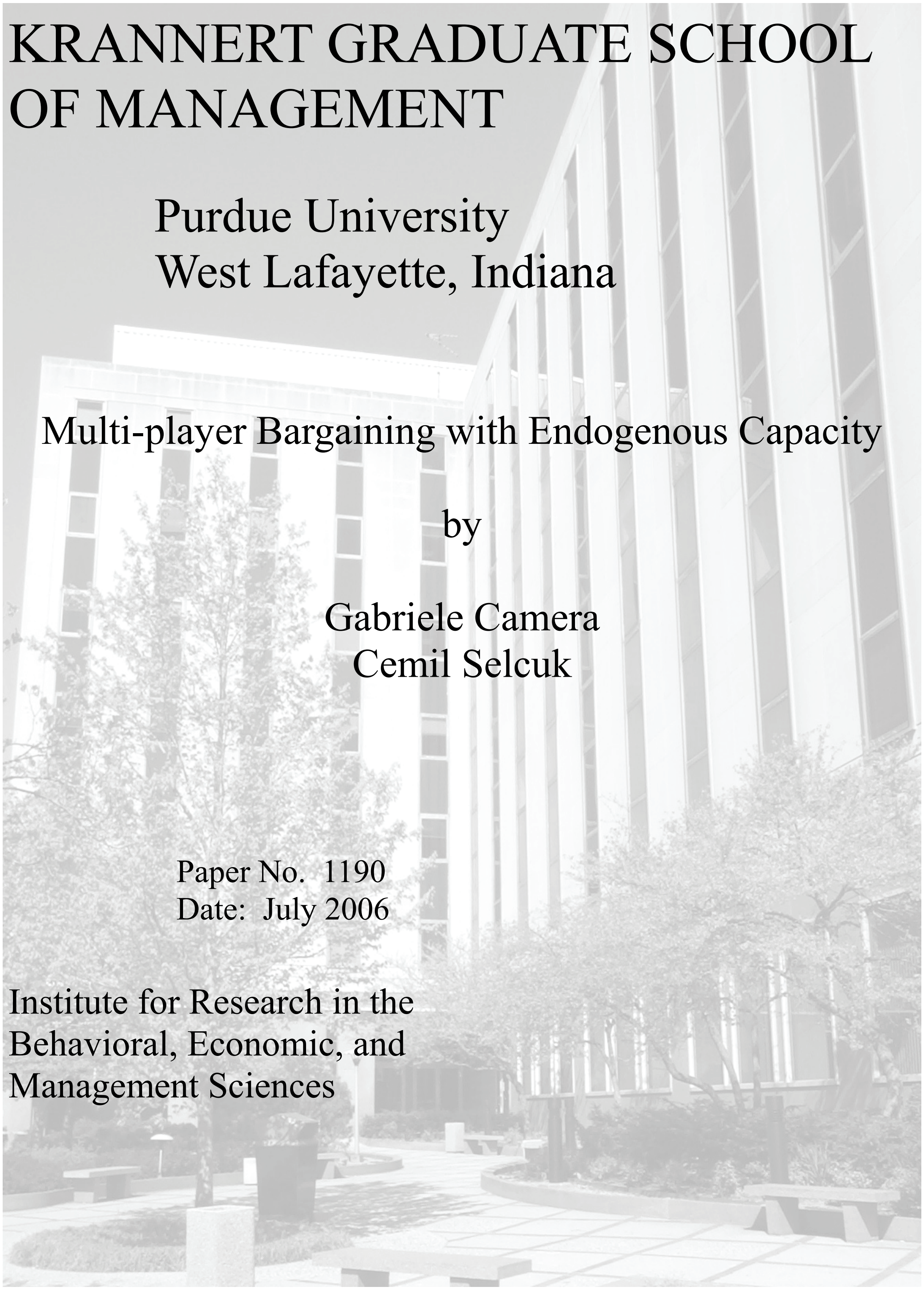

KRANNERT GRADUATE SCHOOL OF MANAGEMENT

Purdue University
West Lafayette, Indiana

Multi-player Bargaining with Endogenous Capacity

by

Gabriele Camera
Cemil Selcuk

Paper No. 1190
Date: July 2006

Institute for Research in the
Behavioral, Economic, and
Management Sciences

Multi-player Bargaining with Endogenous Capacity[1]

Gabriele Camera
Purdue University

Cemil Selcuk
Purdue University

July 12, 2006

Abstract

We study equilibrium prices and trade volume in a market with n identical buyers and a seller who initially commits to some capacity. Sales are sequential and each price is determined by strategic bargaining. A unique subgame perfect equilibrium exists. It is characterized by absence of costly bargaining delays and each trade is settled at a different price. Prices increase with n and fall in the seller's capacity, so if buyers have significant bargaining power, then the seller will strategically constrain capacity to less than n. Thus, despite the efficiency of the bargaining solution, certain distributions of bargaining powers give rise to an allocative inefficiency.

Keywords: Commitment, Inefficiency, Peripheral players, Price heterogeneity, Strategic bargaining
JEL: C78, D20

[1] We thank seminar participants at Purdue, Vanderbilt, the RMM 2005 conference in Toronto, the Midwest Economic Theory Meetings of the Fall 2005 and Spring 2005. This research was partly supported by the NSF grant DMS-0437210.

1 Introduction

This paper studies short-run equilibrium in a market where several buyers desire to trade with a seller who can costlessly produce a fixed number of homogeneous goods. Our main objective is to determine equilibrium volume of trade and sale price(s), when the seller initially commits to a productive capacity and subsequently negotiates over the terms of trade. We also wish to demonstrate how in such situations allocative inefficiency may emerge even if bargaining inefficiencies, externalities or hold-up problems are not present.

Though the model can be applied to a variety of economic situations, the application that we have in mind is that of a labor market. This is a natural setting because in the short run the number of vacancies created by firms is fixed and each firm generally deals with multiple job applicants through a process that often involves wage negotiations. In this context, short-run simply means a situation in which workers cannot apply to another firm, so they are temporarily 'locked-in,' and on the other hand the firm cannot vary the number of vacancies originally advertised. Several interesting questions present themselves in such a setting. Can the firm pay lower wages, and increase profits, by restricting vacancies? Would this lead to bargaining inefficiencies? Are all workers hired at the same wage?

To provide an answer we develop a strategic bargaining game between a central player (a seller or a firm) and n peripheral players (customers or workers) each of whom desires a single indivisible object (a good or a job). The game is of complete information and is sequential, with two stages. In the first stage the seller commits to supply at most $c \leq n$ objects, a choice that we call "capacity." Capacity is costlessly created (so holdup problems are excluded) and production is also costless. In the second stage a discrete-time alternating offers game takes place, in the tradition of [13]. The key features are that customers are served sequentially, selling one good a time, players are randomly selected to make or respond to offers, and there is a cost to bargaining due to discounting in the form of a random stopping rule.

Two main results emerge from the analysis. First, for any given choice of capacity c, the bargaining game has a unique subgame perfect equilibrium. Bargaining equilibrium is characterized by absence of delays, due to discounting. Also, sale prices are decreasing functions of capacity, due to the random offer process. Intuitively, buyers who hesitate to purchase may be excluded from future rounds of negotiations. It follows that if $c < n$, then buyers face consumption risk because there is excess demand. The higher is capacity, the lower is this risk, and so the lower is the customers' reservation price. This leads to a second finding.

Endogenous capacity constraints arise in equilibrium if the seller has a sufficiently limited bargaining power. This means that allocative inefficiency can result even if the bargaining solution is efficient and there are no hold-up problems from investing in capacity. The reason is simple. By restricting capacity below n the seller can raise buyers' reservation prices by forcing them to compete for scarce goods. This bargaining tactic improves the firm's ability to obtain more favorable terms of trade in each transaction, but it is costly in terms of lower trade volume. Hence, equilibrium capacity constraints arise only if the seller is a sufficiently poor negotiator.

Interestingly, there are never bargaining delays, even with capacity constraints, so equilibrium inefficiency is simply tied to the deadweight loss from the firm's choice to supply less than what is demanded.

In the paper we also characterize the 'dynamics' of the terms of trade, i.e., how equilibrium sale prices vary as a function of the remaining inventory. Discounting is a source of sale price heterogeneity but not the only one. Without excess demand, $c = n$, equilibrium prices fall in the order of sale because customers who buy leave the store, which decreases competition among the remaining customers. Hence, prices monotonically fall in the order of sale and early buyers pay a premium simply due to discounting. In fact, this feature changes if there is excess demand, $c < n$. As items are sold, consumption risk increases for the remaining buyers, and this dominates the effect of discounting when many buyers chase few goods. Indeed, we find that with moderate excess demand sale prices are U-shaped in the order of sale, while if excess demand is extreme, then sale prices monotonically rise.

These results complement a literature on bargaining under complete information. In particular, our study extends models in which a central player bargains with many others (e.g., see [2], [8], [9] or [15]), by adding an initial stage of capacity choice. We also contribute to broaden previous studies of bargaining tactics that can be used, prior to negotiations, to strengthen a player's bargaining position.[2] We have found that the ability to restrict capacity, before negotiations start, can be effective in raising the seller's payoff only if his 'bargaining skills' are sufficiently poor. Finally, our study offers an interesting reassessment of the link between bargaining and allocative efficiency. Generally, if an equilibrium without bargaining delays exists, then it is also efficient. Our model, too, generates no equilibrium delays but inefficiency may nevertheless arise if the seller chooses to constrain capacity. A central message is that, though the distribution of bargaining power does not impact bargaining efficiency, it can affect allocative efficiency (see also [3]).

In terms of its applicability, our bargaining model can be readily adopted to determine short-run prices in search and matching frameworks of labor and goods markets. In a labor market context, for instance, in the short-run firms and job seekers are often assumed to be locked-in, in the sense that additional vacancies cannot be created and additional firms cannot be readily contacted (e.g., [1]). Our framework can be used to determine wages at each firm, based on match-specific factors such as the ratio of applicants to available vacancies. This may help explain some of the residual wage heterogeneity that is observed in the data (e.g. see [4])

[2] Examples are found in [7], [11] and [12]. In particular, [7] study the optimal choice of debt by a firm prior to wage negotiations with workers. Choosing a higher debt before bargaining may be advantageous to the firm, as the size of the cake shrinks. The firm employs this tactic when it is not a skilled negotiator. The studies in [11] and [12] consider a two-player one-cake bargaining problem in which both players, before negotiations, make partial commitments (that can be revoked later at some cost) to some share of the cake which they would like to get. In that setting players with higher concession costs obtain a higher share of the pie; thus taking actions –prior to bargaining– that increase the concession cost may improve a player's payoff. The study in [5] obtains a similar qualitative result in a different setting. Other related models are [6] and [10].

The study is organized as follows. Section 2 describes the economic environment and studies the bargaining game. Section 3 studies the choice of capacity. Finally, Section 4 concludes.

2 Model and equilibrium concept

We study a game of complete information between a seller and $n \geq 1$ identical buyers. These buyers are present at the store (or matched to the seller) and each of them desires to consume a single good. There are two stages. In the first stage the seller can costlessly choose capacity $c = 1, ..., n$. This allows the seller to produce, at no cost, up to c units of an indivisible homogeneous good. Alternatively, we can consider c as the initial inventory (costlessly) selected by the seller. In the second stage, goods are offered for sale one at a time, and trading takes place by means of a bargaining mechanism described below. Consumption utility is one for buyers and zero for the seller and since utility is transferable there are gains from trade.

We say that a seller is capacity constrained when $c < n$ and unconstrained otherwise. Also, we define the excess demand to be $n - c$. We adopt subgame perfection as the equilibrium concept, moving backward in our analysis. That is, in the next three subsections we fix c in order to study the bargaining outcome in the second stage of the game. Then, in the remaining section, we move on to study the first stage of the game and the optimal choice c.

2.1 The bargaining game

Consider the second stage of the game, i.e., the bargaining game. Suppose the seller has $c = 1, ..., n$ indivisible goods available. Every player observes c and n and then a trading process starts, which is based on a noncooperative sequential bargaining game of complete information, in the tradition of [13]. Negotiations take place in rounds indexed $t = 1, 2, ...$ In each round t the seller bargains over the sale of a single good as follows. First, a random selection device chooses a buyer with equal probability among all n present. Second, either the seller or the buyer are randomly selected to propose an offer $q \in [0, 1]$. It is assumed that with probability $\gamma \in (0, 1)$ the proposer is the seller and the responder is the buyer, and the converse occurs with probability $1 - \gamma$. We interpret γ as a parameter representing the seller's negotiation skill and denote the elements of the responder's action set by 'accept' or 'reject.'

There is disagreement if the responder rejects the offer q. In that case, the seller keeps the good, all players earn zero utility for the round and remain matched to the seller. If there is agreement, instead, trade occurs so the buyer earns utility $1 - q$ and leaves the store, while the seller earns utility q. The remaining $n - 1$ buyers receive zero utility and remain at the store. At the end of round t, if the seller has no more goods to offer then the game stops, otherwise the game continues with probability $\beta \in (0, 1)$. This random element, which we will refer to as trading risk, makes bargaining delays costly to both buyers and seller. Of course, we can interpret β simply as a discount factor, without loss in generality. As the game progresses to a new round, the seller's initial capacity c falls at most by one unit. Since goods are homogeneous, without loss in generality we let $i = 1, ..., c$ denote the good offered for sale in round $t \geq i$.

Realized payoffs are as follows. Let $s = 0, 1, ..., c$ denote the number of goods sold by the end of the game. If $s = 0$, then every player realizes zero payoff. If $s > 1$ then players that do not trade realize zero payoff. To discuss the payoffs of those who successfully traded assume the game stops at the end of round $t \geq s$. Let q_i denote the sale price of a good when the seller has $c - i + 1$ goods for sale, in round $t \geq i$, with $i = 1, ..., s$. Then $\sum_{i=1}^{s} q_i$ is the seller's realized payoff and $1 - q_i$ is the payoff realized by the buyer of the i^{th} good.

2.2 Bargaining: the main result

Fix $c = 1, ..., n$. We start this section by reporting the main result for the bargaining game, which consists of a full characterization of the subgame perfect equilibrium (SPE) offers and realized payoffs.

Theorem 1 *The bargaining game between $n \geq 1$ buyers and a seller with $c = 1, ..., n$ goods has a unique subgame perfect equilibrium that is characterized as follows. The seller always offers to trade good i at price $q_i^s = q_i(c, n)$ with*

$$q_i(c, n) = 1 - \tfrac{\beta - \alpha}{n - i + 1} \textstyle\sum_{j=i}^{c} \beta^{j-i} \displaystyle\prod_{m=i}^{j} \tfrac{n-m+1}{n-m+1-\alpha} \tag{1}$$

and accepts any offer $q \geq \alpha q_i(c, n)$, where

$$\alpha = \tfrac{\beta\gamma}{\beta\gamma + 1 - \beta}. \tag{2}$$

Each buyer always offers to purchase good i at price

$$q_i^b = \alpha q_i(c, n)$$

and accepts any offer $q \leq q_i(c, n)$.

The theorem tells us that in equilibrium there are no bargaining delays and a good is sold in each bargaining round, until the game stops randomly or the capacity is exhausted. In what follows we prove this theorem via several steps in the form of lemmas. The layout is the following. We will start by conjecturing the existence of a subgame perfect equilibrium that satisfies some basic properties, namely, offers are stationary and are accepted without delay. Then we will calculate such offers and verify that they are indeed subgame perfect. Finally we show that our conjecture is the unique SPE by demonstrating that all SPE of this game satisfy the properties above.

To start, consider an equilibrium characterized by two properties. First, there is no delay, i.e., in equilibrium any offer is accepted in the same round in which it is made. Second, equilibrium offers are stationary, i.e., player types do not modify their offers for a good that went unsold in the previous round.

Suppose the game has reached some round $t \geq i$ and that the seller is offering the i^{th} good. Denote by A_i the set of buyers who are matched to the seller and desire to purchase the good. We have $|A_i| = n - i + 1$ since in previous rounds $i - 1$ buyers have traded with the seller, consumer and the left. Given stationarity, let $q^s_{k,i}(c,n)$ denote the equilibrium offer of the seller to buyer $k \in A_i$ and let $q^b_{k,i}(c,n)$ denote the equilibrium offer of buyer k to the seller, given initial demand n and capacity c. Throughout the paper, we will omit the arguments c and n, when they are understood.

Given no delay in accepting offers, let $\pi_i(c,n)$ denote the seller's expected earnings from bargaining over the i^{th} good, given initial demand n and capacity c. It is defined as

$$\pi_i(c,n) = \sum_{k\in A_i} \frac{\gamma q^s_{k,i} + (1-\gamma) q^b_{k,i}}{n-i+1}.$$

With probability γ the seller gets to make the offer and buyer $k \in A_i$ is selected to receive it with probability $\frac{1}{n-i+1}$. When the offer is accepted without delay, the seller enjoys $q^s_{k,i}$ utility and the buyer $1-q^s_{k,i}$. Similarly with probability $\frac{1-\gamma}{n-i+1}$ some buyer $k \in A_i$ gets to make an offer, in which case the seller's utility is $q^b_{k,i}$ and the buyer is $1-q^b_{k,i}$.

The seller's payoff in the bargaining game is simply the expected utility from selling at most c goods. When there are no delay, we denote it by $\pi(c,n)$ with

$$\pi(c,n) = \sum_{i=1}^{c} \beta^{i-1} \pi_i,$$

and note that the probability of continuation of the game, β, acts as a discount factor.

Now consider buyer k. Let $u_{k,i}(c,n)$ denote his expected utility at the start of some trading round $t \geq i$ in which good i is offered for sale, given initial demand n and capacity c. Considerations similar to those made above imply that when offers are immediately accepted we have

$$u_{k,i}(c,n) = \gamma \frac{1-q^s_{k,i}}{n-i+1} + (1-\gamma)\frac{1-q^b_{k,i}}{n-i+1} + \frac{n-i}{n-i+1}\beta u_{k,i+1}(c,n).$$

The first two terms on the right hand side refer to the contingency in which buyer k is selected to currently receive or make an offer. The third term represents a continuation payoff. Due to random selection, $\frac{n-i}{n-i+1}$ is the probability that the buyer is excluded from this bargaining round. This generates **trading risk** because, even if the seller has some remaining capacity, good $i+1$ will be offered for sale only with probability $\beta < 1$. The notation $u_{k,i+1} \geq 0$ denotes the expected utility from continuing the game, with $u_{k,c+1}(c,n) = u_{c+1}(c,n) = 0$ for all $k \in A_c$ (and note that $A_c \neq \varnothing$ since $c \leq n$). The buyer's payoff in the bargaining game is therefore $u_{k,1}$, which can be obtained by backward iteration.

We are now ready to discuss best responses. Players choose offers on $[0,1]$ to maximize their payoffs and clearly have an incentive to reach agreement as quickly as possible. Indeed, suppose that round $t = i$ results in disagreement between the seller and a buyer $k \in A_i$. Given stationarity and absence of future delays, the seller's continuation payoff is $\beta\pi_i$ and the buyer's is $\beta u_{k,i}$. Therefore any player accepts an offer that is above or his continuation payoff, is indifferent if the offer corresponds to his continuation payoff, and rejects it, otherwise.

Now observe that $u_{k,i}$ is linearly decreasing in $q^b_{k,i}$ and π_i is linearly increasing in q^s_i. Therefore, an offer is individually optimal only if it gives the opponent exactly his continuation payoff, i.e. if it leaves him indifferent to rejecting the offer. It follows that for each good i and each buyer k the expressions

$$q^b_{k,i} = \beta\pi_i \tag{3}$$

$$1 - q^s_{k,i} = \beta u_{k,i}, \tag{4}$$

identify the best responses of buyer k and of the seller.

A first result is that, given no delay and stationarity, the seller's makes identical offers to any buyer k and every buyer k makes the same offer to the seller.

Lemma 2 *Equilibrium offers must be symmetric. That is,* $q^b_{k,i} = q^b_i$ *and* $q^s_{k,i} = q^s_i$ *for each* $i = 1, ..., c$ *and all* $k \in A_i$*. In particular,*

$$q^b_i = \alpha q^s_i \tag{5}$$

$$q^s_i = \tfrac{n-i+1-\beta}{n-i+1-\alpha} - \tfrac{\beta^2(n-i)}{n-i+1-\alpha} u_{i+1} \tag{6}$$

where α *is as in (2). It follows that in equilibrium:*

$$\pi_i = \tfrac{\alpha}{\beta} q^s_i \tag{7}$$

$$u_i = \tfrac{1-\frac{\alpha}{\beta}q^s_i}{n-i+1} + \tfrac{\beta(n-i)}{n-i+1} u_{i+1}. \tag{8}$$

Proof. Consider bargaining over good $i = 1, ..., c$. The right hand side of (3) is not a function of k and so $q^b_{k,i} = q^b_i$ for all $k \in A_i$. This result jointly with the definition of $u_{k,i}$ and (4) imply:

$$q^s_{k,i} = 1 - \tfrac{\beta(1-\gamma)(1-q^b_i)}{n-i+1-\beta\gamma} - \tfrac{\beta^2(n-i)}{n-i+1-\beta\gamma} u_{k,i+1} \tag{9}$$

Use backward induction on i. Start with $i = c$ in which case $u_{k,c+1} = u_{c+1} = 0$ by definition. Thus, we have $q^s_{k,c} = q^s_c = 1 - \frac{\beta(1-\gamma)(1-q^b_i)}{n-i+1-\beta\gamma}$ for all k. For the induction step, suppose $q^s_{k,i+1} = q^s_{i+1}$ for some $i < c - 1$. Then, (4) implies $u_{k,i+1} = u_{i+1}$ for all $k \in A_{i+1}$. Therefore, using (9) we have $q^s_{i,k} = q^s_i$ for all $k \in A_i$.

Having established that offers are symmetric, we have $\pi_i = \gamma q^s_i + (1 - \gamma) q^b_i$. Thus, we can use (3) to obtain (5) and (2). From (5) and symmetry, expression (9) gives us (6). Finally, use (5)-(6) and the definitions of π_i and $u_{k,i}$ to obtain (7)-(8).■

This lemma establishes that, when making offers, the seller treats each buyer identically and vice-versa. Buyers offer a fraction α of what the seller would offer. This fraction is constant across bargaining rounds, it does not depend on i, and grows with γ and β.

We can now obtain an expression for the buyer's payoff and the equilibrium offer a function of parameters. Of course, this generates expressions for π_i and q^b_i as functions of parameters.

Lemma 3 *In equilibrium we have*

$$u_i(c, n) = \tfrac{\Phi_i(c,n)}{n-i+1} \tag{10}$$

and $q_i^s(c,n) = q_i(c,n)$ *with*

$$q_i\left(c,n\right) = 1 - \tfrac{\beta\Phi_i(c,n)}{n-i+1} \tag{11}$$

$$\Phi_i\left(c,n\right) = \tfrac{\beta-\alpha}{\beta}\sum_{j=i}^{c}\beta^{j-i}\prod_{m=i}^{j}\tfrac{n-m+1}{n-m+1-\alpha} \tag{12}$$

for all $i = 1,...,c$. *In particular,* $q_i\left(c,n\right) \in (0,1)$ *for all* i.

Proof. Start by defining

$$\Phi_i\left(c,n\right) = \sum_{j=i}^{c}\beta^{j-i}\left[1 - \tfrac{\alpha}{\beta}q_j^s\right] \tag{13}$$

Clearly, we have

$$\begin{aligned}\Phi_i(c,n) &= 1 - \tfrac{\alpha}{\beta}q_i^s + \beta\sum_{j=i+1}^{c}\beta^{j-(i+1)}\left[1 - \tfrac{\alpha}{\beta}q_j^s\right] \\ &= 1 - \tfrac{\alpha}{\beta}q_i^s + \beta\Phi_{i+1}(c,n).\end{aligned} \tag{14}$$

We will omit the arguments, when understood.

From (7) recall that $\frac{\alpha}{\beta}q_j^s$ is the seller's equilibrium expected surplus in round $j > i$ of bargaining. Thus $1-\frac{\alpha}{\beta}q_i^s$ is the expected surplus to the buyer of good i and Φ_{i+1} is the expected future surplus to buyers, from the sales of goods $i+1$ through c.

To get (10) use backward induction on i. Let $i = c$. From (8) and $u_{c+1} = 0$ we have

$$u_c = \tfrac{1-\frac{\alpha}{\beta}q_c}{n-c+1} = \tfrac{\Phi_c}{n-c+1}.$$

For the induction step suppose $u_{i+1} = \frac{\Phi_{i+1}}{n-i}$ holds for some $i < c-1$. Inserting u_{i+1} into (8), we obtain

$$u_i = \tfrac{1-\frac{\alpha}{\beta}q_i^s+\beta\Phi_{i+1}}{n-i+1} = \tfrac{\Phi_i}{n-i+1}$$

because of (14). This gives us (10).

To find an expression of $\Phi_i\left(c,n\right)$ in terms of the parameters, we use backward induction on i. Let $i = c$. Then, (13)

$$\Phi_c(c,n) = 1 - \tfrac{\alpha}{\beta}q_c^s = \tfrac{\beta-\alpha}{\beta}\tfrac{n-c+1}{n-c+1-\alpha},$$

where we have substituted (6) with $u_{c+1} = 0$ for q_c^s. For the inductive step suppose that for some $i < c-1$ we have

$$\Phi_{i+1}(c,n) = \tfrac{\beta-\alpha}{\beta}\sum_{j=i+1}^{c}\beta^{j-(i+1)}\prod_{m=i+1}^{j}\tfrac{n-m+1}{n-m+1-\alpha}, \tag{15}$$

where we notice that $(\beta-\alpha) \in (0,1)$. From (14) we get

$$\begin{aligned}\Phi_i(c,n) &= 1 - \tfrac{\alpha}{\beta}q_i^s + \beta\Phi_{i+1}(c,n) \\ &= 1 - \tfrac{\alpha}{\beta}[\tfrac{n-i+1-\beta}{n-i+1-\alpha} - \tfrac{\beta^2(n-i)}{n-i+1-\alpha}u_{i+1}] + \beta\Phi_{i+1}(c,n) \\ &= 1 - \tfrac{\alpha(n-i+1-\beta)}{\beta(n-i+1-\alpha)} + \tfrac{\alpha\beta\Phi_{i+1}}{n-i+1-\alpha} + \beta\Phi_{i+1}(c,n) \\ &= \tfrac{n-i+1}{n-i+1-\alpha}[\tfrac{\beta-\alpha}{\beta} + \beta\Phi_{i+1}(c,n)].\end{aligned} \tag{16}$$

where in the second line we have used (6) and in the third we used (10). Inserting Φ_{i+1} from (15) we obtain

$$\Phi_i(c,n) = \tfrac{\beta-\alpha}{\beta}[\tfrac{n-i+1}{n-i+1-\alpha} + \tfrac{n-i+1}{n-i+1-\alpha}\beta \textstyle\sum_{j=i+1}^{c} \beta^{j-(i+1)} \displaystyle\prod_{m=i+1}^{j} \tfrac{n-m+1}{n-m+1-\alpha}],$$

which gives us (12).

Finally, to get q_i^s as a function of the parameters, plug (10) into (4), under symmetry. Note that $\frac{\beta\Phi_i}{n-i+1} \in (0,1)$ for each i, because $0 < \beta - \alpha < 1$. Rearranging (11) and (12) we obtain (1).$\blacksquare$

Lemma 3 characterizes the buyer's expected utility as a function of parameters. The expression Φ_i denotes expected future surplus to buyers due to transactions involving good i through c. Thus the representative buyer's expected utility u_i equals Φ_i divided by $n-i+1$, the number of remaining buyers at the store. The seller's equilibrium offer is $q_i\,(c,n)$ and it leaves the buyer indifferent between accepting and rejecting.

Clearly there is a unique pair (q_i^s, q_i^b) for each i, thus there is a unique SPE satisfying the two properties: stationarity and no-delay. It is easy to check that the strategies described in the Theorem are subgame perfect. The only thing left is to demonstrate is that this is also the unique SPE of this game, i.e., we need to show that every SPE must satisfy stationarity and no-delay.

Lemma 4 *The subgame perfect equilibrium described in Theorem 1 is the unique subgame perfect equilibrium of this game.*

Proof. In the appendix.

To summarize, in any subgame perfect equilibrium, when the seller and some buyers negotiate over the sale of good i we have that if the seller makes the offer, then the good is sold at price q_i, and if the buyer makes the offer then the sale price is αq_i.

2.3 Characterization of equilibrium offers

It is important that we understand how offers respond to changes in capacity c and realized demand n. A first result is that sale prices respond positively to demand pressure, so the seller can exploit strategically his ability to constrain capacity.

Lemma 5 *Equilibrium offers are increasing functions of excess demand $n - c$. Precisely, for each $i = 1, ..., c$ with $c \leq n$, the sequence $\{q_i(\tilde{c}, n)\}_{\tilde{c}=i}^{n}$ is strictly decreasing, $\{q_i\,(c, \tilde{n})\}_{\tilde{n}=c}^{\infty}$ is strictly increasing, and $q_{i+j}(c + j, n + j) = q_i\,(c, n)$ for $j = 1, 2, ...$*

Proof. Consider (12). For all $i \leq c < c' \leq n$ we have

$$\begin{aligned}\Phi_i\left(c', n\right) &= \tfrac{\beta-\alpha}{\beta} \textstyle\sum_{j=i}^{c'} \beta^{j-i} \displaystyle\prod_{m=i}^{j} \tfrac{n-m+1}{n-m+1-\alpha} \\ &= \tfrac{\beta-\alpha}{\beta} \textstyle\sum_{j=i}^{c} \beta^{j-i} \displaystyle\prod_{m=i}^{j} \tfrac{n-m+1}{n-m+1-\alpha} + \tfrac{\beta-\alpha}{\beta} \textstyle\sum_{j=c+1}^{c'} \beta^{j-i} \displaystyle\prod_{m=i}^{j} \tfrac{n-m+1}{n-m+1-\alpha} \\ &> \Phi_i\left(c, n\right)\end{aligned}$$

In the second line we have used the definition of $\Phi_i\left(c, n\right)$. From (11) we have $q_i\left(c, n\right) = 1 - \frac{\beta\Phi_i(c,n)}{n-i+1}$. This implies that $q_i\left(c, n\right) > q_i\left(c', n\right)$ for all $i \leq c < c' \leq n$.

Now consider the effect of n. Let $n' > n \geq c$, then $\frac{n'-i+1}{n'-i+1-\alpha} < \frac{n-i+1}{n-i+1-\alpha}$ for each $i \leq c$. Thus (12) implies $\Phi_i\left(c, n'\right) < \Phi_i\left(c, n\right)$. From (11) we have $q_i\left(c, n'\right) > q_i\left(c, n\right)$ for all $i = 1, ..., c$.

Finally, let $i' = i + j$, $n' = n + j$ and $c' = c + j$ with $j = 0, 1, ...$ Note that $\frac{n'-i'+1}{n'-i'+1-\alpha} = \frac{n-i+1}{n-i+1-\alpha}$, $n' - i' = n - i$, and $\Phi_{i'}\left(c', n'\right) = \Phi_i\left(c, n\right)$, so (11) implies $q_{i'}\left(c', n'\right) = q_i\left(c, n\right)$. ■

When $c < n$ the seller is capacity constrained, so customers face **consumption risk**. This risk grows as c falls and n grows, i.e., as excess demand increases. In both cases buyers are willing to trade at higher prices in any round because they are randomly selected to deal with the seller. When there is excess demand, disagreement is costly to a buyer since it carries the risk of not being able to buy at all in future rounds. This risk increases as $n - c$ rises. The seller does not face this type of risk since he trades in every round that is reached. Therefore, greater demand or lower capacity increase the competition among buyers and so raise prices.

The central consequence is that the seller can effectively increase his bargaining power by restricting capacity, which sorts a positive effect on the intensive margin. Later, we will determine parameters such that this strategic restriction of capacity is advantageous to sellers. In that case, inefficiencies will arise, due to the negative effects on the extensive margin.

The last part of Lemma 5 tells us that what matters for price determination is the number of goods left in inventory. Since, larger capacity and greater demand have opposite effects on excess demand, good i sold at a store that has excess demand $n - c$ is the same as the price of good $i + j$ sold at a store whose capacity and number of customers is also increased by j. This last fact will be used later on. Now, instead, we establish how equilibrium offers respond to changes in β and γ, given $c \leq n$.

Lemma 6 *Let $2 \leq c \leq n$. The function $q_i(c, n)$ is strictly increasing in γ for every $i = 1, ..., c$. If $i = c = n$ then $q_i(c, n)$ is strictly decreasing in β, but if $i \leq c < n$, then the function $q_i(c, n)$ is U-shaped in β for all $i = 1, ..., c$.*

Proof. We start by demonstrating that the function $q_i(c, n)$ is strictly increasing in γ for every $i = 1, ..., c$. Use backward induction on i. Consider $\gamma \in (0, 1)$. Let $i = c$ and demonstrate that

$\frac{\partial q_c(c,n)}{\partial \gamma} > 0$. In equilibrium we have $q_i^s = q_i(c,n)$ from Lemmas 2 and 3. Hence, use (9) with $u_{c+1} = 0$ to get

$$q_c(c,n) = \tfrac{n-c+1-\beta}{n-c+1-\alpha}.$$

From (2) we have $\frac{\partial \alpha}{\partial \gamma} > 0$, so $\frac{\partial q_c(c,n)}{\partial \gamma} > 0$. For the inductive step suppose $\frac{\partial q_{i+1}(c,n)}{\partial \gamma} > 0$ for some $i < c$, and demonstrate that $\frac{\partial q_i(c,n)}{\partial \gamma} > 0$. Using (4) and (9) we can write

$$q_i(c,n) = \tfrac{(n-i+1)(1-\beta)}{n-i+1-\alpha} + \tfrac{\beta(n-i)q_{i+1}(c,n)}{n-i+1-\alpha}. \quad (17)$$

The first term increases with α, and so with γ. Using the inductive step, we see that the second term increases with γ as well, thus the result. Since $q_i^b = \alpha q_i^s$ then every equilibrium offer increases in γ.

Now, we demonstrate the second part of the lemma, on the behavior of offers as β changes. It is simple to verify that if $i = c = n$, then $\beta\Phi_n = \beta(1-\gamma)$ and so

$$q_n(n,n) = 1 - \beta(1-\gamma).$$

Therefore, $\frac{\partial q_n(n,n)}{\partial \beta} < 0$. This is the well-known case where the seller faces a single buyer.

Now consider the case $c < n$ so that $n > m$ in Φ_i for all $i \leq c$. For $\beta \in (0,1)$, we want to prove that, as β grows, we have that $q_i(c,n)$ first decreases and then increases. Referring to (11), this is equivalent to demonstrating that $\frac{\partial \beta\Phi_i(c,n)}{\partial \beta} > 0$ when β small, and $\frac{\partial \beta\Phi_i(c,n)}{\partial \beta} < 0$ when β is sufficiently large. Here, $\Phi_i(c,n)$ is as in (12).

Notice that

$$\beta\Phi_i(c,n) = (\beta-\alpha)\textstyle\sum_{j=i}^{c} \beta^{j-i} \displaystyle\prod_{m=i}^{j} \tfrac{n-m+1}{n-m+1-\alpha}.$$

It follows that

$$\begin{aligned} \tfrac{\partial \beta\Phi_i}{\partial \beta} \quad &= \left(1 - \tfrac{\partial\alpha}{\partial\beta}\right) \textstyle\sum_{j=i}^{c} \beta^{j-i} \displaystyle\prod_{m=i}^{j} \tfrac{n-m+1}{n-m+1-\alpha} \\ &+ (\beta-\alpha) \textstyle\sum_{j=i}^{c} (j-i)\,\beta^{j-i-1} \displaystyle\prod_{m=i}^{j} \tfrac{n-m+1}{n-m+1-\alpha} \\ &+ (\beta-\alpha)\tfrac{\partial\alpha}{\partial\beta} \textstyle\sum_{j=i}^{c} \beta^{j-i} \displaystyle\prod_{m=i}^{j} \tfrac{n-m+1}{n-m+1-\alpha} \textstyle\sum_{k=i}^{j} \tfrac{1}{n-k+1-\alpha}. \end{aligned} \quad (18)$$

From (2) we have $\frac{\partial\alpha}{\partial\beta} = \frac{\gamma}{(1-\beta+\beta\gamma)^2} \in (0,\infty)$.

Using the Intermediate Value Theorem one can establish that $\frac{\partial\alpha}{\partial\beta} \leq 1$ for $\beta \in (0,x] \subset (0,1)$, and $\frac{\partial\alpha}{\partial\beta} > 1$ otherwise. Notice also $\beta \geq \alpha$ from (2). Suppose $\beta \leq x$. Then, every term in (18) is positive and finite. Thus $\frac{\partial\beta\Phi_i}{\partial\beta} > 0$ and so $\frac{\partial q_i(c,n)}{\partial\beta} < 0$.

Now consider $\beta > x$. The main observation is that as $\beta \to 1$ we have $\frac{\partial\beta\Phi_i}{\partial\beta} < 0$. Indeed, we have that $\lim_{\beta\to1}(1 - \frac{\partial\alpha}{\partial\beta}) < 0$ and $\lim_{\beta\to1}(\beta-\alpha) = 0$. Moreover, since $c < n$ the summation terms in (18) are positive and finite as $\beta \to 1$, because $m, k < n$. It follows that $\lim_{\beta\to1}\frac{\partial\beta\Phi_i}{\partial\beta} < 0$ and so $\frac{\partial q_i(c,n)}{\partial\beta} > 0$ for $\beta \in (y,1) \subset (0,1)$ with $y > x$.

Since $\frac{\partial \beta \Phi_i}{\partial \beta} > 0$ for $\beta \in (0, y]$ and $\frac{\partial \beta \Phi_i}{\partial \beta} < 0$ for $\beta \in (y, 1)$, then $q_i(c, n)$ is U-shaped in β. Therefore, q_i^s is U-shaped in β. Since $q_i^b = \alpha q_i(c, n)$ and $\frac{\partial \alpha}{\partial \beta} > 0$, it follows that $\frac{\partial q_i^b}{\partial \beta} > 0$ whenever $\frac{\partial q_i^s}{\partial \beta} > 0$. However, we cannot generally sign $\frac{\partial q_i^b}{\partial \beta}$ when $\frac{\partial q_i^s}{\partial \beta} > 0$.■

There are two implications. First, equilibrium offers are increasing functions of γ. This is true for every player and for every good that is put up for sale because a greater γ improves the seller's bargaining power.

Second, in any round in which the seller faces multiple customers, an increase in the continuation probability β lowers the seller's equilibrium offer if β is small and raises otherwise. Instead, if the seller faces a single customer q_i^s falls with β. This is interesting because a standard result from the bargaining literature is that when there is no excess demand, i.e., when $i = c = n$, the equilibrium offer decreases with β. Instead, when $i \leq c < n$, i.e., when there is excess demand, equilibrium offers of the seller are U-shaped.

Intuitively, as β grows every player discounts less future earnings, all else equal. Thus, the seller lowers his equilibrium offer (and the buyer raises it) when β grows. This is indeed what happens when $i = n$, i.e., when there is only a buyer left to trade with. However, when $i < n$ the seller has excess demand so buyers face consumption risk. A buyer who does not settle the deal today, runs the risk of being left out from the next rounds of negotiations. For example, if there is disagreement today, the seller trades in the next round with probability β but the buyer trades only with probability $\frac{\beta}{n}$. Therefore an increase in β improves the seller's continuation payoff more than the buyer's, especially when n and β are large. In these circumstances, the seller can exploit an increase in β to his advantage, raising the price.

Finally, we establish how sale prices evolve as the remaining inventory shrinks, i.e., we characterize sale prices of each item according to their order of sale.

Lemma 7 *The sequence $\{q_i(c,n)\}_{i=1}^{c}$ can be characterized as follows. It is monotonically decreasing if $\frac{c}{n}$ is sufficiently close to one, it is monotonically increasing if $\frac{c}{n}$ is sufficiently close to zero, and it is U-shaped otherwise.*

Proof. Let $c = 2, .., n$. From the proof of Lemma 5 we see that $\{\Phi_i(c,n)\}_{c=i}^{n}$ is a monotonically increasing positive sequence for all $i = 1, .., c$. From (12) we also see that $\{\Phi_i(c,n)\}_{i=1}^{c}$ is monotonically decreasing.

Define $d_{i+1} = \frac{\beta \Phi_{i+1}}{n-(i+1)+1} - \frac{\beta \Phi_i}{n-i+1}$ and notice that $q_{i+1} - q_i = -d_{i+1}$. Using (16) in the proof of Lemma 3 we get

$$d_{i+1} = -\tfrac{\beta-\alpha}{\beta(n-i)} + \Phi_i(1 - \beta + \tfrac{1-\alpha}{n-i}),$$

noting that $n \geq c \geq i+1$ since $c = 2$.

Notice that $\left\{\frac{1}{n-i}\right\}_{i=1}^{c}$ is increasing and $\{\Phi_i\}_{i=1}^{c}$ is decreasing. Therefore $\{d_{i+1}\}_{i=1}^{c-1}$ is decreasing. Thus, if $d_{i+1} < 0$ then we have $d_j < 0$ for $j > i$. That is to say, if $q_{i+1} > q_i$ then we have $q_{j+1} > q_j$ for all $j \geq i$.

Most importantly, if $d_2 < 0$ then $d_{i+1} < 0$ for all $i = 1, ..., c-1$. It is easy to find conditions such that $d_2 < 0$ (i.e., that $q_2 > q_1$). Substituting for Φ_i from (12) we get

$$d_{i+1} = \tfrac{\beta-\alpha}{\beta}[-\tfrac{1}{n-i} + (1-\beta+\tfrac{1-\alpha}{n-i})\textstyle\sum_{j=i}^{c}\beta^{j-i}\displaystyle\prod_{m=i}^{j}\tfrac{n-m+1}{n-m+1-\alpha}].$$

Recall that $\alpha \in (0,1)$ is independent of i and c, so d_{i+1} increases with c. Thus, consider $c = 2$ in which case we obtain

$$d_2 = \tfrac{\beta-\alpha}{\beta}[-\tfrac{1}{n-1} + (1-\beta+\tfrac{1-\alpha}{n-1})\tfrac{n}{n-\alpha}(1+\tfrac{\beta(n-1)}{n-1-\alpha})].$$

It is easy to see that $\lim_{n\to\infty} d_2 < 0$ since the second term in the square brackets converges to a positive constant, as n grows large. Therefore, if n is sufficiently large and c is sufficiently small, we have $d_2 < 0$ and therefore $d_{i+1} < 0$ for all $i = 1, ..., c-1$, because d_{i+1} falls with i. That is, we need $\frac{c}{n}$ sufficiently close to zero.

Now, consider $i + 1 = c$, which is when d_{i+1} is the smallest, so we have

$$d_c = \tfrac{\beta-\alpha}{\beta}[-\tfrac{1}{n-c+1} + (1-\beta+\tfrac{1-\alpha}{n-c+1})\textstyle\sum_{j=c-1}^{c}\beta^{j-i}\displaystyle\prod_{m=i}^{j}\tfrac{n-m+1}{n-m+1-\alpha}].$$

We have established earlier that, for each i, d_{i+1} increases as c grows from $i+1$ to n. Therefore consider $c = n$, so we have

$$d_n = \tfrac{\beta-\alpha}{\beta}[-1 + (2-\beta-\alpha)\tfrac{2}{2-\alpha}(1+\tfrac{\beta}{1-\alpha})] = \tfrac{\beta-\alpha}{\beta}[1 + \tfrac{2\beta(1-\beta)}{(2-\alpha)(1-\alpha)}] > 0.$$

Since $\{d_{i+1}\}_{i=1}^{c-1}$ is a decreasing sequence, and d_{i+1} increases in c, it follows that $d_{i+1} > 0$ for all $i = 1, .., c-1$, when c is sufficiently close to n. Therefore, we must have $\{q_i\}_{i=1}^{c}$ is a monotonically decreasing sequence when c is sufficiently close to n. That is to say, we need $\frac{c}{n}$ sufficiently close to one.

In between these two extreme cases, there is a case when $d_2 > 0$ but, since d_{i+1} falls with i, $d_c < 0$. In this case, we have $q_i < 0$ for i small and $q_i > 0$ for i large. It follows from our prior discussion that this will occur when $\frac{c}{n}$ is somewhere in between zero and one.■

The main result is that late buyers do not necessarily pay the highest price, when shopping at a capacity-constrained store. Alternatively, job applicants who are hired last do not necessarily receive the lowest wage. Whether "early birds" pay less simply depends on the severity of capacity constraints. Indeed, prices can initially fall and rise only when the few last items are offered for sale.

We illustrate this result in Figure 1 for $n = 15$, $\beta = 0.9$ and $\gamma = 0.1$. It simulates equilibrium sale prices in three economies with different capacity levels, $c = 15$, 14, and 5. On the horizontal axis we have the different goods $i = 1, 2, ..c$, and on the vertical axis the corresponding sale price.

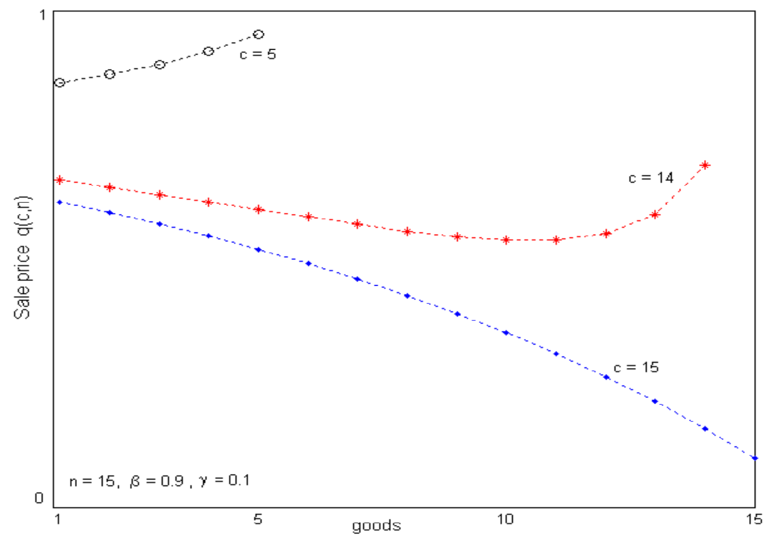

Figure 1 – Sale prices under different inventories

When there is no excess demand, $c = 15$, prices decrease monotonically as items are sold. For small excess demand, $c = 14$, prices initially fall in the order of sale. After the tenth item is sold, prices start to rise. Finally, when excess demand is substantial, $c = 5$, prices increase as the seller's remaining inventory shrinks.

There are two opposing effects that influence prices as items are sold. First, buyers face *trading risk.* Due to the random stopping rule buyers have an incentive to buy sooner rather than later. When good i is offered for sale we have $n-i+1$ customers competing to buy it. The strength of this competition is captured by the probability $\frac{1}{n-i+1}$ of being selected to transact. This competition is fiercer when i small, i.e., when the seller has sold only a small batch of his initial inventory, and it progressively softens as items are sold since some buyers leave the store. On the other hand, when $c < n$ buyers face *consumption risk* since they may not be able to buy even if the game does not stop. This risk increases as items are sold.[3] The opposing influence of trading and consumption risk varies as items are sold with the latter dominating the former when there is severe excess demand.

This is illustrated in Figure 1. When $c = 15$ there is no consumption risk, but only trading risk. As items are sold, trading risk softens and equilibrium prices fall. When $c = 14$ consumption risk is small and so it is dominated by trading risk in the initial stages of the trading sequence.

[3] Suppose players bargain over the i^{th} item. With probability $\prod_{j=i}^{c} \frac{n-j}{n-j+1} \equiv \frac{n-c}{n-i+1}$ a particular buyer will not be selected at all to trade with the seller, even the game does not stop. Notice $\frac{n-j}{n-j+1}$ is the probability that the buyer is not selected by the random device (neither as a proposer nor as a responder) when negotiating over the j^{th} good, for $j = i, .., c$. Clearly $\frac{n-c}{n-i+1}$ increases with i.

So, prices initially fall (until the tenth item) but subsequently rise. When $c = 5$, consumption risk has such a dominant effect that prices increase in the order of sale.

3 Endogenous Capacity

When capacity is exogenous, the bargaining outcome is efficient since there are no wasteful delays, which is in line with the existing literature. In this section, instead, we demonstrate that if the seller can commit to an initial choice of capacity, then this may lead to equilibrium inefficiencies. Therefore, consider the first stage of the game, given the solution to the bargaining problem in the second stage.

Here, the seller solves the problem

$$\max_{c\in\{1,\ldots,n\}} \pi(c,n) \tag{19}$$

given that offers are selected optimally in the second stage of the game, i.e., they satisfy Theorem 1. Let $\widetilde{c}(n)$ denote the set of maximizers, i.e.,

$$\widetilde{c}(n) = \{c : c \in \{1,...,n\} \text{ and } \pi(c,n) \geq \pi(x,n) \text{ for all } x = 1,...,n\}$$

The following theorem characterizes $\widetilde{c}(n)$ as a function of the parameters.

Theorem 8 *Let $\widetilde{c}(n)$ denote the set of maximizers of $\pi(c,n)$ and let $\widehat{c} \in \{2,...,n-1\}$ denote a generic interior capacity choice. We have that*

$$\widetilde{c}(n) = \begin{cases} \{1\} & \text{if } \frac{\beta}{\beta-\alpha} < \varphi(2,n) \\ \{\widehat{c}-1,\ \widehat{c}\} & \text{if } \frac{\beta}{\beta-\alpha} = \varphi(\widehat{c},n) \\ \{\widehat{c}\} & \text{if } \frac{\beta}{\beta-\alpha} \in (\varphi(\widehat{c},n), \varphi(\widehat{c}+1,n)) \\ \{n\} & \text{if } \frac{\beta}{\beta-\alpha} > \varphi(n,n) \end{cases} \tag{20}$$

with

$$\varphi(c,n) = \prod_{m=1}^{c} \frac{n-m+1}{n-m+1-\alpha}. \tag{21}$$

Corollary 9 *If $\frac{\beta}{\beta-\alpha} \leq \varphi(n,n)$, then the equilibrium outcome is inefficient.*

The theorem establishes that the seller generally selects a unique capacity, although knife edge cases exist in which he might be indifferent between two adjacent choices. The intuition is simple. The seller's basic trade-off involves an extensive and an intensive margin because higher capacity raises the expected volume of trade but lowers the unit price, as seen in Lemma 7. This suggests that in general there is a unique capacity choice and that full capacity, $c = n$, is generally not the payoff-maximizing choice. Multiplicity arises due to the discreteness of the seller's choice set.

The corollary, instead, establishes the possibility of allocative inefficiencies, despite the efficiency of the bargaining solution. Allocative inefficiencies depend on how bargaining power is distributed among seller and buyers. To see why, notice that the seller offers less than the market demands, setting $c < n$, whenever he has limited bargaining power. This follows immediately from observation that $\lim_{\gamma\to 0} \frac{\beta}{\beta-\alpha} = \lim_{\gamma\to 0} \varphi(n,n) = 1$ for all n and parameters exists such that $\frac{\beta}{\beta-\alpha} < \varphi(n,n)$ for γ close to zero. The intuition is simple. Lemma 6 has established that every equilibrium price grows with γ, a parameter that captures the seller's bargaining power. Therefore, if γ is sufficiently low, then the seller has an incentive to exploit the intensive margin gains granted by restricting the supply to $c < n$. Of course, this creates an inefficiency because $n-c$ trades will not be realized. Indeed, a social planner would grant the seller enough bargaining power to ensure that $c = n$ is selected.

To prove this theorem we start by deriving the seller's payoff as a function of the model's parameters. In particular, given n, the seller's payoff $\pi(c,n)$ is a step function defined on the discrete set $\{1, ..., n\}$.

Lemma 10 *Given $n \geq 2$ and prices as in Theorem 1, the seller's payoff satisfies*

$$\pi(c,n) = \textstyle\sum_{j=1}^{c} \beta^{j-1} - \Phi_1(c,n), \tag{22}$$

or, equivalently,

$$\pi(c,n) = \pi(c-1,n) + \beta^{c-1}[1 - \tfrac{\beta-\alpha}{\beta}\varphi(c,n)]. \tag{23}$$

Proof. Using the results in Lemma 2 and especially expression (7), we have

$$\pi(c,n) = \tfrac{\alpha}{\beta} \textstyle\sum_{j=1}^{c} \beta^{j-1} q_j(c,n).$$

Notice from expression (13) in the proof of Lemma 3, that for $i = 1$ we have

$$\Phi_1(c,n) = \textstyle\sum_{j=1}^{c} \beta^{j-1} \left[1 - \tfrac{\alpha}{\beta} q_j\right].$$

Therefore, we obtain (22).

Now, for $c = 1, 2, ..., n$, define the function

$$\varphi(c,n) = \prod_{m=1}^{c} \tfrac{n-m+1}{n-m+1-\alpha}.$$

Note that $\varphi(c,n)$ increases in c and falls in n since for all $n \geq 2$ and $c = 2, ..., n$ we have

$$\varphi(c,n) > \varphi(c-1,n) \quad \text{and} \quad \varphi(c,n) > \varphi(c,n+1). \tag{24}$$

The first inequality is proved by noticing that

$$\varphi(c,n) = \varphi(c-1,n) \tfrac{n-c+1}{n-c+1-\alpha} > \varphi(c-1,n).$$

The second inequality is obtained from observing that

$$\varphi(c,n) = \varphi(c,n+1) \tfrac{(n-c+1)(n+1-\alpha)}{(n-c+1-\alpha)(n+1)} > \varphi(c,n+1).$$

From (22) we have

$$\pi\left(c-1,n\right)=\textstyle\sum_{j=1}^{c-1}\beta^{j-1}-\Phi_{1}\left(c-1,n\right). \tag{25}$$

Using (12) with $i=1$, we have

$$\begin{aligned}\Phi_{1}\left(c-1,n\right) &= \tfrac{\beta-\alpha}{\beta}\textstyle\sum_{j=1}^{c-1}\beta^{j-1}\displaystyle\prod_{m=1}^{j}\tfrac{n-m+1}{n-m+1-\alpha}\\ &= \tfrac{\beta-\alpha}{\beta}\textstyle\sum_{j=1}^{c}\beta^{j-1}\displaystyle\prod_{m=1}^{j}\tfrac{n-m+1}{n-m+1-\alpha}-\tfrac{\beta-\alpha}{\beta}\beta^{c-1}\prod_{m=1}^{c}\tfrac{n-m+1}{n-m+1-\alpha}\\ &= \Phi_{1}\left(c,n\right)-\tfrac{\beta-\alpha}{\beta}\beta^{c-1}\varphi\left(c,n\right).\end{aligned}$$

In the last step we have used (12) and the definition of $\varphi\left(c,n\right)$ from (21). Inserting this result into (25) we have

$$\begin{aligned}\pi\left(c-1,n\right) &= \textstyle\sum_{j=1}^{c}\beta^{j-1}-\Phi_{1}\left(c,n\right)-\beta^{c-1}+\tfrac{\beta-\alpha}{\beta}\beta^{c-1}\varphi\left(c,n\right)\\ &= \pi\left(c,n\right)-\beta^{c-1}\left[1-\tfrac{\beta-\alpha}{\beta}\varphi\left(c,n\right)\right]\end{aligned}$$

In the second line we have used (22). This gives (23).■

To complete the proof of the Theorem, we characterize the change in $\pi(c,n)$ for a unit increment in capacity. The main result is that, for every $n\geq 2$, the change in payoff strictly falls with each increment.

Lemma 11 *Let $\Delta\left(c,n\right)=\pi\left(c,n\right)-\pi\left(c-1,n\right)$ denote the payoff change from a unit increment in capacity, for $c=1,...,n$. We have*

$$\Delta\left(c,n\right)=\beta^{c-1}[1-\tfrac{\beta-\alpha}{\beta}\varphi\left(c,n\right)]. \tag{26}$$

Therefore, for all $c=1,..,n-1$ we have

$$\Delta\left(c,n\right)\geq 0\quad\Leftrightarrow\quad\tfrac{\beta}{\beta-\alpha}\geq\varphi\left(c,n\right), \tag{27}$$

with

$$\Delta\left(c,n\right)>\Delta\left(c+1,n\right). \tag{28}$$

This implies that (20) defines the set of maximizer of $\pi(c,n)$.

Proof. From (23) and the definition of $\Delta\left(c,n\right)$ we obtain (26). Clearly, $\Delta\left(1,n\right)=\pi\left(1,n\right)>0$ since $\pi\left(0,n\right)=0$ and (27) is obvious.

To prove that $\Delta\left(c,n\right)$ is strictly decreasing in c notice that β^{c-1} falls in c. From (24) in the proof of Lemma 10 we have that $\varphi\left(c,n\right)$ increases in c, thus $[1-\frac{\beta-\alpha}{\beta}\varphi\left(c,n\right)]$ falls in c. Thus $\Delta\left(c,n\right)>\Delta(c+1,n)$ for $c=1,...,n-1$.

We use (26)-(27) to prove that (20) describes the set of maxima.

- First line of (20). If $\frac{\beta}{\beta-\alpha} < \varphi(2,n)$, then $\Delta(2,n) < 0$ from (27). So, $\Delta(c,n) < \Delta(2,n) < 0$ for all $c > 2$, from (28). Therefore, $c = 1$ is the unique maximizer of $\pi(c,n)$.

- Second line of (20). If $\frac{\beta}{\beta-\alpha} = \varphi(\widehat{c},n)$ for some $\widehat{c} = 2,...,n-1$, then $\Delta(\widehat{c},n) = 0$ from (27). So, (28) implies that $\Delta(c,n) > 0$ for all $c < \widehat{c}$ and $\Delta(c,n) < 0$ for all $c > \widehat{c}$. Since $\pi(\widehat{c},n) = \pi(\widehat{c}-1,n)$ then there are two maximizers, $\{\widehat{c}-1,\widehat{c}\}$.

- Third line of (20). If $\varphi(\widehat{c},n) < \frac{\beta}{\beta-\alpha} < \varphi(\widehat{c}+1,n)$, then $\Delta(\widehat{c}+1,n) < 0 < \Delta(\widehat{c},n)$, from (27). Again, (28) implies that $\Delta(c,n) > 0$ for all $c < \widehat{c}$ and $\Delta(c,n) < 0$ for all $c > \widehat{c}+1$. Therefore, $c = \widehat{c}$ is the unique maximizer of $\pi(c,n)$.

- Fourth line of (20). If $\frac{\beta}{\beta-\alpha} > \varphi(n,n)$, then $< \Delta(c,n) > \Delta(n,n) > 0$ for all $1 \leq c < n$. Therefore, $c = n$ is the unique maximizer of $\pi(c,n)$.$\blacksquare$

To build intuition on how $\widetilde{c}(n)$ responds to changes in the parameters, consider the simple case with $n = 2$. In Figure 2 we illustrate optimal choice of capacity for different values of β and γ.

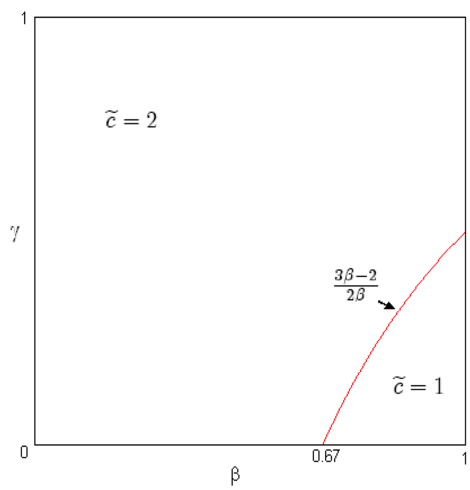

Figure 2 – Optimal capacity when $n = 2$.

Optimal capacity is $\widetilde{c} = 1$ if $\gamma < \frac{3\beta-2}{2\beta}$, $\widetilde{c} = \{1,2\}$ if $\gamma = \frac{3\beta-2}{2\beta}$ and $\widetilde{c} = 2$, otherwise. To see why, fix a low value of γ and vary β. When β is small future consumption is discounted heavily and so the seller does not gain much by constraining capacity, hence $\widetilde{c} = 2$. The opposite occurs when β is high, and so we have $\widetilde{c} = 1$. Now fix β and vary γ. The seller restricts capacity only if his bargaining power is small, i.e., when γ is sufficiently small. This, of course, results in allocative inefficiency since the realized surplus falls by one.

Next we further discuss the allocative efficiency. Given demand n, the seller optimally chooses capacity $\widetilde{c}(n)$ so $\frac{\widetilde{c}(n)}{n}$ is the fraction of buyers who can be served. This ratio is a measure of the level of allocative efficiency, since we have normalized the surplus in each match to one. To examine efficiency with respect to some key parameters, such as the market size and bargaining skill, we simulate different economies. The findings are summarized in Figure 3. We fix $\beta = 0.9$ and let the market size n vary from 2 to 200 (the horizontal axis).The seller's bargaining skill can be either low $\gamma = 0.01$ or it can be high $\gamma = 0.7$.

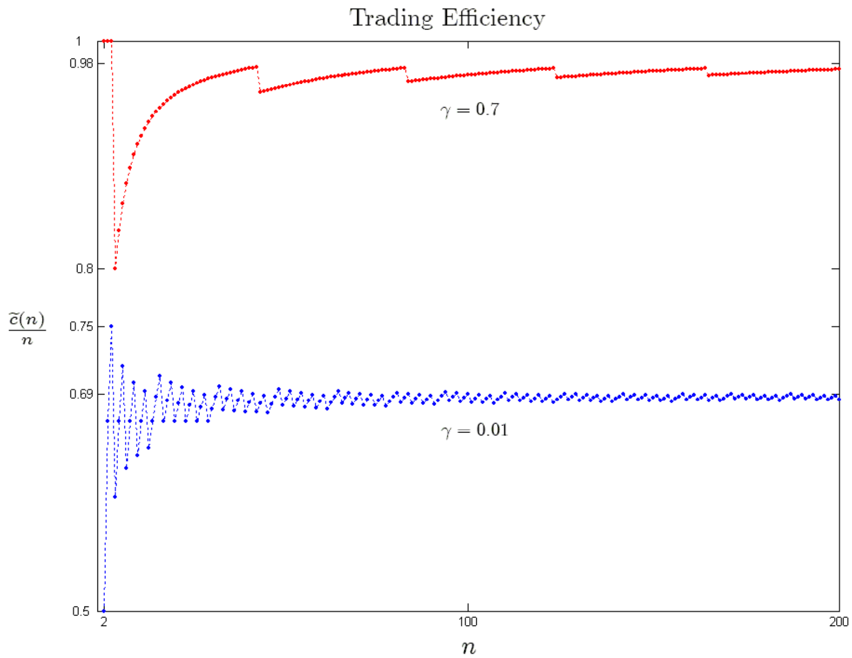

Figure 3 – Trading Efficiency

As n increases, the ratio $\frac{\widetilde{c}(n)}{n}$ converges to a level that depends on seller's bargaining skill γ.[4] For $\gamma = 0.01$ this is about 69% and for $\gamma = 0.7$ about 98%. Recall from earlier discussion that when the seller lacks bargaining skills, i.e., when γ low, he has a stronger incentive to limit his capacity, generating a loss in allocative efficiency. In this stylized economy, the loss from going $\gamma = 0.7$ to $\gamma = 0.01$ is about 30%.

4 Conclusion

This paper has examined short-run equilibrium prices and trade volume in a market with n identical buyers and a seller who commits to some capacity and then sells goods sequentially. To determine sale prices we have developed a strategic process of multilateral bargaining that involves random alternating offers between a central and peripheral players.

We have found that a unique subgame perfect bargaining equilibrium exists and it is efficient since it is characterized by absence of costly delays. We have also demonstrated that the choice of

[4] Non-monotonicity arises because as n increases $\widetilde{c}(n)$ can remain constant initially, and then rise (due to the discreteness of seller's choice set). For example, when $\gamma = 0.7$, we have $\widetilde{c}(2) = 2$, $\widetilde{c}(3) = 3$, $\widetilde{c}(4) = 4$, $\widetilde{c}(5) = 4$ and $\widetilde{c}(6) = 5$. The ratio equals to 1 for $n = 2, 3, 4$, drops to 0.8 for $n = 5$, and rises to 0.83 for $n = 6$, and so on. However this fluctuation dies out as n grows large.

capacity is of strategic relevance because it affects the buyers' reservation prices, hence the bargaining outcome. In a nutshell, restricting capacity allows the seller to obtain more favorable terms of trade because customers must compete with each other for scarce goods. As a consequence, certain distributions of bargaining powers give rise to an allocative inefficiency. In particular, a seller who has limited bargaining power will optimally restrict capacity to increase profit. In this case, some surplus is not realized, which results in inefficiency of equilibrium even if bargaining equilibrium is efficient.

References

[1] Burdett K., Shi, S. and Wright, R. (2001). Pricing and Matching with Frictions. Journal of Political Economy 109 (5), 1060-1085

[2] Calvo-Armengol, A. (1999). A note on three-player noncooperative bargaining with restricted pairwise meetings. Economics Letters 65(1), 47–54

[3] Cai, H. (2001). Uncertainty and Commitment in Multilateral Bargaining. Working Paper

[4] Camera, G. and C. Selcuk (2006). Price dispersion with directed search. Manuscript, Purdue University

[5] Caruana, G., Einav, L., and Quint, D. (2004). Multilateral Bargaining with Concession Costs. Working Paper

[6] Crawford, V. (1982). A Theory of Disagreement in Bargaining. Econometrica 50(3), 6

[7] Dasgupta, S. and Sengupta, K. (1993). Sunk Investment, Bargaining and Choice of Capital Structure. International Economic Review, 34-1, 203-220

[8] Horn, H. and Wolinsky, A. (1988). Worker Substitutability and Patterns of Unionization. Economic Journal 98, 484-497

[9] Jun, B. H. (1987). A Structural Consideration on 3-Person Bargaining. Ph.D. dissertation, University of Pennsylvania

[10] Leventoglu, B. and Tarar, A. (2005). Pre-negotiation Public Commitment in Domestic and International Bargaining. Working Paper

[11] Muthoo, A. (1996). A Bargaining Model Based on the Commitment Tactic. Journal of Economic Theory 69, 134-152

[12] Muthoo, A. (1999). Bargaining Theory with Applications. Cambridge University Press07-637

[13] Rubinstein, A. and Wolinsky, A. (1990). Decentralized Trading, Strategic Behaviour and the Walrasian Outcome. Review of Economic Studies 57(1), 63-78

[14] Shaked, A. and Sutton, J. (1984). Involuntary Unemployment as a Perfect Equilibrium in a Bargaining Model. Econometrica 52(6), 1351-1364

[15] Stole, L. and Zwiebel, J. (1996). Intrafirm bargaining under nonbinding contracts. Review of Economics Studies 63, 375 410.

Appendix

Proof of Lemma 4

To prove uniqueness, we will demonstrate all SPE of this game must satisfy stationarity and no-delay. The proof involves three steps following the method by [14], i.e., showing that the supremum and infimum of the set of SPE payoffs coincide. As described in [12], we exploit the stationary structure of the game. Any two subgames that start with the same player's offer (for the i^{th} good) are strategically equivalent. This means that the sets of subgame perfect equilibria in such subgames are identical. Hence the sets of SPE payoffs to the player making the offer are the same.

Let $\mathcal{B}_{k,i}$ denote the set of SPE payoffs to buyer k in any subgame in which buyer k makes an offer for good $i = 1, ..., c$. Similarly let $\mathcal{S}_{k,i}$ denote the set of SPE payoffs to the seller in any subgame starting with the seller making an offer to some buyer $k \in A_i$. Denote $\underline{b}_{k,i} = \inf \mathcal{B}_{k,i}$, $\overline{b}_{k,i} = \sup \mathcal{B}_{k,i}$, $\underline{s}_{k,i} = \inf \mathcal{S}_{k,i}$, $\overline{s}_{k,i} = \sup \mathcal{S}_{k,i}$.

Notice that the sets of payoffs depend on $k \in A_i$ because in principle different buyers may behave differently. Therefore, since a buyer is selected with uniform probability, we can define the expected infimum and supremum of the set of payoffs for each player as follows. First define the expectations

$$\begin{aligned} \underline{\mu}_i(s) &= \textstyle\sum_{j \in A_i} \frac{\underline{s}_{j,i}}{n-i+1} \\ \overline{\mu}_i(s) &= \textstyle\sum_{j \in A_i} \frac{\overline{s}_{j,i}}{n-i+1} \\ \underline{\mu}_i(b) &= \textstyle\sum_{j \in A_i} \frac{\underline{b}_{j,i}}{n-i+1} \\ \overline{\mu}_i(b) &= \textstyle\sum_{j \in A_i} \frac{\overline{b}_{j,i}}{n-i+1}, \end{aligned}$$

which are conditional on the selection of, respectively, the seller and the buyer to make the offer. Therefore, the unconditional expected infima and suprema of the set of payoffs in the subgame where good i is sold are:

$$\begin{aligned} \underline{w}_i &= \gamma \underline{\mu}_i(s) + (1-\gamma)\left[1 - \overline{\mu}_i(b)\right] \\ \overline{w}_i &= \gamma \overline{\mu}_i(s) + (1-\gamma)[1 - \underline{\mu}_i(b)] \end{aligned} \tag{29}$$

$$\begin{aligned} \underline{u}_{k,i} &= \tfrac{\gamma}{n-i+1}(1 - \overline{s}_{k,i}) + \tfrac{1-\gamma}{n-i+1}\underline{b}_{k,i} + \tfrac{\beta(n-i)}{n-i+1}\underline{u}_{k,i+1} \\ \overline{u}_{k,i} &= \tfrac{\gamma}{n-i+1}(1 - \underline{s}_{k,i}) + \tfrac{1-\gamma}{n-i+1}\overline{b}_{k,i} + \tfrac{\beta(n-i)}{n-i+1}\overline{u}_{k,i+1} \end{aligned} \tag{30}$$

In any subgame in which good i is up for sale, the seller's smallest expected payoff is $\underline{w}_i$. With probability γ he gets to make an offer. The offer is made to buyer $j \in A_i$ with equal probability $\frac{1}{n-i+1}$. The seller's smallest payoff in this case is $\underline{s}_{j,i}$ and the expected smallest payoff is $\underline{\mu}_i(s)$. With probability $1-\gamma$ some buyer makes the offer, and the seller's smallest expected payoff in this case is $1 - \overline{\mu}_i(b)$.

In any subgame in which good i is up for sale, buyer k's smallest expected payoff is $\underline{u}_{k,i}$. With probability $\frac{\gamma}{n-i+1}$ buyer k is in a subgame in which the seller makes him an offer. This gives the buyer at least $1 - \overline{s}_{k,i}$ payoff. With the probability $\frac{1-\gamma}{n-i+1}$ the buyer is in a subgame in which he makes an offer. In this case his smallest payoff is $\underline{b}_{k,i}$. With the complementary probability $\frac{n-i}{n-i+1}$ the buyer is not involved in negotiations. Since good i is sold to some other buyer, and good $i+1$ is put up for sale with probability β, then buyer k's smallest expected payoff is $\beta \underline{u}_{k,i+1}$.

Step 1. For all i and $k \in A_i$ we have

$$\overline{b}_{k,i} \leq 1 - \beta \underline{w}_i \quad \text{and} \quad \underline{b}_{k,i} \geq 1 - \beta \overline{w}_i \tag{31}$$

$$\overline{s}_{k,i} \leq 1 - \beta \underline{u}_{k,i} \quad \text{and} \quad \underline{s}_{k,i} \geq 1 - \beta \overline{u}_{k,i} \tag{32}$$

To prove it start with (31). In any SPE the seller's smallest expected payoff from negotiating over good i is $\underline{w}_i$. Therefore, if buyer k makes an offer, it cannot be less than $\beta \underline{w}_i$ (or the seller would not accept it). Thus, the buyer gets no more than $1 - \beta \underline{w}_i$. The second inequality in (31) can be explained similarly. Now consider (32). In any subgame in which good i is put up for sale, buyer k's minimum expected payoff is $\underline{u}_{k,i}$. Therefore the seller cannot offer less than $\beta \underline{u}_{k,i}$ and so will gets no more than $1 - \beta \underline{u}_{k,i}$.

Step 2. We prove that, for each player, the smallest and highest payoffs coincide. That is, for all i and $k \in A_i$ we have

$$\overline{s}_{k,i} = \underline{s}_{k,i} = q_i(c,n) \quad \text{and} \quad \overline{b}_{k,i} = \underline{b}_{k,i} = 1 - \alpha q_i(c,n),$$

where $q_i(c, n)$ denotes the seller's equilibrium offer.

Start by noticing that, from (31) and (29) we have

$$\overline{b}_{k,i} \leq 1 - \beta \left[\gamma \underline{\mu}_i(s) + (1-\gamma)\left[1 - \overline{\mu}_i(b)\right] \right] \quad \text{for all } k \in A_i. \tag{33}$$

Take the average of both sides of (33) over all buyers in A_i. The left side becomes $\overline{\mu}_i(b)$ since $\sum_{k \in A_i} \frac{\overline{b}_{k,i}}{n-i+1} = \overline{\mu}_i(b)$. The right side is unchanged since it is independent of k, i.e., $\sum_{k \in A_i} \frac{X}{n-i+1} = X$ for X constant, since $|A_i| = n - i + 1$. Then we have

$$\overline{\mu}_i(b) \leq 1 - \beta \left[\gamma \underline{\mu}_i(s) + (1-\gamma)\left[1 - \overline{\mu}_i(b)\right] \right] \quad \Rightarrow \quad \overline{\mu}_i(b) \leq 1 - \alpha \underline{\mu}_i(s)$$

given our definition of α. Using the latter inequality jointly with (33) we obtain

$$\overline{b}_{k,i} \leq 1 - \alpha \underline{\mu}_i(s). \tag{34}$$

We can similarly establish

$$\underline{b}_{k,i} \geq 1 - \alpha \overline{\mu}_i(s). \tag{35}$$

Now use backward induction on i. Let $i = c$. Using (32), (30) and $\underline{u}_{k,c+1} = 0$ we have

$$\overline{s}_{k,c} \leq 1 - \frac{\beta\gamma(1 - \overline{s}_{k,c})}{n-c+1} - \frac{\beta(1-\gamma)\underline{b}_{k,c}}{n-c+1}.$$

Then considering $\underline{b}_{k,c}$ from inequality (35) we have

$$\overline{s}_{k,c} \leq \frac{n-c+1-\beta}{n-c+1-\beta\gamma} + \frac{\alpha\beta(1-\gamma)}{n-c+1-\beta\gamma}\overline{\mu}_c(s). \tag{36}$$

Since this is true for all $k \in A_c$, we take the average of both sides of (36) over all buyers in A_c. The left side becomes $\sum_{k\in A_c} \frac{\overline{s}_{k,i}}{n-i+1} = \overline{\mu}_c(s)$ while the right side is unaffected. Rearranging (36) we get

$$\overline{\mu}_c(s) \leq \frac{n-c+1-\beta}{n-c+1-\alpha} = q_c(c,n).$$

This finding and (36) imply $\overline{s}_{k,c} \leq q_c(c,n)$. We can similarly establish $\underline{s}_{k,c} \geq q_c(c,n)$. Since $\overline{s}_{k,c} \geq \underline{s}_{k,c}$ we have

$$\overline{s}_{k,c} = \underline{s}_{k,c} = q_c(c,n).$$

Then (34) and (35) imply $\overline{b}_{k,c} = \underline{b}_{k,c} = 1 - \alpha q_c(c,n)$ because $\overline{\mu}_c(s) = \underline{\mu}_c(s) = q_c(c,n)$.

For the induction step suppose it is true that $\overline{s}_{k,j} = \underline{s}_{k,j} = q_j(c,n)$ for all $i+1 \leq j \leq c-1$, and $k \in A_j$. Then it is also true that $\overline{b}_{k,j} = \underline{b}_{k,j} = 1 - \alpha q_j(c,n)$, and therefore $\overline{u}_{k,j} = \underline{u}_{k,j} = u_j$. When $j = i+1$, use (11) and (32) to get

$$\beta u_{i+1} = 1 - q_{i+1}(c,n) = \frac{\beta\Phi_{i+1}(c,n)}{n-i}. \tag{37}$$

Now we prove that $\overline{s}_{k,j} = \underline{s}_{k,j} = q_j(c,n)$ and $\overline{b}_{k,j} = \underline{b}_{k,j} = 1 - \alpha q_j(c,n)$ for $j = i$. Using (32), (30) we have

$$\begin{aligned}\overline{s}_{k,i} \quad &\leq 1 - \beta\underline{u}_{k,i} \\ &\leq 1 - \tfrac{\beta\gamma(1-\overline{s}_{k,i})}{n-i+1} - \tfrac{\beta(1-\gamma)\underline{b}_{k,i}}{n-i+1} - \tfrac{\beta^2(n-i)}{n-i+1}u_{i+1} \\ &\leq 1 - \tfrac{\beta\gamma(1-\overline{s}_{k,i})}{n-i+1} - \tfrac{\beta(1-\gamma)(1-\alpha\overline{\mu}_i(s))}{n-i+1} - \tfrac{\beta^2(n-i)}{n-i+1}u_{i+1}.\end{aligned}$$

In the second line we have used the fact that $\underline{u}_{k,i+1} = u_{i+1}$ from the induction step. In the third line we have used (35). Inserting (37) into the last line and rearranging we obtain

$$\overline{s}_{k,i} \leq \tfrac{n-i+1-\beta}{n-i+1-\beta\gamma} + \tfrac{\alpha\beta(1-\gamma)\overline{\mu}_i(s)}{n-i+1-\beta\gamma} - \tfrac{\beta^2\Phi_{i+1}(c,n)}{n-i+1-\beta\gamma}. \tag{38}$$

Since this is true for all $k \in A_i$, we take the average of both sides of (38) over all buyers in A_i. The left side becomes $\sum_{k\in A_i} \frac{\overline{s}_{k,i}}{n-i+1} = \overline{\mu}_i(s)$ while the right side is unchanged. Rearranging (38) we get

$$\overline{\mu}_i(s) \leq \tfrac{n-i+1-\beta}{n-i+1-\alpha} - \tfrac{\beta^2\Phi_{i+1}(c,n)}{n-i+1-\alpha}.$$

Using this and (38) we obtain

$$\begin{aligned}\overline{s}_{k,i} \quad &\leq \tfrac{n-i+1-\beta}{n-i+1-\alpha} - \tfrac{\beta^2\Phi_{i+1}(c,n)}{n-i+1-\alpha} \\ &\leq 1 - \tfrac{\beta\Phi_i(c,n)}{n-i+1} = q_i(c,n).\end{aligned}$$

In the second line we have used (16). Similarly we can establish $\underline{s}_{k,i} \geq q_i(c,n)$. Since $\overline{s}_{k,i} \geq \underline{s}_{k,i}$, we have $\overline{s}_{k,i} = \underline{s}_{k,i} = q_i(c,n)$. Then (34) and (35) imply $\overline{b}_{k,i} = \underline{b}_{k,i} = b_i = 1 - \alpha q_i(c,n)$, because $\overline{\mu}_i(s) = \underline{\mu}_i(s) = q_i(c,n)$.

Using the result in Step 2, we can rearrange (29) and (30) to obtain

$$\begin{aligned}&\underline{w}_i = \overline{w}_i = w_i = \tfrac{\alpha}{\beta}q_i(c,n) \\ &\underline{u}_{k,i} = \overline{u}_{k,i} = u_i = \tfrac{1-\frac{\alpha}{\beta}q_i(c,n)}{n-i+1} + \tfrac{\beta(n-i)}{n-i+1}u_{i+1}.\end{aligned}$$

Compare these two expressions with (7) and (8) respectively. We establish that in any SPE, when the seller and buyers in A_i negotiate over good i, the seller's and every buyer's expected payoffs are

$$w_i = \pi_i (c, n) = \frac{\alpha}{\beta} q_i (c, n)$$
$$u_i = u_i(c, n) = \frac{\Phi_i (c, n)}{n - i + 1}.$$

Step 3. We want to prove that in any SPE offers are accepted without delay and are stationary. We first prove that in any SPE offers are immediately accepted. Suppose we are in a subgame in which the seller is making an offer to some buyer k. The argument above shows that he must offer exactly $q_i (c, n) = 1 - \frac{\beta \Phi_i(c,n)}{n-i+1}$. If the buyer's strategy is to accept any offer $q < q_i (c, n)$ and randomize when $q = q_i (c, n)$, then no best response for the seller exists. Randomization by the buyer is inconsistent with equilibrium. A similar argument applies in any subgame that starts with some buyer's offer. We now prove that offers are stationary. From Step 2 it is obvious that whenever the seller gets to make an offer, he proposes $q_i (c, n)$ and whenever a buyer in A_i makes an offer he proposes $\alpha q_i (c, n)$. This completes the proof of uniqueness.