

The Elastic Provision of Liquidity by Private Agents

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Abstract

I study a model of investment by financially constrained firms that are heterogeneous with respect to their exposure to an aggregate liquidity shock. A firm that is susceptible to the shock will mitigate its exposure by purchasing claims issued by a firm that is not. Liabilities of an unaffected firm may earn a liquidity premium due to their fungibility; and, because they are backed by productive investment, their supply is elastic to the demand. This segmentation implies that an aggregate liquidity shock has different consequences across sectors of the economy.

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1 Introduction

Evidence on the determinants of corporate holdings of cash and marketable securities is consistent with the need of firms to be responsive to investment opportunities or cost shocks in the presence of a wedge between the cost of internal and external funds.¹ Agency theory suggests that such a wedge may arise when a firm's creditors deem that its management must retain a stake in the business in order to induce them to act diligently.² The present paper contributes to an expanding body of research that attempts to assess the consequences for the macroeconomy of this connection between liquidity and the efficiency of firms' investment policies.³

This literature investigates the explanation for and the implications of scarcity of liquidity on the investment of capital-constrained firms facing cash flow uncertainties. Equilibrium in these models is consistent with the findings of Opler et al. (1999) that firms with more uncertain cash flows, those without credit ratings, and small firms maintain larger liquid balances; and the models establish the connection between these characteristics of firms and the difficulties they experience when liquidity becomes scarce. A seemingly related observation is that the debt of higher rated firms offers investors a substantially lower (default-adjusted) rate of return, on average.⁴ A theoretical link established in this paper stems from explicit consideration of the effect of liquidity scarcity on firms positioned to *issue* the claims that function as liquidity.⁵ In this paper, I build a simple general equilibrium model in which the liquidity that is demanded by the capital-constrained firms facing a high degree of un-

¹See Opler et al. (1999).

²Seminal work includes that by Jensen and Meckling (1976), Myers (1977), and Myers and Majluf (1984).

³Important work on macroeconomic implications of agency frictions includes that by Bernanke and Gertler (1989), Holmström and Tirole (1997, 1998, 2001) and Kiyotaki and Moore (1997, 2005).

⁴See Fons (1987) and Altman (1989). Altman (1989) shows that BB-rated debt performed best for a 10-year investment horizon in terms of default-adjusted return; AAA-rated debt earned a 1245 basis point (10-year compounded) premium over Treasury bonds, while the BB premium was 7637 basis points. His data covered the period from 1971-1987.

⁵In Holmström and Tirole (1997), a spread between the default-adjusted returns of high- and low-quality firms exists, because the latter are forced to use (costly) bank finance rather than issuing securities to the market. The explanation to be offered here (while complementary) differs fundamentally, as will become clear.

certainty of future costs (“low quality” firms) is constituted entirely of claims on the output of firms that do not face such shocks (“high quality” firms). When low quality firms’ cost shocks are correlated, liquidity may become scarce; in this case, the claims issued by high quality firms may earn a “liquidity (price) premium”, reducing these firms’ capital costs (and the rate of return on their securities) at the same time that the low quality firms face higher costs. It follows that the supply of liquidity inherits the elasticity properties of the investment of high quality firms with respect to the price of liquidity.

The model I present below draws on the work of Holmström and Tirole (1997, 1998) and Kiyotaki and Moore (2005), wherein liquidity is defined as the means by which wealth can be stored intertemporally and accessed readily. In the models in these papers, the share of firms’ output that is marketable to outside investors is less than the full value of their projects, because of the agency friction described above: investors require managers to retain a share of output proceeds to induce them to act diligently. Firms optimally hold liquidity when their projects face the possibility of a cost shock that would otherwise trigger liquidation of the project while it has positive residual value.

Requiring that debt be collateralized by firms’ holdings of a productive asset, Kiyotaki and Moore (2005) show that the asset, whose supply is fixed in their model, may earn a liquidity premium in the face of an aggregate shock.^{6,7} In their model, the scarcity of the collateral asset (land) is an exogenous property, and its price is determined endogenously. In Holmström and Tirole (1998), firms’ liquidity needs may be met by holding a liability of the government that is supplied perfectly elastically at a price fixed exogenously. They show that such securities may be demanded even if the price is higher than the fundamental one.

I adopt salient features of the model of Holmström and Tirole (1998) into my model, but I depart from them by requiring that liquidity be supplied endogenously, as described

⁶See also Krishnamurthy (2003); he shows that the essential feature of the Kiyotaki and Moore (1997, 2005) mechanism is the scarcity of the collateral, rather than its use as an input of production.

⁷In Kiyotaki and Moore (2005), the “fundamental price” of the asset is the marginal product of the productive asset. In Holmström and Tirole (1998), the appropriate benchmark is the value of the consumption afforded by the government liability.

above. There is no government and no land in my model, so that capital-constrained firms needing liquidity can only hold securities issued by other firms whose projects and financial structures induce diligent performance with respect to those claims. Since these securities are backed by investment, and to the extent that the volume of this investment depends on the rate of interest demanded by the market, the supply of liquid liabilities will be elastic to market conditions, as well. This phenomenon induces an upward-sloping liquidity supply curve that stands in contrast to the horizontal one in Kiyotaki and Moore (2005) and the vertical one in Holmström and Tirole (1998).

The model also shows a link between the balance sheets of the high quality firms and the supply of liquidity. This is because I assume that the marketable share of the high quality firms' output is affected by the same agency friction faced by other firms, linking their issue of securities to their net worth. This result complements the findings of a literature that examines the role of the balance sheets of financial intermediaries on the supply of loanable funds, with several important differences.⁸ First, the firms that supply liquidity need play no explicit role in the financial system – for example, by providing monitoring services as banks do. Second, an increased cost of borrowing experienced by the low quality firms does not induce a deadweight loss as monitoring costs do.

The model also exhibits a surprising rationale for the liquidation of the projects of *high* quality firms. Such a result may obtain when it is allowed that the moral hazard problem may be circumvented by liquidating projects prematurely. Then even though the return on liquidated investment may be low, the (fully marketable) proceeds of liquidation may be greater than the marketable share of the mature project output. In this case, the manager can issue more claims and invest more *ex ante* when he agrees to liquidate after the shock, and this course of action may be optimal if the market price of these securities becomes high.

The rest of the paper is structured as follows. In the next section, I introduce the formal model and assumptions. In the third section, I conduct the analysis of the model. The

⁸The seminal paper is Holmström and Tirole (1997); extensions include Santomero and Seater (2000) and Chen (2001). A related contribution is Van den Heuvel (2007).

fourth section discusses and interprets my findings. I consider some extensions of the basic model in Section 5, and the last section summarizes and concludes.

2 Model Environment

2.1 Time, Preferences, and Endowments

The economy is inhabited by “managers” and a “worker”. Each manager will operate a distinct “firm”, and I will refer synonymously to a manager and his firm. There are three dates $t \in \{0, 1, 2\}$, and there is a single good in each period in the economy. The good is useful for consumption at any date, and the dates 0 and 1 goods are useful also for investment as discussed below. The date t good perishes if it is not consumed or invested by the end of that date. At date 0, manager j has an endowment ω^j of the good. The worker has no endowment of the good at date 0, and no agent is endowed with goods at dates 1 or 2.

The worker can convert units of labor one-for-one into the contemporaneous good at dates 0 and 1. Producing goods in this manner induces a disutility for the worker such that the net gain to working and simply consuming the proceeds is zero. These assumptions will be shown to induce a perfectly elastic demand for a positive quantity of claims to future goods. On the other hand, the worker’s labor is inalienable, so that a promise from the worker to provide labor in the future can be reneged with impunity;⁹ that is, the worker will be unable to issue these claims.

Each manager has a production project that can be used to produce date 2 goods subject to a pattern of investment of goods at date 0 and 1. These projects are discussed in detail in the next subsection.

All agents are risk neutral. Managers evaluate outcomes according to the sum of non-negative consumption at the three dates. The worker evaluates outcomes according to the sum of consumption at each date minus labor expended. At each date, agents act in order

⁹I provide a precise mathematical representation for this concept below.

to maximize the expectation of their payoff at the current and future date(s). Agents do not discount the future.

2.2 Production Projects

Production in the economy is affected by an aggregate shock, a random variable that is realized and observed at date 1. The random variable takes on the value H with probability ϕ_H , and L with probability $\phi_L = 1 - \phi_H$. I will write $s \in S = \{H, L\}$ for a generic outcome. I will refer to a pairing of the date and the realization as the state; that is, state $1s$ indicates date 1 when the realized outcome outcome is s .

At date 0, manager j chooses a level of investment $I^j \in \mathbb{R}_+$. At date 1, the project may need additional investment of goods in order to continue. Precisely, if the outcome from the random variable at date 1 is s , then the project requires additional investment of $(1 - \lambda_s^j) I^j \rho_s^j$, where $\lambda_s^j \in [0, 1]$ is the fraction of the project that the manager chooses to discontinue in state $1s$, and $\rho_s^j \geq 0$. Discontinuance or “liquidation” of the manager’s project yields no residual, so that the fraction that is liquidated is simply lost. At the beginning of date 2, the manager has an opportunity to abscond with a booty of $(1 - \lambda_s^j) I^j \gamma^j$ from the project and consume it, in which case he leaves the remnants of the project valueless. If he chooses rather to allow the project to mature, then it yields an amount $(1 - \lambda_s^j) I^j R^j$, out of which the manager is obligated to service outstanding contractual claims as arranged in previous periods. Note that the other agents have no recourse against the manager when he absconds, but claims against the yield of the project are enforceable once the manager has chosen to allow it to mature.

To facilitate the intuition for the model, I assume that $\rho_L^j = 0$ for all firms j , so that firms’ face no need of additional investment in state $1L$. This framework still allows for heterogeneity of managers’ projects in several dimensions, and I will leave a number of the details about the distribution of their attributes unspecified in this section. I do this to facilitate presentation of some general results in Subsection 3.2. Thereafter, I will focus on

a particular environment with two firms, one of which is acutely affected by the cost shock in state $1H$, while the other is insulated from it.

The following assumption relates to the net value to society of undertaking these projects.

Assumption 1. For each manager j , I assume (i) that the mature yield of the project is greater than the private value obtained by the manager from absconding, $R^j > \gamma^j$; (ii) that the net surplus available from investment in the project is positive in expectation,

$$\sum_s \phi_s (R^j - \rho_s^j) - 1 > 0;$$

and (iii) that the project has positive residual value in each state, $R^j - \rho_s^j > 0$ for each $s \in S$.

In an extension of the basic model, I will also allow that a project may yield some residual, say $L^j \in (0, 1)$, for each unit that is liquidated at date 1. In this case, the manager reaps $\lambda_s^j I^j L^j$ goods in state s . I will assume that the proceeds of liquidation are contractable in the same way that the yield of a mature project at date 2 is assumed to be.

2.3 Market Institutions

I assume that, at each date $t \in \{0, 1\}$, a competitive market exists for state-contingent claims to goods at date $t + 1$. I write q_{1s} for the date 0 price of claims to goods in state $1s$, and I write q_{2s} for the state $1s$ price of claims to goods in state $2s$. I denote the consumption of agent i in state σ by c_σ^i , and I denote the net claims to state ts consumption held by agent i at the end of date $t - 1$ by B_{ts}^i . Since the worker's utility depends only on his consumption net of his labor supply in each period, I will describe his actions only up to this net supply of labor at dates 0 and 1 denoted $x_0 := n_0 - c_0^w$ and $x_{1s} := n_{1s} - c_{1s}^w$, respectively, where n_σ the gross amount of labor that he supplies in state σ .

2.3.1 The Worker's Problem

The worker takes present and future claims prices as given, and chooses net claims holdings, net labor supply at dates 0 and 1, and non-negative consumption at date 2 (x, c_2^w, B^w) to maximize his expected lifetime payoff

$$-x_0 + \sum_{s \in S} \phi_s (-x_{1s} + c_{2s}^w). \quad (1)$$

subject to the budget constraints¹⁰

$$-x_0 + \sum_{s \in S} q_{1s} B_{1s}^w \leq 0 \quad (2)$$

$$-x_{1s} + q_{2s} B_{2s} \leq B_{1s}^w \text{ for each } s \in S \quad (3)$$

$$c_{2s}^w \leq B_{2s}^w \text{ for each } s \in S, \quad (4)$$

and the participation constraints

$$-x_{1s} + c_{2s}^w \geq 0 \text{ for each } s \in S. \quad (5)$$

The budget constraints (2) and (3) require that the net purchases of the worker in the dates 0 and 1 financial markets are no greater than his wealth in the appropriate state. The wealth of the worker at date 0 is zero, and he affords financial assets purchases only by working more than he consumes. Similarly, his wealth in state 1s is given by his accumulation of assets from the previous period B_{1s}^w . The date 2 budget constraints (4) show that the worker consumes no more in state 2s than afforded by his accumulation of claims B_{2s}^w . The last set of constraints (5) reflect the fact that the worker is free to renege on any agreement to provide labor that does not benefit him ex post; this is the mathematical manifestation of inalienable labor.

¹⁰Non-negativity constraints on the worker's consumption in each state 2s, $c_{2s}^w \geq 0$, are left implicit in this formulation.

Notice that the solution to the worker's problem is indeterminate when $q_{1s} = \phi_s$ and $q_{2s} = 1$ for each s . This feature is basic to the inducement of a perfectly elastic residual demand for positive quantities of claims, as mentioned above. The role of the worker in the financial system will be clarified in Subsection 3.1 below.¹¹

2.3.2 A Manager's Problem

Manager j takes claims prices as given and chooses net claims holdings, non-negative consumption and investment, and a project liquidation rule $(c^j, I^j, \lambda^j, B^j)$ to maximize his expected lifetime payoff,

$$c_0^j + \sum_s \phi_s (c_{1s}^j + c_{2s}^j). \quad (6)$$

The manager faces the budget constraints¹²

$$c_0^j + I^j + \sum_{s \in S} q_{1s} B_{1s}^j \leq \omega^j \quad (7)$$

$$c_{1s}^j + (1 - \lambda_s^j) I^j \rho_s^j + q_{2s} B_{2s}^j \leq B_{1s}^j \text{ for each } s \in S \quad (8)$$

$$c_{2s}^j \leq B_{2s}^j + (1 - \lambda_s^j) I^j R^j \text{ for each } s \in S. \quad (9)$$

The date 0 budget constraint (7) says that the manager can apply no more than his endowment to date 0 consumption, investment, and purchase of financial claims. The date 1 budget constraints (8) say that accumulated claims must be used to fund consumption, additional investment, and the portfolio to be held at date 2. Finally, (9) says that date 2 consumption must be funded out of accumulated claims and project dividends.

The last budget constraint is valid under the assumption that the manager does not abscond with the project dividend. Since the manager can always achieve consumption of $(1 - \lambda_s^j) I^j \gamma^j$ at date 2 by his choice to abscond, an additional constraint applies to the

¹¹The problem of the worker in my model is viewed as the formalization of that of the "investors" in Holmström and Tirole (1998), whose precise nature is left somewhat more ambiguous.

¹²Non-negativity constraints on consumption and investment in each state are left implicit in this formulation.

choice problem. To see this, suppose that the manager holds a negative claims position $B_{2s}^j < -I^j (1 - \lambda_s^j) (R^j - \gamma^j) < 0$ for some state at date 2. Then (9) implies that $c_{2s}^j < (1 - \lambda_s^j) I^j \gamma^j$, and the manager can achieve a higher payoff by absconding. Under perfect information, no creditor would buy a quantity of claims from the manager that would induce him to abscond at date 2. Equivalently, it must be that the optimal policy of the manager satisfies the *incentive compatibility constraints*

$$c_{2s}^j \geq (1 - \lambda_s^j) I^j \gamma^j \text{ for each } s \in S. \quad (10)$$

The problem of the manager may now be stated as that of maximizing (6) subject to (7)-(10).

The quantity $\tilde{R}^j := R^j - \gamma^j$, which is positive by Assumption 1, plays an important role in the analysis to follow. The moral hazard problem described above creates a wedge between the internal rate of return available to the manager through investment, and the share that can be pledged to outsiders. As a result, \tilde{R}^j is the *marketable share* of the manager's project, the maximal amount that can credibly be pledged to outside stakeholders per unit of investment. The following assumption implies that, for sufficiently large investment I^j , manager j will be unable to credibly promise to repay $I^j - \omega^j$ to outside creditors, so that he will be unable to finance an arbitrarily large investment.¹³

Assumption 2. The marketable share of each manager j satisfies

$$\sum_s \phi_s (1 - \lambda_s^j) (\tilde{R}^j - \rho_s^j) < 1$$

for all liquidation rules $\lambda^j \in [0, 1]^2$.

In the next section, it will be made clear that the sign of $(\tilde{R}^j - \rho_H^j)$ determines whether firm j will issue claims to state $1H$ goods or purchase them. This dichotomy of firms will

¹³If Assumption 2 were violated, for example, then the problem in (6)-(9) would have no solution for the prices $q_{1s} = \phi_s$ and $q_{2s} = 1$ for each s .

be central to the theory.

2.3.3 Definition of Equilibrium

An *equilibrium* is an allocation of consumption and net labor supply for the worker (x, c_2^w, B^w) ; consumption, investment, and net assets holdings (c^j, I^j, B^j) for each manager j ; and asset prices q such that (i) each agent's problem is solved; and (ii) the markets for contingent claims to consumption clear at dates 0 and 1, i.e., for each $t \in \{1, 2\}$ and $s \in \{H, L\}$, $\sum_i B_{ts}^i = 0$.¹⁴

Each agent's marginal rate of substitution of consumption at date 0 for consumption in state 1s is ϕ_s , and that of consumption in state 1s for consumption in state 2s is 1. Therefore, I will refer to the price system defined by $q_{1s} = \phi_s$ and $q_{2s} = 1$ for each s as the *fundamental* one. As will be seen in the next section, this price system need not support an equilibrium.

3 Analysis

3.1 The Role of the Worker

Inspecting the worker's problem, it is immediate that a finite solution exists only if $q_{1s} \geq \phi_s$ and $q_{2s} \geq 1$. These properties are a manifestation of the worker's infinite capacity to purchase assets that yield a positive return. For example, if it were true that $q_{1s} < \phi_s$, then the worker could profit by converting an arbitrarily large amount of date 0 labor into date 0 goods, and trading the date 0 goods for claims to state 1s consumption. Since none of his constraints are violated by making x_0 arbitrarily positive while setting $x_{1s} = -x_0/q_{1s}$, and since the net contribution of the scheme to his lifetime expected payoff is $\phi_{1s}/q_{1s} - 1 > 0$ per unit of net labor expended, the worker's problem has no solution and there can be no

¹⁴The focus of the analysis to follow will be on the issue and purchase of claims by agents in the model. Thus, I have defined an equilibrium in terms of these markets, rather than those for goods. As usual, imposition of goods market clearing conditions in the definition of equilibrium instead of those for the claims markets would lead to an equivalent concept.

equilibrium.

On the other hand, there is no symmetric argument available that shows that q_{1s} cannot be higher than ϕ_s in equilibrium. To see why, use the first-order conditions and the complementary slackness condition induced by the non-negativity of c_{2s}^w to derive that $q_{2s}c_{2s}^w = c_{2s}^w$, and note that this implies (from (3) and (4)) that

$$B_{1s}^w = -x_{1s} + c_{2s}^w. \quad (11)$$

Now the participation constraints (5) imply that $B_{1s}^w \geq 0$ for each s . Thus, whereas the worker can *buy* assets to take advantage of a *low* price of date 1 consumption, his inability to commit to supply future labor prohibits his *issuing* assets to take advantage of *high* prices.

The following proposition extends this line of argument; the proof is stated in the Appendix.

Proposition 1 *If q is an equilibrium price system, then $q_{1s} \geq \phi_s$ and $q_{2s} \geq 1$ for each s . Furthermore, there is an optimal policy for the worker with claims holdings B^w if and only if the following hold for each s :*

- (i) $B_{1s}^w \geq 0$ with equality if $q_{1s} > \phi_s$; and
- (ii) $B_{2s}^w \geq 0$ with equality if $q_{2s} > 1$.

Next, I show that any equilibrium with positive residual (i.e., unliquidated) investment after state $1s$ must have the one-period price of claims to goods in state $2s$ equal to one, $q_{2s} = 1$. To see this, suppose, on the contrary, that $q_{2s} > 1$. Since the price of consumption in state $2s$ relative to consumption in state $1s$ is higher than agents' marginal rates of substitution for the same goods, managers will reduce consumption at the later date as much as the constraints of their problems will allow. In particular, inspection of the problem of any manager j shows that j will choose

$$c_{2s}^j = B_{2s}^j + (1 - \lambda_s^j) I^j R^j = (1 - \lambda_s^j) I^j \gamma^j;$$

that is, he will structure his portfolio so that his budget constraints (9) and incentive compatibility constraints (10) bind. This argument shows that

$$B_{2s}^j = -I^j (1 - \lambda_s^j) \tilde{R}_s^j \leq 0$$

for each manager j , and the inequality is strict for a manager with positive residual investment after state $1s$, $I^j (1 - \lambda_s^j) > 0$. Then the market clearing condition implies that

$$B_{2s}^w = - \sum_j B_{2s}^j > 0$$

whenever some firm has some positive residual investment after state $1s$; but by Proposition 1, an equilibrium with $q_{2s} > 1$ requires that $B_{2s}^w = 0$, a contradiction. The following lemma summarizes this result, which will be imposed hereafter without discussion.¹⁵

Lemma 1 *In any equilibrium with $I^j (1 - \lambda_s^j) > 0$ for some manager j , it must be that $q_{2s} = 1$.*

3.2 A Firm's Need of Liquidity

From the previous subsection, it can be inferred that the role of the worker in the economy is to provide investment goods to firms at dates 0 and 1 in exchange for claims on the output of those firms. But the possibility of the worker issuing claims that may be desired by firms is ruled out by their inability to commit to make good on such promises. In this section, I will explore the needs and capabilities of firms in the economy with respect to the issue and purchase of claims.

To begin, first note that a manager's budget constraints for states $1s$ and $2s$, (8) and (9),

¹⁵Imposition of the conclusion of Lemma 1 is without loss of generality; more precisely, it can be shown that any equilibrium allocation with no residual investment after state $1s$ can be supported as an equilibrium under a price system with $q_{2s} = 1$.

the non-negativity of agents' consumption, and Lemma 1 imply that

$$B_{1s}^j \geq I^j (1 - \lambda_s^j) \rho_s^j + B_{2s}^j \quad (12)$$

$$\geq -I^j (1 - \lambda_s^j) (\tilde{R}^j - \rho_s^j). \quad (13)$$

It is immediate that firms for which $\tilde{R}^j - \rho_s^j < 0$ will be required to purchase claims to goods in state 1s in order to sustain their projects. On the other hand, firms with $\tilde{R}^j - \rho_s^j > 0$ can issue such claims by (for example) choosing B_{1s}^j and B_{2s}^j so that (12) and (13) hold with equality. The interpretation put forth in this paper will be that firms of the former type *demand liquidity* in state 1s, while the latter firms can *supply liquidity*. The next proposition illuminates the essentiality of firms that supply liquidity.

Proposition 2 *Suppose that there is a firm j with $\tilde{R}^j - \rho_s^j < 0$ and $I^j (1 - \lambda_s^j) > 0$ in equilibrium. Then there must be a firm k with $\tilde{R}^k - \rho_s^k > 0$ and $I^k (1 - \lambda_s^k) > 0$.*

To see the result, first note that the hypothesis of the proposition and the preceding discussion show that $B_{1s}^j > 0$. Next suppose that $\tilde{R}^h - \rho_s^h \leq 0$ holds for all firms h in the economy with $I^h (1 - \lambda_s^h) > 0$; then an analogous argument establishes that $B_{1s}^h \geq 0$. In this case, the market clearing condition shows that

$$B_{1s}^w = - \sum_j B_{1s}^j > 0.$$

This contradicts Proposition 1, proving the claim of Proposition 2.

Proposition 2 follows from the fact that the worker is unable to (credibly) issue the claims demanded by a firm with $\tilde{R}^j - \rho_s^j < 0$, so they can only be issued by another firm. An equivalent statement of Proposition 2 is that, if $\tilde{R}^j - \rho_s^j < 0$ for all firms in the economy, then there can be no residual investment after state 1s in equilibrium.

Before concluding this subsection, I present a related result that will simplify the exposition to follow.

Lemma 2 *Suppose that, for some s , $\tilde{R}^j - \rho_s^j \geq 0$ for all j . Then in any equilibrium with $I^j (1 - \lambda_s^j) > 0$ for some firm j with $\tilde{R}^j - \rho_s^j > 0$, it must be that $q_{1s} = \phi_s$.¹⁶*

The proof of Lemma 2 is very similar to that of Lemma 1; suppose that $q_{1s} > \phi_s$. Invoking Proposition 1, the price of goods in states $1s$ and $2s$ in units of date 0 goods is at least q_{1s} , which is strictly higher than the manager's subjective marginal rate of substitution, ϕ_s . Thus, the manager will optimally choose to reduce consumption at these two dates as much as possible under the constraints he faces. Under the hypotheses of the proposition, it follows that

$$B_{1s}^j = - (1 - \lambda_s^j) I^j (\tilde{R}^j - \rho_s^j) \leq 0$$

for all firms j , and the inequality holds strictly for some j . Thus market clearing implies that

$$B_{1s}^w = - \sum_j B_{1s}^j > 0,$$

contradicting Proposition 1. (This completes the proof of Lemma 2.)

3.3 Supply and Demand of Liquidity in an Economy with Two Firms

For the remainder of the paper, I will consider an environment with two firms. Firm A will be assumed to have $\tilde{R}^A - \rho_H^A > 0$, and firm B has $\tilde{R}^B - \rho_H^B < 0$. For concreteness and for ease of exposition, I assume also that $\rho_H^A = 0$.^{17,18}

Interpreting this environment, the cost shock faced by firm A is less in each state than the marketable share of the firm; thus, from the results of the last subsection, it can be expected that this firm will issue a positive quantity of claims to goods in states $1H$ and $1L$

¹⁶Under the assumptions of Section 2, the hypothesis always holds for $s = L$.

¹⁷A useful mnemonic is to think of firm A as representative of AAA-rated firms, which tend to have stable cash flows and more easily marketed securities; while firm B, like firms with lesser debt ratings or unrated firms, is perceived to face greater risk.

¹⁸Recall that I have already assumed that $\rho_L^A = \rho_L^B = 0$.

in the course of its operations. Firm B's project is risky, and the marketable share of firm B's project is less than the amount of its cost shock in state $1H$. In order to continue its project after state $1H$, firm B will need to have purchased a positive quantity of claims to goods in that state. From the analysis above, we know that these claims will necessarily be those issued by firm A.

Since the marketable share of each firm in the economy is at least as great as its need of liquid claims in state $1L$, Lemma 2 shows immediately that $q_{1L} = \phi_L$. (Hereafter, I will impose this result without further discussion.) On the other hand, the possibility that equilibrium may exhibit a "liquidity price premium" on claims to goods in state $1H$ is left open. That is, the price of such claims may be higher than the fundamental price, ϕ_H .

A fundamental motivation of managers in the model is to invest to take advantage of high return production projects. In the previous subsection, I showed that a manager j with $\tilde{R}^j - \rho_H^j > 0$ may also play a useful role by supplying liquidity in state $1H$. Thus, for the case of firm A, the incentives to invest and to issue credible claims are aligned. In this environment, firm A will find it advantageous to borrow as much as possible against project proceeds and apply all of his marketable lifetime wealth toward investment. The following results comprise the formal statement of this insight; the results are proved in the Appendix.

Lemma 3 *At equilibrium prices, manager A will choose to consume nothing at dates 0 and 1, and he will choose date 2 consumption so that his incentive compatibility constraint is binding in each state; that is, $c_0^A = 0$; and $c_{1s}^A = 0$ and $c_{2s}^A = I^A (1 - \lambda_s^A) \gamma^A$ for each s .*

Corollary 1 *At equilibrium prices, manager A will never liquidate a portion of his project at date 1; he will choose $\lambda_s^A = 0$ for each state s .¹⁹*

Writing $\pi \equiv q_{1H} - \phi_H$ for the liquidity premium applying to price of claims to goods in

¹⁹Recall that I am assuming that there is no possibility of generating a return at date 1; in particular, this result may not hold when $L^j > 0$.

state 1H, the binding constraints can be solved to yield the optimal investment of firm A:

$$I^A = \frac{\omega^A}{1 - (1 + \pi) \tilde{R}^A}. \quad (14)$$

As the focus of the analysis will be on the finance of managers' projects under equilibrium prices, I use Lemma 3 and the budget constraints once more to derive that

$$B_{1s}^A = B_{2s}^A = \beta^A(\pi) \equiv \frac{-\omega^A \tilde{R}^A}{1 - (1 + \pi) \tilde{R}^A} \quad (15)$$

for each s .²⁰ Writing $R \equiv (1 + \pi)^{-1}$ for the market rate of interest on one- (or two-) period non-contingent debt, the expression $\tilde{R}^A \left(1 - \tilde{R}^A/R\right)^{-1}$ in the last equations may be interpreted as the *leverage ratio* of firm A; this is the maximal ratio of the manager's investment to his initial endowment afforded by his budget and incentive compatibility constraints. These equations show that, as the rate of return demanded by lenders in the market is reduced, the firm will issue more and more liabilities, the amount tending to infinity as the market rate approaches the marketable share of the firm; the asymptote occurs as π approaches

$$\bar{\pi}^A \equiv \frac{1}{\tilde{R}^A} - 1.$$

Thus, any finite demand for liabilities can be met by the supply of firm B for some $R > \tilde{R}^A$.

In contrast to the role of firm A in the model, firm B's need of liquidity at date 1 implies that he will be a net buyer of claims to goods in state 1H. Since the price of these claims may be higher than the fundamental price, the need of liquidity may impinge upon the profitability of firm B in a non-fundamental way.

This difference may manifest itself in one of two ways in terms of the response of the investment of firm B as the liquidity premium rises. First, the manager may decide that,

²⁰The hypothesis of "equilibrium prices" implies that the denominator in the expressions on the right-hand-sides on (14) and (15) must be positive. To see this, notice that the manager could finance unlimited consumption if $1 - (1 + \pi) \tilde{R}^A$ were non-positive.

while investment at date 0 is worthwhile, the cost of continuing the project after state $1H$ is too high, and he may choose to liquidate some or all of it in state $1H$ after making a positive investment initially. Alternatively, he may decide that consuming some or all of his endowment at date 0 is preferable to investing as much as the constraints he faces might allow. Which of these modes of behavior is applicable turns out to depend on the veracity of the condition

$$\phi_L R^B - 1 > 0. \tag{16}$$

When (16) holds, it can be seen that manager B's project will be profitable even when it is to be liquidated fully after state $1H$. In contrast, in the polar case, investment cannot be (strictly) profitable ex ante if the project is to be liquidated fully after state $1H$. With respect to the firm's demand for state $1H$ claims, the two modes of behavior are qualitatively similar. Thus, for clarity of exposition, I will assume in the remainder of the text that (16) holds, except in section 5.3, where the alternative case is considered.

When (16) holds, the possibility that firm B may be indifferent about investment in equilibrium has been eliminated. The next lemma shows that the manager invests all he can in this case; the result is proved in the Appendix.

Lemma 4 *Suppose that (16) holds. At equilibrium prices, manager B will choose to consume nothing at dates 0 and 1, and he will choose date 2 consumption so that his incentive compatibility constraint is binding in each state; that is, $c_0^B = 0$, and $c_{1s}^B = 0$ and $c_{2s}^B = (1 - \lambda_s^B) I^B \gamma^B$ for each s .*

Note that this result leaves open the possibility that manager B may decide to liquidate his project after date 1.

At equilibrium prices, Lemma 4 and the binding constraints of manager B's problem can

be seen to yield²¹

$$I^B = \frac{\omega^B}{1 - (1 - \lambda_L^B) \phi_L \tilde{R}^B - (1 - \lambda_H^B) (\phi_H + \pi) (\tilde{R}^B - \rho_H^B)}. \quad (17)$$

Similarly, some algebra shows that the optimal liquidation rule for firm A maximizes

$$\gamma^B \left\{ \frac{\omega^B \sum_s (1 - \lambda_s^B) \phi_s}{1 - (1 - \lambda_L^B) \phi_L \tilde{R}^B - (1 - \lambda_H^B) (\phi_H + \pi) (\tilde{R}^B - \rho_H^B)} \right\}. \quad (18)$$

This objective function has a simple interpretation. The factor γ^B is the manager's date 2 consumption per unit of residual investment at that date. The expression in the curly braces is the expectation over states of the residual investment, which can usefully be dissected further as follows. The factor

$$\kappa^B := \left\{ 1 - (1 - \lambda_L^B) \phi_L \tilde{R}^B - (1 - \lambda_H^B) (\phi_H + \pi) (\tilde{R}^B - \rho_H^B) \right\}^{-1}$$

is the leverage ratio afforded by the incentive compatibility constraints and the given liquidation policy. Therefore $\omega^B \kappa^B$ is the investment of firm B. Finally, the factor $\sum_s (1 - \lambda_s^B) \phi_s$ is the unconditional expectation of the unliquidated portion of manager B's project at date 2 under the chosen policy. Obviously, the manager gets no utility from liquidating a portion of his project.

The function (18) is strictly quasiconcave, and its maximizer λ_H^B admits a simple characterization summarized in the following; the result is proved in the Appendix.

Lemma 5 *Suppose that condition (16) holds. At equilibrium prices, manager B will never liquidate his project in state 1L, and there is a cutoff level $\bar{\pi}$ of the liquidity premium such that liquidation is optimal in state 1H if and only if $\pi \geq \bar{\pi}$. More precisely, optimality has*

²¹Similarly to the case for firm 1 above, the denominator of the expression on the right-hand-side must be positive at equilibrium prices.

$\lambda_L^B = 0$ and $\lambda_H^B \in \Lambda^B(\pi)$, where

$$\Lambda^B(\pi) \equiv \begin{cases} 0, & \text{if } \pi < \bar{\pi}^B \\ [0, 1], & \text{if } \pi = \bar{\pi}^B \\ 1, & \text{if } \pi > \bar{\pi}^B, \end{cases}$$

and

$$\bar{\pi}^B \equiv \frac{\phi_H (1 - \phi_L \rho_H^B)}{\phi_L (\rho_H^B - \tilde{R}^B)}.$$

It follows that firm B's demand for claims to goods in state 1H when the liquidity premium is π is given by

$$B_{1H}^B = \beta^B(\pi, \lambda_H^B) \equiv \frac{\omega^B (1 - \lambda_H^B) (\rho_H^B - \tilde{R}^B)}{1 - \phi_L \tilde{R}^B + (1 - \lambda_H^B) (\phi_H + \pi) (\rho_H^B - \tilde{R}^B)} \quad (19)$$

for some $\lambda_H^B \in \Lambda^B(\pi)$. Its portfolio in the other states of the world is described by

$$B_{1L}^B = \frac{-\omega^B \tilde{R}^B}{1 - \phi_L \tilde{R}^B + (1 - \lambda_H^B) (\phi_H + \pi) (\rho_H^B - \tilde{R}^B)} \quad (20)$$

and

$$B_{1s}^B = \frac{-\omega^B (1 - \lambda_s^B) \tilde{R}^B}{1 - \phi_L \tilde{R}^B + (1 - \lambda_H^B) (\phi_H + \pi) (\rho_H^B - \tilde{R}^B)} \quad (21)$$

for each s .

As suggested above, I will interpret the claims to goods in state 1H issued by firm A at date 0 (that is, the quantity $-B_{1H}^A$) as a function of the liquidity premium as the *supply of liquidity*; and the *demand for liquidity* is the quantity of state 1H claims held by manager B as a function of the liquidity premium. The family of dashed curves in Figure 1 represent the supply of liquidity for three different values of manager A's endowment ω^A , where the other parameters are those specified in Example 1 in the next section. From the analysis above, the supply of liquidity is an increasing function of the liquidity premium with a vertical

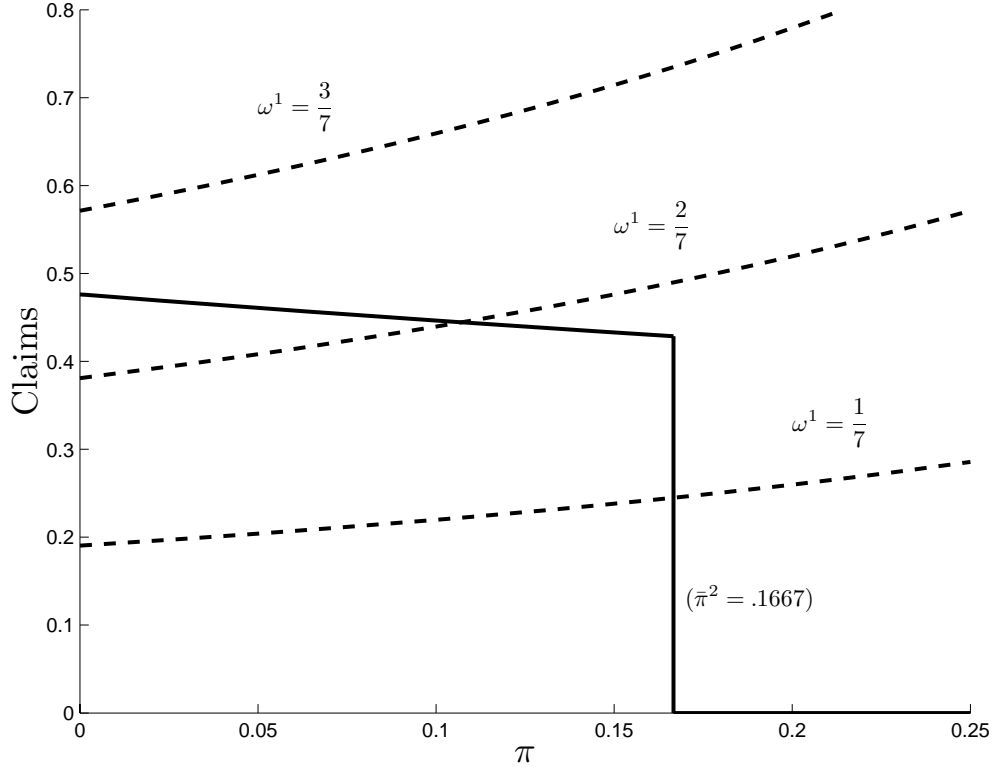


Figure 1: Supply and Demand of Liquidity for the Parameterization in Example 1

asymptote at the value $\bar{\pi}^A$.²²

The demand for liquidity locus for this parameterization is indicated by the solid curve in Figure 1. Formally, this is a decreasing upper-hemicontinuous correspondence.²³ The vertical portion of the demand curve occurs at the value of the liquidity premium $\bar{\pi}^B$ such that the manager of firm B is indifferent with respect to the continuation of his project.²⁴

It should be clear that the elements of the equilibrium have been pinned down by the foregoing analysis up to the determination of the price of claims to goods in state $1H$. The last element may be characterized with reference only to the market for liquidity in state $1H$ (that is, state $1H$ claims) by solving for the unique value of the liquidity premium that will clear this market.

The supply and demand of liquidity represent a convenient device for completing the

²²For the parameters used here, the vertical asymptote of the supply of liquidity occurs at $\bar{\pi}^1 = \frac{3}{4}$.

²³The sense in which the correspondence is decreasing should be apparent from the figure.

²⁴ $\bar{\pi}^2$ is equal to $\frac{1}{6}$ under the parameters of the example.

characterization as follows. First, if the curves do not intersect for some $\pi > 0$, then the supply of liquidity by firm A at the fundamental prices exceeds the demand by firm B.²⁵ This is the case for the highest supply curve, representing the largest level of the endowment of firm A, in the figure. From Proposition 1, it is clear that the worker will be able and willing to purchase excess liquid claims supplied by firm A at the fundamental prices, so that equilibrium liquidity premium for this case is zero.

Next, if the curves intersect for some liquidity premium greater than zero and less than $\bar{\pi}^B$, as they do for the intermediate of the supply curves in the Figure 1, then the equilibrium exhibits a positive liquidity premium given by the value of π at the intersection. In this case, manager B will not liquidate any portion of his project.

The third possibility is that the curves intersect at $\pi = \bar{\pi}^B$, as they do for the lowest supply curve, corresponding to the lowest value of manager A's endowment, in the figure. In this case, manager B is indifferent with respect to the fraction of the project to liquidate in state $1H$, and he behaves in equilibrium to utilize the amount of claims issued by firm A at this price.

This example is examined in greater detail in the next section.

The next proposition describes more rigorously the algorithm by which the equilibrium liquidity premium and an equilibrium liquidation rule for manager B are determined.

Proposition 3 *Suppose that (16) holds. In equilibrium, the claims holdings of the agents in the economy satisfy (15), (19)-(21), and*

$$B_{1H}^w = -B_{1H}^A - B_{1H}^B,$$

where π and λ_H^B are determined from the following conditions.

²⁵The interpretation in the text applies generically. On the other hand, it may be that the curves intersect when the liquidity premium is exactly equal to zero (i.e., $\pi = 0$), in which case it should rather be interpreted that supply of liquidity is *equal* to the demand at the fundamental prices. The proposition states the result more precisely.

1. If $\bar{\pi}^B \leq 0$, then the liquidity premium is equal to zero in equilibrium, and any $\lambda_H^B \in \Lambda^B(0)$ satisfying

$$\beta^A(0) + \beta^B(0, \lambda_H^B) \leq 0 \quad (22)$$

may hold in an equilibrium.

2. If $\bar{\pi}^B > 0$ and

$$\beta^A(0) + \beta^B(0, 0) \leq 0, \quad (23)$$

then the liquidity premium is equal to zero in equilibrium, and firm B will not liquidate any portion of his project at date 1; $\pi = 0$ and $\lambda_H^B = 0$.

3. If $\bar{\pi}^B > 0$ and condition (23) does not hold, then the equilibrium liquidity premium will be positive, and the liquidity premium and the liquidation rule of manager B are given by the unique values satisfying

$$\beta^A(\pi) + \beta^B(\pi, \lambda_H^B) = 0 \quad (24)$$

and $\lambda_H^B \in \Lambda^B(\pi)$.

4 Discussion

4.1 Liquidity Provision, Output, and Investment Across Sectors

Holmström and Tirole (1998), whose modeling devices I have essentially incorporated here, investigate (inter alia) the utility of issue of a government bond to ameliorate the problem of an aggregate liquidity shock. In their model, the liquid security is offered perfectly elastically at a price that is fixed exogenously; this price is interpreted as a per-unit deadweight cost of taxing consumers. Their assumption is analogous to introducing a costly commitment technology for use by the worker (or a government as his proxy) into my model.

In Kiyotaki and Moore (1997, 2005), producers are required to collateralize debt by holding land, a productive asset. Since the supply of land is fixed, liquidity is perfectly elastically supplied in their models. Parenthetically, while the fact that the asset plays a dual role as an input of production may appear to distinguish these models in another dimension, Krishnamurthy (2003) shows that it is rather the incompleteness of markets (or contracts) that induces the distinction. Krishnamurthy allows for trade of state-contingent claims backed by the collateral in a model in the style of Kiyotaki and Moore (1997). In his interpretation, the scarcity of liquidity follows from the shortage of collateral backing for the claims. Krishnamurthy’s “hedging collateral” plays the role of the investment that backs the claims issued by firm A in my model. The supply of the liquid asset remains fixed at an exogenous level in his model, however.

While these papers each examine how liquidity problems affect firms that experience them, the important innovation of the present model is a theory of how liquidity may be provided by the issue of liquid securities by firms that do not. In particular, the model shows that, as long as some firms are unaffected by the shock, and as long as those firms are capable of issuing fungible securities, liquidity problems can have qualitatively different effects in different sectors. The following example illuminates the possibility that some sector may be benefited by such episodes.

Example 1. Consider a numerical example with the following parameterization. Let $R^A = \frac{12}{7}$, $R^B = 2$, $\gamma^A = \gamma^B = \frac{8}{7}$, $\rho_H^B = \frac{8}{7}$, $\phi_H = \frac{1}{4}$, $\omega^A = \frac{2}{7}$, and $\omega^B = \frac{5}{7}$. Thus, it may be calculated that $\tilde{R}^A = \frac{4}{7}$, $\tilde{R}^B = \frac{6}{7}$, and $\tilde{R}^B - \rho_H^B = -\frac{2}{7}$. Note that (16) holds (the left-hand side equals $\frac{1}{2}$), and one can calculate that $\bar{\pi}^A = \frac{3}{4}$ and $\bar{\pi}^B = \frac{1}{6}$.

At the fundamental prices, Lemma 3 and Corollary 1 show that that firm A would invest

$$I^A = \frac{2/7}{1 - 4/7} = \frac{2}{3}$$

in his project (as in (14)). From (15), it can be seen that his investment would be financed

by his issue of $I^A \tilde{R}^A = \frac{2}{3} \cdot \frac{4}{7} = 0.381$ claims for each state. At these prices, Lemmas 4 and 5, and (17) show that the manager of firm B would choose to invest

$$I^B = \frac{5/7}{1 - \frac{3}{4} \left(\frac{6}{7}\right) + \frac{1}{4} \left(\frac{2}{7}\right)} = \frac{5}{3}.$$

To finance this investment, (19) shows that he would like to buy the net amount $-I^B \left(\tilde{R}^B - \rho_H^B\right) = \frac{5}{3} \cdot \frac{2}{7} = 0.476$ of claims for state $1H$.

In the present example, the amount of claims desired by firm B exceeds the amount that would be issued by firm A at the fundamental prices. Since the worker is precluded from issuing claims, the market equilibrium will therefore reflect a premium price on liquid claims. Precisely, Proposition 3 (part 3) shows that the market will clear at the liquidity premium π satisfying

$$\begin{aligned} -\beta^A(\pi) &= \frac{\frac{2}{7} \cdot \frac{4}{7}}{1 - \frac{3}{4} \left(\frac{4}{7}\right) - \left(\frac{1}{4} + \pi\right) \left(\frac{4}{7}\right)} = \frac{8}{21 - 28\pi} \\ = \beta^B(\pi, \lambda_H^B) &= \frac{\frac{5}{7} (1 - \lambda_H^B) \left(\frac{2}{7}\right)}{1 - \frac{3}{4} \left(\frac{6}{7}\right) + \left(\frac{1}{4} + \pi\right) (1 - \lambda_H^B) \left(\frac{2}{7}\right)} = \frac{20 (1 - \lambda_H^B)}{35 + (7 + 28\pi) (1 - \lambda_H^B)} \end{aligned}$$

for some λ_H^B such that

$$\lambda_H^B \in \begin{cases} 0, & \text{if } \pi \in [0, \frac{1}{6}] \\ [0, 1], & \text{if } \pi = \frac{1}{6} \\ 1, & \text{if } \pi > \frac{1}{6}. \end{cases}$$

The solution has $\lambda_H^B = 0$ and $\pi = \frac{3}{28}$. The supply and demand curves are depicted in Figure 1, where the supply of state $1H$ claims is the intermediate of the dashed curves, and the demand curve is the solid curve in the figure. More precisely, the dashed line is the function $\beta^A(\pi)$, and the solid curve depicts the values satisfying $\beta^B(\pi, \lambda_H^B)$ and $\lambda_H^B \in \Lambda^B(\pi)$.

In the equilibrium, manager B's investment can be seen to be $\frac{14}{9}$, less than he would choose in an environment with surplus liquidity. On the other hand, the liquidity price-premium represents an implicit subsidy for the investment of manager A. The latter invests

more $(\frac{7}{9})$ and issues more claims $(\frac{4}{9})$ to goods in each state. Thus, an interesting feature of the model is illuminated: that the liquidity problem of firm B may distort firm A's behavior in the direction of *increased* investment. ■

4.2 Liquidity Supply and Balance Sheet Effects

The effect of a reduction of the endowment of manager A on the market for liquidity is evident by reference to Example 1 and Figure 1. In the figure, it is apparent how the decrease in ω^A shifts the liquidity supply curve down, increasing the equilibrium liquidity premium. Thus, the model shows how productive sectors of the economy may be financially interdependent, and how a shock to the balance sheet of one sector of the economy can spillover to affect investment in another. Moreover, it does not seem necessary that the sector that issues instruments of liquidity have a special role in finance. It is only necessary that that the value of the liabilities issued by firms in this sector constitute the hedge against the shocks that affect another sector.

Such an arrangement can exist directly, as in the model; and it can easily be imagined to exist as an indirect link. The security that fulfills the liquidity function in the model may be likened to one of a number of financial instruments available to corporations in the real world, and the model is probably too abstract to discriminate. Perhaps the most easily recognizable arrangement is that the debt of firm A, intermediated costlessly, constitutes the basis for a loan commitment or a line of credit offered by the intermediary to firm B.²⁶

With respect to these balance sheet effects, the model delivers insights complementary to the literature, following Holmström and Tirole (1997), that explores the inter-dependence between the balance sheets of firms, banks, and lay consumers. These models incorporate an implicit non-trivial elasticity of the supply of loanable funds that can be interpreted to deliver conclusions similar to those of the present study. In particular, the links among the balance sheets of firms in different sectors is central. Whereas they explore the connection

²⁶Holmström and Tirole (1998) seem to prefer such an interpretation in the version of their model with firm heterogeneity.

between the financial conditions of producers and financial intermediaries, it is shown in the present study how the financial conditions of firms in seemingly unrelated sectors of the economy may be as important as the condition of the banks. Another important difference is that the role of banks in monitoring borrowers in Holmström and Tirole (1997) induces a purely deadweight cost on the economy that is not present in the model of this paper.

5 Some Extensions of the Basic Model

5.1 Allowing for Date 1 Scrap Value

In this subsection, I consider the possibility that the project of firm A may be liquidated to yield some residual amount of goods at date 1. Thus, the date 1 budget constraint (8) of manager A is modified to be

$$c_{1s}^A + B_{2s}^A \leq B_{1s}^A + I^A \lambda_s^A L^A \text{ for each } s \in S, \quad (25)$$

where $L^A \in [0, 1)$ is the quantity of goods obtained at date 1 by liquidating one unit of the investment.²⁷

It should be clear that the motivation of manager A to invest is not diminished, so that he will arrange his portfolio to invest as much as possible by choosing consumption at the three dates as specified in Lemma 3. Now solving the date 0 budget constraint, it can be seen that the investment of firm A is

$$I^A = \frac{\omega^A}{1 - \phi_L \left[(1 - \lambda_L^A) \tilde{R}^A + \lambda_L^A L^A \right] - (\phi_H + \pi) \left[(1 - \lambda_H^A) \tilde{R}^A + \lambda_H^A L^A \right]}, \quad (26)$$

and the form of the objective function of manager A (analogous to (18) for manager B)

²⁷Note that I have imposed the assumption that $\rho_s^A = 0$ for each s , and the result from Lemma 1 that $q_{2s} = 1$.

becomes

$$\gamma^A \left\{ \frac{\omega^A [(1 - \lambda_L^A) \phi_L + (1 - \lambda_H^A) \phi_H]}{1 - \phi_L [(1 - \lambda_L^A) \tilde{R}^A + \lambda_L^A L^A] - (\phi_H + \pi) [(1 - \lambda_H^A) \tilde{R}^A + \lambda_H^A L^A]} \right\}. \quad (27)$$

An optimal liquidation policy for manager A is described by the following; the proof is presented in the Appendix.

Lemma 6 *At equilibrium prices, the following are true.*

1. If $\tilde{R}^A \geq L$ then manager A will never liquidate his project in any state in equilibrium.
2. If $\tilde{R}^A < L$, then, in equilibrium, manager A will never liquidate his project in state 1L, but liquidation will occur in state 1H if the liquidity premium is high enough. More precisely, the optimal liquidation policy for manager A has $\lambda_L^A = 0$ and $\lambda_H^A \in \Lambda^A(\pi)$, where

$$\Lambda^A(\pi) \equiv \begin{cases} 0, & \text{if } \pi < \tilde{\pi}^A \\ [0, 1], & \text{if } \pi = \tilde{\pi}^A \\ 1, & \text{if } \pi > \tilde{\pi}^A, \end{cases}$$

and

$$\tilde{\pi}^A \equiv \frac{\phi_H (1 - L^A)}{L^A - \phi_L \tilde{R}^A}.$$

The quantity of state 1H claims held by firm A when the liquidity premium is π is given by

$$B_{1H}^A = \tilde{\beta}^A(\pi, \lambda_H^A) \equiv \frac{-\omega^A [\lambda_H^A L^A + (1 - \lambda_H^A) \tilde{R}^A]}{1 - \phi_L \tilde{R}^A - (\phi_H + \pi) [\lambda_H^A L^A + (1 - \lambda_H^A) \tilde{R}^A]} \quad (28)$$

for some $\lambda_H^A \in \Lambda^A(\pi)$; its portfolio in the other states of the world may be derived easily from the appropriate constraints. The equilibrium liquidity premium can be derived by applying an algorithm similar to that of Proposition 3; the precise statement of the algorithm is omitted.

It may be surprising that the manager of firm A would ever allow his project to be liquidated in this environment, since he can always choose his investment and structure his claims portfolio so that it is not necessary to do so. The important insight is that liquidation of a project at date 1 circumvents the need to provide incentives at date 2. Therefore, even though the overall return from the project is lower when it is liquidated, it is still possible that liquidation may put more value in the hands of outside claims holders than could be achieved by carrying the project through to maturity. Indeed, the lemma shows that this is precisely the necessary and sufficient condition under which the manager of firm A will allow his project to be liquidated when the liquidity premium becomes extreme. Then if the price of liquidity is high enough, the increased investment afforded by promising to liquidate may sufficiently increase the payoff of the manager in state $1L$ to compensate (in expectation) for the sacrifice of payoff in state $1H$.

It should be noted that manager A would prefer to renege on his promise to liquidate the project whenever state $1H$ is realized, since he gets no payoff in the event. Thus, it is necessary that the holders of claims issued by the firm can force it to honor the obligations at date 1. To see that this assumption is consistent with the spirit of the model in other respects, note that, when liquidation occurs, the manager has chosen a volume of state $1H$ claims that creditors will not permit him to roll-over into state $2H$ claims. Thus, his options are to liquidate and reduce the debt or “default”. Then it is equivalent to assume that creditors may seize the project after a default in state $1H$, and liquidate it with a return of L^j per unit of the investment.

Interpreting liberally, it is not necessary that we look for “fire-sale liquidation” of AAA-rated firms’ assets to corroborate this result empirically. It may simply be interpreted that the firm retains the option to divert investment capital at date 1 to a more transparent activity with some net sacrifice of the rate of return. In this light, “liquidation” may simply be interpreted as a (costly) shift of operations to some project more valued by outsiders (and less valued by management).

A second example illustrates the phenomenon numerically.

Example 2. Modify the example of the previous subsection by setting $L^A = \frac{6}{7} > \tilde{R}^A = \frac{4}{7}$. In this case, Lemma 6 shows that liquidation will be optimal for firm A whenever π is at least $\tilde{\pi}^A = \frac{1}{12}$.

Figure 2 depicts the demand for liquidity correspondence of manager B, analyzed in Example 1, together with the implied supply correspondence of manager A with the positive liquidation value L^A . As is apparent from the figure, the new equilibrium liquidity premium is $\frac{1}{12}$, which is lower than it was in the previous example, and manager A now chooses to liquidate a small fraction of his project in state 1H. More precisely, the fraction of firm A to be liquidated in equilibrium satisfies

$$\begin{aligned} -\tilde{\beta}^A \left(\frac{1}{12}, \lambda_H^A \right) &= \frac{\frac{2}{7} [\lambda_H^A (\frac{6}{7}) + (1 - \lambda_H^A) (\frac{4}{7})]}{1 - \frac{3}{4} (\frac{4}{7}) - \frac{1}{3} [\lambda_H^A (\frac{6}{7}) + (1 - \lambda_H^A) (\frac{4}{7})]} \\ &= \beta^B \left(\frac{1}{12}, 0 \right) = \frac{20}{35 + 28 (\frac{1}{3})}, \end{aligned}$$

and it may be calculated that $\lambda_H^A = \frac{2}{29}$. ■

5.2 Incorporating Idiosyncratic Risk

Most of the risk facing firms in an economy, entrepreneurial or otherwise, is probably idiosyncratic, essentially uncorrelated with risk faced by other firms in the economy; and so it is important to understand why it may be appropriate to abstract from such risk as I have done. Without presenting a full treatment, this subsection offers some heuristic discussion of how idiosyncratic risk would interact with the extant mechanisms of the model.

Holmström and Tirole (1998) consider (inter alia) an environment similar to the one specified in Section 2 in which a continuum of firms, identical at date 0, experience purely idiosyncratic cost shocks at date 1. They show that, under a mild assumption and admitting appropriate market institutions, claims on the firms' output can provide sufficient liquidity for the needs of the aggregated sector.

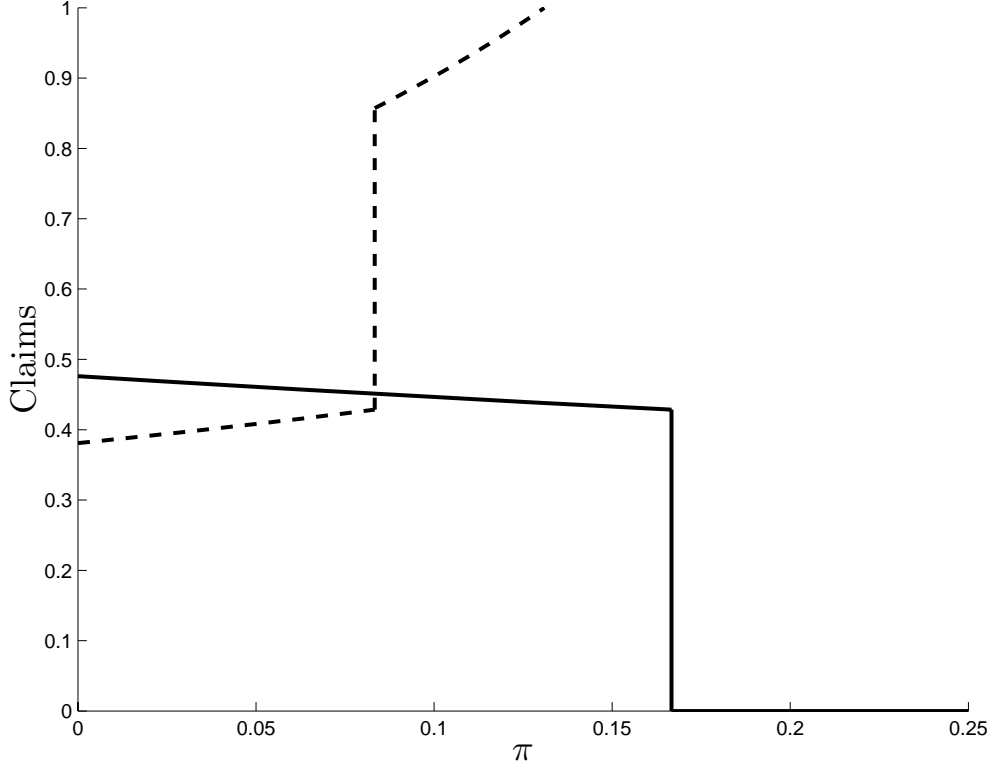


Figure 2: Supply and Demand of Liquidity for the Parameterization in Example 2

To see the result, suppose that there is a unit-continuum of a single *type* of firm, and that the uncertainty facing each firm's production project is as described in Section 2, except that each firm's cost shock is independent of the others. Assume further that

$$\sum_s \phi_s (\tilde{R} - \rho_s) > 0, \quad (29)$$

so that the aggregate quantity of claims that may be issued to outsiders by the sector is positive. Without loss of generality, assume that $(\tilde{R} - \rho_L) > 0$; analogous to the results derived in Section 3, this implies that a firm may issue a positive quantity of claims contingent upon its private shock taking the value L . Now (29) implies that

$$\phi_L (\tilde{R} - \rho_L) > -\phi_H (\tilde{R} - \rho_H),$$

implying (after invoking a Law of Large Numbers) that the aggregate per unit of investment volume of claims that may be issued by firms that experience shock L will be greater than the need of liquidity of the firms that experience shock H .

Considering purely aggregate uncertainty, the case in which identical firms experience costs shocks that are perfectly correlated, Holmström and Tirole (1998) show that the condition (analogous to) $\tilde{R} - \rho_H < 0$ implies that the productive sector requires liquidity from outsiders in state H . It would be straightforward to “tack on” an element of idiosyncratic uncertainty such that the aggregate shock established the mean of the shocks; the same statement would continue to hold in this case. Still, the consequences of the liquidity scarcity would be somewhat mitigated, since any liquidation that is necessitated could be accomplished disproportionately among firms with higher idiosyncratic shocks.

The substantive innovation of the present paper stems from the existence of ex ante heterogeneity among firms (or sectors of firms); without this, for example, there would be no differences in initial investment choices across firms. In other respects, however, the model is essentially the “pure aggregate uncertainty” economy considered by Holmström and Tirole (1998). Thus, the same comments apply about the consequences of adding an idiosyncratic component to the date 1 cost shocks to the model developed here. That is, while the *sector* j for which the mean cost shock satisfies $\tilde{R}^j - \rho_H^j < 0$ in state H would require liquidity from outsiders in this state, that liquidity could be better utilized by discriminating among firms within the sector according to their idiosyncratic shocks.

5.3 The Case That $\phi_L R_L^B - 1 \leq 0$.

In this subsection, I consider the demand for liquidity in the case that (16) does not hold. It will turn out that the result is qualitatively very similar to the case considered in the balance of the text.

According to Proposition 1, equilibrium prices will always be at least as high as agents’ subjective intertemporal marginal rates of substitution. Therefore, it can be seen that there

is no loss of generality in assuming that manager B chooses consumption at dates 1 and 2 to be as small as constraints will allow. Thus, I will impose a priori that $c_{1s}^B = 0$ and $c_{2s}^B = I^B (1 - \lambda_s^B) \gamma^B$. In contrast to the conclusion of Lemma 4, however, it may be that the manager prefers to consume some or all of his date 0 endowment when the liquidity premium becomes extreme; that is, it may be optimal to set $c_0^B > 0$.

Imposing these constraints and substituting the budget constraints into the objective function, the problem faced by the manager is equivalent to that of choosing non-negative investment and a liquidation policy to solve

$$\max \left\{ \omega^B + I^B \left[-1 + \sum_s \phi_s (1 - \lambda_s^B) (R^B - \rho_s^B) + \pi (1 - \lambda_H^B) (\tilde{R}^B - \rho_H^B) \right] \right\} \quad (30)$$

subject to

$$\omega^B - I^B + I^B \sum_s \phi_s (1 - \lambda_s^B) (\tilde{R}^B - \rho_s^B) + I^B \pi (1 - \lambda_H^B) (\tilde{R}^B - \rho_H^B) \geq 0,$$

where the constraint inheres from the non-negativity of date 0 consumption.

Suppose that we were to proceed under the assumption that the constraint is binding at the optimum, and construct the optimal policy for liquidation at date 1 under this hypothesis. We could then solve for investment, deriving equation (17), and substitute it into the objective function to derive the form of the objective function in (18). Next, by maximizing with respect to the liquidation policy, we would arrive at the conclusion of Lemma 5. However in returning to verify the original assumption, we would find that the objective function is non-increasing at when the liquidity premium is at the cutoff value of the liquidity premium for liquidation in state 1H suggested by Lemma 5, and it is strictly decreasing at higher values.

In fact, from (30), it is apparent that the objective function is non-increasing for $\pi \geq \tilde{\pi}^B$, where

$$\tilde{\pi}^B \equiv \frac{\sum_s \phi_s (R^B - \rho_s^B) - 1}{\rho_H - \tilde{R}^B}.$$

Moreover, consistent with the claim of the previous paragraph, it can be verified that $\tilde{\pi}^B \leq \bar{\pi}^B$ holds whenever (16) does not, and that $\tilde{\pi}^B = \bar{\pi}^B$ if and only if $\phi_L R_L^B - 1 = 0$. Thus, it is never optimal to liquidate at date 1 after choosing a positive level of investment at date 0; instead, the optimal response of manager B to an extreme value of the liquidity premium is to reduce his investment in the project ex ante. The following statement summarizes the result; the formal proof is omitted.

Lemma 7 *Suppose that (16) does not hold. Then manager B will never liquidate his project in either state after investing a positive amount at date 0; moreover there is an optimal policy for manager B with investment I^B if and only if*

$$I^B = \frac{\omega^B \chi}{1 - \phi_L \tilde{R}^B - (\phi_H + \pi) (\tilde{R}^B - \rho_H^B)} \quad (31)$$

for some $\chi \in \tilde{\Lambda}^B(\pi)$, where

$$\tilde{\Lambda}^B(\pi) \equiv \begin{cases} 1, & \text{if } \pi < \tilde{\pi}^B \\ [0, 1], & \text{if } \pi = \tilde{\pi}^B \\ 0, & \text{if } \pi > \tilde{\pi}^B. \end{cases}$$

It follows that firm B's demand for claims to goods in state 1H when the liquidity premium is π is given by

$$B_{1H}^B = \tilde{\beta}^B(\pi, \chi) \equiv \frac{\omega^B (\rho_H^B - \tilde{R}^B) \chi}{1 - \phi_L \tilde{R}^B + (\phi_H + \pi) (\rho_H^B - \tilde{R}^B)} \quad (32)$$

for some $\chi \in \tilde{\Lambda}^B(\pi)$. Its portfolio in the other states can be gleaned easily with reference to the binding constraints of the manager's problem. The implications for analysis of equilibrium are qualitatively little changed from the cases considered previously, and the equilibrium liquidity premium can be derived by applying an algorithm similar to that of

Proposition 3; the precise statement of the algorithm is omitted.

6 Conclusions

In the previous sections, I have presented a model of firm finance in which a liquidity need is generated by the confluence of two factors. First, moral hazard induces a wedge between market and internal valuations of firms' projects. Second, the projects face the possibility of an aggregate cost shock at a time when cash flows are low. In this case, firms will need to hoard liquid securities to avoid having to liquidate these (internally) valuable projects. In the model, all securities must be backed by productive assets; there is no government, and the promises of workers can be renegeed with impunity.

The innovation of the paper comes through the introduction of heterogeneity of firms such that some firms' projects may be insulated from the liquidity shock. It is shown how the liabilities of firms in the second category can provide the liquidity valued by other firms. In this case, the supply of liquidity inherits the elasticity properties of the investment projects of these firms. The upward-sloping liquidity supply curve in my model stands in contrast to the vertical one in models with a fixed supply of liquidity (collateral), and the horizontal one in models in which government liquidity is offered at a fixed price. When liquidity is scarce, the price of these liquid liabilities is high, and the firms that need liquidity effectively subsidize the investment of firms that supply it. Thus, cost volatility (i.e., the prospect of a random liquidity shock) in one sector may affect investment in the other sector beneficially.

With the diminishment of banks' role in all aspects of the financial system, it is important to understand what features of the financial contracts that firms value transcend the identity of the counterparty. The degree to which the direct liabilities of corporations actually serve the function of the liquid claims in the model is an empirical question of some interest. The model sheds some light on the degree to which such liabilities are capable of doing so.

Appendix: Proofs of the Results

Proof of Proposition 1. Writing out the first-order conditions for the problem, it can be seen that the Lagrange multiplier on (5) may be expressed as $\xi = q_{1s} - \phi_s$. Then the complementary slackness condition associated with (5) has the form

$$(-x_{1s} + c_{2s}^w)(q_{1s} - \phi_s) = 0. \quad (33)$$

Result (i) then follows from (5), (11), and (33).

Result (ii) follows from the binding constraint (4), the non-negativity of c_{2s}^w , and the condition cited in the text, $q_{2s}c_{2s}^w = c_{2s}^w$. ■

Proof of Lemma 1. The Lemma is proved in the text. ■

Proof of Proposition 2. The Proposition is proved in the text. ■

Proof of Lemma 2. The Lemma is proved in the text. ■

Proof of Lemma 3. First suppose that $(c^A, I^A, \lambda^A, B^A)$ is a feasible policy, and suppose that $c_0^A = \Delta > 0$. Now construct the alternative policy $(\tilde{c}^A, \tilde{I}^A, \tilde{\lambda}^A, B^A)$ as follows. Let $\tilde{c}_0^A = 0$, $\tilde{I}^A = I^A + \Delta$, and $\tilde{\lambda}_s^A = \lambda_s^A I^A / \tilde{I}^A$ and $\tilde{c}_{2s}^A = c_{2s}^A + \Delta R^A$ for each s . Let the remaining elements of the new policy be identical to the old. Now it is easy to check that the new policy is feasible if the old one is. Moreover, subtracting the payoff under the old policy from that generated by the new gives $\Delta (R^A - 1) > 0$, so that the new one is an improvement, a contradiction.

To see that manager A's incentive compatibility constraints must bind, suppose that the feasible policy has

$$c_{2\sigma}^A = I^A (1 - \lambda_\sigma^A) \gamma^A + \Delta$$

for some $\Delta > 0$ for some state σ . Now construct the alternative policy $(\tilde{c}^A, \tilde{I}^A, \tilde{\lambda}^A, B^A)$ as follows. Let $\tilde{I}^A = I^A + q_{1\sigma}q_{2\sigma}\Delta$, and $\tilde{\lambda}_s^A = \lambda_s^A I^A / \tilde{I}^A$ for each s ; and let $\tilde{c}_{2\sigma}^A = c_{2\sigma}^A - \Delta + q_{1\sigma}q_{2\sigma}\Delta R^A$ and $\tilde{c}_{2s}^A = c_{2s}^A + q_{1\sigma}q_{2\sigma}\Delta R^A$ for $s \neq \sigma$. Let the remaining elements of the new policy be identical to the old. Now it is easy to check (using $q_{1\sigma}q_{2\sigma} \geq \phi_\sigma$, which follows from

Proposition 1) that the new policy is feasible if the old one is. Moreover, subtracting the payoff under the old policy from that generated by the new gives

$$\Delta (q_{1\sigma}q_{2\sigma}R^A - \phi_\sigma) \geq \Delta\phi_\sigma (R^A - 1) > 0,$$

where the inequality follows from Assumption 1.

Finally, suppose that the feasible policy has $c_{1\sigma}^A = \Delta > 0$. Then construct the tilde policy with $\tilde{c}_{1\sigma}^A = 0$, $\tilde{I}^A = I^A + q_{1\sigma}\Delta$, $\tilde{\lambda}_s^A = \lambda_s^A I^A / \tilde{I}^A$, and $\tilde{c}_{2s}^A = c_{2s}^A + q_{1\sigma}R^A$; and let the other elements be as in the original policy. Again the feasibility and superiority of the new policy can be verified. ■

Proof of Corollary 1. After imposing the results of Lemma 3, the objective function of manager A becomes

$$\gamma^A \left\{ \frac{\omega^A [\phi_L (1 - \lambda_L^A) + \phi_H (1 - \lambda_H^A)]}{1 - \phi_L (1 - \lambda_L^A) \tilde{R}^A - (\phi_H + \pi) (1 - \lambda_H^A) \tilde{R}^A} \right\}.$$

Under the assumptions of the paper, this function is strictly decreasing in λ_L^A and λ_H^A ; the result follows immediately. ■

Proof of Lemma 4. Suppose that $(c^B, I^B, \lambda^B, B^B)$ is a feasible policy, and that $c_0^B = \Delta > 0$. Construct an alternative policy $(\tilde{c}^B, \tilde{I}^B, \tilde{\lambda}^B, B^B)$ as follows. Let $\tilde{c}_0^B = 0$, $\tilde{c}_{2L}^B = c_{2L}^B + \Delta R^B$, $\tilde{I}^B = I^B + \Delta$, $\tilde{\lambda}_L^B = I^B \lambda_L^B / \tilde{I}^B$, and $\tilde{\lambda}_H^B = 1 - (1 - \lambda_H^B) I^B / \tilde{I}^B$. Define the remaining elements of the new policy as in the old. Now it is easy to check that the new policy is feasible if the old one is. Moreover, subtracting the payoff to the manager of the old policy from that generated by the new one gives

$$\Delta [\phi_L R^B - 1] > 0,$$

where the inequality follows from (16).

(Surpluses for the other states can be excluded in the manner of the proof of Lemma 3.)

■

Proof of Lemma 5. The necessary and sufficient first-order Kuhn-Tucker conditions for a maximum of the strictly quasiconcave objective function (18) are

$$-\frac{q_{1\sigma}}{\phi_\sigma} \left(\tilde{R}^B - \rho_\sigma^B \right) \sum_s \phi_s (1 - \lambda_s^B) - \left[1 - \sum_s q_{1s} (1 - \lambda_s^B) \left(\tilde{R}^B - \rho_s^B \right) \right] \begin{aligned} &\geq 0 \text{ if } \lambda_\sigma^B > 0 \\ &\leq 0 \text{ if } \lambda_\sigma^B < 1 \end{aligned}$$

for each state $\sigma \in \{H, L\}$. I have already argued that the expression in square brackets is positive, so that $\rho_L^B = 0$ implies immediately that $\lambda_L^B = 0$. Imposing this result in the condition for λ_H^B and using the fact that $q_{1L} = \phi_L$ (as shown in the text), the cutoff value of the liquidity premium $\bar{\pi}$ may be derived by simplifying, solving for the value of q_{1H} (say \bar{q}) that makes the left-hand side criterion exactly equal to zero, and using $\bar{\pi} \equiv \bar{q} - \phi_H$. ■

Proof of Lemma 6. The necessary and sufficient first-order Kuhn-Tucker conditions for a maximum of the strictly quasiconcave objective function (27) are

$$\frac{q_{1\sigma}}{\phi_\sigma} \left(L^A - \tilde{R}^A \right) \sum_s \phi_s (1 - \lambda_s^A) - \left\{ 1 - \sum_s q_{1s} \left[\lambda_s^A L^A + (1 - \lambda_s^A) \tilde{R}^A \right] \right\} \begin{aligned} &\geq 0 \text{ if } \lambda_\sigma^A > 0 \\ &\leq 0 \text{ if } \lambda_\sigma^A < 1 \end{aligned}$$

for each state $\sigma \in \{H, L\}$. I have already argued that the term in braces must be positive for all $\lambda^A \in [0, 1]^2$ at equilibrium prices, so result 1 is obvious by inspection.

Now consider the case that $\tilde{R}^A < L^A$; using the result that $q_{1L} = 1 - \phi$, the critical condition for $\lambda_L^A > 0$ to be optimal can be written as $q_{1H} \geq Q_L(\lambda_H^A)$ where

$$Q_L(\lambda_H^A) \equiv \frac{1 - \phi_L L^A - \phi_H \left(L^A - \tilde{R}^A \right) (1 - \lambda_H^A)}{\lambda_H^A L^A + (1 - \lambda_H^A) \tilde{R}^A};$$

and that for $\lambda_H^A > 0$ can be written as $q_{1H} \geq Q_H(\lambda_L^A)$ where

$$Q_H(\lambda_L^A) \equiv \frac{\phi_H \left\{ 1 - \phi_L \left[\lambda_L^A L^A + (1 - \lambda_L^A) \tilde{R}^A \right] \right\}}{\phi_H L^A + \left(L^A - \tilde{R}^A \right) \phi_L (1 - \lambda_L^A)}.$$

It is straightforward to show that $Q_H - Q_L < 0$,²⁸ so that $q_{1H} \geq Q_L(\lambda_H^A)$ implies that $q_{1H} > Q_H(\lambda_L^A)$. Therefore $q_{1H} \geq Q_L(\lambda_H^A)$ implies that $\lambda_H^A = 1$. But inspection of the relation $q_{1H} \geq Q_L(1)$ reveals a contradiction to the equilibrium condition

$$1 - \sum_s q_{1s} \left[\lambda_s^A L^A + (1 - \lambda_s^A) \tilde{R}^A \right] > 0.$$

Thus, it cannot be that $\lambda_L^A > 0$ in equilibrium, proving the first part of result 2. Now evaluating $Q_H(0)$, the critical condition for $\lambda_H^A > 0$ can be restated as $q_{1H} \geq \tilde{q}^A$, and the optimality of the rule $\lambda_H^b \in \Lambda^b(\pi)$ follows directly using $\pi \equiv q_{1H} - \phi_H$ and $\tilde{\pi}^A \equiv \tilde{q}^A - \phi_H$. ■

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²⁸To show this, first show that Q_L is decreasing in λ_H^1 and Q_H is increasing in λ_L^1 . Thus $Q_H(\lambda_L^1) - Q_L(\lambda_H^1) \leq Q_H(1) - Q_L(0)$. Then evaluating the RHS, it is easy to see that it must be negative.

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