Exclusivity and Exclusion on Platform Markets*

Subhasish M. Chowdhury
University of East Anglia
s.modak-chowdhury@uea.ac.uk

Stephen Martin
Purdue University
smartin@purdue.edu

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Abstract

We examine conditions under which an exclusive territorial license granted by the upstream producer of a component that some users regard as essential to one of two firms supplying a platform market can render the other supplier unprofitable, excluding it from the market. We show that the impact of such an exclusive license depends on the strength of consumer preferences for the products of the two downstream firms and the relative size of the market segment for which the complementary consumption good is essential. We also identify conditions under which an exclusive license increases the profit of the other platform, and examine the impact of an exclusive license on market performance.

Keywords: exclusion; essential components; exclusive contract; platform market.

JEL codes: L12, L13, L22

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1 Introduction

We model an upstream firm that supplies what for some consumers is an essential complementary good to a duopoly of downstream firms that supply differentiated platforms to the two sides (advertisers and readers) of the final market. We show that the impact of an exclusive license granted by the upstream firm to one of the downstream firms on market performance depends on the strength of consumer preferences for the products of the two downstream firms and on the relative size of the market segment for which the complementary consumption good is essential. We show that for strong reader preferences (which we model as "transportation cost" in a Hotelling framework), and a sufficiently large fraction of the population that regards the complementary good as essential, an exclusive territorial license can deprive the unlicensed firm of sufficient advertising revenue to make it unprofitable and drive it from the market.[1] However, if the share of such agents in total demand is small and readers regard platforms as sufficiently close substitutes, then an exclusive agreement between one platform and the supplier of the complementary good can increase the profit of the other platform.

An episode from the Dallas, Texas newspaper market motivates the phenomenon we model.[2] The newspaper industry is one of the prototypical examples of a platform market, and as such, it is usually modelled as involving three sets of players: newspapers, readers, and advertisers. In this perspective, readers and advertisers are the two groups that interact on newspaper platforms. A newspaper commonly publishes features, articles, comics, puzzles, etc., along with local and national news and advertisements. Newspaper employees prepare some published material; the remainder is purchased from press syndicates. Press syndicates, upstream firms that sell specialized material to newspapers, are a fourth group in the production of newspapers. Some such syndicates specialize in the distribution of comic strips, acting as agents for cartoonists, often under exclusive territorial contracts.

On August 2,1989, the Dallas Morning News ('Morning News') signed an

[1] For an example of an exclusionary strategy based on loss of advertising revenue in a platform market, see Lorain Journal Co. *v.* United States, 342 U.S. 143 (1951).

[2] See the Appeals Court decision in Times Herald Printing Co. *v.* A. H. Belo Corporation *et al.* (Court of Appeals of Texas, Fourteenth District) 820 S.W.2d 206; 1991 Tex. App. LEXIS 2899; 335-66 Trade Cas. (CCH) P69, 680 (1991). Also see Gelsanliter (1995).

exclusive contract for 26 columns and comic strips provided by the Universal Press Syndicate, offerings that until that time had been available through the Dallas Times Herald ('Times Herald'). The two newspapers had competed in the Dallas area for more than a century. The Universal Syndicate acknowledged that the move was 'predatory', but took the view that the cancellations were required by its contract with the Morning News. The Times Herald suffered a circulation loss of 9,000 to 10,000 weekday deliveries and 15,000 Sunday deliveries. It filed an antitrust lawsuit asking for $33 million in actual damages and up to three times of that amount in punitive damages against the Morning News and its parent company.

A state judge in Texas refused to grant the Times Herald a preliminary injunction to prevent the movement of the syndicated features, on the ground that the Times Herald could be supplied with substitute features supplied by competing syndicates. The Times Herald subsequently lost a District Court jury trial and an appeal of the District Court outcome. However, the Morning News paid $1.5 million to the Times Herald as part of an outside settlement. The Times Herald was unable to recapture its lost reader base and advertising revenue. The Morning News' parent corporation purchased the Times Herald on December 8, 1991 and stopped its publication the next day.[3]

The rise of the internet has made print media a declining industry. The general increase in concentration in the newspaper markets of US cities, and the corresponding reasons and consequences are discussed in Bucklin *et al.*

[3] Another example of the exclusionary effect of a contract giving exclusive access to an essential component took place in the U.S. television industry. Project Runway, a reality show based on fashion design, was shown by the Bravo Network from 2004 to 2008. On July 2006 the show's producers made an exclusive deal to move the show to Lifetime Television starting from 2009. Litigation followed, and was privately settled after Bravo Network prevailed in early stages. Bravo Network subsequently launched a competing program ("The Fashion Show"), which enjoyed about one-quarter Project Runway's number of viewers, and correspondingly less advertising revenue. The switch of Project Runway to a rival network has the potential to exclude the Bravo Network from the market. (See Huff, Richard "'Project Runway' quits Bravo for Lifetime," *NYDailyNews.com* 7 April 2008; Lafayette, Jon "NBCU wins round in 'Project Runway," *TVWeek.com*, 26 September 2008; Associated Press, "'Project Runway' is cleared for move to Lifetime from Bravo," 1 April 2009.) Similarly, T-Mobile's failure to obtain the right to sell Apple iPhones was mentioned as a factor in its proposed March 2011 takeover by AT&T (BBC News, "AT&T and T-Mobile create biggest US firm in $39bn deal," 21 March 2011, http://www.bbc.co.uk/news/business-12802111.) Other examples are provided by "killer apps" available only on a single platform; see Viecens (2009).

(1989) and Genesove (2003). Our stylized model is not meant to imply that the Morning News' exclusive arrangement with the United Press Syndicate was the unique factor responsible for the demise of the Times Herald. But the fact that the Times Herald's otherwise unsuccessful legal action resulted in a $1.5 million private settlement is consistent with the view that the exclusive arrangement was *one* factor in the demise of the Times Herald.

In Section 2 we review the parts of the literatures on exclusionary contracts and two-sided markets that are most closely related to the present study. Section 3 contains the setup of the model, describing assumptions about readers, advertisers, and newspapers. In Section 4, we present results for the monopoly case. Section 5 contains the basic duopoly model. Section 6 discusses equilibrium licensing behavior, and Section 7 examines the welfare consequences of an exclusionary exclusive license. Section 8 concludes. Proofs are given in the Appendix.

2 Literature Review

2.1 Platform markets

Rochet and Tirole (2006) define a two-sided market as a special type of market in which two distinct user groups benefit from the capacity to connect on the platform. The platform charges distinct prices to the two user groups. Examples of this type of market include credit cards, newspapers, radio stations, television channels, travel agencies, video games, and personal computer operating systems.[4]

Rochet and Tirole (2003) introduce a general model of platform competition (closely related to the credit card market) and show how prices and end-user surpluses are determined. In a platform duopoly, end users have to decide whether to transact with only one or with both platforms. Since the decision of end users on one side of the market affects the incentives of end users on the other side of the market, end users face a trade-off. We use Rochet and Tirole's results to justify the assumption that if consumers "single home," reading at most one of all available newspapers, then advertisers will

[4] There is a large theoretical literature on two-sided markets, and here we limit our discussion to the parts of this literature that are directly related to our work. See Rochet and Tirole (2002) and Schmalensee (2002) for applications of models of two-sided markets to the credit card industry, and Rochet and Tirole (2006) and Rysman (2009) for surveys.

advertise on both newspapers.[5]

Caillaud and Jullien (2003) analyze the chicken-egg problem — that failure to capture one side of the market necessarily results in losing the other side — in an intermediate service market. They build a model of imperfect competition among intermediaries and analyze efficient allocations and pricing strategies. When users patronize only one of two intermediaries (the case of single homing), the efficient allocation has all users join the same intermediary. If users are allowed to join both intermediaries, all users are willing to join both intermediaries and the optimal pricing strategy of platforms is to charge a transaction fee rather than a registration fee.

The extensive literature that follows Rochet and Tirole (2003) analyzes different aspects of competition in platform markets. We adapt the Armstrong (2006) "competitive bottlenecks" model (akin to the multi-homing model of Caillaud and Jullien, 2003) to analyze one type of exclusionary conduct in a two-sided market. In the competitive bottlenecks model, platforms compete for a group of single-homing users (Armstrong, 2006, p. 679):

> Here, if it wishes to interact with an agent on the single-homing side, the multi-homing side has no choice but to deal with that agent's chosen platform. Thus, platforms have monopoly power over providing access to their single-homing customers for the multi-homing side.

Using a similar structure, Armstrong and Wright (2007) consider a model of two-sided markets where each side of the market has a different level of product differentiation. Asymmetric product differentiation, if it exists, causes competitive bottlenecks in the market.

2.2 Exclusion

The exclusionary mechanism of the "raising rivals' costs" literature is that (Krattenmaker and Salop, 1986, pp. 223–224, footnote omitted) "a firm may gain the ability to raise price by contracting with input suppliers for the

[5] Choi (2010) models a platform market in which platforms face compatibility issues and only a certain proportion of content is exclusive for a platform. He shows that the possibilities of tying under this structure induces consumers to multihome. See Anderson, Foros and Kind (2010), and Doganoglu and Wright (2010) for other discussions of multihoming in platform markets.

suppliers' agreements not to deal with the purchasing firm's competitors on equal terms." In Segal and Whinston's (2000) reformulation of Rasmusen *et al.* (1991),[6] exclusive dealing contracts deter entry if they deny a potential entrant access to enough buyers to cover sunk entry cost.

The exclusionary mechanism in our model is first-cousin to these approaches. The exclusive license in our model is exclusionary if it denies the unlicensed firm revenue sufficient to cover fixed operating cost. Part of this denial of revenue occurs as the unlicensed newspaper's reader base shrinks, and this corresponds directly to the mechanism at work in Segal and Whinston (2000). Part of the denial of revenue occurs as the unlicensed newspaper's sales of advertising messages shrinks with its reader base. The unlicensed firm is starved of the revenue needed to cover fixed cost through changes in demand on both sides of the platform market.

When exclusion occurs, the licensed firm becomes a monopolist of the platform market. The upstream supplier of the essential component is able to bargain for some or, in the limit, all, of the increased profit. As in Hart and Tirole (1990), an exclusive territorial license is an enabling device that permits the upstream firm to exercise greater market power.

2.3 Exclusion in Two-Sided Markets

Church and Gandal (2004) argue that the direct denial of compatibility, and the restriction of the compatibility of complementary products, are exclusionary in the telecommunications industry.[7] Nocke *et al.* (2007) show that exclusion can reduce welfare if platform effects are weak, but that if platform size is large, exclusion can improve welfare. Hagiu and Lee (2011) discuss exclusionary effects of exclusive contracts between distributors and TV channels.[8] Their model has much in common with ours: platforms are downstream firms; upstream firms (content providers) can either single home or multihome; end users (readers, in our model) single home. But there is no essential component in Hagiu and Lee (2011), and the nature of their results hinges on whether or not the content provider controls its own pricing to end users. Direct control of pricing does not arise in newspaper markets, which we use as a prototype.

[6] See also Fumagalli and Motta (2006).

[7] See also the remarks of Rey and Tirole, (2007, p. 2205).

[8] See also Stennek (2007), Weeds (2009).

Hogendorn and Yuen (2009) analyze a situation in which a player on one side of the market provides a component that (2009, p. 295) "provides sufficient utility to consumers to create a large, discrete indirect network effect when it becomes available on a platform. Thus, its contract with the platform will reflect not only its own attractiveness to consumers but the indirect network effect that it generates as well." Our results obtain if there is a component that is a prerequisite for obtaining utility from a platform, without generating utility in and of itself.[9] They also make assumptions that rule out "tipping" of the platform market to a single supplier. Our model yields conditions under which what had been a duopoly market is supplied by a single firm.

Doganoglu and Wright (2010) model agreements by agents on one side of a platform[10] to supply only one platform firm. Our model examines conditions under which an upstream firm will offer an exclusive territorial license to one platform firm. The Doganoglu and Wright model does not involves essential components, either in the sense either Hogendorn and Yuen (2009) or in the sense of the model developed here.

3 Setup

The basic model is a specialized version of that of Armstrong (2006). There are two newspapers, A located at the left end and B located at the right end of a Hotelling line of length 1. Newspapers sell advertising space to advertisers and print copies of newspapers to readers. We normalize the mass of readers and the mass of advertisers to be one. n_R^i denotes the number of readers of newspaper i, and n_a^i denotes the number of firms that advertise in newspaper i.[11]

We model the incentive of a syndicate to offer an exclusive license and the incentive of a newspaper to accept a license, exclusive or not, if offered. The three stages of the game are shown in Figure 1.

[9] See footnote 13. In models of vertically-differentiated products, it is generally the case that higher-quality varieties have higher equilibrium market shares. Considering for simplicity the case of duopoly, if one variety is of drastically lower quality than another, the low-quality variety will have zero equilibrium market share. The central result of this paper is that exclusion can occur *without* such quality-difference effects.

[10] Their basic model is of a one-sided market.

[11] In what follows, unless otherwise noted, references to "newspaper i" should be understood to carry the qualification "for $i = A, B$."

We treat the syndicate's costs as being entirely sunk before it interacts with newspapers.[12] In stage I, the syndicate offers a license to publish the complementary material to either one (without loss of generality, to firm A) or both newspapers. In stage II, if a newspaper is offered a license, it decides among three options: accept the license, reject the license and remain in the market, or reject the license and exit the market. If a newspaper is not offered a license, it decides to remain in the market or to exit. Newspapers that remain in the market set advertising rates and newspaper prices. In the final stage, advertisers place ads and readers select newspapers.

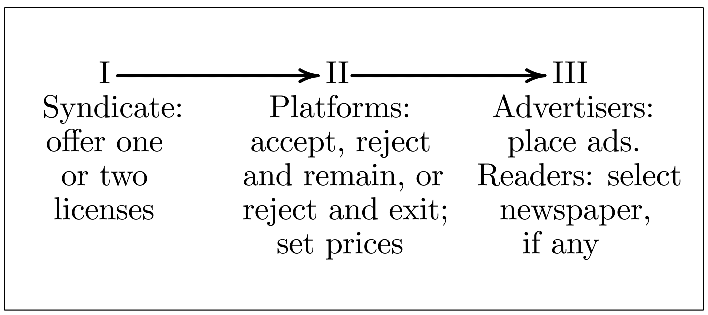

Figure 1: Sequence of decisions.

The terms on which the syndicate offers an exclusive license determine the division of economic profit between the syndicate and the platform that receives the license. Formally, we assume the balance of bargaining power rests with the syndicate. But as discussed below, our welfare results hold whether the balance of bargaining power rests with the syndicate or with the platform.

3.1 Readers

First we derive demand equations for the base case that there is no essential component. These expressions are ingredients for demand equations if there is an essential component.

[12] It would be possible to model the syndicate's arrangements with the authors of the material it markets; this would take us far afield from our topic.

3.1.1 No essential component

The net utility from advertisements of a reader of newspaper i, before allowing for "transportation cost" t is

$$u_R^i = \alpha n_a^i - p^i, \tag{1}$$

where α is marginal utility per advertisement.

We assume readers single-home. For a reader located at x on the Hotelling line, net utilities are

$$u_R^A - tx \tag{2}$$

from newspaper A,

$$u_R^B - t(1-x), \tag{3}$$

from newspaper B.

Boundary readers are at a location that yields the same net utility from either newspaper, $u_R^A - tx^* = u_R^B - t(1-x^*)$, yielding boundary location

$$x^* = \frac{1}{2} + \frac{u_R^A - u_R^B}{2t} = \frac{1}{2} + \frac{\alpha n_a^A - p^A - \left(\alpha n_a^B - p^B\right)}{2t}. \tag{4}$$

Each reader selects the newspaper that offers the greatest net utility, provided that net utility is nonnegative.

The number of readers of each newspaper are

$$n_R^A = \frac{1}{2} + \frac{\alpha\left(n_a^A - n_a^B\right) - p^A + p^B}{2t} \tag{5}$$

$$n_R^B = \frac{1}{2} + \frac{\alpha\left(n_a^B - n_a^A\right) - p^B + p^A}{2t}. \tag{6}$$

3.1.2 Essential component

Dewenter (2003) shows that newspapers, among other media, can form consumer habits that translate into demand for a commodity that becomes an essential component of the media product. Argentesi (2004) shows empirically that weekly supplements (comics, puzzles, etc.) increase readership of (and as a result advertisement in) newspapers. If a newspaper is denied the possibility of supplying habit-forming content, content that a portion of the population regards as essential, the newspaper will see its reader base, and with it advertising revenue, decline. This effect is central to the exclusionary effect of an exclusive territorial license in a platform market.

To model newspaper demand if some portion of the population regards comics as an essential component, we assume that the specification of readers' demand in equations (5) and (6) describes the preferences of a fraction $1-\mu$ of the population, for $0 \leq \mu \leq 1$.[13]

Then quantities demanded of each newspaper from this part of the population are

$$(1-\mu)\left[\frac{1}{2}+\frac{\alpha\left(n_a^A-n_a^B\right)-p^A+p^B}{2t}\right] \tag{7}$$

and

$$(1-\mu)\left[\frac{1}{2}+\frac{\alpha\left(n_a^B-n_a^A\right)-p^B+p^A}{2t}\right] \tag{8}$$

from platforms A and B, respectively.

We assume that the remaining portion μ of the population will read only a newspaper that publishes comics. Otherwise, the utility of this group of consumers is as above. That is, for a consumer who regards comics as an essential component of a newspaper, comics yield no utility in and of themselves, but are a prerequisite for getting utility from a newspaper. This specification minimizes the exclusionary effect of an exclusive license to print comics.[14] A consumer who regards comics as an essential component of a newspaper purchases a newspaper only if it contains comics and if the net utility from reading the newspaper, allowing for transportation cost, is nonnegative.

Suppose newspaper A has an exclusive license to publish comics. The most distant reader from the "comics" group who reads newspaper A is

(a) at the right end of the line if (recall the length of the line is 1)

$$u_R^A-t\,(1)=\alpha n_a^A-p^A-t\geq 0, \tag{9}$$

[13] That is, for readers who regard comics as an essential component, utility is

$$u_R^i=\begin{cases}\alpha n_a^i-p^i-tx & C=1\\ 0 & C=0\end{cases},$$

where $C=1$ if the newspaper has comics, $C=0$ if it does not. This contrasts with the specification of Hogendorn and Yuen, where the number of components enters directly into utility.

[14] See the discussion of Hogendorn and Yuen (2009) in Section 2.3.

or equivalently if p^A is sufficiently low,

$$p^A \leq \alpha n_a^A - t, \tag{10}$$

(b) at distance $x_\mu \leq 1$ that makes net utility zero,

$$u_R^A - tx = \alpha n_a^A - p^A - tx_\mu = 0,$$

$$x_\mu = \frac{\alpha n_a^A - p^A}{t}, \tag{11}$$

if

$$p^A > \alpha n_a^A - t. \tag{12}$$

The number of A readers from the comics group is

$$\begin{array}{ll} \mu & p^A \leq \alpha n_a^A - t \\ \mu x_\mu & p^A \geq \alpha n_a^A - t \end{array}. \tag{13}$$

Quantities demanded of the two newspapers are

$$n_R^A = (1-\mu)\left[\frac{1}{2} + \frac{\alpha\left(\gamma^B - \gamma^A\right) - p^A + p^B}{2t}\right]$$

$$+ \begin{cases} \mu & p^A \leq \alpha\left(1-\gamma^A\right) - t \\ \mu\frac{\alpha\left(1-\gamma^A\right)-p^A}{t} & p^A \geq \alpha\left(1-\gamma^A\right) - t \end{cases}. \tag{14}$$

$$n_R^B = (1-\mu)\left[\frac{1}{2} + \frac{\alpha\left(\gamma^A - \gamma^B\right) - p^B + p^A}{2t}\right]. \tag{15}$$

The number of readers for a firm with an exclusive license differs depending on whether price is low (all consumers who regard comics as essential read the licensed newspaper) or high (consumers who regard comics as essential and who are distant from the licensed newspaper/have a strong preference for the unlicensed newspaper do not read any newspaper).[15,16]

[15] See similarly equation (30), which gives the number of readers of a licensed monopoly newspaper.

[16] For low transportation cost, licensed firms will choose to set low prices, and vice versa. The low price/high price dichotomy therefore translates into a low transportation cost/high transportation cost dichotomy. See footnote 20.

3.2 Advertisers

Let γ^i denote newspaper i's per-reader advertising rate.[17] The cost of placing an ad in newspaper i is

$$\gamma^i n_R^i. \tag{16}$$

Advertisers differ in their profit per sale, β. Following Armstrong (2006), we assume that newspapers do not observe the β of any particular advertiser, but know the distribution of β in the population of advertisers. We assume β is uniformly distributed over $0 \leq \beta \leq 1$.

It will be profitable for an advertiser to place an ad in newspaper i if the profit from placing the ad is greater than or equal to the cost of placing the ad, $\beta n_R^i \geq \gamma^i n_R^i$. The number of ads demanded from newspaper i is therefore

$$n_a^i = 1 - \gamma^i. \tag{17}$$

Substituting (17) in (5) and (6), the number of readers per newspaper become

$$n_R^A = \frac{1}{2t}\left[t + \alpha\left(\gamma^B - \gamma^A\right) - p^A + p^B\right] \tag{18}$$

and

$$n_R^B = \frac{1}{2t}\left[t + \alpha\left(\gamma^A - \gamma^B\right) - p^B + p^A\right], \tag{19}$$

respectively.

Advertisers with $\beta \geq \gamma^A$ make profit $\beta - \gamma^A$ on each of the n_R^A sales they make to readers of platform A. Advertisers' profits on sales to readers of platform A are[18]

$$n_R^A \int_{\beta=\gamma^A}^{\beta=1} \left(\beta - \gamma^A\right) d\beta = \frac{1}{2} n_R^A \left(1 - \gamma^A\right)^2. \tag{20}$$

In the same way, profit on firms' advertisements in platform B are

$$\frac{1}{2} n_R^B \left(1 - \gamma^B\right)^2. \tag{21}$$

Advertisers' total profits are

$$\frac{1}{2} n_R^A \left(1 - \gamma^A\right)^2 + \frac{1}{2} n_R^B \left(1 - \gamma^B\right)^2. \tag{22}$$

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[17] See Rosse (1970) for an estimation of advertising cost in newspaper and Armstrong (2006) for discussion of the case in which the price of placing an advertisement is not proportional to the number of readers.

[18] (20) can more simply be derived as n_R^A times the area of a triangle with base $1 - \gamma^A$, the mass of firms that advertise, and height $1 - \gamma^A$, the profit of firms with the highest β.

3.3 Platforms

Newspapers have a constant marginal cost c to produce a newspaper with n_a advertisements, and fixed cost F.[19] Firm i's payoff function is

$$\pi^i = n^i_R p^i + \gamma^i n^i_R n^i_a - c n^i_R n^i_a - F = n^i_R \left[p^i + \left(\gamma^i - c\right)\left(1 - \gamma^i\right)\right] - F. \quad (23)$$

Let

$$\pi^i_R = p^i + \left(\gamma^i - c\right)\left(1 - \gamma^i\right) \quad (24)$$

denote newspaper i's profit per reader — p^i on the sale of the newspaper to the reader, $\gamma^i - c$ profit per reader per advertisement placed, and $n^i_a = 1 - \gamma^i$ advertisements placed.

Firm i's profit maximization problem is

$$\max_{p^i,\gamma^i} n^i_R \pi^i_R - F. \quad (25)$$

Since β is uniformly distributed on $(0, 1)$, the price per reader of an advertisement cannot be greater than 1. Otherwise no advertisements would be demanded. In principle, in a platform market, the price per reader of an advertisement could be negative. We will assume that prices to advertisers and prices to readers are nonnegative. This gives us

$$0 \leq \gamma^i \leq 1. \quad (26)$$

A second constraint appears in the duopoly version of the model. The usual Hotelling boundary condition ensures that consumers at the boundary location get identical net utility from either newspaper. An additional requirement, if readers at the boundary location are to be served, is that this net utility be nonnegative,

$$\alpha n^i_a - p^i - t x^* \geq 0. \quad (27)$$

Substitute (4) to eliminate x^* and rearrange terms to obtain an expression for the market-coverage constraint,

$$2\alpha - t \geq \alpha\left(\gamma^A + \gamma^B\right) + p^A + p^B, \quad (28)$$

[19] The fixed cost of gathering news to produce the first copy of the paper is typically high, the variable cost to print and sell additional copies of newspaper lower. See Rosse (1970) and Strömberg (2004) for estimation and interpretation of cost structures in the newspaper market.

with choice variables on the right, parameters on the left.

It would be possible to analyze scenarios in which the center of the market is not served in duopoly equilibrium. But we confine our attention to the contrary case.

4 A Monopoly Platform

We examine monopoly equilibrium both to build intuition and because if one firm is excluded from a duopoly market, it is the monopoly payoffs that are divided between the surviving firm and the syndicate.

Suppose there is only one platform, firm A. If firm A is a monopoly supplier, the net utility of a reader located at x is

$$u_R^A = \alpha \left(1 - \gamma^A\right) - p^A - tx. \tag{29}$$

If the monopoly firm has a license for the essential component, its number of readers is

$$n_R^A = \begin{cases} 1 & p^A \leq \alpha \left(1 - \gamma^A\right) - t \\ \frac{\alpha\left(1-\gamma^A\right)-p^A}{t} & p^A \geq \alpha \left(1 - \gamma^A\right) - t \end{cases}. \tag{30}$$

If the monopoly firm does not have a license for the essential component, the expressions for the number of readers in (30) are scaled down by a factor $1 - \mu$.

4.1 Licensed monopoly, low p^A

In the low-price case, firm A's problem is

$$\max_{p^A,\gamma^A} (1) \left[p^A + \left(\gamma^A - c\right) \left(1 - \gamma^A\right)\right] - F \tag{31}$$

such that

$$p^A \leq \alpha \left(1 - \gamma^A\right) - t. \tag{32}$$

As shown in the Appendix, firm A's problem can be analyzed formally using Lagrangian methods. But intuitively, for firm A to maximize profit in the low-price case, the constraint must be binding for the most distant reader,

$$p^{AM} = \alpha \left(1 - \gamma^A\right) - t. \tag{33}$$

It cannot be optimal for firm A to leave the most distant consumer with any surplus.

Given (33), firm A's problem can be reformulated as

$$\max_{\gamma^A} \left(1-\gamma^A\right)\left(\alpha+\gamma^A-c\right)-t-F. \tag{34}$$

The first-order condition to solve (34) is

$$\left(1-\gamma^A\right)-\left(\alpha+\gamma^A-c\right)\equiv 0. \tag{35}$$

A marginal increase in γ^A reduces the number of advertisements sold, $1-\gamma^A$. A marginal increase in γ^A increases profit per advertisement, $\alpha+\gamma^A-c$. Part of the change in profit per advertisement is the decrease in the price readers pay, (33). Part of the increase in profit per advertisement is the increase in profit from sales to advertisers, γ^A-c.

From (35), the monopoly price per reader of an advertisement is

$$\gamma^A=\frac{1}{2}\left[1-(\alpha-c)\right]\equiv\gamma^*. \tag{36}$$

γ^* is the equilibrium price per reader of an advertisement not only for the case of a monopoly platform, but in all the models considered in this paper. This is the "competitive bottleneck" aspect of the basic model: depending on the details (monopoly, duopoly, essential component), a platform's equilibrium number of readers will vary. But it is a monopolist with respect to those readers' access to advertisements, and it charges advertisers the monopoly price.

4.2 High p^A

In the high-price regime, firm A's problem is

$$\max_{p^A,\gamma^A} n_R^A\pi_R^A-F=\frac{\alpha\left(1-\gamma^A\right)-p^A}{t}\left[p^A+\left(\gamma^A-c\right)\left(1-\gamma^A\right)\right]-F, \tag{37}$$

such that $p^A\geq\alpha\left(1-\gamma^A\right)-t$. In the Appendix, we solve the problem without imposing the constraint, then examine conditions for the solution to satisfy the constraint. The consistency condition for the high-price solution to be valid is that transportation cost be sufficiently great,

$$t\geq\frac{1}{2}z^2, \tag{38}$$

	$t \leq \frac{1}{2}z^2$	$\frac{1}{2}z^2 \leq t \leq \frac{2}{3}z^2$
Licensed	$\pi^m_{l1} = z^2 - t - F$	$\pi^m_{l2} = \frac{1}{4t}z^4 - F$
Unlicensed	$\pi^m_{nl1} = (1-\mu)(z^2 - t) - F$	$\pi^m_{nl2} = \frac{1-\mu}{4t}z^4 - F$

Table 1: Monopoly payoffs.

where we write

$$z = 1 - \gamma^* \tag{39}$$

for notational compactness. If this high-transportation-cost constraint is met, the profit-maximizing monopoly price is

$$p^{AM} = \frac{1}{2}z(\alpha + c - \gamma^*). \tag{40}$$

4.3 Monopoly Payoffs

For low levels of transportation cost, $t \leq \frac{1}{2}z^2$, a monopoly supplier sets price so the market is covered, extracting all surplus from the most distant readers. For higher levels of transportation cost, the market is not covered. Row 1 of Table 1 gives the equilibrium payoff of a monopoly newspaper if the firm is licensed (all readers are in the market, although not all readers may be served). Row 2 of Table 1 gives the equilibrium payoffs of an unlicensed monopolist.

5 Newspaper Duopoly

(25) is the generic form of the duopoly maximization problem. The relation between the number of readers and prices differs depending on whether both firms are licensed, one firm is licensed, or neither firm is licensed.

5.1 Both firms licensed

As noted above (see remarks immediately after (36)) and as shown in the Appendix (see (110)), the equilibrium price per reader of an advertisement is $\gamma^* = \frac{1}{2}[1 - (\alpha - c)]$. Equilibrium prices to readers are

$$p^A = p^B = t - z(\gamma^* - c). \tag{41}$$

Licensed	$\pi^d_{ll} = \frac{1}{2}t - F$
Unlicensed	$\pi^d_{nlnl} = \frac{1-\mu}{2}t - F$

Table 2: Duopoly payoffs, symmetric cases (both firms licensed or neither firm licensed).

	$\mu \leq \mu^*$	$\mu \geq \mu^*$
A (licensed)	$\pi^{Ad}_{l1} = \frac{(3+\mu)^2}{1-\mu}\frac{t}{18} - F$	$\pi^{Ad}_{l2} = \left[1 - (1-\mu)\frac{z^2}{4t}\right](z^2 - t) - F$
B (unlicensed)	$\pi^{Bd}_{nl1} = \frac{(3-\mu)^2}{1-\mu}\frac{t}{18} - F$	$\pi^{Bd}_{nl2} = \frac{1-\mu}{8t}z^4 - F$

Table 3: Essential component model payoffs, low transportation cost, firm A licensed, firm B unlicensed.

The market is covered at these prices, as we assume, for

$$t \leq \frac{2}{3}z^2. \tag{42}$$

The corresponding payoff per firm is given in the first row of Table 2.

5.2 Neither firm licensed

If neither firm is licensed, demands are scaled down by the factor $1-\mu$. The resulting payoff per firm is given in the second row of Table 2. From (24), the reduction in profit of an unlicensed firm includes lost advertising revenue, a kind of loss unique to a firm that supplies a platform market.

5.3 Firm A licensed, firm B unlicensed

The analysis of the asymmetric case — one firm licensed, one unlicensed — involves a tedious number of cases, and is relegated to Appendix Section 10.1.5. Payoffs for the low-t and high-t cases are given in Tables 3 and 4, respectively.[20] Payoffs for the case that firm B is licensed and firm A unlicensed are symmetric with the payoffs shown in the tables.

In the low-t case, firm A's payoff rises, and firm B's payoff falls, approaching a positive limit, as μ rises. In the high-t case, firm B's payoff goes to zero as μ rises.

[20] In this context, "low transportation cost" means $t \leq \frac{3}{2}\frac{1-\mu}{3-\mu}z^2$. See inequality (159). μ^* is the value of μ at which $t = \frac{3}{2}\frac{1-\mu}{3-\mu}z^2$; see equation (161).

A (licensed)	$\pi_{l3}^{Ad}=\frac{1+\mu}{2t}\left[\frac{3(1-\mu)t+4\mu z^2}{3+5\mu}\right]^2-F$
B (unlicensed)	$\pi_{nl3}^{Bd}=\frac{1-\mu}{2t}\left[\frac{(3+\mu)t+2\mu z^2}{3+5\mu}\right]^2-F$

Table 4: Essential component model payoffs, high transportation cost, firm A licensed, firm B unlicensed.

6 Exclusion

A short argument (Section 6.1) shows that an exclusive territorial license is *not* exclusionary for the low-t, low-μ case. For the low-t, high-μ and high-t cases, we examine equilibrium payoffs in two cases, first that the syndicate offers a license to one firm (without loss of generality, firm A), and second that the syndicate offers a license to both firms.

6.1 Low t, low μ

Subtraction shows that the payoff of an unlicensed duopolist that competes with a licensed rival is greater, for the low-t, low-μ case, than duopoly profit if both firms are licensed,

$$\frac{(3-\mu)^2}{1-\mu}\frac{t}{18}-F-\left(\frac{1}{2}t-F\right)=\frac{t}{18}\frac{\mu\,(3+\mu)}{1-\mu}>0. \tag{43}$$

Thus for low t, low μ, the unlicensed duopolist's profit satisfies

$$\pi_{nl1}^{Bd}=\frac{(3-\mu)^2}{1-\mu}\frac{t}{18}-F>\frac{1}{2}t-F=\pi_{ll}^{d}. \tag{44}$$

We assume that duopoly is profitable if both firms have licenses, $\pi_{ll}^{d}>0$. This implies $\pi_{nl1}^{Bd}>0$. Then if firm A operates with an exclusive license, firm B will operate, profitably, without a license. Intuitively, firm A raises price somewhat if there are readers who will only read a newspaper with comics. If t is low — readers have weak preferences for one newspaper or the other — some readers who are indifferent toward comics but are unwilling to pay a higher price for newspaper A switch to newspaper B. If μ is sufficiently small, the increase in firm B's market size as readers who are unwilling to pay a higher price switch from newspaper A outweighs the reduction in its market size as consumers who will read only a newspaper with comics switch to firm A. An exclusive territorial license is not exclusionary if consumers regard the two newspapers as close substitutes (low t) and few consumers regard comics as essential (low μ).

6.2 Low t, high μ ; and high t

Theorem 1 *In the low-t, high-μ ; and high-t cases, for μ sufficiently close to 1, and in the high-t case for $F \geq \frac{7}{24}z^2$, it is a subgame perfect equilibrium for the syndicate to offer an exclusive license to firm A for a license fee slightly greater than $2\pi_{ll}^d$, for firm A to accept the offer, and for firm B to exit the market.*

6.2.1 Payoffs

Here we present the argument leading to Theorem 1 for the low-t, high-μ case. Minor changes in the first part of the argument, which are given in the Appendix, lead to the same result for the high-t case.

The inequalities

$$\max\left(\pi_{nlnl}^d, \pi_{nl1}^m, \pi_{nl2}^{Bd}\right) < 0 \leq \min\left(\pi_{ll}^d, \pi_{l1}^m, \pi_{l2}^{Ad}\right) \tag{45}$$

correspond to

$$\max\left[\frac{1-\mu}{2}t, (1-\mu)\left(z^2-t\right), \frac{1-\mu}{8t}z^4\right] < F \leq$$

$$\min\left\{\frac{1}{2}t, z^2-t, \left[1-(1-\mu)\frac{z^2}{4t}\right]\left(z^2-t\right)\right\}. \tag{46}$$

As $\mu \to 1$, (46) approaches

$$\max(0,0,0) = 0 < F \leq \min\left(\frac{1}{2}t, z^2-t, z^2-t\right). \tag{47}$$

Considering the expression on the right,

$$z^2 - t - \frac{1}{2}t = \frac{3}{2}\left(\frac{2}{3}z^2 - t\right) > 0. \tag{48}$$

Hence as $\mu \to 1$ (47) reduces to

$$0 < F \leq \frac{1}{2}t, \tag{49}$$

and the assumption that licensed duopoly is profitable[21] guarantees that (49) is satisfied. Assume μ is large enough so (45) holds. Then it is profitable to be a licensed monopolist ($\pi^m_{l1} > 0$) or duopolist ($\pi^d_{ll} > 0$, $\pi^{Ad}_{l2} > 0$), unprofitable to be an unlicensed monopolist ($\pi^m_{nl1} < 0$) or duopolist ($\pi^d_{nlnl} < 0$, $\pi^{Bd}_{nl2} < 0$).

6.2.2 Exclusive license

Let the syndicate offer A an exclusive contract for a license fee that leaves A a positive payoff. A's options are to reject the contract and exit the market (breaking even), refuse the contract and remain in the market, or accept the contract. If A rejects the contract and continues in the market without a license, B's options are to exit or to continue in the market. If B exits, firm A is an unlicensed monopolist, earning $\pi^m_{nl1} < 0$. If B continues in the market, both firms earn $\pi^d_{nlnl} < 0$. If A accepts the contract, B's options are to exit (breaking even) or to compete as an unlicensed duopolist (earning $\pi^{Bd}_{nl2} < 0$); firm B's payoff-maximizing choice is to exit. If firm B exits, economic profit from the operation of newspaper A is $\pi^m_{l1} > 0$. As the license fee (discussed further below) leaves A with a positive payoff, accepting the offer of a license dominates A's alternative choices. If the syndicate offers firm A an exclusive license, the equilibrium outcome is that A accepts the offer, B exits, newspaper A generates monopoly profit π^m_{l1}, and the license fee determines the division of π^m_{l1} between A and the syndicate.

6.2.3 Dual licenses

We expect that in a market with two suppliers, each would learn the terms of the license offered to the other.[22] Let the syndicate simultaneously and publicly offer licenses to A and B for a license fee that leaves each firm at least a small positive payoff if both firms accept the offer of a license. If A rejects the license and exits, B earns a positive profit (approximately $\pi^m_{l1} - \pi^d_{ll}$) if it accepts the license, which dominates the losses it would make as an unlicensed monopolist or breaking even if it exits the market. If A rejects the license and continues in the market, B makes a positive profit (given the symmetry of payoffs, approximately $\pi^{Ad}_{l2} - \pi^d_{ll}$) if it accepts the license, which

[21] If licensed monopoly is profitable, $\pi^m_{l1} \geq 0$, and licensed duopoly not profitable, $\pi^d_{ll} < 0$, there is one newspaper in equilibrium. But exclusion is not a factor.

[22] Hart and Tirole (1990) examine the different implications of public as opposed to private vertical contracts.

dominates the losses it would make as an unlicensed duopolist or breaking even if it exits the market. If A accepts the license, and B accepts the license as well, B makes a small positive profit, which dominates the losses it would make ($\pi_{nl2}^{Bd} < 0$) competing without a license against a licensed firm A or breaking even if it exits. No matter how firm A responds to the offer of a license, firm B maximizes its payoff by accepting the offer of a license. Firm A's incentives are the same. If the syndicate offers both firms licenses on terms that leave them small positive payoffs if both accept, the equilibrium outcome is for both firms to accept the offer.

6.2.4 Syndicate's payoff and overall outcome

The economic profit generated by newspaper A as a licensed monopolist is $\pi_{l1}^{m} = z^2 - t - F$. The economic profit generated by either newspaper if both firms have licenses is $\pi_{ll}^{d} = \frac{1}{2}t - F$. Monopoly profit exceeds total duopoly profit,

$$\pi_{l1}^{m} - 2\pi_{ll}^{d} = z^2 - t - F - 2\left(\frac{1}{2}t - F\right) = 2\left(\frac{1}{2}z^2 - t\right) + F > 0. \qquad (50)$$

(recall that $t \leq \frac{1}{2}z^2$ in the low-t case).

If the syndicate makes public offers of licenses to both newspapers, asking a license fee slightly less than π_{ll}^{d}, the best alternative for either newspaper is to accept the offer of a license. Neglecting the small reductions in the license fees, the syndicate's payoff would be $2\pi_{ll}^{d}$. Then if the syndicate offers an exclusive license to (say) firm A, firm A could offer to pay the syndicate a license fee slightly greater than $2\pi_{ll}^{d}$, leaving the syndicate strictly better off than if it were to license both firms. Firm A's payoff, slightly less than $\pi_{l1}^{Am} - 2\pi_{ll}^{d} > 0$, would dominate its near-zero payoff as one of two licensed duopolists.[23,24]

[23] The mechanism at work here is essentially the same as that underlying "pay for delay" settlements between patented and generic drug manufacturers in the pharmaceutical industry (on which, see European Commission (2009), Federal Trade Commission (2010)).

[24] Although we have stated Theorem 1 for the case that the balance of bargaining power rests with the syndicate, we do not need to model the bargaining process that determines the division of economic profit between the syndicate and its exclusive licensee to make the welfare comparison that appears in the following section. See Hagiu and Lee (2011) for a model of the division of surplus between duopoly platforms and a continuum of upstream content providers.

	Monopoly		Duopoly
	low-t, high-μ	high-t	
Newspapers' profit	$z^2 - t - F$	$\frac{1}{4t}z^4 - F$	$2\left(\frac{1}{2}t - F\right)$
CS	$\frac{1}{2}t$	$\frac{1}{8t}z^4$	$z^2 - \frac{5}{4}t$
Advertisers' profit	$\frac{1}{2}z^2$	$\frac{1}{4t}z^4$	$\frac{1}{2}z^2$
NSW	$\frac{3}{2}z^2 - \frac{1}{2}t - F$	$\frac{5}{8t}z^4 - F$	$\frac{3}{2}z^2 - \frac{1}{4}t - 2F$

Table 5: Consumer Surplus and Net Social Welfare.

7 Welfare Consequences

We show in the Appendix that profit, consumer surplus, and net social welfare in the various regimes are as reported in Table 5. The "newspapers' profit" given in the first row of the table is the total profit generated by the operation of active newspapers. The license fee determines the division of this surplus between newspaper and syndicate, but does not affect the amount of the surplus.

7.1 Comparison: duopoly and low-t, high-μ monopoly

Comparing duopoly and low-t, high-μ monopoly shows that monopoly profit is greater than total duopoly profit, and duopoly consumer surplus is greater than monopoly consumer surplus, in the low-t, high-μ case:

$$\pi_{l1}^m - 2\pi_{ll}^d = 2\left(\frac{1}{2}z^2 - t\right) + F > 0. \tag{51}$$

$$CS^d - CS_{ltl\mu}^m = \frac{7}{4}\left(\frac{4}{7}z^2 - t\right) > 0. \tag{52}$$

Advertisers' profit is the same under both regimes, since the market is covered in both cases.

Duopoly net social welfare may be greater or less than monopoly net social welfare.

$$NSW^d - NSW_{ltl\mu}^m = \frac{1}{4}t - F. \tag{53}$$

We have assumed that licensed duopoly is profitable for both firms, $\frac{1}{2}t - F \geq 0$. (53) is thus of ambiguous sign. If reader preferences are strong (large t) and fixed cost low, duopoly net social welfare exceeds monopoly net social welfare. If reader preferences are weak and fixed cost high, monopoly net social welfare (which economizes on fixed cost, relative to duopoly) exceeds duopoly net social welfare.

7.2 Comparison: duopoly and high-t monopoly

Monopoly profit exceeds the profit of one duopolist. For duopoly and high-t monopoly, we have

$$\pi_{l2}^{m} - 2\pi_{ll}^{d} = \frac{1}{t}\left(\frac{1}{2}z^2 - t\right)\left(\frac{1}{2}z^2 + t\right) + F. \tag{54}$$

In the high-t case $\frac{1}{2}z^2 \leq t \leq \frac{2}{3}z^2$, for which values of t the first term on the right is nonpositive. $\pi_{l2}^{m} - 2\pi_{ll}^{d} = F > 0$ for $t = \frac{1}{2}z^2$. As t rises from $\frac{1}{2}z^2$ to $\frac{2}{3}z^2$, the first term on the right falls from 0 to $-\frac{7}{24}z^2$.[25] If $F \geq \frac{7}{24}z^2$, $\pi_{l2}^{m} \geq 2\pi_{ll}^{d}$ for all values of t admissible in the high-t case. For F in the range $0 \leq F < \frac{7}{24}z^2$, $\pi_{l2}^{m} - 2\pi_{ll}^{d}$ is positive, zero, or negative as t is less than, equal to, or greater than $\frac{1}{2}F + \frac{1}{2}\sqrt{F^2 + z^4}$.

Consumer surplus,

$$CS^d - CSh_{ht}^{m} = \frac{5}{4}\left[\left(\frac{2}{3}z^2 - t\right) + \frac{2}{15t}z^2\left(t - \frac{1}{10}z^2\right)\right] > 0, \tag{56}$$

and advertisers' profit,

$$\pi_{Ad}^{m} - 2\pi_{Ad}^{d} = \frac{1}{2t}z^2\left(t - \frac{1}{2}z^2\right) > 0, \tag{57}$$

are both greater under duopoly than under high-t monopoly.

The difference in net social welfare,

$$NSW^d - NSW_{ht}^{m} = \frac{1}{4}\left(\frac{2}{3}z^2 - t\right) + \frac{4}{3t}z^2\left(t - \frac{15}{32}z^2\right) - F, \tag{58}$$

is of ambiguous sign (the first two expressions on the right are positive). It is sufficient for duopoly net social welfare to exceed monopoly net social

[25] That is,

$$\frac{\partial}{\partial t}\left(\pi_{l2}^{m} - 2\pi_{ll}^{d}\right) = -\left[1 + \frac{1}{4}\left(\frac{z^2}{t}\right)^2\right] < 0. \tag{55}$$

welfare that monopoly newspaper profit be less than duopoly newspaper profit. Generally, the right-hand side of (58) is more likely to be positive the smaller is fixed cost and the stronger[26] are reader preferences.

8 Conclusion

The literature on one-sided markets suggests (for example, Whinston (1990)) that tying, bundling, and exclusive dealing contracts may, but need not, have exclusionary effect. Our results extend this finding to exclusive territorial licenses in two-sided markets, in which the exclusionary impact of a loss of patronage from one side of the market (readers) is magnified by the resulting loss in revenue (advertising) from the other side of the market.

Many regional markets — regional in physical space, regional in product characteristic space — will support at most a small number of firms. In such markets, an exclusive territorial contract for a complementary product can make unlicensed firms unprofitable, inducing exit, reducing consumer surplus and, in some cases (strong reader preferences, low fixed cost) reducing net social welfare.

Our results hold for the case of a monopoly upstream supplier of an essential component. A logical extension of this framework, and subject for possible future research, is to an upstream duopoly of vertically-differentiated components. It is natural to expect that exclusive territorial licenses will be exclusionary if upstream components differ sharply in quality, otherwise not. One might also view press syndicates as platforms that allow advice columnists, astrologers, and comic strip artists to interact with newspapers. Also a subject for future research, this would lead to a model of an upstream platform market supplying a downstream platform market.

9 References

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[26] $\frac{\partial}{\partial t}\left(NSW^d - NSW^m_{ht}\right) = -\frac{5}{2t^2}\left(\frac{1}{\sqrt{10}}t - \frac{1}{2}z^2\right)\left(\frac{1}{\sqrt{10}}t + \frac{1}{2}z^2\right) > 0$, since in the high-$t$ case $\frac{1}{\sqrt{10}}t - \frac{1}{2}z^2 \leq \left(\frac{2}{3}\frac{1}{\sqrt{10}} - \frac{1}{2}\right)z^2 \approx -0.28918z^2 < 0$.

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10 Appendix

In Section 10.1 we derive payoffs under the various market regimes considered in the paper. In Section 10.2 we derive expressions for consumer surplus and net social welfare for the licensed-monopoly and licensed-duopoly regimes. In Section 10.3 we give steps in the proof of Theorem 1 for the high-t case.

10.1 Payoffs

10.1.1 Licensed Monopoly

Suppose there is only one platform, firm A. If firm A is a monopoly supplier, its objective function is

$$n_R^A \pi_R^A - F. \tag{59}$$

Profit per reader is

$$\pi_R^A = p^A + \left(\gamma^A - c\right)\left(1 - \gamma^A\right). \tag{60}$$

The number of advertisements is

$$n_a^A = 1 - \gamma^A. \tag{61}$$

Net utility of a reader located at x is

$$u_R^A = \alpha\left(1 - \gamma^A\right) - p^A - tx. \tag{62}$$

If firm A has a license, the number of readers is

$$n_R^A = 1 \tag{63}$$

if

$$\alpha \left(1-\gamma^{A}\right)-p^{A}-t\geq 0 \tag{64}$$

or equivalently

$$p^{A}\leq \alpha \left(1-\gamma^{A}\right)-t \tag{65}$$

and

$$x=\frac{\alpha \left(1-\gamma^{A}\right)-p^{A}}{t} \tag{66}$$

if

$$p^{A}\geq \alpha \left(1-\gamma^{A}\right)-t. \tag{67}$$

This gives firm A's licensed monopoly number of readers, (30),

$$n_{R}^{A}=\begin{cases} 1 & p^{A}\leq \alpha \left(1-\gamma^{A}\right)-t \\ \frac{\alpha\left(1-\gamma^{A}\right)-p^{A}}{t} & p^{A}\geq \alpha \left(1-\gamma^{A}\right)-t \end{cases}. \tag{68}$$

Consider the low-price and high-price regimes in turn.

$p^{A}\leq \alpha \left(1-\gamma^{A}\right)-t$ If $p^{A}\leq \alpha \left(1-\gamma^{A}\right)-t$, firm A's problem is

$$\max_{p^{A},\gamma^{A}}(1)\left[p^{A}+\left(\gamma^{A}-c\right)\left(1-\gamma^{A}\right)\right]-F \text{ s.t. } p^{A}\leq \alpha \left(1-\gamma^{A}\right)-t. \tag{69}$$

Set up (69) as a constrained optimization problem. A Lagrangian is

$$\mathcal{L}=p^{A}+\left(\gamma^{A}-c\right)\left(1-\gamma^{A}\right)-F+\lambda\left[\alpha\left(1-\gamma^{A}\right)-t-p^{A}\right]. \tag{70}$$

Kuhn-Tucker first-order conditions are
p^{A}:

$$\frac{\partial \mathcal{L}}{\partial p^{A}}=1-\lambda=0. \tag{71}$$

γ^{A}:

$$\left(\gamma^{A}-c\right)(-1)+(1)\left(1-\gamma^{A}\right)-\lambda\alpha=0 \tag{72}$$

Substituting $\lambda=1$ and rearranging terms gives

$$\gamma^{A}=\frac{1}{2}\left[1-(\alpha-c)\right]=\gamma^{*}. \tag{73}$$

λ:

$$\frac{\partial \mathcal{L}}{\partial \lambda}=\alpha\left(1-\gamma^{A}\right)-t-p^{A}\geq 0 \tag{74}$$

$$\lambda\left[\alpha\left(1-\gamma^{A}\right)-t-p^{A}\right]=0 \tag{75}$$

$$\lambda \geq 0. \tag{76}$$

Then $\lambda = 1$ implies that the constraint is binding, (writing $z = 1 - \gamma^*$, (39))

$$p^{A}=\alpha z-t. \tag{77}$$

This is (33). If it maximizes profit subject to the constraint that the market be covered, firm A sets a price that takes all surplus from the most distant readers.

Firm A's monopoly payoff in the low-price regime is

$$p^{A}+\left(\gamma^{A}-c\right)\left(1-\gamma^{A}\right)-F=\alpha z-t+z\left(\gamma^{*}-c\right)-F$$

(and using $\alpha + \gamma^* - c = 1 - \gamma^* = z$)

$$=z\left(\alpha+\gamma^{*}-c\right)-t-F=z^{2}-t-F. \tag{78}$$

$p^A \geq \alpha\left(1-\gamma^A\right) - t$ If $p^A \geq \alpha\left(1-\gamma^A\right) - t$, firm A's problem is

$$\max_{p^{A},\gamma^{A}} n_{R}^{A}\pi_{R}^{A}-F \text{ s.t. } p^{A}\geq\alpha\left(1-\gamma^{A}\right)-t. \tag{79}$$

We first work out the solution without imposing the constraint, then determine a condition under which the unconstrained solution satisfies the constraint.

First-order conditions for the unconstrained problem are

$$n_{R}^{A}\frac{\partial\pi_{R}^{A}}{\partial p^{A}}+\pi_{R}^{A}\frac{\partial n_{R}^{A}}{\partial p^{A}}=0 \tag{80}$$

and

$$n_{R}^{A}\frac{\partial\pi_{R}^{A}}{\partial\gamma^{A}}+\pi_{R}^{A}\frac{\partial n_{R}^{A}}{\partial\gamma^{A}}=0 \tag{81}$$

with

$$\pi_{R}^{A}=p+\left(\gamma^{A}-c\right)\left(1-\gamma^{A}\right) \tag{82}$$

(so that $\frac{\partial\pi_{R}^{A}}{\partial p}=1$, $\frac{\partial\pi_{R}^{A}}{\partial\gamma}=1-2\gamma+c$) and

$$n_{R}^{A}=\frac{\alpha\left(1-\gamma^{A}\right)-p^{A}}{t}, \tag{83}$$

(so that $\frac{\partial n_R^A}{\partial p} = -\frac{1}{t}$, $\frac{\partial n_R^A}{\partial \gamma} = -\frac{\alpha}{t}$.)

Substituting, the monopoly first-order conditions are

$$n_R^A - \frac{1}{t}\pi_R^A = 0 \tag{84}$$

$$n_R^A (1 - 2\gamma + c) - \frac{\alpha}{t}\pi_R^A = 0. \tag{85}$$

Substitute $\pi_R^A = tn_R^A$ from (84) into (85) to obtain

$$n_R^A (1 - 2\gamma + c - \alpha) = 0, \tag{86}$$

from which

$$\gamma^A = \frac{1}{2}[1 - (\alpha - c)] = \gamma^*. \tag{87}$$

Substituting $\gamma^A = \gamma^*$ into (84) gives

$$\frac{\alpha z - p}{t} - \frac{1}{t}[p + z(\gamma^* - c)] = 0,$$

which yields (omitting several steps)

$$p^{AM} = \frac{1}{2}z(\alpha + c - \gamma^*). \tag{88}$$

This is (40).

The consistency condition for (88) to be a valid solution is that "transportation cost" be sufficiently great:

$$p^{AM} \geq \alpha z - t,$$

which leads to

$$t \geq \frac{1}{2}z^2. \tag{89}$$

Now using (83), firm A's equilibrium number of readers is

$$n_R^A = \frac{\alpha z - p^A}{t} = \frac{z^2}{2t}. \tag{90}$$

Firm A's equilibrium monopoly payoff in the high-t case is

$$n_R^A \pi_R^A - F =$$

(substituting $\pi_R^A = tn_R^A$)

$$t\left(n_R^A\right)^2 - F =$$

(substituting (90))

$$\frac{1}{4t}z^4 - F. \tag{91}$$

The high-price solution is valid for $t \geq \frac{1}{2}z^2$. For $t \leq \frac{1}{2}z^2$, it is the low-price solution that is valid.

10.1.2 Unlicensed monopoly

We need an expression for firm A's payoff as an unlicensed monopolist serving a market with $1-\mu$ readers. The only change from the previous case is that the number of readers is reduced by the scale factor $1-\mu$. Payoffs are

$$\begin{array}{ll} (1-\mu)\left(z^2-t\right)-F & \text{if } \; p^A \leq \alpha\left(1-\gamma^A\right)-t \\ \frac{1-\mu}{4t}z^4 - F & \text{if } \; p^A \geq \alpha\left(1-\gamma^A\right)-t \end{array}. \tag{92}$$

10.1.3 Duopoly, both firms licensed

The first-order conditions for firm A's profit maximization problem, (25) with $i = A$, are

$$\frac{\partial \pi^A}{\partial p^A} = n_R^A \frac{\partial \pi_R^A}{\partial p^A} + \pi_R^A \frac{\partial n_R^A}{\partial p^A} = 0 \tag{93}$$

and

$$\frac{\partial \pi^A}{\partial \gamma^A} = n_R^A \frac{\partial \pi_R^A}{\partial \gamma^A} + \pi_R^A \frac{\partial n_R^A}{\partial \gamma^A} = 0, \tag{94}$$

where from (24)

$$\frac{\partial \pi_R^A}{\partial p^A} = 1 \tag{95}$$

$$\frac{\partial \pi_R^A}{\partial \gamma^A} = \left(\gamma^A - c\right)(-1) + 1 - \gamma^A = 1 + c - 2\gamma^A. \tag{96}$$

and from (18)

$$\frac{\partial n_R^A}{\partial p^A} = -\frac{1}{2t} \tag{97}$$

$$\frac{\partial n_R^A}{\partial \gamma^A} = -\frac{\alpha}{2t}. \tag{98}$$

Substituting (95), (96), (97), and (98) into (93) and (94) gives the first-order conditions

$$\frac{\partial \pi^A}{\partial p^A} = n_R^A - \frac{1}{2t}\pi_R^A \equiv 0 \tag{99}$$

and

$$\frac{\partial \pi^A}{\partial \gamma^A} = n_R^A \left(1 + c - 2\gamma^A\right) - \frac{\alpha}{2t}\pi_R^A \equiv 0. \tag{100}$$

If (99) holds, which it will in equilibrium,

$$\pi_R^A = 2tn_R^A. \tag{101}$$

It follows that in equilibrium, firm A's payoff is

$$\pi^A = n_R^A \pi_R^A = 2t\left(n_R^A\right)^2 - F. \tag{102}$$

Substitute (101) into (99) to eliminate n_R^A, obtaining

$$\frac{\partial \pi^A}{\partial \gamma^A} = \frac{\pi_R^A}{2t}\left(1 + c - 2\gamma^A - \alpha\right) = 0. \tag{103}$$

For a positive equilibrium profit per reader, $\pi_R^A > 0$, (103) gives the equilibrium value of firm A's price-per-reader per advertisement:

$$\gamma^A = \frac{1}{2}\left[1 - (\alpha - c)\right] = \gamma^*. \tag{104}$$

We assume that marginal utility per ad in a newspaper exceeds marginal cost per ad in a newspaper,

$$\alpha - c > 0. \tag{105}$$

The per-reader advertising rate, γ^A, cannot exceed advertisers' profit per reader, the maximum value of which is 1. This gives

$$0 \le \gamma^A \le 1,$$

which implies

$$0 \le 1 - (\alpha - c) \le 2 \tag{106}$$

as a pair of inequalities that must be satisfied by $\alpha - c$.

(105) and (106) give

$$1 \ge \alpha - c \ge 0. \tag{107}$$

In the same way, we obtain for firm B the first-order conditions

$$\frac{\partial \pi^B}{\partial p^B} = n_R^B - \frac{1}{2t}\pi_R^B \equiv 0 \tag{108}$$

and

$$\frac{\partial \pi^B}{\partial \gamma^B} = n_R^B \left(1 + c - 2\gamma^B\right) - \frac{\alpha}{2t}\pi_R^B \equiv 0, \tag{109}$$

and the equilibrium price per reader of an advertisement,

$$\gamma^B = \frac{1}{2}\left[1 - (\alpha - c)\right] = \gamma^*. \tag{110}$$

From (17), the equilibrium number of advertisements (the same for both newspapers) is

$$n_a^A = n_a^B = 1 - \gamma^* = \frac{1}{2}\left(1 + \alpha - c\right). \tag{111}$$

The first-order conditions for p^A and p^B are (99) and (108), respectively. Substituting the equilibrium values of γ^i into (18) and (19) gives expressions for the numbers of readers per newspaper as functions of prices per reader, when $\gamma^A = \gamma^B = \gamma^*$:

$$n_R^A = \frac{1}{2t}\left(t - p^A + p^B\right), \tag{112}$$

$$n_R^B = \frac{1}{2t}\left(t - p^B + p^A\right). \tag{113}$$

Profit-per-reader of newspapers A and B are

$$\pi_R^A = p^A + z\left(\gamma^* - c\right) \tag{114}$$

and

$$\pi_R^B = p^B + z\left(\gamma^* - c\right), \tag{115}$$

respectively.

Using (114) and (115), the first-order conditions (99) and (108) become

$$2p^A - p^B = t - z\left(\gamma^* - c\right) \tag{116}$$

for p^A and

$$-p^A + 2p_R^B = t - z\left(\gamma^* - c\right) \tag{117}$$

for p^B.

The system of first-order equations, which we write in this form to permit comparison with (151) and (207), is

$$\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} p^A \\ p^B \end{pmatrix} = [t - z(\gamma^* - c)] \begin{pmatrix} 1 \\ 1 \end{pmatrix}. \tag{118}$$

Equilibrium prices are

$$p^A = p^B = t - z(\gamma^* - c). \tag{119}$$

This is (41). From (28), for the market to be covered for these prices requires that t not be too great,

$$2\alpha - t \geq \alpha\left(\gamma^A + \gamma^B\right) + p^A + p^B,$$

$$t \leq \frac{2}{3} z(\gamma^* + \alpha - c), \tag{120}$$

or, using $\gamma^* + \alpha - c = 1 - \gamma^* = z$,

$$t \leq \frac{2}{3} z^2. \tag{121}$$

This is (42).

From (102), in equilibrium

$$\pi^A = 2t\left(n_R^A\right)^2 - F.$$

But if the market is covered in symmetric equilibrium, $n_R^A = \frac{1}{2}$ (see also (112)). Hence

$$\pi^A = \pi^B = \frac{1}{2} t - F. \tag{122}$$

10.1.4 Duopoly, A & B unlicensed

The only change from the previous case is that the number of readers is scaled down by the factor $1 - \mu$. Equilibrium payoffs per firm are

$$\pi^A = \pi^B = \frac{1-\mu}{2} t - F. \tag{123}$$

10.1.5 Duopoly, A licensed, B unlicensed

If $p^A \leq \alpha n_a^A - t$, objective functions are

$$\pi^A = n_R^A \pi_R^A - F \tag{124}$$

and

$$\pi^B = n_R^B \pi_R^B - F. \tag{125}$$

$p^A \leq \alpha n_a^A - t$ First analyze the outcome on the assumption that equilibrium values place demand in the low-p^A case. Analyze firm A's profit-maximization problem without imposing

$$p^A \leq \alpha n_a^A - t \tag{126}$$

as a constraint. Find equilibrium prices, and find conditions for (126) to be satisfied.

The number of readers of each firm are

$$n_R^A = (1-\mu)\left[\frac{1}{2} + \frac{\alpha\left(\gamma^B - \gamma^A\right) - p^A + p^B}{2t}\right] + \mu \tag{127}$$

$$= \frac{1}{2}\left[(1+\mu) + (1-\mu)\frac{\alpha\left(\gamma^B - \gamma^A\right) - p^A + p^B}{t}\right]. \tag{128}$$

$$n_R^B = \frac{1}{2}(1-\mu)\left[1 + \frac{\alpha\left(\gamma^A - \gamma^B\right) - p^B + p^A}{t}\right]. \tag{129}$$

The following comparative static derivatives will be used later. For the numbers of readers,

$$\frac{\partial n_R^A}{\partial p^A} = \frac{\partial n_R^B}{\partial p^B} = -\frac{1-\mu}{2t} \tag{130}$$

$$\frac{\partial n_R^A}{\partial \gamma^A} = \frac{\partial n_R^B}{\partial \gamma^B} = -\alpha\frac{1-\mu}{2t} \tag{131}$$

For profitability per reader,

$$\pi_R^A = p^A + \left(\gamma^A - c\right)\left(1 - \gamma^A\right)$$

$$\pi_R^B = p^B + \left(\gamma^B - c\right)\left(1 - \gamma^B\right)$$

$$\frac{\partial \pi_R^A}{\partial p^A} = \frac{\partial \pi_R^A}{\partial p^B} = 1 \tag{132}$$

$$\frac{\partial \pi_R^A}{\partial \gamma^A} = 1 + c - 2\gamma^A \tag{133}$$

$$\frac{\partial \pi_R^B}{\partial \gamma^B} = 1 + c - 2\gamma^B. \tag{134}$$

Firm A Firm A's first-order conditions are
p^A:

$$\frac{\partial \pi^A}{\partial p^A} = n_R^A \frac{\partial \pi_R^A}{\partial p^A} + \pi_R^A \frac{\partial n_R^A}{\partial p^A} = 0$$

$$\frac{\partial \pi^A}{\partial p^A} = n_R^A - \frac{1-\mu}{2t} \pi_R^A = 0. \tag{135}$$

From (135), in equilibrium

$$\pi_R^A = p^A + \left(\gamma^A - c\right)\left(1 - \gamma^A\right) = \frac{2t}{1-\mu} n_R^A. \tag{136}$$

Hence firm A's equilibrium profit satisfies

$$\pi^A = \frac{2t}{1-\mu} \left(n_R^A\right)^2 - F. \tag{137}$$

γ^A:

$$\frac{\partial \pi^A}{\partial \gamma^A} = n_R^A \frac{\partial \pi_R^A}{\partial \gamma^A} + \pi_R^A \frac{\partial n_R^A}{\partial \gamma^A} = 0$$

$$\frac{\partial \pi^A}{\partial \gamma^A} = n_R^A \left(1 + c - 2\gamma^A\right) - \alpha \frac{1-\mu}{2t} \pi_R^A = 0. \tag{138}$$

Substituting (136) into (138), in equilibrium

$$n_R^A \left[\left(1 + c - 2\gamma^A\right) - \alpha\right] = 0 \tag{139}$$

and since $n_R^A > 0$, in equilibrium

$$\gamma^A = \frac{1}{2}\left[1 - (\alpha - c)\right] = \gamma^*. \tag{140}$$

Firm B Firm B's payoff function is

$$\pi^B = n_R^B \pi_R^B - F.$$

The first-order condition with respect to p^B is

$$\frac{\partial \pi^B}{\partial p^B} = n_R^B - \frac{1-\mu}{2t}\pi_R^B = 0. \tag{141}$$

From (141), in equilibrium

$$p^B + \left(\gamma^B - c\right)\left(1 - \gamma^B\right) = \frac{2t}{1-\mu} n_R^B \tag{142}$$

Hence firm B's equilibrium profit satisfies

$$\pi^B = \frac{2t}{1-\mu}\left(n_R^B\right)^2 - F. \tag{143}$$

The first-order condition with respect to γ^B is

$$\frac{\partial \pi^B}{\partial \gamma^B} = n_R^B\left(1 + c - 2\gamma^B\right) - \alpha\frac{1-\mu}{2t}\pi_R^B = 0. \tag{144}$$

Substituting (142) into (144), in equilibrium

$$n_R^B\left(1 + c - 2\gamma^B\right) - \alpha\frac{1-\mu}{2t}\frac{2t}{1-\mu}n_R^B = 0$$

$$n_R^B\left[1 + c - 2\gamma^B - \alpha\right] = 0,$$

and for $n_R^B > 0$ we have

$$\gamma^B = \frac{1}{2}\left[1 - (\alpha - c)\right] = \gamma^*. \tag{145}$$

Equilibrium n_R^A, n_R^B (I) Use the equilibrium values of γ^A and γ^B to rewrite (127) and (129) as

$$n_R^A = \frac{1}{2}\left[(1+\mu) - (1-\mu)\frac{p^A - p^B}{t}\right] \tag{146}$$

and

$$n_R^B = \frac{1}{2}(1-\mu)\left(1 + \frac{p^A - p^B}{t}\right). \tag{147}$$

Equilibrium p^A, p^B Using (146), firm A's first-order condition for p^A, (135), becomes

$$2p^A - p^B = \frac{1+\mu}{1-\mu}t - z\left(\gamma^* - c\right). \tag{148}$$

This is firm A's equilibrium price best-response equation — "equilibrium" because the γs are set at their equilibrium values.

Using (147), firm B's first-order condition for p^B, (141), becomes

$$-p^A + 2p^B = t - z\left(\gamma^* - c\right). \tag{149}$$

This is firm B's equilibrium price best-response equation.

(148) and (149) can be solved for equilibrium prices.

Before doing so, subtract (149) from (148) to obtain an expression for $p^A - p^B$, which is what is needed to find the equilibrium number of readers for each newspaper:

$$p^A - p^B = \frac{2}{3}\frac{\mu}{1-\mu}t. \tag{150}$$

Now write the system of first-order equations in matrix form as

$$\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} p^A \\ p^B \end{pmatrix} = \begin{pmatrix} \frac{1+\mu}{1-\mu} \\ 1 \end{pmatrix} t - z\left(\gamma^* - c\right)\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \tag{151}$$

from which

$$\begin{pmatrix} p^A \\ p^B \end{pmatrix} = \frac{1}{3}\frac{1}{1-\mu}\begin{pmatrix} 3+\mu \\ 3-\mu \end{pmatrix} t - z\left(\gamma^* - c\right)\begin{pmatrix} 1 \\ 1 \end{pmatrix}. \tag{152}$$

$$p^A = \frac{1}{3}\frac{3+\mu}{1-\mu}t - z\left(\gamma^* - c\right) \tag{153}$$

$$p^B = \frac{1}{3}\frac{3-\mu}{1-\mu}t - z\left(\gamma^* - c\right). \tag{154}$$

Subtracting (154) from (153) gives (150).

In a conventional oligopoly model, it would be taken for granted that $p^A \geq 0$, $p^B \geq 0$. In general in a model of a platform market, we cannot automatically assume this. However, if the unconstrained model implies negative prices for newspapers, we would wish to impose zero prices as a constraint and pursue the implications. We therefore assume $p^A \geq 0$, $p^B \geq 0$. See the discussion of Armstrong and Wright (2007, p. 356), who make the same assumption.

Since

$$\frac{\partial p^A}{\partial \mu} = \frac{t}{3}\frac{\partial}{\partial \mu}\left(\frac{3+\mu}{1-\mu}\right) = \frac{4t}{3}\frac{1}{(1-\mu)^2} > 0 \tag{155}$$

$$\frac{\partial p^B}{\partial \mu} = \frac{t}{3}\frac{\partial}{\partial \mu}\left(\frac{3-\mu}{1-\mu}\right) = \frac{2t}{3}\frac{1}{(1-\mu)^2} > 0, \tag{156}$$

p^A and p^B are increasing in μ. It follows that platforms' profits per reader,

$$\pi_R^A = p^A + z(\gamma^* - c) = \frac{1}{3}\frac{3+\mu}{1-\mu}t \tag{157}$$

and

$$\pi_R^B = p^B + z(\gamma^* - c) = \frac{1}{3}\frac{3-\mu}{1-\mu}t \tag{158}$$

are also increasing in μ.

Consistency condition Now examine conditions under which (126), $p^A \leq \alpha n_a^A - t$, will be satisfied.

A preliminary remark is that considering the group that does not regard comics as essential, it must also be that consumers at the boundary location have nonnegative net utility,

$$\alpha n_a^A - p^A - tx \geq 0,$$

for x the boundary distance from the left end of the line. But if a reader at the right end of the line would have nonnegative net utility,

$$0 \leq \alpha n_a^A - p^A - t,$$

then so would a reader located closer to the left end of the line,

$$\alpha n_a^A - p^A - tx = \alpha n_a^A - p^A - t + (1-x) > 0,$$

and this is true in particular if x is the boundary location.

Now examine conditions for (126) to be satisfied:

$$\alpha n_a^A - p^A - t \geq 0.$$

$$\alpha z - \left[\frac{1}{3}\frac{3+\mu}{1-\mu}t - z(\gamma^* - c)\right] - t \geq 0$$

Omitting several steps, the consistency condition becomes

$$t \leq \frac{3}{2}\frac{1-\mu}{3-\mu}z^2. \tag{159}$$

The right-hand side of (159) goes to 0 as $\mu \to 1$. It follows that there is a critical value μ^*, $0 < \mu^* < 1$, such that the consistency condition is satisfied exactly. μ^* is defined by

$$\frac{3}{2}\frac{1-\mu}{3-\mu}z^2 = t. \tag{160}$$

From (160),

$$\mu^* = \frac{3z^2 - 6t}{3z^2 - 2t}. \tag{161}$$

By the argument we made about starting at $\mu = 0$ and increasing μ, μ^* must lie between 0 and 1.

$$p^A = \frac{1}{3}\frac{3+\mu}{1-\mu}t - z\left(\gamma^* - c\right).$$

For $\mu = \mu^*$,

$$p^A = \alpha z - t.$$

$$p^B = \frac{1}{3}\frac{3-\mu}{1-\mu}t - z\left(\gamma^* - c\right).$$

Evaluate this for $\mu = \mu^*$; the result will be used below. Omitting several steps,

$$p^B = \frac{1}{3}\frac{3-\mu^*}{1-\mu^*}t - z\left(\gamma^* - c\right) = \frac{1}{2}z^2 - z\left(\gamma^* - c\right). \tag{162}$$

Equilibrium n_R^A, n_R^B (II) Substituting the expression for $p^A - p^B$, (150), into (146) and (147), the equilibrium numbers of readers per newspaper in the low-p^A regime are

$$n_R^A = \frac{1}{6}\left(3+\mu\right) \tag{163}$$

$$n_R^B = \frac{1}{6}\left(3-\mu\right). \tag{164}$$

In the unconstrained low-p^A regime, n_R^A rises from $\frac{1}{2}$ and n_R^B falls from $\frac{1}{2}$ as μ rises from 0.

Payoffs Substitute from (163) and (164) into (137) and (143), respectively, equilibrium payoffs are

$$\pi^A = \frac{t}{18} \frac{(3+\mu)^2}{1-\mu} - F \tag{165}$$

and

$$\pi^B = \frac{t}{18} \frac{(3-\mu)^2}{1-\mu} - F. \tag{166}$$

Comparative static derivatives with respect to μ are

$$\frac{\partial \pi^A}{\partial \mu} = \frac{t}{18} \frac{(3+\mu)(5-\mu)}{(1-\mu)^2} > 0 \tag{167}$$

$$\frac{\partial \pi^B}{\partial \mu} = \frac{t}{18} \frac{(1+\mu)(3-\mu)}{(1-\mu)^2} > 0. \tag{168}$$

As μ increases, in a comparative static sense, π_R^A and n_R^A both rise, so π^A, their product, certainly rises.

As μ increases, π_R^B rises and n_R^B falls. In the low-p^A regime, the former effect outweighs the latter, and π^B rises as μ rises from 0 to μ^*.

Constrained The equilibrium value

$$p^A = \frac{1}{3} \frac{3+\mu}{1-\mu} t - z(\gamma^* - c),$$

which is obtained by solving firm A's profit maximization problem for the low-p^A case without explicitly imposing the low-p^A constraint,

$$p^A \leq \alpha z - t, \tag{169}$$

satisfies the low-p^A constraint for $\mu \leq \mu^*$. For $\mu \geq \mu^*$, to obtain an equilibrium value of p^A consistent with the condition that defines n_R^A for the low-p^A case requires imposing (169) as an explicit constraint on firm A's profit-maximization problem.

In the low-p^A case, n_R^A is given by (128). A Lagrangian for firm A's constrained optimization problem is

$$\mathcal{L} = n_R^A \pi_R^A - F + \lambda \left[\alpha \left(1 - \gamma^A\right) - t - p^A\right]. \tag{170}$$

Assuming an interior solution, the Kuhn-Tucker first-order conditions are
p^A:

$$n_R^A \frac{\partial \pi_R^A}{\partial p^A} + \pi_R^A \frac{\partial n_R^A}{\partial p^A} - \lambda = 0. \quad (171)$$

γ^A:

$$n_R^A \frac{\partial \pi_R^A}{\partial \gamma^A} + \pi_R^A \frac{\partial n_R^A}{\partial \gamma^A} - \alpha\lambda = 0. \quad (172)$$

λ:

$$\alpha \left(1 - \gamma^A\right) - t - p^A = 0. \quad (173)$$

Substituting (130), (131), (132), (133), and (134), (171) and (172) become

$$n_R^A - \frac{1-\mu}{2t} \pi_R^A - \lambda = 0. \quad (174)$$

and

$$n_R^A \left(1 + c - 2\gamma^A\right) - \alpha \frac{1-\mu}{2t} \pi_R^A - \alpha\lambda = 0, \quad (175)$$

respectively.

$$\alpha \left(1 - \gamma^A\right) - t - p^A = 0. \quad (176)$$

From (174),

$$\frac{1-\mu}{2t} \pi_R^A = n_R^A - \lambda,$$

leading to

$$\pi_R^A = \frac{2t}{1-\mu} \left(n_R^A - \lambda\right). \quad (177)$$

Substitute (177) into (175) to obtain

$$\gamma^A = \frac{1}{2} \left[1 - (\alpha - c)\right] = \gamma^*. \quad (178)$$

Firm B's problem is unaffected by the constraint imposed on firm A. Thus we have

$$\gamma^B = \frac{1}{2} \left[1 - (\alpha - c)\right] = \gamma^*,$$

as before. It follows that the expressions (146) and (147) for n_R^A and n_R^B, respectively, are valid for the constrained optimization case.

Since we know γ, we now have two equations, (174) and (176).

From (174),

$$\lambda = n_R^A - \frac{1-\mu}{2t}\pi_R^A, \tag{179}$$

and substituting for n_R^A and π_R^A, this becomes (omitting several steps)

$$\lambda = \frac{1}{2}(1+\mu) + \frac{1-\mu}{2t}\left[p^B - 2p^A - z(\gamma^* - c)\right]. \tag{180}$$

Rewriting (180) in a form that highlights the relationship to the first-order condition for p^A when the constraint does not bind, (148), gives

$$\lambda = \frac{1-\mu}{2t}\left[\frac{1+\mu}{1-\mu}t - z(\gamma^* - c) - \left(2p^A - p^B\right)\right]. \tag{181}$$

There is further analysis of the equilibrium value of λ below.

From the binding constraint, we get the value of p^A:

$$p^A = \alpha z - t. \tag{182}$$

Firm B's best-response equation is unchanged by the fact that the constraint on firm A's problem is binding; it is

$$-p^A + 2p^B = t - z(\gamma^* - c). \tag{183}$$

Substituting (182) into (183), firm B's equilibrium price when firm A's price is determined by the constraint is

$$p^B = \frac{1}{2}z(\alpha - \gamma^* + c). \tag{184}$$

By definition of μ^*,

$$\frac{1}{3}\frac{3-\mu^*}{1-\mu^*}t = \frac{1}{2}z^2.$$

Hence if $\mu = \mu^*$, firm B's equilibrium price per newspaper when firm A's optimization problem is unconstrained is

$$p^B = \frac{1}{2}z^2 - z(\gamma^* - c).$$

This is identical to (162); firm B's equilibrium price is continuous in μ at the value of μ for which the constraint on firm A's low-p^A optimization problem becomes binding.

When the low-p^A constraint is binding, the difference in equilibrium prices is

$$p^A - p^B = \frac{1}{2}z^2 - t.$$

Above, (159), for consistency in the low-p^A regime with $\mu = 0$, we assumed

$$\frac{1}{2}z^2 \geq t.$$

This implies that in equilibrium in the constrained case

$$p^A - p^B = \frac{1}{2}(1-\gamma^*)^2 - t > 0. \tag{185}$$

Find the equilibrium numbers of readers per platform,

$$n_R^A = \frac{1}{2}\left[1 + \mu + (1-\mu)\frac{p^B - p^A}{t}\right]$$

$$n_R^B = \frac{1}{2}(1-\mu)\left(1 + \frac{p^A - p^B}{t}\right).$$

Using (185), (omitting several steps)

$$n_R^A = 1 - (1-\mu)\frac{z^2}{4t}. \tag{186}$$

$$n_R^B = (1-\mu)\frac{z^2}{4t}. \tag{187}$$

Thus

$$n_R^A + n_R^B = 1.$$

In the low-p^A case, the market is covered.

Find equilibrium firm payoffs.

$$\pi^A = n_R^A \pi_R^A - F$$

$$\pi^B = n_R^B \pi_R^B - F.$$

For firm B, we have as before

$$\pi^B = \frac{2t}{1-\mu}\left(n_R^B\right)^2 - F. \tag{188}$$

When the low-p^A constraint is binding,

$$\pi_R^A = \frac{2t}{1-\mu}\left(n_R^A - \lambda\right),$$

and firm A's equilibrium payoff satisfies

$$\pi^A = \frac{2t}{1-\mu} n_R^A \left(n_R^A - \lambda\right) - F. \tag{189}$$

One of the expressions for λ is (181),

$$\lambda = \frac{1-\mu}{2t}\left\{\frac{1+\mu}{1-\mu}t - \left[2p^A - p^B + z\left(\gamma^* - c\right)\right]\right\}.$$

Consider the expression in brackets; substituting (182) and (184), it is

$$2p^A - p^B + z\left(\gamma^* - c\right) =$$

$$2\left[\alpha z - t\right] - \frac{1}{2}z\left(\alpha - \gamma^* + c\right) + z\left(\gamma^* - c\right) =$$

(omitting several steps)

$$\frac{3}{2}z^2 - 2t.$$

Then

$$\lambda = \frac{1}{2}\left(3 - \mu - \frac{1-\mu}{t}\frac{3}{2}z^2\right) \tag{190}$$

From (186)

$$n_R^A = 1 - (1-\mu)\frac{z^2}{4t}.$$

Then

$$n_R^A - \lambda = \frac{1-\mu}{2}\left(\frac{z^2}{t} - 1\right). \tag{191}$$

Firm A's payoff in the low-p^A regime when the low-p^A constraint is binding is

$$\pi^A = \frac{2t}{1-\mu} n_R^A \left(n_R^A - \lambda\right) - F = \left[1 - (1-\mu)\frac{z^2}{4t}\right]\left(\frac{z^2}{t} - 1\right) t - F. \tag{192}$$

π^A rises as μ rises.

Firm B's equilibrium payoff is

$$\pi^B = \frac{2t}{1-\mu}\left(n_R^B\right)^2 - F = \frac{1-\mu}{8t}z^4 - F. \tag{193}$$

$p^A \geq \alpha n_a^A - t$ The underlying expressions for n_R^B and π_R^B are unchanged from the previous case. Firm B's choice of γ^B is given by (145), and its first-order conditions are as in the low-p^A regime.

Firm A's profit per reader,

$$\pi_R^A = p^A + \left(\gamma^A - c\right)\left(1 - \gamma^A\right),$$

is also as in the low-p^A regime. But in the high-p^A regime (from (14)), platform A's number of readers is

$$\begin{aligned} n_R^A &= (1-\mu)\left[\frac{1}{2} + \frac{\alpha\left(\gamma^B - \gamma^A\right) - p^A + p^B}{2t}\right] + \mu\frac{\alpha n_a^A - p^A}{t} \\ &= (1-\mu)\left[\frac{1}{2} - \frac{\alpha\left(1-\gamma^B\right) - p^B}{2t}\right] + (1+\mu)\frac{\alpha\left(1-\gamma^A\right) - p^A}{2t}. \end{aligned} \quad (194)$$

Firm A's first-order condition with respect to p^A is

$$\frac{\partial \pi^A}{\partial p^A} = n_R^A - \frac{1+\mu}{2t}\left[p^A + \left(\gamma^A - c\right)\left(1-\gamma^A\right)\right] \equiv 0 \quad (195)$$

(compare with (135) for the low-p^A regime).

From (195), in equilibrium

$$\pi_R^A = p^A + \left(\gamma^A - c\right)\left(1-\gamma^A\right) = \frac{2t}{1+\mu} n_R^A \quad (196)$$

and firm A's equilibrium payoff satisfies

$$\pi^A = \frac{2t}{1+\mu}\left(n_R^A\right)^2 - F. \quad (197)$$

Firm A's first-order condition with respect to γ^A is

$$\frac{\partial \pi^A}{\partial \gamma^A} = n_R^A \frac{\partial \pi_R^A}{\partial \gamma^A} + \pi_R^A \frac{\partial n_R^A}{\partial \gamma^A} \equiv 0 \quad (198)$$

or

$$\frac{\partial \pi^A}{\partial \gamma^A} = n_R^A\left(1 + c - 2\gamma^A\right) - \alpha\frac{1+\mu}{2t}\pi_R^A \equiv 0. \quad (199)$$

Substituting (196) into (199), in equilibrium

$$n_R^A\left(1 + c - 2\gamma^A - \alpha\right) \equiv 0$$

and

$$\gamma^A = \frac{1}{2}(1 + c - \alpha) = \gamma^*. \quad (200)$$

Equilibrium n_R^A, n_R^B (I) Substituting $\gamma^A = \gamma^B = \gamma^*$ in (194) and (129), the equilibrium numbers of readers satisfy

$$n_R^A = \frac{1}{2}(1-\mu) + \mu\frac{\alpha(1-\gamma)}{t} - (1+\mu)\frac{p^A}{2t} + (1-\mu)\frac{p^B}{2t} \tag{201}$$

and

$$n_R^B = \frac{1}{2}(1-\mu)\left(1 + \frac{p^A - p^B}{t}\right). \tag{202}$$

Equilibrium p^A, p^B Using (201), firm A's first-order condition for p^A, (195),

$$\frac{\partial \pi^A}{\partial p^A} = n_R^A - \frac{1+\mu}{2t}\left[p^A + z(\gamma^* - c)\right] \equiv 0,$$

becomes (omitting several steps)

$$2(1+\mu)p^A - (1-\mu)p^B = t - z(\gamma^* - c) - \{t - 2\alpha z + z(\gamma^* - c)\}\mu. \tag{203}$$

The first-order condition for p^B is

$$-p^A + 2p^B = t - z(\gamma^* - c). \tag{204}$$

Write the system of equations is

$$\begin{pmatrix} 2(1+\mu) & -(1-\mu) \\ -1 & 2 \end{pmatrix}\begin{pmatrix} p^A \\ p^B \end{pmatrix} = [t - z(\gamma^* - c)]\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$
$$- [t - 2\alpha z + z(\gamma^* - c)]\mu\begin{pmatrix} 1 \\ 0 \end{pmatrix}. \tag{205}$$

The system of first-order conditions can be solved for prices,

$$(3+5\mu)\begin{pmatrix} p^A \\ p^B \end{pmatrix} = [t - z(\gamma^* - c)]\begin{pmatrix} 2 & 1-\mu \\ 1 & 2(1+\mu) \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$
$$- [t - 2\alpha z + z(\gamma^* - c)]\mu\begin{pmatrix} 2 & 1-\mu \\ 1 & 2(1+\mu) \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}. \tag{206}$$

Instead of looking at the solutions written in this form, it is useful to multiply both sides of (206) by

$$\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix},$$

obtaining a transformed system of equations

$$(3+5\mu)\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} p^A \\ p^B \end{pmatrix} = [t - z(\gamma^* - c)]\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 1-\mu \\ 1 & 2(1+\mu) \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$-[t - 2\alpha z + z(\gamma^* - c)]\,\mu\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 1-\mu \\ 1 & 2(1+\mu) \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

Coefficient matrices on the right are

$$\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 1-\mu \\ 1 & 2(1+\mu) \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3-4\mu \\ 3+5\mu \end{pmatrix}$$

and

$$\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 1-\mu \\ 1 & 2(1+\mu) \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}.$$

The transformed system of equations is (omitting several steps)

$$\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} p^A \\ p^B \end{pmatrix} = [t - z(\gamma^* - c)]\begin{pmatrix} 1 \\ 1 \end{pmatrix} - \frac{12\mu}{3+5\mu}\left(t - \frac{1}{2}z\right)\begin{pmatrix} 1 \\ 0 \end{pmatrix}. \tag{207}$$

The first equation in (207) is a linear combination of the first-order conditions of the two platforms. It is clear from (207) that if $\mu = 0$, the system of first-order conditions of the essential component model corresponds to the system of first-order conditions of the basic model.

Solving (207) gives equilibrium prices

$$\begin{pmatrix} p^A \\ p^B \end{pmatrix} = [t - z(\gamma^* - c)]\begin{pmatrix} 1 \\ 1 \end{pmatrix} - \frac{4\mu}{3+5\mu}\left(t - \frac{1}{2}z^2\right)\begin{pmatrix} 2 \\ 1 \end{pmatrix}. \tag{208}$$

$$p^A = t - z(\gamma^* - c) - \frac{8\mu}{3+5\mu}\left(t - \frac{1}{2}z^2\right). \tag{209}$$

$$p^B = t - z(\gamma^* - c) - \frac{4\mu}{3+5\mu}\left(t - \frac{1}{2}z\right). \tag{210}$$

Numbers of readers We use (209) and (210) to evaluate the numbers of readers of each platform, (201) and (202).

Considering first platform B, from (209) and (210),

$$p^A - p^B = -\frac{4\mu}{3+5\mu}\left(t - \frac{1}{2}z^2\right) < 0. \tag{211}$$

Substituting (211) into (202) and rearranging terms gives

$$n_R^B = \frac{1-\mu}{2t}\frac{(3+\mu)\,t + 2\mu z^2}{3+5\mu}. \tag{212}$$

Now turn to platform A. We need to evaluate

$$-(1+\mu)\,p^A + (1-\mu)\,p^B = -\left(p^A - p^B\right) - \mu\left(p^A + p^B\right). \tag{213}$$

From (208),

$$-(1+\mu)\,p^A + (1-\mu)\,p^B =$$

(omitting several steps)

$$-2\mu\left[\frac{1-\mu}{3+5\mu}t - z\left(\gamma^* - c\right) + \frac{1+3\mu}{3+5\mu}z^2\right]. \tag{214}$$

Then

$$n_R^A = \frac{1}{2}(1-\mu) + \mu\frac{\alpha z}{t} + \frac{-(1+\mu)\,p^A + (1-\mu)\,p^B}{2t} =$$

(omitting several steps)

$$= \frac{1+\mu}{3+5\mu}\left[\frac{3}{2}(1-\mu) + \frac{2z^2}{t}\mu\right]. \tag{215}$$

The total number of readers is

$$n_R^A + n_R^B = \frac{1}{3+5\mu}\left[(1-\mu)(3+2\mu) + \mu(3+\mu)\frac{z^2}{t}\right]. \tag{216}$$

Consistency The consistency condition is

$$p^A \geq \alpha n_a^A - t.$$

Rewrite (209) to collect terms in t and obtain

$$p^A = 3\frac{1-\mu}{3+5\mu}t - z\left(\gamma^* - c\right) + \frac{4\mu}{3+5\mu}z^2.$$

Then

$$p^A - \alpha n_a^A + t = 2\frac{3+\mu}{3+5\mu}\left(t - \frac{1}{2}z^2\right).$$

In the high-p^A case, consistency requires

$$t \geq \frac{1}{2} z^2.$$

In the unconstrained low-p^A case, consistency requires the opposite relationship (see (159) for $\mu = 0$):

$$\frac{1}{2} z^2 \geq t.$$

Payoffs From (197) and (143), equilibrium payoffs are

$$\pi^A = \frac{2t}{1+\mu} \left(n_R^A\right)^2 - F$$

and

$$\pi^B = \frac{2t}{1-\mu} \left(n_R^B\right)^2 - F.$$

Using (215), firm A's equilibrium payoff is

$$\pi^A = 2t \frac{1+\mu}{(3+5\mu)^2} \left[\frac{3}{2}(1-\mu) + \frac{2z^2}{t}\mu\right]^2 - F. \tag{217}$$

Using (212), firm B's equilibrium payoff is

$$\pi^B = \frac{1-\mu}{2t} \left[\frac{(3+\mu)\,t + 2\mu z^2}{3+5\mu}\right]^2 - F. \tag{218}$$

As $\mu \to 1$, π^B becomes negative.

10.2 Welfare

10.2.1 Monopoly, low-t

In the low-price regime, the monopoly supplier sets a price that makes consumers at the right end of the line indifferent between purchasing and not purchasing a newspaper. Consumers whose preferences place them closer to the left end of the line enjoy positive surplus if they buy. Consumer surplus is the area of the shaded triangle in Figure 2,

$$\frac{1}{2} t. \tag{219}$$

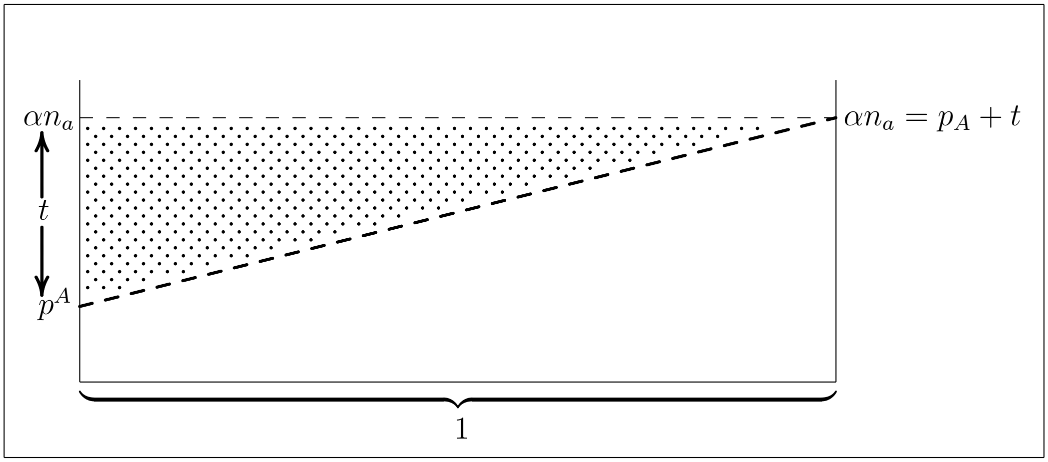

Figure 2: Consumer surplus, monopoly, low-t regime.

Economic profit generated by newspaper A in the low-t regime is $\pi_{l1}^m = z^2 - t - F$.[27] Adapting equation (20) to the present case, advertisers' profit in the low-price licensed-monopoly regime is

$$\frac{1}{2} n_R^A \left(1 - \gamma^A\right)^2 = \frac{1}{2} z^2. \tag{220}$$

Net social welfare in the low-price regime is the sum of profits and consumer surplus,

$$z^2 - t - F + \frac{1}{2} z^2 + \frac{1}{2} t = \frac{3}{2} z^2 - \frac{1}{2} t - F. \tag{221}$$

10.2.2 Monopoly, high t

Consumer surplus in the high-t regime is the area of the triangle in Figure 3,

$$\frac{\left(\alpha n_a - p_A\right)^2}{2t}. \tag{222}$$

Substituting $\alpha n_a = \alpha z$ and $p^{AM} = \frac{1}{2} z \left(\alpha + c - \gamma^*\right)$, and using $z = \gamma^* + \alpha - c$ gives

$$\alpha n_a - p_A = \frac{1}{2} z^2. \tag{223}$$

[27] The amount of the license fee determines the division of this profit between firm A and the syndicate; this division does not affect net social welfare.

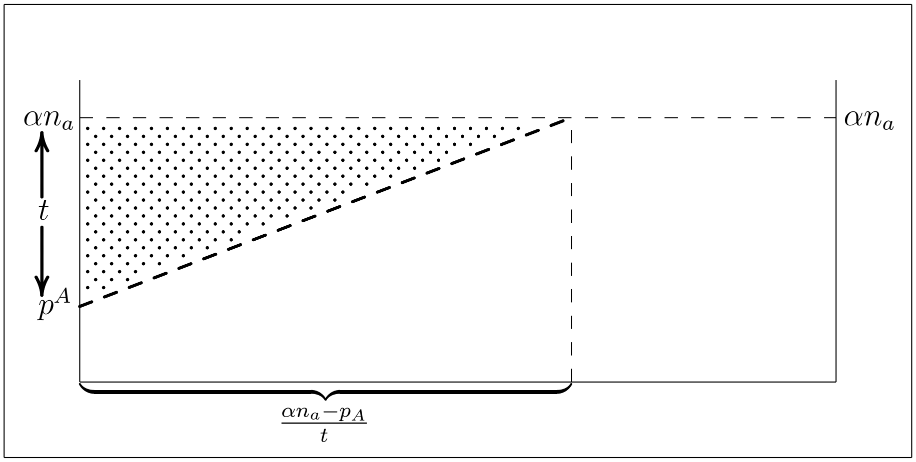

Figure 3: Consumer surplus, monopoly, high-t regime.

Then consumer surplus in the firm A monopoly, high-price regime is

$$\frac{1}{2t}\left(\alpha n_a - p_A\right)^2 = \frac{1}{8t}z^4. \tag{224}$$

Economic profit from the operation of newspaper A in the high-price regime is $\pi_{l2}^m = \frac{1}{4t}z^4 - F$. Advertisers' profit is (using $n_R^A = \frac{1}{2t}z^2$)

$$\frac{1}{2}n_R^A z^2 = \frac{1}{4t}z^4. \tag{225}$$

Net social welfare in the high-t licensed-monopoly regime is

$$\frac{1}{4t}z^4 - F + \frac{1}{4t}z^4 + \frac{1}{8t}z^4 = \frac{5}{8t}z^4 - F. \tag{226}$$

10.2.3 Licensed-firm Duopoly

Net utility at either extreme of the line (the location for which transportation cost is zero) is (omitting superscripts since we consider symmetric equilibrium values)

$$\begin{gathered}\alpha n_a - p = \\ \alpha z - t + \left(\gamma^* - c\right)z = z\left(\gamma^* + \alpha - c\right) - t = \\ z^2 - t.\end{gathered} \tag{227}$$

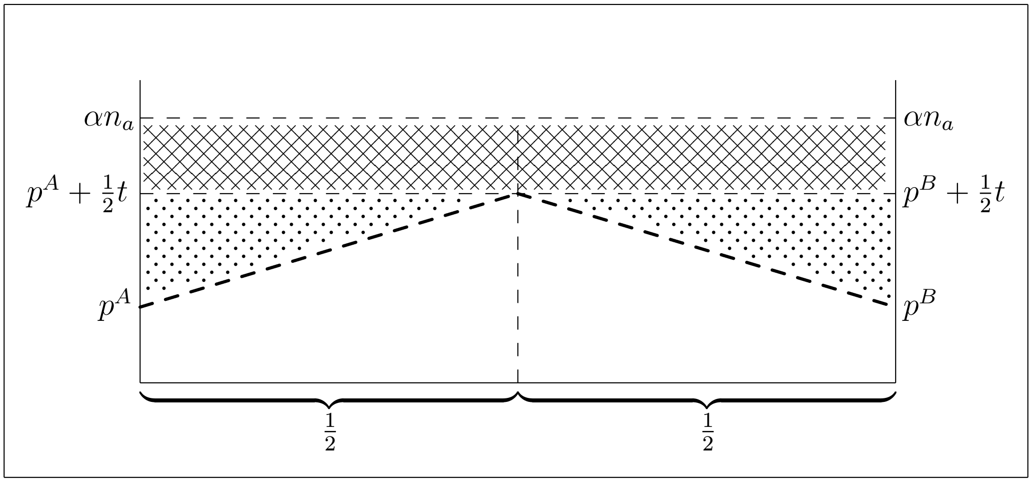

Figure 4: Consumer surplus, duopoly, both firms licensed.

Net utility at the center of the line (transportation cost $\frac{1}{2}t$) is

$$\alpha n_a - p - \frac{1}{2}t = z\left(\gamma^* + \alpha - c\right) - t - \frac{1}{2}t = z^2 - \frac{3}{2}t. \tag{228}$$

By the assumption that the market is covered, (121), we have assumed (228) is nonnegative.

Consumer surplus is the shaded area shown in Figure 4, one-half of which is

$$\int_{x=0}^{1/2} \left[\alpha n_a - (p + tx)\right] dx = \int_{x=0}^{1/2} \left[(\alpha n_a - p) - tx\right] dx =$$

$$\left[(\alpha n_a - p)\, x - \frac{1}{2}tx^2\right]_{x=0}^{1/2} = (\alpha n_a - p)\left(\frac{1}{2}\right) - \frac{1}{2}t\left(\frac{1}{2}\right)^2. \tag{229}$$

Total consumer surplus is twice (229),

$$\alpha n_a - p - t\left(\frac{1}{2}\right)^2 = z^2 - t - \frac{1}{4}t = z^2 - \frac{5}{4}t. \tag{230}$$

From equation (20), advertisers' profits are

$$\frac{1}{2}\left(n_R^A + n_R^B\right) z^2,$$

and in equilibrium for this model, $n_R^A = n_R^B = \frac{1}{2}$; hence advertisers' equilibrium profits are

$$\frac{1}{2}z^2, \tag{231}$$

Equilibrium net social welfare is the sum of the profits generated by the two newspapers, advertisers' profits, and consumer surplus,

$$2\left(\frac{1}{2}t - F\right) + \frac{1}{2}z^2 + z^2 - \frac{5}{4}t$$

$$2\left(\frac{1}{2}t - F\right) + \frac{1}{2}z^2 + z^2 - \frac{5}{4}t = \frac{3}{2}z^2 - \frac{1}{4}t - 2F. \tag{232}$$

10.3 High-t

Here we show that Theorem 1 holds for the high-t case. The inequalities

$$\max\left(\pi_{nlnl}^{d}, \pi_{nl2}^{m}, \pi_{nl3}^{Bd}\right) < 0 \leq \min\left(\pi_{ll}^{d}, \pi_{l2}^{m}, \pi_{l3}^{Ad}\right) \tag{233}$$

correspond to

$$\max\left\{\frac{1-\mu}{2}t, \frac{1-\mu}{4t}z^4, \frac{1-\mu}{2t}\left[\frac{(3+\mu)\,t + 2\mu z^2}{3+5\mu}\right]^2\right\} < F \leq$$

$$\min\left\{\frac{1}{2}t, \frac{1}{4t}z^4, \frac{1+\mu}{2t}\left[\frac{3\,(1-\mu)\,t + 4\mu z^2}{3+5\mu}\right]^2\right\}. \tag{234}$$

As $\mu \to 1$, (234) approaches

$$\max(0,0,0) = 0 < F \leq \min\left(\frac{1}{2}t, \frac{1}{4t}z^4, \frac{1}{4t}z^4\right). \tag{235}$$

By subtraction,

$$\frac{1}{4t}z^4 - \frac{1}{2}t = \frac{1}{2t}\left(\frac{1}{2}z^4 - t^2\right) = \frac{1}{2t}\left(\frac{1}{\sqrt{2}}z^2 - t\right)\left(\frac{1}{\sqrt{2}}z^2 + t\right), \tag{236}$$

which is positive in the high-t case since

$$\frac{1}{\sqrt{2}}z^2 - t > \frac{2}{3}z^2 - t \geq 0. \tag{237}$$

Then (235) reduces to (49). From this point, the argument is as for the low-t, high-μ case.