Econ 371 Fall 2010

Answer Key for Problem Set 3 (Chapters 17)

Instructor: Kanda Naknoi

November 8, 2010

ANSWER 1.a (1 point)

There is a one-time drop in output \((Q)\) from 50 to 39 in period 0. Let the new consumption be \(C_0\). Since the consumer desires to smooth consumption, the consumption is \(C_0\) for all periods \(t>0\).

The present value of consumption must be the same as the present value of output:

\[
PV(C) = PV(Q) \tag{1}
\]

\[
C_0 + C_0/1.1 + C_0/1.1^2 + C_0/1.1^3 + \ldots = 39 + 50/1.1 + 50/1.1^2 + 50/1.1^3 + \ldots
\]

\[
C_0(1.1)/0.1 = 39-50 + (50 + 50/1.1 + 50/1.1^2 + 50/1.1^3 + \ldots)
\]

\[
C_0(11) = -11 + 50(1.1)/0.1
\]

\[
C_0 = -1 + 50
\]

\[
C_0 = 49
\]

ANSWER 1.b (1 point)

(i) \(TB(t) = Q(t) - C(t)\)

For \(t = 0\), \(TB(0) = 39 - 49 = -10\)

For \(t > 0\), \(TB(t) = 50 - 49 = 1\)

(ii) \(NFIA(t) = r W(t-1)\), where \(CA(t) = TB(t) + NFIA(t)\) and \(W(t) = W(t-1) + CA(t)\).

For \(t = 0\), \(NFIA(0) = 0.1(0) = 0\). Thus, \(CA(0) = -10 + 0 = -10\), and \(W(0) = -10\).

For \(t = 1\), \(NFIA(1) = 0.1(-10) = -1\). Thus, \(CA(1) = -1 + 0 = -1\), and \(W(1) = -10 - 0 = -10\).

For \(t > 1\), \(NFIA(t) = NFIA(1) = -1\), \(CA(t) = CA(1) = 0\) and \(W(t) = W(1) = -10\).

Note that for \(t > 0\), \(NFIA(t) = -TB(t)\). In other words, trade surplus from period 1 is used to finance interest payments to the creditor. For this reason, current account is balanced from period 1. We summarize the path of these variables as follows.

| Time | 0  | 1  | 2  | 3  | ...
|------|----|----|----|----|----
| Q    | 39 | 50 | 50 | 50 | ...
| C    | 49 | 49 | 49 | 49 | ...
| TB = Q-C | -10 | 1 | 1 | 1 | ...
| NFIA = 0.1 W | 0 | -1 | -1 | -1 | ...
| CA = TB + NFIA | -10 | 0 | 0 | 0 | ...
| W(t) = W(t-1) + CA(t) | -10 | -10 | -10 | -10 | ...
ANSWER 2.a (1 point)

The criterion for investing in a project is that:

\[
P(V\text{Cost of investment}) < P(V\text{Return on investment}) \quad (2)
\]

Since there is no new investment from Year 1, \(P(V\text{Cost of Investment}) = \$40\) billion. The present value of return on investment now depends on the rate of return on investment and the world interest rate. Every period this project pays 6% of \(\$40\) billion = 0.06(40) = \$2.4 billion. Hence, \(P(V\text{Return on Investment}) = 0 + 2.4/1.04 + 2.4/1.04^2 + \ldots = 2.4/0.04 = \$60\) billion > \(P(V\text{cost of investment})\). As a result, this investment project should be undertaken.

Note that an alternative way to answer this question is to compare the interest rate on borrowing and the return on investment. The interest rate on borrowing is 4%, and lower than the 6% return on investment. Hence, this project should be undertaken because it pays higher return than the cost of borrowing.

ANSWER 2.b (1 point)

The home country can borrow as much as it desires. Thus, the home residents can smooth consumption and finance its investment from borrowing. They choose constant consumption path \(C_0\) such that: \(P(VQ) = P(VC) + P(VI)\), where \(Q = 100\) in period 0 and \(Q(t) = 102.4\) for \(t>0\), and \(I = 40\) in Period 0 and \(I(t) = 0\) for \(t>0\). Substitute the paths of \(Q\) and \(I\) into the lifetime budget constraint:

\[
P(VQ) = P(VC) + P(VI)
\]

\[
100 + 102.4/1.04 + 102.4/1.04^2 + \ldots = (C_0 + C_0/1.04 + C_0/1.04^2 + \ldots) + 40
\]

\[
100 - 102.4 + (102.4 + 102.4/1.04 + 102.4/1.04^2 + \ldots) = C_0(1.04)/0.04 + 40
\]

\[
-2.4 + 102.4(1.04)/0.04 = 26C_0 + 40
\]

\[
C_0 = 102.4 - (40+2.4)/26
\]

\[
C_0 = 100.77
\]

ANSWER 2.c (1 point)

(i) \(T(B(t)) = Q(t) - C(t) - I(t)\)

For \(t = 0\), \(T(B(0)) = 100 - 100.77 - 40 = -40.77\)

For \(t > 0\), \(T(B(t)) = 102.4 - 100.77 - 0 = 1.63\)

(ii) \(NFIA(t) = r W(t-1)\) where \(CA(t) = TB(t) + NFIA(t)\) and \(W(t) = W(t-1) + CA(t)\).

For \(t = 0\), \(NFIA(0) = 0.04(0) = 0\). Thus, \(CA(0) = -40.77 + 0 = -40.77\) and \(W(0) = 0 - 40.77 = -40.77\).

For \(t = 1\), \(NFIA(1) = 0.04(-40.77) = -1.63\). Thus, \(CA(1) = 1.63 - 1.63 = 0\), and \(W(1) = -40.77 - 0 = -40.77\).

For \(t > 1\), \(NFIA(t) = NFIA(1) = -1.63\), \(CA(t) = CA(1) = 0\) and \(W(t) = W(1) = -40.77\).
Note that for $t > 0$, $\text{NFIA}(t) = -\text{TB}(t)$. In other words, trade surplus from period 1 is used to finance interest payments to the creditor. For this reason, current account is balanced from period 1.

We summarize the path of these variables as follows.

<table>
<thead>
<tr>
<th>Time</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q</td>
<td>100</td>
<td>102.40</td>
<td>102.40</td>
<td>102.40</td>
<td>...</td>
</tr>
<tr>
<td>I</td>
<td>40</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>...</td>
</tr>
<tr>
<td>C</td>
<td>100.77</td>
<td>100.77</td>
<td>100.77</td>
<td>100.77</td>
<td>...</td>
</tr>
<tr>
<td>$\text{TB} = Q - I - C$</td>
<td>-40.77</td>
<td>1.63</td>
<td>1.63</td>
<td>1.63</td>
<td>...</td>
</tr>
<tr>
<td>$\text{NFIA} = -0.04 W$</td>
<td>0</td>
<td>-1.63</td>
<td>-1.63</td>
<td>-1.63</td>
<td>...</td>
</tr>
<tr>
<td>$\text{CA} = \text{TB} + \text{NFIA}$</td>
<td>-40.77</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>...</td>
</tr>
<tr>
<td>$W(t) = W(t-1) + \text{CA}(t)$</td>
<td>-40.77</td>
<td>-40.77</td>
<td>-40.77</td>
<td>-40.77</td>
<td>...</td>
</tr>
</tbody>
</table>

In period 0, the consumers borrow to finance both investment and an increase in consumption. This is because they foresee “permanent increases” in their output and lifetime income. Thus, they can afford to increase consumption even before the investment begins to pay off. This is possible because the home country has more advanced technology than the rest of the world.

**ANSWER 3.a (1 point)**

Same as ANSWER 2.a

**ANSWER 3.b-3.c (2 points)**

In this case, the home country faces the borrowing constraint that limits the borrowing to $35 billion. Hence, the investment project must be partially financed by the domestic fund for $5 billion. The only way to do this is to cut consumption in Year 0 by $5 billion. Thus, the static budget constraint binds in Year 0. From Year 1, the static budget constraint also binds because the home residents do not need to borrow any more. The home residents use income to consume and to service debt. Thus, from Year 1 the home residents will run trade surplus only to service the interest rate payment $= 0.04(35) = $1.4 billion, and consume the rest of output.

<table>
<thead>
<tr>
<th>Time</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q</td>
<td>100</td>
<td>102.40</td>
<td>102.40</td>
<td>102.40</td>
<td>...</td>
</tr>
<tr>
<td>I with foreign fund</td>
<td>35</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>...</td>
</tr>
<tr>
<td>I with home fund</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>...</td>
</tr>
<tr>
<td>$C = Q - I - \text{TB}$</td>
<td>95</td>
<td>101</td>
<td>101</td>
<td>101</td>
<td>...</td>
</tr>
<tr>
<td>$\text{TB} =$ borrowing limit, $t = 0$</td>
<td>35</td>
<td>1.4</td>
<td>1.4</td>
<td>1.4</td>
<td>...</td>
</tr>
<tr>
<td>$\text{NFIA} =$ -0.04(35), $t &gt; 0$</td>
<td>0</td>
<td>-1.4</td>
<td>-1.4</td>
<td>-1.4</td>
<td>...</td>
</tr>
<tr>
<td>$\text{CA} =$ $\text{TB} + \text{NFIA}$</td>
<td>-35</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>...</td>
</tr>
<tr>
<td>$W(t) = W(t-1) + \text{CA}(t)$</td>
<td>-35</td>
<td>-35</td>
<td>-35</td>
<td>-35</td>
<td>...</td>
</tr>
</tbody>
</table>
ANSWER 4.a (1 point)

National income is 100% distributed to capital income, and labor income is 0. Payoffs for each state from firms in each country are given by Case 1 as follows.

Case 1: No foreign direct investment

<table>
<thead>
<tr>
<th>State</th>
<th>Home income</th>
<th>Foreign income</th>
<th>World income</th>
</tr>
</thead>
<tbody>
<tr>
<td>QK</td>
<td>QL</td>
<td>Q</td>
<td>QK* QL* Q</td>
</tr>
<tr>
<td>1</td>
<td>100</td>
<td>0</td>
<td>110</td>
</tr>
<tr>
<td>2</td>
<td>110</td>
<td>0</td>
<td>110</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>State</th>
<th>Home income</th>
<th>Foreign income</th>
<th>World income</th>
</tr>
</thead>
<tbody>
<tr>
<td>QK</td>
<td>QL</td>
<td>Q</td>
<td>QK* QL* Q</td>
</tr>
<tr>
<td>1</td>
<td>105</td>
<td>0</td>
<td>105</td>
</tr>
<tr>
<td>2</td>
<td>105</td>
<td>0</td>
<td>105</td>
</tr>
</tbody>
</table>

Case 2: Now suppose that 50% of shares of domestic firms are owned by foreign residents.

<table>
<thead>
<tr>
<th>State</th>
<th>Home income</th>
<th>Foreign income</th>
<th>World income</th>
</tr>
</thead>
<tbody>
<tr>
<td>QK</td>
<td>QL</td>
<td>Q</td>
<td>QK* QL* Q</td>
</tr>
<tr>
<td>1</td>
<td>105</td>
<td>0</td>
<td>105</td>
</tr>
<tr>
<td>2</td>
<td>105</td>
<td>0</td>
<td>105</td>
</tr>
</tbody>
</table>

In case 2, difference of national income across states is 105-105 = 0 for both countries.

Case 3: Now suppose that 100% of shares of domestic firms are owned by foreign residents.

<table>
<thead>
<tr>
<th>State</th>
<th>Home income</th>
<th>Foreign income</th>
<th>World income</th>
</tr>
</thead>
<tbody>
<tr>
<td>QK</td>
<td>QL</td>
<td>Q</td>
<td>QK* QL* Q</td>
</tr>
<tr>
<td>1</td>
<td>110</td>
<td>0</td>
<td>110</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>0</td>
<td>110</td>
</tr>
</tbody>
</table>

In Case 3, difference of national income across states is 110-100 = 10 for both countries.

The portfolio in Case 2 dominates that in Case 3, because the portfolio in Case 2 has removed differences of national income across states for both countries. Case 2 is the best for the purpose of risk diversification in a sense that it guarantees 105 of income regardless of the state.

ANSWER 4.b (1 point)

When capital income is 40% of national income, we get the same payoff structure as the example in the handout: http://www.krannert.purdue.edu/faculty/knaknoi/Econ371/handout_ch17.pdf.

The difference of income across states is 6 in Case 2, and 2 in Case 3. Hence, the portfolio in Case 3 dominates that in Case 2 for the purpose of risk diversification.