Equilibrium State Aid in Integrating Markets

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Recommended Citation
Available at: http://www.bepress.com/bejeap/vol8/iss1/art33

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Stephen Martin and Paola Valbonesi

Abstract

We present a model of the impact of state aid on equilibrium market structure and on market performance in an integrating market when the process of integration is driven by consumer inertia. In a partial equilibrium model, it is an equilibrium for governments to grant state aid, even though this reduces common market welfare.

KEYWORDS: state aid, exit, market integration

*We thank Sergio Vergalli for research assistance, David Collie, Gianni De Fraja, Vincenzo Denicòlò, Michele Moretto, Massimo Motta and Patrick Rey for comments on a much earlier version of this paper, and appreciate comments received at the University of Bern, the University of Padova, the University of East Anglia, the Jornadas de Economía Industrial of Bilbao, Michigan State University, the 2° CSEF-IGIER Symposium and at the 2006 Earie Conference in Amsterdam. We gratefully acknowledge financial support from the MIUR (Cofin 2004 - No. 2004134814_005 and Cofin 2006 - No. 2006130472_003 grants). Responsibility for errors is our own.
1 Introduction

Despite the prohibition of state aid that distorts competition contained in Article 87(1) of the Treaty establishing the European Community, state aid has been an enduring feature of the EC economic landscape. Although state aid has decreased since the end of the 1990s, in 2004 state aid overall in the fifteen Member States amounted to around €56 billion, representing about 0.6 per cent of EU GDP. In relative terms, aid ranged from 0.3 per cent of GDP in the United Kingdom to 1.5 per cent in Finland. State aid policy seems certain to remain the subject of controversy as the accession of economically less-developed Member States shifts the standards for permissibility of aid throughout the Community.

One reason for the persistence of state aid is the mandatory and discretionary exceptions to the Article 87(1) prohibition that appear in Articles 87(2) and 87(3). But there are fundamental economic forces at work, connected with the process of market integration itself, that create incentives for Member States to take advantage of those exceptions.

The increased competition that accompanies market integration improves market performance by reducing firms’ abilities to hold price above marginal cost and by eliminating waste (reducing X-inefficiency). It is less commonly noted that the increase in rivalry that comes with market integration may, and in general will, result in the exit of less efficient firms. Such exit, and the concentration of production in the hands of fewer, more efficient firms that goes with it, is one source of improved performance in the integrated market. But the prospect of exit can drive less-efficient, rent-seeking firms to seek assistance from their home-country governments. Thus the process of market integration

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1 Article 87(1) provides that “any aid granted by a Member State or through State resources in any form whatsoever which distorts or threatens to distort competition by favouring certain undertakings or the production of certain goods shall, in so far as it affects trade between Member States, be incompatible with the common market.” Article 88(2) provides that incompatible aid is to be altered or abolished.

2 For up-dated statistics on state aid, see: http://ec.europa.eu/comm/competition/state_aid/scoreboard/indicators/k1.html#total

3 See, for example, Ricard (2005). The 10 new members of the European Union devote a larger percentage of their GDP per capita to state subsidies to business than do the 15 older Member States (respectively about 1.35% of new member versus 0.45% of the older Member States in the period 2002-04). In absolute terms, the new member states granted €6,274 billion aid compared with €42,717 billion for the EU-15 in the period 2002-04 (EU State Aid Scoreboard, Spring 2006, p.11).

4 See, for example, Vickers (1995), Nickell (1996), and Hay and Liu (1997).

5 See, however, Symeonidis’ (2000) discussion of the impact on market structure of an unanticipated toughening of UK competition policy.
itself generates continuing pressure for the granting of state aid.

There is a large policy-oriented literature on European Union state aid policy.\(^6\) Formal treatments are rare. Collie (2000, 2003), in the work most closely related to the present discussion of which we are aware, models the impact of state aid on market performance in an integrated market. In this paper, in addition to examining the impact of state aid on market performance, we investigate the transition period in the run-up to full integration and consider the impact of state aid on market structure.

The analysis presented in this paper is based on the observation that the economics of equilibrium market structure in an integrating market has elements in common with the analysis of exit from a declining industry.\(^7\) This insight develops from the analysis of demand curves characterized by consumer inertia — a preference for the product of domestic producers that persists for a limited period even after formal barriers to trade have been eliminated. In such markets, and in the absence of government intervention, shifts in the residual demand curves facing individual firms in imperfectly competitive integrating markets dictate a reduction in the equilibrium number of firms. We show formally that state aid, by frustrating such reductions, neutralizes an efficiency effect of competition in an integrating market, and blocks the way to realization of an efficient specialization of production and division of labor in the common market.

That market integration may induce exit, absent state aid, is without doubt. An example from the early history of EU market integration is that of the Belgian coal industry in the European Coal and Steel Community. Belgian costs were so high that coal suppliers in the Ruhr would have been able to undersell Belgian mines in Belgium without engaging in freight absorption (Lister, 1960, p. 296; Meade et al., 1962, p. 292). The history of EU competition policy is replete with examples of Member States granting aid to their firms that was generally admitted to be contrary to Treaty provisions by all parties involved except the legal representatives of the aid-granting member states, who argued in defense of the aid before the European Court of Justice.

In contrast to the general literature on subsidies, which relies mainly on models similar to those found in the strategic trade, tax competition and rent-seeking literatures,\(^8\) our model develops the idea that the incentive to supply

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\(^6\)For references, see Martin and Evans (1991), Besley and Seabright (1999), Martin and Valbonesi (2000), and Friederiszick et al. (2005).


\(^8\)In economics, the roots of this literature go back to Adam Smith; see Elmslie (2004) and for a survey, Martin and Valbonesi (2006).
state aid is endogenously created by the process of market integration itself. In Section 2 we set up the analytical framework by considering the case of integration of two identical markets, first leaving aside and then taking into account the requirement that the equilibrium number of firms be an integer. In Section 3 we outline the demand inertia specification that holds while the market integration process is underway. In Section 4 we discuss the impact of market integration on firms’ payoffs and analyze the implications for state aid for performance in the component and integrated markets. Section 5 concludes. Proofs are given in the Appendices.

2 Market integration and market structure

Our model is of two countries that integrate their markets for a single product. Following a long tradition in applied theoretical work, we assume quantity-setting behavior with identical linear demands in each pre-integration market and constant marginal cost technologies.9 To highlight country-specific differences in cost in the simplest possible way, we model firms that differ in fixed costs. Following common practice in the strategic trade policy literature, we consider the case of market integration between countries that are identical in the sense that market demand, constant marginal cost, and the equilibrium number of firms are the same in both countries before integration.

2.1 Ignoring the integer constraint

2.1.1 Pre-integration market

Before integration, there are two countries, each home to a Cournot oligopoly with inverse demand equation:

$$p_i = a - bQ_i,$$

9See d’Aspremont and Jacquemin (1988), Kamien et al. (1992), Anis et al. (2002), and Hinloopen (2003). For some problems, the distinction between price-setting and quantity-setting behavior is critical. Given the result that equilibrium price-cost margins are smaller, all else equal, if firms set prices rather than quantities, the reduction in the equilibrium number of firms with market integration will occur whether firms set prices or quantities; the quantity-setting framework with a homogeneous product is chosen for simplicity.
for $i = 1, 2$. In this section, we assume that all firms have identical cost functions, with constant marginal cost $c$ and fixed cost $F$. The firm cost function is then

$$c(q) = F + cq.$$ 

If the number of firms adjusts to make Cournot equilibrium profit zero, and leaving aside for the moment the fact that the number of firms must be an integer, the pre-integration equilibrium number of firms in each country is

$$n^* = \frac{\frac{a-c}{b}}{\sqrt{\frac{F}{b}}} - 1. \quad (2)$$

The numerator of the fraction on the left, $\frac{a-c}{b}$, is the quantity that would be demanded in either of the component markets if price were equal to marginal cost, and is one way to measure market size. The denominator rises as fixed cost rises. (2) therefore says that the equilibrium number of firms in a Cournot market is larger, the larger is the market and the smaller is fixed cost.

### 2.1.2 Post-integration market

Suppose now that the two countries form a common market, in which firms cannot price discriminate based on nationality. In the completely integrated market, firms face a demand equation that is the horizontal sum of the single-market demand equations,

$$p = a - \frac{1}{2}bQ. \quad (3)$$

Substituting $\frac{b}{2}$ for $b$ in (2), the equilibrium number of firms in the integrated market $m^*$ satisfies

$$m^* + 1 = \sqrt{2\frac{a-c}{b}} = \sqrt{2} (n^* + 1).$$

---

10 The slope parameter $b$ can be normalized to some convenient value (usually taken to be 1) by appropriate redefinition of the units in which output is measured. Having normalized $b$ for a single-country market, the slope of the integrated-market inverse demand curve cannot then be normalized again. With this in mind, we write the slope parameter explicitly in (1).

11 In Section 4 we allow fixed costs to differ across countries.

12 Details are in the working paper versions of this paper, Martin and Valbonesi (2006).

13 In one perspective, this may be regarded as a definition of market integration. Nationality-based price discrimination may also be prohibited by competition policy, as indeed it is in the European Union.
The ratio \((m^* + 1)/(n^* + 1)\) equals the square root of the number of equally-sized markets that integrate to form a single market — in this case, two. The equilibrium number of firms in the integrated market exceeds the equilibrium number of firms in a single component market, but is less than the total number of firms in the pre-integration markets.

![Figure 1: Pre- and post-integration equilibrium number of firms, ignoring integer constraint; \(a = 110, c = 10, b = 1\); fixed cost falls moving left to right.](image)

This relationship is illustrated in Figure 1, which depicts the pre- and post-integration equilibrium number of firms as a function of fixed cost.\(^{14}\) The relationship is illustrated in Figure 1, which depicts the pre- and post-integration equilibrium number of firms as a function of fixed cost.\(^{14}\) The

\(^{14}\)Of necessity, Figure 1 is drawn for specific parameter values, but the relationships it
Graph is drawn with the vertical axis cutting the horizontal axis at the level of fixed cost equal to pre-integration monopoly profit. As fixed cost falls from this level (contrary to usual practice, moving from left to right along the horizontal axis), the equilibrium number of firms (pre- and post-integration) rises. The post-equilibrium number of firms in the single market is less than the pre-integration equilibrium number of firms in each single market. Market integration implies that some firms must exit.

Although market integration leads to a reduction in the number of firms, it means an improvement in market performance. Pre-integration long-run equilibrium price and post-equilibrium long-run equilibrium price are

\[ p(n^*) = c + \sqrt{bF} \]

and

\[ p(m^*) = c + \frac{1}{\sqrt{2}} \sqrt{bF} < p(n^*) , \]

respectively. Because integration induces exit, it economizes on fixed costs. Continuing firms produce at larger scale, reducing average cost, equilibrium price, and increasing consumer surplus.

### 2.2 Considering the integer constraint

Writing \( \pi_i(n) \) for pre-integration Cournot equilibrium per-firm profit and \( \pi^I(m) \) for post-integration Cournot equilibrium profit, the pre- and post-integration equilibrium numbers of firms satisfy the inequalities

\[ \pi_i(n) \geq 0 > \pi_i(n+1) \]  \hspace{1cm} (4)

(for \( i = 1, 2 \)) and

\[ \pi^I(m) \geq 0 > \pi^I(m+1) , \]  \hspace{1cm} (5)

respectively.

The relation between the pre- and post-integration equilibrium number of firms, taking the integer constraint into account, as fixed cost falls from a high level, is given in Table 1 for up to \( n = 8 \) pre-integration firms in each market. The general relationships of Table 1 are illustrated for particular parameter values in Figure 2.

The table and the figure both show that there is a range of fixed costs (row 3 in Table 1) for which market integration implies an increase in the
<table>
<thead>
<tr>
<th>row</th>
<th>Fixed cost</th>
<th>( n )</th>
<th>( m )</th>
<th>( 2n - m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \frac{1}{2} x ) (^2)</td>
<td>( 2 \left( \frac{1}{2} \right) x ) (^2)</td>
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<td>1</td>
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<td>( \left( \frac{1}{3} \right) x ) (^2)</td>
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<td>( \left( \frac{1}{5} \right) x ) (^2)</td>
<td>( 2 \left( \frac{1}{2} \right) x ) (^2)</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>( 2 \left( \frac{1}{8} \right) x ) (^2)</td>
<td>( \left( \frac{1}{2} \right) x ) (^2)</td>
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<td>6</td>
<td>( \left( \frac{1}{5} \right) x ) (^2)</td>
<td>( 2 \left( \frac{1}{2} \right) x ) (^2)</td>
<td>3</td>
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<tr>
<td>7</td>
<td>( 2 \left( \frac{1}{7} \right) x ) (^2)</td>
<td>( \left( \frac{1}{3} \right) x ) (^2)</td>
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<td>5</td>
</tr>
<tr>
<td>8</td>
<td>( 2 \left( \frac{1}{9} \right) x ) (^2)</td>
<td>( 2 \left( \frac{1}{2} \right) x ) (^2)</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>9</td>
<td>( \left( \frac{1}{5} \right) x ) (^2)</td>
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<td>11</td>
<td>( \frac{1}{7} x ) (^2)</td>
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<td>15</td>
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<tr>
<td>16</td>
<td>( \frac{1}{5} x ) (^2)</td>
<td>( \left( \frac{1}{10} \right) x ) (^2)</td>
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<td>10</td>
</tr>
<tr>
<td>17</td>
<td>( \frac{1}{9} x ) (^2)</td>
<td>( \left( \frac{1}{3} \right) x ) (^2)</td>
<td>7</td>
<td>11</td>
</tr>
<tr>
<td>18</td>
<td>( \frac{1}{7} x ) (^2)</td>
<td>( \left( \frac{1}{10} \right) x ) (^2)</td>
<td>8</td>
<td>11</td>
</tr>
</tbody>
</table>

Table 1: Pre- and post-integration numbers of firms, numerical example, \( a=110, c=10, b=1 \). \( x = (a - c)^2/b \).
Figure 2: Pre- and post-integration equilibrium integer number of firms; \( a = 110, c = 10, b = 1 \); fixed cost falls moving left to right.
equilibrium integer number of firms. Specifically, if the pre-integration number of firms in each national market is one, equilibrium market structure in the fully-integrated market falls in one of three cases. For high levels of fixed cost, the equilibrium number of firms in the integrated market is one (row 1). For intermediate levels of fixed cost, the equilibrium number of firms in the integrated market is two (row 2). For a narrow range\(^{15}\) of low fixed cost, the equilibrium number of firms in the fully-integrated market is three. For fixed cost in this range, the equilibrium integer number of firms in each national market is one, the equilibrium integer number of firms in the integrated market is three.\(^{16}\) One can show analytically that some exit must follow market integration (that is, one can show that \(2n - m \geq 1\) for \(n \geq 5\), and that the number of exiting firms weakly rises as fixed cost falls. Direct computations from Table 1 show that the lower bound \(n \geq 5\) is too strict; \(2n - m \geq 1\) (some exit must follow market integration) for \(n \geq 4\).

3 Market demand during the integration period

(1) and (3) are, respectively, the equation of the pre-integration inverse demand curve in country \(i\), \(i = 1, 2\), and equation of the inverse demand curve in the fully-integrated market. We assume that consumer behavior is described by a continuous, well-behaved integration function \(\iota(t)\), with

\[
0 \leq \iota(t) \leq 1
\]

\[
\iota(0) = 0 \quad \iota'(t) > 0 \quad \iota(T) = 1.
\]

A time period of length \(T\) is required to complete the integration process. At time \(t\) during the integration period, a fraction \(\iota(t)\) of consumers in each market are “in” the integrated market and these consumers consider firms in either country as potential sources of supply. Consumers in each country in the complementary fraction \(1 - \iota(t)\) consider only their domestic suppliers

\(^{15}\)In rows 1-3 of Table 1, fixed cost falls in the range \(\frac{1}{2} - \frac{1}{b} = \frac{5}{36}\) times the scale factor \(x\). In row 3, fixed cost falls in the range \(\frac{1}{b} - \frac{1}{b} = \frac{7}{17}\) times the scale factor \(x\). The range of fixed cost in which row 3 holds is thus one-tenth of the range in which rows 1, 2, and 3 hold.

\(^{16}\)One other such case should be mentioned, as it shows a potential benefit of market integration: for fixed cost in the range \(b(\frac{1}{2} \frac{a-c}{b})^2 \leq F < 2b(\frac{1}{2} \frac{a-c}{b})^2\), the pre-integration equilibrium number of firms is zero, the post-integration equilibrium number of firms is one. If fixed cost is very high, market integration can mean that it is profitable for a single firm to supply a market that would not otherwise be served. One would not expect state aid to be an issue in such cases.
as potential suppliers. The higher is $\iota$, the smaller is the fraction of consumers still buying only on the pre-integration national market and the more integrated is the common market.$^{17}$

The equation of the inverse demand curve facing each firm in the partially-integrated market is a weighted average of the national pre-integration demand curve and the full-integration residual demand curve,

$$Q_i(t) = [1 - \iota(t)] \frac{a - p_i(t)}{b} + \iota(t) \left[ 2 \frac{a - p_i(t)}{b} - Q_j(t) \right],$$

for $i, j = 1, 2$ and $j \neq i$. In what follows, we suppress the time argument where this is possible without confusion. It will then be natural simply to write of “integration level $\iota$.”

In the “exit from declining markets literature” (Ghemawat and Nalebuff 1985, 1990; Brainard, 1994; others), it is typical to assume that demand declines monotonically to zero over time in a well-behaved way. The assumptions we make about the integration function, which are rooted in assumptions about consumer behavior, correspond to such declining demand assumptions. Scitovsky (1950), Waterson (2003), search models of imperfectly competitive markets, and the literature on consumer switching costs all emphasize the importance of consumer behavior for market performance. The European Commission, in its First Report on Competition Policy, referred (1972, p. 14) to “differences in the habits of consumers” as one reason for persistent price differences across Member States in the Common Market. The same point is made in the 1997 Green Paper on Vertical Restraints, which notes that “there are huge discrepancies as to national preferences and local tastes of the consumers which are unlikely to fade quickly.” Goldberg and Verboven (2001, p. 840) conclude that “the existence of a strong bias for domestic brands” is one of three main factors explaining differences in automobile prices across five EU Member States.$^{18}$ The formulation given by (6) is one way of modelling the consumer inertia on demand side of an integrating market.

In a Hotelling model, Schultz (2005) obtains a comparable effect by allowing for two classes of consumers, those who are informed of the prices of both suppliers and those who are informed of the price of only one supplier. Schultz’s transparency parameter, the fraction of consumers informed of both

$^{17}$If the integration process is linear, we would have $\iota(t) = t/T$. Alternatively, the integration process might follow the kind of logistic pattern that is common in diffusion models. Results for logistic integration are qualitatively similar, and are available on request from the authors.

$^{18}$The other two are differences across Member States in the impact of quotas on imports from Japan and country-specific costs. See also Goldberg and Verboven (2005).
prices, corresponds conceptually to our integration parameter, although the
details of the models are quite different. Reinhard Selten has explored the
impact of demand inertia on market performance in experimental markets.
Consumer heterogeneity is one of the standard explanations for equilibrium
price heterogeneity, and the two groups of consumers in our markets may be
thought of as “shoppers” (buying in the integrated market) and “nonshoppers”
(buying only in the nonintegrated market).

Inverting (6), the inverse demand equation facing country \( i \) firms at integra-
tion level \( \iota \) is

\[
p_i = a - b \frac{Q_i + \iota Q_j}{1 + \iota},
\]

for \( i, j = 1, 2 \) and \( i \neq j \).

In what follows, we embed the partial-integration inverse demand equations
in a dynamic game featuring an integrating market from time 0 to time \( T \) and a
fully-integrated market thereafter. Firms in one country have higher fixed cost
than firms in the other country. At the start of the game, the government of
the high-cost country decides whether or not to subsidize its firms’ losses. If it
commits to subsidies, the government of the other country decides whether or
not to subsidize its firms’ losses. Firms maximize present-discounted payoffs.
Countries maximize present-discounted net social welfare.

## 4 Equilibrium withdrawal integration levels

Empirically, it is known that there are persistent cost differences across plants
in the same industry (Roberts and Supina, 1996, 1997). Country-specific differ-
ences in cost (which for simplicity we treat as differences in fixed cost) might
reflect locational differences or, for natural resource industries, differences in
the quality of mineral deposits (the case of Belgian coal, mentioned in the
introduction, is an example). We now assume that costs differ systematically
across countries. For simplicity, we assume that costs differ only in fixed cost,
and without loss of generality, we assume that country 2 firms have lower fixed
cost than country 2 firms:

\[
F_2 < F_1.
\]

We also suppose that \( F_1 \) and \( F_2 \) are such that the pre-integration equilibrium
number of firms is the same in both countries.

\(^{19}\) (7) bears a family resemblance to the Bowley (1924) specification for the inverse demand
equation of one variety of a differentiated product group. We ought to expect, therefore,
that a partially integrated market for a homogeneous product behaves in some ways like a
completely integrated market for a differentiated product.
Before the integration process begins, the equilibrium number of firms in each country is $n$. If there are $n_1 \leq n$ firms active in country 1 and $n_2 \leq n$ firms active in country 2, straightforward calculation shows that equilibrium outputs per firm are

$$q_1 = \frac{(1 + \iota) \left[ 1 + (1 - \iota) n_2 \right]}{n_1 + n_2 + 1 + (1 - \iota^2) n_1 n_2} \frac{a - c}{b} \quad (9)$$

and

$$q_2 = \frac{(1 + \iota) \left[ 1 + (1 - \iota) n_1 \right]}{n_1 + n_2 + 1 + (1 - \iota^2) n_1 n_2} \frac{a - c}{b} \quad (10)$$

for country 1 and country 2 firms, respectively.

### 4.1 No state aid

In the “declining industry” literature, the driving assumption is that the demand curve moves continuously toward the origin. In this model (see Section 6.2), the residual demand curve facing country 1 firms flattens as integration increases, with the price-axis intercept moving toward the origin. If the relation between fixed cost and market size means that $m$ is strictly less than $2n$, there is an integration level at which the residual demand curve facing a single country 1 firm is tangent to its average cost curve, and it makes zero economic profit. For greater integration levels, country 1 firms make economic losses if they remain in the market.

The impact of market integration on firm profit is illustrated in Figure 3 for the case $n = m = 1$. Both firms make nonnegative (and in general positive) profit as integration begins, $\iota = 0$. As integration increases, residual demand curves flatten and fall toward the origin. By our assumptions about the ranking of fixed costs, the high-fixed cost country 1 firm sees its profit go to zero earlier in the integration process than does the low-fixed cost country 2 firm. For the parameter values used to draw Figure 3, the country 1 firm’s profit equals zero at integration level $\iota = 0.5$, when its residual demand curve is tangent to its average cost curve. If both firms remain in the market, firm, the country 2 firm’s residual demand curve is tangent to its average cost curve for integration level $\iota = 0.925$. For greater integration levels, both firms lose money if both remain active.

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20 To minimize visual clutter, residual marginal revenue curves and the marginal cost curve are omitted from Figure 3. However, $q_1 = 60$ is firm 1’s noncooperative duopoly equilibrium output for integration level $\iota = 0.5$; $q_2 = 65.8$ is firm 2’s noncooperative duopoly equilibrium output for integration level $\iota = 0.925$. 

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http://www.bepress.com/bejeap/vol8/iss1/art33
Withdrawal integration levels, Cournot duopoly, partially integrated market. Each residual demand curve is drawn for the other firm producing its equilibrium output at the indicated integration level. \( a = 110, c = 10, b = 1, F_1 = 2400, F_2 = 2250 \).

By way of notation, let \( \pi_1 (n + 1 - j, n) \) denote the profit of a country 1 firm in the partially-integrated market if \( n + 1 - j \) country 1 firms and \( n \) country 2 firms are active, for \( j = 1, 2, 3, ..., 2n - m - 1, 2n - m \).\(^{21}\) Define \( \iota_j \) as the integration level at which \( \pi_1 (n + 1 - j, n) = 0 \). The mixed-withdrawal strategy subgame perfect equilibrium without state aid is given by Proposition 1, which is proven in the Appendix.

**Proposition 1:** With firms competing as Nash-Cournot oligopolists throughout, it is a subgame perfect equilibrium for all firms to produce from integration level 0 to integration level \( \iota_1 \), for one country 1 firm to withdraw when integration level \( \iota_1 \) is reached, for a second country 1 firm to withdraw when integration level \( \iota_2 \) is reached,

\(^{21}\)As \( j \) rises from 1 to \( 2n - m \), the number of active country 1 firms falls from \( n \) to \( m - n + 1 \).
..., for an \((m - n)\)th country 1 firm to withdraw when integration level \(\tau_{2n-m}\) is reached, and for no further exit to occur.

The equilibrium is unique up to permutations of country 1 firms.

### 4.2 State aid

We measure welfare as the sum of consumer surplus and producer surplus (economic profit). For cases in which subsidies are granted, we assume that the social cost of a euro of aid is one euro. It may well be — particularly in view of the self-imposed budget constraints of the Stability and Growth Pact — that the opportunity cost of granting aid to a firm is more than one euro. If the welfare cost of a euro of aid is more than a euro, then it is less likely that the net impact of aid will be beneficial, all else equal.

#### 4.2.1 By country 1

In the spirit of the early strategic trade policy literature, suppose first that country 1 and only country 1 can commit to giving its home firms lump-sum aid in the amount of any losses home firms might sustain. The out-of-pocket cost of this policy to country 1 is the discounted value of the subsidies. With subsidies, country 1 firms are guaranteed at least a normal flow rate of return on investment, and would not exit the market. The loss-minimizing action for country 2 firms is then for \(n - m\) of them to exit at zero-profit integration levels defined analogously to the country 1-firm exit integration levels of Proposition 1.

The benefits to country 1 are the (appropriately discounted) economic profits of its firms, which includes profit on sales made in country 2, as well as additional consumer surplus to those country 1 consumers who are not in the integrated market during the integration period but who purchase in an oligopoly submarket supplied by a larger number of country 1 firms. The cost to country 1 is the discounted value of the subsidies, which end once \(n - m\) country 2 firms have withdrawn from the market. Unless discount rates are very high, the net benefit to country 1 will be positive, and granting the subsidy will be beneficial for country 1.

A subsidy by country 1 imposes costs on country 2: the profits that some country 2 firms would otherwise earn are lost after they exit, and some surplus that country 2 consumers would otherwise enjoy in the partially integrated market is lost. A subsidy also reduces the overall economic benefit from integration. The globally-efficient outcome is that all country 2 firms, which have
4.2.2 By both countries

If country 2 can also commit to loss-neutralizing subsidies for its home firm, it can avoid the losses that would be inflicted by a unilateral country 1 subsidy. Subsidies would continue forever. Consumers would be better off, but net welfare, taking subsidies into account, would be reduced, compared with the no-subsidy case. Further, all potential welfare gains from integration would be lost, since there would be no saving of fixed cost.

We focus on welfare flows in the fully-integrated market, which are given in Table 2.22,23 Proposition 2, which is proven in Appendix 6.4, summarizes incentives for country 2 to grant subsidies.

**Proposition 2**: (a) If \( n = m = 1 \) (pre- and post-integration markets are natural monopolies), if country 1 commits to subsidizing lower fixed cost, supply the integrated market. Subsidies granted by country 1 to its firms impose higher fixed cost on the integrated market forever.

### Table 2: Alternative welfare welfare flows per unit time interval, fully-integrated market

The first element in each sum is firms’ profits. the second is consumer surplus. The upper expression in each cell refers to country 1, the second expression in each cell refers to country 2. \( x = (a - c)^2/b \); see text for definition of parameters.

<table>
<thead>
<tr>
<th>Country</th>
<th>No subsidy</th>
<th>Subsidy</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( (m-n) \left( \frac{2}{(m+1)^2} x - F_1 \right) + \frac{1}{2} \left( \frac{m}{m+1} \right)^2 x )</td>
<td>( (m-n) \left( \frac{2}{(m+1)^2} x - F_1 \right) + \frac{1}{2} \left( \frac{m}{m+1} \right)^2 x )</td>
</tr>
<tr>
<td>2</td>
<td>( n \left( \frac{2}{(m+1)^2} x - F_2 \right) + \frac{1}{2} \left( \frac{m}{m+1} \right)^2 x )</td>
<td>( n \left( \frac{2}{(m+1)^2} x - F_2 \right) + \frac{1}{2} \left( \frac{m}{m+1} \right)^2 x )</td>
</tr>
</tbody>
</table>

22 The expressions for discounted welfare in the partially-integrated market must be evaluated numerically. If the interest rate used for discounting is sufficiently low, qualitative results for discounted values must be the same as the qualitative relationships of welfare flows in the fully-integrated market, which can be evaluated analytically. Numerical evaluations of present-discounted welfare comparisons for the \( n = m = 1 \) case are reported in the working paper version of this paper.

23 The elements of the upper-right cell of the table are shown for completeness. Given our assumptions, it the high-fixed-cost country does not commit to subsidizing its firms, the low-fixed-cost country would have no occasion to actually grant subsidies, since its firms would never suffer losses; flow welfare values are the same as those in the upper-left cell.
losses of its firms and country 2 can commit to subsidizing losses of its home firm, it will increase its flow welfare in the fully-integrated market by doing so. In this case, subsidies continue forever; consumers are better off, and overall net welfare is reduced, compared with the no-subsidy case.

(b) For larger values of \( n \) and \( m \), if country 1 commits to subsidizing losses of its firms, and country 2 can commit to subsidizing losses of its home firms, it would decrease its flow welfare in the fully-integrated market by doing so. If country 2 maximizes its net social welfare, short-run subsidies by country 1 leave flow consumer welfare in the fully-integrated market as in the no-subsidy case, increase country 1 net social welfare at the expense of country 2, and permanently shift some production to higher-fixed-cost firms.

Except for the case in which the pre- and post-integration markets are natural monopolies \((n = m = 1)\), country 2 would not increase its welfare by subsidizing its own firms, given subsidies by country 1.

This result may be one explanation for the persistence of EU Member State attempts to grant aid that contravenes the competition policy provisions of the EC Treaty: granting aid by at least one country during the integration process is the noncooperative welfare-maximizing equilibrium outcome.

Two aspects of the situation, both outside our formal model, may explain Member State agreement on those Treaty provisions. The first is that, given government budget constraints, the social welfare cost of each euro used to subsidize a private firm is very likely more than one euro (Neary, 1994; Collie, 2003). Then the welfare gains from subsidizing the home country firm are less than those used for Proposition 2.

Second, one may note that for a common market, it is the welfare effects in all markets, not any one market, that are of interest. In one market, country 1 may better itself at the expense of country 2 by granting aid to home country firms. In another market, it is country 2 that will come out ahead if both countries subsidize. Taking all markets into account restores a situation in which a commitment by both countries not to grant aid, thus maximizing overall welfare, can be a noncooperative equilibrium.

5 Conclusion

Paradoxically, market integration, which expands the size of the market available to each firm, has some economic implications in common with those of
declining markets. In declining markets, the equilibrium number of firms falls over time. In integrating markets, if the integer constraint is ignored, the equilibrium number of firms falls with market integration. If the integer constraint is taken into account, then except for a narrow range of fixed cost, the equilibrium number of firms does not rise, and in general falls, with market integration. Most often, realization of the full efficiency benefits that flow from market integration dictates that some firms must go out of business. In such cases, it is a result of the integration process that it is in the national interest of some component market governments to grant state aid to their own firms, even though such aid reduces or eliminates the economic benefits that flow from integration. This in turn justifies binding inter-state agreements that prohibit distortionary state aid.

The observation that EU member states, having agreed to control of state aid, have a history of granting aid that is regularly found to violate Treaty provisions, invites explanation. One part of such an explanation lies in time inconsistency, as national governments find it convenient to deal with conjunctural crises, especially in the run-up to national elections, with policy choices that will (under the Treaty) be neutralized later. Another part of the explanation may be that penalties have not always been sufficient (it is only relatively recently that aid found to violate guidelines has been recovered). But another part of the explanation may be that when it is consumer behavior that spreads the integration process over time, if attention is confined to individual product markets, granting of subsidies is an equilibrium outcome, even though this reduces common market welfare for that product market.

6 Appendices

6.1 Table 1

Expressing per-firm payoffs in terms of the underlying parameters of the model, the conditions for the pre-integration equilibrium number of firms to be \( n \), (4), becomes

\[
b \left( \frac{1}{n+1} \frac{a-c}{b} \right)^2 \geq F_i > b \left( \frac{1}{n+2} \frac{a-c}{b} \right)^2. \tag{11}\]

Similarly, the conditions for the post-integration equilibrium number of firms to be \( m \), (5), is

\[
2b \left( \frac{1}{m+1} \frac{a-c}{b} \right)^2 \geq F_i > 2b \left( \frac{1}{m+2} \frac{a-c}{b} \right)^2. \tag{12}\]
Evaluation of the inequalities for \( n = 1, \ldots, 8 \) and \( m = 1, \ldots, 11 \) gives the upper and lower limits of fixed cost indicated in the table.

Subtraction shows that each row identifies a proper interval:\(^{24}\) row 1: \( \left( \frac{1}{3} \right)^2 - 2 \left( \frac{1}{3} \right)^2 = \frac{1}{36} > 0 \); row 2: \( 2 \left( \frac{1}{3} \right)^2 - 2 \left( \frac{1}{4} \right)^2 = \frac{7}{72} > 0 \); row 3: \( 2 \left( \frac{1}{4} \right)^2 - \left( \frac{1}{5} \right)^2 = \frac{1}{72} \); row 4: \( \left( \frac{1}{3} \right)^2 - 2 \left( \frac{1}{5} \right)^2 = \frac{7}{225} > 0 \); row 5: \( 2 \left( \frac{1}{5} \right)^2 - \left( \frac{1}{7} \right)^2 = \frac{7}{400} > 0 \); row 6: \( \left( \frac{1}{3} \right)^2 - 2 \left( \frac{1}{7} \right)^2 = \frac{7}{117} > 0 \); row 7: \( 2 \left( \frac{1}{7} \right)^2 - 2 \left( \frac{1}{8} \right)^2 = \frac{10}{882} > 0 \); row 8: \( 2 \left( \frac{1}{7} \right)^2 - \left( \frac{1}{8} \right)^2 = \frac{1}{1225} > 0 \); row 9: \( \left( \frac{1}{7} \right)^2 - 2 \left( \frac{1}{8} \right)^2 = \frac{7}{800} > 0 \); row 10: \( 2 \left( \frac{1}{8} \right)^2 - \left( \frac{1}{9} \right)^2 = \frac{1}{288} > 0 \); row 11: \( \left( \frac{1}{6} \right)^2 - 2 \left( \frac{1}{9} \right)^2 = \frac{1}{324} > 0 \); row 12: \( 2 \left( \frac{1}{9} \right)^2 - \left( \frac{1}{10} \right)^2 = \frac{17}{3600} > 0 \); row 13: \( \left( \frac{1}{6} \right)^2 - 2 \left( \frac{1}{10} \right)^2 = \frac{2450}{1250} > 0 \); row 14: \( 2 \left( \frac{1}{10} \right)^2 - 2 \left( \frac{1}{11} \right)^2 = \frac{7}{660} > 0 \); row 15: \( 2 \left( \frac{1}{11} \right)^2 - \left( \frac{1}{12} \right)^2 = \frac{7}{744} > 0 \); row 16: \( \left( \frac{1}{8} \right)^2 - 2 \left( \frac{1}{12} \right)^2 = \frac{1}{576} > 0 \); row 17: \( 2 \left( \frac{1}{12} \right)^2 - \left( \frac{1}{13} \right)^2 = \frac{7}{1389} > 0 \); row 18: \( \left( \frac{1}{9} \right)^2 - 2 \left( \frac{1}{13} \right)^2 = \frac{7}{648} > 0 \).

Combining (11) and (12), the conditions for \( n \) to be the pre-integration number of firms and \( m \) to be the post-integration number of firms are

\[
\min \left[ \frac{1}{(n+1)^2}, \frac{2}{(m+1)^2} \right] x \geq F_i > \max \left[ \frac{1}{(n+2)^2}, \frac{2}{(m+2)^2} \right] x
\]

(writing \( x = b \left( \frac{a-c}{b} \right)^2 \) for notational compactness).

In principle, there are four possible cases,

A:

\[
\frac{1}{(n+1)^2} x \geq F_i > \frac{1}{(n+2)^2} x.
\]

B:

\[
\frac{1}{(n+1)^2} x \geq F_i > \frac{2}{(m+2)^2} x.
\]

C:

\[
\frac{2}{(m+1)^2} x \geq F_i > \frac{1}{(n+2)^2} x.
\]

D:

\[
\frac{2}{(m+1)^2} x \geq F_i > \frac{2}{(m+2)^2} x.
\]

Case A cannot occur, which can be shown by assuming it does, which implies

\[
\frac{2}{(m+1)^2} \geq \frac{1}{(n+1)^2} \geq \frac{F_i}{x} > \frac{1}{(n+2)^2} \geq \frac{2}{(m+2)^2}.
\]

Manipulation shows that the first and last inequalities, along with \( 2n - m \geq 0 \), are mutually inconsistent. Thus in the proof of Proposition 2, attention can be confined to cases B, C, and D.

\(^{24}\)We omit the term \((a-c)^2/b\) that is common to all expressions.
6.2 Residual demand in the partially-integrated market

What follows justifies the discussion of Figure 3.

6.2.1 Country 1 firms

Substituting the expressions for equilibrium outputs in the partially-integrated market, (9) and (10), into the demand equation facing country 1 firms, (7), the equation of the residual demand curve facing a single country 1 firm in the partially-integrated market if all other firms produce their equilibrium values is

\[ p_1 = c + 2 \frac{1 + (1 - \iota) n_2}{1 + n_1 + n_2 + (1 - \iota^2) n_1 n_2} (a - c) - \frac{b}{1 + \iota} q_1. \]  

(13)

As integration proceeds (as \( \iota \) increases from 0 to 1), the slope of the residual demand curve decreases (in absolute value) from \(-1\) to \(-1/2\): the demand curve flattens, rotating in a counterclockwise direction about the price-axis intercept.

The part of the price-axis intercept that is affect as integration goes forward is

\[ \frac{1 + (1 - \iota) n_2}{1 + n_1 + n_2 + (1 - \iota^2) n_1 n_2}. \]

The derivative with respect to \( \iota \) is

\[ \frac{\partial}{\partial \iota} \left( \frac{1 + (1 - \iota) n_2}{1 + n_1 + n_2 + (1 - \iota^2) n_1 n_2} \right) = -n_2 \frac{1 + n_1 + n_2 + n_1 n_2 - \iota n_1 [2 + (2 - \iota) n_2]}{(1 + n_1 + n_2 + (1 - \iota^2) n_1 n_2)^2}. \]

The numerator on the right-hand side,

\[ 1 + n_1 + n_2 + n_1 n_2 - \iota n_1 [2 + (2 - \iota) n_2], \]

is positive for \( \iota = 0 \) and positive for \( \iota = 1 \) provided \( n_2 \geq n_1 \), which is the case in the absence of subsidies. The coefficient of \( \iota \), \( n_1 [2 + (2 - \iota) n_2] \), is positive. Hence the numerator is positive for \( \iota = 0 \), becomes smaller as \( \iota \) rises, but remains positive for \( \iota = 1 \). It follows that the derivative is negative: the price-axis intercept of the residual demand curve facing a single country 1 firm moves toward the origin as integration increases.
6.2.2 Country 2 firms

Similarly, the residual demand equation facing a single country 2 firm in the partially-integrated market is

\[ p_2 = c + 2 \frac{1 + (1 - \iota) n_1}{1 + n_1 + n_2 + (1 - \iota^2) n_1 n_2} (a - c) - \frac{b}{1 + \iota} q_2. \]  

(14)

The residual demand curve flattens as integration increases. The derivative of the price-axis intercept with respect to \( \iota \) is

\[ \frac{\partial}{\partial \iota} \left( \frac{1 + (1 - \iota) n_1}{1 + n_1 + n_2 + (1 - \iota^2) n_1 n_2} \right) = \]

\[ - \frac{n_1 (1 + n_1 + n_2 + n_1 n_2 - \iota n_2 [2 + (2 - \iota) n_1])}{[1 + n_1 + n_2 + (1 - \iota^2) n_1 n_2]^2}. \]

The numerator on the right, 

\[ 1 + n_1 + n_2 + n_1 n_2 - \iota n_2 [2 + (2 - \iota) n_1], \]

is positive for \( \iota = 0 \). For \( \iota = 1 \), it is positive for \( n_2 = n_1 \), zero for \( n_1 = n_2 - 1 \) (after the first country 1 firm drops out) and positive thereafter. For low levels of integration, the price-axis intercept of the country 2 firm residual demand curve falls as integration increases. For high levels of integration, in the absence of subsidies, the price-axis intercept of country 2 firms rises as integration proceeds.

6.3 Proof of Proposition 1

The argument of the proof is, with one difference, that of Brainard (1994, Section 2.A). Brainard derives a subgame perfect equilibrium strategy for her declining industry model by working backward from times at which either firm would earn zero profit even as a monopolist. In the model considered here, the analysis works backward from integration levels at which firms would have zero values playing mixed withdrawal strategies.

Define \( \iota_j = \) integration level at which \( \pi_1 (n + 1 - j, n) = 0 \) for all \( n + 1 - j \) country 1 firms, with all \( n \) country 2 firms active. Then along the candidate equilibrium path, for \( j = 1 \), the first country 1 firm withdraws; for \( j = 2 \), the second country 1 firm withdraws, \ldots, for \( j = 2n - m - 1 \), the \((2n - m - 1)\)th country 1 firm withdraws, and for \( j = 2n - m \), the \((2n - m)\)th country 1 firm withdraws.

Write \( \tau_j \) for the time at which integration level \( \iota_j \) is reached.

The proof is by induction.
6.3.1 Step 1: suppose integration level $\iota_{2n-m}$ has been reached.

There are $m - n + 1$ country 1 firms active and $n$ country 2 firms active.

(A) At any point in time from $\tau_{2n-m}$ onward,

$\sigma_{1j} =$ withdrawal probability of country 1 firm $j$, $j = 1, 2, 3, ..., m - n + 1$,

$\sigma_{2j} =$ withdrawal probability of country 2 firm $j$, $j = 1, 2, 3, ..., n$.

Equilibrium withdrawal strategies vary with time during the integration period and are constant in the fully-integrated market.

For notational compactness, omit to write the functional dependence of withdrawal probabilities on time during the integration period if this is possible without confusion.

If one firm exits, all remaining firms make nonnegative profit; no further exit occurs.

If all firms are in the market at time $T$ or any time thereafter, it is a sub-game perfect equilibrium for the firms to play the mixed withdrawal strategies given below. Equilibrium expected values playing these strategies are zero.

6.3.2 Derive equilibrium withdrawal probabilities in the fully integrated market.

In this section, derive withdrawal probabilities for the fully-integrated market that make firms’ expected values equal to zero if $m + 1$ firms are active.

The fully integrated market inverse demand equation is

$$p_i = a - \frac{1}{2}b(Q_1 + Q_2).$$

If $m - n + 1$ country 1 firms are active in the fully integrated market, flow duopoly profits per firm are

$$\pi_1^f(m + 1) = 2b \left( \frac{1}{m + 2} \frac{a - c}{b} \right)^2 - F_1 < 0$$

and

$$\pi_2^f(m + 1) = 2b \left( \frac{1}{m + 1} \frac{a - c}{b} \right)^2 - F_2 < 0$$

for country 1 and country 2 firms, respectively. Profit rates depend on the total number of active firms $(m + 1)$, not on the distribution of active firms across countries.

Let $\sigma_{ij}$ be country $i$ firm $j$’s probability of withdrawal at time $t \geq T$, for $i = 1, 2$, and $j$ as above.
Withdrawal probabilities are constant for any \( t \geq T \) at which both firms are in the market, since their expected payoffs in different states of the world are the same at all such times. Withdrawal strategies follow an exponential distribution; the probability that firm 1\( j \) drops out before time \( t \), given that no other firm has dropped out, is

\[
1 - e^{-\sigma_{1j} t}.
\]

With probability density \( \left[ \exp - \left( \sum_{j=1}^{m-n+1} \sigma_{1j} + \sum_{j=1}^n \sigma_{2j} \right) t \right] dt \), no firm has dropped out at time \( t \), and any country 1 firm’s payoff is \( \pi^I_1 (m + 1) < 0 \).

With probability density \( \sigma_{1k} \left[ \exp - \left( \sum_{j=1}^{m-n+1} \sigma_{1j} + \sum_{j=1}^n \sigma_{2j} \right) t \right] dt \), country 1 firm 1\( k \) drops out at time \( t \), no other firm has dropped out, and the value of the exiting country 1 firm from that time onward is 0.

With probability density

\[
\left( \sum_{j=1, j \neq k}^{m-n+1} \sigma_{1j} + \sum_{j=1}^n \sigma_{2j} \right) \left[ \exp - \left( \sum_{j=1}^{m-n+1} \sigma_{1j} + \sum_{j=1}^n \sigma_{2j} \right) t \right] dt,
\]

it is one of the other firms (country 1 or country 2) that drops out first, and country 1 firm 1\( k \)’s value from that point onward is

\[
\frac{\pi^I_1 (m)}{r}.
\]

Integrating over all future time, firm 1\( k \)’s discounted expected value at time \( t \geq T \) if \( m + 1 \) firms are in the market is

\[
V^I_1 = \int_0^\infty e^{-(r + \sum_{j=1, j \neq k}^{m-n+1} \sigma_{1j} + \sum_{j=1}^n \sigma_{2j}) t} \left[ \sigma_1 (0) + \left( \sum_{j=1, j \neq k}^{m-n+1} \sigma_{1j} + \sum_{j=1}^n \sigma_{2j} \right) \frac{\pi^I_1 (m)}{r} + \right] dt.
\]

(15)

Evaluate this integral and consider symmetric equilibria, so \( \sigma_{1j} = \sigma_1 \) for all country 1 firms and \( \sigma_{2j} = \sigma_2 \) for all country 2 firms.

Then the value of a typical country 1 firm in the fully-integrated market is

\[
V^I_1 = \frac{[(m - n) \sigma_1 + n \sigma_2] \frac{\pi^I_1 (m)}{r} + \pi^I_1 (m + 1)}{r + (m - n) \sigma_1 + n \sigma_2}.
\]

In the same way, the value of a typical country 2 firm from \( t \geq T \) with \( n-m+1 \) country 1 firms in the market and \( n \) country 2 firms in the market is

\[
V^I_2 = \frac{[(m - n + 1) \sigma_1 + (n - 1) \sigma_2] \frac{\pi^I_2 (m)}{r} + \pi^I_2 (m + 1)}{r + (m - n) \sigma_1 + n \sigma_2}.
\]

(16)
For firms to be willing to play random strategies, these values must be zero. Then $\sigma_1$ and $\sigma_2$ are obtained by solving the equations

$$ (m - n) \sigma_1 + n \sigma_2 = \frac{-\pi^f_1 (m + 1)}{\pi^f_1 (m) / r} $$

$$ (m - n + 1) \sigma_1 + (n - 1) \sigma_2 = \frac{-\pi^f_2 (m + 1)}{\pi^f_2 (m) / r}. $$

The solutions are

$$ \sigma_1 = \frac{1}{m} \left[ n \frac{-\pi^f_2 (m + 1)}{\pi^f_2 (m) / r} - (n - 1) \frac{-\pi^f_1 (m + 1)}{\pi^f_1 (m) / r} \right] $$

and

$$ \sigma_2 = \frac{1}{m} \left[ (m - n + 1) \frac{-\pi^f_1 (m + 1)}{\pi^f_1 (m) / r} - (m - n) \frac{-\pi^f_2 (m + 1)}{\pi^f_2 (m) / r} \right]. $$

The withdrawal probabilities are positive for $n = m = 1$. For larger values of $n$, limit attention to parameter values for which the withdrawal probabilities are nonnegative; this is the economically interesting case. Given this assumption, withdrawal probabilities are less than one for $r$ sufficiently small. Assume this to be the case.

### 6.3.3 Next steps

If the fully-integrated market is reached with $m + 1$ firms active, playing the withdrawal probabilities defined in the previous section makes firms’ expected values equal to zero.

At integration level $\iota_{2n-m}$, the profits of all $2n - m + 1$ country 1 firms equal zero.

The profit of each country 2 firm is positive.

If there are $m + 1$ firms operating in the fully integrated market, all firms make negative profit.

By the intermediate value theorem, between $\iota = \iota_{2n-m}$ and $\iota = 1$, there is an integration level $\iota (m - n + 1, n)$ at which the profit of any country 2 firm equals zero.

The logic of the argument that follows is to find time-varying withdrawal probabilities over the range of integration levels

$$ \iota (m - n + 1, n) \leq \iota \leq 1 $$

that make firms’ values zero.
Then argue that for integration levels in the range
\[ \nu_{2n-m} \leq \nu \leq \nu (m - n + 1, n), \]
all country 2 firms make positive profit, hence will not withdraw, while the country 1 firms make losses. Hence it will be value-maximizing for one of the \( n - m + 1 \) country 1 firms will withdraw, to avoid a period of losses followed by periods over which its value is zero.

Then continue the proof by induction.

So consider integration levels within the range
\[ \nu (m - n + 1, n) \leq \nu \leq 1, \]
and suppose \( m + 1 \) firms are active.

Let \( \sigma_{1j}(t) \) be the probability that country 1 firm \( j \) withdraws at time \( t \) during the integration period, conditional on not having withdrawn before.\(^{25}\)

For notational convenience, write
\[
S_{1j}(t) = \int_{\tau=0}^{t} \sigma_{1j}(\tau) \, d\tau. \tag{17}
\]

With probability density
\[
\left[ \exp - \left( \sum_{j=1}^{m-n+1} S_{1j} + \sum_{j=1}^{n} S_{2j} \right) t \right] dt,
\]
no firm has dropped out at time \( t \), and any country 1 firm’s payoff is \( \pi_1^I (m + 1) < 0 \).

With probability density
\[
\sigma_{1k}(t) \left[ \exp - \left( \sum_{j=1}^{m-n+1} S_{1j} + \sum_{j=1}^{n} S_{2j} \right) t \right] dt,
\]
firm 1\( k \) drops out at time \( t \), no other firms have yet dropped out, and firm 1\( j \)’s value from that point onward is 0.

With probability density
\[
\left( \sum_{j=1, j \neq k}^{m-n+1} \sigma_{1j} + \sum_{j=1}^{n} \sigma_{2j} \right) \left[ \exp - \left( \sum_{j=1}^{m-n+1} S_{1j} + \sum_{j=1}^{n} S_{2j} \right) t \right] dt,
\]
\(^{25}\)For a similar formulation in another context, see Fudenberg et al. (1983).
it is one of the other firms that drops out first, and firm 1$k$’s value from that moment onward is

\[ V_{1k}(m,t) = \int_{T}^{T} \pi_{1j}(m-n,n;\tau) e^{-r\tau} d\tau + e^{-rT} \frac{\pi_{1j}(m-n,n;1)}{r}. \]

Firm 1$k$’s expected value at time $t$ during integration levels in the range $\nu(m-n+1,n) \leq \nu \leq 1$ if $m+1$ firms are active is

\[ V_{1k}(m+1,t) = \int_{t}^{\infty} e^{-[r\tau+\sum_{j=1}^{m-n+1} S_{1j}+\sum_{j=1}^{n} S_{2j}]} \times \left[ \sigma_{1k}(t)(0) + \left( \sum_{j=1, j \neq k}^{m-n+1} \sigma_{1j} + \sum_{j=1}^{n} \sigma_{2j} \right) V_{1k}(m,t) + \pi_{1k}(m-n+1,n) \right] d\tau = \int_{t}^{\infty} e^{-[r\tau+\sum_{j=1}^{m-n+1} S_{1j}+\sum_{j=1}^{n} S_{2j}]} \times \left[ \left( \sum_{j=1, j \neq k}^{m-n+1} \sigma_{1j} + \sum_{j=1}^{n} \sigma_{2j} \right) V_{1k}(m,t) + \pi_{1k}(m-n+1,n) \right] d\tau. \]

Now pass to symmetric equilibria: $\sigma_{1j} = \sigma_{1}$ for all $j$, $\sigma_{2j} = \sigma_{2}$ for all $j$.

The typical country 1 firm’s symmetric equilibrium value is

\[ V_{1}(m+1,t) = \int_{t}^{\infty} e^{-[r\tau+(m-n+1)S_{1}(\tau)+nS_{2}(\tau)]} \times \left[ \left( (m-n) \sigma_{1} + n \sigma_{2} \right) V_{1}(m,\tau) + \frac{\pi_{1}(m-n+1,n,\tau)}{\pi_{1}} \right] d\tau. \]

Similarly, the typical country 2 firm’s symmetric equilibrium value over the same range of integration levels with $m+1$ firms active is

\[ V_{2}(m+1,t) = \int_{t}^{\infty} e^{-[r\tau+(m-n+1)S_{1}(\tau)+nS_{2}(\tau)]} \times \left[ \left( (m-n+1) \sigma_{1} + (n-1) \sigma_{2} \right) V_{2}(m,\tau) + \frac{\pi_{2}(m-n+1,n,\tau)}{\pi_{2}} \right] d\tau. \]

In order for the firms to be willing to play mixed strategies, these expected values must be zero, which requires that the expressions in brackets under the integral signs be zero,

\[ ((m-n) \sigma_{1} + n \sigma_{2}) V_{1}(m,t) + \pi_{1}(m-n+1,n,t) = 0 \]

and

\[ ((m-n+1) \sigma_{1} + (n-1) \sigma_{2}) V_{2}(m,t) + \pi_{2}(m-n+1,n,t) = 0. \]
Rewrite these equations as

\[(m - n) \sigma_1 + n \sigma_2 = -\frac{\pi_1 (m - n + 1, n, t)}{V_1 (m, t)}\]

and

\[(m - n + 1) \sigma_1 + (n - 1) \sigma_2 = -\frac{\pi_2 (m - n + 1, n, t)}{V_2 (m, t)}\].

The (now, time-dependent) withdrawal probabilities are the solutions to the system of equations.

As before, limit attention to parameter values for which the withdrawal probabilities are positive. If \( r \) is sufficiently small, the withdrawal probabilities will be less than one.

Between integration levels \( \iota_{2n-m} \) and \( \iota (m - n + 1, n) \), all country 1 firms lose money, all country 2 firms make money. The country 2 firms will not withdraw.

After integration level \( \iota (m - n + 1, n) \), all firms just break even. Hence there are \( m - n + 1 \) symmetric equilibria, in each of which one of the \( m - n + 1 \) country 1 firms withdraws at integration level \( \iota_{2n-m} \) and all other firms remain in the market.

Next step in the proof by induction: suppose it has been shown that for integration level \( \iota_{a+1} \), for \( a + 1 \geq 2 \), that it has been shown that there are \( n - a \) symmetric equilibria in which one of the country 1 firms withdraws.

Show that if there are \( n - a + 1 \) country 1 firms and \( n \) country 2 firms remaining at integration level \( \iota_a \), then there are \( n - a + 1 \) symmetric equilibria in which one of the \( n - a + 1 \) country 1 firms withdraws at integration level \( \iota_a \).

Sketch of a proof:

(a) at integration level \( \iota_a \), all country 1 firms lose money, all country 2 firms make positive profit.

(b) at integration level 1, all firms lose money if there are \( n - a + 1 + n = 2n - a + 1 \) firms active.

(c) by the intermediate value theorem, there is some integration level \( \iota (n - a + 1, n) \), with

\[\iota_a < \iota (n - a + 1, n) < 1,\]

at which country 2 firms just break even.

(d) as above, define time-dependent withdrawal probabilities that make firms’ expected values equal to zero for integration levels between \( \iota (n - a + 1, n) \) and 1 and time-independent withdrawal probabilities for the \( 2n - a + 1 \) firms in the fully-integrated market that make expected values equal to zero.
(e) then country 1 firms at integration level \( \iota_a \) face a period of losses followed by expected values equal to zero. Hence it is an equilibrium for one of them to withdraw, and there are \( n - a + 1 \) such equilibria. This completes the proof.

6.4 Proof of Proposition 2

6.4.1 Fully-integrated market, no subsidies

In the fully-integration market with \( n_1 \) country 1 firms active and \( n_2 \) country 2 firms active, the integrated market is an \((n_1 + n_2)\)-firm Cournot oligopoly in a market with inverse demand equation

\[
p = a - \frac{1}{2}bQ.
\]

Cournot equilibrium output per firm is

\[
q = \frac{1}{n_1 + n_2 + 1} \frac{a - c}{b/2} = \frac{2}{n_1 + n_2 + 1} \frac{a - c}{b}.
\]

Writing \( x = b \left( \frac{a-c}{b} \right)^2 \) for notational compactness, equilibrium profit per firm is (where the subscript \( i \) now denotes the country of origin of the firm)

\[
\pi_i = \frac{2}{(n_1 + n_2 + 1)^2} x - F_i.
\]

In the fully-integrated market, if there is no state aid that distorts the exit process, there are \( n \) country 2 firms active and \( m - n \) country 1 firms active. The total profit of the \( m - n \) surviving country 1 firms is

\[
(m - n) \left[ \frac{2}{(m + 1)^2} x - F_1 \right].
\]

The total profit of the \( n \) country 2 firms is

\[
n \left[ \frac{2}{(m + 1)^2} x - F_2 \right].
\]

Price is

\[
p = c + \frac{1}{m + 1} (a - c).
\]

Consumer surplus in each country is

\[
\frac{1}{2} \left( \frac{m}{m + 1} \right)^2 x.
\]
Net social welfare for country 1 if there are no subsidies is the sum of profit and consumer surplus,

\[(m - n) \left[ \frac{2}{(m + 1)^2} x - F_1 \right] + \frac{1}{2} \left( \frac{m}{m + 1} \right)^2 x.\]

The two terms could be combined but there is no advantage to doing so. Similarly, net social welfare for country 2 if there are no subsidies is

\[n \left[ \frac{2}{(m + 1)^2} x - F_2 \right] + \frac{1}{2} \left( \frac{m}{m + 1} \right)^2 x.\]

6.4.2 Country 1 subsidies only

As long as \(m\) firms in all operate in the fully-integrated market, consumer surplus is unaffected by the identity of the \(m\) firms that survive. Flow consumer surplus is therefore as in the previous case.

Net social welfare, country 1:

\[n \left[ \frac{2}{(m + 1)^2} x - F_1 \right] + \frac{1}{2} \left( \frac{m}{m + 1} \right)^2 x.\]

Net social welfare, country 2:

\[(m - n) \left[ \frac{2}{(m + 1)^2} x - F_2 \right] + \frac{1}{2} \left( \frac{m}{m + 1} \right)^2 x.\]

6.4.3 Both countries subsidize

The fully-integrated market is a 2n-firm Cournot oligopoly.

Output per firm is

\[q = \frac{2}{2n + 1} \frac{a - c}{b}.\]

Total output: there are 2n firms,

\[Q = \frac{4n}{2n + 1} \frac{a - c}{b}.\]

Price:

\[p = c + \frac{a - c}{2n + 1}.\]
Equilibrium profit per firm is (where the subscript $i$ now denotes the country of origin of the firm)

$$\pi = \frac{1}{2} bq^2 - F = \frac{1}{2} b \left( \frac{2a - c}{2n + 1} \right)^2 - F_i$$

$$= \frac{2}{(2n + 1)^2} x - F_i,$$

and by our assumptions, profit per firm is negative.

Each country grants its firms a subsidy equal to the firms’ losses; the flow rate of subsidy by country $i$ is

$$-n \left[ \frac{2}{(2n + 1)^2} x - F_i \right] > 0.$$  

Consumer surplus: for all consumers

$$\frac{1}{2} \left( a - c \right) \left( \frac{4n}{2n + 1} b \right) = \left( \frac{2n}{2n + 1} \right)^2 x.$$

Consumers in each country get half this.

### 6.4.4 Welfare comparisons

Two comparisons are of primary interest:

- the change in country 1’s flow welfare in the fully-integrated market if it alone grants subsidies, compared with the no-subsidy case;

- the change in country 2’s flow welfare in the fully-integrated market if it grants subsidies, given that country 1 grants subsidies.

It is these values that determine countries’ incentives to grant subsidies.

**First comparison** Since the rate of consumer surplus is the same in both cases, the impact of country 1 subsidies on net social welfare is only the change in the flow of profit to firms based in the different countries.

Country 1: the change in profit is

$$n \left[ \frac{2}{(m + 1)^2} x - F_1 \right] - (m - n) \left[ \frac{2}{(m + 1)^2} x - F_1 \right] =$$

$$(2n - m) \left[ \frac{2}{(m + 1)^2} x - F_1 \right] > 0.$$
Country 2: the change in profit is
\[
(m - n) \left[ \frac{2}{(m + 1)^2} x - F_2 \right] - n \left[ \frac{2}{(m + 1)^2} x - F_2 \right] = \\
- (2n - m) \left[ \frac{2}{(m + 1)^2} x - F_2 \right] < 0.
\]

The overall change in net welfare (both countries together) is the sum of the changes for each country,
\[
- (2n - m) (F_1 - F_2) < 0.
\]

Hence if country 1 grants subsidies and country 2 does not, country 1 welfare increases, country 2 welfare decreases, and overall welfare increases.

**Second comparison**

**Consumers** Consumers are better off — the rate of consumer surplus is greater, since the number of firms is greater. The change in welfare — for both countries — is
\[
\frac{(2n - m) (m + 2n + 4mn)}{(2n + 1)^2 (m + 1)^2} x.
\]

**Country 1 firms** n country 1 firms operate whether country 2 grants subsidies or not. The change in “profit” per firm is
\[
- 2 \frac{2n - m}{(2n + 1) (m + 1)} \left( \frac{1}{2n + 1} + \frac{1}{m + 1} \right) x < 0.
\]

If country 2 grants a subsidy, in addition to country 1, country 1 firms swing from profit to loss.

The change in country 1 flow profit for all n country 1 firms is
\[
- \frac{2n (2n - m) (2n + m + 2)}{(2n + 1)^2 (m + 1)^2} x < 0.
\]

**Country 2 firms** The change in profit of country 2 firms from the country-1-subsidy only case to the both-countries subsidize case is the difference between a negative term (losses when both countries subsidize) and a positive term (profit to $m - n$ firms if only country 1 subsidizes). The difference in profit is
\[
(2n - m) \left[ \frac{2 \frac{2n^2 + 2n + 1 - mn}{(2n + 1)^2 (m + 1)^2} x - F_2 }{2n - m} \right].
\]

We know the change must be negative.
Country 2, overall  The overall change in country 2 welfare is the change in surplus of country 2 consumers (positive) and the change in profit of country 2 firms (negative): $2n - m$ times

$$x - F_2$$

Three cases  We know from Section 6.1 that there are three cases to consider:

B:

$$\frac{1}{(n+1)^2}x \geq F_i > \frac{2}{(m+2)^2}x.$$  

C:

$$\frac{2}{(m+1)^2}x \geq F_i > \frac{1}{(n+2)^2}x.$$  

D:

$$\frac{2}{(m+1)^2}x \geq F_i > \frac{2}{(m+2)^2}x.$$  

Cases B&D and case C must be considered separately, and separately for $m$ odd and $m$ even.

Possible $n$ for each $m$

For cases where there is some exit,

$$2n - m > 0.$$  

$$2n > m > 0.$$  

$$n > \frac{1}{2}m > 0.$$  

$m$ is either odd or even.

If $m$ is odd, $m + 1$ is even.

$m$ even: the possible values of $n$ are $\frac{1}{2}m + 1, \frac{3}{2}m + 2, \ldots, m - 1$.

$m$ odd: the possible values of $n$ are $\frac{1}{2}(m + 1) + 1, \frac{1}{2}(m + 1) + 2, \ldots, m - 1$.

$n$ and the change in country 2 welfare

The derivative of the change in country 2 welfare with respect to $n$ is negative:

$$\frac{\partial}{\partial n} \left( \frac{1}{2} \frac{m + 2n + 4mn}{(2n + 1)^2 (m + 1)^2} + \frac{2}{2n + 2n^2 + 1 - mn} \right) =$$

$$- \frac{2m + 2n + 3}{(2n + 1)^3 (m + 1)^2} < 0.$$
The change in country 2 welfare falls as \( n \) rises. Hence it suffices to sign this expression for the smallest value of \( n \) — if it is negative for the smallest value, it is negative for larger values of \( n \) as well.

**Cases B & D**

\( m \) even; let \( n = \frac{1}{2}m + 1 \). The change in country 2 welfare is

\[
\frac{1}{2} \left( \frac{1}{2}m + 1 \right) + 4m \left( \frac{1}{2}m + 1 \right) + \frac{2}{2} \left( \frac{1}{2}m + 1 \right) + 2 \left( \frac{1}{2}m + 1 \right)^2 + 1 - m \left( \frac{1}{2}m + 1 \right) \]

\[
\frac{2}{( \frac{1}{2}m + 1 )^2 (m + 1)^2},
\]

or \( 2n - m \) times

\[
\frac{7m + m^2 + 11}{(m + 3)^2 (m + 1)^2} x - F_2
\]

In cases B and D,

\[-F_i < \frac{2}{(m + 2)^2 x} < 0.\]

Add \( \frac{7m + m^2 + 11}{(m + 3)^2 (m + 1)^2} x \) to both sides:

\[
\frac{7m + m^2 + 11}{(m + 3)^2 (m + 1)^2} x - F_i < \frac{7m + m^2 + 11}{(m + 3)^2 (m + 1)^2} x - \frac{2}{(m + 2)^2 x}.
\]

Evaluate the right-hand side:

\[
\frac{7m + m^2 + 11}{(m + 3)^2 (m + 1)^2} x - \frac{2}{(m + 2)^2 x} = \frac{26 + 24m - m^2 - 5m^3 - m^4}{(m + 3)^2 (m + 2)^2 (m + 1)^2}.
\]

The numerator on the right, \( 26 + 24m - m^2 - 5m^3 - m^4 \), has real roots at \( m = -1.4395, 2.1712 \) and has negative derivative for \( m \geq 2.1712 \), hence for \( m \geq 3 \). These are the cases of interest (see Table 1). Hence in cases B and D and for even \( m \), if country 1 has committed to subsidize its firms’ losses, country 2 would reduce its flow welfare in the fully-integrated market by committing to subsidize its firms’ losses.

\( m \) odd: let \( n \) take its minimum value, \( n = \frac{1}{2} (m + 1) + 1 = \frac{1}{2}m + \frac{3}{2} \). The change in country 2 flow welfare is then \( 2n - m \) times

\[
\frac{1}{2} \left( \frac{18m + 2m^2 + 37}{(m + 4)^2 (m + 1)^2} x - F_2
\]

http://www.bepress.com/bejeap/vol8/iss1/art33
As in the $m$ even case, from

$$-F_i < -\frac{2}{(m+2)^2}x < 0.$$  

we obtain

$$\frac{1}{2} \frac{18m + 2m^2 + 37}{(m+4)^2 (m+1)^2} x - F_i < \left( \frac{1}{2} \frac{18m + 2m^2 + 37}{(m+4)^2 (m+1)^2} - \frac{2}{(m+2)^2} \right) x$$

$$= -\frac{1}{2} (m-2) \frac{42 + 51m + 18m^2 + 2m^3}{(m+4)^2 (m+2)^2 (m+1)^2},$$

and this is negative for $m \geq 3$.

**Case C**

Here

$$-F_i < -\frac{1}{(n+2)^2}x < 0.$$  

Instances of case C in which exit occurs have $m \geq 6$.

$m$ even: let $n = \frac{1}{2}m + 1$.

From above, the change in country 2’s welfare is $2n - m$ times

$$\frac{7m + m^2 + 11}{(m+3)^2 (m+1)^2} x - F_2$$

From

$$-F_2 < -\frac{1}{\left( \frac{1}{2}m + 1 + 2 \right)^2}x$$

we obtain

$$\frac{7m + m^2 + 11}{(m+3)^2 (m+1)^2} x - F_2 < \left[ \frac{7m + m^2 + 11}{(m+3)^2 (m+1)^2} - \frac{1}{\left( \frac{1}{2}m + 1 + 2 \right)^2} \right] x$$

$$= -\frac{-288m - 43m^2 + 13m^3 + 3m^4 - 360}{(m+6)^2 (m+3)^2 (m+1)^2} x.$$  

The numerator on the far right, $-288m - 43m^2 + 13m^3 + 3m^4 - 360$, has its largest real root at $m = 4.5893$, and positive derivative for larger values of $m$. Subsidies by country 2 in cases of interest would therefore reduce country 2 welfare.

$m$ odd: let $n$ take its minimum value, $n = \frac{1}{2} (m+1) + 1 = \frac{1}{2}m + \frac{3}{2}$.
In this case,

$$-F_2 < -\frac{1}{\left(\frac{3}{2}m + \frac{3}{2} + 2\right)^2} x < 0.$$ 

From the consideration of the $m$ odd for cases B and D, the change in country 2 welfare is $2n - m$ times

$$\frac{1}{2} \frac{18m + 2m^2 + 37}{(m + 4)^2 (m + 1)^2} x - F_2$$

Hence

$$\frac{1}{2} \frac{18m + 2m^2 + 37}{(m + 4)^2 (m + 1)^2} x - F_2 < \left[ \frac{1}{2} \frac{18m + 2m^2 + 37}{(m + 4)^2 (m + 1)^2} - \frac{1}{\left(\frac{3}{2}m + \frac{3}{2} + 2\right)^2} \right] x$$

$$= -\frac{1}{2} \frac{16m^4 + 34m^3 - 123m^2 - 1080m - 1685}{(m + 4)^2 (m + 7)^2 (m + 1)^2}.$$ 

The numerator on the far right has its largest real root at $m = 5.5422$ and positive derivative for larger values of $m$. This suffices for the proof.

References


