⁰ An important difference, which will be seen below to affect the quality of the models' forecasts, is the value of the trend term τ as estimated for the 1900-1973 sub-sample.³¹

The model in (38) was also estimated without constraints. The maximized likelihoods reported in Column (3) of Table 4 show once again that the restrictions imposed by the theory are not rejected by the data in either sample.³²

The four rows under the label "Forecast MSE" in Table 4 display the mean squared error of out-of-sample forecasts of the M1-income ratio from each model for 1-, 5-, 10-, and 20-year forecast horizons. The models are parameterized using estimates obtained from the 1900-1973 sub-sample, and forecasts are constructed

²⁹Note that these are precisely the exclusion restrictions implied by the model.

³⁰The test statistics

$$2(155.70 - 155.25) = 0.90$$

and

$$2(119.61 - 111.46) = 16.30$$

for the full sample and the sub-sample, respectively, are draws from a $\chi^2(14)$ distribution under the null hypothesis that the restrictions are valid. The 5% critical value for the test is 23.7.

³¹The trend τ is equal to $\tilde{\gamma}_q$ under the restrictions of the theory.

³²The test statistics

$$2(164.86 - 155.25) = 19.22$$

and

$$2(128.94 - 111.46) = 34.96$$

for the full sample and the sub-sample, respectively, are draws from a $\chi^2(23)$ distribution under the null hypothesis that the restrictions are valid. The 5% critical value for the test is 35.2.

	1900-2007		1900-1973	
	(1a)	(1b)	(2a)	(2b)
$\Phi_{0,12}$	-0.097	-0.0994 (0.020)	-0.146	-0.139 (0.024)
$\Phi_{1,11}$	1.806	1.805 (0.060)	1.462	1.379 (0.094)
$\Phi_{1,12}$	0.146	0.167 (0.044)	0.187	0.197 (0.052)
$\Phi_{1,22}$	1.138	1.150 (0.085)	1.041	1.008 (0.105)
$\Phi_{2,11}$	-0.813	-0.813 (0.058)	-0.491	-0.482 (0.083)
$\Phi_{2,12}$	-0.047	-0.0822 (0.045)	-0.0540	-0.068 (0.054)
$\Phi_{2,22}$	-0.221	-0.226 (0.0832)	-0.130	-0.061 (0.105)
$\Phi_{3,12}$	0.006	0.0107 (0.020)	0.0020	0.028 (0.029)
Θ_{11}	-1.000	-1.000 (0.024)	-0.9999	-1.000 (0.039)
$\Sigma_{e,11}$	0.053	0.0528 (0.004)	0.0522	0.0476 (0.0044)
$\Sigma_{e,22}$	0.254	0.255 (0.020)	0.238	0.232 (0.022)
μ_m	-0.990	0.997 (0.120)	-1.001	-1.124 (0.141)
μ_r	-3.185	-3.185 (0.280)	-3.201	-3.251 (0.418)
τ	-0.0106	-0.0104 (0.0014)	-0.0107	-0.0065 (0.0005)
LL	155.25	155.70	111.46	119.61

Table 3: Estimates of coefficients of a reduced from structural VARMAX model. Approximate standard errors are reported in parentheses. Columns “a” report coefficients implied by estimates of the theoretical model.

	<i>Theory</i>	<i>VARMAX Models</i>	
	(1)	(2)	(3)
		1900-2007	
<i>LL</i>	155.25	155.70	164.86
		1900-1973	
<i>LL</i>	111.46	119.61	128.94
Forecast MSE			
1-year	0.010	0.098	0.117
5-year	0.041	0.142	0.035
10-year	0.028	0.064	0.038
20-year	0.007	0.076	0.331
Simulation MSE			
1-year	0.010	0.100	0.335
5-year	0.032	0.149	0.103
10-year	0.031	0.085	0.064
20-year	0.025	0.127	0.129

Table 4: Model Estimates. Approximate standard errors are reported in parentheses. Columns (2)-(3) reports results for the structural VARMAX(3,1) model and the unconstrained VARMAX(3,1) model, respectively.

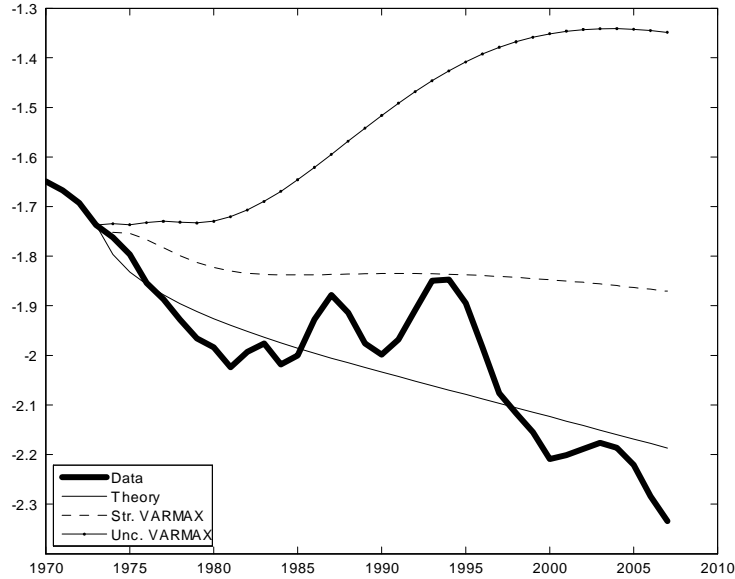


Figure 4: Forecasts of M1-income ratio after 1973 from various models.

for 1974-2007 period.³³ Figure 4 shows the 1973 forecasts of the subsequent years together with the realization from the data. From the figure, it may be gleaned that the most important aspect of the quality of long-horizon forecasts is the accuracy of the estimate of the trend. In this respect, it may be interpreted that imposition the theoretical restrictions is crucial to efficient identification. With the minor exception of the 5-year forecast from the unconstrained model, forecasts obtained from the theoretical model dominate those from the other models at each horizon.

The four rows under the label “Simulation MSE” display the mean squared error of simulations in the spirit of Goldfeld’s (1973, 1976) exercises from each model for the same time horizons, and Figure 5 plots simulations from from each model analogous to the forecasts shown in Figure 4.^{34,35} The simulations offer a picture

³³This is subject to the obvious limitations required by feasibility of constructing forecasts of the appropriate length. More precisely, 1-, 5-, 10-, and 20-year forecasts are obtained for the periods starting in 1974, 1978, 1983, and 1993, respectively.

³⁴Forecasts of the type reviewed previously cannot be constructed for single-equation frameworks like those studied by Goldfeld, because these models do not incorporate a framework of prediction for the value of the opportunity cost variable on the right-hand side.

³⁵Formally, these “simulations” of the M1-income ratio are estimates of missing (i.e., omitted) data points obtained by application of the Kalman filter. See, for example, Section 4.8 of Durbin and Koopman (2001, Chapter 4), or Terceiro et al. (2000).

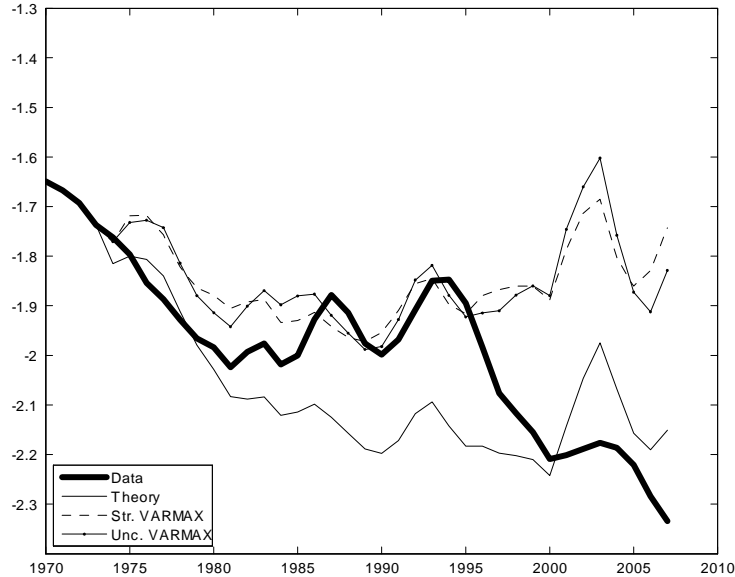


Figure 5: Simulations of M1-income ratio after 1973 from various models.

similar to that from the forecasts: simulations obtained from the theoretical model dominate those from the less-constrained models.

5.5 Estimates of Unobserved Determinants of the Demand for Money

Figure 6 shows an estimate of \hat{z}_t , the log-deviation from trend of the transactions skill Z_t , together with the demeaned log-opportunity cost.³⁶ The juxtaposition conveys the intuition for the primary driver of transactions skill accumulation, the incentive to economize on the use of money induced by the opportunity cost of holding it. The series begins the century above trend, falling at a moderate pace, under an episode characterized by moderate nominal interest rates, until the 1930s. Thereafter, transactions skill falls at a fast pace following with a lagged response the plummeting of the opportunity cost. The variable begins to rise again after the opportunity cost rebounds in the late 1940s, and continues in parallel as rates rise. The rate of growth of transactions skill stabilizes in the 1990s, and eventually falls below trend in the 2000s as interest rates have fallen and have remained low.

³⁶The estimate is obtained by applying the Kalman smoothing algorithm to the data. (See Hamilton (1994), for example.)

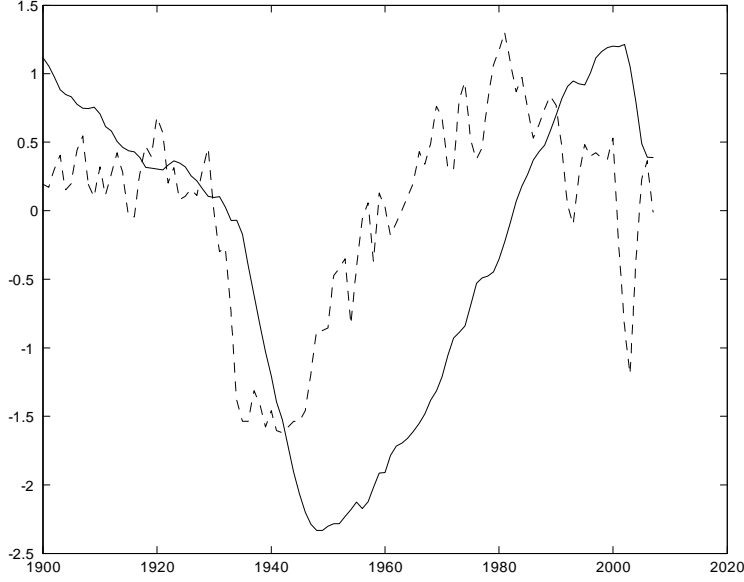


Figure 6: Log-transactions skill (log-deviation from trend, solid line) and demeaned log-opportunity cost (dashed line).

Figure 7 depicts the estimate of the stochastic component u_t of the substitution term q_t in log-deviation from its mean, along with detrended time series construct

$$\left[1 + \left(\frac{M_t^2}{M_t^1} \right)^\alpha \right]^{\frac{1}{\alpha}},$$

where M_t^1 is M1 and M_t^2 is the non-M1 component of M2. The constructed variable represents the theoretical value of the substitution term q_t when $\kappa_t \equiv 1$ (c.f., (10) and (11)) under the assumption that M2 is a reasonable proxy measure of broad money. The degree to which the movement of the estimate of u_t agrees with the movement of the construct is unclear. For the 1940s, for example, little correlation is apparent. On the other hand, there seems to be significant low frequency comovement during other episodes, and particularly for the period after 1960. If this interpretation is apt, then the residual correlation can be ascribed to fluctuation of the relative efficiency of making transactions with the two types of money as captured by κ_t .

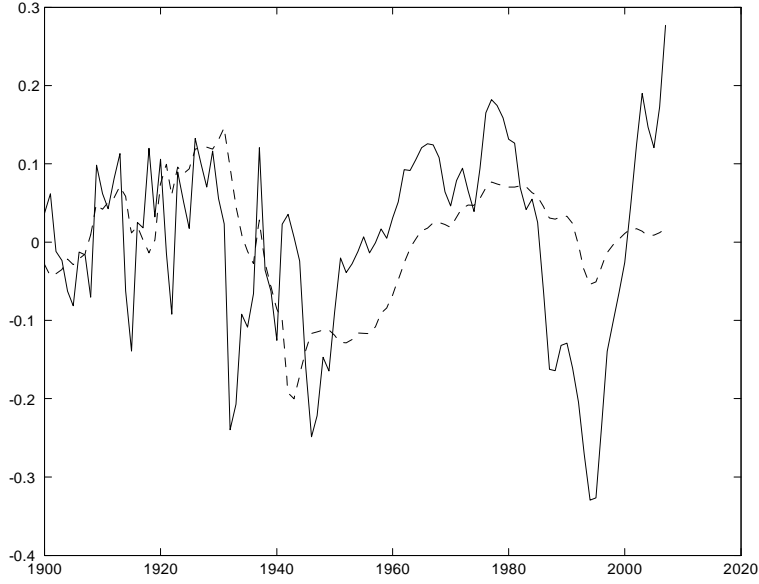


Figure 7: Estimated substitution term and related construct from the data (all detrended).

5.6 Assessing Implications for Economic Welfare

The welfare cost of making transactions accrues from the necessity of spending time transacting, and that of spending time improving transactions skill. To construct meaningful and useful measures of these costs, notice that (from (7) and (14)), and constructing a linear approximation)³⁷

$$\begin{aligned}
 S_t &\approx \frac{\nu}{\eta} \left(\frac{1}{\bar{\lambda}} - \bar{N} \right) - \frac{\nu}{\eta} \bar{N} \hat{N}_t \\
 &= \bar{S} - \frac{\nu}{\eta} \bar{N} \hat{N}_t
 \end{aligned}$$

³⁷This approximation also uses the restriction from the benchmark parameterization that $\Psi = 0$, which implies that $\hat{\lambda}_t = 0$.

and (from (8) and a linear approximation)

$$\begin{aligned} X_t &\approx \left(\frac{\bar{\gamma}_z}{\phi} - \frac{1-\delta}{\phi} \right) + \frac{\bar{\gamma}_z}{\phi} \hat{\gamma}_{z,t} \\ &= \bar{X} + \frac{\bar{\gamma}_z}{\phi} \hat{\gamma}_{z,t}. \end{aligned}$$

Thus, we can write

$$\Delta_{S,t} \equiv \frac{S_t - \bar{S}}{\bar{N}} \approx -\frac{\nu}{\eta} \hat{N}_t$$

and

$$\Delta_{X,t} \equiv \frac{X_t - \bar{X}}{\bar{N}} \approx \frac{\bar{\gamma}_z}{\bar{N}\phi} \hat{\gamma}_{z,t}.$$

The statistics $\Delta_{S,t}$ and $\Delta_{X,t}$ can be interpreted as the *extraordinary* time costs of shopping and improving transactions skill (respectively) as ratios to the time cost of labor at a point in time.

These are the useful measures of the welfare cost in the present context, at least if one is interested in implications for policy, for the following reasons. First, without an estimate of the parameter ψ_0 , the standard approach (following Lucas (1987)) of casting the cost directly into terms of consumption-equivalent cannot be followed. On the other hand, comparing the cost to that of time spent in productive work seems intuitive and equally interpretable. Second, the absolute level of the amount of time invested in improving transactions skill depends on the depreciation parameter δ , which is not identified. Whatever this absolute level, however, the model suggests that it cannot be affected by “policy variables”. To see this, recall the result from Subsection 4.1 that the level of this time investment is independent of the level of the opportunity cost along a balanced growth path. Finally and similarly, since the mean level of time spent shopping \bar{S} is a parameter calibrated a priori, the dynamic properties of $\Delta_{S,t}$ and its relative magnitude are just as informative as the estimates of S_t that accrue from the model.

Estimates of these welfare cost measures are shown in Figure 8. It is apparent that fluctuations of the cost of skill accumulation dwarf those of shopping time; the standard deviation of the former is 0.034, while that of the latter is 0.0055. Thus, the sum of these measures of the time cost reflect primarily the movements of the time devoted to skill accumulation. One may be skeptical about the time series of $\Delta_{S,t}$, as it is so closely related to what is probably an unreliable measure of fluctuations in productive activity and employment. Given the domination of cost induced by skill accumulation in total time cost, however, it may be surmised that

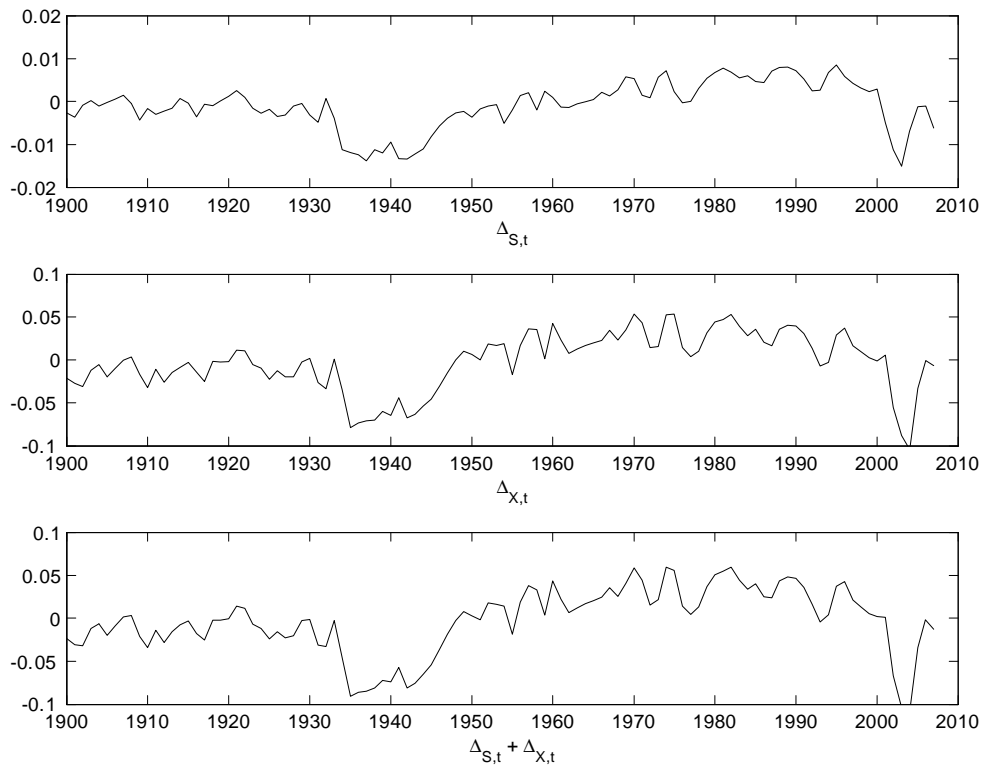


Figure 8: Welfare measures: extraordinary time costs of shopping and skill accumulation as ratio of labor time.

the fluctuations of the opportunity cost that induce extraordinary expense of such time take a substantial toll in terms of welfare.

6 Conclusions

I introduce two novel elements into an otherwise standard shopping time model of the use of money in equilibrium. The first, which constitutes the fundamental theoretical innovation of the paper, is dependence of transactions efficiency upon the stock of accumulated human capital or “transactions skill”. The second is a framework allowing for substitution between alternative forms of money. The first innovation induces different long- and short-run elasticities of the money-income ratio to the opportunity cost of holding money, the perception of which has been at the center of much controversy in the empirical literature. The second innovation affords consistency of the model with the empirical phenomenon of a declining ratio of M1 to income along a balanced growth path; the necessity of such a device is documented carefully in Section 2. To the best of my knowledge, this is the first model to explain how these empirical phenomena can be consistent with a stationary general equilibrium in a fully-articulated economic model.

In the stationary equilibrium, transactions skill is accumulated, on average, at a rate that is independent of the process generating the money opportunity cost. Thus, for example, a permanent increase of the opportunity cost affects the level of that skill, but not its long-run rate of growth. Since the rate of accumulation determines the cost of the investment (as in Lucas’s (1988b) “Uzawa-Rosen formulation” of the mechanism of human capital accumulation), a permanent increase of the level of the opportunity cost has no permanent effects on welfare. An implication is that, to the extent that the effect of a higher rate of price inflation is merely to increase the nominal interest rate, households’ steady-state welfare is independent of the rate of inflation.

I derive from the theory a reduced form equation relating the money-income ratio to lags of itself, current and lagged values of the opportunity cost, and a moving average of shocks to the efficiency of the transactions technology. This equation allows reinterpretation of much work in the empirical literature, and provides insight into its shortcomings.

The model is estimated using annual data on the M1-income ratio, a commercial paper rate, and net national product for the years 1900-2007. Estimated parameters show a high degree of stability across subsamples. The model is nested in a VAR-MAX(3,1) representation, and the implied restrictions are not rejected against more general specifications. Moreover, imposition of the restrictions markedly improves

out-of-sample forecasts and simulations relative to more general models.

It emerges from this exercise that substitution from M1 to broader forms of money appears to be a robust long-run phenomenon, not one isolated to the post-1965 economy. Additionally, the substitution effect appears to be adequately described by a deterministic growth term together with a moderately persistent stationary component.

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