1) **Constant returns to scale** means that if we increase all the factor inputs by the same percentage, then output increases by exactly the same percentage.

1. a) (1 point) Suppose we double both capital stock and labor input: \(K'=2K\) and \(L'=2L\). Then, \(Y' = F(K',L') = F(2K, 2L) = A(2(2K) + 3(2L)) = A(2(2K+3L)) = 2A(2K+3L) = 2F(K, L) = 2Y\). Clearly, doubling inputs doubles output. Therefore, the production function in this question exhibits constant returns to scale.

1. b) (1 point) Suppose we double both capital stock and labor input: \(K'=2K\) and \(L'=2L\). Then, \(Y' = F(2K, 2L) = A(2K)(2L) = 4AKL = 4F(K, L) = 4Y\). It is obvious that when we double inputs, we get four times of output. This result implies that the production function in this question does not exhibit constant returns to scale.

2) 2. a) (1 point) From 2005 to 2011, the real GDP has grown to $13 trillion but the nominal GDP has grown to $14 trillion. The $1 billion difference between the nominal GDP and the real GDP is clearly from inflation or an increase in the price level. We can calculate the price level in 2011 \((P_{2011})\) from the definition of GDP deflator:

\[
P_{2011} = \text{GDP deflator} = \frac{\text{Nominal GDP}}{\text{Real GDP}} \times 100
\]

\[
= \frac{14}{13} \times 100
\]

\[
= 107.69
\]

(1)

Notice that the GDP deflator of the base year is always 100. Thus, the GDP inflator has increased by 7.69\% \((= (107.69-100)/100)\) from 2005 to 2011. In other words, the inflation rate over 6 years is 7.69\%. To find the average rate of inflation per annum, we assume that the rate of inflation is the same every year. By definition of inflation rate (\(\pi\)), inflation increases the price level in period \(t\) \((P_t)\):

\[
\pi = \frac{P_t}{P_{t-1}} - 1
\]

(2)

Thus,

\[
P_t = (1+\pi) P_{t-1}.
\]

(3)

In other words, the GDP inflator in this period is the product of gross inflation and the GDP deflator in the last period. We are interested in period \(t = 2006, \ldots, 2011\). Hence,

\[
P_{2011} = (1+\pi) P_{2010},
\]

(4)

\[
P_{2010} = (1+\pi) P_{2009},
\]

(5)

\[
P_{2009} = (1+\pi) P_{2008},
\]

(6)

\[
P_{2008} = (1+\pi) P_{2007},
\]

(7)

\[
P_{2007} = (1+\pi) P_{2006},
\]

(8)

\[
P_{2006} = (1+\pi) P_{2005}.
\]

(9)

We can substitute (9) into (8), (8) into (7), \ldots, (5) into (4) to relate \(P_{2011}\) and \(P_{2005}\):

\[
P_{2011} = (1+\pi)^6 P_{2005}
\]

(10)
Substitute (1) and $P_{2005}=100$ into (10):

$$107.69 = (1+\pi)^6100$$

From (11),

$$1 + \pi = 1.0769^{1/6} = 1.012428.$$

Therefore, $\pi = 0.012428 = 1.2428\%$. This is the inflation rate per annum.

2.b) (1 point)

The income paid to labor in real terms, or real labor income, is the real wage times the quantity of labor. We also know that the real wage equals the marginal product of labor, $w=MPL$. Thus, real labor income = $wL = MPLxL$. Also, $MPL = \Delta Y/\Delta L = \Delta( AK^{0.3}L^{0.7})/\Delta L = (0.7AK^{0.3}L^{0.7})/L = 0.7Y/L$. Hence, real labor income = $(0.7Y/L)L = 0.7Y = 0.7\times$13 trillion = $9.1$ trillion.

Similarly, the income paid to owners of capital in real terms, known as real capital income, is the real rental price of capital times the quantity of capital, $r_cK$. The real rental price of capital equals the marginal product of capital, $r_c = MPK$. Also, $MPK = \Delta Y/\Delta K = \Delta( AK^{0.3}L^{0.7})/\Delta K = (0.3AK^{0.3}L^{0.7})/K = 0.3Y/K$. Hence, real capital income = $(0.3Y/K)K = 0.3Y = 0.3\times$13 trillion = $3.9$ trillion.

Alternatively, we can solve this question easily by noticing that, for the Cobb-Douglas production function the share of each factor in national income is simply the power of each factor in the production function. Thus, the real labor income = $0.7Y = 0.7\times$13 trillion = $9.1$ trillion. Likewise, the real capital income = $0.3Y = 0.3\times$13 trillion = $3.9$ trillion.

3) 3.a) (1 point)

Suppose initially Mexico runs a trade surplus. Since trade balance is the same as saving and investment gap, then $NX1 = S_1 - I_1 > 0$ or $S_1 > I_1$. This implies that initially, national saving is larger than domestic investment. The initial equilibrium is illustrated by 2 points in Figure 1. The first point is the intersection of the world real interest rate $r_w$ and the national saving curve $S$. The other point is the intersection between the world
real interest rate $r_w$ and the domestic investment curve $I$.

If the government reduces the consumption tax, there will be an incentive for more consumption. More consumption means less saving, so this policy shifts the national saving curve to the left. This has the following effects on Mexico’s capital market:

(i) Given the old world real interest rate and old investment curve, investment remains the same: $I_2 = I_1$

(ii) Given the old real interest rate and new saving curve, national saving decreases: $S_2 < S_1$

(iii) Since trade balance is the gap between saving and investment, the decrease of saving and unchanged investment result in a deterioration of the trade balance or a fall in the net exports. In equilibrium, the trade balance is equal to the net capital outflows. For this reason, the net capital outflows must fall too. The new level of net exports or net capital outflows is measured by the distance between $I_2$ and $S_2$ in Figure 1.

3.b) (1 point)

No, the answer does not depend on whether Mexico initially has a trade deficit or a trade surplus. We have already illustrated the situation where Mexico initially has a trade surplus. Let us look at the other situation. Suppose that initially Mexico runs a trade deficit. Since trade balance is the same as saving and investment gap, then $NX_1 = S_1 - I_1 < 0$, so $S_1 < I_1$ as in Figure 2.

![Figure 2](image)

If the government reduces the consumption tax, there will be an incentive for more consumption. More consumption means less saving, so it shifts the national saving curve to the left. This has the same effects on Mexico’s capital market as in part (a). Since trade balance is the gap between saving and investment, the decrease of saving and unchanged investment result in a larger deficit. In other word, there will be a fall in net exports. In equilibrium, the scale of net exports is equal to the next capital outflows. For this reason, the scale of net capital outflows falls too. The new level of net exports or net capital outflows is measured by the distance between $I_2$ and $S_2$ in Figure 2. Graphically, this distance is longer than the distance between $I_1$ and $S_1$. However, the distance has a negative sign, thus a longer distance implies a larger negative value, which represent a
fall of a negative number. For example, this distance may fall from -100 to -120, i.e. a larger trade deficit.

4.

4.a) (1 point)
Japan is one of the large economies that drive the world real interest rate. For this reason, the world real interest rate must change. We display the effect of a decrease in Japan’s saving on the world real interest rate in Figure 3. In Figure 3, the world capital market as a whole is a closed economy, thus the world capital market equilibrium is the intersection between the world saving $Sw$ and the world investment $Iw$. A decrease in Japan’s saving decreases the world saving and thus $Sw$ shifts to the left, from $Sw$ to $Sw1$. As a result, the world real interest rate rises.

![Figure 3](image)

4.b) (1 point)
There are two shifts in Japan’s capital market. The national saving curve will shift to the left due to the decrease in Japan’s national saving. The other shift is the upward shift in the world interest rate. The effects on Japan’s national saving, investment and net capital outflows are as follows. (Figure 4)

- The rise in the world real interest rate decreases the domestic investment.
- Despite the rise in the world real interest rate, the domestic saving will still decrease as it is the original.
- The net export will decrease, if the decrease in national saving is larger than the decrease in investment. (In theory, it is possible that investment is so elastic that it falls more than national saving. But empirically this does not happen.) The decrease in net export implies a decrease of net capital outflows from Japan.
4.c) (1 point)
We can look at the right panel of Figure 4. On the right side, it shows the effects on the ROW. The rise in the world real interest rate increases saving and decreases investment from the ROW. Hence, the scale of capital inflows to the ROW decreases, as measured by the distance NX2ROW. Note that the scale of capital inflows to the ROW is the scale of capital outflows from Japan in part (b). Therefore, we can now conclude that the scale of capital outflows from Japan must fall too. This is something we cannot conclude if we look at only Japan’s capital market without taking into account the general equilibrium effect on the ROW’s capital market, as in part (b).

4.d) (1 point)
The US is one of the countries in the ROW. For this reason, its capital inflows will decrease as in part (c). To avoid a decrease in capital inflows, the Congress can implement policies which aim to increase national saving or decrease domestic investment, in order to reduce the real world interest rate from \( r_w2 \) to the original level or \( r_w1 \). For example, the Congress can raise the consumption tax to encourage saving, or it can raise the tax on equipments to discourage investment. Both policies have a negative impact on the world real interest rate.