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Abstract In 2005, the Democratic National Committee adopted the 50-state strategy in lieu of the strategy of focusing solely on battleground states. The rationale given for this move is that campaign expenditures are durable outlays that impact both current and future campaigns. This paper investigates the optimality of the 50-state strategy in a simple dynamic game of campaign resource allocation in which expenditures act as a form of investment. Neither the 50-state nor the battleground-states strategy is likely to arise in equilibrium. Instead, parties employ a modified battleground-states strategy in which they stochastically target non-battleground states.

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1 Introduction

One of the defining attributes of Howard Dean's leadership of the Democratic National Committee (DNC) is the 50-state strategy. In essence, the 50-state strategy commits campaign resources to all 50 states rather than concentrating on only the swing or battleground states. This strategy is not without critics. In fact, both the Democratic Congressional Campaign Committee and the Democratic Senatorial Campaign Committee openly opposed the 50-state strategy (see Bai (2006), Edsell (2006), Gilgoff(2006)). Even after the large Democratic gains in the 2006 midterm elections this strategy has drawn criticism (see Lizza (2006)). This paper utilizes a simple dynamic game of campaign resource allocation to analyze both sides of the controversy surrounding this strategy.

The rationale typically given for the 50-state strategy is that campaign expenditures are durable outlays which impact not only the current campaign but also strengthen the party in future campaigns. As stated by Dean in a 2006 e-mail sent to Democrats nationwide,[1] "our 50-state strategy has already laid a nationwide foundation for victory this year, in 2008 and beyond." To illustrate the intuitively appealing logic of this strategy, consider for example a race in which the democratic candidate has little chance of winning. If current campaign expenditures persist into future campaigns, then committing resources to such a race may indeed be optimal, even if the candidate goes on to lose the race, since the expenditure is an investment that will help make that race more competitive in the future.

Opponents of the 50-state strategy argue that races in which the democratic candidate is either a strong favorite or a strong underdog have essentially been decided (won and lost respectively), and campaign resources will only have an impact, and therefore should only be committed, in the swing or battleground races (battleground-states strategy). According to this line of reasoning, the 50-state strategy is clearly suboptimal in a one-shot environment or in the absence of persistent campaign expenditures. Remarkably, critics of the strategy appear to argue that the 50-state strategy is suboptimal even if intertemporal considerations are taken into account. As stated by Rep. Rahm Emanuel (Ill.), Chair of the Democratic Congressional Campaign Committee for the 2006 election cycle, "The way you build long-term is to succeed short-term." (Edsell 2006)

[1] This e-mail appears in its entirety on the DNC's website, `www.democrats.org/a/2006/06/50-state_strate_1.php`.

To examine both sides of the controversy surrounding the 50-state strategy, this paper utilizes a simple two-period campaign resource allocation game in which campaign expenditures in the first period state contests serve as a form of investment with benefits that persist into the second period contests. The game has a unique subgame perfect equilibrium, in which parties employ nondegenerate mixed local strategies in each state in each period.

In the second (final) period, subgame equilibria are consistent with a modified battleground-states strategy in which swing states are hotly contested, but parties stochastically target non-battleground states, each allocating zero resources to a state with a probability that increases with the strength of the incumbent party in that state and decreases in the value of the state. Although the investment effect leads to increased effective stakes for the first period contests, equilibrium first period strategies are still consistent with a modified battleground-states strategy in which non-battleground states are stochastically targeted.

Because parties randomize in each state in each period, we may compute an explicit probability that a 50-state strategy will be followed by either party in either period. Although, under our assumptions, this probability is non-zero, it will generally be quite small. We conclude that a 50-state strategy is unlikely to be optimal.

2 Related Literature

This paper extends Snyder's (1989) static analysis of campaign resource allocation[2] to examine the nature of the incentives arising in an intertemporal model of campaign resource allocation with persistent campaign expenditures that act as a form of investment. That paper models a static campaign between two political parties competing in a set of independent, simultaneous, and probabilistic contests (with a contest success function adapted from Rosen (1986)). Under a probabilistic contest success function, the party that allocates an effectively higher level of resources in a particular state has a higher probability of winning that state but does not win with certainty. Within each period, our formulation of the political campaign resource allocation game differs from Snyder (1989) in that the competition within each state is assumed to be deterministic. (More formally, we utilize an all-pay auction contest success function with affine hand-

[2] See Robson (2005) and Strömberg (2008) for a closely related games. See also the early contributions of Brams and Davis (1973,1974) and Colantoni, Levesque, and Ordeshook (1975).

icapping.) That is, the party that allocates the effectively higher level of resources to a particular state wins that state with certainty. Our use of a deterministic success function is motivated by its analytical appeal and its widespread use in political applications including the literatures on political lobbying (for a recent example see Polborn (2006)), political campaigns (see for example Meirowitz (2008)), and redistributive competition (see for example the literature following Myerson (1993)).

Our result is also relevant to the theoretical literature on dynamic contests. In the context of a contest, the term “dynamic” covers a wide range of potential approaches. In the single contest environment, dynamic games of sunk investment allowing for simultaneous moves have been examined by Harris and Vickers (1987), Budd, Harris and Vickers (1993), Fudenberg et al. (1983), Klumpp and Polborn (2006), Konrad and Kovenock (2005, 2006), McAfee (2000) and Agastya and McAfee (2006).[3] Most of these papers examine what in the Harris-Vickers taxonomy of dynamic structures would be called either a “race” or a “tug-of-war” in which the contestants compete over a single prize. More closely related to our formulation is Mehlum and Moene (2007) who also examine a dynamic model with incumbency advantages in which the status of incumbent may change from period to period depending on the outcome of each period’s contest.[4] However, in contrast to this paper, Mehlum and Moene (2007) examine a game in which the incumbency advantage is exogenous.[5] In our formulation the incumbency advantage is endogenously determined by persistent campaign expenditures. In particular, we allow for a portion of the campaign expenditures in each state to persist into the subsequent period with a proportional decay. This formulation of persistent campaign expenditures is reminiscent of the role of advertising as a form of investment in the optimal advertis-

[3] In the single contest environment alternating move models of sunk expenditure following and expanding upon the logic of the Shubik (1971) “Dollar Auction Game” have been examined by O’Neill (1986), Leininger (1989, 1991), and Demange (1992). (Closely related is Harris and Vickers’ (1985) single dimensional alternating move model of a race.) Multidimensional versions of these types of games have been examined by Dekel, Jackson, and Wolinsky (2006a,b) and applied to the issue of vote buying.

[4] See also Konrad (2006), Polborn (2006), and Stephan and Ursprung (1998) who examine models in which a challenger repeatedly attacks an incumbent until the incumbent loses, at which point the game ends.

[5] That paper also utilizes a probabilistic contest success function and focuses on a single contest in each period.

ing literature.[6] Our paper extends the dynamic simultaneous–move contest literature by providing an intertemporal contest framework that allows for expenditures to be durable outlays. In this setting we find that in the first period of the model, the incentives arising from the persistence of contest expenditures induces an extension of the combination all-pay auction/war of attrition (Hirshleifer and Riley (1978) and Riley (1998)).

Section 3 presents the two-stage intertemporal political campaign resource allocation game. Section 4 characterizes the unique subgame perfect equilibrium of the intertemporal game and explores the properties of the equilibrium in each stage. Section 5 concludes.

3 The Model

We examine a two-stage intertemporal campaign resource allocation game in which in each stage $t = 1,2$, as in the static analysis of Snyder (1989), two parties, A and B, simultaneously allocate costly campaign resources across the individual states. There are n states which are indexed by $j = 1,\dots,n$. Each state is won by the party that runs the most effective campaign. The value of winning the campaign in state j is denoted by v_j. Two possible objectives for the parties include: (1) maximizing the expected sum of the payoffs from each of the state campaigns and (2) maximizing the probability of winning a majority of the available payoffs. Due to the fact that with a deterministic success function and a finite number of states the solution to the majority objective is still an open question,[7] we restrict our attention to the first of these objectives which is consistent with a proportional system in which the parties share power in proportion to the value of the states in which they win.

In addition to the set of state valuations $\{v_j\}_{j=1}^n$, each state has an incumbent party with a potential investment advantage that is determined by campaign expenditures in the prior period. Let $\mathscr{N}_i^t$ denote the set of states in which party i is the incumbent in period t. The investment advantage is modeled as a head-start advantage. Let $a_j^t \geq 0$

[6] To the best of our knowledge this literature originates with Nerlove and Arrow (1962). For a survey see Feichtinger, Hartl, and Sethi (1994). For a recent application see Marinelli (2007).

[7] See for example Szentes and Rosenthal (2003) which examines the all-pay auction by committee problem with a super-majority rule which requires a player to win $n-1$ of n contests each with equal value. See also Van Cayseele, Deneckere, and de Vries (2001) which examines a version of this game which requires unanimity.

denote the head-start advantage in state j in period t; a_j^t represents the number of units of the campaign resource that the challenging party in state j must spend in period t in order to make voters indifferent between the two parties when the incumbent party spends zero units of the resource in state j in period t.

The state of the campaign game, denoted by S^t, is given by the stage t and the investment advantage and identity of the incumbent party in each of the n states: $S^t = \{t, \{a_j^t\}_{j=1}^n, \{\mathcal{N}_i^t\}_{i\in\{A,B\}}\}$. In period 2 the investment advantage is a function of the two parties' period 1 campaign expenditures. Figure 1 illustrates the investment advantage dynamics. Let I denote the incumbent party and C the challenging party in state j in period t. If the incumbent party allocates $x_{j,I}^t$ resources to campaigning in state j in period t, then the incumbent's effective campaign expenditure in state j is $x_{j,I}^t + a_j^t$. If the challenging party allocates $x_{j,C}^t$ resources to campaigning in state j in period t, then the challenger's effective campaign expenditure in state j is $x_{j,C}^t$. Each state is won by the party that runs the most effective campaign. Thus, the incumbent party I wins the campaign in state j in period 1 if

$$x_{j,I}^1 + a_j^1 \geq x_{j,C}^1.$$

In this case, the party that is the state j incumbent in period 1 will remain the incumbent in period 2, and the investment effect in period 2 is defined as a proportion of the difference between the incumbent's expenditure in state j in period 1 minus the effective expenditure of the challenger,

$$a_j^2 = \rho(x_{j,I}^1 + a_j^1 - x_{j,C}^1)$$

where $\rho \in (0,1]$ is the constant per period rate of decay of prior period's effective expenditures. Similarly, the incumbent party I loses the campaign in state j in period 1 if

$$x_{j,I}^1 + a_j^1 < x_{j,C}^1.$$

In this case, the party that is the state j challenger in period 1 will become the incumbent in period 2, and the investment effect in period 2 is defined as a proportion of the difference in the effective expenditures in state j in period 1,

$$a_j^2 = \rho(x_{j,C}^1 - x_{j,I}^1 - a_j^1)$$

where again ρ is the constant per period rate of decay of prior period's effective expenditures.

[Insert Figure 1]

Although we have assumed that the investment advantage is a linear handicap, this type of effectiveness advantage dates back to Lein (1990) and is frequently used in the literature on unfair contests (see for instance: Clark and Riis (2000), Konrad (2002), Meirowitz (2008), Polborn (2006), and Sahuguet and Persico (2006)). In order to highlight the basic incentives driving the campaign investment dynamics, we have also abstracted from any additional sources of incumbency advantage.

The parties maximize the sum of the discounted payoffs across the two periods, where $\delta \in (0,1)$ denotes the common discount factor employed by the two parties. The payoff in a given period is the expected sum of the values of the states won net of the expected campaign expenditures. In maximizing the intertemporal payoffs, the parties take into account that the first period's expenditures are durable outlays which generate the investment effect described above. Our focus on two periods is motivated by two factors. First is the observation that on average the national committee chairs of both of the major political parties serve for two election cycles.[8] Throughout its history the DNC has only had 7, out of a total of 50, chairs who served for more than 4 years.[9] The Republican National Committee has only had 3, out of a total of 62, chairs serve for more than 4 years.[10] Given the short tenure of most national committee chairs this seems like a reasonable modeling choice. Second, our two-period model is the simplest possible setup that allows us to examine how the persistence of campaign expenditures changes the nature of campaign resource allocation.[11]

We characterize the unique subgame perfect equilibrium of the two-stage game. The equilibrium behavior strategy profiles require non-degenerate randomization at each stage. A local strategy, which we label a *campaign resource schedule* for party i, is

[8] The average tenure for the Republican National Committee Chair is 3.02 years and for the Democratic National Committee Chair is 3.40 years.

[9] The last one was Robert S. Strauss who served from 1972 to 1977.

[10] The last one was Marcus A. Hanna who served from 1896 to 1904.

[11] While not usually given as an argument for the 50-states strategy, our analysis also abstracts from the issue of endogenous budget constraints. It is important to note that the results of the one-shot game remain largely unchanged if the objective of each party is to maximize the sum of the payoffs from each of the state campaigns subject to a budget constraint. (See for example Kovenock and Roberson (2008), Kvasov (2007), and Roberson (2006) who examine the role of budget constraints in simultaneous contests.)

a set of cumulative distribution functions, $\{F^t_{j,i}\}^n_{j=1}$, one distribution function for each state j, which depends on the state S^t. The only restriction that is placed on the set of feasible strategies is that each state must receive a nonnegative amount of campaign resources.

We make the following assumptions on the rate of decay ρ, the discount factor δ, and the initial state of the game S^1.

Assumption 1 In S^1, $a^1_j < v_j$ for all states j.

Assumption 1 rules out cases in which the initial incumbency advantages in one or more of the states are so large that the challenger optimally drops out of the race in period 1.

Assumption 2 The rate of decay ρ and the discount factor δ satisfy $\frac{1}{1+\delta} > \rho$.

Assumption 2 rules out cases in which it is optimal for the incumbent to make a period 1 campaign expenditure that is so large that the resulting period 2 investment advantage induces the challenger to drop out of the race in period 2.

4 Optimal Strategies

We begin our analysis in the final stage and move back through the game tree. The period 2 equilibrium campaign resource schedules are given in Theorem 1. We intially restrict our attention to the case in which $a^2_j < v_j$ for all states j, and then show that given Assumption 2, this holds in the unique subgame perfect equilibrium.

Theorem 1 *In period* 2 *with state of the game* S^2 *such that* $a^2_j < v_j$ *for all* j*, the unique subgame Nash equilibrium is for each party i to choose the following campaign resource schedules: for each state* j *in which party i is the incumbent party*

$$\overline{F}^2_{j,i}(x) = \frac{a^2_j}{v_j} + \frac{x}{v_j} \quad x \in \left[0, v_j - a^2_j\right]$$

and for each state j *in which party i is the challenging party*

$$\underline{F}^2_{j,i}(x) = \begin{cases} \frac{a^2_j}{v_j} & x \in \left[0, a^2_j\right) \\ \frac{x}{v_j} & x \in \left[a^2_j, v_j\right] \end{cases}$$

In equilibrium, party A's period 2 payoff is $\sum_{j \in \mathcal{N}^2_A} a^2_j$ *and party B's period 2 payoff is* $\sum_{j \in \mathcal{N}^2_B} a^2_j$.

See Konrad (2002) for a discussion of the single state case of Theorem 1. The proof of uniqueness follows from Baye, Kovenock, and De Vries (1996).

The equilibrium strategies given in Theorem 1 appear to conform to the standard short-run electoral gains arguments against the 50-state strategy. In each state both the advantaged and disadvantaged parties rationally forgo allocating resources with positive probability (a_j/v_j) and the more advantaged or disadvantaged a party is in a given state the more likely that party is to forgo allocating resources to that state. This is essentially a stochastic guerilla warfare strategy. The challenger has incentive to concede the state and allocate zero resources with positive probability. However, when the challenger contests the state he randomizes over the same effective support as the incumbent. Conversely, the incumbent knowing that the challenger will concede the state with positive probability, optimally chooses to leave the state undefended (allocate zero resources) with positive probability and to rely only on the built-up investment advantage.

Observe that each party's period 2 payoff depends critically on the outcome in period 1. In particular, each party only receives a positive expected payoff from the states in which the party is the incumbent and carries over a positive investment advantage from period 1. In the states in which a party is the challenger, the expected payoff is zero. Additionally, for the incumbent the expected payoff is exactly equal to the built-up investment advantage.

We now solve for the subgame perfect equilibrium local strategies in the first period, which are unique for a given initial state S^1.

Theorem 2 *Suppose Assumptions 1 and 2 hold. In the unique subgame perfect equilibrium local strategies in the first period each party i chooses the following campaign resource schedules:*

For each state j in which party i is the incumbent party

$$\overline{F}^1_{j,i}(x) = \frac{a^1_j}{v_j} + \left(\frac{1}{\delta\rho} - \frac{a^1_j}{v_j}\right)\left(1 - e^{-\frac{\delta\rho}{v_j}(x)}\right) \quad x \in \left[0, -\frac{v_j}{\delta\rho}\ln\left(\frac{1-\delta\rho}{1-\frac{\delta\rho a^1_j}{v_j}}\right)\right]$$

and for each state j in which party i is the challenging party

$$\underline{F}^1_{j,i}(x) = \begin{cases} \frac{a^1_j}{v_j} & x \in \left[0, a^1_j\right) \\ \frac{a^1_j}{v_j} + \left(\frac{1}{\delta\rho} - \frac{a^1_j}{v_j}\right)\left(1 - e^{-\frac{\delta\rho}{v_j}\left(x - a^1_j\right)}\right) & x \in \left[a^1_j, a^1_j - \frac{v_j}{\delta\rho}\ln\left(\frac{1-\delta\rho}{1-\frac{\delta\rho a^1_j}{v_j}}\right)\right] \end{cases} \tag{1}$$

In equilibrium, party A's total payoff is $\sum_{j \in \mathcal{N}_A^1} [a_j^1 + (a_j^1)^2(\delta\rho/v_j)]$ *and party B's total payoff is* $\sum_{j \in \mathcal{N}_B^1} [a_j^1 + (a_j^1)^2(\delta\rho/v_j)]$.

Proof For the proof that these strategies form the unique first-stage local strategies of a subgame perfect equilibrium, we begin by establishing the payoffs that result from following the equilibrium strategies in each period. Then, applying the one-stage-deviation principle for finite horizon games, we move on to the examination of deviations from the supports of the equilibrium strategies in period 1, given the (unique) induced equilibrium strategies in period 2. The proof of uniqueness of the first-stage strategies is given in the appendix. Let π_i^t denote the payoff to player i in period t.

Suppose that in period 1 and state S^1 player i uses the equilibrium strategy. We show that if player $-i$ uses any pure strategy $\mathbf{x}_{-i}^1|S^1$ in period 1 that is contained in his equilibrium support, then the expected payoff to player $-i$ for the two-period intertemporal game is

$$E(\pi_{-i}^1) + \delta E(\pi_{-i}^2) = \sum_{j \in \mathcal{N}_{-i}^1} [a_j^1 + (a_j^1)^2(\delta\rho/v_j)].$$

From Theorem 1 we know that in each state j the period 2 payoff is 0 for the challenger and a_j^2 for the incumbent. It follows directly that the payoff from winning state j in period 1 is equal to the value of state j plus the discounted expected value from being the incumbent in state j in period 2, $v_j + \delta \left[a_j^2\right]$, where a_j^2 is the induced investment advantage carried over from the first period.

Suppose player A follows his equilibrium strategy in period 1 and that both players conform to their equilibrium strategies in period 2 given the resulting state S^2. If player B uses any pure strategy contained in the support of the period 1 equilibrium local strategy, the expected payoff to player B in the intertemporal game is

$$\begin{gathered} E(\pi_B^1) + \delta E(\pi_B^2) = \\ \sum_{j \in \mathcal{N}_B^1} \left[v_j + \delta \left(a_j^2\right)\right] \underline{F}_{j,A}^1(0) + \sum_{j \in \mathcal{N}_B^1} \left[\int_0^{x_{j,B}^1 + a_j^1} \left[v_j + \delta \left(a_j^2\right)\right] d\underline{F}_{j,A}^1\left(x_{j,A}^1\right)\right] \\ + \sum_{j \notin \mathcal{N}_B^1} \left[v_j + \delta \left(a_j^2\right)\right] \overline{F}_{j,A}^1(0) + \sum_{j \notin \mathcal{N}_B^1} \left[\int_0^{x_{j,B}^1 - a_j^1} \left[v_j + \delta \left(a_j^2\right)\right] d\overline{F}_{j,A}^1\left(x_{j,A}^1\right)\right] \\ - \sum_{j=1}^n x_{j,B}^1 \end{gathered} \tag{2}$$

The first two summands on the right-hand side of this equation represent party B's expected winnings in the states in which party B is intially the incumbent, while the second two summands represent party B's expected winnings from states in which B is initially the challenger. The final term is the total cost of party B's campaign expenditures in period 1.

If party B is initially the incumbent in state j, $j \in \mathcal{N}_B^1$, and party B wins the campaign in period 1 then the investment advantage that party B enjoys in period 2 is given by $a_j^2 = \rho(x_{j,B}^1 + a_j^1 - x_{j,A}^1)$. Similarly, if party B is initially the challenger in state j, $j \notin \mathcal{N}_B^1$, and party B wins the campaign in period 1, then the investment advantage that party B enjoys in period 2 is given by $a_j^2 = \rho(x_{j,B}^1 - x_{j,A}^1 - a_j^1)$.

Inserting these two expressions, equation (2) may now be written as

$$\begin{array}{c} E(\pi_B^1) + \delta E(\pi_B^{2t}) = \\ \sum_{j\in\mathcal{N}_B^1}[(v_j + \delta\rho\left(x_{j,B}^1 + a_j^1\right))\underline{F}_{j,A}^1\left(x_{j,B}^1 + a_j^1\right) - x_{j,B}^1] \\ +\sum_{j\in\mathcal{N}_B^1}[\int_{a_j^1}^{x_{j,B}^1+a_j^1} -\delta\rho x_{j,A}^1 d\underline{F}_{j,A}^1\left(x_{j,A}^1\right)] \\ +\sum_{j\notin\mathcal{N}_B^1}[(v_j + \delta\rho\left(x_{j,B}^1 - a_j^1\right))\overline{F}_{j,A}^1\left(x_{j,B}^1 - a_j^1\right) - x_{j,B}^1] \\ +\sum_{j\notin\mathcal{N}_B^1}[\int_0^{x_{j,B}^1-a_j^1} -\delta\rho x_{j,A}^1 d\overline{F}_{j,A}^1\left(x_{j,A}^1\right)] \end{array} \tag{3}$$

Inserting in the equilibrium distributions for $\overline{F}_{j,A}^1(\cdot)$ and $\underline{F}_{j,A}^1(\cdot)$ from (1) and simplifying yields $E(\pi_B^1) + \delta E(\pi_B^2) = \sum_{j\in\mathcal{N}_B^1}[a_j^1 + (a_j^1)^2(\delta\rho/v_j)]$.

To complete the proof of the theorem, we now show that neither player can increase his expected payoff by unilaterally deviating to an expenditure off of the equilibrium support (given in (1)) in period 1, given the resulting subgame equilibrium arising in period 2.

To demonstrate this, we break down the examination of potential deviations into two parts: (i) deviations above the upper bound of the support that are small enough that the period 1 margin of victory does not induce the challenger to drop out of the race in period 2, and (ii) deviations above the upper bound that are sufficiently large that the challenger is induced to drop out in period 2.

We begin with case (i). In order for the challenger to not drop out of the race in period 2, it must be the that $a_j^2 < v_j$. (Note that if S^1 satisfies assumptions 1 and 2 and both players are following the equilibrium strategy in period 1 then $a_j^2 < v_j$ with certainty.[12]) Thus, in case (i) the relevant payoff to check is given by equation (3).

From (3) it follows that in any state j in which party i is the incumbent in period 1 player i's expected payoff in state j from using any pure strategy $x_{j.i}^1|S^1$ contained in the

[12] In particular, if both players are following the equilibrium strategy then the maximal value of the period 2 investment advantage, denoted $\overline{a}_j^2$, occurs at the point at which the period 1 challenger allocates zero resources and the incumbent allocates an amount

support of the equilibrium strategy in period 1 is

$$E_j(\pi_i^1)+\delta E_j(\pi_i^2)=[(v_j+\delta\rho\left(x_{j,i}^1+a_j^1\right))\underline{F}_{j,-i}^1\left(x_{j,i}^1+a_j^1\right)-x_{j,i}^1] \\ +[\int_{a_j^1}^{x_{j,i}^1+a_j^1}-\delta\rho x_{j,-i}^1 d\underline{F}_{j,-i}^1\left(x_{j,-i}^1\right)] \quad (4)$$

For the incumbent the support of the equilibrium strategy is given by $[0,-(v_j/(\delta\rho))\ln((1-\delta\rho)/(1-\frac{\delta\rho a_j^1}{v_j}))]$. If player i chooses a pure strategy above the upper bound of the support of the equilibrium strategy in some state j (and this strategy results in $a_j^2<v_j$) then $\underline{F}_{j,-i}^1(x_{j,i}^1+a_j^1)=1$ and from (4) player i's expected payoff in state j is less than $a_j^1+(a_j^1)^2(\delta\rho/v_j)$, i.e., the payoff from not deviating from the support. A similar result applies to states in which party i is the challenger, establishing that no player i has an incentive to deviate from the support of the equilibrium strategy if the deviation does not induce the period 2 challenger to drop out of the race.

In case (ii), the margin of victory in period 1 is large enough (this condition is given by $a_j^2>v_j$) that the challenger drops out of the race in period 2. We will now show that this case is ruled out if S^1 satisfies Assumption 1 and Assumption 2 holds. Observe that if player i is the incumbent and wins in period 1 then $a_j^2=\rho(x_{j,i}^1-x_{j,-i}^1+a_j^1)$ and $a_j^2>v_j$ implies that $x_{j,i}^1>\frac{v_j}{\rho}+x_{j,-i}^1-a_j^1$. At a minimum, for the margin of victory to satisfy the conditions for case (ii) it must be that $x_{j,i}^1>\frac{v_j}{\rho}-a_j^1$.

The payoff to the incumbent from choosing a pure strategy in period 1 which induces the challenger to drop out of the race in period 2 is equal to the value of winning the state in period 1 plus the discounted value of winning the state in period 2 minus the period 1 expenditure $v_j+\delta v_j-x_{j,i}^1$. For this to be a profitable strategy it must be the case that this payoff is greater than the equilibrium payoff in this state $a_j^1+(a_j^1)^2(\delta\rho/v_j)$. As previously noted, for the margin of victory to satisfy case (ii) it must at least be the case that $x_{j,i}^1>\frac{v_j}{\rho}-a_j^1$. The following condition rules out the possibility that any case (ii) deviation is profitable:

$$v_j+\delta v_j-\frac{v_j}{\rho}+a_j^1\leq a_j^1+(a_j^1)^2(\delta\rho/v_j) \quad (5)$$

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equal to the upper bound,

$$\overline{a}_j^2=\rho\left(-\frac{v_j}{\delta\rho}\ln\left(\frac{1-\delta\rho}{1-\frac{\delta\rho a_j^1}{v_j}}\right)+a_j^1\right)$$

but from assumption 2 it follows that $\overline{a}_j^2<v_j$.

Clearly this condition holds under Assumption 2 ($\frac{1}{1+\delta} > \rho$). A similar result applies in the case that player i is the challenger in period 1.

This completes the proof that the strategies given in Theorem 2 form a subgame perfect equilibrium. The proof of uniqueness is given in the appendix. □

The intuition for Theorem 2 is straightforward. In period 1 each party's resource allocation impacts not only the current campaign but also the subsequent campaign. The strategic differences between periods 1 and 2 may be interpreted as reflecting the differences between midterm and presidential election cycles. In this context, the period 1 strategy coincides with a midterm campaign strategy that uses current expenditures to make an investment in the upcoming presidential campaign. Similarly, the period 2 strategy may be interpreted as a presidential campaign strategy of cashing-in on the built-up investment advantages. Clearly, these additional strategic considerations result in discrepancies between optimal short- and long-run campaign strategies.

More formally, the strategic difference between periods 1 and 2 corresponds directly to the difference between the all-pay auction and the combination all-pay auction/war of attrition. In the all-pay auction, each bidder submits a bid, the high bid wins, and all bids are forfeited. The combination all-pay auction/war of attrition differs in that the bidders care not only about winning but also the margin of victory. In particular, for a two-player combination all-pay auction/war of attrition with a common prize worth $v \geq 0$ the payoff function for each player i is given by

$$u_i(x_1, x_2) = \begin{cases} v - x_i + \beta(x_i - x_{-i}) & \text{if } x_i > x_{-i} \geq 0 \\ -x_i & \text{if } x_{-i} > x_i \geq 0 \\ \frac{v}{2} - x_i & \text{if } x_i = x_{-i} \geq 0 \end{cases}$$

The equilibrium strategies in period 1 correspond directly to an extension of the combination all-pay auction/war of attrition, examined earlier by Hirshleifer and Riley (1978) and Riley (1998), to allow for discrimination, in the form a head-start advantage, on the part of the auctioneer. Thus, in period 1, or the midterm election cycle, the parties take into account the margin of victory and its impact on the build up of the investment advantages. The equilibrium strategy in period 2 corresponds directly to an all-pay auction with discrimination in the form of a head-start advantage, as analyzed by Konrad (2002). Thus, in period 2, or the presidential election cycle, the parties do not take into account the margin of victory, but instead cash-in on the built-up investment advantages.

4.1 Discussion

Given Theorems 1 and 2 we now examine the qualitative nature of the equilibrium campaign resource schedules and the optimality of the 50-state strategy. Proposition 1 examines the effects that contest asymmetry and the value of the state have on both parties' expected expenditures.

Proposition 1 *In each period and in each state, the equilibrium expected expenditures of the incumbent and challenger are both increasing in the value of the state (v_j) and decreasing in the investment advantage (a_j^t).*

The period 1 expected expenditure in state j for the incumbent $E_{\overline{F}^1_{j,i}}(x)$, calculated as $E_{\overline{F}^1_{j,i}}(x) = \int_0^\infty x d\overline{F}^1_{j,i}$, is

$$E_{\overline{F}^1_{j,i}}(x) = \left(\frac{v_j}{\delta^2\rho^2}\right)\left[(1-\delta\rho)\ln\left(\frac{1-\delta\rho}{1-\frac{\delta\rho a_j^1}{v_j}}\right) + \delta\rho - \frac{\delta\rho a_j^1}{v_j}\right].$$

It follows directly that the incumbent's expected expenditure is increasing in the value of the state v_j ($dE_{\overline{F}^1_{j,i}}(x)/dv_j > 0$) and decreasing in size of the investment advantage a_j^1 ($dE_{\overline{F}^1_{j,i}}(x)/da_j^1 < 0$). Note that as the investment advantage a_j^1 decreases the race in state j becomes more symmetric. That is, as the race becomes more symmetric the incumbent's expected expenditure increases.

For the challenger, the period 1 expected expenditures in state j, $E_{\underline{F}^1_{j,i}}(x)$, are given by,

$$E_{\underline{F}^1_{j,i}}(x) = \left(\frac{v_j}{\delta^2\rho^2}\right)\left[(1-\delta\rho)\left(\frac{-\delta\rho a_j^1}{v_j} + \ln\left(\frac{1-\delta\rho}{1-\frac{\delta\rho a_j^1}{v_j}}\right)\right) + \delta\rho - \left(\frac{\delta\rho a_j^1}{v_j}\right)^2\right].$$

As with the incumbent, the challenger's expected expenditure is decreasing in the investment advantage ($dE_{\underline{F}^1_{j,i}}(x)/da_j^1 < 0$) and increasing in the value v_j of the state ($dE_{\underline{F}^1_{j,i}}(x)/dv_j > 0$).

For any given value of $\delta\rho$ which satisfies Assumption 2 representative iso-expected expenditures for the incumbent and the challenger are given in Figure 2 below. The combinations of (v_j, a_j^1) that satisfy Assumption 1 lie below the 45^o line. The solid lines correspond to level curves of expected expenditures, which are increasing as you move southwest from any $(v_j, a_j^1) \in \mathbb{R}^2_{++}$ which satisfies Assumption 1.

[Insert Figure 2]

The period 2 expected expenditure in state j for the incumbent is $E_{\overline{F}^2_{j,i}}(x) = (v_j^2 - (a_j^2)^2)/(2v_j)$, and the period 2 expected expenditure in state j for the challenger is $E_{\underline{F}^2_{j,i}}(x) = (v_j - a_j^2)^2/(2v_j)$. Clearly, both of these expressions are also increasing in v_j and decreasing[13] in a_j^2.

Proposition 2 examines the optimality of the 50-state strategy. Recall that the basic argument for the 50-state strategy is that campaign expenditures are durable outlays that build the party up for future campaigns and, thus, strictly positive levels of campaign resources should be allocated to each of the states. Conversely, the basic argument against the 50-state strategy is that "the way you build long-term is to succeed short-term" and short-term success requires that you focus on the battleground states.

Since equilibrium in our model requires randomization at each stage, we may compare the likelihood that a party chooses the 50-state strategy in the case that campaign expenditures *are* durable outlays (period 1) and in the case that they *are not* durable outlays (period 2).

Proposition 2 *Regardless of whether or not campaign expenditures are durable outlays, the likelihood that a party chooses the 50-state strategy is equal to* $\prod_{j=1}^{n}\left(1 - \frac{a_j^t}{v_j}\right)$ *for* $t = 1, 2$.

In each state both advantaged and disadvantaged parties may rationally forgo allocating resources to a state with positive probability. The likelihood that a party forgoes allocating campaign resources to a state is increasing in its advantage, or disadvantage, in that state. That is the battleground states, in which the parties' are the most symmetric, are the most likely to receive a positive level of resources. Since the randomization employed by a party in its equilibrium strategy is independent across states, the probability of employing a 50-state strategy is simply the product of the respective probabilities of allocating a positive level of the resource to each state j. The probability that each party allocates a positive level of the resource to state j is $(1 - (a_j^t/v_j^t))$ for $t = 1, 2$. Thus, if parties behave strategically and optimize given the behavior of their rival, a 50-state strategy is a seemingly unlikely outcome.

[13] Note that Assumption 2 implies that $a_j^2 < v_j$ with certainty in any equilibrium realization of a_j^2.

To summarize, equilibrium expected expenditures for both parties are increasing in the value of a state and decreasing in the incumbent head-start advantage. The incidence of zero expenditure is identical for both parties, increasing in the incumbency advantage, and decreasing in the value of the state. States with no incumbency advantage receive positive allocations with certainty from both parties regardless of the value of the state.

These predictions appear to be consistent with evidence appearing in Figure 1 and Table 3 of Strömberg (2008), which provide data on the parties' presidential and vice presidential candidates' post-convention campaign visits during the 2000 and 2004 elections. As noted by Strömberg, and also predicted by our model, large states with close forecasted vote shares tend to receive a larger number of campaign visits by both parties. Smaller states in which the parties are close in vote share are likely to obtain a smaller, but positive, number of campaign visits from the two parties. Moreover, the data show that, in both election campaigns, states with large forecasted vote share differences were quite likely to receive zero campaign visits by both parties and that several states were visited by one party but not the other.[14] These data appear inconsistent with the model examined by Strömberg, who assumes an interior equilibrium in each state in which the parties expend identical positive levels of the resource within the state.[15] However, the data appear consistent with our model, in which parties allocate zero resources to a state with positive probability (unless the two parties contest the state symmetrically) and, due to the equilibrium mixed strategies, generally allocate different levels of the resource to a given state.

5 Conclusion

The standard argument for the 50-state strategy is that campaign expenditures constitute a long-run investment that will build up the party for future election cycles. This paper examines the optimality of this strategy in a simple intertemporal model of political campaign resource allocation with persistent campaign expenditures. The equi-

[14] In the 2000 elections, 24 states received no post-convention visits by both parties' candidates and two states received no visits by one party's candidates. In the 2004 elections, 20 states received no post-convention visits by both parties' candidates and 13 states received no visits by only one party's candidates.

[15] Strömberg (2008) claims that in his model a unique interior pure strategy equilibrium always exists. However, it is easily verified that this is not the case for sufficiently small variance of his state and national popularity parameters.

librium in period 2 illustrates the standard short-run gains arguments against the 50-state strategy. Each party plays a modified battleground-states strategy in which they stochastically forgo allocating resources to states in which they are either advantaged or disadvantaged and more highly contest the battleground states. In the first period, parties optimally utilize the persistence of campaign expenditures to invest in the period 2 campaign, at which time built-up investment advantages are cashed-in. However, even in period 1 the basic structure of the modified battleground-states strategy arises. That is, even with persistent campaign expenditures, the short-term electoral gains from focusing (stochastically) on the battleground states outweigh the long-term party building gains from investing in all of the states.

Appendix

The following lemmas establish the uniqueness of the period 1 subgame perfect equilibrium campaign resource allocation schedules for $a_j^1 > 0$.[16] Let $\overline{F}_j^1$ denote the incumbent's period 1 campaign resource allocation schedule in state j and let $\overline{s}_{j,I}$ and $\underline{s}_{j,I}$ denote the upper and lower bounds, respectively, of the support of $\overline{F}_j^1$. Let $\underline{F}_j^1$, $\overline{s}_{j,C}$, and $\underline{s}_{j,C}$ be similarly defined for the challenger.

The first two lemmas characterize the necessary conditions that arise in all of the possible configurations of the lower bound of the supports.

Lemma (A.1) *If $\underline{s}_{j,C} < \underline{s}_{j,I} + a_j^1$ then (1) $\underline{F}_j^1$ is constant over the half-open interval $(0, \underline{s}_{j,I} + a_j^1]$, (2) $\underline{s}_{j,C} = 0$, (3) $\underline{F}_j^1(0) > 0$, and (4) $\underline{s}_{j,I} = 0$.*

Proof Recall that in period 1 when the incumbent spends $\underline{s}_{j,I}$ in state j the effective expenditure is $\underline{s}_{j,I} + a_j^1$. Suppose there exists an equilibrium in which $\underline{s}_{j,C} < \underline{s}_{j,I} + a_j^1$. For any campaign expenditure at or below $\underline{s}_{j,I} + a_j^1$ the challenger loses in state j with certainty. Furthermore, the period 2 payoff is zero in any state which was lost in period 1. Thus, it is suboptimal for the challenger to choose any period 1 expenditure in the half-open interval $(0, \underline{s}_{j,I} + a_j^1]$, where $\underline{s}_{j,I} + a_j^1$ is included due to the tie-breaking rule.

To demonstrate (2) and (3) note that if $\underline{s}_{j,C} < \underline{s}_{j,I} + a_j^1$, then $\underline{F}_j^1(0) > 0$ since $\underline{F}_j^1$ is constant over $(0, \underline{s}_{j,I} + a_j^1]$.

To prove (4) note that $\underline{F}_j^1$ is constant over the half-open interval $(0, \underline{s}_{j,I} + a_j^1]$. Thus, if $\underline{s}_{j,I} > 0$ the incumbent can increase his payoff by setting $\underline{s}_{j,I} = 0$. □

[16] See Riley (1998) for the uniqueness argument with no head-start.

Lemma (A.2) *If $\underline{s}_{j,C} \geq \underline{s}_{j,I} + a_j^1$ then (1) $\underline{s}_{j,I} = 0$, (2) $\overline{F}_j^1(0) > 0$, (3) $\underline{s}_{j,C} = a_j^1$, and (4) $\underline{F}_j^1(a_j^1) = 0$.*

Proof Suppose $\underline{s}_{j,C} \geq \underline{s}_{j,I} + a_j^1$. For any campaign expenditure below $\underline{s}_{j,C} - a_j^1$ the incumbent loses in state j with certainty. As previously noted, the period 2 payoff is zero in any state which was lost in period 1. Thus, it is suboptimal for the incumbent to choose any period 1 expenditure in the interval $(0, \underline{s}_{j,C} - a_j^1)$.

For (1) and (2), note that if $\underline{s}_{j,C} \geq \underline{s}_{j,I} + a_j^1$, then $\underline{s}_{j,I} = 0$ and $\overline{F}_j^1(0) > 0$, since $\overline{F}_j^1$ is constant over $(0, \underline{s}_{j,C} - a_j^1)$.

For (3), $\overline{F}_j^1$ is constant over the interval $(0, \underline{s}_{j,C} - a_j^1)$. Thus, if $\underline{s}_{j,C} > a_j^1$ the challenger can increase his payoff by slightly lowering $\underline{s}_{j,C}$ towards a_j^1.

For (4), note that with any campaign expenditure at or below a_j^1 the challenger loses with certainty. Thus, it is suboptimal for the challenger to place positive mass on a_j^1. $\square$

Lemmas (A.1) and (A.2) provide the only two possible cases. The remaining parts of the proof establish that it must be Lemma (A.1) that applies, $\underline{s}_{j,C} < \underline{s}_{j,I} + a_j^1$, and that in this case there is a unique equilibrium.

In the following lemmas we will restrict our attention to the case that the incumbent does not choose a period 1 pure strategy that is large enough that the period 1 challenger not only loses in period 1 but also drops out of the period 2 race. Earlier arguments in the proof of Theorem 2 showed that Assumption 2 rules out the optimality of any such strategies.

Lemma (A.3) $\overline{s}_{j,I} = \overline{s}_{j,C} - a_j^1$.

Proof Suppose that the incumbent chooses to spend $x_{j,I}^1 > \overline{s}_{j,C} - a_j^1$ in state j. From equation (2), the incumbent's expected payoff in state j is

$$v_j + \delta\rho\left(x_{j,I}^1 + a_j^1\right) - \delta\rho E_{\underline{F}_j^1}(x) - x_{j,I}^1$$

Since $\delta\rho < 1$, in any equilibrium the incumbent sets $\overline{s}_{j,I} \leq \overline{s}_{j,C} - a_j^1$.

Similarly, suppose that the challenger chooses to spend $x_{j,C}^1 > \overline{s}_{j,I} + a_j^1$ in state j. From equation (2), the challenger's expected payoff in state j is

$$v_j + \delta\rho\left(x_{j,C}^1 - a_j^1\right) - \delta\rho E_{\overline{F}_j^1}(x) - x_{j,C}^1$$

Since $\delta\rho < 1$, in any equilibrium the challenger sets $\overline{s}_{j,C} \leq \overline{s}_{j,I} + a_j^1$. $\square$

Lemma (A.4) *Neither player $i = C, I$ places positive mass on any strictly positive point in the support of their campaign resource allocation schedule.*

Lemma (A.4) follows directly from Lemma 1 in Riley (1998).

Lemma (A.5) $\underline{F}_j^1$ *is strictly increasing over* $[a_j^1, \bar{s}_{j,C}]$ *and* $\overline{F}_j^1$ *is strictly increasing over* $(0, \bar{s}_{j,I}]$.

Proof By way of contradiction, suppose that there exists an equilibrium in which $\underline{F}_j^1$ is constant over the interval $[\alpha, \beta) \subset [a_j^1, \bar{s}_{j,C}]$ and strictly increasing above β in its support. For this to be an equilibrium, it must be the case that $\overline{F}_j^1$ is also constant over the interval $[\alpha - a_j^1, \beta - a_j^1)$. Otherwise, the incumbent could increase his payoff.

If $\overline{F}_j^1(\alpha - a_j^1) = \overline{F}_j^1(\beta - a_j^1)$, then for any $\varepsilon > 0$ spending $\beta + \varepsilon$ in state j cannot be optimal for the challenger. Indeed, from Lemma (A.4) discretely reducing expenditure from $\beta + \varepsilon$ to $\alpha + \varepsilon$ would strictly increase the challenger's payoff. Consequently, if $\underline{F}_j^1$ is constant over $[\alpha, \beta)$ it is constant over $[\alpha_j^1, \bar{s}_{j,C}]$, a contradiction to the definition $\bar{s}_{j,C}$. $\square$

The following two lemmas utilize the following properties of the equilibrium expected payoffs. Recall that for the incumbent the expected payoff in state j from using any pure strategy $x_{j.I}^1$ contained in the support of the equilibrium strategy in period 1 is

$$E_j(\pi_I^1) + \delta E_j(\pi_I^2) = [(v_j + \delta\rho\left(x_{j,I}^1 + a_j^1\right))\underline{F}_j^1\left(x_{j,I}^1 + a_j^1\right) - x_{j,I}^1] + [\int_{a_j^1}^{x_{j,I}^1 + a_j^1} -\delta\rho x_{j,C}^1 d\underline{F}_j^1\left(x_{j,C}^1\right)] \quad (6)$$

Equilibrium payoffs must be attained over the support of the incumbent's strategy. From Lemmas A.4 and A.5, the players randomize continuously on the intervals $[a_j^1, \bar{s}_{j,C}]$ and $(0, \bar{s}_{j,I}]$, respectively. Thus, differentiating (5) with respect to $x_{j.I}^1$ it follows that for $x_{j,I}^1 \in (0, \bar{s}_{j,I}]$,

$$d\underline{F}_j^1\left(x_{j,I}^1 + a_j^1\right) = \frac{1}{v_j} - \frac{\delta\rho}{v_j}\underline{F}_j^1\left(x_{j,I}^1 + a_j^1\right). \quad (7)$$

A similar argument establishes that for $x_{j,C}^1 \in [a_j^1, \bar{s}_{j,C}]$,

$$d\overline{F}_j^1\left(x_{j,C}^1 - a_j^1\right) = \frac{1}{v_j} - \frac{\delta\rho}{v_j}\overline{F}_j^1\left(x_{j,C}^1 - a_j^1\right). \quad (8)$$

Equations (6) and (7) are first order linear differential equations with solutions:

$$\underline{F}_j^1(x) = \frac{1}{\delta\rho} - \frac{K_1}{\delta\rho}\exp\left(-\frac{\delta\rho}{v}(x)\right) \quad (9)$$

for $x \in [a_j^1, \overline{s}_{j,C}]$ and constant K_1 and

$$\overline{F}_j^1(x) = \frac{1}{\delta\rho} - \frac{K_2}{\delta\rho} \exp\left(-\frac{\delta\rho}{v}(x)\right) \tag{10}$$

for $x \in [0, \overline{s}_{j,I}]$ and constant K_2.

Lemma (A.6) $\underline{s}_{j,C} < \underline{s}_{j,I} + a_j^1$.

Proof By way of contradiction, suppose that there exists an equilibrium in which $\underline{s}_{j,C} \geq \underline{s}_{j,I} + a_j^1$. From Lemma (A.2), $\underline{s}_{j,C} = a_j^1$ and $\underline{F}_j^1(a_j^1) = 0$. Combining this with $\underline{F}_j^1(\overline{s}_{j,C}) = 1$, it follows from (A.4), (A.5), and (9) that $\overline{s}_{j,C} = a_j^1 - \frac{v_j}{\delta\rho}\ln(1-\delta\rho)$.

From Lemma (A.4), $\overline{s}_{j,I} = \overline{s}_{j,C} - a_j^1$ and $\overline{s}_{j,C} - a_j^1 = -\frac{v_j}{\delta\rho}\ln(1-\delta\rho)$. From $\overline{F}_j^1(\overline{s}_{j,C} - a_j^1) = 1$, it follows from (10) that $K_2 = \exp\left(\frac{\delta\rho}{v_j}\right)$ or equivalently $\overline{F}_j^1(x) = \frac{1}{\delta\rho} - \frac{1}{\delta\rho}\exp\left(-\frac{\delta\rho}{v_j}(x)\right)$. Thus, $\overline{F}_j^1(0) = 0$ which contradicts point (2) of Lemma (A.2). That is, $\overline{F}_j^1(0) > 0$ and consequently the conditions of Lemma (A.1) hold. $\square$

Lemma (A.7) *There exists a unique set K_1, K_2, and $\overline{s}_{j,C}$ which forms an equilibrium.*

Proof From Lemma (A.1), $\underline{F}_j^1(0) > 0$. Since the challenger earns an expected payoff of zero by setting $x_{j,C}^1 = 0$, his expected payoff is zero for each point in the support of his equilibrium campaign resource allocation schedule (except possibly at the point $x_{j,C}^1 = a_j^1$ for which the incumbent has a mass point at 0). As $x_{j,C}^1$ converges to a_j^1 from above, the challenger's expected payoff converges to $v_j\overline{F}_j^1(0) - a_j^1$. Setting this equal to zero, $\overline{F}_j^1(0) = \frac{a_j^1}{v_j}$.

From $\overline{F}_j^1(0) = \frac{a_j^1}{v_j}$, it follows from (10) that $K_2 = (1 - \frac{\delta\rho a_j^1}{v_j})\exp(\frac{\delta\rho}{v_j})$ or equivalently $\overline{F}_j^1(x) = \frac{1}{\delta\rho} - \left(\frac{1}{\delta\rho} - \frac{a_j^1}{v_j}\right)\exp\left(-\frac{\delta\rho}{v_j}(x)\right)$. Thus, from (9) $\overline{s}_{j,C} - a_j^1 = -\frac{v_j}{\delta\rho}\ln\left(\frac{1-\delta\rho}{1-\frac{\delta\rho a_j^1}{v_j}}\right)$.

From (A.3), setting $\underline{F}_j^1(\overline{s}_{j,C}) = 1$, it follows that $K_1 = (1 - \frac{\delta\rho a_j^1}{v_j})\exp(\frac{\delta\rho a_j^1}{v_j})$ or equivalently $\underline{F}_j^1(x) = \frac{1}{\delta\rho} - \left(\frac{1}{\delta\rho} - \frac{a_j^1}{v_j}\right)\exp\left(-\frac{\delta\rho}{v_j}(x - a_j^1)\right)$. $\square$

To complete the proof note that the K_1, K_2, and $\overline{s}_{j,C}$ characterized in Lemma (A.7) result in the unique equilibrium distributions given in Theorem 2, and from arguments made in the proof of Theorem 2, it is suboptimal for either player to deviate from the support of this strategy.

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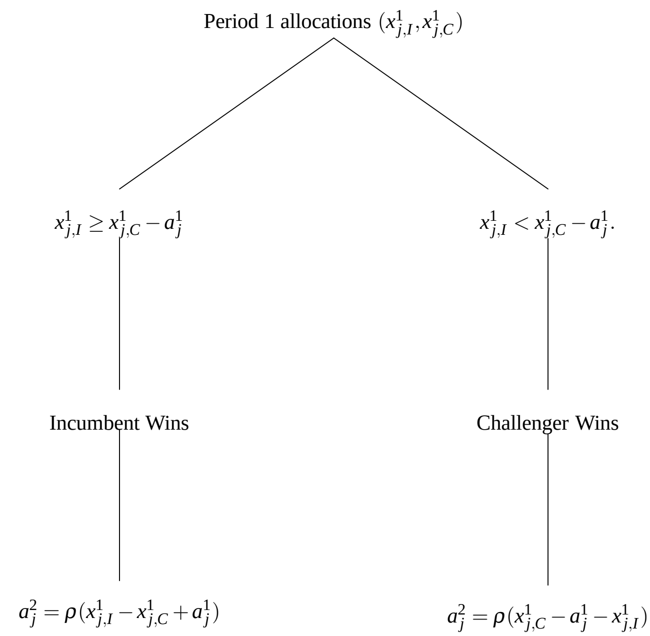

Fig. 1 Investment Advantage Dynamics

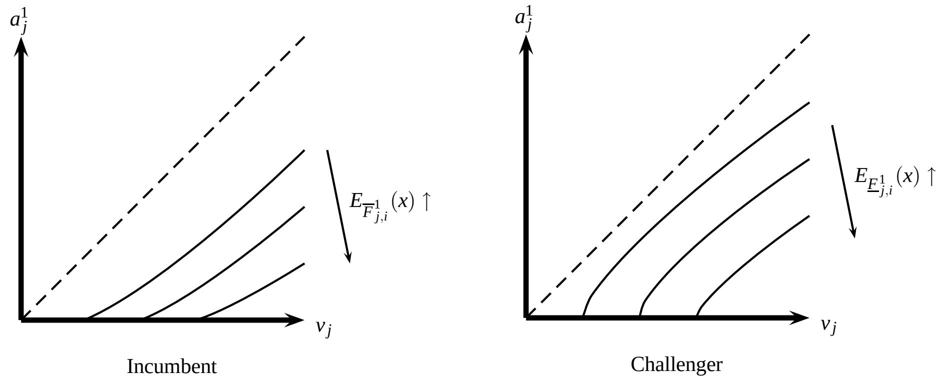

Fig. 2 Period 1 iso-expected expenditures for the incumbent and challenger