• **Consumption smoothing**

Idea: Budget constraint is defined over lifetime in the presence of financial globalization. Thus, total expenditure in each period can deviate from income in each period. But over lifetime expenditure cannot exceed income:

\[
\text{Lifetime income} = \text{Lifetime expenditure} \]

\[
\text{PV (income)} = \text{PV (expenditure)} \]

\[
\text{PV(Q)} = \text{PV(C)} + \text{PV(I)}
\]

\[
Q_0 + \frac{Q_1}{(1+r)} + \frac{Q_2}{(1+r)^2} + \cdots = \left( C_0 + \frac{C_1}{(1+r)} + \frac{C_2}{(1+r)^2} + \cdots \right) + \left( I_0 + \frac{I_1}{(1+r)} + \frac{I_2}{(1+r)^2} + \cdots \right)
\]

These conditions characterize the lifetime budget constraint or long-run budget constraint (LRBC). They are sometimes called no-Ponzi-scheme condition.

• **Gains from consumption smoothing in an open economy**

Assume for now that investment is zero. To understand the benefits of financial globalization, consider the following 3 cases.

**Case 1: Closed economy**

Financial globalization is absent. Thus, budget constraint binds in every period.

\( Q_t = 100 \Rightarrow C_t = Q_t = 100 \)

**Case 2: Closed economy with a temporary fall in output**

\( Q_0 = 79, Q_t = 100 \) for \( t > 0 \)

Again, \( C_t = Q_t \) for all \( t \).

**Case 3: Open economy with a temporary fall in output**
Q0 = 79, Qt = 100 for t>0

To find C, use the LBC condition: PV(Q) = PV(C). Let the world interest rate is 5%.

\[ Q0 + \frac{Q1}{(1 + r)} + \frac{Q2}{(1 + r)^2} + \ldots = \left( C0 + \frac{C1}{(1 + r)} + \frac{C2}{(1 + r)^2} + \ldots \right) \]

With consumption smoothing, the consumer adjusts the consumption only once: \( Ct = C0 \) for all \( t \).

\[ 79 + \frac{100}{(1.05)} + \frac{100}{(1.05)^2} + \ldots = \left( C0 + \frac{C0}{(1.05)} + \frac{C0}{(1.05)^2} + \ldots \right) \]

We can add -100 + 100 to the left hand side of the above equation.

\[ (79 - 100) + \left( 100 + \frac{100}{(1.05)} + \frac{100}{(1.05)^2} + \ldots \right) = \left( C0 + \frac{C0}{(1.05)} + \frac{C0}{(1.05)^2} + \ldots \right) \]

\[ -21 + 100 \left( \frac{1.05}{0.05} \right) = C0 \left( \frac{1.05}{0.05} \right) \]

\[ C0 = 2079 \left( \frac{0.05}{1.05} \right) = 99 \]

Trade balance: \( TBt = Qt - Ct \)

<table>
<thead>
<tr>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( TB0 = Q0 - C0 = 79-79 = 0 )</td>
<td>( 79-99 = -20 )</td>
</tr>
<tr>
<td>( TB1 = Q1 - C1 = 100-100 = 0 )</td>
<td>( 100-99 = 1 )</td>
</tr>
<tr>
<td>( TBs = TB1, for s &gt; 1 )</td>
<td>( TBs = TB1, for s &gt; 1 )</td>
</tr>
</tbody>
</table>

Since \( TBt = Qt - Ct \), \( PV(TB) = PV(Q) - PV(C) = 0 \).

To verify, substitute the numerical example above to calculate \( PV(TB) \). Alternatively, we show that present value of debt burden is exactly the same as present value of debt payment.

\[ PV(\text{debt burden}) = PV(\text{debt payment}) \]

\[ 20 = \frac{1}{1.05} + \frac{1}{1.05^2} + \frac{1}{1.05^3} + \ldots \]

\[ = \frac{1}{0.05} \]

\[ = 20 \]

What happens to current account and external wealth in Case 3?

\( CA = TB + NFIA \)
Where, NFIA = Net factor income flows from abroad. We assume no labor mobility across border, thus NFIA is net capital income paid to external assets or net capital income paid to external debts.

Suppose that this country is neither a debtor nor a creditor in Period -1, i.e. $W(-1) = 0$.

\[
CA_0 = TB_0 + NFIA_0 = -20 + 0 = -20 \quad \Rightarrow \quad W_0 = W(-1) + CA_0 = 0 - 20 = -20
\]

\[
CA_1 = TB_1 + NFIA_1 = TB_1 + r(W_0) = 1 + 0.05(-20) = 0 \quad \Rightarrow \quad W_1 = W_0 + CA_1 = -20 + 0 = -20
\]

$CA_s = CA_1$, for $s > 1$. As a result, $W_s = W_1$ for $s > 1$.

Note that in this case both $TB < 0$ and $CA < 0$ when income falls. Thus, the theory suggests a positive correlation between $TB$ and income. This correlation contradicts the empirical evidence. The correlation between $TB$ and income in the short run is negative in the data, and this finding is quite robust to sample periods and sample countries.

- **Gain from efficient investment in an open economy**

Consider an investment project which costs 16 units of consumption today. This project will pay 5 units of consumption from the next period forever. Assume that in the absence of investment income is 100. Should the entrepreneur fund this project?

\[
PV(I) = 16
\]

\[
PV(Investment \ return) = 0 + 5/1.05 + 5/1.05^2 + 5/1.05^3 + \ldots = 1/1.05 \ PV(5) = 5/0.05 = 100
\]

Thus, $PV(Investment \ return) > PV(I)$. This project is profitable and should be funded. We can use LBC condition to solve for the path of consumption.

\[
Q_0 + \frac{Q_1}{1+r} + \frac{Q_2}{(1+r)^2} + \ldots = \left(C_0 + \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + \ldots\right) + \left(I_0 + \frac{I_1}{1+r} + \frac{I_2}{(1+r)^2} + \ldots\right)
\]

\[
100 + \frac{105}{1.05} + \frac{105}{(1.05)^2} + \ldots = \left(C_0 + \frac{C_0}{1.05} + \frac{C_0}{(1.05)^2} + \ldots\right) + \left(16 + \frac{0}{1.05} + \frac{0}{(1.05)^2} + \ldots\right)
\]

\[
100 - 105 + \left(105 + \frac{105}{1.05} + \frac{105}{(1.05)^2} + \ldots\right) = C_0\left(\frac{1.05}{0.05}\right) + 16
\]

\[
-5 + 2205 = C_0\left(\frac{1.05}{0.05}\right) + 16
\]

\[
C_0 = 2184\left(\frac{0.05}{1.05}\right) = 104
\]

1. What happens to trade balance?

\[
TB_0 = Q_0 - C_0 - I_0 = 100 - 104 - 16 = -20 < 0
\]
TB1 = Q1 – C1 – I1 = 105 – 104 -0 = 1 > 0

TBs = TB1, for s > 1.

In other words, the short run TB deficit is financed by TB surplus in the future. TB deficit is associated with subsequent periods of output expansion. In contrast, the previous example without investment predicts that TB deficit arises when output falls. In the data, we usually observe TB deficit when output expands. This implies that it is very critical to study investment in a model of consumption smoothing.

2. The paths of CA and W in this case are the same as Case 3 in the previous section.

3. What is the rate of return to capital? This is theoretically the same as the marginal product of capital (MPK). It can be calculated as 5/16 = 0.3125 = 31.25% per annum. This is much higher than the world interest rate 5%. It makes sense that capital flows into the U.S. thanks to such larger return differentials.

- Gains from diversification of risk

Assume Q and Q* are perfectly negatively correlated. Suppose income is 60% from labor income (QL) and 40% from capital income (QK):

\[ Q = QK + QL \]

**Case 1: No foreign direct investment: Domestic residents own 100% of shares of domestic firms.**

<table>
<thead>
<tr>
<th>State</th>
<th>Home income</th>
<th>Foreign income</th>
<th>World income</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>QK  QL  Q</td>
<td>QK*  QL*  Q*</td>
<td>QK+QK*  QL+QL*  Q+Q*</td>
</tr>
<tr>
<td>1</td>
<td>40  60 100</td>
<td>44  66  110</td>
<td>84  126 210</td>
</tr>
<tr>
<td>2</td>
<td>44  66 110</td>
<td>40  60  100</td>
<td>84  126 210</td>
</tr>
</tbody>
</table>

*Reason: QK in State 1 = 42 = 0.5(40) + 0.5(44), where 40 and 44 are capital income from production in each country in State 1 from Case 1.*

**Case 2: 50% home portfolio and 50% foreign portfolio**

<table>
<thead>
<tr>
<th>State</th>
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<th>Foreign income</th>
<th>World income</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>QK  QL  Q</td>
<td>QK*  QL*  Q*</td>
<td>QK+QK*  QL+QL*  Q+Q*</td>
</tr>
<tr>
<td>1</td>
<td>42  60 102</td>
<td>42  66  108</td>
<td>84  126 210</td>
</tr>
<tr>
<td>2</td>
<td>42  66 108</td>
<td>42  60  102</td>
<td>84  126 210</td>
</tr>
</tbody>
</table>

*Reason: QK in State 2 = 42 = 0.5(44) + 0.5(40), where 44 and 40 are capital income from production in each country in State 2 from Case 1.*
Volatility of income (= difference of Q across states) in Case 2 (102 vs. 108) is lower than that in Case 1 (100 vs. 110) for both countries.

What happens to trade balance?

State 1: TB = Q-C = Original Q in Case 1 – Q in Case 2 = 100 - 102 = -2

State 1: NFIA = Net investment income transfer = 0.5(44) - 0.5(40) = 2.

State 1: CA = TB + Net investment income transfer = -2+2 = 0!
Investment income finances shortfall in income.

We can show similar numbers for State 2 and we will find that CA = 0 for State 2 too.

Case 3: 100% foreign portfolio

<table>
<thead>
<tr>
<th>State</th>
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<th>Foreign income</th>
<th>World income</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>QK  QL  Q</td>
<td>QK* QL* Q*</td>
<td>QK+QK* QL+QL* Q+Q*</td>
</tr>
<tr>
<td>1</td>
<td>44 60 104</td>
<td>40 66 106</td>
<td>84 126 210</td>
</tr>
<tr>
<td>2</td>
<td>40 66 106</td>
<td>44 60 104</td>
<td>84 126 210</td>
</tr>
</tbody>
</table>

(Reason: QK in State 1 = 44 = 0(40) + 1(44), where 40 and 44 are capital income from production in each country in State 1 from Case 1.)

(Reason: QK in State 2 = 40 = 0(44) + 1(40), where 44 and 40 are capital income from production in each country in State 2 from Case 1.)

Volatility of Q (= difference in Q across states) in case 3 is even lower (104 vs. 106) than that in Case 2 (102 vs. 108), but Q in the good state and Q in the bad state will never be the same, i.e. risk sharing is not perfect. Reason: we cannot diversify labor income risks.