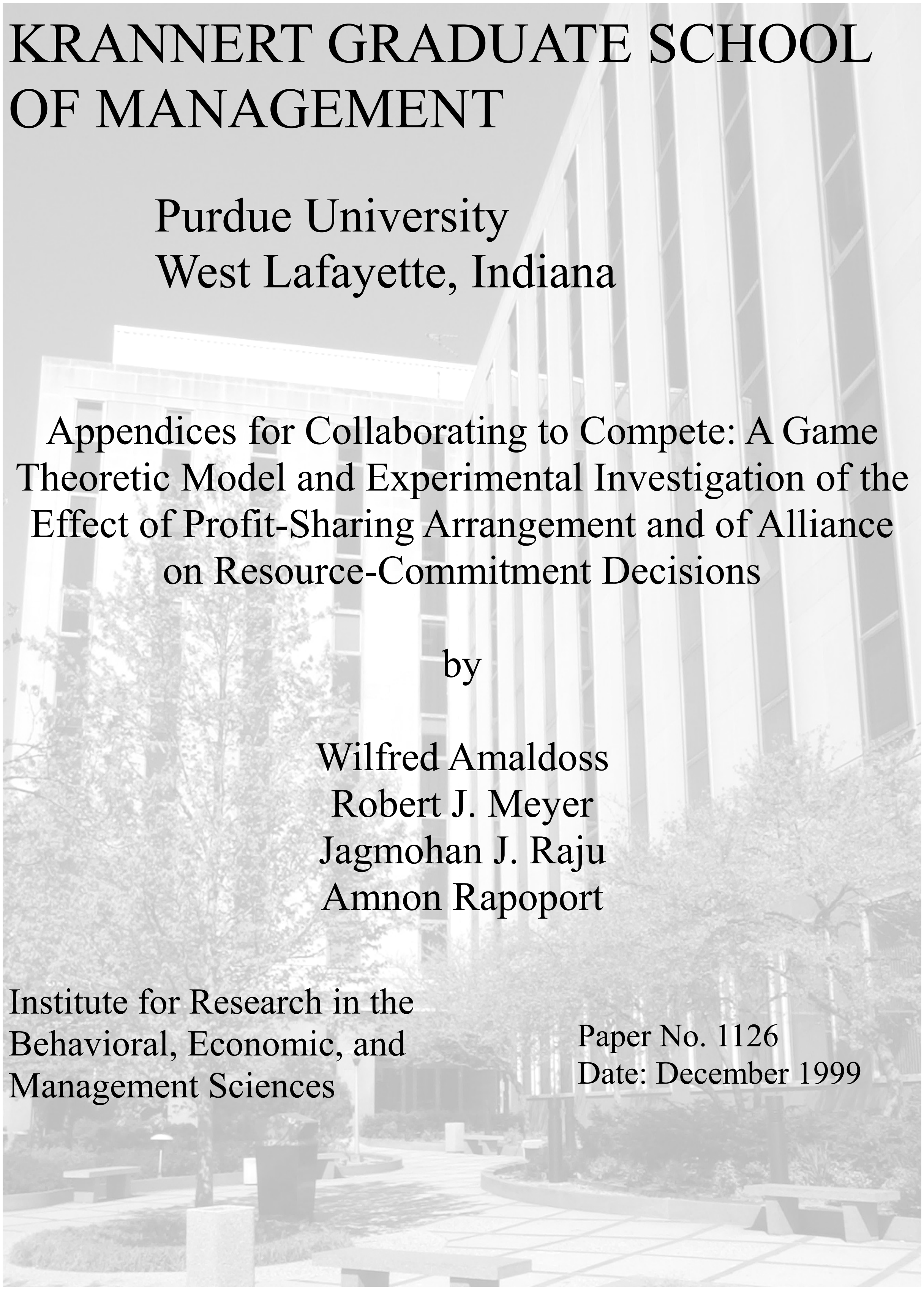

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Appendices for Collaborating to Compete: A Game Theoretic Model and Experimental Investigation of the Effect of Profit-Sharing Arrangement and of Alliance on Resource-Commitment Decisions

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Appendices

for

Collaborating to Compete
A Game-Theoretic Model and Experimental Investigation of the Effect of Profit-Sharing Arrangement and Type of Alliance on Resource-Commitment Decisions

Appendix 1:

Proofs of Theoretical Results

Appendix 2:

Instructions for Subjects,

Trends in Aggregate Investment Pattern, and

Additional Analysis on Ties and Zero-Order Behavior

Appendix 1: Proofs of Theoretical Results

Section 1.1

Claim: If $s = 0$ and $m > 2c > 0$, then the inter-alliance competition between two same-function alliances has only a symmetric mixed strategy equilibrium.

Proof: If each firm in alliance i and alliance j invests 0, then the payoff of player k in alliance i is 0. If firm k deviates unilaterally and invests either $c/2$ or c, the firm's payoff is $m/2 - c/2$ or $m/2 - c$, respectively.

If investing 0 is an equilibrium, then

$$0 - (m/2 - c/2) \geq 0$$

and

$$0 - (m/2 - c) \geq 0.$$

The second inequality implies that $2c \geq m$. But this violates our assumption that $m > 2c$. Therefore, investing 0 is not a symmetric equilibrium

We next show that investing $c/2$ is not a symmetric equilibrium. Suppose that each firm in alliance i and alliance j invests $c/2$. Then the payoff for firm k is $- c/2$. But if firm k unilaterally deviates and invests 0 or c, then its payoff is 0 or $m/2 - c$ respectively. For investing $c/2$ to be a symmetric equilibrium, the following two equations need to be satisfied:

$$- c/2 \geq 0$$

and

$$- c/2 \geq m/2 - c.$$

The second inequality is violated if $m > 2c$. Hence contributing $c/2$ is not a symmetric equilibrium.

Now suppose that each player invests c. Then in equilibrium the following 2 inequalities must be satisfied for investing c to be a symmetric equilibrium:

$$- c \geq 0$$

and

$$- c \geq - c/2.$$

Again the second inequality is violated if $2c > 0$. Hence, contributing c is not a symmetric equilibrium.

Thus, the inter-alliance competition does not have a symmetric pure strategy equilibrium when $s = 0$ and $m > 2c > 0$.

Section 1.2

Claim: If $s = m/2$ and $4c > m > 2c > 0$, then the inter-alliance competition between two same-function alliances has only a symmetric mixed strategy equilibrium. (Note: If $m > 4c$, then the pure strategy equilibrium is to invest c as $m/4 > c$).

Proof: If each firm in alliance i and alliance j invests 0, then the payoff of player k in alliance i is $m/4$. If firm k deviates unilaterally and invests either $c/2$ or c, the firm's payoff is $m/2 - c/2$ or $m/2 - c$ respectively.

If investing 0 is an equilibrium, then

$$m/4 \geq (m/2 - c/2),$$

and

$$m/4 \geq (m/2 - c).$$

The first inequality implies that $2c \geq m$. But this violates our assumption that $m > 2c$. Therefore investing 0 is not a symmetric equilibrium

We next show that investing $c/2$ is not a symmetric equilibrium. Suppose that each firm in alliance i and alliance j invest $c/2$. Then the payoff for firm k is $m/4 - c/2$. But if partner k unilaterally deviates and invests 0 or c, then its payoff is 0 or $m/2 - c$. For investing $c/2$ to be a symmetric equilibrium, the following two equations need to be satisfied:

$$m/4 - c/2 \geq 0,$$

and

$$m/4 - c/2 \geq m/2 - c.$$

The second inequality is violated if $m > 2c$. Hence contributing $c/2$ is not a symmetric equilibrium.

Now suppose that each player invests c. Then in equilibrium the following 2 inequalities must be satisfied for investing c to be a symmetric equilibrium:

$$m/4 - c \geq 0$$

and

$$m/4 - c \geq - c/2.$$

The first inequality is violated if $m < 4c$. Hence, contributing c is not a symmetric equilibrium.

Thus, the inter-alliance competition does not have a symmetric pure strategy equilibrium when $s = m/2$ and $4c > m > 2c > 0$.

<u>Section 1.3</u>

This section derives the system of equations that provide the equilibrium solution when partners in a same function alliance share profits equally.

Consider players i_1 and i_2 in alliance i. Player i_1 could be involved with player i_2 in any of the 5 games presented below, depending on the sum of inputs of the partners in alliance j, $U(j)$ ($U(j) = \{0, c/2, c, 3c/2, 2c\}$).

If $U(j) = 0$:

Player i_2's investment

Player i_1's Investment		c	c/2	0
	c	m/2 - c , m/2 - c	m/2 - c, m/2 - c/2	m/2 - c, m/2
	c/2	m/2 - c/2, m/2 - c	m/2 - c/2, m/2 - c/2	m/2 - c/2, m/2
	0	m/2, m/2 -c	m/2, m/2 - c/2	s/2, s/2

If $U(j) = c/2$:

Player i_2's investment

Player i_1's Investment		c	c/2	0
	c	m/2 - c , m/2 - c	m/2 - c, m/2 - c/2	m/2 - c, m/2
	c/2	m/2 - c/2, m/2 - c	m/2 - c/2, m/2 - c/2	s/2 - c/2, m/2
	0	m/2, m/2 -c	s/2, s/2 - c/2	0, 0

If $U(j) = c$:

Player i_2's investment

Player i_1's Investment		c	c/2	0
	c	m/2 - c , m/2 - c	m/2 - c, m/2 - c/2	s/2 - c, s/2
	c/2	m/2 - c/2, m/2 - c	s/2 - c/2, s/2 - c/2	- c/2, 0
	0	s/2, s/2 -c	0, - c/2	0, 0

If $U(j) = 3c/2$:

Player i_2's investment

Player i_1's Investment		c	c/2	0
	c	m/2 - c , m/2 - c	s/2 - c, s/2 - c/2	- c, 0
	c/2	m/2 - c/2, m/2 - c	- c/2, - c/2	- c/2, 0
	0	0, - c	0, - c/2	0, 0

If $U(j) = 2c$:

Player i_2's investment

Player i_1's		c	c/2	0
	c	s/2 - c , s/2 - c	- c, - c/2	- c, 0
Investment	c/2	- c/2, - c	- c/2, - c/2	- c/2, 0
	0	0 , - c	0, - c/2	0, 0

To construct the expected value of investing *0, c/2,* or *c* by firm i_1 in the collaboration, we need to consider partner i_2's investment behavior ($I_{i2} = \{0, c/2, c\}$) along with the behavior of the competing alliance, $U(j)$ ($U(j) = \{0, c/2, c, 3c/2, 2c\}$). In a symmetric equilibrium, the distribution of strategies is identical for all the players. Recall that we denote the probability of a player investing *0*, *c*, and *c/2* units of capital by p_1, p_2, and p_3, respectively. The joint probability of all players, except player i_1, is provided in Table 5 below.

Table 5: Same-Function Alliance

The Joint Probabilities (All players except player i_1 of alliance *i*)

$U(j)$

		0	c/2	c	3c/2	2c
	c	$p_3 p_1^2$	$2 p_1 p_2 p_3$	$p_3 (2 p_1 p_3 + p_2^2)$	$2 p_3^2 p_2$	p_3^3
Player i_2	c/2	$p_2 p_1^2$	$2 p_1 p_2^2$	$p_2 (2 p_1 p_3 + p_2^2)$	$2 p_2^2 p_3$	$p_2 p_3^2$
	0	p_1^3	$2 p_1^2 p_2$	$p_1 (2 p_1 p_3 + p_2^2)$	$2 p_1 p_2 p_3$	$p_1 p_3^3$
	Marginal	p_1^2	$2 p_1 p_2$	$(2 p_1 p_3 + p_2^2)$	$2 p_2 p_3$	p_3^2

We denote the expected value of contributing *0*, *c/2*, and *c* by *EV(0)*, *EV(c/2)*, and *EV(c)*, respectively. Using Table 5, we compute the expected values of investing *0*, *c/2*, and *c*.

$$EV(0) = s/2\,(p_1^3) + m/2\,(p_2 p_1^2) + m/2\,(p_3 p_1^2) + s/2\,(2\,p_1 p_2^2) + m/2\,(2\,p_1 p_2 p_3)$$
$$+ s/2\,(p_3)\,(2\,p_1 p_3 + p_2^2) + c.$$

$$EV(c/2) = m/2\,(p_1^3) + m/2\,(p_2 p_1^2) + m/2\,(p_3 p_1^2) + s/2\,(2\,p_1^2\,p_2)$$
$$+ m/2\,(2\,p_1 p_2^2) + m/2\,(2\,p_1 p_2 p_3) + s/2\,(p_2)\,(2\,p_1 p_3 + p_2^2)$$

$$+ m/2\,(p_3)\,(2\,p_1\,p_3 + p_2^2) + s/2\,(2\,p_3^2\,p_2) + c/2$$

$$EV(c) = m/2\,(p_1^3) + m/2\,(p_2\,p_1^2) + m/2\,(p_3\,p_1^2) + m/2\,(2\,p_1^2 p_2) + m/2\,(\,2\,p_1\,p_2^2)$$
$$+ m/2\,(2\,p_1\,p_2\,p_3) + s/2\,(p_1)\,(2\,p_1\,p_3 + p_2^2) + m/2\,(p_2)\,(2\,p_1\,p_3 + p_2^2)$$
$$+ m/2\,p_3\,(2\,p_1\,p_3 + p_2^2) + s/3\,(2\,p_2^2\,p_3) + m/2\,(2\,p_3^2\,p_2) + s/2\,(p_3^3).$$

As $EV(c/2) - EV(0) = 0$, we have:

$$(s/2)\,(2\,p_1^{\,2}\,p_2 + p_2^{\,3} + 2\,p_1\,p_2\,p_3 + 2\,p_2\,p_3^{\,2} - p_1^{\,3} - 2\,p_1\,p_2^{\,2}$$
$$- p_2^{\,2}\,p_3 - 2\,p_1\,p_3^{\,2}) + (m/2)\,(p_1^{\,3} + 2\,p_1\,p_2^{\,2} + p_2^{\,2}\,p_3 + 2\,p_1\,p_3^{\,2}) \;=\; c/2$$

As $EV(c) - EV(c/2) = 0$, we have:

$$(s/2)\,(p_1\,p_2^{\,2} + 2\,p_1^{\,2}\,p_3 + p_3^{\,3} - 2\,p_1^{\,2}\,p_2 - p_2^{\,3} - 2\,p_2\,p_3^{\,2} + 2\,p_2^{\,2}\,p_3$$
$$-2\,p_1\,p_2\,p_3) + (m/2)\,(2\,p_1\,p_2 + p_2^{\,3} + 2\,p_2\,p_3^{\,2} + 2\,p_1\,p_2\,p_3) \;=\; c/2$$

If $s = 0$, then after some algebra we cab show that the equilibrium probabilities are the solution of the following system of equations:

$$p_1^{\,3} + 2\,p_1\,p_2^{\,2} + p_2^{\,2}\,p_3 + 2\,p_1\,p_3^{\,2} \;=\; c/m, \qquad (A1)$$

$$2\,p_1^{\,2}\,p_2 + p_2^{\,3} + 2\,p_2\,p_3^{\,2} + 2\,p_1\,p_2\,p_3 \;= c/m, \qquad (A2)$$

$$p_1 + p_2 + p_3 = 1. \qquad (A3)$$

Section 1.4

Claim: In same-function alliances, if partners share profits equally, then:

$p_1 > p_2 > p_3$ if $0.375 > c/m > 0.244$

$p_1 > p_3 > p_2$ if $0.244 > c/m > 0.228$

$p_3 > p_1 > p_2$ if $0.288 > c/m > 0.$

Proof: First we show that $p_1 > p_2$ *for* $c/m < 0.375.$

Substituting $p_3 = 1 - p_1 - p_2$ in Equation A1, we obtain:

$$p_1^{\,3} + 2\,p_1 p_2^{\,2} + p_2^{\,2}(1 - p_1 - p_2) + 2\,p_1\,(1 - p_1 - p_2)^{2} - c/m = 0.$$

Let $p_1 = p_2 + \delta_1$. Therefore,

$$(p_2 + \delta_1)^3 + 2\,(p_2 + \delta_1)\,p_2^{\,2} + p_2^{\,2}(1 - p_2 - \delta_1 - p_2) + 2\,(p_2 + \delta_1)\,(1 - p_2 - \delta_1 - p_2)^{2}$$

$$- \; c/m = 0 \qquad (A4)$$

Similarly, we substitute $p_3 = 1 - p_1 - p_2$ and $p_1 = p_2 + \delta_1$ in Equation A2 to obtain:

$$2(p_2 + \delta_1)^2\,p_2 + p_2^{\,3} + 2\,p_2\,(1 - p_2 - \delta_1 - p_2)^{2} - 2\,(p_2 + \delta_1)\,(p_2 + \delta_1)\,(1 - p_2 - \delta_1 - p_2)$$

$$- c/m = 0 \qquad (A5)$$

Using Equations (A4) and (A5), we solve for the value of c/m when $\delta_1 = 0$.

If $\delta_1 = 0$, then $c/m = 0.375$ and this solution is unique. Note that if $c/m = 0.4$, then $\delta_1 = -0.029 < 0$. But, if $c/m = 0.05$, then $\delta_1 = 0.0004 > 0$. Therefore, $p_1 > p_2$ *for* $c/m < 0.375$.

Second, we show that $p_3 > p_1$ *for* $c/m < 0.228$.

Let $p_1 = (p_3 + \delta_2)$. Substituting $p_2 = 1 - p_1 - p_3$ and $p_1 = (p_3 + \delta_2)$ in Equation A1, we obtain:

$$(p_3 + \delta_2)^{3} + 2\,(p_3 + \delta_2)\,(1 - 2p_3 - \delta_2)^{2} + (1 - 2p_3 - \delta_2)^{2}\,(p_3 + \delta_2) + 2\,(p_3 + \delta_2)\,p_3^{\,2}$$

$$- c/m = 0 \qquad (A6)$$

Similarly, we substitute $p_2 = 1 - p_1 - p_3$ and $p_1 = p_3 + \delta_2$ in Equation A2 to obtain:

$$1 - 3p_3 + 5\,p_3^{2} - 3\,p_3^{3} - 3\,(p_3 + \delta_2) + 4\,p_3(p_3 + \delta_2) - 3\,p_3^{3}\,(p_3 + \delta_2) + 5\,(p_3 + \delta_1)^{2}$$

$$- 3\,p_3^{3}\,(p_3 + \delta_2)^2 - 3\,(p_3 + \delta_2)^{3} - c/m = 0 \qquad (A7)$$

Using Equations A6 and A7, we solve for the value of c/m when $\delta_2 = 0$. If $\delta_2 = 0$, then $c/m = 0.228$ and this solution is unique. Further, if $c/m = 0.2$, then $\delta_2 = -\,0.399 < 0$. But if $c/m = 0.25$, then $\delta_2 = 0.193 > 0$. Therefore, $p_3 > p_1$ *for* $c/m < 0.228$.

Finally, we show that $p_3 > p_2\ \forall\, c/m < 0.244$: Let $p_2 = (p_3 + \delta_3)$. We substitute $p_1 = 1 - p_2 - p_3$ and $p_2 = p_3 + \delta_3$ in Equation A1 to obtain:

$$1 - 3\,p_3 + 5\,p_3^{\,2} - 3\,p_3^{\,2} - 3\,(p_3 + \delta_3) + 6\,p_3\,(p_3 + \delta_3) - 5\,p_3^{\,2}\,(p_3 + \delta_3)$$

$$+ 5 (p_3 + \delta_3)^2 - 4 (p_3 + \delta_3)^2 - .3 (p_3 + \delta_3)^3 - c/m = 0 \qquad (A8)$$

Similarly, we substitute $p_1 = 1 - p_2 - p_3$ and $p_2 = p_3 + \delta_2$ in Equation A2 to obtain:

$$2 (p_3 + \delta_3) - 6 p_3 (p_3 + \delta_3) + 6 p_3^2 (p_3 + \delta_3) - 4 (p_3 + \delta_3)^2$$

$$+ 6 p_3 (p_3 + \delta_3)^2 + 3 (p_3 + \delta_3)^3 \qquad (A9)$$

Using Equation A8 and A9, we solve for the value of c/m when $\delta_3 = 0$.

If $\delta_3 = 0$, then $c/m = 0.244$ and this solution is unique. We also find that if $c/m = 0.25$, then $\delta_3 = 0.03 > 0$.

Also, if $c/m = 0.2$, then $\delta_3 = -0.433 < 0$. Therefore, $p_3 > p_2$ *for* $c/m < 0.244$.

Using the three results above, we conclude that:

$p_1 > p_2 > p_3$ if $0.375 > c/m > 0.244$

$p_1 > p_3 > p_2$ if $0.244 > c/m > 0.228$

$p_3 > p_1 > p_2$ if $0.288 > c/m > 0.$

Section 1.5

Claim: If partners in same-function alliances share profits equally, then: $\frac{dp_3}{d\frac{c}{m}} < 0$; $\frac{dp_2}{d\frac{c}{m}} > 0$; and $\frac{dp_1}{d\frac{c}{m}} > 0 \; \forall p_1 \in (0.084, 0.475)$.

Proof: First, we establish that $\frac{dp_2}{d\frac{c}{m}} > 0$ and $\frac{dp_3}{d\frac{c}{m}} < 0$.

From Equations A1 and A2, we have:

$$F^1 (p_1, p_2, p_3; c/m) = p_1^3 + 2 p_1 p_2^2 + p_2^2 p_3 + 2 p_1 p_3^2 - c/m \equiv 0$$

$$F^2 (p_1, p_2, p_3; c/m) = 2 p_1^2 p_2 + p_2^3 + 2 p_2 p_3^2 + 2 p_1 p_2 p_3 - c/m \equiv 0$$

As $p_1 = 1 - p_2 - p_3$, we have:

$$F^1 (p_2, p_3; c/m) = 1 - 3 p_2 + 5 p_2^2 - 3 p_2^3 - 3 p_3 + 6 p_2 p_3 - 4 p_2^2 p_3 + 5 p_3^2 - 5 p_2 p_3^2$$

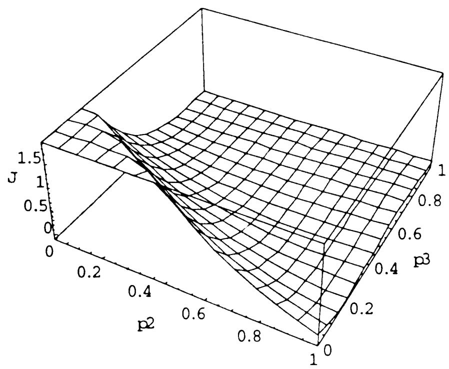
$$- 3 p_3^3 - c/m \equiv 0$$

$$F^2 (p_2, p_3; c/m) = 3 p_2^3 + 2 p_2^2 (p_3^2 - 2) + 2 p_2 (1 - p_3 + p_3^2) - c/m \equiv 0.$$

$$|J| = \begin{vmatrix} \dfrac{\partial F^1}{\partial p_2} & \dfrac{\partial F^1}{\partial p_3} \\ \dfrac{\partial F^2}{\partial p_2} & \dfrac{\partial F^2}{\partial p_3} \end{vmatrix}$$

$$= 6 - 30p_2 + 57p_2^2 - 48p_2^3 + 18p_2^4 - 26p_3 + 100p_2p_3 - 134p_2^2p_3 + 54p_2^3p_3 + 44p_3^2 - 110p_2p_3^2 + 87p_2^2p_3^2 - 38p_3^3 + 36p_2p_3^3 + 18p_3^4$$

As $|J|$ involves terms raised to the power of 4, we plot $|J|$ for $p_2 \in (0,1)$ and $p_3 \in (0,1)$. In Plot 1, it is evident that $|J| > 0$ for $p_2 \in (0,1)$ and $p_3 \in (0,1)$.



Plot 1.

We have

$$|J_{p_2}| = \begin{vmatrix} 1 & \dfrac{\partial F^1}{\partial p_3} \\ 1 & \dfrac{\partial F^2}{\partial p_3} \end{vmatrix}$$

$$= 3 - 8p_2 + 6p_2^2 - 10p_3 + 14p_2p_3 + 9p_3^2$$

$We\ know\ that\ Minimun\left|J_{p_2}\right| = \underset{p_2,p_3}{Min}\left|J_{p_2}\right| = \underset{p_2}{Min}\left[\underset{p_3(p_2)}{Min}\left|J_{p_2}\right|\right].$

$$\frac{\partial\left|J_{p_2}\right|}{\partial p_3} = -10 + 14p_2 + 18p_3.$$

Hence the min *value of* $\left|J_{p_3}\right|$ *with respect to* p_3 *will be when* $p_3 = \frac{1}{9}(5 - 7p_2)$.

$$\underset{p_3(p_2)}{Min}\left|J_{p_2}\right| = \frac{1}{9}\left(2 - 2p_2 + 5p_2^{\ 2}\right) > 0,\ \forall\, p_2 \in (0,1)$$

Therefore, $\left|J_{p_2}\right| > 0$

Hence, $\dfrac{dp_2}{d\frac{c}{m}} = \dfrac{\left|J_{p_2}\right|}{|J|} > 0,\ \forall\, p_2 \in (0,1).$

$$\left|J_{p_3}\right| = \begin{vmatrix} \dfrac{\partial F^1}{\partial p_2} & 1 \\ \dfrac{\partial F^2}{\partial p_2} & 1 \end{vmatrix}$$

$$= -5 + 18p_2 - 18p_2^{\ 2} + 8p_3 - 12p_2p_3 - 7p_3^{\ 2}$$

$Maximum\left|J_{p_3}\right| = \underset{p_2,p_3}{Max}\left|J_{p_3}\right| = \underset{p_2}{Max}\left[\underset{p_3(p_2)}{Max}\left|J_{p_3}\right|\right].$

$$\frac{\partial\left|J_{p_3}\right|}{\partial p_3} = -8 - 12p_2 - 14p_3.$$

Hence the max *value of* $\left|J_{p_3}\right|$ *with respect to* p_3 *will be when* $p_3 = -\frac{2}{7}(3p_2 - 2)$.

$$\underset{p_3(p_2)}{Max}\left|J_{p_3}\right| = \frac{1}{7}\left(78p_2 - 90p_2^{\ 2} - 19\right) < 0\ \forall\, p_2 \in (0,1)$$

Therefore, $\left|J_{p_3}\right| < 0$

Hence, $\dfrac{dp_3}{d\frac{c}{m}} = \dfrac{\left|J_{p_3}\right|}{|J|} < 0,\ \forall\, p_2 \in (0,1).$

Now, we prove that $\frac{dp_1}{d\frac{c}{m}} > 0$. Again from equations A1 and A2, we have:

$$F^1(p_1, p_2, p_3; c/m) = p_1^{\,3} + 2\,p_1\,p_2^{\,2} + p_2^{\,2}\,p_3 + 2\,p_1\,p_3^{\,2} - c/m = 0$$

$$F^2(p_1, p_2, p_3; c/m) = 2\,p_1^{\,2}\,p_2 + p_2^{\,3} + 2\,p_2\,p_3^{\,2} + 2\,p_1\,p_2\,p_3 - c/m = 0$$

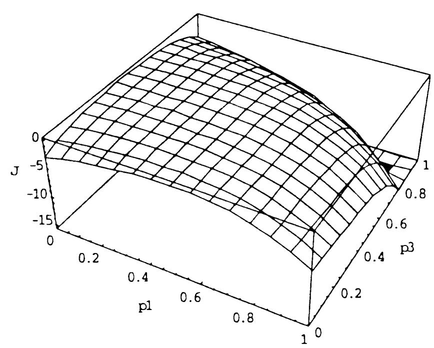
As $p_2 = 1 - p_1 - p_3$, we have:

$$F^1(p_1, p_3; c/m) = p_1^{\,3} + 2\,p_1\,p_3^{\,2} + 2\,pl\,(p_1 + p_3 - 1)^2 + p_3\,(p_1 + p_3 - 1)^{\,2} - c/m = 0$$

$$F^2(p_1, p_3; c/m) = 1 - 3\,p_1 + 5\,p_1^{\,2} - 3\,p_1^{\,2} - 3\,p_3 + 8\,p_1\,p_3 - 7\,p_1^{\,2}\,p_3 + 5\,p_3^{\,2}$$
$$- 7\,pl\,p_3^{\,2} - 3p_3^{\,3} - c/m = 0$$

$$|J| = \begin{vmatrix} \frac{\partial F^1}{\partial p_1} & \frac{\partial F^1}{\partial p_3} \\ \frac{\partial F^2}{\partial p_1} & \frac{\partial F^2}{\partial p_3} \end{vmatrix}$$
$$= -3 + 12p_1 - 21p_1^{\,2} + 24p_1^{\,3} - 18p_1^{\,4} + 18p_3 - 48p_1p_3 + 44p_1^{\,2}p_3 - 18p_1^{\,3}p_3 -$$
$$48p_3^{\,2} + 80p_1p_3^{\,2} - 33p_1^{\,2}p_3^{\,2} + 62p_3^{\,3} - 48p_1p_3^{\,3} - 33p_3^{\,4}.$$

As $|J|$ involves terms raised to the power of 4, we plot $|J|$ for $p_1 \in (0,1)$ and $p_3 \in (0,1)$. We find that $|J| < 0$ in Plot 2.



Plot 2.

Next, we have

$$|J_{p_1}| = \begin{vmatrix} 1 & \frac{\partial F^1}{\partial p_3} \\ 1 & \frac{\partial F^2}{\partial p_3} \end{vmatrix}$$

$$= -2(2 - 7p_1 + 6p_1^2 - 7p_3 + 13p_1p_3 + 6p_3^2$$

We know that $\underset{p_1}{Max}\, J_{p_1} = \underset{p_1, p_3}{Max}[J_{p_1}] = \underset{p_1}{Max}\left[\underset{p_3(p_1)}{Max}\, J_{p_1}\right]$

$\frac{\partial J_{p_1}}{\partial p_1} = -2(-7 + 12p_1 + 13p_3).$

Therefore, the max *value of* J_{p_1} *with respect to* p_3 *will be when* $p_3 = \frac{1}{12}(7 - 13p_3)$.

$$\underset{p_3(p_1)}{Max}|J_{p_1}| = \frac{1}{12}(1 - 14p_1 + 25p_1^2)$$

$$\underset{p_3(p_1)}{Max}|J_{p_1}| < 0,\ \forall\ p_1 \in (0.084, 0.475)$$

Therefore, $\frac{dp_1}{d\frac{c}{m}} = \frac{|J_{p_1}|}{|J|} > 0, \forall\ p_1 \in (0.084, 0.475)$

Section 1.6

This section outlines the system of equations that provides the equilibrium solution for the proportional profit-sharing arrangement in a same-function alliance. Our purpose is to compare the investment behavior under equal and proportional profit-sharing arrangement. We denote the expected value of investing *0*, *c/2*, and *c* by *EV(0)*, *EV(c/2)*, and *EV(c)* respectively. Using Table 5 above, we compute the expected values of investing *0*, *c/2*, and *c*.

$EV(0) = s/2\ (p_1^3) + c$

$EV(c/2) = m\ (p_1^3) + m/2\ (p_2 p_1^2) + m/3\ (p_3 p_1^2) + s\ (2\ p_1^2\ p_2) + m/2\ (2\ p_1 p_2^2)$
$+ m/3\ (2\ p_1 p_2 p_3) + s/2\ (p_2)\ (2\ p_1 p_3 + p_2^2) + m/3\ (p_3)\ (2\ p_1 p_3 + p_2^2)$
$+ s/3\ (2\ p_3^2\ p_2) + c/2$

$EV(c) = m\ (p_1^3) + 2\ m/3\ (p_2 p_1^2) + m/2\ (p_3 p_1^2) + m\ (2\ p_1^2 p_2) + 2\ m/3\ (2\ p_1 p_2^2)$
$+ m/2\ (2\ p_1 p_2 p_3) + s\ (p_1)\ (2\ p_1 p_3 + p_2^2) + 2\ m/3\ (p_2)\ (2\ p_1 p_3 + p_2^2)$

$$+ m/2\, p_3 (2\, p_1 p_3 + p_2^2) + 2\, s/3\, (2\, p_2^2\, p_3) + m/2\, (2\, p_3^2\, p_2) + s/2\, (p_3^3).$$

Simplifying, we obtain that the equilibrium solution is given by the following system of three equations:

$$EV(c/2) - EV(0) = 0,$$
$$EV(c) - EV(c/2) = 0, \text{ and}$$
$$p_1 + p_2 + p_3 = 1$$

.

By setting $s=0$, we obtain the following system of three equations:

$$2(p_1^3 + 1/2\, (p_1^2 p_2) + p_1 p_2^2 + 1/3\, (p_1^2 p_3) + 2/3\, (p_1 p_2 p_3) + 1/3\, (p_3)(p_2^2 + 2\, p_1 p_3)) - c/m = 0 \quad (A10)$$

$$2(p_1^3 + 8/3\, (p_1^2 p_2) + 4/3\, p_1 p_2^2 + 1/2\, (p_1^2 p_3) + p_1 p_2 p_3 + 1/3\, (p_2 p_3)(p_2^2 + 2\, p_1 p_3)) - 2(p_1^3 + 1/2\, (p_1^2 p_2) + p_1 p_2^2 + 1/3\, (p_1^2 p_3) + 2/3\, (p_1 p_2 p_3) + 1/3\, (p_3)(p_2^2 + 2\, p_1 p_3)) - c/m = 0 \quad (A11)$$

$$p_1 + p_2 + p_3 = 1. \quad (A12)$$

Equations A10-A12 provide the equilibrium solution for the inter-alliance game where partners share the profits proportionally.

Section 1.7

Claim: In same-function alliances if partners share profits proportionally, then:

$p_3 > p_1 > p_2$ *for* $0 < c/m < 0.5$.

Proof: First we show that $p_1 > p_2$ *for* $0 < c/m < 0.5$.

Let $p_1 = p_2 + \delta_1$. We substitute $p_3 = 1 - p_1 - p_2$, and $p_1 = p_2 + \delta_1$ in Equation A10 and obtain:

$$2\,(1/3\, p_2^2 - 1/3\, p_2^3 + 2/3\, (p_2 + \delta_1) - 2/3\, p_2 (p_2 + \delta_1) + 2/3\, p_2^2 (p_2 + \delta_1) - (p_2 + \delta_1)^2 + 5/6\, p_2 (p_2 + \delta_1)^2 + 4/3\, (p_2 + \delta_1)^3) - c/m = 0 \quad (A13)$$

Similarly, we substitute $p_3 = 1 - p_1 - p_2$ and $p_1 = p_2 + \delta_1$ in Equation A11 to obtain:

$$2\,(p_2 - 7/3\,p_2^2 + 4/3\,p_2^3 + 1/3\,p_2^5 - 1/3\,p_2^6 - 2/3\,(p_2 + \delta_1) - 1/3\,(p_2 + \delta_1)$$

$$+ 5/3\,p_2^2\,(p_2 + \delta_1)^2 + 4/3\,p_2^3\,(p_2 + \delta_1) - 8/3\,p_2^4\,(p_2 + \delta_1) + p_2^5\,(p_2 + \delta_1)$$

$$+ 3/2\,(p_2 + \delta_1)^2 + 8/3\,p_2\,(p_2 + \delta_1)^2 - 4\,p_2^2\,(p_2 + \delta_1)^2 + 4/3\,p_2^3\,(p_2 + \delta_1)^2$$

$$+ 4/3\,p_2^4\,(p_2 + \delta_1)^2 - 5/6\,(p_2 + \delta_1)^3 - 4\,p_2\,(p_2 + \delta_1)^3$$

$$+ 8\,p_2^2\,(p_2 + \delta_1)^3 - 8/3\,p_2^3\,(p_2 + \delta_1)^3 + 4\,p_2\,(p_2 + \delta_1)^4$$

$$4\,p_2^2\,(p_2 + \delta_1)^4 - 4/3\,p_2\,(p_2 + \delta_1)^5) - c/m = 0 \qquad (A14)$$

Using Equations A13 and A14, we solve for the value of c/m when $\delta_1=0$. If $\delta_1=0$, then $c/m = 0.625$ and this solution is unique. Note that by definition $m > 2c$, and hence only $c/m < 0.5$ is feasible. We know that as $m > 0$ and $c > 0$, and so $c/m > 0$. Therefore, p_1 and p_2 do not intersect if $0 < c/m < 0.5$. For instance, if $c/m = 0.25$, then $\delta_1 = 0.16 > 0$; and if $c/m = 0.125$, then $\delta_1 = 0.074 > 0$.

Hence, $p_1 > p_2$ *for* $0 < c/m < 0.5$.

Second, we show that $p_3 > p_1$ *for* $c/m < 0.5$.

Let $p_1 = p_3 + \delta_2$. Substituting $p_2 = 1 - p_1 - p_3$ and $p_1 = p_3 + \delta_2$ in Equation A10 we obtain:

$$2\,(1/3\,p_3 - 2/3\,p_3^2 + 1/3\,p_3^3 + (p_3 + \delta_2) - 2\,p_3\,(p_3 + \delta_2) + 5/3\,p_3^2\,(2p_3 + \delta_2)$$

$$- 3/2\,(p_3 + \delta_2)^2 + 3/2\,p_3\,(p_3 + \delta_2)^2 + 3/2\,(p_3 - \delta_2)^3) - c/m = 0. \qquad (A15)$$

Similarly, substituting $p_2 = 1 - p_1 - p_3$ and $p_1 = p_3 + \delta_2$ in Equation A11 we obtain:

$$2(2\,p_3^3 - 10/3\,p_3^4 + 5/3\,p_3^5 - 1/3\,p_3^6 + 1/3\,(p_3 + \delta_2) - 4/3\,p_3\,(p_3 + \delta_2)$$

$$+ 17/3\,p_3^2\,(p_3 + \delta_2)^2 - 14\,p_3^3\,(p_3 + \delta_2) + 32/3\,p_3^4\,(p_3 + \delta_2)$$

$$- 3\,p_3^5\,(p_3 + \delta_2)^2 + 3/2\,(p_3 + \delta_2)^2 + 4/3\,p_3\,(p_3 + \delta_2)^2$$

$$- 14\,p_3^2\,(p_3 + \delta_2)^2 + 58/3\,p_3^3\,(p_3 + \delta_2)^2 - 26/3\,p_3^4\,(p_3 + \delta_2)^2$$

$$- 11/6\,(p_3 + \delta_2)^3 - 10/3\,p_3\,(p_3 + \delta_2)^3 + 32/3\,p_3^3\,(p_3 + \delta_2)^3$$

$$- 26/3\,p_3^3\,(p_3 + \delta_2)^3 + 5/3\,p_3\,(p_3 + \delta_2)^4 - 3\,p_3^2\,(p_3 + \delta_2)^4$$

$$- 1/3\,p_3\,(p_3 + \delta_2)^5) - c/m = 0 \qquad (A16)$$

Using Equations A15 and A16, we solve for the value of c/m when $\delta_1=0$. We find that $\delta_1=0$ only if $c/m= 0$. But as $c > 0$, and $m > 0$, $c/m = 0$ is not feasible. Therefore, p_1 and p_2 do not intersect if $0 < c/m < 0.5$. For example, if $c/m = 0.25$, then $\delta_2 = - 0.36 < 0$; and if $c/m = 0.125$, then $\delta_2 = -0.739 < 0$. Hence, $p_3 > p_1$ *for* $0 < c/m < 0.5$.

Third, we can show $p_3 > p_2$ *for* $c/m < 0.5$.

Let $p_2 = (p_3 + \delta_3)$. Substituting $p_1 = 1 - p_2 - p_3$ and $p_2 = p_3 + \delta_3$ in Equation A10, we obtain:

$$2(8/3\, p_3 - 4/3\, p_3^{\,3} - 5/2\,(p_3 + \delta_3) + 5\, p_3 (p_3 + \delta_3) - 19/6\, p_3^{\,2} (p_3 + \delta_3)$$

$$+ 3\,(p_3 + \delta_3)^2 - 3\, p_3 (p_3 + \delta_3)^2 - 3/2\,(p_3 + \delta_3)^3\,) - c/m = 0 \qquad (A17)$$

Similarly, we substitute $p_1 = 1 - p_2 - p_3$ and $p_2 = p_3 + \delta_2$ in Equation A11 to obtain:

$$2\,(1/6\, p_3 - 5/2\, p_3^{\,3} + 13/6\,(p_3 + \delta_3) - 13/3\, p_3 (p_3 + \delta_3) + 23/6\, p_3^{\,2} (p_3 + \delta_3)$$

$$+ 4/3\, p_3^{\,3} (p_3 + \delta_3) - 8/3\, p_3^{\,4} (p_3 + \delta_3) + 4/3\, p_3^{\,5} (p_3 + \delta_3) - 4\,(p_3 + \delta_3)^2$$

$$+ 7/2\, p_3 (p_3 + \delta_3)^2 - 8/3\, p_3^{\,3} (p_3 + \delta_3)^2 + 8/3\, p_3^{\,4} (p_3 + \delta_3)^2 + 11/6\,(p_3 + \delta_3)^3$$

$$+ 4/3\, p_3^{\,2} (p_3 + \delta_3)^3 - 4/3\, p_3^{\,2} (p_3 + \delta_3)^4 + 1/3\, p_3 (p_3 + \delta_3)^5 - c/m = 0 \qquad (A18)$$

Using Equation A17 and A18 we solve for the value of c/m when $\delta_3=0$. Again, we find that p_3 and p_2 do not intersect if $0 < c/m < 0.5$. Note that if $c/m = 0.25$, then $\delta_2 = - 0.523 < 0$; and if $c/m = 0.125$, then $\delta_2 = -0.813 < 0$. Hence, $p_3 > p_2$ *for* $0 < c/m < 0.5$.

Taken together these results prove that $p_3 > p_1 > p_2$ *for* $0 < c/m < 0.5$.

Section 1.8

Claim: In same-function alliances, if partners share profits proportionally, then:

$$\frac{dp_3}{d\frac{c}{m}} < 0;\ \frac{dp_2}{d\frac{c}{m}} > 0;\ \frac{dp_1}{d\frac{c}{m}} > 0.$$

Proof: First, we prove that $\frac{dp_3}{d\frac{c}{m}} < 0$ and $\frac{dp_2}{d\frac{c}{m}} > 0$.

From Equations A10 and A11, we have:

$F^1(p_1, p_2, p_3; c/m) = 2(p_1^3 + 1/2\,(p_1^2 p_2) + p_1 p_2^2 + 1/3\,(p_1^2 p_3)$

$+ 2/3\,(p_1 p_2 p_3) + 1/3\,(p_3)(p_2^2 + 2 p_1 p_3)) - c/m \equiv 0$

$F^2(p_1, p_2, p_3; c/m) = 2(p_1^3 + 8/3\,(p_1^2 p_2) + 4/3\, p_1 p_2^2 + 1/2\,(p_1^2 p_3)$

$+ p_1 p_2 p_3 + 1/3\,(p_2 p_3)(p_2^2 + 2 p_1 p_3))$

$- 2(p_1^3 + 1/2\,(p_1^2 p_2) + p_1 p_2^2 + 1/3\,(p_1^2 p_3) + 2/3\,(p_1 p_2 p_3)$

$+ 1/3\,(p_3)(p_2^2 + 2 p_1 p_3)) - c/m \equiv 0$

As $p_1 = 1 - p_2 - p_3$, we have:

$F^1(p_2, p_3; c/m) = 2 - 3 p_2^3 - 6 p_2^2 (p_3 - 1) - 16/3\, p_3 + 6 p_3^2 - 8/3\, p_3^3$

$+ p_2 (10 p_3 - 19/3\, p_3^2 - 5) \equiv 0$

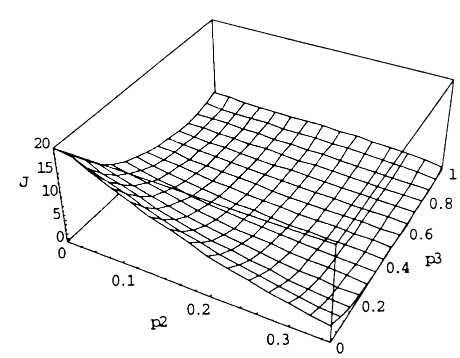
$F^2(p_2, p_3; c/m) = 1/3\,(p_3 + 2 p_2^5 p_3 - 6 p_3^2 - 8 p_2^4 p_3^2 + 5 p_3^2 + p_2^3 (11 + 8 p_3^2)$

$+ p_2^2 (21 p_3 - 16 p_3^3 + 16 p_3^4 - 24) + p_2 (13 - 26 p_3 + 23 p_3^2 + 8 p_3^3 - 16 p_3^4 + 8 p_3^5)$

$- 3\, c/m \equiv 0$

As these implicit functions involve terms raised to the power of 4, we use plots to draw inferences.

$$|J| = \begin{vmatrix} \dfrac{\partial F^1}{\partial p_2} & \dfrac{\partial F^1}{\partial p_3} \\ \dfrac{\partial F^2}{\partial p_2} & \dfrac{\partial F^2}{\partial p_3} \end{vmatrix}.$$

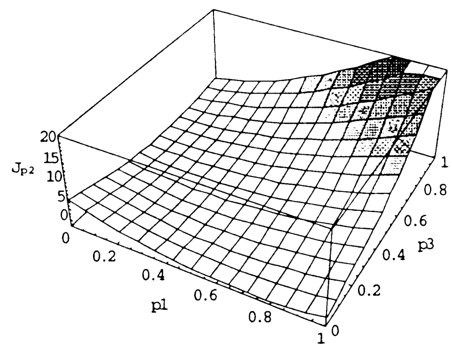


Plot 3.

We know that $m > 2c > 0$. Therefore, $0.5 > c/m > 0$. Using A10, A11, and A12, we find that $p_2 < 0.35$ for $c/m < 0.5$. Plot 3 below presents the value of $|J| \forall\, p_2 \in (0{,}0.35), p_3 \in (0{,}1)$. We observe that $|J| > 0$.

Next, we have

$$|J_{p_2}| = \begin{vmatrix} 1 & \dfrac{\partial F^1}{\partial p_3} \\ 1 & \dfrac{\partial F^2}{\partial p_3} \end{vmatrix}.$$



Plot 4.

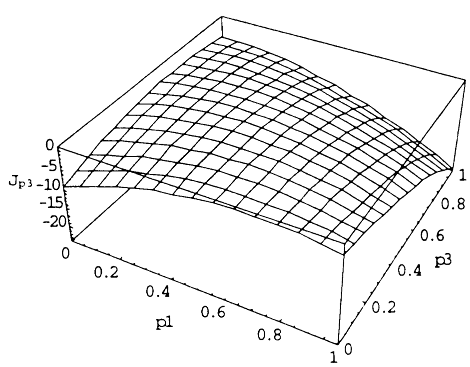
In Plot 4, we again notice that $|J_{p_2}| > 0 \forall\, p_2 \in (0{,}1), p_3 \in (0{,}1)$.

Hence, $\dfrac{dp_2}{d\dfrac{c}{m}} = \dfrac{|J_{p_2}|}{|J|} > 0.$

Finally,

$$|J_{p_3}| = \begin{vmatrix} \dfrac{\partial F^1}{\partial p_2} & 1 \\ \dfrac{\partial F^2}{\partial p_2} & 1 \end{vmatrix}$$

Plot 5.

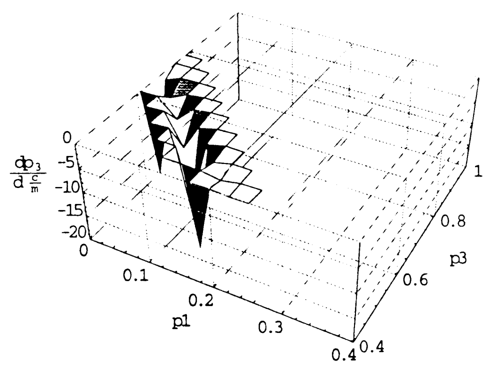


From Equations A15 and A16, we find that $p1 < 0.4$. Plot 5 shows that $|J_{p_3}| < 0, \forall\, p_2 \in (0,1), p_3 \in (0,1)$.

Hence, $\dfrac{dp_3}{d\frac{c}{m}} = \dfrac{|J_{p_3}|}{|J|} < 0$.

Similarly, we computed $\dfrac{dp_1}{d\frac{c}{m}} = \dfrac{|J_{p_1}|}{|J|}$. We know from Equations A15 and A16 that $p_1 < 0.5$ and $p_3 > 0.3$

In Plot 6 below, we present the comparative static. The region that takes negative values is not part of the equilibrium solution. For instance, if $p_1 = 0.361$ then $p_3 = 0.442$ $(c/m = 0.33)$; if $p_1 = 0.161$ then $p_3 = 0.786$ $(c/m = 0.17)$; if $p_1 = 0.087$ then $p_3 = 0.885$ $(c/m = 0.1)$.



Plot 6.

Hence, $\dfrac{dp_1}{d\frac{c}{m}} > 0$, if p_1 and p_3 constitute equilibrium probabilities.

Section 1.9

Claim: When partners develop new products in parallel and agree to share profits equally, the inter-alliance game has only a symmetric mixed strategy equilibrium if $m > 2c$, $s = 0$, and $c > 0$.

Proof: If each firm in alliance *i* and alliance *j* invests *0*, then the payoff of player *k* in alliance *i* is *0*. If firm *k* deviates unilaterally and invests either *c/2* or *c*, the firm's payoff is *m/2 - c/2* or *m/2 – c*, respectively.

If investing *0* is a symmetric equilibrium, then

$0 \geq m/2 - c/2,$

and

$0 \geq m/2 - c.$

The second inequality implies that $2c \geq m$. But this violates our assumption that $m > 2c$. Therefore, investing 0 is not a symmetric equilibrium

We next show that investing $c/2$ is not a symmetric equilibrium. Suppose that each firm in alliance i and alliance j invest $c/2$. Then the payoff for firm k is $- c/2$. But if k unilaterally deviates and invests 0 or c, then its payoff is 0 or $m/2 - c$. For investing $c/2$ to be a symmetric equilibrium, the following two equations need to be satisfied:

$- c/2 \geq 0,$

and

$- c/2 \geq - c + m/2.$

The second inequality is violated if $m > 2c$. Hence, contributing $c/2$ is not a symmetric equilibrium.

Now suppose that each player invests c. Then in equilibrium the following two inequalities must be satisfied for investing c to be a symmetric equilibrium:

$- c \geq 0$

and

$- c \geq - c/2.$

These inequalities are violated if $c > 0$. Hence, contributing c is not a symmetric equilibrium.

Thus, this inter-alliance competition does not a symmetric pure strategy equilibrium when $s = 0$, $m > 2c$, and $c > 0$.

<u>Section 1.10</u>

Claim: When partners develop new products in parallel and share profits equally, the inter-alliance game has only a symmetric mixed strategy equilibrium if $s = m/2$, $4c > m > 2c > 0$.

Proof: If each firm in alliance i and alliance j invests 0, then the payoff of player k in alliance i is $m/4$. If firm k deviates unilaterally and invests either $c/2$ or c, then the firm's payoff is $m/2 - c/2$ or $m/2 - c$, respectively

If investing 0 is an equilibrium, then

$m/4 \geq m/2 - c/2,$

and

$m/4 \geq m/2 - c.$

The first inequality implies that $2c \geq m$. But this violates our assumption that $m > 2c$. Therefore, investing 0 is not a symmetric equilibrium

We next show that investing $c/2$ is not a equilibrium. Suppose that each firm in alliance i and alliance j invest $c/2$. Then the payoff for firm k is $m/4 - c/2$. But if k unilaterally deviates and invests 0 or c, then its payoff is 0 or $m/2 - c$. For investing $c/2$ to be a symmetric equilibrium, the following two equations need to be satisfied:

$m/4 - c/2 \geq 0,$

and

$m/4 - c/2 \geq m/2 - c.$

The second inequality is violated if $m > 2c$. Hence contributing $c/2$ is not a symmetric equilibrium.

Now suppose that each player invests c. Then in equilibrium the following 2 inequalities must be satisfied for investing c to be a symmetric equilibrium:

$m/4 - c \geq 0,$

and

$m/4 - c \geq - c/2.$

Again these inequalities will be violated if $4c > m$. Hence, contributing c is not a symmetric equilibrium.

Thus, this inter-alliance competition does have not a symmetric pure strategy equilibrium when $s = m/2$, and $4c > m > 2c > 0$.

Section 1.11

In this section we derive the system of equations used to solve for the equilibrium solution when partners develop products in parallel and share profits equally.

The joint behavior of the competing alliance, j, and partner i_2 is given in Table 6 below. Using this information, as we did earlier, we compute the expected value of investing 0, $c/2$, or c by partner i_1 in alliance i.

Table 6: Parallel Development of New Products

Joint Probabilities (All players except partner i_1 in alliance i)

$Max\{I_{j1}, I_{j2}\}$

Player i_2's Investment		0	c/2	C
	c	$p_3(p_1^2)$	$p_3(2p_1p_2 + p_2^2)$	$p_3(2p_1p_3 + 2p_2p_3 + p_3^2)$
	c/2	$p_2(p_1^2)$	$p_2(2p_1p_2 + p_2^2)$	$p_2(2p_1p_3 + 2p_2p_3 + p_3^2)$
	0	p_1^3	$p_1(2p_1p_2 + p_2^2)$	$p_1(2p_1p_3 + 2p_2p_3 + p_3^2)$
	Marginal	p_1^2	$2p_1p_2 + p_2^2$	$2p_1p_3 + 2p_2p_3 + p_3^2$

Using these probabilities we computed the $EV(0)$, $EV(c/2)$, and $EV(c)$.

$$EV(0) = (s/2)\,p_1^3 + m/2\,(p_2p_1^2 + p_3p_1^2) + (s/2)\,(p_2)\,(2p_1p_2 + p_2^2)$$
$$+\ (m/2)\,(p_3)\,(2p_1p_2 + p_2^2) + (s/2)\,(p_3)\,(2p_1p_3 + 2p_2p_3 + p_3^2) + c.$$

$$EV(c/2) = (m/2)\,p_1^3 + m/2\,(p_2p_1^2 + p_3p_1^2) + (s/2)\,(p_1 + p_2)\,(2p_1p_2 + p_2^2)$$
$$+\ (m/2)\,(p_3)\,(2p_1p_2 + p_2^2) + (s/2)\,(p_3)\,(2p_1p_3 + 2p_2p_3 + p_3^2) + c/2.$$

$$EV(c) = (m/2)\,p_1^2 + (m/2)\,(2p_1p_2 + p_2^2) + (s/2)\,(2p_1p_3 + 2p_2p_3 + p_3^2) + c/2.$$

As $EV(c/2) - EV(0) = 0$, we get:

$$(m/2 - s/2)\,p_1^3 + s/2\,(p_1)\,(2p_1p_2 + p_2^2) = c/2$$

Similarly, as $EV(c) - EV(c/2) = 0$, we get:

$$(m/2 - s/2)\,(p_1 + p_2)\,(2p_1p_2 + p_2^2) + (s/2)\,(p_1 + p_2)\,(2p_1p_3 + 2p_2p_3 + p_3^2) = c/2$$

By setting $s = 0$ we obtain the following system of three equations that provide the equilibrium solution:

$$(m/2)\,p_1^3 = c/2 \qquad (A19)$$

$$(m/2)\,(p_1 + p_2)\,(2p_1p_2 + p_2^2) = c/2 \qquad (A20)$$

$$p_1 + p_2 + p_3 = 1 \qquad (A21)$$

Section 1.12

Claim: The mixed strategy equilibrium solution for the competition between parallel alliances, if partners share profits equally, is: $p_1 = \sqrt[3]{\frac{c}{m}}$, $p_2 = \beta.\sqrt[3]{\frac{c}{m}}$ and $p_3 = 1-(1+\beta).\sqrt[3]{\frac{c}{m}}$, where $\beta = 0.32471$.

Proof: From Equation A19, we obtain that $p_1 = \sqrt[3]{\frac{c}{m}}$.

Now substituting this result in Equation A20, we obtain:

$$\left(\sqrt[3]{\frac{c}{m}} + p_2\right) + \left(2(\sqrt[3]{\frac{c}{m}})p_2 + p_2^2\right) - \frac{c}{m} = 0.$$

Solving this equation, we find that $p_2 = \beta.\sqrt[3]{\frac{c}{m}}$ where $\beta = 0.32471$.

Then substituting the values of p_1 and p_2 in Equation A21, we find that $p_3 = 1-(1+\beta).\sqrt[3]{\frac{c}{m}}$.

Section 1.13

Claim: Under equal profit-sharing arrangement, partners developing products in parallel free ride ($p_1 = \sqrt[3]{\frac{c}{m}}$) more often than those in same-function alliance ($p_1 < \sqrt[3]{\frac{c}{m}}$).

Proof: We have already shown that in parallel alliances partners sharing profits equally will invest 0 units of capital in proportions given by $p_1 = \sqrt[3]{\frac{c}{m}}$ (see Section 1.12).

Now we proceed to show that $p_1 < \sqrt[3]{\frac{c}{m}}$ in the case of same-function alliances, where partners share profits equally. We know from Equation A1 that

$$p_1^{\ 3} + 2\, p_1 p_2^{\ 2} + p_2^{\ 2} p_3 + 2\, p_1 p_3^{\ 2} = c/m$$

Therefore,

$$p_1^{\ 3} = c/m - (2\, p_1 p_2^{\ 2} + p_2^{\ 2} p_3 + 2\, p_1 p_3^{2}).$$

As $\{p_1, p_2, p_3\} \in (0,1)$, we have:

$$p_1^{\,3} < c/m.$$

Hence, $p_1 < \sqrt[3]{\frac{c}{m}}$ in case of parallel alliances where partners share profits equally.

Section 1.14

In this section we present the system of equations for studying the equilibrium behavior if products are developed in parallel and profits are shared in proportion. Denote the expected value of contributing 0, $c/2$, and c by $EV(0)$, $EV(c/2)$, and $EV(c)$, respectively.

Using Table 6, we compute the expected values of investing 0, $c/2$, and c.

$$EV(0) = s/2\,(p_1^3) + c$$

$$EV(c/2) = (m)\,p_1^3 + (m/2)\,(p_2\,p_1^2) + (m/3)\,(p_3\,p_1^2) + (s)\,p_1\,(2\,p_1\,p_2 + p_2^2)$$
$$+ (s/2)\,p_2\,(2\,p_1\,p_2 + p_2^2) + (m/3)\,p_3\,(2\,p_1\,p_2 + p_2^2) + (s/3)\,p_3\,(2\,p_1\,p_3 + 2\,p_2\,p_3 + p_3^2) + c/2$$

$$EV(c) = (m)\,p_1^3 + (2\,m/3)\,(p_2\,p_1^2) + (m/2)\,(p_3\,p_1^2) + (m)\,p_1\,(2\,p_1\,p_2 + p_2^2)$$
$$+ (2\,m/3)\,p_2\,(2\,p_1\,p_2 + p_2^2) + (m/2)\,p_3\,(2\,p_1\,p_2 + p_2^2) + (s)\,p_1\,(2\,p_1\,p_3 + 2\,p_2\,p_3 + p_3^2)$$
$$+ (2\,s/3)\,p_2\,(2\,p_1\,p_3 + 2\,p_2\,p_3 + p_3^2) + (s/2)\,p_3\,(2\,p_1\,p_3 + 2\,p_2\,p_3 + p_3^2)$$

The system of three equations that provides the equilibrium solution is:

$$EV(c/2) - EV(0) = 0$$
$$EV(c) - EV(c/2) = 0$$
$$p_1 + p_2 + p_3 = 1$$

After setting $s = 0$, we obtain the following system of equations:

$$p_1^3 + 1/2\,(p_2\,p_1^2) + 1/3\,(p_3\,p_1^2) + (1/3)\,p_3\,(2\,p_1\,p_2 + p_2^2) = c/m \qquad (A23)$$

$$((2/3)\,(p_2\,p_1^2) + (1/2)\,(p_3\,p_1^2) + p_1\,(2\,p_1\,p_2 + p_2^2)$$
$$+ (2/3)\,p_2\,(2\,p_1\,p_2 + p_2^2) + (1/2)\,p_3\,(2\,p_1\,p_2 + p_2^2))$$
$$-((1/2)\,(p_2\,p_1^2) + (1/3)\,(p_3\,p_1^2) + (1/3)\,p_3\,(2\,p_1\,p_2 + p_2^2)) = c/m \qquad (A24)$$

$p_1 + p_2 + p_3 = 1$ (A25)

Section 1.15

Claim: In case of parallel alliances where partners share profits proportionally, we observe that:

$p_1 > p_3 > p_2$ if $0.5 > c/m > 0.26$,

$p_3 > p_1 > p_2$ if $0.26 > c/m > 0.042$,

$p_3 > p_2 > p_1$ if $0.042 > c/m > 0$.

Proof: First, we show that $p_1 > p_2$ *for* $c/m > 0.0837$.

Let $p_1 = p_2 + \delta_1$. We substitute $p_3 = 1 - p_1 - p_2$, and $p_1 = p_2 + \delta_1$ in Equation A23 and obtain:

$$1/3\, p_2^2 - 1/3\, p_2^3 + 2/3\,(p_2 + \delta_1) - p_2^2 (p_2 + \delta_1) + 1/3\,(p_2 + \delta_1)^2$$

$$- 1/2\, p_2 (p_2 + \delta_1)^2 + 2/3\,(p_2 + \delta_1)^3 - c/m = 0 \qquad (A26)$$

Similarly, we substitute $p_3 = 1 - p_1 - p_2$ and $p_1 = p_2 + \delta_1$ in Equation A24 to obtain:

$$1/6\, p_2^2 + 1/2\, p_2^3 + 1/3\, p_2 (p_2 + \delta_1) + 11/6\, p_2^3 (p_2 + \delta_1) + 1/6\, p_2^4 (p_2 + \delta_1)$$

$$+ 5/3\, p_2 (p_2 + \delta_1)^2 - 1/6\, p_2 (p_2 + \delta_1)^3 - c/m = 0 \qquad (A27)$$

Using Equations A23 and A24, we solve for the value of p_2 when $\delta_1 = 0$. If $\delta_1 = 0$, then $c/m = 0.042$ and this solution is unique. Further, if $c/m = 0.08$, then $\delta_1 = 0.067 > 0$. But, if $c/m = 0.01$ then $\delta_1 < 0$. Hence, $p_1 > p_2$ *for* $c/m > 0.042$.

Second, we show that $p_3 > p_1$ *for* $c/m < 0.26$.

Let $p_1 = p_3 + \delta_2$. Substituting $p_2 = 1 - p_1 - p_3$ and $p_1 = p_3 + \delta_2$ in Equation A23, we obtain:

$$2\,(1/3\, p_3 - 2/3\, p_3^2 + 1/3\, p_3^3 + 1/2\,(p_3 + \delta_2)^2 - 1/2\, p_3 (p_3 + \delta_2)^2$$

$$+ 1/2\,(2p_3 + \delta_2)^2) - c/m = 0 \qquad (A28)$$

Similarly, substituting $p_2 = 1 - p_1 - p_3$ and $p_1 = p_3 + \delta_2$ in Equation A24, we obtain:

$$2\,(2/3 - 11/6\, p_3 + 5/3\, p_3^2 + 1/2\, p_3^3 + 1/3\,(p_3 + \delta_2) - 2/3\, p_3 (p_3 + \delta_2)$$

$$+ 1/3\, p_3^2 (p_3 + \delta_2) - 1/2\, (p_3 + \delta_2)^2 + 1/2\, p_3 (p_3 + \delta_2)^2 - 1/2\, (p_3 + \delta_2)^3)$$

$$- c/m = 0 \quad (A29)$$

Using Equations A27 and A28, we solve for the value of c/m when $\delta_2=0$.

If $\delta_2=0$, then $c/m = 0.26$ and this solution is unique. Further, if $c/m = 0.286$, then $\delta_2 = 0.044 > 0$. But, if $c/m = 0.25$ then $\delta_2 = - 0.017 < 0$. Hence, $p_3 > p_1$ *for* $c/m < 0.26$

Third, we can show $p_3 > p_2$ *for* $0 < c/m < 0.5$.

Let $p_2 = (p_3 + \delta_3)$. Substituting $p_1 = 1 - p_2 - p_3$ and $p_2 = p_3 + \delta_3$ in Equation A23, we obtain:

$$2\, (7/3\, p_3^{\ 2} - 8/3\, p_3 + 7/3\, p_3^{\ 2} - 2/3\, p_3^{\ 3} - 5/2\, (p_3 + \delta_3) - 5\, p_3 (p_3 + \delta_3)$$

$$-\quad 5/2\, p_3^{\ 2} (p_3 + \delta_3) + 2\, (p_3 + \delta_3)^2 - 2\, p_3 (p_3 + \delta_3)^2 - 1/2\, (p_3 + \delta_3)^3)$$

$$- c/m = 0 \quad (A30)$$

Similarly, we substitute $p_1 = 1 - p_2 - p_3$ and $p_2 = p_3 + \delta_2$ in Equation A24 to obtain:

$$2\, (1/6\, p_3 - 1/2\, p_3^{\ 2} + 1/6\, p_3^{\ 2} + 13/6\, (p_3 + \delta_3) - 13/3\, p_3 (p_3 + \delta_3) + 13/6\, p_3^{\ 2} (p_3 + \delta_3)$$

$$- 2\, (p_3 + \delta_3)^2 + 2\, p_3 (p_3 + \delta_3)^2 + 1/2\, (p_3 + \delta_3)^3) - c/m = 0 \quad (A31)$$

Using Equation A29 and A30, we solve for the value of c/m when $\delta_3=0$. We find that p_3 and p_2 do not intersect if $c/m < 0.5$. For instance, if $c/m = 0.4$, then $\delta_3 = - 0.072 < 0$; and if $c/m = 0.071$ then $\delta_2 = - 0.516 < 0$. We know that as $m > 0$ and $c > 0$, and so $c/m > 0$. Therefore, $p_3 > p_2$ *for* $0 < c/m < 0$. Taken together these results prove that:

$p_1 > p_3 > p_2$ if $0.5 > c/m > 0.26$,

$p_3 > p_1 > p_2$ if $0.26 > c/m > 0.042$,

$p_3 > p_2 > p_1$ if $0.042 > c/m > 0$.

Section 1.16

Claim: For parallel alliances where partners share profits proportionally: $\frac{dp_3}{d\frac{c}{m}} < 0$; $\frac{dp_2}{d\frac{c}{m}} > 0$; $\frac{dp_1}{d\frac{c}{m}} > 0$.

Proof: First, we prove that $\frac{dp_3}{d\frac{c}{m}} < 0, \forall\, p_3 \in (0,1)$ and $\frac{dp_2}{d\frac{c}{m}} > 0, \forall\, p_2 \in (0,1)$.

From Equations A23 and A24, we have:

$$F^1(p_1, p_2, p_3; c/m) = p_1^3 + 1/2\,(p_2\,p_1^2) + 1/3\,(p_3\,p_1^2) + (1/3)\,p_3\,(2\,p_1\,p_2 + p_2^2) - c/m \equiv 0$$

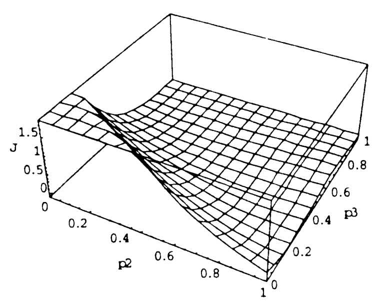
$$F^2(p_1, p_2, p_3; c/m) = ((2/3)\,(p_2\,p_1^2) + (1/2)\,(p_3\,p_1^2) + p_1\,(2\,p_1\,p_2 + p_2^2) + (2/3)\,p_2\,(2\,p_1\,p_2 + p_2^2) + (1/2)\,p_3\,(2\,p_1\,p_2 + p_2^2)) - ((1/2)\,(p_2\,p_1^2) + (1/3)\,(p_3\,p_1^2) + (1/3)\,p_3\,(2\,p_1\,p_2 + p_2^2)) - c/m \equiv 0$$

Substituting $p_1 = 1 - p_2 - p_3$ in these equations we obtain $F^1(p_2, p_3; c/m)$ and $F^2(p_2, p_3; c/m)$.

$$|J| = \begin{vmatrix} \frac{\partial F^1}{\partial p_2} & \frac{\partial F^1}{\partial p_3} \\ \frac{\partial F^2}{\partial p_2} & \frac{\partial F^2}{\partial p_3} \end{vmatrix}$$

$$= 1/36\,(p_3 - 1)(360 p_2 - 207 p_2^2 + 36 p_2^3 + 497 p_3 - 576 p_2 p_3 + 153 p_2^2 p_3 - 415 p_3^2 - 216 p_2 p_3^2 + 111 p_3^3 - 193)$$

In Plot 7 below we present $|J|$ for $p_2 \in (0,1)$ and $p_3 \in (0,1)$. We observe that $|J| > 0$.

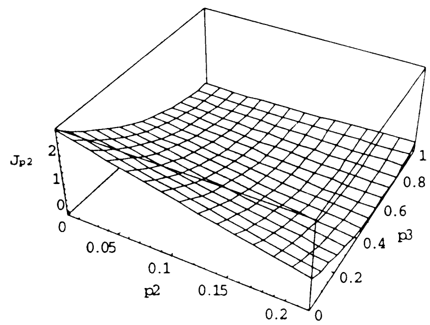


Plot 7.

$$\left|J_{p_2}\right| = \begin{vmatrix} 1 & \dfrac{\partial F^1}{\partial p_3} \\ 1 & \dfrac{\partial F^2}{\partial p_3} \end{vmatrix}$$

$$= \frac{1}{6}(17 - 56p_2 + 24p_2^2 - 32p_3 + 56p_2p_3 + 15p_3^2)$$

Using Equations A13 and A14, we find that if *c/m* < *0.5,* p_2 < *0.21*. In Plot 8 below we present $\left|J_{p_2}\right|$ for $p_2 \in (0,0.21)$ and $p_3 \in (0,1)$. Note that $\left|J_{p_2}\right| < 0$.



Plot 8.

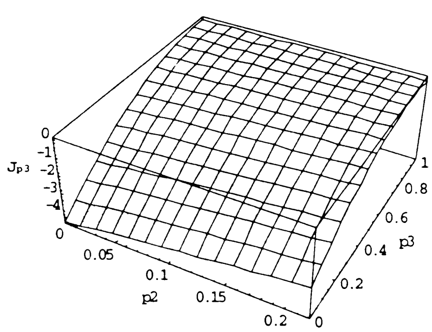
We already know that $\left|J\right| > 0$. Hence, $\dfrac{dp_2}{d\dfrac{c}{m}} = \dfrac{\left|J_{p_2}\right|}{\left|J\right|} > 0$.

$$\left|J_{p_3}\right| = \begin{vmatrix} \dfrac{\partial F^1}{\partial p_2} & 1 \\ \dfrac{\partial F^2}{\partial p_2} & 1 \end{vmatrix}$$

$$= \frac{1}{3}\left(24p_2 - 9{p_2}^2 + 28p_3 - 24p_2p_3 - 14{p_3}^2 - 14\right)$$

In Plot 9 below we present $\left|J_{p_3}\right|$ for $p_2 \in (0,0.21)$ and $p_3 \in (0,1)$. Note that $\left|J_{p_3}\right| < 0$.

Plot 9.



Hence, $\dfrac{dp_3}{d\frac{c}{m}} = \dfrac{|J_{p_3}|}{|J|} < 0$

Next, we prove that $\dfrac{dp_1}{d\frac{c}{m}} > 0$. As $p_2 = 1 - p_1 - p_3$, we have:

$$F^1 (p_1, p_3; c/m) = p_1^{\,3} + 1/2\, p_2^{\,2} (1- p_1 - p_3) + 1/3\, (p_1^{\,2} p_3)$$

$$+ 1/3\, (2 p_1 (1- p_1 - p_3) + (1- p_1 - p_3)^2) p_3 - c/m \equiv 0$$

$$F^2 (p_1, p_3; c/m) = p_1 (2 p_1 (1- p_1 - p_3) + (1 - p_1 - p_3)^2)$$

$$+1/6\, p_1^{\,2} (1- p_1 - p_3) + 2/3\, (2 p_1 (1- p_1 - p_2) + (1- p_1 - p_3)^2) (1- p_1 - p_3)$$

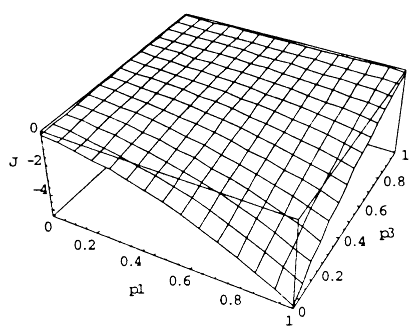
$$+ 1/6\, (p_1^{\,2} p_3) + 1/6\, (2 p_1 (1- p_1 - p_3) + (1- p_1 - p_3)^2) p_3 - c/m \equiv 0$$

$$|J| = \begin{vmatrix} \dfrac{\partial F^1}{\partial p_1} & \dfrac{\partial F^1}{\partial p_3} \\ \dfrac{\partial F^2}{\partial p_1} & \dfrac{\partial F^2}{\partial p_3} \end{vmatrix}$$

$$= \frac{1}{36}(p_3 - 1)(4 + 54p_1 + 99p_1^{\,2} + 36p_1^{\,3} - 20p_3 - 72p_1p_3 - 45p_1^{\,2}p_3 +$$

$$28p_3^{\,2} + 18p_1p_3^{\,2} - 12p_3^{\,3})$$

Plot 10 below presents the value of $|J| \forall\, p_1 \in (0,1, p_3 \in (0,1)$. Note $|J| < 0$.

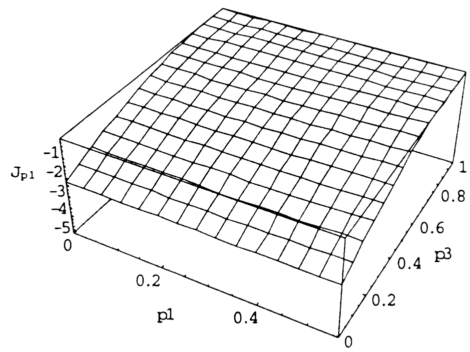


Plot 10.

$$\left|J_{p_1}\right| = \begin{vmatrix} 1 & \dfrac{\partial F^1}{\partial p_3} \\ 1 & \dfrac{\partial F^2}{\partial p_3} \end{vmatrix}$$

$$= \frac{1}{6}\left(-13 - 4p_1 + 6p_1^2 + 28p_3 + 4p_1p_3 - 15p_3^2\right)$$

Using Equations A15 and A16, we find that if $c/m < 0.5$, then $p_1 < 0.56$. In Plot 11 we present $\left|J_{p_1}\right|$ for $p_1 \in (0,0.56)$ and $p_3 \in (0,1)$. Note that $\left|J_{p_1}\right| < 0$.



Plot 11.

Hence, $\dfrac{dp_1}{d\frac{c}{m}} = \dfrac{\left|J_{p_1}\right|}{|J|} > 0$

Section 17

Section 1.17 has three parts: Section 1.17a proves that the asymmetric game has only a mixed strategy equilibrium, if partners share profits equally; Section 1.17b derives the system of equations that provide the corresponding equilibrium solution; Section 1.17c presents the system of equations that provide the equilibrium solution, if partners share profits proportionally.

Section 1.17a:

Claim: If $s = 0$, $m > 2c > 0$, $c_{i1} = c_{j1} = c/2$, and $c_{i2} = c_{j2} = c$, then the inter-alliance competition between two same-function alliances, where partners share profits equally, only has a symmetric mixed strategy equilibrium.

Proof: First, we establish that there is no symmetric pure strategy equilibrium for the weak players irrespective of the symmetric pure strategies played by the strong players. Later, we show that there exists no symmetric pure strategy equilibrium for the strong players irrespective of the symmetric pure strategy played by the weak players.

Suppose that the strong players in both alliances *i* and *j* invest c units of capital ($I_{i2} = I_{j2} = c$). Also suppose that the weak players in both alliances invest *c/2*. Now the payoff for the weak player in alliance *i* is - *c/2*. But if the weak player in alliance *i* unilaterally deviates and invests *0* or *c/4*, then its payoff is *0* or – *c/4,* respectively.

If investing *c/2* is a symmetric equilibrium for the weak players, then

$$- c/2 \geq 0$$

and

$$- c/2 > - c/4.$$

Both these inequalities violate the assumption that $c > 0$. Therefore, investing *c/2* is not a symmetric equilibrium for the weak players.

Now suppose that the weak players in both alliances invest *c/4*. Also suppose that the strong players in both alliances *i* and *j* invest *c* units of capital. The payoff for the weak player in alliance *i* is - *c/4*. But if the weak player in alliance *i* unilaterally deviates and invests *0* or *c/2*, then its payoff is *0* or *m/2* - *c/2* respectively.

If investing *c/4* is a symmetric equilibrium for the weak players, then

$$- c/4 \geq 0$$

and

$$- c/4 > m/2 - c/4.$$

The second inequality violates the assumption that $m \geq 2c$. Therefore, investing *c/4* is not a symmetric equilibrium for the weak players.

Now suppose that the weak players in both alliances invest *0*. Also suppose that the strong players in both alliances *i* and *j* invest *c* units of capital. Now the payoff for the weak player on alliance *i* is *0*. But if the weak player in alliance *i* unilaterally deviates and invests *c/4* or *c/2*, then its payoff is *m/2* – *c/4* or *m/2* - *c/2* respectively.

If investing *0* is a symmetric equilibrium for the weak players, then

$$0 \geq m/2 - c/4,$$

and

$$0 > m/2 - c/2.$$

Both these inequalities violate the assumption that $m \geq 2c$. Therefore, investing 0 is not a symmetric equilibrium for the weak players.

Next suppose that the strong players in both alliances i and j invest $3c/4$ units of capital. Also suppose that the weak players in both alliances invest $c/2$. Now the payoff for the weak player in alliance i is $- c/2$. But if the weak player in alliance i unilaterally deviates and invests 0 or $c/4$, then its payoff is 0 or $– c/4$ respectively.

If investing $c/2$ is a symmetric equilibrium for the weak players, then

$$- c/2 \geq 0$$

and

$$- c/2 > - c/4.$$

Both these inequalities violate the assumption that $c > 0$. Therefore, investing $c/2$ is not a symmetric equilibrium for the weak players.

Now suppose that the weak players in both alliances invest $c/4$. Suppose that the strong players in both alliances i and j invest $3c/4$ units of capital. Now the payoff for the weak player in alliance i is $-c/4$. But if the weak player in alliance i unilaterally deviates and invests 0 or $c/2$, then its payoff is 0 or $m/2 - c/2$ respectively.

If investing $c/4$ is a symmetric equilibrium for the weak players, then

$$- c/4 \geq 0$$

and

$$- c/4 > m/2 - c/4.$$

The second inequality violates the assumption that $m \geq 2c$. Therefore, investing $c/4$ is not a symmetric equilibrium for the weak players.

Next, suppose that the weak players in both alliances invest 0. Suppose that the strong players in both alliances i and j invest $3c/4$ units of capital. Now the payoff for the weak player in alliance i is 0. But if the weak player in alliance i unilaterally deviates and invests $c/4$ or $c/2$, then its payoff is $m/2 – c/4$ or $m/2 - c/2$ respectively.

If investing 0 is a symmetric equilibrium for the weak players, then

$$0 \geq m/2 – c/4,$$

and

$$0 > m/2 - c/2.$$

Both these inequalities violate the assumption that $m \geq 2c$. Therefore, investing 0 is not a symmetric equilibrium for the weak players.

Similarly, we find that there is no symmetric pure strategy equilibrium for the weak players, when both the strong players invest *c/2, c/4* or *0* units of capital. Hence, there is no symmetric pure strategy equilibrium for the weak players, irrespective of whether both the strong players invest *c, 3c/4, c/2 c/4* and *0* units of capital.

Second, we proceed to establish that there is no symmetric pure strategy equilibrium for the strong players. Suppose that the weak players in both alliances *i* and *j* invest c/2 units of capital ($I_{i1} = I_{j1} = c/2$). Also suppose that the strong players in both alliances invest *c*. Now the payoff for the strong player in alliance *i* is - *c*. But, if the strong player in alliance *i* unilaterally deviates and invests *0, c/4, c/2,* or *3c/4* then its payoff is *0, - c/4, - c/2,* or *-3c/4.*

If investing *c* is a symmetric equilibrium for the strong players, then

$- c \geq 0,$

$- c \geq -c/4,$

$- c \geq -c/2,$ and

$- c \geq -3c/4.$

These inequalities violate the assumption that $c > 0$. Therefore, investing *c* is not a symmetric equilibrium for the strong players.

Now suppose that the strong players in both alliances invest *3c/4*. Suppose that the weak players in both alliances *i* and *j* invest *c/2* units of capital. So, the payoff for the strong player in alliance *i* is – *3c/4*. But if the strong player in alliance *i* unilaterally deviates and invests *0, c/4, c/2,* or *c* then its payoff is *0, - c/4, - c/2,* or *m/2 - c.*

If investing *3c/4* is a symmetric equilibrium for the strong players, then

$- 3c/4 \geq 0,$

$- 3c/4 \geq -c/4,$

$- 3c/4 \geq -c/2,$ and

$- 3c/4 \geq m/2 - c.$

The last inequality violates the assumption that $m \geq 2c$. Therefore, investing *3c/4* is not a symmetric equilibrium for the strong players.

Now suppose that the strong players in both alliances invest *c/2*. Suppose that the weak players in both alliances *i* and *j* invest *c/2* units of capital. So, the payoff for the strong player in alliance *i* is – *c/2*. But if the strong player in alliance *i* unilaterally deviates and invests *0, c/4, 3c/4,* or *c* then its payoff is *0, - c/4, - c/2, m/2 - 3c/4, m/2 – c.*

If investing *c/2* is a symmetric equilibrium for the strong players, then

$- c/2 \geq 0,$

$- c/2 \geq -c/4,$

$- c/2 \geq m/2 - 3c/4,$ and

$- c/2 \geq m/2 - c.$

The last two inequalities violate the assumption that $m \geq 2c$. Therefore, investing $c/2$ is not a symmetric equilibrium for the strong players.

Now suppose that the strong players in both alliances invest $c/4$. Suppose that the weak players in both alliances i and j invest $c/2$ units of capital. Therefore, the payoff for the strong player in alliance i is $- c/4$. But if the strong player in alliance i unilaterally deviates and invests $0, c/2, 3c/4$ or c then its payoff is $0, - m/2 - c/2, m/2- 3c/4,$ or $m/2 - c$.

If investing $c/4$ is a symmetric equilibrium for the strong players, then

$- c/4 \geq 0,$

$- c/4 \geq m/2 - c/2,$

$- c/4 \geq m/2 - 3c/4,$ and

$- c/4 \geq m/2 - c.$

The last three inequalities violate the assumption that $m \geq 2c$. Therefore, investing $c/4$ is not a symmetric equilibrium for the strong players.

Now suppose that the strong players in both alliances invest 0. Suppose that the weak players in both alliances i and j invest $c/2$ units of capital. Therefore, the payoff for the strong player in alliance i is 0. But if the strong player in alliance i unilaterally deviates and invests $c/4, c/2, 3c/4,$ or c then its payoff is $m/2 - c/4, m/2 - c/2, m/2 - 3c/4,$ or $m/2 - c$.

If investing 0 is a symmetric equilibrium for the strong players, then

$0 \geq m/2 - c/4,$

$0 \geq m/2 - c/2,$

$0 \geq m/2 - 3c/4,$ and

$0 \geq m/2 - c.$

The last inequality violates the assumption that $m \geq 2c$. Therefore, investing 0 is not a symmetric equilibrium for the strong players.

Next suppose that the weak players in both alliances i and j invest $c/4$ units of capital ($I_{i1} = I_{j1} = c/4$). Also suppose that the strong players in both alliances invest c. Now the payoff for the strong player on alliance i is $- c$. But if the strong player in alliance i unilaterally deviates and invests 0, $c/4$, $c/2$, or $3c/4$ then its payoff is 0, $- c/4$, $- c/2$, or $-3c/4$.

If investing c is a symmetric equilibrium for the strong players, then

$- c \geq 0$,

$- c \geq -c/4$,

$- c \geq -c/2$, and

$- c \geq -3c/4$.

These inequalities violate the assumption that $c \geq 0$. Therefore, investing c is not a symmetric equilibrium for the strong players.

Now suppose that the strong players in both alliances invest $3c/4$. Suppose that the weak players in both alliances i and j invest $c/4$ units of capital. So, the payoff for the strong player in alliance i is – $3c/4$. But if the strong player in alliance i unilaterally deviates and invests 0, $c/4$, $c/2$, or c then its payoff is 0, $- c/4$, $- c/2$, or $m/2 - c$.

If investing $3c/4$ is a symmetric equilibrium for the strong players, then

$- 3c/4 \geq 0$,

$- 3c/4 \geq -c/4$,

$- 3c/4 \geq -c/2$, and

$- 3c/4 \geq m/2 - c$.

The last inequality violates the assumption that $m \geq 2c$. Therefore, investing $3c/4$ is not a symmetric equilibrium for the strong players.

Now suppose that the strong players in both alliances invest $c/2$. Suppose that the weak players in both alliances i and j invest $c/4$ units of capital. So, the payoff for the strong player in alliance i is – $c/2$. But if the strong player in alliance i unilaterally deviates and invests 0, $c/4$, $3c/4$, or c then its payoff is 0, $- c/4$, $- c/2$, $m/2 - 3c/4$, $m/2 - c$.

If investing $c/2$ is a symmetric equilibrium for the strong players, then

$- c/2 \geq 0$,

$- c/2 \geq -c/4$,

$- c/2 \geq m/2 - 3c/4$, and

$- c/2 \geq m/2 - c$.

The last two inequalities violate the assumption that $m \geq 2c$. Therefore, investing $c/2$ is not a symmetric equilibrium for the strong players.

Now suppose that the strong players in both alliances invest $c/4$. Suppose that the weak players in both alliances i and j invest $c/4$ units of capital. So, the payoff for the strong player in alliance i is $-c/4$. But if the strong player in alliance i unilaterally deviates and invests 0, $c/2$, $3c/4$ or c then its payoff is 0, $-m/2 - c/2$, $m/2 - 3c/4$, or $m/2 - c$.

If investing $c/4$ is a symmetric equilibrium for the strong players, then

$-c/4 \geq 0$,

$-c/4 \geq m/2 - c/2$,

$-c/4 \geq m/2 - 3c/4$, and

$-c/4 \geq m/2 - c$.

The last three inequalities violate the assumption that $m \geq 2c$. Therefore, investing $c/4$ is not a symmetric equilibrium for the strong players.

Now suppose that the strong players in both alliances invest 0. Suppose that the weak players in both alliances i and j invest $c/4$ units of capital. Therefore, the payoff for the strong player in alliance i is 0. But if the strong player in alliance i unilaterally deviates and invests $c/4$, $c/2$, $3c/4$ or c then its payoff is $m/2 - c/4$, $m/2 - c/2$, $m/2 - 3c/4$, or $m/2 - c$.

If investing 0 is a symmetric equilibrium for the strong players, then

$0 \geq m/2 - c/4$,

$0 \geq m/2 - c/2$,

$0 \geq m/2 - 3c/4$, and

$0 \geq m/2 - c$.

The last inequality violates the assumption that $m \geq 2c$. Therefore, investing 0 is not a symmetric equilibrium for the strong players.

Similarly, we find that there is no symmetric pure strategy equilibrium for the strong players, if both the weak players invest 0 units of capital. Hence, there is no symmetric pure strategy equilibrium for the strong players, irrespective of whether both the weak players invest $c/2$ $c/4$ and 0 units of capital.

Therefore, in this game with asymmetric investment capital there is no symmetric pure strategy equilibrium.

Section 1.17b

In this section we derive the system of equations that provide the equilibrium solution for the competition between two same-function alliance, if players share profits equally but $c_{i1} = c_{j1} = c/2$, and $c_{i2} = c_{j2} = c$.

We denote the probability of the strong player contributing *0, c/4, c/2, 3c/4* and *c* by p_1, p_2, p_3, p_4, and p_5, respectively. Similarly, we denote the probability of the weak players investing *0, c/4,* and *c/2* by q_1, q_2, and q_3, respectively. Using this notation we present in the table below the joint probabilities.

Table 7: Same-function Alliance (Asymmetric Players within an alliance)

Joint Probabilities (All players except i_1 in alliance i)

$U(j)$

		0	c/4	c/2	3c/4	c	1¼ c	1½ c
	0	$p_1 (p_1q_1)$	$p_1 (p_1q_2 + p_2q_1)$	$P_1 (p_1q_3 + p_3q_1 + p_2q_2)$	$p_1 (p_4q_1 + p_3q_2 + p_2q_3)$	$p_1 (p_5q_1 + p_4q_2 + p_3q_3)$	$p_1 (p_4q_3 + p_5q_2)$	$p_1 (p_5q_3)$
	c/4	$p_2 (p_1q_1)$	$p_2 (p_1q_2 + p_2q_1)$	$P_2 (p_1q_3 + p_3q_1 + p_2q_2)$	$p_2 (p_4q_1 + p_3q_2 + p_2q_3)$	$p_2 (p_5q_1 + p_4q_2 + p_3q_3)$	$p_2 (p_4q_3 + p_5q_2)$	$p_2 (p_5q_3)$
Player i_2's Investment	c/2	$p_3 (p_1q_1)$	$p_3 (p_1q_2 + p_2q_1)$	$P_3 (p_1q_3 + p_3q_1 + p_2q_2)$	$p_3 (p_4q_1 + p_3q_2 + p_2q_3)$	$p_3 (p_5q_1 + p_4q_2 + p_3q_3)$	$p_3 (p_4q_3 + p_5q_2)$	$p_3 (p_5q_3)$
	3c/4	$p_4 (p_1q_1)$	$p_4 (p_1q_2 + p_2q_1)$	$P_4 (p_1q_3 + p_3q_1 + p_2q_2)$	$p_4 (p_4q_1 + p_3q_2 + p_2q_3)$	$p_4 (p_5q_1 + p_4q_2 + p_3q_3)$	$p_4 (p_4q_3 + p_5q_2)$	$p_4 (p_5q_3)$
	c	$p_5 (p_1q_1)$	$p_5 (p_1q_2 + p_2q_1)$	$P_5 (p_1q_3 + p_3q_1 + p_2q_2)$	$p_5 (p_4q_1 + p_3q_2 + p_2q_3)$	$p_5 (p_5q_1 + p_4q_2 + p_3q_3)$	$p_5 (p_4q_3 + p_5q_2)$	$p_5 (p_5q_3)$
	Marginal	(p_1q_1)	$(p_1q_2 + p_2q_1)$	$(p_1q_3 + p_3q_1 + p_2q_2)$	$(p_4q_1 + p_3q_2 + p_2q_3)$	$(p_5q_1 + p_4q_2 + p_3q_3)$	$(p_4q_3 + p_5q_2)$	(p_5q_3)

Using these joint probabilities, we next compute the expected value of contributing *0, c/4,* and *c/2* by the weak player in alliance *i*. We denote these expected values by $EV^{weak}(0)$, $EV^{weak}(c/4)$, and $EV^{weak}(c/2)$, respectively. We also know that $p_1 + p_2 + p_3 + p_4 + p_5 = 1$.

$$EV^{weak}(0) = (1-p_1)\,(p_1q_1)\,(m/2) + (1-p_1-p_2)\,(p_1q_2 + p_2q_1)\,(m/2)$$

$$+ (1-p_1-p_2-p_3)(m/2)(p_1q_3 + p_3q_1 + p_2q_2)$$
$$+ (1-p_1-p_2-p_3-p_4)(m/2)(p_4q_1 + p_3q_2 + p_2q_3) + c/2.$$

$$EV^{weak}(c/4) = (p_1q_1)(m/2) + (1-p_1)(p_1q_2 + p_2q_1)(m/2)$$
$$+ (1-p_1-p_2)(m/2)(p_1q_3 + p_3q_1 + p_2q_2)$$
$$+ (1-p_1-p_2-p_3)(m/2)(p_4q_1 + p_3q_2 + p_2q_3)$$
$$+ (1-p_1-p_2-p_3-p_4)(m/2)(p_5q_1 + p_4q_2 + p_3q_3) + c/4.$$

$$EV^{weak}(c/2) = (p_1q_1)(m/2) + (p_1q_2 + p_2q_1)(m/2) + (1-p_1)(m/2)(p_1q_3 + p_3q_1 + p_2q_2)$$
$$+ (1-p_1-p_2)(m/2)(p_4q_1 + p_3q_2 + p_2q_3) + (1-p_1-p_2-p_3)(m/2)(p_5q_1 + p_4q_2 + p_3q_3)$$
$$+ (1-p_1-p_2-p_3-p_4)(m/2)(p_4q_3 + p_5q_2).$$

Similarly, in Table 8 below we present the joint probabilities for all players except player i_2 in alliance i

Table 8: Same-function Alliance (Asymmetric Players within an alliance)

Joint Probabilities (All players except i_2 in alliance i)

U(j)

		0	c/4	c/2	3c/4	c	1¼ c	1½ c
Player i_1's Investment	0	$q_1(p_1q_1)$	$q_1(p_1q_2 + p_2q_1)$	$q_1(p_1q_3 + p_3q_1 + p_2q_2)$	$q_1(p_4q_1 + p_3q_2 + p_2q_3)$	$q_1(p_5q_1 + p_4q_2 + p_3q_3)$	$q_1(p_4q_3 + p_5q_2)$	$q_1(p_5q_3)$
	c/4	$q_2(p_1q_1)$	$q_2(p_1q_2 + p_2q_1)$	$q_2(p_1q_3 + p_3q_1 + p_2q_2)$	$q_2(p_4q_1 + p_3q_2 + p_2q_3)$	$q_2(p_5q_1 + p_4q_2 + p_3q_3)$	$q_2(p_4q_3 + p_5q_2)$	$q_2(p_5q_3)$
	c/2	$q_3(p_1q_1)$	$q_3(p_1q_2 + p_2q_1)$	$q_3(p_1q_3 + p_3q_1 + p_2q_2)$	$q_3(p_4q_1 + p_3q_2 + p_2q_3)$	$q_3(p_5q_1 + p_4q_2 + p_3q_3)$	$q_3(p_4q_3 + p_5q_2)$	$q_3(p_5q_3)$
	Marginal	(p_1q_1)	$(p_1q_2 + p_2q_1)$	$(p_1q_3 + p_3q_1 + p_2q_2)$	$(p_4q_1 + p_3q_2 + p_2q_3)$	$(p_5q_1 + p_4q_2 + p_3q_3)$	$(p_4q_3 + p_5q_2)$	(p_5q_3)

Using these joint probabilities, we next compute the expected value of contributing *0, c/4, c/2 3c/4, c* by the strong player in alliance *i*. We denote these expected values by $EV^{strong}(0)$, $EV^{strong}(c/4)$, $EV^{strong}(c/2)$, $EV^{strong}(3c/4)$, and $EV^{strong}(c)$, respectively. We also know that $q_1 + q_2 + q_3 = 1$.

$$EV^{strong}(0) = (1-q_1)(p_1q_1)(m/2) + (1-q_1-q_2)(p_1q_2 + p_2q_1)(m/2) + c.$$

$$EV^{strong}(c/4) = (p_1q_1)(m/2) + (1-q_1)(p_1q_2 + p_2q_1)(m/2)$$

$+ (1-q_1-q_2)(m/2)(p_1q_3 + p_3q_1 + p_2q_2) + 3c/4.$

$$EV^{strong}(c/2) = (p_1q_1)(m/2) + (p_1q_2 + p_2q_1)(m/2) + (1-q_1)(m/2)(p_1q_3 + p_3q_1 + p_2q_2)$$
$$+ (1-q_1-q_2)(m/2)(p_4q_1 + p_3q_2 + p_2q_3) + c/2.$$

$$EV^{strong}(3c/4) = (p_1q_1)(m/2) + (p_1q_2 + p_2q_1)(m/2) + (m/2)(p_1q_3 + p_3q_1 + p_2q_2)$$
$$+ (1-q_1)(m/2)(p_4q_1 + p_3q_2 + p_2q_3) + (1-q_1-q_2)(m/2)(p_5q_1 + p_4q_2 + p_3q_3) + c/4.$$

$$EV^{strong}(c) = (p_1q_1)(m/2) + (p_1q_2 + p_2q_1)(m/2) + (m/2)(p_1q_3 + p_3q_1 + p_2q_2)$$
$$+ (m/2)(p_4q_1 + p_3q_2 + p_2q_3) + (1-q_1)(m/2)(p_5q_1 + p_4q_2 + p_3q_3)$$
$$+ (1-q_1-q_2)(m/2)(p_4q_3 + p_5q_2).$$

The following system of system of 8 equations provides the equilibrium solution (namely, p_1, p_2, p_3, p_4, p_5, q_1, q_2, and q_3):

$EV^{weak}(c/4) - EV^{weak}(0) = 0,$
$EV^{weak}(c/2) - EV^{weak}(c/4) = 0,$
$EV^{strong}(c/4) - EV^{strong}(0) = 0,$
$EV^{strong}(c/2) - EV^{strong}(c/4) = 0,$
$EV^{strong}(3c/4) - EV^{strong}(c/2) = 0,$
$EV^{strong}(c) - EV^{strong}(3c/4) = 0.$
$q_1 + q_2 + q_3 = 1,$
$p_1 + p_2 + p_3 + p_4 + p_5 = 1$

Section 1.17c

In this section we present the system of equations that provide the equilibrium solution for the competition between two same-function alliance, if players share profits proportionally but $c_{i1} = c_{j1} = c/2$, and $c_{i2} = c_{j2} = c$.

We use the joint probability tables (Tables 7 and 8) presented in section 1.17b to derive the following expected values.

$EV^{weak}(0) = c/2.$

$EV^{weak}(c/4) = \quad p_1 (p_1q_1)(m) + (p_2)(p_1q_1)(m/2) + (p_3)(p_1q_1)(m/3) + (p_4)(p_1q_1)(m/4)$
$+ (p_5)(p_1q_1)(m/5) + (p_2)(p_1q_2 + p_2q_1)(m/2) + (p_3)(p_1q_2 + p_2q_1)(m/3) + (p_4)(p_1q_2$
$+ p_2q_1)(m/4) + (p_5)(p_1q_2 + p_2q_1)(m/5) + (p_3)(m/3)(p_1q_3 + p_3q_1 + p_2q_2)$
$+ (p_4)(m/4)(p_1q_3 + p_3q_1 + p_2q_2) + (p_5)(m/5)(p_1q_3 + p_3q_1 + (p_4)(m/4)(p_4q_1 + p_3q_2$
$+ p_2q_3) + (p_5)(m/5)(p_4q_1 + p_3q_2 + p_2q_3)(p_5)(m/5)(p_5q_1 + p_4q_2 + p_3q_3) + c/4.$

$EV^{weak}(c/2) = \quad p_1 (p_1q_1)(m) + (p_2)(p_1q_1)(2/3\ m) + (p_3)(p_1q_1)(m/2)$
$+ (p_4)(p_1q_1)(2/5\ m) + (p_5)(p_1q_1)(m/3) + (p_1)(p_1q_2 + p_2q_1)(m)$
$+ (p_2)(p_1q_2 + p_2q_1)(2/3\ m) + (p_3)(p_1q_2 + p_2q_1)(m/2) + (p_4)(p_1q_2 + p_2q_1)(2/5\ m)$
$+ (p_5)(p_1q_2 + p_2q_1)(m/3) + (p_2)(2/3\ m)(p_1q_3 + p_3q_1 + p_2q_2)$
$+ (p_3)(m/2)(p_1q_3 + p_3q_1 + p_2q_2) + (p_4)(2/5\ m)(p_1q_3 + p_3q_1 + p_2q_2)$
$+ (p_5)(m/3)(p_1q_3 + p_3q_1 + p_3)(m/2)(p_4q_1 + p_3q_2 + p_2q_3) + (p_4)(2/5\ m)(p_4q_1 + p_3q_2$
$+ p_2q_3) + (p_5)(m/3)(p_4q_1 + p_3q_2 + p_2q_3)(p_4)(2/5\ m)(p_5q_1 + p_4q_2 + p_3q_3)$
$+ (p_5)(m/3)(p_5q_1 + p_4q_2 + p_3q_3) + (p_4)(m/3)(p_4q_3 + p_5q_2).$

$EV^{strong}(0) = \quad c$

$EV^{strong}(c/4) = q_1 (p_1q_1)(m) + (q_2)(p_1q_1)(m/2) + (q_3)(p_1q_1)(m/3)$
$+ (q_2)(p_1q_2 + p_2q_1)(m/2) + (q_3)(p_1q_2 + p_2q_1)(m/3)$
$+ (q_3)(m/3)(p_1q_3 + p_3q_1 + p_2q_2) + 3\ c/4.$

$EV^{strong}(c/2) = q_1 (p_1q_1)(m) + (q_2)(p_1q_1)(2/3\ m) + (q_3)(p_1q_1)(m/2)$
$+ (q_1)(p_1q_2 + p_2q_1)(m) + (q_2)(p_1q_2 + p_2q_1)(2/3\ m) + (q_3)(p_1q_2 + p_2q_1)(m/2)$
$+ (q_2)(2/3\ m)(p_1q_3 + p_3q_1 + p_2q_2) + (q_3)(m/2)(p_1q_3 + p_3q_1 + p_2q_2)$
$+ (q_3)(m/2)(p_4q_1 + p_3q_2 + p_2q_3) + c/2.$

$EV^{strong}(3c/4) = q_1 (p_1q_1)(m) + (q_2)(p_1q_1)(3/4\ m) + (q_3)(p_1q_1)(3/5\ m)$
$+ (q_1)(p_1q_2 + p_2q_1)(m) + (q_2)(p_1q_2 + p_2q_1)(3/4\ m) + (q_3)(p_1q_2 + p_2q_1)(3/5\ m)$
$+ (q_1)(m)(p_1q_3 + p_3q_1 + p_2q_2) + (q_2)(3/4\ m)(p_1q_3 + p_3q_1 + p_2q_2)$
$+ (q_3)(3/5\ m)(p_1q_3 + p_3q_1 + p_2q_2)$
$+ (q_2)(3/4\ m)(p_4q_1 + p_3q_2 + p_2q_3) + (q_3)(3/5\ m)(p_4q_1 + p_3q_2 + p_2q_3)$
$+ (q_3)(3/5\ m)(p_5q_1 + p_4q_2 + p_3q_3) + c/4.$

$EV^{strong}(c) = \quad q_1 (p_1q_1) (m) + (q_2) (p_1q_1) (4/5\ m) + (q_3) (p_1q_1) (4/6\ m)$

$+(q_1) (p_1q_2 + p_2q_1) (m) + (q_2) (p_1q_2 + p_2q_1) (4/5\ m) +(q_3) (p_1q_2 + p_2q_1) (4/6\ m)$

$+ (q_1) (m) (p_1q_3 + p_3q_1 + p_2q_2) + (q_2) (4/5\ m) (p_1q_3 + p_3q_1 + p_2q_2)$

$+ (q_3) (4/6\ m) (p_1q_3 + p_3q_1 + p_2q_2) + (q_1) (m) (p_4q_1 + p_3q_2 + p_2q_3)$

$+ (q_2) (4/5m) (p_4q_1 + p_3q_2 + p_2q_3) + (q_3) (4/6\ m) (p_4q_1 + p_3q_2 + p_2q_3)$

$+ (q_2) (4/5\ m) (p_5q_1 + p_4q_2 + p_3q_3) + (q_3) (4/6\ m) (p_5q_1 + p_4q_2 + p_3q_3)$

$+ (q_3) (4/6\ m) (p_4q_3 + p_5q_2).$

The following system of system of 8 equations provides the equilibrium solution (namely, p_1, p_2, p_3, p_4, p_5, q_1, q_2, and q_3):

$EV^{weak}(c/4) - EV^{weak}(0) = 0,$

$EV^{weak}(c/2) - EV^{weak}(c/4) = 0,$

$EV^{strong}(c/4) - EV^{strong}(0) = 0,$

$EV^{strong}(c/2) - EV^{strong}(c/4) = 0,$

$EV^{strong}(3c/4) - EV^{strong}(c/2) = 0,$

$EV^{strong}(c) - EV^{strong}(3c/4) = 0,$

$q_1 + q_2 + q_3 = 1,$

$p_1 + p_2 + p_3 + p_4 + p_5 = 1$

Section 1.18

In this section we derive the equilibrium solution for the competition between two same-function alliances where each player can invest 0, c/4, c/2, 3c/4, or c units of capital.

We denote the probability of investing 0, c/4, c/2, 3c/4, or c units of capital by p_1, p_2, p_3, p_4, and p_5, respectively. In the table below we present the joint probabilities.

Table 8: Same-function Alliance (Asymmetric Players within an alliance)

Joint Probabilities (All players except i_1 in alliance i)

$U(j)$

		0	c/4	c/2	3 c/4	c	1 ¼ c	1 ½ c	1 ¾ c	2 c
	0	p_1 (p_1^2)	P_1 $(2p_1p_2)$	p_1 $(p_2^2 + 2p_1p_3)$	p_1 $(2p_1p_4 + 2p_2p_3)$	p_1 $(p_3^2 + 2p_1p_5 + 2p_1p_2)$	p_1 $(2p_2p_5 + 2p_3p_4)$	p_1 $(p_4^2 + 2p_3p_5)$	p_1 $(2p_4p_5)$	p_1 (p_5^2)
	c/4	p_2 (p_1^2)	P_2 $(2p_1p_2)$	p_2 $(p_2^2 + 2p_1p_3)$	p_2 $(2p_1p_4 + 2p_2p_3)$	p_2 $(p_3^2 + 2p_1p_5 + 2p_1p_2)$	p_2 $(2p_2p_5 + 2p_3p_4)$	p_2 $(p_4^2 + 2p_3p_5)$	p_2 $(2p_4p_5)$	p_2 (p_5^2)
Player i_2's Investment	c/2	p_3 (p_1^2)	P_3 $(2p_1p_2)$	p_3 $(p_2^2 + 2p_1p_3)$	p_3 $(2p_1p_4 + 2p_2p_3)$	p_3 $(p_3^2 + 2p_1p_5 + 2p_1p_2)$	p_3 $(2p_2p_5 + 2p_3p_4)$	p_3 $(p_4^2 + 2p_3p_5)$	p_3 $(2p_4p_5)$	p_3 (p_5^2)
	3 c/4	p_4 (p_1^2)	P_4 $(2p_1p_2)$	p_4 $(p_2^2 + 2p_1p_3)$	p_4 $(2p_1p_4 + 2p_2p_3)$	p_4 $(p_3^2 + 2p_1p_5 + 2p_1p_2)$	p_4 $(2p_2p_5 + 2p_3p_4)$	p_4 $(p_4^2 + 2p_3p_5)$	p_4 $(2p_4p_5)$	p_4 (p_5^2)
	c	p_5 (p_1^2)	P_5 $(2p_1p_2)$	p_5 $(p_2^2 + 2p_1p_3)$	p_5 $(2p_1p_4 + 2p_2p_3)$	p_5 $(p_3^2 + 2p_1p_5 + 2p_1p_2)$	p_5 $(2p_2p_5 + 2p_3p_4)$	p_5 $(p_4^2 + 2p_3p_5)$	p_5 $(2p_4p_5)$	p_5 (p_5^2)
	Marginal	(p_1^2)	$(2p_1p_2)$	$(p_2^2 + 2p_1p_3)$	$(2p_1p_4 + 2p_2p_3)$	$(p_3^2 + 2p_1p_5 + 2p_1p_2)$	$(2p_2p_5 + 2p_3p_4)$	$(p_4^2 + 2p_3p_5)$	$(2p_4p_5)$	(p_5^2)

Using these joint probabilities, we next compute the expected value of contributing *0, c/4, c/2, 3c/4, or c* by the player i_1 in alliance i. We denote these expected values by *EV(0)*, *EV(c/4)*, and *EV(c/2)*, *EV(3c/4)*, *EV(c)*, respectively.

$$EV(0) = (1 - p_1)(p_1^2)(m/2) + (1 - p_1 - p_2)(2p_1p_2)(m/2)$$

$$+ (1 - p_1 - p_2 - p_3)(p_2^2 + 2p_1p_3)(m/2) + (1 - p_1 - p_2 - p_3 - p_4)(2p_1p_4 + 2p_2p_3)(m/2) + c.$$

$$EV(c/4) = (p_1^2)(m/2) + (1 - p_1)(2p_1p_2)(m/2) + (1 - p_1 - p_2)(p_2^2 + 2p_1p_3)(m/2)$$
$$+ (1 - p_1 - p_2 - p_3)(2p_1p_4 + 2p_2p_3)(m/2)$$
$$+ (1 - p_1 - p_2 - p_3 - p_4)(p_3^2 + 2p_1p_5 + 2p_1p_2)(m/2) + 3c/4.$$

$$EV(c/2) = (p_1^2)(m/2) + (2p_1p_2)(m/2) + (1 - p_1)(p_2^2 + 2p_1p_3)(m/2)$$
$$+ (1 - p_1 - p_2)(2p_1p_4 + 2p_2p_3)(m/2) + (1 - p_1 - p_2 - p_3)(p_3^2 + 2p_1p_5$$
$$+ 2p_1p_2)(m/2) + (1 - p_1 - p_2 - p_3 - p_4)(2p_2p_5 + 2p_3p_4)(m/2) + c/2.$$

$$EV(3c/4) = (p_1^2)(m/2) + (2p_1p_2)(m/2) + (p_2^2 + 2p_1p_3)(m/2)$$
$$+ (1 - p_1)(2p_1p_4 + 2p_2p_3)(m/2) + (1 - p_1 - p_2)(p_3^2 + 2p_1p_5 + 2p_1p_2)(m/2)$$
$$+ (1 - p_1 - p_2 - p_3)(2p_2p_5 + 2p_3p_4)(m/2) + (1 - p_1 - p_2 - p_3 - p_4)(p_4^2 + 2p_3p_5)(m/2) + c/4.$$

$$EV(c) = (p_1^2)(m/2) + (2p_1p_2)(m/2) + (p_2^2 + 2p_1p_3)(m/2) + (2p_1p_4 + 2p_2p_3)(m/2)$$
$$+ (1 - p_1)(p_3^2 + 2p_1p_5 + 2p_1p_2)(m/2) + (1 - p_1 - p_2)(2p_2p_5 + 2p_3p_4)(m/2)$$
$$+ p_1 - p_2 - p_3)(p_4^2 + 2p_3p_5)(m/2) + (1 - p_1 - p_2 - p_3 - p_4)(2p_4p_5)(m/2).$$

The following system of system of five equations provides the equilibrium solution (namely, p_1, p_2, p_3, p_4, and p_5):

$$EV(c/4) - EV(0) = 0,$$
$$EV(c/2) - EV(c/4) = 0,$$
$$EV(3c/4) - EV(c/2) = 0,$$
$$EV(c) - EV(3c/4) = 0,$$
$$p_1 + p_2 + p_3 + p_4 + p_5 = 1.$$

Appendix 2: Instructions for Subjects and Figures on Empirical Distribution

***Section 2.1a*: Same-Function Alliance with equal profit-sharing arrangement**

You will participate today in a decision making experiment concerning competition between two alliances (groups of firms). Each alliance is comprised of two partners.

We are interested in studying how two alliances compete with each other in the development of a new product. We are simulating this common situation in the laboratory. You will represent a firm, which is a member of one of the competing alliances. Three other subjects will represent the other three firms (the other member of your alliance and the two firms in the competing alliance).

The experiment involves many trials, and all of them have the same structure. At the beginning of each trial, you will be provided with some investment capital and then asked how much of it you wish to invest in the new product development project. The other three firms (the other member of your alliance and the two firms in the competing alliance) will also be provided the same amount of investment capital and asked to make similar investment decisions

The rules of this investment game are simple. The capital invested by each firm in the development of the new product is non-recoverable. Therefore once invested the money is lost irrespective of the outcome of the competition. The alliance that invests more capital in the product development research will succeed in developing the new product, and each member of the winning alliance will receive a fixed reward. The reward does not depend on the relative investments made by each member of the winning alliance, so both the members of the alliance will receive the same fixed reward. The fixed reward represents profit that each member of the successful alliance earns from marketing the new product. Each member of the losing alliance receives nothing.

Experimental Procedure: As discussed above, there are 2 groups of firms (alliances), and each group is comprised of two players (members). At the beginning of each trial each player will be given some investment capital. All the players will receive the same investment capital and it remains unchanged from trial to trial. The investment capital will be stated in terms of a fictitious currency called "francs", and at the end of the experiment your earnings will be converted to US dollars.

Once each player is allotted (endowed) some investment capital, he/she must decide how much to invest in his/her group's new product development research. You may invest any number of francs (including zero), provided your investment does not exceed your endowment (investment capital allotted for the trial). After all the four players have made their investment decisions, privately and anonymously, the computer will compare the total investments made by the two groups of players. Members of the group that invests the larger amount will succeed in developing the new technology product, and they will receive a reward of known size (in francs). Members of the losing group will receive nothing. In this game ties will be counted as loses, as no alliance can be considered a winner. In other words, if the investments made by both groups are equal, then no reward will be given to any of the four players.

As you can see from the game description, the individual payoffs for a trial are computed as follows:

Payoff to a member of the winning group = endowment for the trial - investment made by the firm in the trial + reward

Payoff to a member of the losing group = endowment for the trial - investment made by the firm in the trial

At the end of each trial the computer will display the following information:

1) The total investments made by the winning and the losing groups

2) The group winning the competition,

3) Your payoff for the trial.

It is important to note that only you know your investment decisions, and you are taking these decisions under complete anonymity. Group membership will vary from trial to trial. On each trial you will be paired with a different person in this room, and both of you will compete as a group against another new group of two players.

We are providing below an example to help you understand how your payoff is computed at the end of each trial.

Example: Suppose the capital endowed to each subject at the beginning of a trial is 2 francs, and the reward for winning the competition is 3 francs to each member of the successful group. Also suppose that you invest 2 francs and your partner invests 1 franc in the new product development research. Let the other group of players make a total investment of 2 francs in the development of their new product. Your group has invested more for developing the new product, so your group wins the competition. Each member of your group gets a reward of 3 francs.

Your payoff in this trial will be:

Your payoff = endowment - your investment + reward = 2 - 2 + 3 = 3 francs.

Your partner's payoff = endowment - your partner's investment + reward

= 2 - 1 + 3 = 4 francs.

Now imagine that the other group of players, who are competing against your group, invest 3 francs as well. In this case there is a tie and the reward will not be awarded to members of either groups. Your payoff in case of a tie will be as follows:

Your payoff = endowment - your investment + reward = 2 - 2 + 0 = 0 francs.

Your partner's payoff = endowment - your partner's investment + reward

= 2 - 1 + 0 = 1 francs.

This concludes the description of the decision task ahead of you. Paper and pencil are placed beside the computer terminal so that you may record the investments made by your group and the other group. At the end of the experiment, your accumulated payoff will be converted to US dollars at the conversion rate of 100 francs = 3 dollars. You will be asked to sign a receipt for the money, and complete a brief questionnaire before leaving the lab. We are required to retain some biographical information about you, as we are paying you for participating in this experiment. However, during the course of this experiment you will remain anonymous. If you have any questions, please raise your hand and the supervisor will assist you.

After all the participants have understood the instructions, we will start the computerized experiment. In order to help you become familiar with the decision task, you will go through five practice trials.

Section 2.1b: Same-function alliance with proportional profit sharing

You will participate today in a decision making experiment concerning competition between two alliances (groups of firms). Each alliance is comprised of two partners.

We are interested in studying how two alliances compete with each other in the development of a new product. We are simulating this common situation in the laboratory. You will represent a firm, which is a member of one of the competing alliances. Three other subjects will represent the other three firms (the other member of your alliance and the two firms in the competing alliance).

The experiment involves many trials, and all of them have the same structure. At the beginning of each trial, you will be provided with some investment capital and then asked how much of it you wish to invest in the new product development project. The other three firms (the other member of your alliance and the two firms in the competing alliance) will also be provided the same amount of investment capital and asked to make similar investment decisions

The rules of this investment game are simple. The capital invested by each firm in the development of the new product is non-recoverable. Therefore, once invested the money is lost irrespective of the outcome of the competition. The alliance that invests more capital in the product development research will succeed in developing the new product, and the winning alliance will receive a fixed reward. The fixed reward represents the profit that the successful alliance earns from marketing the new product. The members of the winning alliance will share the reward in *proportion* to their investments in the development of the new product. So the member of the winning alliance who invests more will get a larger share of the fixed reward. Each member of the losing alliance receives nothing.

Experimental Procedure: As discussed above, there are 2 groups of firms (alliances), and each group is comprised of two players (members). At the beginning of each trial each player will be given some investment capital. All the players will receive the same investment capital and it remains unchanged from trial to trial. The investment capital will be stated in terms of a fictitious currency called "francs", and at the end of the experiment your earnings will be converted to US dollars.

Once each player is allotted (endowed) some investment capital, he/she must decide how much to invest in his/her group's new product development research. You may invest any number of francs (including zero), provided your investment does not exceed your endowment (investment capital allotted for the trial). After all the four players have made their investment decisions, privately and anonymously, the computer will compare the total investments made by the two groups of players. The group that invests the larger amount will succeed in developing the new technology product, and it will receive a reward of known size (in francs). The members of the winning group will share the reward in proportion to their relative investments. The losing group will receive nothing. In the case of ties, both groups will receive nothing.

As you can see from the game description, the individual payoffs for a trial are computed as follows:

Payoff to a member of the winning group = endowment for the trial
- investment made by the firm in the trial
+ reward (investment made by the firm in the trial / total investment of the winning alliance)

Payoff to a member of the losing group = endowment for the trial
- investment made by the firm in the trial

At the end of each trial the computer will display the following information:

1) The total investments made by the winning and the losing groups

2) The group winning the competition,

3) Your payoff for the trial.

It is important to note that only you know your investment decisions, and you are taking these decisions under complete anonymity. Group membership will vary from trial to trial. On each trial you will be paired with a different person in this room, and both of you will compete as a group against another new group of two players.

We are providing below an example to help you understand how your payoff is computed at the end of each trial.

Example: Suppose the capital endowed to each subject at the beginning of a trial is 2 francs, and the reward for winning the competition is 6 francs. Also now suppose that you invest 2 francs and your partner invests 1 franc in the new product development research. Let the other group of players make a total investment of 2 francs in the development of their new product. Your group has invested more for developing the new product, so your group wins the competition and your group gets the reward of 6 francs. The payoff for the members of your group in this trial is as follows:

Your payoff = your endowment for the trial
- your investment in the trial
+ reward (your investment / total investment of your alliance)
= 2 - 2 + 6 (2/3)
= 4 francs.

Your partner's payoff = your partner's endowment for the trial
- your partner's investment in the trial
+ reward (your partner's investment / total investment of your alliance)
= 2 - 1 + 6 (1/3)
= 3 francs.

Now imagine that the other group of players, who are competing against your group, invest 3 francs as well. In this case there is a tie and both the groups do not get any reward. The payoff in the case of a tie will be as follows:

Your payoff = your endowment - your investment + reward = 2 - 2 + 0 = 0 francs.

Your partner's payoff = your partner's endowment - your partner's investment + reward
= 2 - 1 + 0 = 1 francs.

This concludes the description of the decision task ahead of you. Paper and pencil are placed beside the computer terminal so that you may record the investments made by your group and the other group. At the end of the experiment, your accumulated payoff will be converted to US dollars at the conversion rate of 100 francs = 3 dollars. You will be asked to sign a receipt for the money, and complete a brief questionnaire before leaving the lab. We are required to retain some biographical information about you, as we are paying you for participating in this experiment. However, during the course of this experiment you will remain anonymous. If you have any questions, please raise your hand and the supervisor will assist you.

After all the participants have understood the instructions, we will start the computerized experiment. In order to help you become familiar with the decision task, you will go through five practice trials.

***Section 2.1c*: Parallel development of new products with equal profit-sharing arrangement**

You will participate today in a decision making experiment concerning competition between two alliances (groups of firms). Each alliance is comprised of two partners.

We are interested in studying how two alliances compete with each other in the development of a new product. We are simulating this common situation in the laboratory. You will represent a firm, which is a member of one of the competing alliances. Three other subjects will represent the other three firms (the other member of your alliance and the two firms in the competing alliance).

The experiment involves many trials, and all of them have the same structure. At the beginning of each trial, you will be provided with some investment capital and then asked how much of it you wish to invest in the new product development project. The other three firms (the other member of your alliance and the two firms in the competing alliance) will also be provided the same amount of investment capital and asked to make similar investment decisions

The rules of this investment game are simple. The capital invested by each firm in the development of the new product is non-recoverable. Therefore once invested the money is lost irrespective of the outcome of the competition. The utility (value) of the new product developed by an alliance depends on the maximum investment made by a partner in the alliance. For example, if the partners A and B in an alliance invest 2 and 1 units of resources respectively for developing the new product then the utility of the new product so developed will be 2 utils. The alliance offering the better product will win the competition, and each member of the winning alliance will receive a fixed reward. The reward does not depend on the relative investments made by each member of the winning alliance, so both the members of the alliance will receive the same fixed reward. The fixed reward represents profit that each member of the successful alliance earns from marketing the new product. Each member of the losing alliance receives nothing.

Experimental Procedure: As discussed above, there are 2 groups of firms (alliances), and each group is comprised of two players (members). At the beginning of each trial each player will be given some investment capital. All the players will receive the same investment capital and it remains unchanged from trial to trial. The investment capital will be stated in terms of a fictitious currency called "francs", and at the end of the experiment your earnings will be converted to US dollars.

Once each player is allotted (endowed) some investment capital, he/she must decide how much to invest in his/her group's new product development research. You may invest any number of francs (including zero), provided your investment does not exceed your endowment (investment capital allotted for the trial). After all the four players have made their investment decisions, privately and anonymously, the computer will compute the value of the new product developed by each of the two competing alliances. The utility (value) of the new product developed by an alliance depends on the maximum investment made by a partner in the alliance. The alliance offering a better product will win the competition. Members of the winning alliance will receive a reward of known size (in francs). Members of the losing group will receive nothing. In this game ties will be counted as loses, as no alliance can be considered a winner. In other words, if the maximum investments made by both groups are equal, then no reward will be given to any of the four players.

As you can see from the game description, the individual payoffs for a trial are computed as follows:

Payoff to a member of the winning group = endowment for the trial - investment made by the firm in the trial + reward

Payoff to a member of the losing group = endowment for the trial - investment made by the firm in the trial

At the end of each trial the computer will display the following information:

1) The total investments made by the winning and the losing groups

2) The group winning the competition,

3) Your payoff for the trial.

It is important to note that only you know your investment decisions , and you are taking these decisions under complete anonymity. Group membership will vary from trial to trial. On each trial you will be paired with a different person in this room, and both of you will compete as a group against another new group of two players.

We are providing below an example to help you understand how your payoff is computed at the end of each trial.

Example: Suppose the capital endowed to each subject at the beginning of a trial is 2 francs, and the reward for winning the competition is 3 francs to each member of the successful group. Also suppose that you invest 2 francs and your partner invests 1 franc in the new product development research. The maximum investment made by your group is 2 francs, and the value of the new product developed by your group is 2 utils. Let each player in the competing alliance make an investment of 1 franc each. The competing alliance has made a maximum investment of 1 franc, and the value of the new product developed by the competing alliance is 1 util. So your group has developed a better and it wins the competition. Each member of your group gets a reward of 3 francs each.

Your payoff in this trial will be:

Your payoff = endowment - your investment + reward = 2 - 2 + 3 = 3 francs.

Your partner's payoff = endowment - your partner's investment + reward

= 2 - 1 + 3 = 4 francs

Now imagine that the two players in the competing alliance invest 2 francs each. So the value of the new product so developed by the competing alliance is 2 utils. In this case there is a tie and the reward will not be awarded to members of either groups. Your payoff in case of a tie will be as follows:

Your payoff = endowment - your investment + reward = 2 - 2 + 0 = 0 francs.

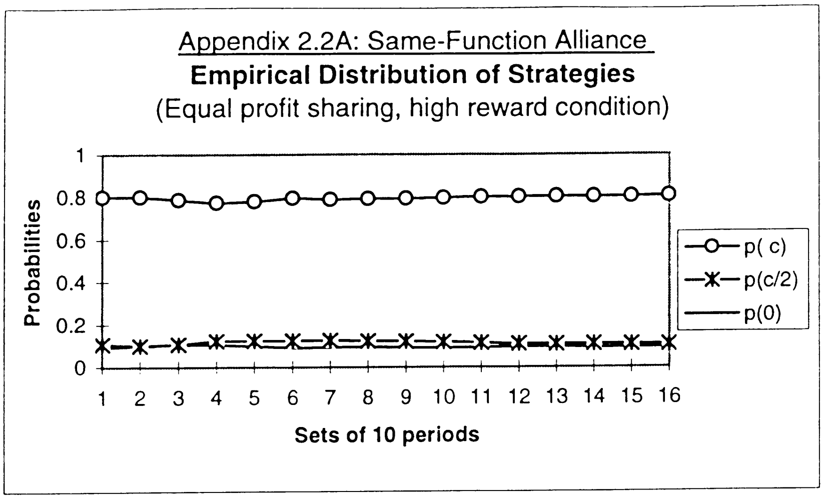
Your partner's payoff = endowment - your partner's investment + reward

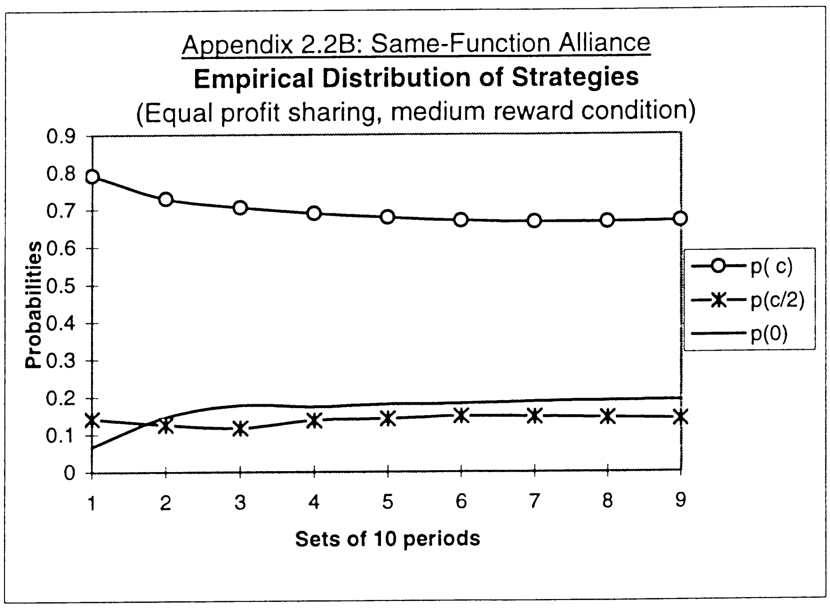
= 2 - 1 + 0 = 1 franc.

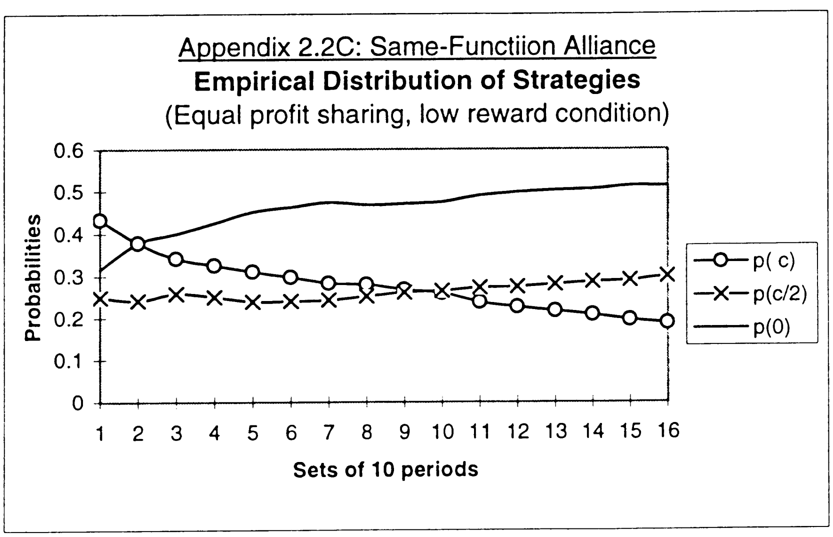
This concludes the description of the decision task ahead of you. Paper and pencil are placed beside the computer terminal so that you may record the investments made by your group and the other group. At the end of the experiment, your accumulated payoff will be converted to US dollars at the conversion rate of 100 francs = 3 dollars. You will be asked to sign a receipt for the money, and complete a brief questionnaire before leaving the lab. We are required to retain some biographical information about you, as we are paying you for participating in this experiment. However, during the course of this experiment you will remain anonymous. If you have any questions, please raise your hand and the supervisor will assist you.

After all the participants have understood the instructions, we will start the computerized experiment. In order to help you become familiar with the decision task, you will go through five practice trials.

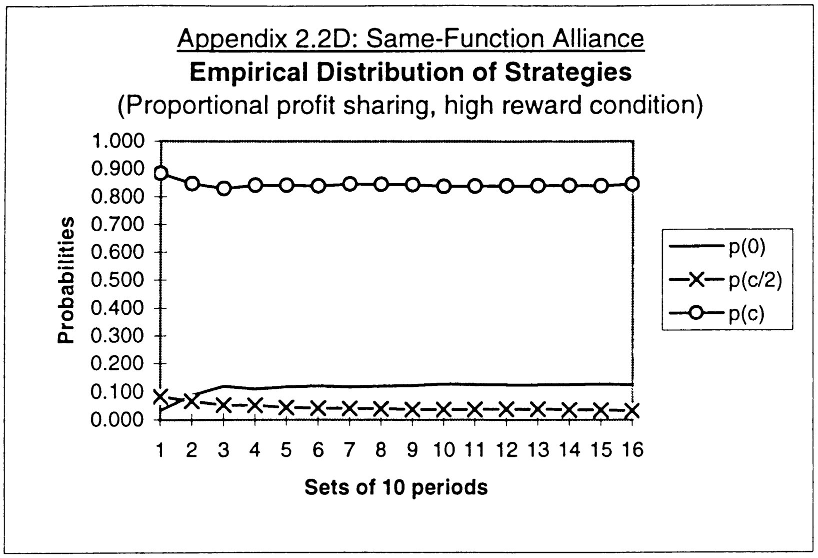
Section 2.2 Trends in the aggregate investment pattern of subjects is presented in the following pages.

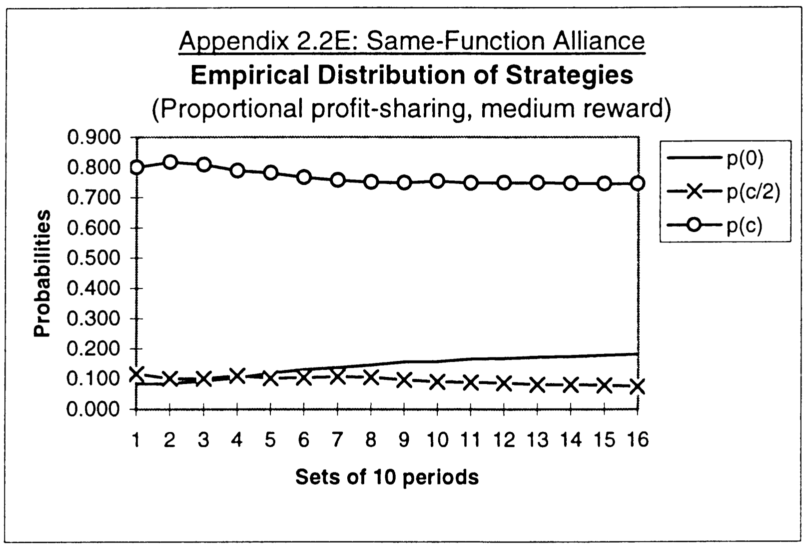


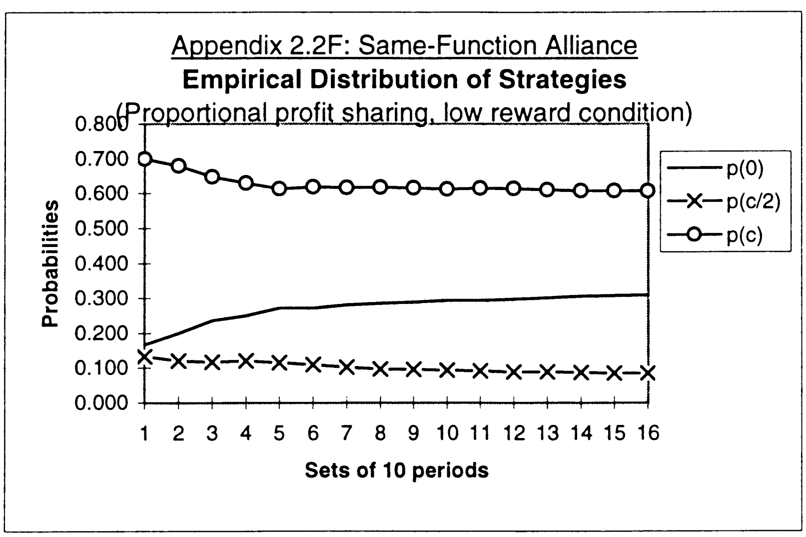




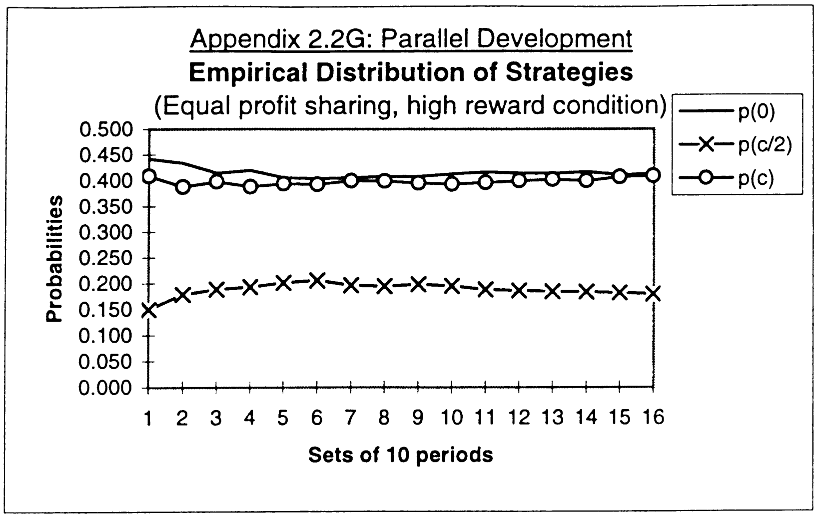
Note: The empirical distribution was computed across subjects in blocks of 10 trials

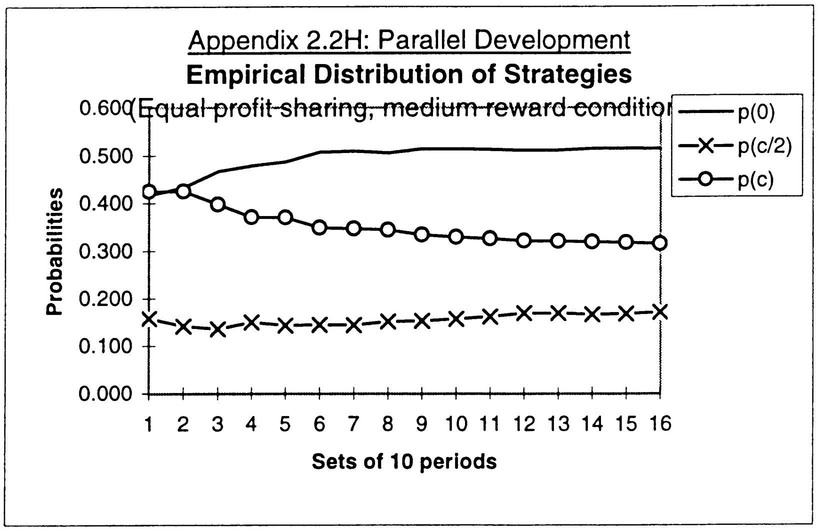


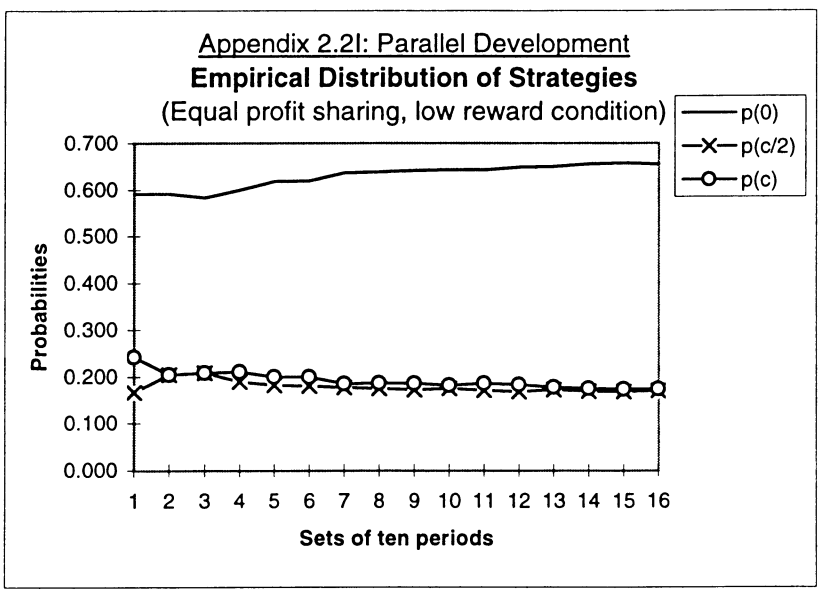




Note: The empirical distribution was computed across subjects in blocks of 10 trials







Note: The empirical distribution was computed across subjects in blocks of 10 trials

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