Chapter 6

Competition Policy: Publicity vs. Prohibition & Punishment

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6.1 Introduction

The competition policies of the European Union and the United States are based on the principle of prohibition and punishment. Certain types of strategic business behavior are outlawed and firms (in some circumstances, individuals) found to have engaged in such conduct are subject to penalties. Exceptions may be possible,¹ and the enthusiasm with which the rules are enforced is subject to cycles, but the underlying approach is clear.

Well-developed lines of decisions in both jurisdictions limit the exchange of information among competitors. Trade associations are typically permitted to collect information from single firms and report it in aggregate form, but any practice that allows the identification of prices and quantities of particular firms or for particular transactions is viewed with suspicion. Oligopoly theory suggests that uncertainty about what rivals are doing may induce a firm to behave more competitively, and may prevent rivals from retali-

¹Many European Commission block exemptions permit firms with market shares that do not exceed specified levels to engage in various types of behavior that is denied to larger firms. The Commission may set aside the Article 92 prohibition of state aid if such aid is found to serve other EU purposes. In the United States, an early 1950s antitrust challenge to the conduct of major oil companies floundered on the shoals of the foreign relations.
ing against such behavior. Exchanges of detailed information about specific transactions are thought to facilitate tacit collusion by allowing rivals to react rapidly to selective price cuts or to other behavior that might destabilize a cozy oligopolistic equilibrium.²

The competition policy of Denmark, in contrast, is based on the principle of abuse control. Interfirm agreements are not prohibited. Specified types of interfirm agreements must, however, be notified to the Competition Council, which has the power and responsibility to investigate arrangements that are thought to have anticompetitive potential. One of the primary policy instruments available to the Competition Council to deal with such arrangements is publicity, the public reporting of the results of investigations into market conditions. The goal is to promote transparency of information about market conditions, and this can involve official circulation of precisely the sort of information that EU and US competition policy forbids firms to make arrangements to distribute.

The genesis of this policy is not entirely clear. One motivation may be that complete and perfect information is part of the classroom model of perfect competition. It does seem to reflect a hostility toward price discrimination, and the expectation that consumers would press for the best prices available if they were made aware of price differences.³

To formally analyze the effectiveness of a transparency policy, however, one needs to specify the means by which a transparency policy is expected to affect firm conduct. Hypotheses about such mechanisms were advanced in US debates during the early years after the passage of the Sherman Act, when for a time publicity was part of US antitrust policy.

Early Supreme Court decisions interpreting the Sherman Act raised the fear that it would be ineffective, and in any event early enforcement efforts were not vigorous.⁴ A long debate over the strengthening of antitrust policy

²Experimental evidence suggests that the ability of rivals to identify and punish an individual defector is an important element in maintaining tacit collusion (Holt, 1995, p. 407). Transparency would contribute to making such targeted reactions possible.

³There may also be elements of industry capture behind the transparency approach; representatives of the business community are traditionally involved in the development of Danish competition policy. For further discussion, see Albæk, Møllgaard, and Overgaard (1996a, 1996b). At this writing, there are proposals to amend the Danish Competition Act by introducing some elements of prohibition and punishment and moving Denmark toward the EU approach. This is discussed OECD (1996) and at the Competition Council’s world wide web site, http://www.ks.dk/.

⁴See, for example, Letwin(1965), Chapter 5.
ensued, and this led to the establishment (in 1903) of the Bureau of Corporations, which was charged with gathering information about industrial practices and publicizing abuses.

Publicity was expected to serve several purposes. A major target was corrupt financial practices, which would not now be regarded as falling within the sphere of competition policy (Jenks, 1900, p. 222)\textsuperscript{5}

Publicity regarding the organization of a business, which should compel promoters to show clearly to investors the basis on which a large corporation or a combination of corporations is organized, would certainly put careful investors in a position to protect themselves.

But publicity also aimed to improve market performance by raising the threat of entry or inducing actual entry (Jenks, 1900, pp. 223-4):\textsuperscript{6}

\begin{quote}
publicity ...which should show with a reasonable degree of detail the profits of the larger combinations would, in case of the abuse of their power, so stimulate competition against them, either actually or potentially, that consumers would to a great degree be protected against excessive increase in prices.
\end{quote}

\textsuperscript{5}Most references to publicity in the influential Preliminary Report of the Industrial Commission on Trusts and Industrial Combinations (1900) occur with respect to financial abuse.

\textsuperscript{6}An additional element was prominent in a somewhat later British debate on publicity as an element of competition policy — that publicity given to monopolistic practices would so shame the businessmen concerned that such behavior be abandoned (Hilton, 1918, p. 30, quoted in Dennison, 1980, p. 230):

\begin{quote}
by ensuring that extortion should be publicly pilloried, [publicity] would do much to prevent it being practised. In quite respectable business conduct, as in other human affairs, many things are done which would not be done if there were a greater probability of their being made public.
\end{quote}

Marshall (1923, p. 442) makes a similar argument:

upright men are often half-way converted towards removing such just grounds as there may be for complaint against their conduct, by reading a well-informed and well-balanced statement of those grounds; and knowing that an impartial public is forming its judgment on them. In this and many other ways a careful authoritative inquiry, with publication of the evidence taken, goes a long way towards removing sources of social harm...
Publicity was endorsed by Governor Theodore Roosevelt in a message to the New York state legislature. Roosevelt thought of publicity as a possible first step toward eventual adoption of a more severe policy (January 1900; emphasis added; quoted by Thorelli, 1954, p. 415):

Where a trust becomes a monopoly the State has an immediate right to interfere. Care should be taken not to stifle enterprise or disclose any facts of a business that are essentially private; but the State for the protection of the public should exercise the right to inspect, to examine thoroughly all the workings of great corporations just as is now done with banks; and wherever the interests of the public demand it, it should publish the results of its examination. Then, if there are inordinate profits, competition or public sentiment will give the public the benefit in lowered prices; and if not, the power of taxation remains. It is therefore evident that publicity is the one sure and adequate remedy which we can now invoke. There may be other remedies, but what these others are we can only find out by publicity, as the result of investigation. The first requisite is knowledge, full and complete.

After several years experience with the information-disseminating activities of the Bureau of Corporations, the Sherman Act was reinforced by passage of the Federal Trade Commission Act and the Clayton Act. These laws committed US antitrust to the prohibition and punishment approach, but information-gathering was by no means abandoned. The need for such activity had been highlighted by President Wilson in a 1914 speech to Congress (Baker and Dodd, 1925, pp. 85-6)

And the business men of the country desire something more than that the menace of legal process ...be made explicit and intelligible. They desire the advice, the definite guidance, and information which can be supplied by an administrative body, an interstate trade commission.

[The opinion of the country] demands such a commission only as an indispensable instrument of information and publicity, as a clearing house for the facts by which both the public mind and the managers of the great business undertakings should be guided....
6.2. MONOPOLY WITH A $P^2$ COMPETITION POLICY

Information gathering was part of the mandate of the Federal Trade Commission, along with its prohibition and punishment responsibilities (Scherer, 1990).

I develop here a model of firm behavior under alternative competition policies, prohibition and punishment (henceforth, $P^2$) and publicity, where publicity affects firm conduct because of its impact on the possibility of entry. This permits a comparison of the impacts of the two types of policy on market performance.

For simplicity, I examine an industry that is supplied by a single firm. I begin with the case of a monopolist that does not face the threat of entry but must deal with a $P^2$ competition policy. Then I turn to the contrasting case of a monopolist that faces the threat of entry without any competition policy at all. Then two cases are then combined compare the impact of a $P^2$ policy with the threat of entry and a publicity policy on market performance.

### 6.2 Monopoly with a $P^2$ competition policy

#### 6.2.1 Enforcement

If publicity is to affect firm behavior, information about the market must in some way be imperfect. I assume that inverse demand curve has a random intercept,

$$ p = a + \varepsilon - q $$

where the random term $\varepsilon$ is uniformly distributed over the interval $(-\alpha, \alpha)$:

$$ -\alpha \leq \varepsilon \leq \alpha. $$

Marginal cost is a constant $c$ per unit. I will also assume that there is an entry cost $K$, although this becomes a factor only in the following section.

Three parameters describe the $P^2$ policy: the threshold price $p_G$, the probability $\gamma$ of prosecution and conviction if a price greater than $p_G$ is observed, and the fine $F$ that is imposed if conviction takes place.

The threshold price is a device to model the decision-making process of the enforcement agency: if a price greater than or equal to $p_G$ is observed,

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7Assume also that $a - \alpha \geq 0$, so that a sufficiently small quantity will always bring a positive price.
the government investigates the activities of the firm.\footnote{In practice, competition authorities receive a variety of signals, including complaints from rivals and customers. By extending the model developed here to include two industries and a budget constraint that limits the investigative possibilities of the enforcement agency, one could explicitly model resource allocation by the enforcement agency. While the economist’s natural starting point in such a study would be to suppose that decisions are made to maximize expected consumers’ surplus or expected net social welfare, other objective functions have been ascribed to enforcement agencies. See Posner (1976, pp. 230-1); Dorman (1980, p. 1115); Schwartz (1980, p. 1093); Shughart (1996).}

Competition policy typically does not prohibit monopoly as such: what it prohibits are certain types of conduct by firms with some degree of market power that are thought to be against the public interest. “Against the public interest” may mean that some measure of welfare (such as consumers’ surplus or consumers’ plus producers’ surplus) is reduced, or that the conduct raises the cost of actual or potential rivals. It may refer to some noneconomic objective function. If investigation leads competition authorities to conclude that such misbehavior has occurred, they must decide whether or not to initiate an enforcement proceeding, the exact nature of which will vary across legal regimes. If there is such a proceeding, competition authorities may or may not succeed in obtaining a finding that the firm has violated competition policy. I use a single parameter to describe the ambiguity, from the firm’s point of view, concerning the possibilities of prosecution and conviction: $\gamma$ is the probability that a firm is found to have violated competition policy, if a price greater than $p_G$ is observed.

Third, if there is a finding that an antitrust violation has taken place, let $F$ be the resulting fine.\footnote{It is possible to allow for structural remedies—nationalization or the breakup of the firm—along the lines developed in the section on entry deterrence. Although provision for such penalties is a part of competition policy in many jurisdictions, they are rarely applied. I owe to John Baldwin the observation that the threat of nationalization may have appeared quite tangible at certain times and in certain countries.} $\gamma F$, the expected fine if an investigation takes place, is an index of the severity of competition policy.

### 6.2.2 The incumbent’s payoff function

The probability that the government observes a price at or above the threshold level is

$$\tau_G \equiv \Pr(p \geq p_G) = \Pr(a - q + \varepsilon \geq p_G)$$
6.2. MONOPOLY WITH A $P^2$ COMPETITION POLICY

Output range  \[ \tau_G \]  Expected payoff function
\[ 0 \leq q \leq a - \alpha - p_G \]  1  \( (a - c - q)q - \gamma F \)
\[ a - \alpha - p_G \leq q \leq a + \alpha - p_G \]  \( \frac{a + \alpha - p_G - q}{2\alpha} \)  \( (a - c - q)q - \frac{a + \alpha - p_G - q}{2\alpha} \gamma F \)
\[ a + \alpha - p_G \leq q \]  0  \( (a - c - q)q \)

Table 6.1: $P^2$ Competition Policy and Monopolist’s Single Period Payoff Function

\[
= \Pr[\varepsilon \geq p_G - (a - q)] = \frac{\alpha - [p_G - (a - q)]}{2\alpha} = \frac{a + \alpha - p_G - q}{2\alpha}, \quad (6.3)
\]
provided this lies between 0 and 1. This is illustrated in Figure 6.1.

If \( q \geq a + \alpha - p_G \), realized price is certainly less than \( p_G \) and the probability of investigation for competition policy violations is zero (\( \tau_G = 0 \)). If \( q \leq a - \alpha - p_G \), realized price is certainly greater than \( p_G \), and investigation is certain (\( \tau_G = 1 \)).

Table 1 describes the three segments of the monopolist’s payoff function.

6.2.3 Market performance with a $P^2$ policy

Extreme values of $p_G$

In the absence of a competition policy, the incumbent’s profit-maximizing output would be

\[
q_0 = \frac{1}{2}(a - c), \quad (6.4)
\]
sold at expected price\(^{10}\)

\[
p_0 = c + q_0 \quad (6.5)
\]
and yielding expected payoff

\[
\pi_0 = (q_0)^2 \quad (6.6)
\]

The output \( q_0 \) would yield the range of possible prices

\[
p_0 - \alpha \leq p \leq p_0 + \alpha. \quad (6.7)
\]

If \( p_G > p_0 + \alpha \), then the unconstrained monopoly output could not lead to a realized price high enough to trigger a government investigation. As shown
$\tau_G = \Pr(p \geq p_G) = \Pr(a + \varepsilon - q \geq p_G) = \frac{a + \alpha - q - p_G}{2\alpha}$

(for $0 \leq \tau_G \leq 1$)

Figure 6.1: Demand, threshold price $p_G$
6.2. MONOPOLY WITH A $P^2$ COMPETITION POLICY

Figure 6.2: Payoff function, $a + \alpha - p_G \leq q_0$
in Figure 6.2, the incumbent’s profit-maximizing output for such a threshold price is simply $q_0$.

At the other extreme, if $p_G$ is very large, the $P^2$ policy has no impact on the firm’s output decision because the output expansion required to avoid or reduce the probability of investigation is so great that the firm prefers simply to produce output $q_0$ and face the certainty of investigation. The typical payoff function in such cases, shown in Figure 6.3, has a global maximum at $q = q_0$. The threshold price $p_G$ is so low relative to the expected penalty $\gamma F$ that the firm maximizes its expected payoff by producing monopoly output $q_0$ and accepting the inevitability of investigation. If the threat of punishment is to alter behavior, it must be possible for the firm to avoid punishment at a reasonable opportunity cost.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{payoff_function.png}
\caption{Payoff function, $a + \alpha - p_G \geq q_0$}
\end{figure}

\footnote{For notational simplicity I omit the expectations operator.}
Intermediate values of $p_G$

Intermediate values of $p_G$ raise the possibility that the firm’s profit-maximizing output choice will be in the range $a - \alpha - p_G \leq q \leq a + \alpha - p_G$, where $0 \leq \tau_G \leq 1$ and the payoff function is

$$\pi = (a - c - q)q - (q_G - q)\frac{\gamma F}{2\alpha}.$$  \hfill (6.8)

Figure 6.4: Payoff function, $q_0 \leq a + \alpha - p_G \leq q_0 + \frac{\gamma F}{4\alpha}$

The payoff function may have three configurations. If $p_G$ is only slightly below $p_0 + \alpha$, the incumbent maximizes its expected payoff by expanding output from $q_0$ to $a + \alpha - p_G$ and driving $\tau_G$ to zero (Figure 6.4). If $p_G$ is somewhat lower, the firm will expand output beyond $q_0$, but not enough to make $\tau_G = 0$. As $p_G$ falls, the payoff function develops two local maxima (as
in Figure 6.5). If $p_G$ is sufficiently small, the local maximum that yields the greatest payoff is at $q = q_0$.

\[ \pi = \pi_0 - (q - q_0)^2 - \gamma F \]

\[ \pi = \pi_0 - (q - q_0)^2 \]

\[ \text{Figure 6.5: Payoff function, } q_0 + 2\alpha \leq a + \alpha - p_G \leq q_0 + 2\alpha + \frac{\gamma F}{4\alpha} \]

Theorem 1 (proven in the Appendix) gives the relationship between the threshold price $p_G$ and the incumbent’s equilibrium output level. There are two cases, depending on the relationship between the expected fine $\gamma F$ and the demand uncertainty parameter $\alpha$. In both cases, the threat of antitrust investigation induces output expansion and therefore improved market performance, provided the threshold price that triggers investigation is not too low relative to the expected antitrust penalty. If the incumbent must expand output too much to avoid investigation, it will simply take the greatest short-run profit it can and risk a fine.

**Theorem 1** The relationship between $p_G$ and expected-profit maximizing output $q_1$ is

(a) $0 \leq \gamma F \leq 16\alpha^2$
6.3 Entry deterrence without competition policy

6.3.1 Probabilistic evaluation of the threat of entry

Entry games often explicitly model the interaction between one or more incumbents and one or more known potential entrants (see, for example, Martin, 1995). This approach to modelling entry can be traced to Friedman’s (1979) influential criticism of naive limit price models. Considering a situation in which (Friedman, 1979, p. 237) “there are no stochastic elements

If \( \alpha \) is sufficiently large, \( q_0 + 2\alpha \) will equal the competitive output level \( a - c \). The incumbent would shut down rather than produce above this output level.

It is usual to associate the limit price model with the work of Joe S. Bain, although it appears as well in Kaldor (1935) and Clark (1940). Bain (1949, pp. 452-3) makes essentially the same criticism of the naive limit price model as Friedman, which is most accurately thought of as targeting the Sylos-Labini postulate (for discussion of which, see Modigliani, 1958).
Figure 6.6: Theorem 2.1

\[ a + \alpha \]
\[ a \]
\[ p_0 + \alpha \]
\[ p_0 \]
\[ p_0 + \alpha - \frac{\gamma F}{4\alpha} \]
\[ p_0 + \alpha - \frac{\gamma F}{8\alpha} \]

(a) \( 0 \leq \gamma F \leq 16\alpha^2 \)

\[ p_0 + \alpha - \sqrt{\gamma F} \]

(b) \( 16\alpha^2 \leq \gamma F \)

Figure 6.6: Theorem 2.1
... and both the established firm and the entrant know the demand and cost functions which prevail before and after entry,” Friedman observes that the notion of limit price is simply out of place (1979, p. 237):

in the models examined below an entrant has no direct interest in an established firm’s pre-entry price policy. What matters to him is the price pattern which would emerge after he were to come in. That is, he wants to know what equilibrium behavior in the market would be if he were in, and he wants to know what profits he would have under such an equilibrium.

Friedman’s arguments are cited by Milgrom and Roberts (1982, p. 44) in their own seminal work; they note that his conclusions depend on the assumption of complete information. They relax this assumption by examining a situation in which there is one incumbent and one potential entrant, each uncertain about the other’s costs.

Here I relax the assumption of complete and perfect information in another direction, modelling a situation in which an incumbent anticipates the possibility of entry without being able to specifically identify all potential entrants or fully model their actions. I suppose that the incumbent regards entry as a two-stage process. If realized price reaches or exceeds a threshold level $p_E$, the incumbent expects that potential entrants will actively investigate the possibility of coming into the market. If investigation takes place, no attempt at entry may result. If entry is attempted, it may fail. Let $\eta$ be the probability that successful entry occurs, given that $p \geq p_E$.

The assumption that entry is investigated only if price exceeds some threshold level may be motivated with reference to recent empirical findings on entry. Although it is common in models of entry games to assume that entry will occur if expected post-entry profitability is positive, Geroski (1995, p. 427) notes that actual entry “seems to be slow to react to high profits.” A theoretical rationale for this empirical regularity may be found in the work of Dixit and Pindyck (1994), which suggests that entrants, expecting positive post-entry profit, would nonetheless defer entry until it is sufficiently unlikely that no greater profit opportunity will emerge elsewhere.

\footnote{For a case study, see the account of decisions about entry that appears in the decision U.S. v. Penn-Olin Chemical Co. 378 U.S. 158 (1964). For analyses of entrants’ post-entry experiences, see Geroski (1991), Geroski and Schwalbach (1991).}
For an incumbent that operates in many geographic markets (as in the chain store game), the probability $\eta$ may be thought of as reflecting the accumulated experience of the firm. Alternatively, one may think of “investigation” as a process by which a potential entrant learns its value of $K$, with $\eta$ the entry probability implied by the distribution of entry costs across potential entrants. The assumption that incumbents expect successful entry to occur with only a certain probability, if price is high enough to induce investigation, may also be thought of describing the behavior of incumbents who believe they face (possibly unknown) potential entrants playing mixed strategies.

Let $\tau_E$ denote the probability that a potential entrant investigates the possibility of coming into the market. If $\tau_E$ lies between zero and one, it is given by

$$\tau_E \equiv \Pr(p \geq p_E) = \frac{\alpha - [p_E - (a - q)]}{2\alpha} = \frac{a + \alpha - q - p_E}{2\alpha},$$

(6.9) provided this lies between zero and one.

The present discounted value of an incumbent that produces output $q$ is

$$V_M = \frac{(a - c - q)q + (1 - \eta \tau_E)V_M + \eta \tau_E V_2}{1 + r},$$

(6.10)

where

$$V_2 = \frac{1}{r} \left( \frac{a - c}{3} \right)^2$$

(6.11)

is the expected present-discounted value of a Cournot duopolist in the market.\(^{14}\)

The first term on the right in (6.10) is the income received by the monopolist at the end of the period, appropriately discounted. At the start of the next period, one of two events occurs. With probability $(1 - \tau E \eta)$, entry does not occur, and the incumbent’s value from that point forward is $V_M$. This explains the second term in the numerator on the right. With probability $\tau E \eta$, entry does occur, and from that point forward the incumbent earns Cournot duopoly profit in each period, yielding present value $V_2$. This explains the third term in the numerator on the right.

\(^{14}\)Assume also that entry cost $K$ is greater than the noncooperative equilibrium value of a Cournot triopolist, so that at most one episode of entry would take place.
Combining terms, the incumbent’s present discounted value is

\[ V_m = V_2 + \frac{(a - c - q)q - \pi_2}{r + \eta\tau_E} \]  

(6.12)

\[ = V_2 + \frac{\pi_0 - \pi_2 - (q - q_0)^2}{r + \eta\tau_E} \]  

(6.13)

\[ = V_2 + \frac{\Delta - (q - q_0)^2}{r + \eta\tau_E}, \]  

(6.14)

writing

\[ \Delta = \pi_0 - \pi_2 \]  

(6.15)

for the difference between monopoly and duopoly profit.

The expected present discounted value of the incumbent exceeds the expected present discounted value of a duopolist by a term that is proportional to the excess of the monopolist’s single period payoff over the single period payoff of a duopolist. The modified discount factor that appears in the final term on the right includes the endogenous probability that entry occurs.

Let

\[ p_B = p_0 + \alpha - \left( \sqrt{\Delta + \left( \frac{2\alpha r}{\eta} \right)^2} - \frac{2\alpha r}{\eta} \right) \]  

(6.16)

\[ p_C = p_0 + \alpha - \left( \sqrt{\Delta + \left( \frac{2\alpha r + \frac{2\alpha r}{\eta}}{\eta} \right)^2} - \frac{2\alpha r}{\eta} \right) \]  

(6.17)

(6.16) and (6.17) are the end-points of the range of threshold values \( p_E \) where the first-order condition to maximize (6.14) results in a value of \( \tau_E \) that lies between 0 and 1. It is shown in the Appendix that

**Theorem 2** the relationship between threshold price \( p_E \) and the incumbent’s expected-profit maximizing output \( q_2 \) is

(a) for \( a + \alpha \geq p_E \geq p_0 + \alpha \), \( q_2 = q_0 \);

(b1) for \( p_0 + \alpha \geq p_E \geq \max(p_B, p_0 - \alpha) \), \( q_2 = q_0 + p_0 + \alpha - p_E \);

(b2) \( p_0 - \alpha \geq p_E \geq p_B \), if such a range exists,

\[ q_2 = \begin{cases} 
q_0 + p_0 + \alpha - p_E & \text{if } p_0 - \alpha \geq p_E \geq p_0 - \sqrt{\frac{\eta}{r + \eta}} \Delta \\
q_0 & \text{if } p_0 - \sqrt{\frac{\eta}{r + \eta}} \Delta \geq p_E \geq p_B 
\end{cases} \]
(c) for \( p_B \geq p_E \geq p_C \)

\[
q_2 = q_0 + p_0 + \alpha + \frac{2\alpha r}{\eta} - p_E - \sqrt{\left( p_0 + \alpha + \frac{2\alpha r}{\eta} - p_E \right)^2 - \Delta};
\]  
(6.18)

(d) for \( p_C \geq p_E \), \( q_2 = q_0 \).

Figure 6.7: Theorem 3.1

Theorem 2 is illustrated in Figure 6.7. The threat of entry, and the resulting loss of value, induce some expansion of output above the unconstrained monopoly level. But output expansion itself implies a loss of value. If the threat of entry is to induce output expansion, the price reduction required to eliminate or reduce the probability of investigation by potential entrants cannot be too great. Otherwise, the profit-maximizing incumbent will simply produce the unconstrained monopoly output.
6.4 Competition policy with the threat of entry

6.4.1 Monopoly entry deterrence with \( P^2 \) competition policy

If an incumbent monopolist faces both the possibility of entry and the threat of antitrust enforcement, its expected present discounted value is

\[
V_m = \frac{\pi_0 - (q - q_0)^2 - \tau_G \gamma F + (1 - \eta \tau_E)V_m + \eta \tau_E V_2}{1 + r}.
\]  

(6.19)

The interpretation of (6.19) is similar to that of (6.10). Current income now includes an expected antitrust fine. Future expected value is a weighted average of values if entry does not and does occur. Collecting terms, (6.19) can be rewritten

\[
V_m = V_2 + \Delta - \frac{(q - q_0)^2 - \tau_G \gamma F}{r + \eta \tau_E}.
\]  

(6.20)

6.4.2 Monopoly entry deterrence with a publicity-based competition policy

The interpretation of \( \tau_E \) and the entry probability \( \eta \) are unchanged. Now, however, let \( \tau_G \), defined as before with regard to a threshold price \( p_G \) denote the probability that the government investigates the industry, and \( \phi \) the probability that the government issues a report detailing industry circumstances. Entry now takes place, if at all, either after the firm investigates the industry or after the government issues a report, bringing the industry to the attention of potential entrants. I assume that government and private investigation are independent events, in a probabilistic sense.

The probability of states of the world in which potential entrants make a decision about entry is\(^{15}\)

\[
A = \tau_E + (1 - \tau_E)\phi \tau_G.
\]  

(6.21)

\(^{15}\)Equivalently, one may write

\[
A = \phi \tau_G + \tau_E (1 - \phi \tau_G).
\]

This is the probability that there is a government report plus the probability that there is not a government report times the probability that entrants investigate the industry.
The first term on the right is the probability that the entrant investigates the industry. The second term on the right is the probability that the entrant does not investigate the industry, the government does, and the government issues a report about the industry.

The expected value of the incumbent is

\[
V_m = \frac{\pi_0 - (q - q_0)^2 + (1 - \eta A)V_m + \eta AV_2}{1 + r}
\]

\[
= V_2 + \frac{\Delta - (q - q_0)^2}{r + \eta A}
\]

(6.22)

6.4.3 Comparative market performance

To compare the impact of the $P^2$ and publicity policies on market performance, I present a series of numerical examples.\(^{16}\) Some parameter values are common to all the examples:

- $a = 101, c = 1$;
- $\alpha = 15$;
- $\pi_0 = 2500, \pi_2 = 1111\frac{1}{7}$;
- $r = 1/10$;
- $\phi = 1/2$;
- $\gamma F = \Delta$.

The final parameter choice is made in the interest of comparability: the expected lost profit after investigation, under a $P^2$ policy, equals the lost profit if entry occurs.

For these parameter values, the unconstrained monopoly output is $q_0 = 50$ and the unconstrained monopoly price is $p_0 = 1 + 50 = 51$. Duopoly output and price would be $66\frac{2}{3}$ and $34\frac{1}{3}$ respectively.

\(^{16}\)Four basic cases must be considered to derive results like those of Theorems 2.1 and 3.1. One is where $q_G \leq q_E - 2\alpha$ and another is where $q_E \leq q_G - 2\alpha$. In these cases, the intervals over which $0 \leq \tau_G \leq 1$ and $0 \leq \tau_E \leq 1$ do not overlap. The remaining cases are $q_E - 2\alpha \leq q_G \leq q_E$ and $q_G - 2\alpha \leq q_E \leq q_G$, for which the intervals over which $0 \leq \tau_G \leq 1$ and $0 \leq \tau_E \leq 1$ do overlap. In each of these four cases, the incumbent’s payoff function is defined differently over five segments of output space. Derivation of analytic results requires analysis of the shape of the incumbent’s payoff function. Each of the four basic cases has numerous subcases, depending on values of $\gamma F, \eta, \phi$, and $a - c$.\]
Case 1: High threshold prices

First suppose $p_E = 61, p_G = 56$. Threshold prices are relatively close to the unconstrained monopoly level.

The incumbent’s payoff functions under the alternative competition regimes are shown in Figure 6.8, for $\eta = 1/2$. The incumbent’s value-maximizing output under either competition policy is $q = 60$, implying $p = 41$. By expanding output only slightly above the unconstrained monopoly level, the incumbent makes the probability of entry and the probability of antitrust investigation equal to zero. This output expansion improves market performance, compared with the unconstrained monopoly case.

The classic policy conundrum posed by limit pricing appears: while mar-
ket performance is improved compared with the unconstrained monopoly case, market performance remains worse than in the noncooperative duopoly equilibrium of a one-shot game.

Case 2: Low threshold prices

![Graph showing the comparison between Publicity policy and $P^2$-policy](image)

Notes: $a = 101$; $c = 1$; $\alpha = 15$; $\pi_0 = 2500$; $\gamma F = \Delta = 1388 \frac{8}{9}$; $\phi = 1/2$.

Figure 6.9: $\eta = 1/2$; $p_G = 36$; $p_E = 41$.

Now keep all parameters from the previous example except the threshold prices unchanged, and let $p_E = 41$, $p_G = 36$. Both policies result in improved market performance, as shown in Figure 6.9: $q_1 = 75$ under either policy. Here $\tau_E = 0$, while $\tau_G = 1/6$, so the possibility of government intervention is not completely foreclosed. Under the publicity policy, $A = 0 + (1/2)(1/6) = 1/12$. $\eta A$, the probability of states of the world in which entry occurs, is $1/24 = 0.042$. 
6.4. COMPETITION POLICY WITH THE THREAT OF ENTRY

Case 3: Low entry probability

![Graph showing competition policy with the threat of entry]

Notes: \( a = 101; \ c = 1; \ \alpha = 15; \ \pi_0 = 2500; \ \gamma F = \Delta = 1388 \frac{8}{9}; \ \phi = 1/2. \)

Figure 6.10: \( \eta = 1/10; \ \ p_G = 36; \ p_E = 41. \)

What the first two examples do not bring out is the sensitivity of publicity policy to the probability of entry. Keep all parameters of the previous case fixed, but let \( \eta = 1/10. \) The resulting value functions are shown in Figure 6.10. The \( P^2 \) policy remains almost as effective as before: \( q_1 = 75, \ p_1 = 26; \) the probability of entry is driven to zero, and the probability of antitrust investigation is 1/6. Under a publicity policy, \( q_1 = 63.9, \ p_1 = 37.1. \) In this case, \( \tau_E = 0.37, \ \tau_G = 0.536, \) and \( A = 0.539. \) \( \eta A \) equals 0.054. When the probability of entry is very low, a publicity policy is ineffective.
6.5 Conclusion

The relative efficiency of punishment-based and publicity-based policies depends on the relative impact of actual and potential competition on market performance. For a publicity-based competition policy to be effective, incumbents must believe that entry is a genuine possibility. In terms of the model developed here, neither $\eta$ nor market size $(a - c)$ can be too small, and entry cost $K$ cannot be too large.

If the market will support only a limited number of firms in noncooperative equilibrium, then a publicity policy is unlikely to be effective. Paradoxically, it is small economies like Denmark that should rely on a punishment-based policy, while continent-wide economies like the EU and the US might in principle find a publicity policy effective. This point is not without importance from the point of view of an eventual World Competition Agency, any manifestation of which is unlikely to be endowed with punishment powers in its initial form.

At the same time, for a punishment-based policy to be effective, incumbents must believe that there is a realistic possibility of a fine. If the political establishment lacks the will to support a credible prohibition-and-punishment policy, then a publicity policy may be better than nothing.

Considering the relative impact of $P^2$ and publicity policies on market performance, publicity should be expected to chill rivalry among actual competitors, while increasing the impact of potential competition. If there are a relatively large number of actual competitors, publicity will be a less effective competition policy than prohibition and punishment. But if there is only a small number of actual competitors, the threat of publicity will increase the power of potential competition, and the reality of publicity may induce entry.

Finally, one should not lose sight of the fact that $P^2$ and publicity policies are not mutually exclusive. Publicity could be the first stage of a policy that included the possibility of punishment in a second stage if publicity did not result in improved market performance. Further, enforcement proceedings initiated under a $P^2$ policy would as a practical matter generate publicity that would come to the attention of potential entrants. In this sense, a $P^2$ policy is also a publicity policy, while the converse is not the case.\footnote{Allowing for this in the formal model, (6.19) becomes}

$$V_m = \frac{\pi_0 - (q - q_0)^2 - \tau \sigma \gamma F + (1 - \eta A)V_m + \eta AV_2}{1 + r}.$$
6.6 Appendix

6.6.1 Proof of Theorem 2.1

The payoff function

For notational convenience, define

\[ q_G = a + \alpha - p_G. \] (6.23)

The payoff function has three regions.

For

\[ 0 \leq q \leq q_G - 2\alpha, \] (6.24)

antitrust investigation is certain. The payoff function is

\[ \pi = (a - c - q)q - \gamma F = \pi_0 - (q - q_0)^2 - \gamma F. \] (6.25)

For

\[ q_G - 2\alpha \leq q \leq q_G, \] (6.26)

\[ \tau_G \] lies between zero and one. The payoff function is

\[ \pi = (a - c - q)q - \frac{q_G - q}{2\alpha} \gamma F \]

\[ = \pi_0 - \left( q_G - q_0 - \frac{\gamma F}{8\alpha} \right) \frac{\gamma F}{2\alpha} - \left( q - q_0 - \frac{\gamma F}{4\alpha} \right)^2 \] (6.27)

For

\[ q_G \leq q, \] (6.28)

\[ \tau_G = 0 \] and the payoff function is

\[ \pi = (a - c - q)q = \pi_0 - (q - q_0)^2 \] (6.29)

For purposes of the proof, one must consider three ranges for the expected penalty:

\[ = \frac{\pi_0 - (q - q_0)^2 - \tau_G \gamma F + \eta AW_2}{r + \eta A}. \]
(A) \(0 \leq \gamma F \leq 8\alpha^2;\)
(B) \(8\alpha^2 \leq \gamma F \leq 16\alpha^2;\)
(C) \(16\alpha^2 \leq \gamma F.\)

Cases (A) and (B) correspond to case (a) in the statement of the theorem. The results for cases (A) and (B) are the same, although details of the proof differ. Case (C) corresponds to case (b) of the theorem.

(A) \(0 \leq \gamma F \leq 8\alpha^2\)

Note that in case (A)
\[
\frac{\gamma F}{4\alpha} \leq 2\alpha
\]  
(6.30)
(A1) \(q_0 \leq q_G \leq q_0 + (\gamma F/4\alpha).\)

(6.30) and \(q_G \leq q_0 + (\gamma F/4\alpha)\) imply \(q_G \leq q_0 + 2\alpha,\) so \(q_G - 2\alpha \leq q_0.\)

Over the output range \(0 \leq q \leq q_G - 2\alpha,\) the payoff function (6.25) is a parabola that slopes upward to the right, with local maximum at \(q = q_G - 2\alpha.\)

Since \(q_G \leq q_0 + (\gamma F/4\alpha),\) the payoff function (6.27) is a parabola that slopes upward to the right, with local maximum on \(q_G - 2\alpha \leq q \leq q_G\) at \(q = q_G.\)

For \(q_G \leq q,\) the payoff function (6.29) is a parabola that slopes downward to the right, with local maximum \(q = q_G.\) The global maximum of the payoff function in case (A2) is at \(q = q_G.\)

(A2) \(q_0 + (\gamma F/4\alpha) \leq q_G \leq q_0 + 2\alpha.\)

By inequality (6.30), such a region exists.

By subtraction, \(q_G - 2\alpha \leq q_0,\) so \(q_G - 2\alpha \leq q_0 \leq q_G + (\gamma F/4\alpha) \leq q_G,\) and \(q_0 + (\gamma F/4\alpha)\) lies within the range \((q_G - 2\alpha, q_G).\)

Over the output range \(0 \leq q \leq q_G - 2\alpha,\) the payoff function (6.25) is an upward sloping parabola, with local maximum at \(q = q_G - 2\alpha.\) Over the output range \(q_G - 2\alpha \leq q \leq q_G,\) the payoff function is (6.27), which has a local maximum at \(q = q_0 + (\gamma F/4\alpha).\)

Over the output range \(q_G \leq q,\) the payoff function (6.29) is a downward
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sloping parabola, with local maximum at \( q = q_G \). The global maximum in case (A3) is \( q = q_0 + (\gamma F/4\alpha) \).

(A4) \( q_0 + 2\alpha \leq q_G \leq q_0 + (\gamma F/4\alpha) + 2\alpha \).

By subtraction, \( q_0 \leq q_G - 2\alpha \leq q_0 + (\gamma F/4\alpha) \). Over the output range \( 0 \leq q \leq q_G - 2\alpha \), the payoff function (6.25) is a parabola with local maximum at \( q = q_0 \). The firm’s payoff at \( q = q_0 \) is

\[
\pi(q_0) = \pi_0 - \gamma F. \tag{6.31}
\]

Over the output range \( q_G - 2\alpha \leq q \leq q_G \), the payoff function is (6.27), which has a local maximum within this range at output level \( q_0 + (\gamma F/4\alpha) \). The firm’s payoff at this output level is

\[
\pi \left( q_0 + \frac{\gamma F}{4\alpha} \right) = \pi_0 - \left( q_G - q_0 - \frac{\gamma F}{8\alpha} \right) \frac{\gamma F}{2\alpha}. \tag{6.32}
\]

Comparing (6.31) and (6.32), the global maximum is at output level \( q_0 + (\gamma F/4\alpha) \) if

\[
q_G \leq q_0 + \frac{\gamma F}{8\alpha} + 2\alpha; \tag{6.33}
\]

otherwise, the global maximum is at \( q = q_0 \).

(A5) \( q_0 + (\gamma F/4\alpha) + 2\alpha \leq q_G \).

By subtraction, \( q_0 + (\gamma F/4\alpha) \leq q_G - 2\alpha \). Over the output range \( 0 \leq q \leq q_G - 2\alpha \), the payoff function (6.25) is a parabola with local maximum at \( q = q_0 \). Over the output range \( q_G - 2\alpha \leq q \leq q_G \) the payoff function (6.27) is a downward sloping parabola with local maximum at \( q = q_G - 2\alpha \). Over the output range \( q_G \leq q \leq q_G \) the payoff function (6.29) is a downward sloping parabola with local maximum at \( q = q_G \). The global maximum in case (A5) is at \( q = q_0 \).

Results for case (A) may be summarized as

(a1) for \( q_G \leq q_0 \), \( q_1 = q_0 \);
(a2) for \( q_0 \leq q_G \leq q_0 + (\gamma F/4\alpha) \), \( q_1 = q_G \);
(a3) for \( q_0 + (\gamma F/4\alpha) \leq q_G \leq q_0 + (\gamma F/8\alpha) + 2\alpha \), \( q_1 = q_0 + (\gamma F/4\alpha) \);
(a4) for \( q_0 + (\gamma F/8\alpha) + 2\alpha \leq q_G \), \( q_1 = q_0 \).

If these results are rewritten in terms of \( p_G \) and \( p_0 \), one obtains the statement of case (a) given in the text.
(B) $8\alpha^2 \leq \gamma F \leq 16\alpha^2$

In case (B)

$$2\alpha \leq \frac{\gamma F}{4\alpha} \leq 4\alpha \quad (6.34)$$

(B1) $q_G \leq q_0$. The arguments given for case (A1) establish that the global maximum is at $q = q_0$.

(B2) For $q_0 \leq q_G \leq q_0 + 2\alpha$.

By subtraction, $q_G - 2\alpha \leq q_0$. This and (6.34) establish that in case (B2)

$$q_G - 2\alpha \leq q_0 \leq q_G \leq q_0 + 2\alpha \leq q_0 + \frac{\gamma F}{4\alpha} \leq q_0 + 4\alpha. \quad (6.35)$$

Over the output range $0 \leq q \leq q_G - 2\alpha$, the payoff function (6.25) is a parabola that slopes upward to the right, with local maximum at $q = q_G - 2\alpha$.

Since $q_G \leq q_0 + (\gamma F/4\alpha)$, the payoff function (6.27) is a parabola that slopes upward to the right, with local maximum on $q_G - 2\alpha \leq q \leq q_G$ at $q = q_G$.

For $q_G \leq q$, the payoff function (6.29) is a parabola that slopes downward to the right, with local maximum at $q = q_G$. The global maximum of the payoff function in case (B2) is at $q = q_G$.

(B3) $q_0 + 2\alpha \leq q_G \leq q_0 + (\gamma F/4\alpha)$.

By subtraction, $q_0 \leq q_G - 2\alpha$; there is one local maximum at $q = q_0$, with payoff (6.31), and another at $q = q_G$, with payoff

$$\pi(q_G) = \pi_0 - (q_G - q_0)^2. \quad (6.36)$$

Comparing payoffs, the global maximum is at $q_G$ if

$$q_G \leq q_0 + \sqrt{\gamma F}. \quad (6.37)$$

This condition is met if

$$q_0 + \frac{\gamma F}{4\alpha} \leq q_0 + \sqrt{\gamma F} \quad (6.38)$$

or

$$\gamma F \leq 16\alpha^2 \quad (6.39)$$

This condition is always met in case (B3), so the global maximum value in case (B3) is at $q = q_G$.

(B4) $q_0 + (\gamma F/4\alpha) \leq q_G \leq q_0 + (\gamma F/4\alpha) + 2\alpha$. 
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By subtraction, $q_0 \leq q_0 + (\gamma F/4\alpha) - 2\alpha \leq q_0 + (\gamma F/4\alpha) \leq q_G$. The arguments of case (A4) apply, with the same result.

(B5) $q_0 + (\gamma F/4\alpha) + 2\alpha \leq q_G$.

The arguments of case (A5) apply, with the same result.

The results for case (B) are the same as those for case (A).

(C) $16\alpha^2 \leq \gamma F$

In case (C),

$$4\alpha \leq \frac{\gamma F}{4\alpha}$$

and

$$4\alpha \leq \sqrt{\gamma F}.$$  

(C1) For $q_G \leq q_0$, $q_1 = q_0$. See discussion of case (A1).

(C2) $q_0 \leq q_G \leq q_0 + 2\alpha$.

By subtraction, $q_G - 2\alpha \leq q_0$; hence $q_G - 2\alpha \leq q_0 \leq q_G \leq q_0 + 2\alpha \leq q_0 + 4\alpha \leq q_0 + (\gamma F/2\alpha)$. The payoff function is an upward sloping parabola over output range $0 \leq q \leq q_G - 2\alpha$, with local maximum at $q = q_G - 2\alpha$.

The payoff function over output range $q_G - 2\alpha \leq q \leq q_G$ is also an upward sloping parabola, with local maximum at $q = q_G$. The payoff function is a downward sloping parabola over the range $q = q_G$, with local maximum at $q = q_G$. The global maximum of the payoff function in case (C2) is at $q = q_G$.

(C3) $q_0 + 2\alpha \leq q_G \leq q_0 + (\gamma F/4\alpha)$.

By subtraction, $q_0 \leq q_G - 2\alpha$. Hence $q_0 \leq q_0 - 2\alpha \leq q_G \leq q_0 + (\gamma F/4\alpha)$. There is one local maximum at $q_0$, with payoff (6.31), and another at $q_G$, with payoff (6.36). (6.37) is the condition for the global maximum to be at $q_G$.

(C4) $q_0 + (\gamma F/4\alpha) \leq q_G \leq q_0 + (\gamma F/4\alpha) + 2\alpha$.

Here $q_0 \leq q_0 + (\gamma F/4\alpha) - 2\alpha \leq q_G - 2\alpha \leq q_0 + (\gamma F/4\alpha) \leq q_G$. There is one local maximum at $q_0$, with payoff (6.31), and another at $q_0 + (\gamma F/4\alpha)$, with payoff (6.32). The condition for the global maximum to be at $q_0 + (\gamma F/4\alpha)$ is (6.33), but this condition is never met in case (C). The global maximum of the payoff function in case (C4) is $q = q_G$.

(C5) $q_0 + (\gamma F/4\alpha) + 2\alpha \leq q_G$.

The arguments of case (A5) apply, with the same result.

The results for case (C) may be summarized as

(c1) for $0 \leq q_G \leq q_0$, $q_1 = q_0$;
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\[(c2) \text{ for } q_0 \leq q_G \leq q_0 + \sqrt{\gamma F}, \ q_1 = q_G;\]
\[(c3) \ q_0 + \sqrt{\gamma F} \leq q_G, \ q_1 = q_G.\]
If these results are rewritten in terms of \( p_G \) and \( p_0 \), one obtains the statement of case (b) given in the text.
\[\square\]

### 6.6.2 Proof of Theorem 3.1

Let
\[q_E = a + \alpha - p_E.\]  \hspace{1cm} (6.42)

Written in terms of \( q_E \), the root (6.18) is
\[q_2 = q_E + \frac{2\alpha r}{\eta} - \sqrt{\left(\frac{2\alpha r}{\eta} + q_E - q_0\right)^2 - \Delta}.\]  \hspace{1cm} (6.43)

For notational convenience, write
\[q_B = q_0 - \frac{2\alpha r}{\eta} + \sqrt{\Delta + \left(\frac{2\alpha r}{\eta}\right)^2},\]  \hspace{1cm} (6.44)
\[q_C = q_0 - \frac{2\alpha r}{\eta} + \sqrt{\Delta + \left(2\alpha + \frac{2\alpha r}{\eta}\right)^2}.\]  \hspace{1cm} (6.45)

so that
\[p_B = a + \alpha - q_B\]  \hspace{1cm} (6.46)
and
\[p_C = a + \alpha - q_B.\]  \hspace{1cm} (6.47)

**The value function**

The proof involves the following properties of the value function when \( 0 \leq \tau_E \leq 1 \) (for outputs in the range \( q_E - 2\alpha \leq q \leq q_E \)):

1. \( q_0 \leq q_E \leq q_B \) implies \( dV_m/dq \geq 0 \) for \( q_E - 2\alpha \leq q \leq q_E \);
2. \( q_B \leq q_E \leq q_C \) implies \( V_m \) has a local maximum within the range \( q_E - 2\alpha \leq q \leq q_E \);
3. \( q_C \leq q_E \) implies \( dV_m/dq \leq 0 \) for \( q_E - 2\alpha \leq q \leq q_E \).
Over the output range $q_E - 2\alpha \leq q \leq q_E$ the value function is

$$V_m = V_2 + \frac{2\alpha \Delta - (q - q_0)^2}{\eta \frac{2\alpha r}{\eta} + q_E - q},$$

(6.48)

with slope

$$\frac{dV_m}{dq} = \frac{2\alpha \left(\frac{2\alpha r}{\eta} + q_E - q\right)^2 - \left(\frac{2\alpha r}{\eta} + q_E - q_0\right)^2 + \Delta}{\left(\frac{2\alpha r}{\eta} + q_E - q\right)^2}. \quad (6.49)$$

The proof of (1) falls in two parts.

For the root (6.43) to be valid, two conditions must be met. First the discriminant

$$DISC = \left(\frac{2\alpha r}{\eta} + q_E - q_0\right)^2 - \Delta \quad (6.50)$$

must be nonnegative. This condition is met if

$$q_E \geq q_0 - \frac{2\alpha r}{\eta} + \sqrt{\Delta} = q_A. \quad (6.51)$$

If $q_E < q_A$, then $DISC < 0$ and the slope of the value function is positive:

$$\frac{dV_m}{dq} = \frac{2\alpha \left(\frac{2\alpha r}{\eta} + q_E - q\right)^2 - DISC}{\left(\frac{2\alpha r}{\eta} + q_E - q\right)^2} > 0. \quad (6.52)$$

The other condition that must be met for the root (6.43) to be valid is that the implied value of $\tau_E$ lie between 0 and 1. This condition is met for

$$q_B \leq q_E \leq q_C. \quad (6.53)$$

Consider now $q_E$ in the interval $q_A \leq q_E \leq q_B$, from which

$$q_0 - \frac{2\alpha r}{\eta} + \sqrt{\Delta} \leq q_E \leq q_0 - \frac{2\alpha r}{\eta} + \sqrt{\Delta + \left(\frac{2\alpha r}{\eta}\right)^2}$$

---

18 There is another output range over which $DISC$ is nonnegative, but this is ruled out by the assumption that $q_E \geq q_0$.

19 There is another output range over which $\tau_E$ lies between 0 and 1, but this is ruled out by the assumption that $q_E \geq q_0$. 

\[\sqrt{\Delta} \leq q_E - q_0 + \frac{2\alpha r}{\eta} \leq \sqrt{\Delta + \left(\frac{2\alpha r}{\eta}\right)^2}\]

\[\Delta \leq \left(q_E - q_0 + \frac{2\alpha r}{\eta}\right)^2 \leq \Delta + \left(\frac{2\alpha r}{\eta}\right)^2\]

\[-\Delta \geq -\left(q_E - q_0 + \frac{2\alpha r}{\eta}\right)^2 \geq -\Delta - \left(\frac{2\alpha r}{\eta}\right)^2\]

\[0 \geq -\left(q_E - q_0 + \frac{2\alpha r}{\eta}\right)^2 + \Delta \geq -\left(\frac{2\alpha r}{\eta}\right)^2\]  
(6.54)

Now since \(q_E - 2\alpha \leq q \leq q_E\) we have

\[-2\alpha \leq q - q_E \leq 0\]

\[-\left(2\alpha + \frac{2\alpha r}{\eta}\right) \leq q - q_E - \frac{2\alpha r}{\eta} \leq -\frac{2\alpha r}{\eta}\]

\(\left(2\alpha + \frac{2\alpha r}{\eta}\right)^2 \geq \left(\frac{2\alpha r}{\eta} + q_E - q\right)^2 \geq \left(\frac{2\alpha r}{\eta}\right)^2\)  
(6.55)

Adding (6.54) and (6.55) gives

\(\left(2\alpha + \frac{2\alpha r}{\eta}\right)^2 \geq \left(\frac{2\alpha r}{\eta} + q_E - q\right)^2 - \left(q_E - q_0 + \frac{2\alpha r}{\eta}\right)^2 + \Delta \geq 0,\)  
(6.56)

so the slope of the value function is nonnegative. This establishes (1).

(2) This follows by substituting the root (6.43) into

\[0 \leq \frac{q_E - q}{2\alpha} \leq 1.\]  
(6.57)

(3) \(q_C \leq q_E.\) From

\[q_0 - \frac{2\alpha r}{\eta} + \sqrt{\Delta + \left(2\alpha + \frac{2\alpha r}{\eta}\right)^2} \leq q_E,\]  
(6.58)

we obtain

\[\Delta + \left(2\alpha + \frac{2\alpha r}{\eta}\right)^2 \leq \left(q_E + \frac{2\alpha r}{\eta} - q_0\right)^2\]
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\[-\left( 2\alpha + \frac{2\alpha r}{\eta} \right)^2 \geq -\left( q_E + \frac{2\alpha r}{\eta} - q_0 \right)^2 + \Delta \quad (6.59)\]

We are interested in the slope of the value function over the interval

\[ q_E - 2\alpha \leq q \leq q_E \]

\[-2\alpha \leq q - q_E \leq 0 \]

\[-\left( 2\alpha + \frac{2\alpha}{\eta} \right) \leq q - q_E - \frac{2\alpha}{\eta} \leq -\frac{2\alpha}{\eta} \]

\[ \left( 2\alpha + \frac{2\alpha}{\eta} \right)^2 \geq \left( \frac{2\alpha r}{\eta} + q_E - q \right)^2 \geq \left( \frac{2\alpha}{\eta} \right)^2 \quad (6.60)\]

Add (6.59) and (6.60) to obtain

\[ 0 \geq \left( \frac{2\alpha r}{\eta} + q_E - q \right)^2 - \left( \frac{2\alpha r}{\eta} + q_E - q_0 \right)^2 + (\Delta), \quad (6.61)\]

so the slope of the value function is negative.

Now return to the proof of the theorem proper.

(A) \( 0 \leq q_E \leq q_0 \).

\( q_0 \) is the global maximum of the unconstrained value function, and in case (a) the value function and the unconstrained value function coincide over the region that includes output \( q_0 \). Greater outputs mean lower profit; higher outputs mean lower profit and may involve the possibility of entry. \( q_2 = q_0 \) is the global maximum in case (a).

(B) \( q_0 \leq q_E \leq q_B \).

Remark: the condition for

\[ q_B \geq q_0 + 2\alpha \quad (6.62) \]

is

\[ \left( \frac{a - c}{2\alpha} \right)^2 \geq \frac{6}{5} \left( 1 + \frac{2r}{\eta} \right) \quad (6.63) \]

This will be satisfied unless \( \alpha \) is large relative to \( a - c \).

First consider \( q_0 \leq q_E \leq \min(q_0 + 2\alpha, q_B) \).
This implies $q_E - 2\alpha \leq q_0$. Over the output range $0 \leq 1 \leq q_E - 2\alpha$, the value function is a parabola that slopes upward to the right, with local maximum at $q = q_E - 2\alpha$. By property (1) of the value function, it is upward sloping over the intermediate output range $q_E - 2\alpha \leq q \leq q_E$, with local maximum at $q = q_E$. For $q \leq q_E$ the value function is a downward sloping parabola, also with local maximum at $q = q_E$. Now consider $q_0 + 2\alpha \leq q_E \leq q_B$, if this range exists. Here $q_0 \leq q_E - 2\alpha$; the value function has one local maximum at $q = q_0$ in the output range $q_0 \leq q_E - 2\alpha$, yielding value

\[ V_m(q_0) = V_2 + \frac{\Delta}{r + \eta}, \quad (6.64) \]

and another (by the arguments given for case (b)) at $q = q_E$, yielding value

\[ V_m(q_E) = V_2 + \frac{\Delta - (q_E - q_0)^2}{r}. \quad (6.65) \]

Comparing (6.64) and (6.65), the condition for the global maximum to be at $q_E$ is

\[ q_E \leq q_0 + \sqrt{\frac{\eta}{r + \eta}} \Delta \quad (6.66) \]

Analysis shows that $q_B$ may be greater or less than the right-hand side of inequality (6.66). Hence if the range $q_0 + 2\alpha \leq q_E \leq q_B$ exists,

\[ q_2 = \begin{cases} 
q_E & q_E \leq q_0 + \sqrt{\frac{\eta}{r + \eta}} \Delta \\
q_0 & q_E \geq q_0 + \sqrt{\frac{\eta}{r + \eta}} \Delta 
\end{cases} \quad (6.67) \]

(C) $q_B \leq q_E \leq q_C$

The global maximum is found by setting (6.49) equal to zero, yielding the root (6.43).

(D) $q_C \leq q_E$, $q_2 = q_0$

There is a local maximum at $q = q_0$ over the output range $q_0 \leq q \leq q_E - 2\alpha$. By property (3) the value function is downward sloping over the intermediate
output range $q_E - 2\alpha \leq q \leq q_E$, and the value function is a downward-sloping parabola over the right-hand output range $q_E \leq q$. Hence the global maximum is at $q = q_0$.

Rewriting these results in terms of $p_E$ and $p_0$ gives Theorem 3.1.

6.7 References


— “The Danish Competition Act and barriers to entry,” October 1996b.


OECD Competition Policy in OECD Countries. 1996.


