



Competition Policy for High Technology Industries

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Abstract. I present a model of optimal product-market competition policy when industries differ in the potential for quality-improving technological advance. In a two-period, model, a competition authority with limited resources administers a deterrence-based competition policy toward two industries. In one of the industries, an incumbent firm chooses the level of resources to invest in a quality-improving R&D project. In the other industry, product quality is constant. Optimal policy requires the competition authority to administer a tougher competition policy before innovation, all else equal, the greater the potential quality improvement. I derive basic results for the case of one-time innovation, and extend them to the cases of sequential innovation and patent protection that confers limited antitrust immunity.

Keywords: competition policy, antitrust policy, innovation

1. Introduction

Recognition of the issues in the debate over the application of traditional antitrust policy to high-technology industries may be traced to Schumpeter (1943, p. 84),¹ but the debate itself has moved to center stage with the internationalization and restructuring of telecommunications industries, with applications of biotechnology in both agriculture and pharmaceuticals, and with *U.S. v. Microsoft*.² In this paper, I present a model of the impact of deterrence-based product market competition policy on firms' incentives to invest in quality-improving innovation.

In the model of competition policy, a Competition Authority with limited investigatory resources monitors performance in two markets, each supplied by a monopolist.^{3,4} Demand in each market has a random element. The Competition Authority sets a threshold price for each industry and investigates an industry if realized price exceeds the industry's investigation threshold.

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1 "Economists are at long last emerging from the stage at which price competition was all they saw."

2 On which, see Evans et al. (2000).

3 For surveys of the formal literature on competition policy, see Rey (2001) and Motta (2003). Rey (p. 3) notes that "Compared with regulation theory, the theory of competition policy is still in its early stage of development...."

4 The assumption of monopoly suppliers is made for simplicity, and is not essential for the results. See Martin (2000b) for an extension of the model of competition policy to oligopoly.

As demand has a random part, realized price may be high enough to trigger an investigation either because the monopoly supplier has restricted output or because the random part of demand has pushed up the demand curve. If the supplier has restricted output, the resulting exercise of monopoly power may or may not be legal. I make the realistic assumption that neither the firm nor the Competition Authority can predict the workings of the legal system with certainty. Instead, if an investigation occurs, there is a known probability that the firm will be tried and convicted of violating competition law, and a known fine imposed if conviction takes place.

The profit-maximizing response of a firm facing such a competition policy is to expand output above the unconstrained monopoly level, lowering expected price and reducing the expected value of fines.

Quality-improving innovation is possible in industry 1 but not in industry 2. In the basic model, there are two periods of random length. In the first period the Competition Authority sets threshold prices for both industries and firm 1 (the supplier of industry 1) invests to develop a quality-improving innovation. The time to discovery is random, and depends on firm 1's R&D effort. When the innovation takes place, period one ends, period two begins, and the Competition Authority adjusts investigation threshold prices optimally, taking into account the higher post-innovation quality product of industry 1.

The Competition Authority cannot commit in advance to the investigation thresholds it will set in period two. All parties know that at the start of the second period the Competition Authority will set investigation thresholds to maximize the expected present discounted value of net social welfare from the moment of innovation forward.⁵

At the start of the first period, the Competition Authority sets investigation thresholds to maximize the expected present discounted value of net social welfare over all future time, taking into account the adjustment of thresholds that will occur at the moment of innovation.

At the start of the first period, firm 1 selects its output and its R&D investment level, knowing its expected payoff before innovation (which depends on the first-period investigation threshold) and knowing its expected payoff after innovation (which depends on the higher quality level and the optimal investigation threshold that will be set after innovation).

The firm's incentive to invest in innovation is the expected present discounted value of the incremental profit that is expected with a higher-quality product. As the Competition Authority cannot commit in advance to second-period investigation levels, it cannot encourage innovation by promising a high investigation threshold (a low probability of investigation) in the second period. Section 2 shows that the optimal first-period investigation threshold price for the high-technology industry is lowered by possibility of quality improvement: the Competition Authority toughens competition policy to increase the incentive to innovate. Section 3 sketches an extension of the basic model to a sequence of two innovations. Section 4 extends the basic model in a different direction and

5 Qualitatively, the results of the paper hold if the competition authority can commit in advance to a second-period threshold price.

examines the impact on market performance of a patent policy that grants the successful innovator the legal right to extract full monopoly profit for a specified period. This profit is an incentive to innovate, and with such a patent policy, optimal competition policy toward the high technology industry is not as tough as would otherwise be the case. When the investigation threshold is set optimally taking patent protection into account, the result may be less R&D and worse market performance, all else equal.

2. A basic model

2.1. The second period

2.1.1. Competition policy and the firm's output decision

For notational compactness in this section, I omit time subscripts and write χ_i for the quality level of firm i 's product. For firm 1, the quality level is χ_{11} in period 1 and $\chi_{12} > \chi_{11}$ in period 2. The quality level for firm 2's product is χ_2 in both periods.

The inverse demand curve for product i is

$$p_i = p_i(q_i, \chi_i) + \varepsilon_i. \quad (1)$$

I assume that the demand curve is downward sloping ($\partial p_i / \partial q_i \leq 0$) and that price rises with quality but at a decreasing rate,

$$\frac{\partial p_i}{\partial \chi_i} > 0, \quad \frac{\partial^2 p_i}{\partial \chi_i^2} \leq 0. \quad (2)$$

The random term ε_i has zero mean, continuous density function $f_i(x)$ and is defined on

$$\underline{\varepsilon}_i \leq \varepsilon_i \leq \bar{\varepsilon}_i \leq \infty. \quad (3)$$

I also assume that if output is sufficiently small, price is above marginal cost even for low realized values of ε_i :

$$p_i(0) + \underline{\varepsilon} \geq c_i. \quad (4)$$

This ensures that the market is not expected to go out of existence and simplifies the calculation of expected payoffs.

The Competition Authority sets an investigation threshold price g_i for industry i .⁶ The probability of investigation τ_i is

$$\begin{aligned} \tau_i[g_i - p_i(q_i, \chi_i)] &= \Pr[p_i(q_i, \chi_i) + \varepsilon_i \geq g_i] \\ &= \Pr[\varepsilon_i \geq g_i - p_i(q_i, \chi_i)] \\ &= \int_{g_i - p_i(q_i, \chi_i)}^{\bar{\varepsilon}} f_i(x) dx. \end{aligned} \quad (5)$$

6 See Souam (2001) for a model in which this type of monitoring is optimal.

τ_i has a negative first derivative,

$$\tau_i' = -f_i[g_i - p_i(q_i, \chi_i)] < 0, \quad (6)$$

and I assume that τ_i has a positive second derivative (which amounts to assuming that $f_i' < 0$ in the neighborhood of equilibrium):

$$\tau_i'' = -f_i'[g_i - p_i(q_i, \chi_i)] > 0. \quad (7)$$

Holding output and quality constant, a lower investigation threshold increases the probability of investigation:

$$\frac{\partial \tau_i}{\partial g_i} = -f_i[g_i - p_i(q_i, \chi_i)] < 0. \quad (8)$$

Holding the investigation threshold and quality constant, greater output reduces the probability of investigation, all else equal, and at a decreasing rate:

$$\frac{\partial \tau_i}{\partial q_i} = -f_i[g_i - p_i(q_i, \chi_i)] \left[-\frac{\partial p_i(q_i, \chi_i)}{\partial q_i} \right] = f_i \frac{\partial p_i}{\partial q_i} < 0 \quad (9)$$

$$\frac{\partial^2 \tau_i}{\partial q_i^2} = f_i \frac{\partial^2 p_i}{\partial q_i^2} + \frac{\partial p_i}{\partial q_i} f_i' \left(-\frac{\partial p_i}{\partial q_i} \right) = f_i \frac{\partial^2 p_i}{\partial q_i^2} - \left(\frac{\partial p_i}{\partial q_i} \right)^2 f_i' > 0. \quad (10)$$

Equation (9) follows from the assumption that the inverse demand curve is downward sloping. Equation (10) is true for linear demand, and I assume it holds here.

Holding output and the investigation threshold constant, a higher quality level increases the probability of investigation:⁷

$$\frac{\partial \tau_i}{\partial \chi_i} = -f_i[g_i - p_i(q_i, \chi_i)] \left[-\frac{\partial p_i}{\partial \chi_i} \right] = f_i \frac{\partial p_i}{\partial \chi_i} > 0. \quad (11)$$

If realized price is greater than or equal to g_i the firm is investigated and with probability γ_i is subject to a fine F_i . I assume that whether or not the firm is convicted is uncertain to reflect the fact that neither firm nor Competition Authority can predict the workings of the judicial system with certainty.⁸

This specification may be particularly appropriate for common-law systems, in which legal standards develop in an evolutionary way as decisions are rendered in individual cases (Priest, 1977; Rubin, 1977). It is clear that competition enforcement agencies lose cases which they bring expecting to win and that defendants decline to settle cases that

7 $\partial^2 \tau_i / \partial \chi_i^2$ is positive if demand is linear in quality and otherwise of ambiguous sign.

8 It would be possible to considerably elaborate the enforcement game, by allowing for the possibility of settlement and by making the probability of conviction and the amount of an eventual fine dependent on resources committed by the parties to prosecution and defense. These aspects of competition policy are not central to the question addressed here.

they expect to win but subsequently lose. Such outcomes would not occur if the workings of the judicial system could be foreseen with certainty.

Tirole's (1999, pp. 761–762) remarks on contract enforcement apply with equal force to the application of competition law:

Judges are subject to both moral hazard (they may not put enough effort into reading and understanding the details of the case) and adverse selection (they may not have the proper background to understand what the parties tell them; they may also have their own preferences, in the form of legal precedents and principles, which they may embody in their decisions . . .).

The idea that the judicial system has its own preferences, its own mores and traditions, which develop over time, motivates the specification used here.

Taking expected fines into account, the firm's instantaneous expected payoff in the second period is⁹

$$\begin{aligned}\pi_i &= [p_i(q_i, \chi_i) - c_i]q_i - \tau_i \gamma_i F_i \\ &= [p_i(q_i, \chi_i) - c_i]q_i - \gamma_i F_i \int_{g_i - p_i(q_i, \chi_i)}^{\bar{e}_i} f_i(x) dx.\end{aligned}\quad (12)$$

The firm picks its output level to maximize expected profit. The first-order condition for profit maximization is

$$\frac{\partial E(\pi_i)}{\partial q_i} = p_i(q_i, \chi_i) - c_i + q_i \frac{\partial p_i(q_i, \chi_i)}{\partial q_i} - \gamma_i F_i \frac{\partial \tau_i [g_i - p_i(q_i, \chi_i)]}{\partial q_i} \equiv 0. \quad (13)$$

The profit-maximizing firm picks an output level at which expected marginal revenue is less than marginal production cost,

$$p_i + q_i \frac{\partial p_i}{\partial q_i} = c_i + \gamma_i F_i \frac{\partial \tau_i}{\partial q_i}, \quad (14)$$

expanding output above the level that would maximize profit in the absence of competition policy to reduce the probability of investigation and the expected value of fines.

The first-order condition implicitly defines the firm's output $q_i(g_i, \chi_i)$ as a function of the investigation threshold and of quality. Lemmas 1 and 2 present some comparative static properties of the firm's profit-maximizing output.¹⁰

Lemma 1: (Competition policy and firm behavior in the absence of technological progress)

9 In what follows, I omit the expectations operator for notational simplicity.

10 Proofs are given in the Appendix.

- a. if $\gamma_i F_i$ is sufficiently great, the firm picks an output that makes expected price less than the investigation threshold,

$$g_i - p_i > 0;$$

- b. a lower investigation threshold induces greater output, all else equal:

$$\frac{\partial q_i}{\partial g_i} < 0;$$

- c. greater expected fines induce greater output, all else equal:

$$\frac{\partial q_i}{\partial \gamma_i F_i} > 0;$$

- d. a sufficient condition for an increase in quality to increase equilibrium output, all else equal, is¹¹

$$\frac{\partial^2 p_i}{\partial \chi_i \partial q_i} = \frac{\partial}{\partial \chi_i} \left(\frac{\partial p_i}{\partial q_i} \right) \geq 0.$$

Lemma 2: (The probability of investigation) Assume that if the Competition Authority raises g_i , the resulting output decrease (Lemma 1) does not increase price so much that $g_i - p_i$ falls, *i.e.*,¹²

$$\frac{\partial(g_i - p_i)}{\partial g_i} > 0. \quad (15)$$

Then a decrease in g_i increases the probability of investigation:

$$\frac{d\tau_i}{dg_i} = \tau_i \frac{\partial(g_i - p_i)}{\partial g_i} = -f_i(g_i - p_i) \left(1 - \frac{\partial p_i}{\partial q_i} \frac{\partial q_i}{\partial g_i} \right) < 0. \quad (16)$$

I assume that $d^2\tau_i/dg_i^2 > 0$; successive decreases in g_i bring smaller increases in the probability of investigation.

11 Since $\partial p_i/\partial q_i < 0$, $\partial/\partial \chi_i(\partial p_i/\partial q_i) \geq 0$ means that an increase in quality does not steepen, and in general flattens, the inverse demand curve.

12 Directly from $\partial(g_i - p_i)/\partial g_i = 1 - (\partial p_i/\partial q_i)(\partial q_i/\partial g_i)$, this means that the output response to a change in the investigation threshold is smaller in magnitude than the slope of the demand curve, $-dq_i/dp_i > -\partial q_i/\partial g_i > 0$.

2.1.2. The Competition Authority's problem

Let I_i denote the cost of investigating industry i , for $i = 1, 2$. The budget constraint is that expected investment costs cannot exceed the Competition Authority's budget B :

$$\tau_1[g_1 - p_1(q_1, \chi_1)]I_1 + \tau_2[g_2 - p_2(q_2, \chi_2)]I_2 \leq B. \quad (17)$$

This way of formulating the budget constraint implies that expected fines are receipts to the government in general, not to the Competition Authority. Furthermore, since it is expected investigation costs that must not exceed the budget, the Competition Authority must have reserves that allow it to cover realized investigation expenses in states of the world for which both industries are investigated.

The expected fine that follows investigation is $\gamma_i F_i$; the probability of investigation is τ_i . Overall, the expected fine is $\tau_i \gamma_i F_i$. This is an expected transfer from the firm to the government, and drops out of the expression for net social welfare. Net social welfare from industry i is the sum of consumers' and producers' surplus,

$$W_i[q_i(g_i, \chi_i), \chi_i] = \int_0^{q_i(g_i, \chi_i)} [p_i(x_i, \chi_i) - c_i] dx_i. \quad (18)$$

A reduction in g_i increases W_i , all else equal:

$$\frac{\partial W_i}{\partial g_i} = [p_i(q_i, \chi_i) - c_i] \frac{\partial q_i}{\partial g_i} < 0. \quad (19)$$

The Competition Authority's problem is to maximize social welfare, net of enforcement cost,

$$\max_{g_1, g_2} W_1[q_1(g_1, \chi_1), \chi_1] + W_2[q_2(g_2, \chi_2), \chi_2] - \tau_1 I_1 - \tau_2 I_2, \quad (20)$$

subject to the budget constraint (17).

To render the problem interesting from an economic point of view, assume that the budget constraint is binding: the Competition Authority does not have enough resources to investigate all industries for all realized prices. That the budget constraint binds links the Competition Authority's choice of investigation thresholds for the two industries: a lower value of g_1 implies a higher value of g_2 and vice-versa.

If the budget constraint is binding, we can reformulate the Competition Authority's problem as

$$\max_{g_1, g_2} W_1[q_1(g_1, \chi_1), \chi_1] + W_2[q_2(g_2, \chi_2), \chi_2] - B \quad (21)$$

such that

$$\tau_1 I_1 + \tau_2 I_2 \leq B. \quad (22)$$

Conditions characterizing the solution to the Competition Authority's problem can be found by maximizing the Lagrangian

$$L_2 = W_1[q_1(g_1, \chi_1), \chi_1] + W_2[q_2(g_2, \chi_2), \chi_2] - B + \lambda_2 [B - \tau_1 I_1 - \tau_2 I_2]. \quad (23)$$

First-order conditions. The first-order conditions for the constrained optimization problem are

$$\frac{\partial L_2}{\partial \lambda} = B - \tau_1 I_1 - \tau_2 I_2 \equiv 0, \quad (24)$$

$$\frac{\partial L_2}{\partial g_1} = \frac{\partial W_1}{\partial g_1} - \lambda_2 I_1 \frac{\partial \tau_1}{\partial g_1} \equiv 0, \quad (25)$$

and

$$\frac{\partial L_2}{\partial g_2} = \frac{\partial W_2}{\partial g_2} - \lambda_2 I_2 \frac{\partial \tau_2}{\partial g_2} \equiv 0. \quad (26)$$

Conditions (25) and (26) imply that at the optimum the marginal increase in welfare per marginal increase in investigation costs is the same in both industries, and is the shadow value of budget funding for the Competition Authority:

$$\lambda_2 = \frac{\partial W_1 / \partial g_1}{I_1 (\partial \tau_1 / \partial g_1)} = \frac{\partial W_2 / \partial g_2}{I_2 (\partial \tau_2 / \partial g_2)}. \quad (27)$$

The solution to the second-period welfare maximization problem can be characterized in a familiar way. In the equation

$$\tau_1 I_1 + \tau_2 I_2 = \bar{B}, \quad (28)$$

τ_1 and τ_2 are functions of g_1 and g_2 , respectively. For given industry investigation costs I_1 and I_2 , equation (28) identifies all pairs (g_1, g_2) that generate a total expected investigation cost \bar{B} . In other words, (28) is the equation of an isobudget curve in (g_1, g_2) -space.

The slope of an isobudget curve is

$$\left. \frac{dg_2}{dg_1} \right|_{\bar{B}} = - \frac{I_1}{I_2} \frac{\partial \tau_1 / \partial g_1}{\partial \tau_2 / \partial g_2} < 0. \quad (29)$$

A greater budget allows the Competition Authority to set lower investigation thresholds for both industries, if it chooses to do so, and corresponds to an isobudget curve that is closer to the origin in (g_1, g_2) -space.

In this formulation, investigation costs play the same role as the prices of goods in the standard consumer choice model, and as factor prices in the standard model of firm cost minimization.

Figure 1 shows three isobudget curves for a particular set of functional forms and parameter values. The inverse demand curve is linear,

$$p_i = \chi_i + \varepsilon_i - q_i, \quad (30)$$

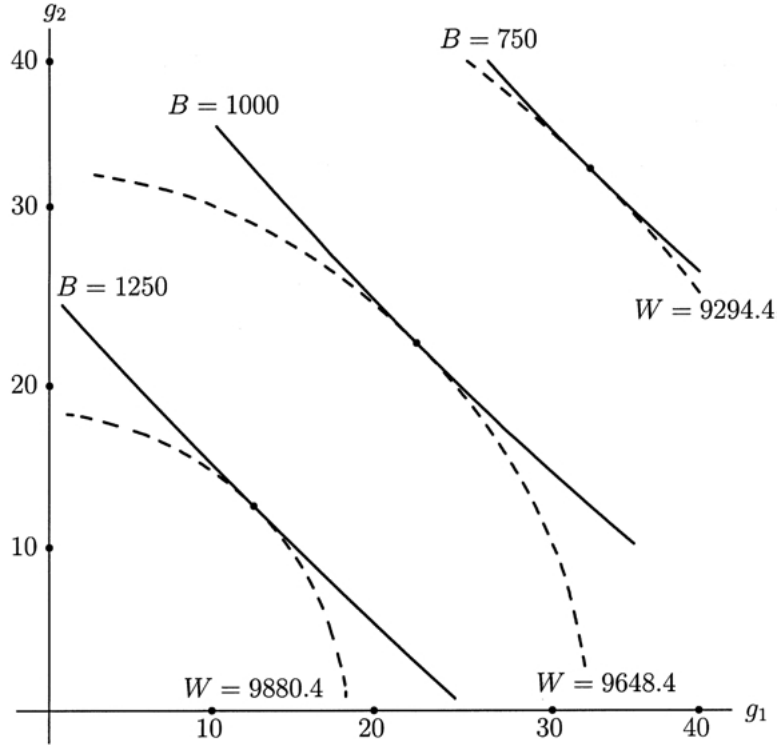


Figure 1. Equilibrium threshold levels, alternative budget levels ($\chi_1 = \chi_2 = 100, c_1 = c_2 = 0, \sigma_1 = \sigma_2 = 10, \gamma_1 F_1 = \gamma_2 F_2 = 2500, I_1 = I_2 = 2000$). Indicated welfare levels are gross of expected investigation cost.

and the random term ε_i has the negative exponential distribution

$$1 - \exp\left(-\frac{\varepsilon_i + \sigma_i}{\sigma_i}\right). \tag{31}$$

Unit cost is assumed constant and without loss of generality normalized to be zero in both industries. To highlight the impact of differences in potential for quality improvement, I assume that the two industries are identical in the pre-innovation period.

The equation

$$W_1(q_1, \chi_1) + W_2(q_2, \chi_2) = \bar{W} \tag{32}$$

identifies all combinations of g_1 and g_2 that generate a given gross welfare level \bar{W} . It is the equation of an isowelfare curve in (g_1, g_2) -space. Isowelfare curves closer to the origin represent higher levels of welfare; the slope of an isowelfare curve is

$$\left. \frac{dg_2}{dg_1} \right|_{\bar{W}} = -\frac{\partial W_1 / \partial g_1}{\partial W_2 / \partial g_2} < 0. \tag{33}$$

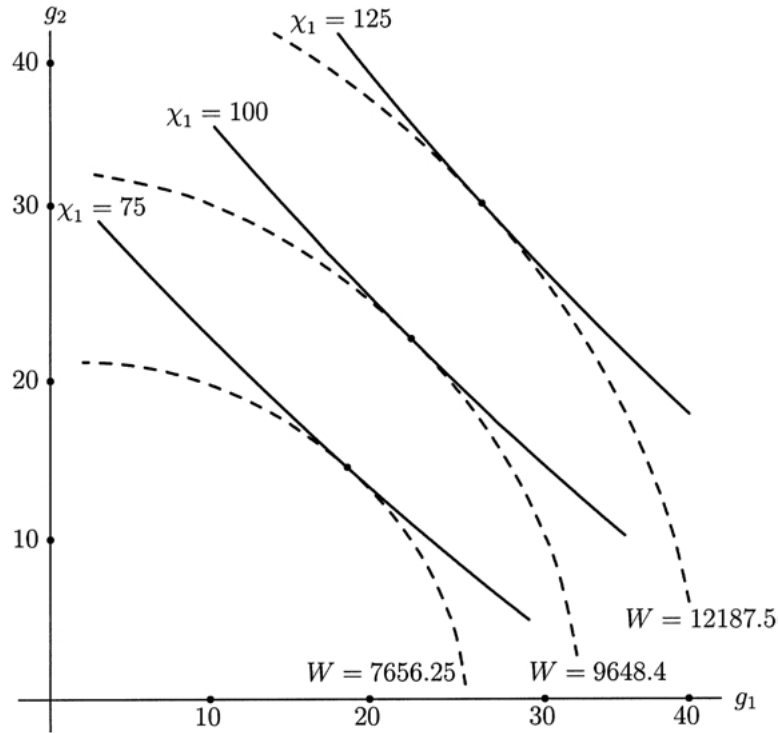


Figure 2. Equilibrium threshold levels, alternative product 1 quality levels ($B = 1000, \gamma_2 = 100, c_1 = c_2 = 0, \sigma_1 = \sigma_2 = 10, \gamma_1 F_1 = \gamma_2 F_2 = 2500, I_1 = I_2 = 2000$). Indicated welfare levels are gross of expected investigation cost.

Rewriting equation (27) as

$$-\frac{\partial W_1 / \partial g_1}{\partial W_2 / \partial g_2} = -\frac{I_1}{I_2} \frac{\partial \tau_1 / \partial g_1}{\partial \tau_2 / \partial g_2}, \quad (34)$$

the first-order conditions for constrained welfare optimization imply that welfare is maximized where the isobudget curve for the available budget is tangent to the isowelfare curve that is as close to the origin as possible.¹³

Three such tangencies, for different budget levels, as shown in Figure 1. Connecting the points of tangency gives a budget contraction path (not shown in the diagram) that corresponds to the income expansion path of the standard two-good consumer problem.

Investigation probabilities and welfare are all functions of product quality. If quality changes, the maps of isobudget curves and isowelfare curves both shift. Figure 2 shows

¹³ From equation (21), net welfare is the difference between the level of gross welfare associated with the isowelfare curve and the budget level associated with the isobudget curve.

isobudget and isowelfare curves for $B = 1000$ and $\chi_1 = 75, 100, 125$ and other parameters as for Figure 1.¹⁴

In the normal case (and this is what is shown in Figure 2) an increase in quality in one industry raises investigation thresholds in both industries. As χ_1 rises, the optimal threshold price g_1 rises as well, but less than proportionately, so that the expected threshold-price gap $g_1 - p_1$ falls. A smaller $g_1 - p_1$ means a higher probability of investigation in industry 1 and a greater expected investigation cost for industry 1. Given the budget constraint, the optimal g_2 must rise to reduce expected investigation cost for industry 2.

2.2. The first period

2.2.1. Firm 1

In the first period, firm 1's instantaneous profit is

$$\pi_{11}(\chi_{11}, g_{11}) = [p_{11}(\chi_{11}, g_{11}) - c_1]q_{11} - \gamma_1 F_1 \int_{\varepsilon_1 = g_{11} - p_{11}}^{\bar{\varepsilon}_1} f_1(\varepsilon_1) d\varepsilon_1. \quad (35)$$

Firm 1 produces a product of quality χ_{11} and undertakes a research project of intensity h at cost $z(h)$ (with $z'(h), z''(h) > 0$); the research project leads to development of a higher-quality product χ_{12} , which firm 1 produces in the second period.¹⁵ The time T of expected success in bringing the higher-quality product to market is random, with negative exponential distribution

$$\Pr(T \leq t) = 1 - \exp(-ht). \quad (36)$$

Firm 1 picks output q_{11} and research intensity h to maximize its expected value

$$\begin{aligned} V_1 &= \int_{t=0}^{\infty} e^{-(r+h)t} \left\{ \pi_{11}(\chi_{11}, g_{11}) - z(h) + h \frac{\pi_{12}(\chi_{12}, \chi_2)}{r} \right\} dt \\ &= \frac{\pi_{11}(\chi_{11}, g_{11}) - z(h) + h(\pi_{12}(\chi_{12}, \chi_2)/r)}{r + h} \end{aligned} \quad (37)$$

This is a standard innovation race formulation.¹⁶ Before innovation, the firm collects its instantaneous payoff and pays for R&D; the probability density that innovation has not occurred is proportional to e^{-ht} . The probability density that innovation occurs at time t is proportional to he^{-ht} , and from the moment of innovation firm 1's value is

14 One may think of an isobudget surface in (g_1, g_2, χ_1) -space. Then the three isobudget curves in Figure 2 are level curves of this surface. Similarly for the three isowelfare curves.

15 See Martin (2000a) for an application to cost-saving innovation.

16 The model of R&D can be extended to allow for spillovers and endogenous absorptive capacity (Martin, 2002). For simplicity, such extensions are not considered here.

$\pi_{12}(\chi_{12}, \chi_2)/r$, where $\pi_{12}(\chi_{12}, \chi_2)$ is firm 1's payoff with quality $\chi_{12} > \chi_{11}$ under an optimally administered competition policy. Expected payoffs are discounted at rate r .

Firm 1 picks q_{11} to maximize π_{11} . This decision has been analyzed in Section 2.1.1.¹⁷

Firm 1 picks h to maximize V_1 . The first-order condition for value maximization with respect to h is

$$z(h) - (r + h)z'(h) + \pi_{12}(\chi_{12}, \chi_2) - \pi_{11}(\chi_{11}, g_{11}) \equiv 0. \quad (38)$$

It is convenient to rewrite (38) as

$$z(h) - (r + h)z'(h) + \Delta = 0, \quad (39)$$

where

$$\Delta = \pi_{12}(\chi_{12}, \chi_2) - \pi_{11}(\chi_{11}, g_{11})$$

is the difference in instantaneous profit before and after innovation. Then differentiating equation (39) with respect to Δ gives the comparative static result

$$\frac{dh}{d\Delta} = \frac{1}{(r + h)z''(h)} > 0. \quad (40)$$

Any change that increases Δ increases h . It follows that

Lemma 3:

- a. Tougher competition policy in the first period increases equilibrium R&D intensity, all else equal:

$$\frac{\partial h}{\partial g_{11}} < 0; \quad (41)$$

- b. higher post-innovation quality increases equilibrium R&D intensity, all else equal:

$$\frac{\partial h}{\partial \chi_{12}} > 0. \quad (42)$$

A tougher first-period competition policy (lower g_{11}) lowers π_{11} and increases Δ ; a higher second-period quality increases π_{12} and increases Δ . Both types of changes increase equilibrium R&D intensity.

17 I follow the literature and assume that financial constraints are not binding, so that the firm is able to fund the level of R&D effort specified by its first-order condition (38). This may be an unreasonable assumption for small firms, but small firms are unlikely to attract the attention of a Competition Authority (or to receive it, in the kind of model of optimal competition policy considered here; see Martin, 2000a). See Jensen and Showalter (2001) for a racing model of innovation with financial market-product market interaction.

2.2.2. *The competition authority*

Let

$$M_1(g_{11}, g_{21}) = W_{11}[q_1(g_{11}, \chi_{11}), \chi_{11}] + W_{21}[q_2(g_{21}, \chi_2), \chi_2] - B \quad (43)$$

denote instantaneous welfare in the first period, gross of the cost of R&D, where W_{11} is instantaneous net social welfare in industry 1,

$$W_{11}[q_1(g_{11}, \chi_{11}), \chi_{11}] = \int_0^{q_{11}(g_{11}, \chi_{11})} [p_1(x_1, \chi_{11}) - c_1] dx_1, \quad (44)$$

and W_{21} is instantaneous net social welfare in industry 2,

$$W_{21}[q_2(g_{21}, \chi_2), \chi_2] = \int_0^{q_{21}(g_{21}, \chi_2)} [p_2(x_2, \chi_2) - c_2] dx_2. \quad (45)$$

Let M_2 denote instantaneous welfare in the second period. M_2 is the result of the Competition Authority's second-period optimization and depends on all second-period parameters of the model. For notational simplicity and to emphasize the variable of interest in the present context, write $M_2(\chi_{12})$.

The Competition Authority picks the first-period investigation thresholds g_{11} and g_{21} to maximize the expected present discounted value of net social welfare,

$$M = \frac{M_1(g_{11}, g_{21}) - z[h(g_{11}; \chi_{12})] + h(g_{11}; \chi_{12})M_2(\chi_{12})/r}{r + h(g_{21}; \chi_{21})} \quad (46)$$

subject to the first-period budget constraint

$$\tau_{11}I_1 + \tau_{21}I_2 \leq B, \quad (47)$$

a constraint which we will assume to be binding. Once again, the budget constraint links the Competition Authority's choice of investigation thresholds for the two industries: a lower value of g_{11} implies a higher value of g_{21} , and vice-versa.

Necessary conditions that must be satisfied by optimal thresholds are obtained from the Lagrangian

$$L_1 = \frac{M_1(g_{11}, g_{21}) - z[h(g_{11}; \chi_{12})] + h(g_{11}; \chi_{12})M_2(\chi_{12})/r}{r + h(g_{11}; \chi_{12})} + \lambda_1(B - \tau_{11}I_1 - \tau_{21}I_2). \quad (48)$$

The first-order necessary conditions are

$$\frac{\partial L_1}{\partial \lambda_1} = B - \tau_{11}I_1 - \tau_{21}I_2 \equiv 0 \quad (49)$$

$$\begin{aligned} \frac{\partial L_1}{\partial g_{11}} = & \frac{(r+h)(\partial M_1/\partial g_{11}) + [M_2(\chi_{12}) - M_1(g_{11}, g_{21}) + z(h) - (r+h)z'(h)](\partial h/\partial g_{11})}{(r+h)^2} \\ & - \lambda_1 I_1 \frac{\partial \tau_{11}}{\partial g_{11}} \equiv 0 \end{aligned} \quad (50)$$

$$\frac{\partial L_1}{\partial g_{21}} = \frac{\partial M_1(g_{11}, g_{21})/\partial g_{21}}{r+h(g_{11}; \chi_{12})} - \lambda_1 I_2 \frac{\partial \tau_{21}}{\partial g_{21}} \equiv 0. \quad (51)$$

Comparing equations (49)–(51) with the corresponding conditions for the Competition Authority's second-period problem, equations (24)–(26), the factor that distinguishes the first-period problem is that by its choice of g_{11} the Competition Authority influences h and thus the expected duration of the first period.

Differentiating the first-order conditions, a sufficient condition for optimal competition policy to be tougher for the higher-technology sector and more relaxed for the low-technology industry,

$$\frac{\partial g_{11}}{\partial \chi_{21}} < 0, \quad \frac{\partial g_{21}}{\partial \chi_{21}} > 0, \quad (52)$$

is that

$$\frac{\partial^2 L_1}{\partial \chi_{12} \partial g_{11}} < 0. \quad (53)$$

This condition is met if an increase in second-period quality increases the magnitude of the impact of a reduction in g_{11} on optimum constrained welfare.¹⁸

Figure 3 illustrates the comparative static impact of increasing second-period quality on first-period thresholds and R&D intensity for the linear demand, constant marginal production cost, negative exponential uncertainty, quadratic R&D cost example of Figures 1 and 2.

In the absence of competition policy, monopoly profit is 2500. This is also the expected fine in the event of investigation, so this example is one of relatively high penalties.

When there is no technological advance ($\chi_{11} = \chi_{12} = 100$), firm 1 does not invest in R&D. The Competition Authority sets identical investigation thresholds for both industries ($g_{11} = g_{21} = 33.302$). Firms expand output above the unconstrained monopoly level (73.4375 instead of 50) and expected prices (26.5625) are below the investigation threshold.

As the quality of firm 1's post-innovation product rises, the welfare-maximizing Competition Authority shifts enforcement resources to industry 1 and away from industry

18 See the Appendix.

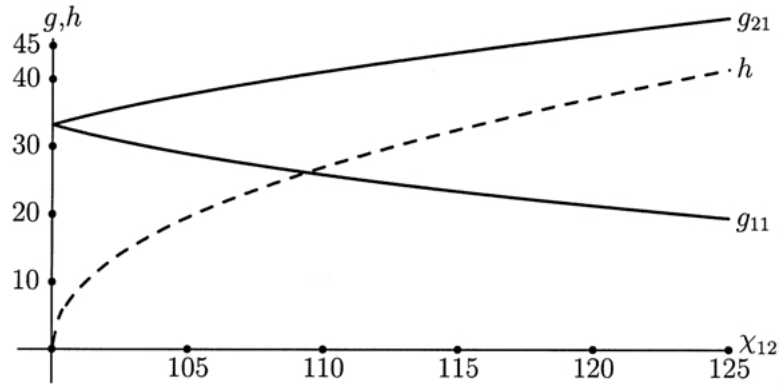


Figure 3. First-period equilibrium threshold levels and R&D intensity, alternative χ_{12} levels; ($\chi_{11} = \chi_2 = 100$, $\sigma_1 = \sigma_2 = 10$, $\gamma_1 F_1 = \gamma_2 F_2 = 2500$, $I_1 = I_2 = 2000$, $B = 750$, $r = 1/10$).

2. As χ_{12} rises, g_{11} falls, to 19.499 (recall that marginal cost is normalized at zero) for a 25% quality improvement. At the same time, g_{21} rises, to 49.202 to a 25% quality improvement of product 1.

The two firms respond in a profit-maximizing way to these changes: firm 1 expands output and also increases R&D intensity as χ_{12} rises and g_{11} falls. Firm 2 reduces output as g_{21} rises. As firm 1 faces a tougher and tougher competition policy, its first-period equilibrium threshold-expected price gap ($g - p$) falls. As firm 2 faces a gentler and gentler competition policy, its first-period threshold-expected price gap rises.

In the basic model, optimal competition policy is tougher, all else equal, toward high-technology industries. The greater the potential quality improvement, the greater the welfare payoff from private investment in innovation. Tougher competition policy increases the difference between pre-innovation and post-innovation payoffs, making innovation privately more desirable, increasing equilibrium R&D intensity and shortening the expected time to discovery. The greater the potential quality improvement, the greater the net social benefit from shortening the time to discovery (net of the cost of accepting worse market performance in low technology sectors).

3. Sequential innovation¹⁹

The results presented to this point do not depend on the assumption that innovation is a once-and-for-all event. To illustrate by example,²⁰ suppose that for the parameter values

19 I am grateful to an anonymous referee for suggesting that I consider the case treated in this section.

20 General results can be obtained along the lines of Section 2.2.2 (in period 1 the Competition Authority sets investigation thresholds that maximize the expected present discounted value of net social welfare over all future time, taking into account the optimal adjustment of investigation thresholds when the first stage innovation is realized), and subject to a condition that corresponds to equation (53).

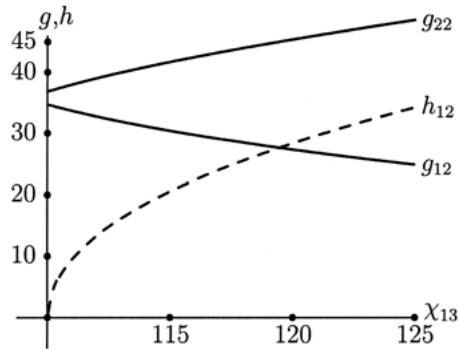


Figure 4. Second-period equilibrium threshold levels and R&D intensity, alternative χ_{13} levels; ($\chi_{12} = 110$, $\chi_2 = 100$, $\sigma_1 = \sigma_2 = 10$, $\gamma_1 F_1 = \gamma_2 F_2 = 2500$, $I_1 = I_2 = 2000$, $B = 750$, $r = 1/10$).

of Figure 3 innovation takes place in two steps, a first-stage innovation from $\chi_{11} = 100$ to $\chi_{12} = 110$ and a second-stage innovation to a quality level $\chi_{13} \geq \chi_{12}$.

Figure 4 shows socially optimal second-period investigation thresholds and privately optimal R&D intensity for the second period, as a function of the quality level to be realized by successful innovation. If third-period quality is $\chi_{13} = 110$, there is no innovation after the first quality increment is realized. Industry 1 is then somewhat larger, in the sense of greater potential net social welfare, and for that reason receives a somewhat lower optimal investigation threshold ($g_{13} = 34.789$) than industry 2 ($g_{23} = 36.930$).²¹ The greater the quality level to be realized in period 3, the greater the R&D intensity chosen by the monopoly supplier of the high-technology industry, and the lower the optimal investigation threshold set by the Competition Authority.

Figure 5 shows socially optimal first-period investigation thresholds and privately optimal R&D intensity for the first period, as a function of the quality level to be realized by successful innovation at the end of the second period. With $\chi_{11} = 100$, $\chi_{12} = \chi_{13} = 110$, there is only one innovation stage, and optimal investigation thresholds are those shown for $\chi_{12} = 110$ in Figure 3: $g_{11} = 25.918$, $g_{21} = 41.218$ (and the value-maximizing R&D level is $h_1 = 27.043$). As χ_{13} rises optimal g_{11} falls, optimal g_{21} and equilibrium h_1 rise, all else equal (to 19.481, 49.226, and 41.482, respectively, for $\chi_{13} = 125$).

The second-period innovation cannot be realized until after the first-period innovation is realized. The greater the quality improvement that can be obtained in the second stage, the greater the social value of realizing the first-stage innovation, and the greater the social payoff to lowering the first-period investigation threshold for the high-technology industry (which implies, by dint of the budget constraint, raising the investigation threshold for the low-technology industry). By increasing incentives for the high-tech

21 That it is socially optimal to set lower investigation thresholds for markets that are larger in this sense, all else equal, is a general result; see Martin (2000b).

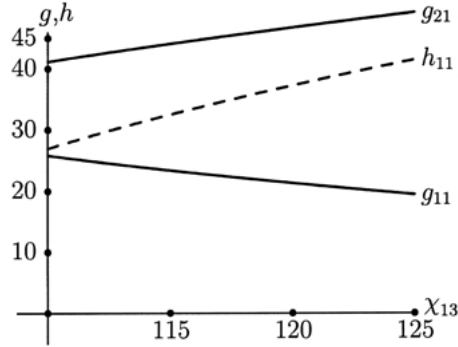


Figure 5. First-period equilibrium threshold levels and R&D intensity, alternative χ_{13} levels; ($\chi_{12} = 110$, $\chi_{11} = \chi_2 = 100$, $\sigma_1 = \sigma_2 = 10$, $\gamma_1 F_1 = \gamma_2 F_2 = 2500$, $I_1 = I_2 = 2000$, $B = 750$, $r = 1/10$).

firm to invest resources in realizing the first-stage quality improvement, the Competition Authority brings closer the realization of the second-stage quality improvement.

4. Patent protection, competition policy, and market performance²²

In the model of Section 2, the Competition Authority cannot commit to a relaxed enforcement policy in the post-discovery world. A patent, however, not only confers property rights in the subject product or process but also grants specific types of immunity from antitrust prosecution.²³

Here I model this aspect of patent policy by assuming that a patent confers complete immunity from antitrust prosecution for the life of the patent. For this specification, if firm 1 obtains a patent of duration L when it innovates, its value from the moment of discovery is

$$V_L = (1 - e^{-rL}) \frac{\pi_m}{r} + e^{-rL} \frac{\pi_{12}(\chi_{12}, \chi_2)}{r}. \tag{54}$$

22 For references to the broader literature on patent design, see Waterson (1990), Green and Scotchmer (1995), Chang (1995), Matutes et al. (1996), Scotchmer (1996), and O’Donoghue (1998).

23 Under U.S. (U.S. v. General Electric Company 272 US 426 (1926)) and E.U. (Parke Davis v. Probel [1968] ECR 55) competition policy a patent conveys the right to exercise market power over the patented item, and it is in this specific sense that “immunity” should be understood. A patent does not convey the right to extend the grant of monopoly to other markets. Early decisions concerning abuse of a dominant position under EU competition policy suggest that exercise of legally acquired monopoly power is in and of itself an abuse of a dominant position (see Brattle Group, 2000, p. 53, for an argument that this interpretation continues to apply). It is not unambiguously clear that this interpretation is valid; in any case, under EU competition policy, a patent does not convey the right to use distribution methods that have the effect of dividing the common market.

The first term on the right is the value of monopoly rents, discounted back to the moment of discovery. The second term on the right is the discounted value of economic profit after the patent expires and the firm is once again subject to an optimally administered competition policy.

At the start of the first period, firm 1 picks output q_{11} to maximize instantaneous profit and R&D intensity h to maximize its value

$$V = \frac{\pi_{11}(\chi_{11}, g_{11}) - z(h) + hV_L(\chi_{12}, \chi_2, L)}{r + h}. \quad (55)$$

The first-order condition for R&D intensity is

$$z(h) - (r + h)z'(h) + rV_L(\chi_{12}, \chi_2, L) - \pi_{11}(\chi_{11}, g_{11}) = 0, \quad (56)$$

from which an increase in patent length increases the firm's profit-maximizing R&D intensity, all else equal:

$$\frac{\partial h}{\partial L} = -\frac{1}{(r + h)z''(h)} \frac{\partial}{\partial L} [rV_L(\chi_{12}, \chi_2, L) - \pi_{11}(\chi_{11}, g_{11})] > 0. \quad (57)$$

The comparative static result (57) is illustrated in Figure 6, for the parameterization of Figure 1 and $\chi_{12} = 125$, a 25% quality increase. The investigation threshold is fixed at $g_{11} = 19.499$, the equilibrium level without patent protection. Without patent protection, equilibrium R&D intensity $h = 41.551$ and the expected time to discovery is $1/h = 0.02407$. I adopt this as the unit of time: $L = 1$ means patent protection lasts 0.02407 periods; $L = 2$ means patent protection lasts $2(0.02407)$ periods; and so on. As L rises, holding g_{11} and χ_{12} constant, $h(L)$ rises.

However, it is not optimal for the Competition Authority to hold g_{11} constant as L rises.

While the patent is in force, the Competition Authority devotes all resources to the monitoring industry 2. The present value of welfare from the moment of discovery is therefore

$$\frac{1}{r}M_2 = \frac{1}{r} [(1 - e^{-rL})(W_{1m} + W_{22a} - B) + e^{-rL}M_2(\chi_{12})]. \quad (58)$$

The first-term in the brackets on the right is the present value of welfare while the patent is in force, with unconstrained monopoly in industry 1 and the full competition authority budget devoted to monitoring industry 2. The second term in brackets on the right is the present value of welfare once the patent expires. The Competition Authority selects the investigation thresholds g_{11} and g_{21} to maximize

$$G_L = \frac{M_1(g_{11}, g_{21}) - z(h) + (h(g_{11}, L; \chi_{12})/r)M_2}{r + h(g_{11}, L; \chi_{21})} \quad (59)$$

subject to the budget constraint (17).

A longer patent length and a lower investigation threshold both act to induce greater private investment in R&D. But longer L reduces the social payoff to innovation because it means a longer period of legal monopoly and consequent reduced consumer welfare.

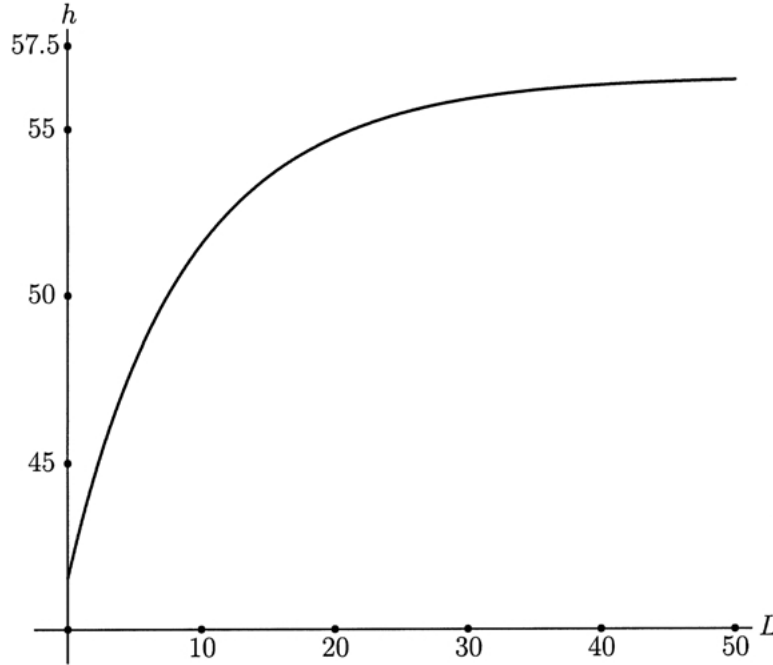


Figure 6. Patent length (L) and R&D intensity (h), investigation threshold constant, $\chi_{12} = 125$.

With longer L , the Competition Authority shifts investigation resources away from industry 1 (higher g_{11}) and toward industry 2 (lower g_{21}) in the first period. The higher value of g_{11} tends to lower h , and the net impact of an increase in L on h is ambiguous. That is,

$$\frac{dh}{dL} = \frac{\partial h}{\partial L} + \frac{\partial h}{\partial g_{11}} \frac{\partial g_{11}}{\partial L}, \tag{60}$$

with $\partial h/\partial L > 0$, $\partial h/\partial g_{11} < 0$, and $\partial g_{11}/\partial L > 0$. The sign of dh/dL is thus ambiguous. Table 1 shows that the cases $dh/dL < 0$ and $dh/dL > 0$ may both arise.

As L increases, first-period welfare rises as the Competition Authority acts to equalize the toughness of competition policy toward industries that are, in the first period, identical. Second-period welfare falls as L increases, because of greater losses of consumer welfare while the patent is valid. For the parameterization of Figure 1 and $\chi_{12} = 125$, increases in patent length reduce net social welfare.

Although there have been some steps to tailor patent length to the circumstances of particular industries (*i.e.* pharmaceuticals), this remains the exception rather than the rule. It therefore seems appropriate to treat patent length as a factor that competition law enforcement agencies must take as given. If patent length is considered a variable, however, it is worth noting that for the parameterization of Table 1, however, optimal patent length is $L = 0$.

Table 1. Equilibrium characteristics, alternative patent lengths; $L = 1$ is expected time to discovery without patent protection.

L	g_{11}	g_{21}	q_{11}	q_{21}	h	M_1	M_2	$G_L (\times 10^5)$
0	19.499	49.202	83.63	63.245	41.551	8440.5	10,986	1.0976
1	19.555	49.126	83.587	63.288	41.548	8441.4	10,984	1.0973
2	19.611	49.051	83.544	63.331	41.545	8442.3	10,982	1.0971
3	19.667	48.975	83.500	63.375	41.542	8443.2	10,980	1.0969
10	20.056	48.455	83.202	63.673	41.525	8449.1	10,965	1.0955
100	24.463	42.921	79.856	67.019	41.556	8503.2	10,797	1.0788
∞	39.217	27.687	69.409	77.466	46.495	8528.2	10,104	1.0096

5. Conclusion

Technological advances do not fall like manna from heaven: they result from investments of profit-seeking firms. The levels of firms' investments in new products and processes depends on the difference in expected profit before and after innovation. Tough pre-innovation product market competition policy reduces pre-innovation profit relative to post-innovation profit and stimulates private investment in innovation. This is so even when the Competition Authority reoptimizes the allocation of enforcement resources after innovation.

Like the stick of competition policy, the carrot of patent protection encourages private investment in innovation. Unlike competition policy, patent protection reduces consumer welfare after innovation. When competition policy is optimally administered, the net effect increased patent protection may be to reduce R&D intensity and social welfare.

Appendix

Proof of Lemma 1: Lemma 1(a)–(c) are Theorem 1 of Martin (2000a); proofs are given here for completeness.

The first-order condition for firm i 's profit-maximization problem is

$$\frac{\partial E(\pi_i)}{\partial q_i} = p_i - c_i + q_i \frac{\partial p_i}{\partial q_i} - \gamma_i F_i \frac{\partial \tau_i}{\partial q_i} = p_i - c_i + (q_i - \gamma_i F_i f_i) \frac{\partial p_i}{\partial q_i} \equiv 0, \quad (61)$$

which implies

$$q_i - \gamma_i F_i f_i > 0; \quad (62)$$

this is used in the proof of (d).

The second-order condition is

$$\frac{\partial^2 E(\pi_i)}{\partial q_i^2} = 2 \frac{\partial p_i}{\partial q_i} + q_i \frac{\partial^2 p_i}{\partial q_i^2} - \gamma_i F_i \frac{\partial \tau_i}{\partial q_i} < 0 \tag{63}$$

or equivalently

$$\frac{\partial^2 E(\pi_i)}{\partial q_i^2} = 2 \frac{\partial p_i}{\partial q_i} + \gamma_i F_i f_i' \left(\frac{\partial p_i}{\partial q_i} \right)^2 + (q_i - \gamma_i F_i f_i) \frac{\partial^2 p_i}{\partial q_i^2} < 0. \tag{64}$$

a. Substituting $p_i = g_i - \varepsilon_i$, the first-order condition can be written as

$$\frac{1}{\gamma_i F_i} \left(\frac{g_i - c_i - \varepsilon_i}{(\partial p_i / \partial q_i)} + q_i \right) = f_i(\varepsilon_i). \tag{65}$$

On the left-hand side, q_i is evaluated at $g_i - \varepsilon_i^*$; $\partial p_i / \partial q_i < 0$ is evaluated at $q_i(g_i - \varepsilon_i^*)$.

Fix g_i , and consider the left-hand and right-hand sides as separate functions of ε_i . The functions can be graphed, as in Figure 7 (which is drawn for an exponential density and linear demand) and the intersection of the two curves gives the equilibrium value ε_i^* .

Since $f_i(\varepsilon_i)$ falls to the right of $\varepsilon_i = 0$, the first condition for $\varepsilon_i^* = g_i - p_i^* > 0$ is that $\gamma_i F_i$ be sufficiently great:

$$\frac{1}{\gamma_i F_i} \left(\frac{g_i - c_i}{(dp_i/dq_i)} + q_i \right) < f_i(0). \tag{66}$$

Then the left-hand side of equation (65) is below the right-hand side at $\varepsilon_i = 0$.

The second requirement for $\varepsilon_i^* = g_i - p_i^* > 0$ is that the function on the left slope upward. The slope of this function is

$$-\frac{1}{\gamma_i F_i} \left[2 \frac{dq_i}{dp_i} + (p_i - c_i) \frac{d^2 q_i}{dp_i^2} \right]. \tag{67}$$

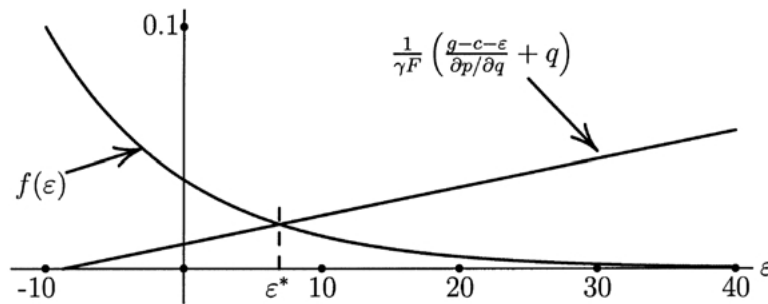


Figure 7. Equilibrium $g - \varepsilon$.

The second condition for $\varepsilon_i^* = g_i - p_i^* > 0$ is that the term in brackets be negative; this is satisfied for linear demand, and is assumed to hold.

b. Differentiating the first-order condition with respect to g_i yields

$$\frac{\partial q_i}{\partial g_i} = \frac{1}{[-\partial^2 E(\pi_i)/\partial q_i^2]} \frac{\partial^2 E(\pi_i)}{\partial g_i \partial q_i}. \quad (68)$$

The second-order condition implies that the term in brackets on the right is positive. Differentiating the first-order condition gives

$$\frac{\partial^2 E(\pi_i)}{\partial g_i \partial q_i} = -\gamma_i F_i f_i' \frac{dp_i}{dq_i} < 0, \quad (69)$$

where the sign depends on $\varepsilon_i^* = g_i - p_i^* > 0$ and the assumption that $f_i' < 0$ for $\varepsilon > 0$.

c. Since

$$\frac{\partial^2 E(\pi_i)}{\partial (\gamma_i F_i) \partial q_i} = -f_i \frac{dp_i}{dq_i} > 0, \quad (70)$$

$$\frac{\partial q_i}{\partial (\gamma_i F_i)} = \frac{1}{[-(\partial^2 E(\pi_i)/\partial q_i^2)]} \frac{\partial^2 E(\pi_i)}{\partial (\gamma_i F_i) \partial q_i} > 0.$$

d. Differentiating the first-order condition,

$$\frac{\partial q_i}{\partial \chi_i} = \frac{1}{[-(\partial^2 E(\pi_i)/\partial q_i^2)]} \left[\frac{\partial p_i}{\partial \chi_i} + \gamma_i F_i f_i' \frac{\partial p_i}{\partial \chi_i} \frac{\partial p_i}{\partial q_i} + (q_i - \gamma_i F_i f_i) \frac{\partial^2 p_i}{\partial \chi_i \partial q_i} \right].$$

The denominator on the right is positive by the second-order condition. The first two terms in the numerator on the right are positive. $q_i - \gamma_i F_i f_i > 0$ by Equation (62). Then $(\partial^2 p_i/\partial \chi_i \partial q_i) \geq 0$ is sufficient to make the numerator, and the entire right-hand side, positive.

Basic model: the Competition Authority's first-period problem

Equation (48) is the Lagrangian for the Competition Authority's first-period problem. Differentiating the first-order necessary conditions (49)–(51) with respect to χ_{12} gives the system of equations

$$\begin{pmatrix} 0 & -I_1 \frac{\partial \tau_{11}}{\partial g_{11}} & -I_2 \frac{\partial \tau_{21}}{\partial g_{21}} \\ -I_1 \frac{\partial \tau_{11}}{\partial g_{11}} & \frac{\partial^2 L_1}{\partial g_{11}^2} & \frac{\partial^2 L_1}{\partial g_{11} \partial g_{21}} \\ -I_2 \frac{\partial \tau_{21}}{\partial g_{21}} & \frac{\partial^2 L_1}{\partial g_{11} \partial g_{21}} & \frac{\partial^2 L_1}{\partial g_{21}^2} \end{pmatrix} \begin{pmatrix} \frac{\partial \lambda}{\partial \chi_{12}} \\ \frac{\partial g_{11}}{\partial \chi_{12}} \\ \frac{\partial g_{21}}{\partial \chi_{12}} \end{pmatrix} = - \begin{pmatrix} 0 \\ \frac{\partial^2 L_1}{\partial \chi_{12} \partial g_{11}} \\ \frac{\partial^2 L_1}{\partial \chi_{12} \partial g_{21}} \end{pmatrix}. \quad (71)$$

Second-order conditions require that the determinant

$$D_1 = 2 \left(I_1 \frac{\partial \tau_{11}}{\partial g_{11}} \right) \left(I_2 \frac{\partial \tau_{21}}{\partial g_{21}} \right) \left(\frac{\partial^2 L_1}{\partial g_{11} \partial g_{21}} \right) - \left(I_1 \frac{\partial \tau_{11}}{\partial g_{11}} \right)^2 \frac{\partial^2 L_2}{\partial g_{21}^2} - \left(I_2 \frac{\partial \tau_{21}}{\partial g_{21}} \right)^2 \times \frac{\partial^2 L_1}{\partial g_{11}^2} \quad (72)$$

of the coefficient matrix on the left be positive.

In the first term on the right, $\partial \tau_{11} / \partial g_{11} < 0$, $\partial \tau_{21} / \partial g_{21} < 0$, and

$$\frac{\partial^2 L_1}{\partial g_{11} \partial g_{21}} = - \frac{1}{(r+h)^2} \frac{\partial M_1}{\partial g_{21}} \frac{\partial h}{\partial g_{11}} < 0 \quad (73)$$

(noting that $\partial M_1 / \partial g_{21}$ is not a function of g_{11}).

One set of sufficient conditions for $D_1 > 0$ is that

$$\frac{\partial^2 L_1}{\partial g_{11}^2} < 0, \quad \frac{\partial^2 L_2}{\partial g_{21}^2} < 0 \quad (74)$$

and sufficiently large in magnitude. I assume that the second-order condition is satisfied.

Then from Equation (71) we obtain the comparative static derivatives

$$D_1 \frac{\partial g_{11}}{\partial \chi_{12}} = \left(I_2 \frac{\partial \tau_{21}}{\partial g_{21}} \right)^2 \frac{\partial^2 L_1}{\partial \chi_{12} \partial g_{11}} - \left(I_1 \frac{\partial \tau_{11}}{\partial g_{11}} \right) \left(I_2 \frac{\partial \tau_{21}}{\partial g_{21}} \right) \frac{\partial^2 L_1}{\partial \chi_{12} \partial g_{21}} \quad (75)$$

$$D_1 \frac{\partial g_{21}}{\partial \chi_{12}} = - \left(I_1 \frac{\partial \tau_{11}}{\partial g_{11}} \right) \left(I_2 \frac{\partial \tau_{21}}{\partial g_{21}} \right) \frac{\partial^2 L_1}{\partial \chi_{12} \partial g_{11}} + \left(I_1 \frac{\partial \tau_{11}}{\partial g_{11}} \right)^2 \frac{\partial^2 L_1}{\partial \chi_{12} \partial g_{21}} \quad (76)$$

Here

$$\frac{\partial^2 L_1}{\partial \chi_{12} \partial g_{21}} = - \frac{1}{(r+h)^2} \frac{\partial M_1}{\partial g_{21}} \frac{\partial h}{\partial \chi_{12}} > 0. \quad (77)$$

It follows from equations (75) and (76) that a sufficient condition for $\partial g_{11} / \partial \chi_{12} < 0$, $\partial g_{21} / \partial \chi_{12} > 0$ is equation (53),

$$\frac{\partial^2 L_1}{\partial \chi_{12} \partial g_{11}} = \frac{\partial}{\partial \chi_{12}} \left\{ \frac{(r+h)(\partial M_1 / \partial g_{11}) + [M_2(\chi_{12}) - M_1(g_{11}, g_{21}) + z(h) - (r+h)z'(h)](\partial h / \partial g_{11})}{(r+h)^2} - \lambda_1 I_1 \frac{\partial \tau_{11}}{\partial g_{11}} \right\}$$

(using equation (38) to substitute $z(h) - (r+h)z'(h) = -(\pi_{12} - \pi_{11}) = -\Delta$)

$$= \frac{\partial}{\partial \chi_{12}} \left\{ \frac{(r+h)(\partial M_1 / \partial g_{11}) + [M_2(\chi_{12}) - M_1(g_{11}, g_{21}) - \Delta](\partial h / \partial g_{11})}{(r+h)^2} - \lambda_1 I_1 \frac{\partial \tau_{11}}{\partial g_{11}} \right\} < 0. \quad (78)$$

The first-order condition for g_{11} is

$$\frac{1}{(r+h)} \frac{\partial M_1}{\partial g_{11}} + \frac{M_2(\chi_{12}) - M_1(g_{11}, g_{21}) - \Delta}{(r+h)^2} \frac{\partial h}{\partial g_{11}} \equiv \lambda_1 I_1 \frac{\partial \tau_{11}}{\partial g_{11}}. \quad (79)$$

Since $\partial \tau_{11} / \partial g_{11} < 0$, we also have that the left-hand side of equation (79) is negative.

Substituting from equation (79) to eliminate λ_1 gives as a necessary condition for inequality (53) that the proportional impact of an increase in χ_{12} on $\partial \tau_{11} / \partial g_{11} < 0$ be less than the proportional impact of an increase in χ_{12} on $\partial M / \partial g_{11}$,

$$\frac{\partial / \partial \chi_{12} \left\{ ((\partial M_1 / \partial g_{11}) / r + h(g_{11}; \chi_{12})) + (M_2(\chi_{12}) - M_1(g_{11}, g_{21}) - \Delta / [r + h(g_{11}; \chi_{12})])^2 (\partial h / \partial g_{11}) \right\}}{(1/r + h)(\partial M_1 / \partial g_{11}) + (M_2(\chi_{12}) - M_1(g_{11}, g_{21}) - \Delta / (r+h)^2)(\partial h / \partial g_{11})} > \frac{1}{(\partial \tau_{11} / \partial g_{11})} \frac{\partial}{\partial \chi_{12}} \left(\frac{\partial \tau_{11}}{\partial g_{11}} \right), \quad (80)$$

noting that division by a negative number reverses the direction of the inequality.

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