Influence activities, coalitions, and uniform policies

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Abstract

This paper examines the effects of endogenous coalition formation in a setting where agents lobby a policy-maker. Our motivating example is of insiders lobbying for weaker regulation to allow for privately beneficial but socially-wasteful diversion. Policy uniformity (e.g., one-size-fits-all rules) cause agents to free ride on each other's lobbying and gives them an incentive to form lobbying coalitions, i.e., lobbies. We show that the coalition formation mechanism influences whether lobbies are formed by similar or dissimilar agents. Additionally, endogenous lobby formation causes the effects of policy uniformity and lobbying costs on aggregate lobbying activity and policy strength to be non-monotonic.

Keywords: lobbies, coalitions, one-size-fits-all, regulation

JEL Codes: D72, G38, L51, M40

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1 Introduction

Policies (e.g., regulations) are usually set with a degree of uniformity, applying similar treatments to diverse agents. These agents, in turn, often have the ability to influence the policies that they are exposed to and can organize into coalitions or lobbies that coordinate their influence activities. In this paper, we explore the agents' choices of how to organize into lobbies, and how agents' ability to organize endogenously influences their lobbying activities and a policy-maker's choices.

We focus on a setting in which firm insiders have the ability to expropriate value from outsiders. Regulation reduces insiders' opportunities to expropriate value and can improve welfare, because expropriation causes deadweight losses. In our model, a regulator or enforcement agency decides about the regulation or enforcement policies for a set of heterogeneous firms. The regulator is interested in reducing the deadweight losses from expropriation, but can be influenced by insiders, who lobby for weaker regulation. While the regulator is constrained to treat the firms similarly, the firms differ in their insiders' abilities to take actions that are socially harmful. We model the constraint on the regulator's choice as costs associated with regulatory heterogeneity. As a result, different firms are exposed to similar policies. This causes a free rider problem among firms when they can lobby the regulator to apply weaker standards (Friedman and Heinle [2016]). To overcome the free-rider problem and, thus, to more effectively reduce the extent of regulation or enforcement, firms can organize into coalitions or lobbies. Firms within a coalition choose their lobbying efforts to maximize the joint utility of all firms in the coalition. That is, firms mitigate the free rider problem because they internalize the effect of their lobbying on the other firms in the coalition. We endogenize the formation of lobbies and analyze the characteristics of the coalitions that form.

The baseline model consists of three firms. We assume that there is a cost of forming a coalition that increases in the size of the coalition. This cost, in essence, represents the cost that firms face to overcome the free-rider problem. Absent the cost, the grand coalition (i.e., the lobby including all insiders) would always be optimal, due to the positive externalities that insiders enjoy from each other's lobbying. Furthermore, we initially assume that specialists (or lobbyists) are necessary to form coalitions. For simplicity, we assume that lobbyists extract the full gains from coalition formation, holding the insiders at what amounts to their reservation utility, and precluding lobbyists from making positive-utility offers to insiders to deviate from existing coalitions.[1] In equilibrium, before coalition formation costs, lobbyists always prefer to form coalitions of firms with greater divertable amounts, as these have the greatest gains from formation.

As an alternative, we explore the implications of allowing insiders to form coalitions themselves, without lobbyists. In this setting, we introduce the notion of offer-stable coalitions, defined essentially as coalitions that, if a member is made an offer to deviate, can prevent such a deviation by making a credible counteroffers to that coalition member. Interestingly, we find that this coalition formation mechanism can support the grand coalition, the large and medium firms colluding, or the small and large firms colluding. The only two-member coalition that is never stable is the one that includes the small and medium firms. In other words, offer stable coalitions can be between either similar firms (i.e., medium and large), or between dissimilar firms (small and large). In contrast, lobbyist-formed coalitions always include similar firms.

Regardless of the coalition formation mechanism, we show that regulatory uniformity has non-monotonic effects on aggregate lobbying and policy strength when coalitions form endogenously. This occurs because the primary effect of uniformity in promoting free-riding between lobbying insiders, also encourages them to form coalitions. That is, uniformity tends to decrease lobbying and strengthen regulatory policies, except around thresholds that cause insiders to coalesce into new or larger lobbying coalitions. These coalitions cause discrete jumps in the insiders' ability to overcome free-rider problems, leading to increased lobbying and, in turn, weaker policies.

[1] This is similar in spirit to the bargaining approach to coalition formation described in Ray and Vohra [1999].

Extending the model, we allow for a continuum of firms, rather than only three. In the continuum-of-firms variant, we only allow for lobbyist-formed coalitions, as this avoids the curse of dimensionality associated with coalition formation as the number of potential coalition members grows (see, e.g., Ray and Vohra [2014]). We show that the intuition derived in the three-firm model substantially carries over to a setting with a continuum of firms. We also present conditions for signing the effect of regulatory uniformity on lobby sizes and aggregate lobbying, based on the relation between coalition formation costs and coalition size.

Our motivating example is one of securities regulation and enforcement related to asset diversion or misappropriation. For example, U.S. regulators recently proposed expanded compensation clawback and deferred bonus terms for executives at financial firms, ostensibly to prevent managers from benefitting from potentially excess compensation (Borak, Ackerman, and Rexrode [2016]).[2] While the proposed rules apply to all financial institutions, specific provisions would vary with firm size. For firms with over $250 billion in assets, the new restrictions would apply to senior executives and the employees in the top 5% of the firm's pay distribution, while for firms with $50 to $250 billion, only senior executives and employees in the top 2% of the pay distribution would be affected. From a modeling perspective, the proposed regulation reflects a degree of uniformity, since all financial firms face clawback and deferred bonus restrictions, as well as a degree of sized-based discrimination, as the provisions at larger firms would apply to a broader swath of employees. With respect to lobbying, the financial industry features many examples of lobbying coalitions, e.g., the Financial Services Roundtable and the Circa coalition of activist investors (Benoit [2016]).

Beyond the example of securities regulation, and the specific example of executive pay at financial institutions, the features in our model carry over to several settings. The key tensions are a policy maker that is constrained in some way to set similar policies for a

[2] The regulation was proposed jointly by six regulators, including the Securities and Exchange Commission (SEC), National Credit Union Administration, Federal Deposit Inusrance Corporation, Federal Reserve, Office of the Comptroller of the Currency, and the Federal Housing Finance Agency.

set of heterogeneous agents, where the agents differ in their abilities to take actions that are socially harmful. For example, the insiders in our model could be lobbying for weaker employee-protection regulations, or for fewer restrictions on investments that impose negative externalities, e.g., involving pollution. Similarly, with slight modification, our model can be interpreted as representing a single industry seeking trade protection or a set of countries seeking to influence product standards set out in a trade deal (e.g., Charlemagne [2016]).[3]

The foundation for our study is the literature on regulatory choice in economics (e.g., Arrow [1950]), which helps explain observed choices by highlighting how lobbying and regulatory capture cause regulators to choose non-welfare-maximizing rules and transfers (e.g., Stigler [1971]; Grossman and Helpman [1996]). Our model is most closely related to Friedman and Heinle [2016], who present a similar two-firm model involving a regulator who can probabilistically prevent asset diversion through regulation but is subject to regulatory capture and lobbying pressure. However, the model in Friedman and Heinle [2016] precludes the formation of lobbying coalitions, leaving firms no way to overcome the free-riding problem generated by uniform policies. Bebchuk and Neeman [2010] investigate a model in which different groups lobby the regulator over the level of investor protection in a perfectly uniform regulatory regime. Although this is a regime in which coalitions would be most valuable (which we show in our model here), lobbying coalitions are assumed impossible. In Bertomeu and Magee [2014], regulatory outcomes are chosen by a combination of a majoritarian vote by firms and the standard setter's bliss point. In their model, as in most that feature voting as the policy selection tool, there is limited scope for the type of collusion via coalitions that we explore here.

Related studies in the trade literature also explore endogenous lobby formation. Mitra [1999], for example, allows firms to form lobbies endogenously to coordinate their efforts on lobbying for trade protections or subsidies. Lobbies pay an exogenous cost of organizing, but

[3] As a related example, Rodrik [1986] focuses on trade-offs between industry-wide tariffs and firm-specific subsidies in a setting in which industry-wide tariffs promote free-riding on firms' tariff-seeking. Damania and Fredriksson [2000] explore coalition formation and lobbying in a model focused on environmental protections.

only firms within the same industry will organize with each other. In this way, the bounds of a lobby are essentially exogenous, as firms are exogenously assigned to industries and the regulator, by definition, treats firms within an industry homogeneously. Furthermore, lobbies in different industries compete for trade subsidies, generating negative externalities, in contrast to the lobbies in our model which have positive externalities on each other. Magee [2002], building on Pecorino [1998], models a single industry with homogeneous firms and a focus on the interplay between industry concentration and the ease of overcoming free-riding between firms. Drazen, Limão, and Stratmann [2007] examine the effect of caps on political spending, finding potentially unintended consequences because caps can improve the per-dollar efficiency of lobbying. In a model with endogenous lobby formation, this increase in efficiency can cause greater entry of lobbyists, leading to greater equilibrium lobbying and increasing the consequent policy distortions.

2 Model setup

Our baseline model extends the model in Friedman and Heinle [2016] to three firms that can endogenously form lobbying coalitions with each other. Later, we extend the model to a continuum of firms, but focus first on a three-firm model in which the key tradeoffs are clearer. Allowing for endogenous coalition formation introduces substantial complication into the model. Primarily, the complication relates to issues that arise in any model of endogenous coalition formation, i.e., the combinatorial problems of cooperative game theory. Furthermore, two important features of our model limit our ability to use well-known cooperative game theory solution techniques or apply the results of earlier studies.

First, the value of a given coalition to its insiders depends on the overall coalition structure, meaning that we cannot write a function for the value of a coalition that depends only the characteristics of that coalition and ignores the overall coalition structure. This means that solutions based on characteristic functions are not applicable (e.g., those based on the core and Shapley value; see Myerson [2013]). However, Thrall and Lucas [1963], and Myerson [1978] analyze games in partition function form, wherein the value to a coalition depends on the entire coalition structure. Thrall and Lucas [1963] present results primarily for 2- and 3-player games, and allow for transfers across coalitions, which we prohibit. Myerson [1978] explores the role of commitments to threat strategies, although these threats may not be sequentially rational *ex post*. Ray and Vohra [1997, 1999] and Yi [1997] also explore games with externalities in which, by definition, a player's utility is influenced by the coalition structure as long as the coalition structure influences other players' equilibrium actions. Yi [1997] develops rules for stable coalition structures in the presence of positive externalities, but assumes (Yi's condition P.2) that per-member payoffs are decreasing in coalition size. In our model, the payoff structure emerges as a function of the regulatory environment, and is not in general characterized by per-member payoffs that decrease in coalition size.

Second, by the nature of our focus on the importance of regulatory uniformity, it is crucial for the players, in our model firms or insiders, to be heterogeneous. In a model with homogeneous firms, restricting the regulator to any degree of regulatory uniformity is free, as the regulator optimally desires to set homogeneous regulation across the cross-section of firms. The lack of homogeneity means that we care about which firms are members of which lobbies or coalitions, and cannot simply use coalition size as an outcome variable of interest, as in the model of Bloch [2002]. A benefit of allowing for heterogeneity is that we can derive predictions on which firms find it optimal to associate with each other, and which associations can be sustained in equilibrium.

In our model there are three firms, indexed by i, and a regulatory agency. Again, we use firms with agency problems related to asset diversion as our central example, but note that the key forces apply generally in settings featuring coalitions of agents attempting to influence uniform policies. Because of the similarity of the lobbying-diversion game, we borrow large parts of the model description from Friedman and Heinle [2016]. Firms are composed of risk-neutral insiders and outsiders, between whom there is an agency conflict. While outsiders have a claim on the assets or cash flows of the firm, insiders have an opportunity to pursue private benefits, which represent, for example, diversion of funds or consumption of perks and slack. To fix language, we refer to the personally beneficial action that the insider takes as diversion of funds. We use the index, i, also to indicate the insider associated with firm i.

Specifically, when the insider diverts funds, she gains $D_i > 0$, but this imposes a cost on outsiders of $A_i = (1 + \lambda) D_i$, where $\lambda > 0$.[4] Insider diversion of funds is therefore socially inefficient and imposes a net welfare loss of $D_i\lambda > 0$. Our assumption of $D_i > 0$ implies that the insider always prefers to take the personally beneficial action.[5] Furthermore, we assume that outsiders cannot infer in a timely fashion whether the insider has diverted funds, and thus cannot write an effective contract. We denote firms by s, m, and ℓ, for small, medium, and large, respectively, with $D_s < D_m < D_\ell$ and, for ease of exposition, we refer below to firms with higher D_i as larger or worse firms and firms with lower D_i as smaller or better firms, as the heterogeneity captures potential diversion that might be related to firm size.

Regulation limits the insider's opportunity to divert. Specifically, we model the intensity of regulation governing each firm i as the probability, π_i, that an insider is unable to divert. The insider can therefore divert with probability $(1 - \pi_i)$. Finally, before the regulator specifies the regulatory intensities, each insider can exert effort B_i to lobby the regulator to relax the regulatory intensity for his firm. Insiders and outsiders cannot contract on the type of lobbying activity we model, nor can insiders commit *ex ante* not to lobby *ex post*. The inability to contract or commit on this dimension of influence seems plausible, as, for example, it would be difficult for arms-length investors to determine what exact policies managers were promoting in private meetings with regulators, i.e., whether managers were pursuing beneficial trade protections or harmful regulatory slack.

Insiders benefit only from potential diversion. Each insider incurs a personal cost of lobbying the regulator, $\frac{c}{2}B_i^2$. The parameter $c > 0$ captures the ability of outsiders to effectively

[4]Firms in our model are heterogeneous in the size of the potential diversion, although they are homogeneous in the proportional costs of diversion, $1 + \lambda$.

[5]We assume the cost, A_i is borne only by the outsiders.

monitor and deter insiders' lobbying. A higher value of c reflects a less severe insider-outsider agency problem on lobbying that facilitates the subsequent diversion problem. Each insider's expected utility is given by

$$U_i = (1 - \pi_i) D_i - \frac{c}{2} B_i^2. \tag{1}$$

With probability $(1 - \pi_i)$, the insider is able to take the personally beneficial action and consume D_i. Insiders always bear the cost of lobbying because they lobby the regulator before potential diversion occurs. We assume that insiders cannot commit to "share the spoils" with the regulator and do not use their diverted resources to extract regulatory concessions.[6] Outsiders in our model are passive players, who either lose A_i or do not.

When the regulator decides on the regulatory intensity, the costs of lobbying, $\frac{c}{2}B_i^2$, are sunk. Therefore, the aggregate utility that can be influenced by the regulator is given by the expected losses from diversion:

$$L(\pi, D, \lambda) = -\lambda \sum_{i \in \{s,m,\ell\}} D_i (1 - \pi_i). \tag{2}$$

The welfare-interested regulator is only concerned about diversion because of the welfare loss, $\lambda \bar{D}$, that it imposes on society.[7] This welfare loss occurs with probability $(1 - \pi_i)$, for each firm i. The regulator wants to minimize this welfare loss subject to the costs of regulation.[8]

Regulation is costly for three reasons. First, regulation is costly in and of itself, with a convex cost of regulation, $\frac{1}{2}\pi_i^2$ for each firm. Second, each insider can influence the regulator through lobbying activity B_i, which increases the cost of regulatory intensity by $B_i \pi_i$. Third, defining a different regulatory intensity for different firms imposes additional costs, which we model as $\frac{k}{2}\sum_i \left(\pi_i - \bar{\pi}_{\setminus i}\right)^2$, where $\bar{\pi}_{\setminus i} = \frac{1}{2}\sum_{i' \neq i} \pi_{i'}$. Given that heterogeneous regulation

[6] Relaxing this assumption would not substantially change our results.

[7] Note that this implies that the regulator would optimally allow insiders to divert when $\lambda = 0$. In a more general model, allowing insiders to divert could reduce outsiders' ex ante investment incentives, leading to a welfare-destroying under-investment problem.

[8] See Friedman and Heinle (2016) for further discussion of interpretations of c, λ, and π_i.

plausibly requires greater care in drafting and increased expenditures in enforcement (e.g., staff costs), k can be interpreted as a technical constraint on the regulator. Alternatively, k could be an institutional commitment (for example, a mission statement) to regulate different firms in a similar fashion. When $k = 0$, the regulator is free to choose individualized regulation without incurring any penalty (IR), while as $k \to \infty$, the regulator will set the same regulatory intensity for all firms, enacting a one-size-fits-all uniform regime (UR).

Thus, the total cost of regulation is given by

$$C(\pi, B, k) = \sum_{i \in \{s,m,\ell\}} \left(\frac{\pi_i^2}{2} + B_i \pi_i + \frac{k}{2} \left(\pi_i - \bar{\pi}_{\setminus i} \right)^2 \right), \tag{3}$$

and the regulator's expected utility is

$$U_R = L(\pi, D, \lambda) - C(\pi, B, k). \tag{4}$$

The regulator chooses the regulatory intensity for all firms to minimize the loss of diversion subject to the cost of regulation. Insiders, in turn, can form coalitions to coordinate their lobbying efforts. A coalition is defined as a set of firms, denoted by $l_j \equiv \{i : i \text{ is a member of coalition } j\}$, where j indicates a particular coalition. We impose an increasing cost to forming larger lobbies, defined as χ_j, where j is the size of the lobby, with $0 < \chi_2 < \chi_3$. Within a lobby, utility is transferable. The insiders who are joined in a particular lobby choose lobbying efforts to maximize the joint surplus of all firms in the lobby,

$$U_{l_j} = \textstyle\sum_{i \in l_j} U_i. \tag{5}$$

We use the terms lobby and coalition interchangeably. Before lobbying efforts are chosen, coalitions can be formed. We assume that coalitions are formed by lobbyists, who extract the net gains from coalition formation, which we define below. Table 1 shows the timeline.

$t=0$	$t=1$	$t=2$	$t=3$
Lobbyists form coalitions l_j	Insiders choose influence activities B_i	Regulator chooses regulatory intensities π_i	Insider diversion may or may not occur

Table 1
Timeline of events with managerial influence activities

3 Regulation and Influence activities

Given lobbying efforts and the timing of the game, in period $t=2$, the regulator chooses regulatory intensities as $\underset{\pi_i, i\in\{s,m,\ell\}}{\arg\min} \sum_i \left[\lambda(1-\pi_i) D_i + \frac{\pi_i^2}{2} + B_i\pi_i + \frac{k}{2}\left(\pi_i - \bar{\pi}_{\setminus i}\right)^2\right]$. The first-order-conditions (FOC) are a set of three equations that imply,

$$\hat{\pi}_i = \frac{4(\lambda D_i - B_i) + 3k\left(\lambda \bar{D} - \bar{B}\right)}{4+9k} = \frac{(4+3k)(\lambda D_i - B_i) + 3k\left(\lambda \bar{D}_{\setminus i} - \bar{B}_{\setminus i}\right)}{4+9k}, \tag{6}$$

where $\bar{D}_{\setminus i} = \sum_{i' \neq i} D_{i'}$ and $\bar{B}_{\setminus i} = \sum_{i' \neq i} B_{i'}$; e.g., $\bar{B}_{\setminus s} = B_m + B_\ell$ and $\bar{D}_{\setminus \ell} = D_s + D_m$.

In what follows, we derive the optimal lobbying efforts, conditional on the coalition structure. Table 2 displays the five possible coalition structures. We assume throughout

Structure name	Notation	Coalition structure
Independent firms	I	$\{\{s\},\{m\},\{\ell\}\}$
Small-medium lobby	sm	$\{\{s,m\},\{\ell\}\}$
Small-large lobby	$s\ell$	$\{\{s,\ell\},\{m\}\}$
Medium-large lobby	$m\ell$	$\{\{s\},\{m,\ell\}\}$
Grand lobby	G	$\{\{s,m,\ell\}\}$

Table 2
Coalition Structures

that the exogenous parameters are such that regulation is defined by (6) and $\hat{\pi}_i \in (0,1)\,\forall i$.

In each setting, a coalition (which, without loss of generality, can be a one-firm coalition) chooses lobbying to maximize the expected utility of its members, which is $\sum_{i\in l_j} U_i|_{\pi_i=\hat{\pi}_i}$, where U_i is defined in (1) as $U_i = (1-\pi_i) D_i - \frac{c}{2}B_i^2$, and $\hat{\pi}_i$ is defined in (6). We do not specify transfers within lobbies, restricting them only to be feasible (i.e., the sum of the individual utilities of the lobby members equals the total utility of the members of the lobby). In the following three subsections, we present the optimal lobbying, regulatory strengths, and expected insider utilities for each of the coalition structures.

3.1 Firms act independently

When the firms act independently, lobbying and regulatory strengths are given by

$$\hat{B}_{i,I} = D_i \frac{4+3k}{c(4+9k)} \text{ and} \tag{7a}$$

$$\hat{\pi}_{i,I} = \frac{(\lambda c(4+9k) - (4+3k))\left(4D_i + 3k\bar{D}\right)}{c(4+9k)^2}, \tag{7b}$$

where $\bar{B}_I = \sum_i \hat{B}_i$ is the total lobbying. The above implies that firm utility is given by

$$\begin{aligned}\hat{U}_{i,I} &= D_i\left(1 - \frac{(\lambda c(4+9k) - (4+3k))\,3k\bar{D}}{c(4+9k)^2}\right) \\ &\quad -D_i^2\left(\frac{8(\lambda c(4+9k) - (4+3k))}{2c(4+9k)^2} + \frac{(4+3k)^2}{2c(4+9k)}\right).\end{aligned} \tag{8}$$

3.2 Two-firm lobbies

We present the results for each of the three potential two-firm lobbies in the appendix. Generally, in the two-firm setting, if firms g and h form a lobby or coalition, leaving firm j out, we have the following: $\hat{B}_{j,gh} = \hat{B}_{j,I} = D_j \frac{4+3k}{c(4+9k)}$; $\hat{B}_{g,gh} = \hat{B}_{g,I} + \frac{3kD_h}{c(4+9k)} = \frac{4D_g+3k\bar{D}_{gh}}{c(4+9k)}$; and $\bar{B}_{gh} = \hat{B}_I + \frac{3k\bar{D}_{gh}}{c(4+9k)}$, where $\bar{D}_{gh} = D_g + D_h$. Regulatory strengths are: $\hat{\pi}_{j,gh} = \hat{\pi}_{j,I} - \frac{9k^2\bar{D}_{gh}}{c(4+9k)^2}$ and $\hat{\pi}_{g,gh} = \hat{\pi}_{g,I} - \frac{3k\left(4D_h+3k\bar{D}_{gh}\right)}{c(4+9k)^2}$. The insider's expected utilities are: $\hat{U}_{gh,gh} = \hat{U}_{g,I} + \hat{U}_{h,I} + \frac{9k^2\overline{D^2_{gh}}}{2c(4+9k)^2}$ and $\hat{U}_{j,gh} = \hat{U}_{j,I} + \frac{9k^2D_j\bar{D}_{gh}}{c(4+9k)^2}$, where $\overline{D^2_{gh}} = D_g^2 + D_h^2$ and $\hat{U}_{gh,gh}$ is the total utility of the insiders in the gh coalition. Note that $\hat{U}_{j,gh} \geq \hat{U}_{j,I}$, implying that the non-coalition member benefits from the other firms forming a coalition.

3.3 Grand coalition/three-firm lobby

When all firms join together in the three-firm lobby, also referred to as the grand coalition, lobbying is $\hat{B}_{i,G} = \frac{4D_i + 3k\bar{D}}{c(4+9k)}$ for each firm, which implies that total lobbying is given by $\bar{B}_G = \frac{\bar{D}}{c}$. Regulatory strengths are:

$$\hat{\pi}_{i,G} = \frac{4D_i\left(\lambda - \frac{4}{c(4+9k)}\right) + 3k\bar{D}\left(\lambda - \frac{1}{c} - \frac{4}{c(4+9k)}\right)}{4+9k}$$

and the total expected utility of all insiders in the grand coalition is

$$\hat{U}_G = \hat{U}_{s,I} + \hat{U}_{m,I} + \hat{U}_{\ell,I} + \frac{9k^2\left(\overline{D^2} + D_s D_m + D_s D_\ell + D_m D_\ell\right)}{c\left(4+9k\right)^2},$$

where $\overline{D^2} = \sum_i D_i^2$, in contrast to $\bar{D}^2 = \left(\sum_i D_i\right)^2$.

3.4 Discussion of coalitions

Whenever the regulator is at least somewhat constrained to enact uniform regulation, i.e., whenever $k > 0$, the free-riding problem between insiders provides an opportunity for gains from forming coalitions. Whenever insiders form coalitions, they lobby more, because they internalize the externality effect of their lobbying on the other insider(s) in the coalition. This drives regulatory strength down for all firms. Although lobbying efforts are greater, each insider's expected utility is also greater, and not just for the members of the coalition. Regulatory uniformity means that the non-coalition member (e.g., insider ℓ in the presence of the sm coalition) faces weaker regulation due only to the coalition members increasing their lobbying.

Total expected utility is greatest for the three-firm coalition, and lowest when each firm lobbies independently. The gains from forming coalitions, furthermore, are increasing in the degree of regulatory uniformity, k, as $\frac{d}{dk}\left(\frac{9k^2}{c(4+9k)^2}\right) = \frac{72k}{c(9k+4)^3} > 0$.

4 Coalition formation

In this subsection, we explore the formation of coalitions or lobbies in period $t = 0$. First, note that the grand coalition yields the greatest total utility for the three insiders, as

$$\hat{U}_G > \hat{U}_{j,gh} + \hat{U}_{gh,gh} > \hat{U}_{s,I} + \hat{U}_{m,I} + \hat{U}_{\ell,I} \tag{9}$$

Therefore, absent coalition costs, in equilibrium the grand coalition will be stable.

As noted above, we assume that lobbies can only be formed by lobbyists, whose only role is in coalition formation. Essentially, only lobbyists possess the technology to facilitate collusion on lobbying between insiders. Because there can be no more than one coalition in equilibrium with three firms, we assume that there is only one lobbyist. This abstracts from potential competition between lobby-forming specialists, but substantially simplifies the problem. Ray and Vohra [1999], focusing on endogenous coalition formation, label some players as "proposers", who, by proposing coalition structures and within-coalition transfers, play essentially the same role as the lobbyist here. Bloch [2002] similarly presents a sequential coalition formation game featuring proposers.

We assume that the lobbyist can extract all of the gains that accrue to insiders from forming a coalition, net of the coalition formation costs.. That is, the lobbyist's utility is given by

$$L\left(l_j\right) = \sum_{i \in l_j} \left(\hat{U}_{i,l_j} - \hat{U}_{i,I}\right) - \chi_{|l_j|}, \tag{10}$$

where $|l_j|$ is the size (the number of firms in) the lobby. The lobbyist, maximizing (10), always prefers to form the $m\ell$-lobby over either the $s\ell$- or the sm-lobby. Furthermore, the net gains of forming the $m\ell$ coalition and the grand coalition are given by $\frac{9k^2\left(D_\ell^2+D_m^2\right)}{2c(4+9k)^2} - \chi_2$ and $\frac{9k^2\left(\overline{D^2}+D_sD_m+D_sD_\ell+D_mD_\ell\right)}{c(4+9k)^2} - \chi_3$, respectively. Since the lobbyist will maximize the net gain, the equilibrium coalition structure is mainly determined by two thresholds, denote $\Xi_1 = \frac{9k^2\left(D_\ell^2+D_m^2\right)}{2c(4+9k)^2}$ and $\Xi_2 = \frac{9k^2\left(\overline{D^2}+D_sD_m+D_sD_\ell+D_mD_\ell\right)}{c(4+9k)^2}$. Furthermore, denote $\Delta\chi = \chi_3 - \chi_2$

the extra cost of a three-firm coalition, relative to a two-firm coalition. The equilibrium lobbies are given by the following theorem.

Theorem 1 (Lobbyist-based coalitions) *When lobbyists form coalitions:*

1. *If $\chi_2 > \Xi_1$ and $\chi_3 > \Xi_2$, then the lobbyist will not form a coalition.*
2. *If $\chi_2 > \Xi_1$ and $\chi_3 < \Xi_2$, then the lobbyist will optimally form the grand lobby, in which all insiders collude on lobbying efforts.*
3. *If $\chi_2 < \Xi_1$ and $\Delta\chi > \Xi_2 - \Xi_1$, then the lobbyist will optimally form the $m\ell$ coalition, in which insiders m and ℓ collude on lobbying.*
4. *If $\chi_2 < \Xi_1$ and $\Delta\chi < \Xi_2 - \Xi_1$, then the lobbyist will optimally form the grand lobby, in which all insiders collude on lobbying efforts.*

4.1 Model Analysis

As the analysis above shows, the two possible equilibrium coalitions are $m\ell$ and G. In either coalition, insiders group by similarity. That is, either no firms, the higher types, or all firms form a coalition. This result is similar to much of the prior literature on endogenous lobbying (e.g., Mitra [1999]), in which only the most similar firms organize into coalitions.

Proposition 1 *When lobbyists form coalitions, coalitions consist of similar insiders.*

Next, we turn to the influence of coalition costs, χ, the cost to insiders of lobbying efforts, c, and the degree of regulatory uniformity, k, on coalition formation, lobbying, B, and regulatory strength, π. To facilitate the discussion, we first introduce "effective coalition formation costs", $\chi_2^E = \chi_2 \frac{2c(4+9k)^2}{9k^2}$ and $\chi_3^E = \chi_3 \frac{2c(4+9k)^2}{9k^2}$, allowing us to express each of the inequalities in Theorem 1 as comparisons between an effective coalition formation cost and a function of divertable amounts, D_i, only. For example, $\chi_2 < \Xi_1$ can be expressed as $\chi_2^E < D_\ell^2 + D_m^2$. This will prove useful because it subsumes lobbying costs and uniformity into effective coalition formation costs.

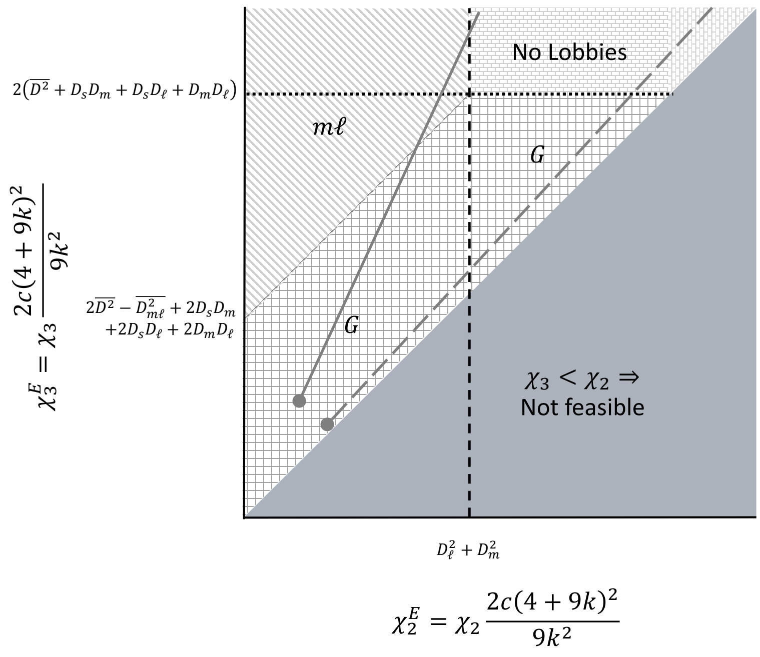

Figure 1

Coalition structures as functions of effective coalition-formation costs, χ_2^E and χ_3^E. The grey line segment in each subfigure indicates a nexus of points such that $\chi_3^E = \chi_2^E \frac{\chi_3}{\chi_2}$. The grey dot at the lower-left end of each line segment is at $(\chi_2^E, \chi_3^E) = (18c\chi_2, 18c\chi_3)$.

Figure 1 illustrates the topography of coalitions for the lobbyist-formed coalitions, respectively. The figure maps out regions of coalitions as functions of effective coalition formation costs, χ_2^E and χ_3^E. The area in the bottom-right is not feasible, as it is defined by $\chi_2^E < \chi_3^E \Leftrightarrow \chi_2 < \chi_3$. The remaining area, in the upper-left of the figure, is divided into regions in which different types of coalitions will exist in equilibrium, either no lobbies, the grand lobby, or the $m\ell$ lobby. These regions correspond to the regions described in Theorem 1. Note that the areas of the regions can change, but the shapes defining the regions hold generally.

In addition to these regions, the figure features two rays that starts at a gray dot and proceeds up and to the right. These rays are useful for thinking about how the coalition structure changes if we keep D_i's fixed but allow k, c, and χ's to change. Specifically, each ray traces out the nexus of points defined by $\chi_3^E = \chi_2^E \frac{\chi_3}{\chi_2}$ for $\chi_2^E \geq 18c\chi_2$. The slope is given by the proportional relation between χ_2 and χ_3. For a given c, the lowest and left-most point on the ray, at the gray dot, is defined by the point $\left(\chi_2^E, \chi_3^E\right) = (18c\chi_2, 18c\chi_3)$, because $\lim_{k\to\infty} \frac{2c(4+9k)^2}{9k^2} = 18c$. If c is very small, the gray dot will be close to the origin, but if c is large, the gray dot will be far, and effective coalition costs cannot be made small. In general, if the ray were extended down and to the left, it would intersect the origin for any χ_2 and χ_3, i.e., for any slope. Via the $\frac{2c(4+9k)^2}{9k^2}$ term in the χ^E's, c and k can be thought of as determining the location on the ray. Increasing k or decreasing c causes a shift down or to the left. Decreasing k or increasing c, in contrast, cause shifts in the other direction. Overall, the χ's determine the slope of the ray, while c determines the point closest to the origin, and c and k jointly determine the relevant point on the ray that defines the equilibrium coalition structure, i.e., where we fall on the plot.

Proposition 2 ***Cost of coalition formation***

1. *An increase in χ_3 causes weakly smaller lobbies, with potential transitions from the grand coalition to the $m\ell$ lobby or to no lobbies.*
2. *An increase in χ_2 can cause lobbies to grow (by moving from $m\ell$ lobby to the grand coalition) or to disband (by moving from an $m\ell$ coalition to no lobbies).*
3. *Concurrent proportional increases in χ_2 and χ_3 cause weakly smaller lobbies, with potential transitions from the grand coalition to the $m\ell$ lobby or to no lobbies.*

From Figure 1, an increase in χ_3 will tend to steepen the gray rays, leading to the results described in Proposition 2, part 1. For example, all else equal, the solid gray ray in Figure 1 has higher χ_3 than the dashed gray ray. Not surprisingly, higher costs can lead to smaller lobbies, as is always the case with the cost of the three-firm coalition, χ_3. An increase in χ_2 will tend to flatten the gray rays in Figure 1 (e.g., a transition from the solid gray ray to the dashed gray ray). Holding c and k constant, this can cause the coalition shifts described in part 2 of the proposition. The $m\ell$ coalition can become unstable with an increase in χ_2, causing a transition either to the grand coalition or to no coalitions as χ_2^E moves from below to above $D_\ell^2 + D_m^2$. Finally, increasing χ_2 and χ_3 proportionately at the same time causes a shift up and to the right along the gray rays. As in Proposition 2, part 1 this can cause a transition from G to $m\ell$, from G to I, or from $m\ell$ to I.

Proposition 3 ***Lobbying costs and uniformity***
Increases in lobbying costs, c, and decreases in regulatory uniformity, k, cause weakly smaller lobbies.

First note that lobbying costs, c, and regulatory uniformity, k, determine the relevant location on a given $\chi_3^E = \chi_2^E \frac{\chi_3}{\chi_2}$ ray drawn in Figure 1; the particular gray lines in the figure are examples. First, higher lobbying costs decrease the benefit of lobbying, and thereby decrease the benefits of forming a coalition that helps overcome the free-rider problem on lobbying effort. At one extreme, as $c \to \infty$, firms have no reason to form lobbies. At the other extreme, as $c \to 0$, insiders exert significant lobbying effort, making the grand coalition highly desirable. At intermediate levels of c, increasing c causes a shift up and to the right along a given gray ray, which can cause a transition from G to $m\ell$, from G to I, or from $m\ell$ to I, similar to the effect of a concurrent proportional increase of the costs to form a coalition.

Increases in regulatory uniformity, k, exacerbate the free-rider problem on lobbying and, thus, tend to promote the formation of lobbies. When k is very low, the free-rider problem is insignificant, giving insiders little incentive to bear the costs of forming coalitions. Note that $\lim_{k\to 0} \chi_j^E \to \infty$ which corresponds to a location on the gray rays in Figure 1 that is in the "No Lobbies" region. Increasing k causes a shift down and to the left along a given ray. When k is sufficiently high, insiders can benefit from coordinating their lobbying efforts, which would lead to a transition from I to either G or $m\ell$. Further increases can then cause a transition from $m\ell$ to G. However, even with high k, though, lobbying can be prohibitively costly to facilitate beneficial coalitions.

We next turn to the effects of parameter changes on the lobbying efforts and regulatory strength. Note that while changes in χ_2 and χ_3 only affect lobbying efforts and regulatory strengths through their effects on the equilibrium coalition structure, changes in c and k directly affect both through their effects on insiders and the regulator.

Corollary 1 ***Cost of coalition formation***

1. *An increase in χ_3, all else equal, causes weakly lower lobbying, B, higher regulatory strength, π, and lower expected losses from diversion.*

2. *An increase in χ_2, all else equal, can cause either weakly lower lobbying, B, higher regulatory strength, π, and lower expected losses from diversion; or greater lobbying, B, lower regulatory strength, π, and greater expected losses from diversion.*

Corollary 1 is a result of larger coalitions leading to more lobbying, which in turn weakens regulation and increases insiders' chances to inefficiently divert resources. Coalition formation costs influence lobbying and regulatory strength indirectly, that is, only through their influence on the coalition structure. In other words, in a regression of total lobbying on formation costs and observed coalition structures, for instance, formation costs should have no explanatory power.

As noted above, unlike coalition formation costs, both lobbying costs and regulatory uniformity have direct effects on equilibrium lobbying behavior and regulatory strengths. In fact, as shown in Friedman and Heinle [2016], absent lobbying coalitions, k and c have monotonic effects on lobbying and regulatory strength. In the presence of endogenously-formed coalitions, both regulatory uniformity and lobbying costs influence whether and which insiders organize into lobbying coalitions, causing the effects of c and k to be non-monotonic.

Corollary 2 ***Lobbying costs and uniformity***
An increase in lobbying costs, c, or regulatory uniformity, k, has non monotonic effects on lobbying effort, B, and regulatory strength, π.

Figure 2 plots total lobbying, $\bar{B}$, and average regulatory strength, $\pi^{ave} = \frac{1}{3}\sum \pi_i$, as functions of regulatory uniformity, k, for two sets of parameters that differ only in χ_2. The solid gray curves have $\chi_2 = 0.7$, while the dashed black curves have $\chi_2 = 2$. In all cases, $\chi_3 = 2.65$, meaning that coalition formation costs are convex when $\chi_2 = 0.7$ and concave when $\chi_2 = 2$.[9]

[9] Other parameters are set as $D_s = 1$, $D_m = 15$, $D_\ell = 30$, $c = 1$, and $\lambda = 1.2$.

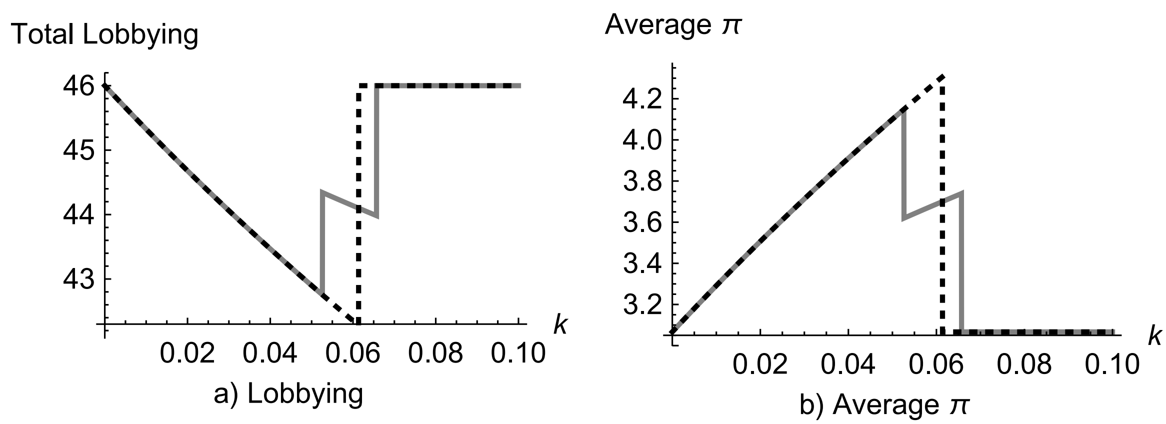

Figure 2
Total lobbying and average regulatory strength as functions of regulatory uniformity, k. Parameters are set as $D_s = 1$, $D_m = 15$, $D_\ell = 30$, $\chi_3 = 2.65$, $c = 1$, and $\lambda = 1.2$. For the solid gray line, $\chi_2 = 0.7$. For the dotted black line, $\chi_2 = 2$.

When $\chi_2 = 2$, coalition formation costs are concave, corresponding to the relatively flat $\chi_3^E = \chi_2^E \frac{\chi_2}{\chi_3}$ solid ray in Figure 1. Starting from $k = 0$, increasing k tends to reduce lobbying and increase regulatory strength. These continue monotonically in k until we reach a threshold level of k that makes the benefit of forming a three-firm coalition sufficiently large. At this point, as the coalition is formed, we see a discrete jump in total lobbying and a drop in average regulatory strength. Further increases in regulatory uniformity have no more effects, as k is moot in the presence of the grand coalition and increases in k maintain the dominance of the grand coalition.

When $\chi_2 = 0.7$, coalition costs are convex, corresponding to the steeper $\chi_3^E = \chi_2^E \frac{\chi_2}{\chi_3}$ dashed ray in Figure 1. Total lobbying and average regulatory strength mostly behave as they do when coalition costs are concave. There is, however, a key difference in the intermediate range of k. In this region, as we increase k from about 0.05, we first see a transition from the grand coalition to the $m\ell$ coalition around $k = 0.055$. This first transition occurs when the $m\ell$ coalition becomes feasible, as χ_2^E drops below $D_\ell^2 + D_m^2$. As k continues to increase, the

$m\ell$ coalition remains optimal, but total lobbying decreases and average regulatory strength increase, as the free-rider problem between the $m\ell$ coalition and insider s worsen. Note the weaker influence of uniformity on total lobbying and average regulatory strength in the presence of the $m\ell$ coalition, reflecting the respective insiders' ability to overcome the free-rider problem within their lobby. As k increases past about 0.067, χ_3^E becomes less than $\chi_2^E + 2\overline{D^2} - \overline{D^2_{m\ell}} + D_m D_\ell + D_s D_\ell + D_s D_m$, and, as the gain from the grand coalition starts to dominate the gain from the $m\ell$ coalition, we see a shift to the high lobbying and low regulatory strength associated with the grand coalition. (Recall that when costs are convex, χ_3^E will decrease faster in k than χ_2^E will.)

Overall, Figure 2 corroborates and elaborates on the effects of regulatory uniformity described in Corollary 2. Results would be similar for changes in lobbying costs, c, as increases in c have similar effects as decreases in k in terms of shifting along the gray rays in Figure 1. Figure 2 shows that increases in k have non-monotonic effects on lobbying and regulatory strength when insiders can organize into lobbies to overcome free-rider problems on lobbying. Almost everywhere, the effects of k on lobbying (regulatory strength) are negative (positive), but these effects can be significantly outweighed by discrete jumps or drops as changes in k cause insiders to change how they organize into coalitions.

5 Coalitions formed by insiders

In the analysis above we assumed that only external lobbyists can form a coalition. Essentially, the assumption reflects a setting where lobbyists are necessary to lobby for multiple firms *and* where lobbyists have all the bargaining power. In this section, we explore the opposite setting, either lobbyists are not necessary or they do not have any bargaining power. In such a situation, insiders will be able to form coalitions. Necessarily, the coalitions that emerge must be offer stable, i.e., immune to deviations by coalition-members that can be prevented by feasible counteroffers.[10] In forming coalitions, we assume that insiders seek to maximize their gains from coalition formation. We do not model the allocation of such gains within a coalition, as these are irrelevant in the later stages involving lobbying and regulatory strength. Instead, we focus on the formation of offer-stable coalitions. While we use the terms "stable" and "offer-stable" interchangeably, note that our concept of offer-stable coalitions is somewhat different from coalition stability as defined in Von Neumann and Morgenstern [1944] (see also Ray and Vohra [2014]).

Definition 1 *A coalition is offer-stable if each coalition member, i, can be made better-off inside the coalition than outside of it. That is, the coalition l_j is stable if each $i \in l_j$ could be made to decline any feasible offers from other insiders to join alternative coalitions, $l_{j'} \neq l_j$. A feasible offer is one that (a set of) insiders would be willing to make based on the expected utility of coalition $l_{j'}$ relative to the utility these insiders would expect in the presence of coalition l_j.*

Our notion of stability is based on the idea that if insider i was considering leaving a given coalition, then the remaining insiders in the coalition could offer insider i any amount up to the amount by which those insiders gain from being members of the coalition including i. The logic extends naturally if we replace insider i with a set of insiders. Offer-stability is similar in spirit to several fundamental types of coalition stability described in Ray and Vohra [2014], based on whether various coalition structures are blocked by alternatives.[11] Fundamentally, offer-stability relies on credible counteroffers to individual (sets of) potentially-deviating coalition members.

To illustrate the notion of offer-stable coalitions, consider the grand coalition. If $\Xi_2 > \chi_3$, then the gain from forming the grand coalition, relative to not forming any coalition, outweighs the costs. The grand coalition is then preferred to the independence arrangement (i.e., each insider lobbying independently) and the grand coalition is partially stable (i.e.,

[10] This is similar to the notion of "blocking" in Ray and Vohra [1997, 2014]. Additionally, our offer-stable coalitions are unique in each region of the parameter space, allowing for clean predictions with an intuitive stability criterion.

[11] Our offer-stability differs from the stability notion used in Ray and Vohra [1997], in which subsets of l_j can deviate, but deviations always make the coalition structure finer, as the deviants are precluded from joining with insiders who were not initially in l_j. In contrast, we allow for the deviating insiders to form coalitions with new partners.

stable relative to the case in which $\chi_2 >> 0$), because insiders j and k can offer up to $\Xi_2 - \chi_3$ to insider i to prevent her from leaving. However, it could be that two members of the grand coalition prefer to form a two-firm coalition over the grand coalition. That is, it may be that the extra costs of having a three-firm coalition (relative to a two-firm coalition) outweigh the benefit. For example, with $\chi_2 = 0$ and $\chi_3 > \Xi_2$, a two-firm coalition may be optimal. Clearly, there are gains to be had from a two-firm coalition, but we have not yet shown whether any two-firm coalition is offer stable. To develop two-firm coalition stability, we define a maximum deviation offer.

Definition 2 (Maximum deviation offer) *If insiders g and h are members of the gh coalition, the maximum deviation offer from insider j to insider g, $MDO_{j,g}^{gh}$ is the greatest amount that insider j would offer insider g to leave the gh coalition and join the new jg coalition.*

We can calculate the maximum deviation offers that a given insider would be willing to make as

$$MDO_{j,g}^{gh} = \hat{U}_{jg,jg} - \hat{U}_{j,gh}, \tag{11}$$

which is the utility that the jg coalition would achieve minus the utility that insider j expects to achieve in the presence of the gh coalition. The $MDO_{j,g}^{gh}$ is the maximum offer, because insider j in coalition jg faces a budget constraint (in terms of transferable utility) of $\hat{U}_{jg,jg}$. Furthermore, insider j's outside option is to remain independent in a structure featuring the gh coalition, meaning that she should be willing to offer insider g no more than the gain achieved from g's deviation from gh to jg. For example,

$$MDO_{s,\ell}^{m\ell} = \hat{U}_{s\ell,s\ell} - \hat{U}_{s,m\ell} = \hat{U}_{\ell,I} + \frac{9k^2\left(\overline{D_{s\ell}^2} - 2D_s\bar{D}_{m\ell}\right)}{2c\left(4+9k\right)^2}, \text{ and} \tag{12}$$

$$MDO_{m,\ell}^{s\ell} = \hat{U}_{m\ell,m\ell} - \hat{U}_{m,s\ell} = \hat{U}_{\ell,I} + \frac{9k^2\left(\overline{D_{m\ell}^2} - 2D_m\bar{D}_{s\ell}\right)}{2c\left(4+9k\right)^2}. \tag{13}$$

The following definition illustrates the offer-stability concept within the set of two-firm coalitions.

Lemma 1 (Offer stability for a two-firm coalition) *A two-firm coalition gh consisting of the insiders from firms g and h is offer-stable if* $MDO^{gh}_{j,g} < MDO^{gj}_{h,g}$ *and* $MDO^{gh}_{j,h} < MDO^{hj}_{g,h}$*, i.e., if: 1) insider j's maximum deviation offer to g conditional on coalition gh is lower than insider h's maximum deviation offer to g conditional on coalition jg; and 2) insider j's maximum deviation offer to h conditional on coalition gh is lower than insider g's maximum deviation offer to h conditional on coalition jh. A coalition structure containing a stable two-firm coalition is two-firm stable.*

An offer-stable two-firm coalition, as defined, cannot be broken up by the insider who is not a member of the coalition. That is, the most that insider is willing to offer either of the coalition members is less than either coalition member would offer the other coalition member to leave the coalition that would form if the independent insider's initial offer was successful. Offer-stability implies that the outsider's utility in the presence of a two-firm coalition is relevant for determining the surviving coalition. As such, it is useful to define the following additional relevant thresholds: $\Xi_3 = \frac{9k^2\left(D_\ell^2+D_s^2\right)}{2c(4+9k)^2} = \hat{U}_{s\ell,s\ell} - \hat{U}_{s,I} - \hat{U}_{\ell,I}$; $\Xi_4 = \frac{9k^2\left(2\overline{D^2}-\overline{D^2_{m\ell}}+D_mD_\ell\right)}{2c(4+9k)^2} = \Xi_2 - \Xi_1 + \hat{U}_{s,I} - \hat{U}_{s,m\ell}$; and $\Xi_5 = \frac{9k^2\left(2\overline{D^2}-\overline{D^2_{s\ell}}+D_sD_\ell\right)}{2c(4+9k)^2} = \Xi_2 - \Xi_3 + \hat{U}_{m,I} - \hat{U}_{m,s\ell}$. The following theorem summarizes the offer-stable coalitions.

Theorem 2 (Offer-stable coalitions) *The following coalition structures are offer-stable.*

1. *If* $\chi_2 > \Xi_1$ *and*
   (a) $\chi_3 > \Xi_2$*, then no coalitions will form and all insiders will lobby individually;*
   (b) $\chi_3 < \Xi_2$*, then the offer-stable coalition structure consists of the grand lobby.*
2. *If* $\chi_2 \in (\Xi_3, \Xi_1)$ *and*
   (a) $\chi_3 - \chi_2 > \Xi_4$*, then the offer-stable coalition structure consists of the* $m\ell$ *lobby.*
   (b) $\chi_3 - \chi_2 < \Xi_4$*, then the offer-stable coalition structure consists of the grand lobby.*
3. *If* $\chi_2 < \Xi_3$ *and*
   (a) $\chi_3 - \chi_2 > \Xi_5$*, then the offer-stable coalition structure consists of the* $s\ell$ *lobby;*
   (b) $\chi_3 - \chi_2 < \Xi_5$*, then the offer-stable coalition structure consists of the grand lobby.*

Note that when the costs to forming a two-firm coalition are sufficiently low, $\chi_2 < \Xi_3$, and the extra cost of forming the grand coalition is sufficiently large, then the only offer-stable coalition is the $s\ell$ lobby. This is in stark contrast to the setting where a lobbyist forms the coalition. When firms form coalitions, the equilibrium coalition has to be stable against an outside offer. In the setting of Theorem 2 Part 3 a, the $m\ell$ lobby is not stable because the m insider's gain from the $s\ell$ lobby is relatively high. For that reason, the s insider's gain from joining the lobby exceeds the m insider's loss from leaving the lobby and the $s\ell$ lobby persists.

5.1 Analysis of offer-stable coalitions

From the above analysis it is easy to see that the coalition formation process plays a role in determining whether coalitions are characterized by similarity or polarization. If specialists (i.e., lobbyists) are necessary for lobbying coalitions to form, then the two possible equilibrium coalitions are either $m\ell$ or G. In either, insiders group by similarity. That is, either no firms, the higher types, or all firms form a coalition. In contrast, if specialists are not necessary and insiders can organize themselves into offer-stable coalitions, then there is scope for a diverse coalition with polarized membership, in that the s and ℓ insiders are able to form an offer-stable coalition. This result in particular contrasts with much of the prior literature on endogenous lobbying (e.g., Mitra [1999]), in which only the most similar firms organize into coalitions, but is similar in spirit to the results of Baccara and Yariv [2016], who find potential for polarization or similarity of membership in peer-selected groups organized to produce public goods. Our result is summarized in the following proposition.

Proposition 4 *If insiders form offer-stable coalitions, then coalitions can be polarized, featuring dissimilar insiders. In contrast, if lobbyists form coalitions, then only coalitions of similar insiders are possible.*

Similar to Figure 1, Figure 3 illustrates the equilibrium coalition structure, in this case for the offer-stable equilibrium. Note that in addition to the possible coalition structures with lobbyist formed coalitions, the $s\ell$ coalition is also feasible under offer-stable coalition formation. The dashed ray in Figure 3 shows that a shift to the bottom and left (e.g., with

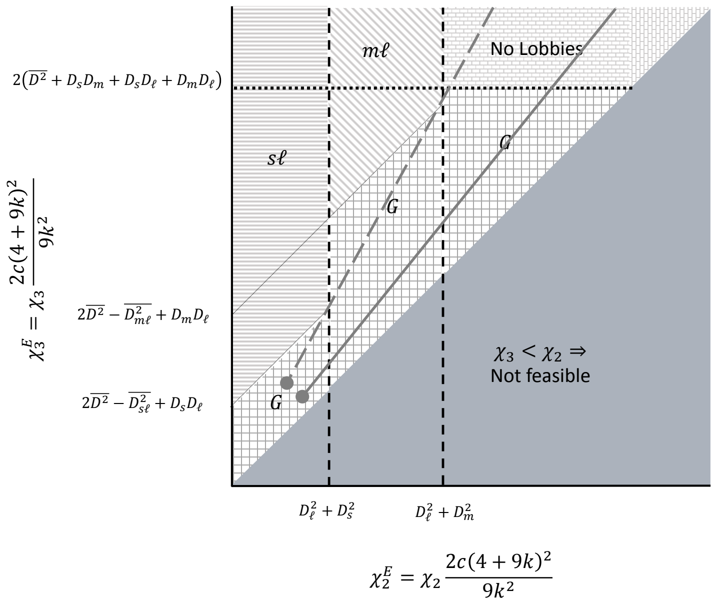

Figure 3

Coalition structures as functions of effective coalition-formation costs, χ_2^E and χ_3^E. The grey line segment in each subfigure indicates a nexus of points such that $\chi_3^E = \chi_2^E \frac{\chi_3}{\chi_2}$. The grey dot at the lower-left end of the line segment is at $(\chi_2^E, \chi_3^E) = (18c\chi_2, 18c\chi_3)$.

an increase in uniformity, k) can cause a transition from the grand lobby to the $m\ell$ lobby and back to the grand lobby. Similarly, on the solid ray, a shift to the bottom and left can cause a transition from the $m\ell$ coalition to the grand lobby to the $s\ell$ coalition and back to the grand lobby. The reason is that when the equilibrium moves from a three-firm coalition (where all firms are in the coalition) to a two-firm coalition, the utility of the non-coalition member matters. Specifically, because the coalition members are able to overcome the free-rider problem and each insider's lobbying decreases all insiders' regulatory strength, the non-coalition member's utility increases (relative to the non-coalition case). This externality of the two-firm lobby can make it profitable for two insiders in the grand coalition to offer the other insider to leave the coalition when, for example, k increases and causes a shift on a ray that moves the offer-stable coalition from the grand lobby to a two-firm lobby.

While the results regarding individual changes in χ_3 and χ_2 from the insider-induced coalitions still apply (albeit with the additional possibility of an increase in χ_2 causing a

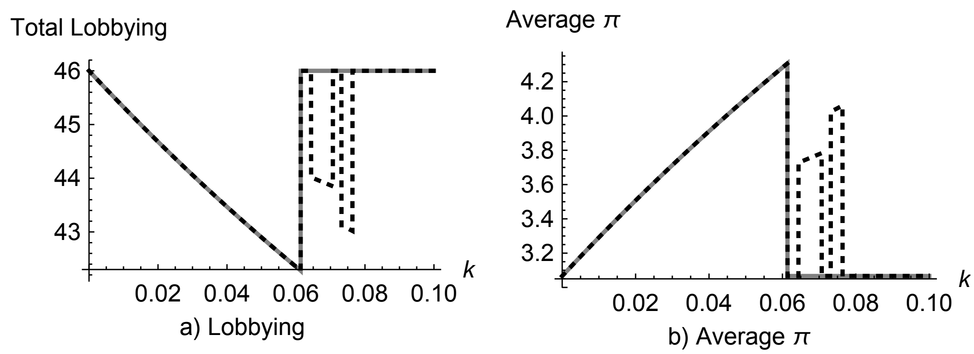

Figure 4

Total lobbying and average regulatory strength as functions of regulatory uniformity, k. Parameters are set as $D_s = 1$, $D_m = 15$, $D_\ell = 30$, $\chi_3 = 2.65$, $c = 1$, and $\lambda = 1.2$. For the solid gray line, $\chi_2 = 1$. For the dotted black line, $\chi_2 = 2$.

shift from the $s\ell$ lobby to either the $m\ell$ or G structure), changes in parameters that causes a shift along the rays in Figure 3 can cause non-monotonic changes in the size of the coalitions.

Corollary 3 *Concurrent proportional increases in χ_2 and χ_3, increases in lobbying costs, c, and increases in regulatory uniformity, k, can lead to larger or smaller lobbies.*

For example, a change in parameters that causes a shift up and to the right along the gray rays (increasing χ_2 and χ_3, increasing c, or decreasing k) can cause a transition from G to $s\ell$, from $s\ell$ to G, from G to $m\ell$, from $m\ell$ to G, or from G to I. As we note above, the reason is that the non-coalition member's utility is important for determining the offer-stable coalition.

Because there are parameter values where an increase in k or an decrease in c can break up the grand coalition and, instead, lead to a two firm coalition, lobbying and regulatory strength as functions exhibit discrete jumps both up and down in k as Figure 4 shows. Recall that increases in k have similar effects as decreases in c.

The following corollary summarizes the comparative statics.

Corollary 4 *An increase in lobbying costs, c, or regulatory uniformity, k, all else equal, can* *cause either weakly lower lobbying, B, and higher regulatory strength, π; or greater lobbying, B, and lower regulatory strength, π.*

When $\chi_2 = 2$, coalition formation costs are concave, corresponding to a relatively flat $\chi_3^E = \chi_2^E \frac{\chi_2}{\chi_3}$ solid ray in Figure 3 and the solid plots in Figure 4. Starting from $k = 0$, increasing k tends to reduce lobbying and increase regulatory strength. These continue monotonically in k until we reach a threshold level of k that makes the benefit of forming a three-firm coalition sufficiently large. At this point, as the coalition is formed, we see a discrete jump in total lobbying and a drop in average regulatory strength. Further increases in regulatory uniformity have no more effects, as k is moot in the presence of the grand coalition and increases in k maintain the dominance of the grand coalition.

When $\chi_2 = 1$, coalition costs are convex, corresponding to a steeper $\chi_3^E = \chi_2^E \frac{\chi_2}{\chi_3}$ dashed ray in Figure 3 and the dashed plots in Figure 4. Total lobbying and average regulatory strength mostly behave as they do when coalition costs are concave. There is, however, a key difference in the intermediate range of $k \in (0.06, 0.08)$. In this region, as we increase k from 0.06, we first see a transition from the grand coalition to the $m\ell$ coalition around $k = 0.061$. This first transition occurs when the $m\ell$ coalition becomes feasible, as χ_2^E drops below $D_\ell^2 + D_m^2$. As k continues to increase, the $m\ell$ coalition remains optimal, but total lobbying decreases and average regulatory strength increase, as the free-rider problem between the $m\ell$ coalition and insider s worsen. As k increases past about 0.07, χ_3^E becomes less than $\chi_2^E + 2\overline{D^2} - \overline{D_{m\ell}^2} + D_m D_\ell$, and, as the gain from the grand coalition starts to dominate the gain from the $m\ell$ coalition, we see a shift back to the high lobbying and low regulatory strength associated with the grand coalition. (Recall that when costs are convex, χ_3^E will decrease faster in k than χ_2^E will.) Next, as k increases further, the $s\ell$ coalition becomes feasible with χ_2^E dipping below $D_\ell^2 + D_s^2$. At $k \approx 0.074$, the $s\ell$ coalition is preferable to the grand coalition, but at $k \approx 0.076$, the grand coalition again becomes preferable, as χ_3^E drops below $\chi_2^E + 2\overline{D^2} - \overline{D_{s\ell}^2} + D_s D_\ell$. With k between 0.074 and 0.076, increases in regulatory uniformity again cause decreases in lobbying and increases in regulatory strength, as the free-rider problem between the $s\ell$ coalition and insider m gets worse. As k increases beyond 0.076, the grand coalition is again optimal, and further increases in regulatory uniformity cease to play a significant role.

6 Continuum-of-firms model

In this section we extend the model to a continuum-of-firms variant. This allows us to explore the implications of having more than one lobbying coalition, which is precluded in the three-firm setting. We analyze a continuum of firms rather than a finite set of more than three firms because the continuum facilitates comparative statics analysis and approximates a large-economy limit.

Specifically, we assume that $i \in [0,1]$ and, without loss of generality, we order firms by D_i, and assume that we can write $D(i) : [0,1] \to (0,\infty)$, where $D(i) = D_i$ is a continuously differentiable, increasing function. We use bars to denote sums, such as $\bar{D} = \int_0^1 D_i di$, and variables without subscripts to denote vectors, where appropriate, such as $D = \{D_i\}_{i\in[0,1]}$. The regulator's utility is again given by $U_R = L(\pi, D, \lambda) - C(\pi, B, k)$, where L and C are the continuum-based analogues of their three-firm versions:

$$L(\pi, D, \lambda) = -\lambda \int_0^1 D_i (1-\pi_i)\, di, \text{ and} \tag{14}$$

$$C(\pi, B, k) = \int_0^1 \left(\frac{\pi_i^2}{2} + B_i \pi_i + \frac{k}{2} \left(\pi_i - \bar{\pi}_{\setminus i}\right)^2 \right) di. \tag{15}$$

In the analysis, we focus on the case where lobbyists form coalitions; insiders cannot organize themselves. In the setting with a continuum of firms, (i) any firm is measure zero such that a single firm can never make a profitable offer to join a coalition, and (ii) any lobby will be a continuum of firms such that enforcing collusion on lobbying within the coalition is likely to require special technology. Finally, lobbyist coalition formation helps us avoid the curse of dimensionality that arises in coalition formation games. That is, if there are N insiders, then there are $2^N - 1$ potential coalitions to consider.

More specifically, we assume that lobbyists arrive one-by-one during period $t = 0$. Each lobbyist can organize a group of insiders into a lobby. We define $U_i \left(l_j; \Lambda_{i \notin l_j}\right)$ as the expected value of U_i from (1) conditional on insider i being a member of lobby l_j, and the overall coalition structure of the non-l_j firms, $\Lambda_{i \notin l_j}$. We define $U_i \left(\emptyset; \Lambda_{i \notin l_j}\right)$ similarly, but under the assumption that insider i is not a member of a lobbying coalition. Given this notation, we assume that the lobbyist can extract the net benefit of lobbying from the insiders, which is

$$V_j = \int_{i \in l_j} \left[U_i \left(l_j; \Lambda_{i \notin l_j}\right) - U_i \left(\emptyset; \Lambda_{i \notin l_j}\right)\right] di - \chi \left(|l_j|\right),$$

where $\chi \left(|l_j|\right)$ is the cost of forming a lobby whose size is $l_j = \int_{i \in l_j} di$. Assuming that the lobbyist extracts the full net benefit is a convenient assumption, as it makes each insider strictly indifferent between joining or not joining a given lobby, conditional on the remaining coalition structure, $\Lambda_{i \notin l_j}$.[12]

6.1 Regulation and lobbying efforts

Given lobbying, B, the regulator chooses regulation, $\hat{\pi} = \arg\max_{\pi} U_R$. For a given i, the regulator's FOC is $\frac{dU_R}{d\pi_i} = 0$, or $0 = \lambda D_i - \left(\pi_i \left(1 + k\right) + B_i - k\bar{\pi}_{\emptyset}\right)$. Solving the FOC for π_i and integrating both sides over all firms yields the regulator's choice,

$$\hat{\pi}_i = \frac{\lambda D_i - B_i + k\left(\lambda \bar{D} - \bar{B}\right)}{1 + k}. \tag{16}$$

While the structure of the regulator's choice of $\hat{\pi}_i$ is similar to that in the three firm setting, any firm's regulatory intensity does not affect the average intensity, such that the regulator

[12] We would achieve results of a similar form if we posited a fixed sharing rule between lobbyists and the coalitions they form. (Note that our assumption that lobbyists extract the full net gain is one such sharing rule.) Absent a fixed sharing rule, lobbyists likely would engage in a sort of poaching behavior, similar to how, in the three-firm case, insider s was able to poach insider ℓ from the $m\ell$ coalition. Poaching between lobbyists, in turn, likely causes a coalition formation problem at least as complicated as the one involving a continuum of insiders seeking stable coalitions, with the added complication of lobbyists intermediating.

simply weights the average deadweight loss and lobbying by $\frac{1}{1+k}$ and $\frac{k}{1+k}$, respectively.[13] A given coalition l_j's problem is to choose optimal levels of lobbying effort for each of its members,

$$\hat{B} \in \arg \max_{B(i), i \in l_j} \int_{i \in l_j} U_i di. \tag{17}$$

Given our structure the choice of lobbying effort for each of the coalition members is tantamount to choosing the optimal amount of total lobbying effort, $\bar{B}_{l_j} = \int_{i \in l_j} B_i di$, and the allocation of that effort among the coalition members, conditional on total lobbying effort. The following Lemma establishes firm i's optimal lobbying effort, the optimal total lobbying effort of coalition j, and total lobbying effort in the economy.

Lemma 2 *With a continuum of firms and coalitions formed by lobbyists, the optimal lobbying effort for firm i in coalition j is*

$$\hat{B}\left(i; l_j\right) = \frac{D_i + \bar{D}_{l_j} k}{c\left(1+k\right)}. \tag{18}$$

From (18), the optimal lobbying effort for insider i when she is not part of any coalition is $\hat{B}\left(i; \emptyset\right) = \frac{D_i}{c(1+k)}$. The following corollary investigates the total lobbying effort in the economy in the knife edge cases of the grand lobby and no coalitions.

Corollary 5 *With a continuum of firms and lobbyist-formed coalitions, when no coalitions are formed, the total lobbying effort in the economy goes to zero as the regulatory system becomes perfectly uniform.*

With perfect uniformity, only the average lobbying and deadweight loss of diversion matter for the optimal regulatory intensity. Since any firm's lobbying does not affect the average lobbying with a continuum of firms, the free-rider problem is maximized and no unorganized firms lobby. However, since a positive-measure coalition's collective lobbying effort does affect the average lobbying, firms that are organized in a coalition provide lobbying efforts. Specifically, all firms in coalition j act as if they were one firm with the lobby-specific aggregate potential for diversion and, thus $\lim_{k\to\infty} \hat{B}\left(i; l_j\right) = \frac{1}{c}\bar{D}_{l_j}$.

[13] Note that this result also follows from taking the limit of the N-firm case as $N \to \infty$. Supporting analysis is available from the authors.

Corollary 5 requires that no coalitions are formed; however, changes in uniformity, as shown in the three-firm case, affect insiders' incentives to form coalitions. Specifically, insiders' incentives to form coalitions are strongest as the regulatory system approaches perfect uniformity (i.e., $k \to \infty$).

Before we investigate the endogenous formation of coalitions, we first establish the utility gain for firm i from joining a coalition, which will in turn be extracted by the lobbyist who forms said coalition. The expected utility for a coalition, U_{l_j}, conditional on an optimal lobbying strategy and the regulator's choice of regulation, $(\hat{U}_{l_j})$ is given by

$$\hat{U}_{l_j} = \bar{D}_{l_j} \left(1 + \frac{k\left(\bar{B}_{i \notin l_j} - \lambda \bar{D}\right)}{1+k} \right) + \frac{\left(\bar{D}_{l_j}\right)^2 k\left(2 + k\left|l_j\right|\right)}{2c\left(1+k\right)^2} + \overline{D^2_{l_j}} \left(\frac{1}{2c\left(1+k\right)^2} - \frac{\lambda}{1+k} \right) \quad (19)$$

where $\overline{D^2_{l_j}} = \int_{i \in l_j} \left(D_i\right)^2 di$ and $\left(\bar{D}_{l_j}\right)^2 = \left(\int_{i \in l_j} D_i di\right)^2$. The utility gain for the coalition members is $\Delta\hat{U}_{l_j} = \int_{i \in l_j} \left[U_i\left(l_j; \Lambda_{i \notin l_j}\right) - U_i\left(\emptyset; \Lambda_{i \notin l_j}\right)\right] di$ and is given by

$$\Delta\hat{U}_{l_j} = \frac{k^2\left(\bar{D}_{l_j}\right)^2 \left|l_j\right|}{2c\left(1+k\right)^2}. \quad (20)$$

Interestingly, $\Delta\hat{U}_{l_j} = \frac{k^2\left(\bar{D}_{l_j}\right)^2 |l_j|}{2c(1+k)^2}$ is strictly positive, increasing in $\bar{D}_{l_j}$, the size of the lobby, $|l_j|$, and regulatory uniformity, k. The coalition gain is decreasing in lobbying costs. That is, larger coalitions overcome the free-rider problem for a larger set of firms and, because of the externality of each insider's lobbying effort, this benefits all insiders. Since insiders with larger divertable amounts prefer higher lobbying efforts, they benefit more from coalition formation, such that the value of the coalition increases in the aggregate divertable amount of the coalition members. Finally, higher regulatory uniformity increases the free-rider problem such that coalition formation becomes more beneficial. The expression for $\Delta\hat{U}_{l_j}$ in (20) further implies that the coalition gain is greater when $\bar{D}_{l_j}$ is higher, implying the following proposition.

Proposition 5 *All else equal, a coalition prefers to add insiders with higher D_i.*

Proposition 5 suggests that when a lobbyist looks to form a coalition, he will invite the insiders with the highest potential for diversion, which is similar to the result from the three-firm analysis with lobbyist-formed coalitions.

6.2 Formation of lobbies

In period $t = 0$, lobbyists arrive and form lobbies, extracting the coalition benefits, $\Delta \hat{U}_{l_j}$, and bearing the coalition formation costs, $\chi(|l_j|)$. Ray and Vohra [1999] assume a pre-specified order of proposers, similar to our arrival of lobbyists. The first lobbyist who arrives will choose l_1 to maximize $V_1 = \Delta U_{l_1} - \chi(|l_1|)$, formalized as

$$\hat{l}_1 \in \arg\max_{l_1 \subset [0,1]} \Delta U_{l_1} - \chi(|l_1|). \tag{21}$$

ΔU_{l_1} is increasing in $\bar{D}_{l_1}$, and $\chi(|l_1|)$ depends only on the size of the coalition. The lobbyist will therefore, as in the three-firm case, pick a convex set of insiders with the highest D_i.

Lemma 3 *In the continuum-of-firms model, earlier lobbies contain insiders with greater divertable funds.*

The lobbyists's problem is therefore to pick the smallest D_i to admit into the coalition. To facilitate analysis, we assume that the following second-order condition (SOC) is generally satisfied:

$$\frac{k^2}{c(1+k)^2}\left(2D_{\gamma_j}\bar{D}_{l_j} + D_{\gamma_j}^2|l_j| - \bar{D}_{l_j}|l_j|\frac{dD_{\gamma_j}}{d\gamma_j}\right) - \chi''(|l_j|) < 0 \tag{22}$$

where $\bar{D}_{l_t} = \int_{\gamma_t}^{\gamma_{t-1}} D_i di$ and $|l_t| = \int_{\gamma_t}^{\gamma_{t-1}} di$. The following proposition establishes the coalition formation equilibrium.

Proposition 6 *In the continuum-of-firms model, lobbyist t either forms no lobby or chooses $\hat{l}_t = (\hat{\gamma}_t, \hat{\gamma}_{t-1}]$, where $D_{\hat{\gamma}_t}$ is the smallest D_i for which $i \in l_t$, with $\hat{\gamma}_0 = 1$. Conditional on $t-1$ lobbies having already formed and assuming the condition in (22) holds:*

a) *$\hat{\gamma}_t$ either solves the first-order condition (FOC), $G(\hat{\gamma}_{t-1}, \hat{\gamma}_t) = 0$, where*

$$G(\hat{\gamma}_{t-1}, \hat{\gamma}_t) = -\frac{k^2}{2c(1+k)^2}\left(\left(\bar{D}_{l_t}\right)^2 + 2\bar{D}_{l_t}D_{\hat{\gamma}_t}\left|\hat{l}_t\right|\right) + \chi'\left(\left|\hat{l}_t\right|\right) \tag{23}$$

for some $\hat{\gamma}_t \in [0, \hat{\gamma}_{t-1})$*;*

b) $G(\hat{\gamma}_{t-1}, \hat{\gamma}_t) < 0 \forall \hat{\gamma}_t \in [0, \hat{\gamma}_{t-1})$ *and we have* $\hat{l}_t = [0, \hat{\gamma}_{t-1}]$ *and no further lobbies form; or*

c) $G(\hat{\gamma}_{t-1}, \hat{\gamma}_t) > 0 \forall \hat{\gamma}_t \in [0, \hat{\gamma}_{t-1})$*, in which case no further lobbies form.*

The FOC in (23) can be used to iteratively construct the equilibrium lobbies. For the first lobby, $j = 1$, and $\hat{\gamma}_j$ solves (23) for $\gamma_0 = 1$. For the second lobby, $j = 2$, $\gamma_{j-1} = \hat{\gamma}_1$, and $\hat{\gamma}_2$ solves (23) for $\gamma_{j-1} = \hat{\gamma}_1$. This continues until either lobbyists prefer not to form lobbies (which can happen initially), or until there are no insiders left to organize into lobbies, as represented by cases (b) and (c) in Proposition 6. Higher uniformity, k, all else equal, makes it more likely that lobbies will form, since $\frac{d}{dk}\left(\frac{k^2}{2c(1+k)^2}\right) = \frac{k}{c(k+1)^3} > 0$ implies that the first term in (23) gets more negative as k increases, and this term must be sufficiently negative to offset the marginal costs of increasing coalition size, $\chi'\left(\left|\hat{l}_j\right|\right)$.

Proposition 6 implies that the equilibrium is a partitioning of the continuum of firms. The first lobbyist forms a coalition of the firms with the highest potential for diversion. If it is not worthwhile to form the first lobby, then no lobbies form. If the first lobby is the grand coalition, i.e., $\hat{l}_1 = [0, 1]$, then clearly that is the only lobby that forms. If $\hat{\gamma}_1 \in (0, 1)$, the second lobbyist will form a coalition of firms with the highest potential for diversion that are not part of the first coalition, etc. The following proposition further characterizes the equilibrium.

Proposition 7 *In the continuum-of-firms model, earlier coalitions include more insiders as members.*

Proposition 7 follows from greater gains to lobby formation when the lobby includes higher-D_i members. Since earlier lobbies include higher-D_i members, by Lemma 3, these lobbies will also provide greater gains to their members and in turn to lobbyists, at least before coalition formation costs.

We next turn to how regulatory uniformity influences lobbies. First, we define a dense coalition structure.

Definition 3 (Dense Coalition Structure) *A coalition structure is dense if all firms are members of coalitions. A coalition structure is not dense if there exists at least one firm that is not a member of a positive-measure coalition.*

Proposition 6 provides iterative conditions for whether a parameter-vector, $(D, \chi(\cdot), c, k)$, will result in a dense coalition structure. The effect of regulatory uniformity on coalition structure depends on the market structure (i.e., the distribution of D), as well as the cost of lobbying function. The following condition on the coalition formation cost will be useful.

Condition 1 (Relatively Concave Coalition Costs) *Coalition formation costs, $\chi(\cdot)$ are relatively concave if $\chi''\left(\left|\hat{l}_j\right|\right) < \Omega_1$ for each $\hat{l}_j$ in the coalition structure, where*

$$\begin{aligned}\Omega_1 &= \frac{k^2}{c(1+k)^2}\left(2\bar{D}_{l_j}D_{\hat{\gamma}_j} + D_{\hat{\gamma}_j}^2\left|\hat{l}_j\right| - \bar{D}_{l_j}\left|\hat{l}_j\right|\frac{dD_{\hat{\gamma}_j}}{d\hat{\gamma}_j}\right) \\ &\quad -\frac{1}{\frac{d\hat{\gamma}_{j-1}}{dk}}\frac{\frac{1}{k(k+1)}\bar{D}_{l_j}\left(\bar{D}_{l_j}+2\left|\hat{l}_j\right|D_{\hat{\gamma}_j}\right)}{\left(D_{\hat{\gamma}_{j-1}}-D_{\hat{\gamma}_j}\right)\left(\bar{D}_{l_j}+\left|\hat{l}_j\right|D_{\hat{\gamma}_j}\right)+\bar{D}_{l_j}\left|\hat{l}_j\right|\frac{dD_{\hat{\gamma}_j}}{d\hat{\gamma}_j}}.\end{aligned}$$

Relatively concave coalition costs imply that the marginal cost of forming a coalition does not increase too quickly in coalition size. If, on the other hand, coalition formation costs are very convex, i.e., $\chi''(\cdot) >> 0$, then coalition size will be relatively unresponsive to changes in other parameters, such as k, c, and D.

Proposition 8 *Assume there are $J \geq 1$ lobbies.*

1. *Lobby membership for the first lobby, $\left|\hat{l}_1\right|$, is increasing in regulatory uniformity, k.*
2. *If coalition formation costs are relatively concave, as in Condition 1, then for coalitions $j \in \{2, ..., J-1\}$, lobby membership, $\left|\hat{l}_j\right|$ is increasing in regulatory uniformity.*
3. *If the coalition structure is dense or $\hat{\gamma}_J$ is close to zero, then $\left|\hat{l}_J\right|$ is decreasing in regulatory uniformity, while if the coalition structure is not dense and $\hat{\gamma}_J$ is not close to zero, then $\left|\hat{l}_J\right|$ is increasing in regulatory uniformity.*

For the first coalition, an increase in regulatory uniformity has the pure effect of increasing the benefits to coalition formation, because it becomes more important to overcome the free-rider problem driven by regulatory uniformity. This increases the size of the first lobby, which, naturally, implies a decrease in the lower-bound of the first lobby, $\hat{\gamma}_1$. For subsequent lobbies, there are countervailing effects. First, the direct influence of the increase in uniformity still applies to lobby $j > 1$, as it does for lobby 1, as all lobbies desire to get larger to overcome the more significant free-rider problem. However, when earlier lobbies get larger, they cannibalize insiders from later lobbies. For example, suppose $\hat{\gamma}_1$ decreases to $\hat{\gamma}'_1$with the increase in k. This implies that insiders with $i \in (\hat{\gamma}'_1, \hat{\gamma}_1)$ shift from being in lobby 2 to being in lobby 1. Now, lobby 2 has lower-D_i members, who benefit less from coalition formation, and this drives a shrinking effect on lobby 2, consistent with Proposition 7. Condition 1 is sufficient for the former effect to dominate, meaning that $\hat{\gamma}_2$ will also shift down to $\hat{\gamma}'_2$, and that the difference in differences $(\hat{\gamma}_1 - \hat{\gamma}'_1) - (\hat{\gamma}_2 - \hat{\gamma}'_2)$ will be positive such that lobby 2 also grows with the increase in k. Similar logic implies iteratively that $\hat{\gamma}'_j < \hat{\gamma}'_j$ for all $j \in \{3, ..., J-1\}$. For lobby J, there are two possibilities. If we are in a dense coalition structure, then $\hat{\gamma}_J$ is fixed at 0 and the decrease in $\hat{\gamma}'_{J-1}$ implies that lobby J shrinks. If we are not in a dense coalition structure, then $\hat{\gamma}_J$ is free to decrease, implying a marginal increase in the size of lobby J. A third case is possible, if the coalition structure is not dense, but $\left(\hat{\gamma}_{J-1} - \hat{\gamma}'_{J-1}\right) - \hat{\gamma}_J < 0$, then coalition J will shrink with the increase in k that moves $\hat{\gamma}_{J-1}$ to $\hat{\gamma}'_{J-1}$.

Corollary 6 *If there are $J > 1$ coalitions, the coalition structure is dense or $\hat{\gamma}_J$ is close to zero, and Condition 1 is satisfied for coalitions $j \in \{2, J-1\}$, then the number of lobbies is weakly decreasing in regulatory uniformity, k. Otherwise, the number of lobbies is weakly increasing in regulatory uniformity.*

The intuition for Corollary 6 follows from Proposition 8. In the dense coalition structure, if each lobby grows with an increase in k, then it is possible for lobby J to get squeezed out of existence through all its members joining earlier lobbies. If the coalition structure is not dense, then an increase in k, by increasing the benefits from lobby formation, can make it profitable for a lobbyist to organize a group of previously-unorganized insiders.

Mathematically, increasing k can allow (23) to be satisfied for a previously nonexistent lobby.

Figure 3 shows the equilibrium coalitions for three values of regulatory uniformity, $k = 0.8$, $k = 1$, and $k = 1.2$. The cost function is $\chi(|l_x|) = \frac{1}{2}(|l_x| + 0.1)^6$, $D_i = 1 + \frac{i}{10}$, and $c = 1$. As k increases, moving from the left-most bars to the right-most bars, the size of the lobby with the largest firms increases. This lobby is the first to form, and the lobbyist will optimally choose to include more insiders as the benefit to overcoming the free-rider problem on lobbying increases with k. Other lobbies, conditional on formation and the lower-bound, are larger as well. That is, the second lobby to form is larger with higher k, even though it spans a set of insiders with less resources per capita to divert. As we move from $k = 0.8$ to $k = 1.2$, Lobby 4 disappears, as its members are absorbed into Lobby 3. Additionally, Lobby 3 shrinks as we move from $k = 1$ to $k = 1.2$, as it becomes constrained by the lower-bound of $i = 0$ with $k = 1.2$.

6.3 Analysis of the continuum-of-firms model

In the proof to Corollary 5, we establish that with a continuum of firms, total lobbying in the economy is given by $\bar{B} = \frac{\bar{D} + k \sum_j \left[\bar{D}_{l_j} |l_j|\right]}{c(1+k)}$ (see equation 47). Similar to above, total lobbying is independent of regulatory uniformity when the grand lobby is optimal and decreases in regulatory uniformity when firms are not organized in coalitions at all. When firms are organized in more than one coalition, $J > 1$, an increase in uniformity has competing effects on total lobbying. The direct effect is to decrease total lobbying, as in the case with no lobbies. The indirect effect, as suggested in Proposition 8, can be to increase lobby size, which tends to increase lobbying because it allows more insiders to overcome the free-rider problem. Intuition might suggest that based on $\frac{d\bar{B}}{dk}$ at the boundaries of $|l_1| = 1$ and $J = 0$, respectively, that lobbies, while tending to mitigate the negative effect of regulatory uniformity on total lobbying, only completely eliminate the effect when $|l_1| = 1$, i.e., when there is a single, grand lobby of insiders. This, however, ignores the potentially nonlinear effect of regulatory

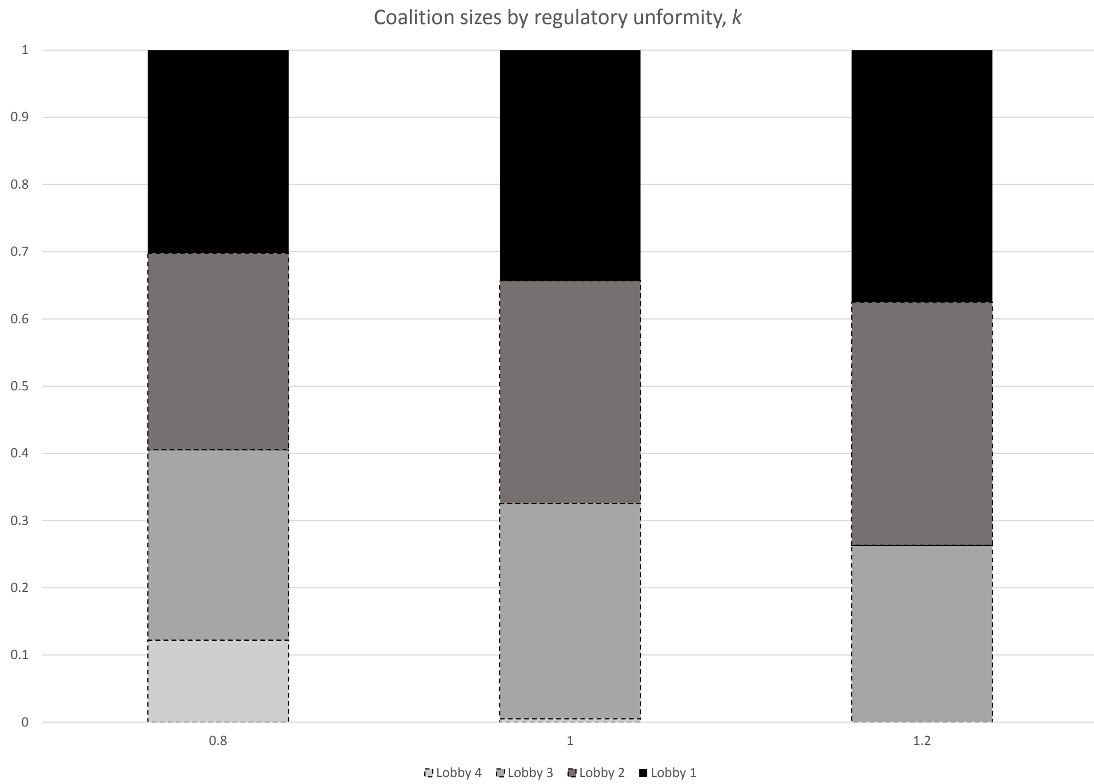

Figure 5

This figure shows equilibrium coalitions for three values of regulatory uniformity, $k = 0.8$, $k = 1$, and $k = 1.2$. The cost function is $\chi(|l_x|) = \frac{1}{2}(|l_x| + 0.1)^6$, divertable funds are given by $D_i = 1 + \frac{i}{10}$, and lobbying effort costs are $c = 1$. Darker shades indicate earlier lobbies.

uniformity on lobby formation and growth, which can cause large countervailing effects, as illustrated in the following proposition.

Proposition 9 *Total lobbying is increasing in regulatory uniformity if coalition structure contains lobbies, is not characterized by the grand lobby, and for each lobby, j,*

$$\chi''\left(\left|\hat{l}_j\right|\right) < \Omega_2. \tag{24}$$

where

$$\Omega_2 = \frac{k^2}{c(1+k)^2}\left(2\bar{D}_{l_j}D_{\hat{\gamma}_j} + D_{\hat{\gamma}_j}^2\left|\hat{l}_j\right| - \bar{D}_{l_j}\left|\hat{l}_j\right|\frac{dD_{\hat{\gamma}_j}}{d\hat{\gamma}_j}\right) - \frac{\bar{D}_{l_j}\left(\bar{D}_{l_j} + 2\left|\hat{l}_j\right|D_{\hat{\gamma}_j}\right)}{(k+1)k\hat{X}(D,\chi)} \quad \textit{and}$$

$$\hat{X}(D,\chi) = \frac{c(1+k)^2}{k^2}\frac{d\gamma_{j-1}}{dk}\left(\frac{D_{\gamma_{j-1}} - D_{\gamma_j}}{D_{\hat{\gamma}_j} + \bar{D}_{l_j}/|l_j|}\right) - (1-|l_j|)\frac{c(1+k)}{k^3}\frac{\bar{D}_{l_j}/|l_j|}{D_{\hat{\gamma}_j} + \bar{D}_{l_j}/|l_j|}$$
$$+\frac{d\gamma_{j-1}}{dk}\left(\left(D_{\gamma_{j-1}} - D_{\gamma_j}\right)\left(\bar{D}_{l_j} + \left|\hat{l}_j\right|D_{\hat{\gamma}_j}\right) + \bar{D}_{l_j}\left|\hat{l}_j\right|\frac{dD_{\hat{\gamma}_j}}{d\hat{\gamma}_j}\right).$$

Proposition 9 shows that the intuition based on the $\frac{d\bar{B}}{dk}$ at the extremes of $|l_1| = 1$ and $J = 0$ only hold when coalition formation costs are not too convex. We show in the Proof of Proposition 9 that (24) is a more restrictive condition than (52). That is, it is possible for all lobbies to be increasing in size as regulatory uniformity increases, but for total lobbying not to be increasing.

7 Conclusion

In this study we investigate the propensity of agents to form coalitions that lobby against stricter policies, focusing on the motivating example of firms' insiders lobbying against stronger securities regulation and enforcement. In our model, firms form coalitions because regulation is, at least partly, uniform across firms. This uniformity, in turn, implies that one firm's lobbying has effects on all other firm's regulation, such that a free-rider problem on lobbying arises among firms. The benefit to forming coalitions arises because we assume that a coalition is able to overcome the free-rider problem among the firms in the coalition, which increases the lobbying efforts of all firms in the coalition. As the free-rider problem is caused by uniformity, a more uniform regulatory process increases the benefits to forming a lobbying coalition.

We analyze both three-firm and continuum-of-firms versions of the model. In the three-firm version, we allow for coalitions to be formed by external lobbyists or by the firms themselves. In this setting, the coalition formation influences the potential lobbies that can form, with dissimilar firms potentially joining together only when lobbies are formed by the insiders themselves. With a continuum of firms, we only allow for lobbyist-formed coalitions. We find that the coalition structure consists of convex intervals of firms, implying that similar types of firms join together, as in the three-firm case with lobbyists. Lobbies with more problematic insiders (i.e., those with more to potentially divert) tend to be larger. Insiders with little potential to divert may not even form into coalitions.

We further find that endogenous coalition formation causes the effects of regulatory uniformity and lobbying costs on total lobbying and average policy strength to be non-monotonic. Increasing the degree of uniformity both: (i) increases the free-rider problem, which decreases lobbying and increase average policy strength; and (ii), increases the benefit of forming coalitions, which increases lobbying and decreases average policy strength. In an environment with a fixed coalition structure, the first effect outweighs the second. However, since the coalition structure reacts to changes in policy uniformity, we find non-monotonic effects of increasing the extent of uniformity. In the three-firm setting, these manifest as jumps. In the continuum-of-firms setting, the effects are smooth. Similarly, increasing the cost insiders bear personally for lobbying, has a direct effect that decreases lobbying but has an indirect effect on the benefit of forming a lobby. We show, in the continuum-of-firms setting, that the net impact of the two effects of changing regulatory uniformity depends on the relation between (i.e., curvature of the function relating) coalition size and coalition formation costs.

References

Arrow, K. J. "A difficulty in the concept of social welfare." *Journal of Political Economy* (1950): 328–346.

Baccara, M., and L. Yariv. "Choosing Peers: Homophily and Polarization in Groups." *Journal of Economic Theory* 165 (2016): 152–178.

Bebchuk, L., and Z. Neeman. "Investor protection and interest group politics." *Review of Financial Studies* 23 (2010): 1089–1119.

Benoit, D. "Activist Investors Have a New Target: Washington." *The Wall Street Journal* (2016), accessed at http://www.wsj.com/articles/new-rules-curbing-wall-street-pay-announced-1461247600 on April 27, 2016.

Bertomeu, J., and R. P. Magee. "Political pressures and the evolution of disclosure regulation." *Review of Accounting Studies* 20 (2014): 775–802.

Bloch, F. "Coalitions and networks in industrial organization." *The Manchester School* 70 (2002): 36–55.

Borak, D., A. Ackerman, and C. Rexrode. "New rules curbing Wall Street pay proposed." *The Wall Street Journal* (2016), accessed at http://www.wsj.com/articles/new-rules-curbing-wall-street-pay-announced-1461247600 on April 27, 2016.

Charlemagne "Trading places." *The Economist* (2016), accessed at http://www.economist.com/news/europe/21697877-what-aversion-global-trade-says-about-europe-and-america-trading-places on April 25, 2016.

Damania, R., and P. G. Fredriksson. "On the formation of industry lobby groups." *Journal of Economic Behavior & Organization* 41 (2000): 315–335.

Drazen, A., N. Limão, and T. Stratmann. "Political contribution caps and lobby formation: Theory and evidence." *Journal of Public Economics* 91 (2007): 723–754.

Friedman, H. L., and M. S. Heinle. "Lobbying and uniform disclosure regulation." *Journal of Accounting Research* 54 (2016): 863–893.

Grossman, G., and E. Helpman. "Electoral competition and special interest politics." *The Review of Economic Studies* 63 (1996): 265–286.

Magee, C. "Endogenous trade policy and lobby formation: an application to the free-rider problem." *Journal of International Economics* 57 (2002): 449–471.

Mitra, D. "Endogenous lobby formation and endogenous protection: a long-run model of trade policy determination." *American Economic Review* 89 (1999): 1116–1134.

Myerson, R. B. "Threat equilibria and fair settlements in cooperative games." *Mathematics of Operations Research* 3 (1978): 265–274.

——— *Game theory* . Harvard University Press, 2013.

PECORINO, P. "Is there a free-rider problem in lobbying? Endogenous tariffs, trigger strategies, and the number of firms." *American Economic Review* 88 (1998): 652–660.

RAY, D., AND R. VOHRA. "Equilibrium binding agreements." *Journal of Economic Theory* 73 (1997): 30–78.

——— "A theory of endogenous coalition structures." *Games and Economic Behavior* 26 (1999): 286–336.

——— "Coalition formation." *Handbook of Game Theory* 4 (2014): 239–326.

RODRIK, D. "Tariffs, subsidies, and welfare with endogenous policy." *Journal of International Economics* 21 (1986): 285–299.

STIGLER, G. "The theory of economic regulation." *The Bell Journal of Economics* 2 (1971): 3–21.

THRALL, R. M., AND W. F. LUCAS. "N-person games in partition function form." *Naval Research Logistics Quarterly* 10 (1963): 281–298.

VON NEUMANN, J., AND O. MORGENSTERN. *Theory of games and economic behavior* . Princeton University Press, Princeton, New Jersey, 1944.

YI, S.-S. "Stable coalition structures with externalities." *Games and Economic Behavior* 20 (1997): 201–237.

Appendix

Section 3.2: Two-firm coalition results

Small-medium firm coalition: When the small (s) and medium (m) firms form a lobby together, lobbying is as follows:

$$\begin{aligned}
\hat{B}_{s,sm} &= \hat{B}_{s,I} + \frac{3kD_m}{c\,(4+9k)} = \frac{4D_s + 3k\bar{D}_{sm}}{c\,(4+9k)} \\
\hat{B}_{m,sm} &= \hat{B}_{m,I} + \frac{3kD_s}{c\,(4+9k)} = \frac{4D_m + 3k\bar{D}_{sm}}{c\,(4+9k)} \\
\hat{B}_{\ell,sm} &= \hat{B}_{\ell,I} = D_\ell \frac{4+3k}{c\,(4+9k)} \\
\bar{B}_{sm} &= \hat{B}_I + \frac{3k\bar{D}_{sm}}{c\,(4+9k)}
\end{aligned}$$

where $\bar{D}_{sm} = D_m + D_s$. Regulatory strengths are:

$$\begin{aligned}
\hat{\pi}_{s,sm} &= \frac{4\left(\lambda D_s - \hat{B}_{s,I} - \frac{3kD_m}{c(4+9k)}\right) + 3k\left(\lambda\bar{D} - \hat{B}_I - \frac{3k\bar{D}_{sm}}{c(4+9k)}\right)}{4+9k} \\
&= \hat{\pi}_{s,I} - \frac{3k\left(4D_m + 3k\bar{D}_{sm}\right)}{c\,(4+9k)^2}, \\
\hat{\pi}_{m,sm} &= \hat{\pi}_{m,I} - \frac{3k\left(4D_s + 3k\bar{D}_{sm}\right)}{c\,(4+9k)^2}, \text{ and} \\
\hat{\pi}_{\ell,sm} &= \hat{\pi}_{\ell,I} - \frac{9k^2\bar{D}_{sm}}{c\,(4+9k)^2}.
\end{aligned}$$

The utilities are:

$$\begin{aligned}
\hat{U}_{sm,sm} &= \frac{c(9k+4)\left((9k+4)\bar{D}_{sm} - \lambda\left(3k\bar{D}_{sm}\bar{D} + 4\overline{D^2_{sm}}\right)\right) + 3k\bar{D}_{sm}\bar{D}\,(3k+4) + 8\overline{D^2_{sm}}}{c(9k+4)^2} \\
&= \hat{U}_{s,I} + \hat{U}_{m,I} + \frac{9k^2\overline{D^2_{sm}}}{2c\,(4+9k)^2}, \text{ and} \\
U_{\ell,sm} &= \frac{D_\ell\left(4D_\ell\lambda - 4 - 2c(9k+4)(3k(\lambda\bar{D} - 3)) + D_\ell(3k+4)^2 + 12k(3k+2)\bar{D}_{sm}\right)}{2c(9k+4)^2} \\
&= \hat{U}_{\ell,I} + \frac{9k^2 D_\ell \bar{D}_{sm}}{c\,(4+9k)^2}
\end{aligned}$$

where $\overline{D_{sm}^2} = D_s^2 + D_m^2$.

Small-large firm coalition: When the small (s) and large (ℓ) firms form a lobby together, lobbying is as follows:

$$\begin{aligned}
\hat{B}_{s,s\ell} &= \hat{B}_{s,I} + \frac{3kD_\ell}{c\,(4+9k)} = \frac{4D_s + 3k\bar{D}_{s\ell}}{c\,(4+9k)} \\
\hat{B}_{m,s\ell} &= \hat{B}_{m,I} = D_m \frac{4+3k}{c\,(4+9k)} \\
\hat{B}_{\ell,s\ell} &= \hat{B}_{\ell,I} + \frac{3kD_s}{c\,(4+9k)} = \frac{4D_\ell + 3k\bar{D}_{s\ell}}{c\,(4+9k)} \\
\bar{B}_{sm} &= \hat{B}_I + \frac{3k\bar{D}_{s\ell}}{c\,(4+9k)}
\end{aligned}$$

where $\bar{D}_{s\ell} = D_s + D_\ell$. Regulatory strengths are:

$$\begin{aligned}
\hat{\pi}_{s,s\ell} &= \hat{\pi}_{s,I} - \frac{3k\left(4D_\ell + 3k\bar{D}_{s\ell}\right)}{c\,(4+9k)^2}, \\
\hat{\pi}_{m,s\ell} &= \hat{\pi}_{m,I} - \frac{9k^2\bar{D}_{s\ell}}{c\,(4+9k)^2}, \text{ and} \\
\hat{\pi}_{\ell,s\ell} &= \hat{\pi}_{\ell,I} - \frac{3k\left(4D_s + 3k\bar{D}_{s\ell}\right)}{c\,(4+9k)^2}.
\end{aligned}$$

and utilities are:

$$\begin{aligned}
\hat{U}_{s\ell,s\ell} &= \frac{c(9k+4)\left((9k+4)\,\bar{D}_{s\ell} - \lambda\left(4\overline{D_{s\ell}^2} + 3k\bar{D}_{s\ell}\bar{D}\right)\right) + 8\overline{D_{s\ell}^2} + 3k\bar{D}_{s\ell}\bar{D}\,(3k+4)}{c(9k+4)^2} \\
&= \hat{U}_{s,I} + \hat{U}_{\ell,I} + \frac{9k^2\overline{D_{s\ell}^2}}{2c\,(4+9k)^2}, \text{ and} \\
\hat{U}_{m,s\ell} &= \frac{D_m\left(-2c(9k+4)(3k(\lambda\bar{D}-3) + 4D_m\lambda - 4) + 12k(3k+2)\bar{D}_{s\ell} + D_m(3k+4)^2\right)}{2c(9k+4)^2} \\
&= \hat{U}_{m,I} + \frac{9k^2 D_m\bar{D}_{s\ell}}{c\,(4+9k)^2}
\end{aligned}$$

where $\overline{D_{s\ell}^2} = D_s^2 + D_\ell^2$.

Medium-large firm coalition: When the medium (m) and large (ℓ) firms form a lobby together, lobbying is as follows:

$$\begin{aligned}
\hat{B}_{s,m\ell} &= \hat{B}_{s,I} = D_s \frac{4+3k}{c\,(4+9k)} \\
\hat{B}_{m,m\ell} &= \hat{B}_{m,I} + \frac{3kD_\ell}{c\,(4+9k)} = \frac{4D_m + 3k\bar{D}_{m\ell}}{c\,(4+9k)} \\
\hat{B}_{\ell,m\ell} &= \hat{B}_{\ell,I} + \frac{3kD_m}{c\,(4+9k)} = \frac{4D_\ell + 3k\bar{D}_{m\ell}}{c\,(4+9k)} \\
\bar{B}_{sm} &= \hat{B}_I + \frac{3k\bar{D}_{m\ell}}{c\,(4+9k)},
\end{aligned}$$

where $\bar{D}_{m\ell} = D_m + D_\ell$. Regulatory strengths are:

$$\begin{aligned}
\hat{\pi}_{s,m\ell} &= \hat{\pi}_{s,I} - \frac{9k^2\bar{D}_{m\ell}}{c\,(4+9k)^2}, \\
\hat{\pi}_{m,m\ell} &= \hat{\pi}_{m,I} - \frac{3k\left(4D_\ell + 3k\bar{D}_{m\ell}\right)}{c\,(4+9k)^2}, \text{ and} \\
\hat{\pi}_{\ell,m\ell} &= \hat{\pi}_{\ell,I} - \frac{3k\left(4D_m + 3k\bar{D}_{m\ell}\right)}{c\,(4+9k)^2}.
\end{aligned}$$

and utilities are:

$$\begin{aligned}
\hat{U}_{m\ell,m\ell} &= \frac{c(9k+4)\left((9k+4)\,\bar{D}_{m\ell} - \lambda\left(4\overline{D^2_{m\ell}} + 3k\bar{D}_{m\ell}\bar{D}\right)\right) + 8\overline{D^2_{m\ell}} + 3k\bar{D}_{m\ell}\bar{D}\,(3k+4)}{c(9k+4)^2} \\
&= \hat{U}_{m,I} + \hat{U}_{\ell,I} + \frac{9k^2\overline{D^2_{m\ell}}}{2c\,(4+9k)^2}, \text{ and} \\
\hat{U}_{s,m\ell} &= \frac{D_s\left(-2c(9k+4)(3k(\lambda\bar{D}-3) + 4D_s\lambda - 4) + 12k(3k+2)\bar{D}_{m\ell} + D_s(3k+4)^2\right)}{2c(9k+4)^2} \\
&= \hat{U}_{s,I} + \frac{9k^2 D_s\bar{D}_{m\ell}}{c\,(4+9k)^2},
\end{aligned}$$

where $\overline{D^2_{m\ell}} = D^2_m + D^2_\ell$.

Proof of Theorem 2: Offer-stable coalitions

To derive Theorem 2, we first prove the following lemma:

Lemma 4 *Coalition $s\ell$ is the only offer-stable two-firm coalition when two-firm coalition* *costs are sufficiently low.*

Proof of Lemma 4. In general, the maximum deviation offer, or MDO,

$$MDO_{g,h}^{jh} = \hat{U}_{h,I} + \frac{9k^2\left(\overline{D_{gh}^2} - 2D_g\bar{D}_{jh}\right)}{2c\left(4+9k\right)^2}$$

First, $MDO_{s,\ell}^{sm} > MDO_{m,\ell}^{s\ell}$:

$$\begin{aligned}
MDO_{s,\ell}^{sm} &> MDO_{m,\ell}^{s\ell} \\
D_s^2 + D_\ell^2 - 2D_sD_m - 2D_sD_\ell &> D_m^2 + D_\ell^2 - 2D_mD_s - 2D_mD_\ell \\
2D_mD_\ell - 2D_sD_\ell &> D_m^2 - D_s^2 \\
2D_\ell\left(D_m - D_s\right) &> \left(D_m + D_s\right)\left(D_m - D_s\right) \\
D_\ell + D_\ell &> D_m + D_s
\end{aligned}$$

which implies that insider s can make a credible offer to insider ℓ to leave the $m\ell$ coalition. Second, $MDO_{\ell,s}^{sm} > MDO_{m,s}^{s\ell}$

$$\begin{aligned}
MDO_{\ell,s}^{sm} &> MDO_{m,s}^{s\ell} \\
\hat{U}_{s,I} + \frac{9k^2\left(\overline{D_{\ell s}^2} - 2D_\ell\bar{D}_{ms}\right)}{2c\left(4+9k\right)^2} &> \hat{U}_{s,I} + \frac{9k^2\left(\overline{D_{ms}^2} - 2D_m\bar{D}_{\ell s}\right)}{2c\left(4+9k\right)^2} \\
D_\ell^2 + D_s^2 - 2D_\ell\left(D_m + D_s\right) &> D_s^2 + D_m^2 - 2D_m\left(D_\ell + D_s\right) \\
D_\ell + D_m &> 2D_s
\end{aligned}$$

which implies that insider ℓ can make a credible offer to insider s to leave the ms coalition. These jointly imply that, absent coalition costs, $\chi_2 \approx 0$, and with prohibitively high three-firm costs, $\chi_3 >> 0$, the $s\ell$ coalition is the only stable two-firm coalition. If an ms coalition forms, then insider ℓ will pick off insider s. If the $m\ell$ coalition forms, then insider s will pick off insider ℓ. If coalition $s\ell$ forms, then insider m cannot make a profitable deviation offer to either insider s or insider ℓ, because all insiders know that such an offer would be dominated. ■

If $\chi_2 > \frac{9k^2\left(D_\ell^2+D_s^2\right)}{2c(4+9k)^2}$, then the s and ℓ insiders will choose to disband the lobby, even though it is stable relative to other two-firm coalitions. In what follows, the gain from the $s\ell$ coalition can affect the equilibrium coalition structure, to simply exposition, we denote $\Xi_3 = \frac{9k^2\left(D_\ell^2+D_s^2\right)}{2c(4+9k)^2}$. Therefore, if $\chi_2 \in (\Xi_3, \Xi_1)$, then insiders ℓ and m will find it profitable to form a coalition. In the presence of these coalition costs,

$$\chi MDO_{s,l}^{ml} = \hat{U}_{sl,sl} - \hat{U}_{s,ml} - \chi_2 = \hat{U}_{\ell,I} - \frac{9k^2\left(D_sD_m + D_sD_\ell\right)}{c\left(4+9k\right)^2} - \varepsilon, \text{ where}$$

$\chi_2 = \frac{9k^2\left(D_\ell^2+D_s^2\right)}{c(4+9k)^2} + \varepsilon$, with $\varepsilon > 0$. Clearly, $\chi MDO_{s,l}^{ml}$ is less than $\hat{U}_{\ell,I}$. Furthermore, if insider ℓ defects and joins the $s\ell$ coalition, we know that the instant later, that coalition will fall apart. However, if $\chi_2 < \Xi_1$, then insider m can offer up to $\hat{U}_{\ell,I} + \Xi_1 - \chi_2 > \hat{U}_{\ell,I}$ to insider ℓ, which is better than what insider ℓ would obtain from defecting to the doomed $s\ell$ coalition. In this case, then, if $\chi_2 \in (\Xi_3, \Xi_1)$, the $m\ell$ coalition is stable. Finally, if $\chi_2 > \Xi_1$, which is the maximum gain from a two-firm coalition, then there is no two-firm stable coalition structure.

If $\chi_2 < \Xi_3$ and $\chi_3 < \Xi_2$, then the insiders will rationally organize into a three-firm coalition if and only if

$$\begin{aligned} & \hat{U}_G - \chi_3 > \hat{U}_{s\ell,s\ell} - \chi_2 + \hat{U}_{m,s\ell} \\ \Leftrightarrow \quad & \frac{9k^2\left(2\overline{D^2} - \overline{D_{s\ell}^2} + 2D_sD_\ell\right)}{2c\left(4+9k\right)^2} + \chi_2 > \chi_3. \end{aligned} \tag{25}$$

The condition in (25) also implies that neither m nor $s\ell$ (jointly) will find it profitable to unilaterally leave the grand coalition. We know that

$$\begin{aligned}\frac{9k^2\left(2\overline{D^2}-\overline{D^2_{s\ell}}+2D_sD_\ell\right)}{2c\left(4+9k\right)^2}+\chi_2 &< \frac{9k^2\left(2\overline{D^2}+2D_sD_\ell\right)}{2c\left(4+9k\right)^2}\\ &< \frac{9k^2\left(\overline{D^2}+D_sD_m+D_sD_\ell+D_mD_\ell\right)}{c\left(4+9k\right)^2}.\end{aligned}$$

So, we can write,

$$\frac{9k^2\left(\overline{D^2}+D_sD_m+D_sD_\ell+D_mD_\ell\right)}{c\left(4+9k\right)^2} > \frac{9k^2\left(2\overline{D^2}-\overline{D^2_{s\ell}}+2D_sD_\ell\right)}{2c\left(4+9k\right)^2}+\chi_2>\chi_3$$

which shows that the upper-bound on insiders organizing into three lobbies is, in fact, lower than what it would be if two-firm coalitions were prohibited.

Alternatively, if $\chi_2 \in \left(\frac{9k^2\left(D_\ell^2+D_s^2\right)}{2c(4+9k)^2}, \frac{9k^2\left(D_\ell^2+D_m^2\right)}{2c(4+9k)^2}\right)$, then the insiders will rationally organize into a three-firm coalition if and only if

$$\hat{U}_G-\chi_3>\hat{U}_{m\ell,m\ell}-\chi_2+\hat{U}_{s,m\ell}\Leftrightarrow\frac{9k^2\left(2\overline{D^2}-\overline{D^2_{m\ell}}+2D_mD_\ell\right)}{2c\left(4+9k\right)^2}+\chi_2>\chi_3. \tag{26}$$

The condition in (26) also implies that neither s nor $m\ell$ (jointly) will find it profitable to unilaterally leave the grand coalition when two-firm coalition costs are too high to support the $s\ell$ coalition, but low enough for the $m\ell$ coalition to survive. It is straightforward to show that when $\chi_2 \in \left(\frac{9k^2\left(D_\ell^2+D_s^2\right)}{2c(4+9k)^2}, \frac{9k^2\left(D_\ell^2+D_m^2\right)}{2c(4+9k)^2}\right)$ and condition in (26) is satisfied,

$$\frac{9k^2\left(\overline{D^2}+D_sD_m+D_sD_\ell+D_mD_\ell\right)}{c\left(4+9k\right)^2} > \frac{9k^2\left(2\overline{D^2}-\overline{D^2_{m\ell}}+2D_mD_\ell\right)}{2c\left(4+9k\right)^2}+\chi_2>\chi_3,$$

which shows that the upper-bound on the cost to insiders of organizing into three lobbies is, in fact, lower than what it would be if two-firm coalitions were prohibited.

Proof of Lemma 2. First, set $\bar{B}_{l_j} = \int_{i\in l_j} B_i di$ and find

$$\hat{B} \in \arg\max_{B(i),i\in l_j} \int_{i\in l_j} U_i di \text{ s.t. } \bar{B}_{l_j} = \int_{i\in l_j} B_i di.$$

Let μ_{l_j} be the Lagrange multiplier on the $\bar{B}_{l_j} = \int_{i\in l_j} B_i di$ constraint. Substituting (16) and rearranging terms yields the Lagrangian

$$\hat{B} \in \arg\max_{B(i),i\in l_j} \int_{i\in l_j} \left(\frac{D_i B(i)}{1+k} - \frac{c}{2} B(i)^2 - \mu_{l_j} B(i) \right) di - \mu_{l_j} \bar{B}_{l_j}. \tag{27}$$

Solving the Euler-Lagrange equation yields the optimal firm specific lobbying efforts as

$$\hat{B}(i) = \frac{D_i - \mu_{l_j}(1+k)}{c(1+k)}. \tag{28}$$

Integrating (28) over $i \in l_j$ yields

$$\begin{aligned}
\int_{i\in l_j} \hat{B}(i)\, di &= \int_{i\in l_j} \frac{D_i - \mu_{l_j}(1+k)}{c(1+k)} di \\
\Rightarrow \bar{B}_{l_j} &= \frac{\int_{i\in l_j} D_i di}{c(1+k)} - \frac{\int_{i\in l_j} \mu_{l_j} di}{c} \\
\Rightarrow \bar{B}_{l_j} &= \frac{\bar{D}_{l_j}}{c(1+k)} - \frac{\mu_{l_j} |l_j|}{c} \\
\Rightarrow \mu_{l_j} &= \frac{\bar{D}_{l_j} - c\bar{B}_{l_j}(1+k)}{(1+k)|l_j|} \qquad (29)
\end{aligned}$$

where $\bar{D}_{l_j} = \int_{i\in l_j} D_i di$ is the size of divertable funds associated with the coalition, and $|l_j| = \int_{l_j} di$ is a measure of the size of the lobby. From (28) and (29), we have

$$\hat{B}(i) = \frac{\left(D_i |l_j| - \bar{D}_{l_j}\right) + c\bar{B}_{l_j}(1+k)}{c|l_j|(1+k)} = \frac{D_i |l_j| - \bar{D}_{l_j}}{c|l_j|(1+k)} + \frac{\bar{B}_{l_j}}{|l_j|} \tag{30}$$

Conditional on an optimal allocation, the coalition will choose $\hat{B}_{l_j}$ as the optimal total level of lobbying,

$$\hat{B}_{l_j} \in \arg\max_{\bar{B}_{l_j}} \int_{i\in l_j} U_i di \text{ s.t. } \hat{B}(i) = \frac{D_i |l_j| - \bar{D}_{l_j}}{c|l_j|(1+k)} + \frac{\bar{B}_{l_j}}{|l_j|}. \tag{31}$$

Denote $\bar{B}_{i\notin l_j} = \int_{i\notin l_j} B_i di$, which allows us to rewrite the maximization problem as

$$\begin{aligned} \hat{B}_{l_j} \in & \arg\max_{\bar{B}_{l_j}} \int_{i\in l_j} \left(\left(1 - \frac{(\lambda D_i - B_i) + k\left(\lambda \bar{D} - \bar{B}\right)}{1+k} \right) D_i - \frac{c}{2} B_i^2 \right) di \\ & \text{s.t. } \hat{B}(i) = \frac{D_i |l_j| - \bar{D}_{l_j}}{c|l_j|(1+k)} + \frac{\bar{B}_{l_j}}{|l_j|}. \end{aligned}$$

Substituting the constraint allows us to write

$$\hat{B}_{l_j} \in \arg\max_{\bar{B}_{l_j}} \frac{\bar{D}_{l_j}\bar{B}_{l_j}}{1+k}\left(\frac{1}{|l_j|} + k\right) - \frac{c}{2}\frac{\left(\bar{B}_{l_j}\right)^2}{|l_j|}. \tag{32}$$

The FOC on the problem in (32) gives

$$\hat{B}_{l_j} = \frac{\bar{D}_{l_j}}{c} \times \frac{1+k|l_j|}{1+k} \tag{33}$$

which is increasing in $\bar{D}_{l_j}$, decreasing in c, increasing in the size of the lobby, $|l_j|$, and decreasing in k, as

$$\frac{d}{dk}\left(\frac{\bar{D}_{l_j}}{c}\frac{1+k|l_j|}{1+k}\right) = \frac{\bar{D}_{l_j}}{c}\frac{|l_j|-1}{(k+1)^2} < 0$$

for all lobbies smaller than the grand coalition in which all firms lobby collectively. We can now write

$$\hat{B}(i;l_j) = \frac{D_i + \bar{D}_{l_j}k}{c(1+k)} \tag{34}$$

which is, except for the $\bar{D}_{l_j} = \int_{i\in l_j} D_i di$ term, independent of the size of the lobby. If the insider lobbies independently, then she will set $\hat{B}(i;\emptyset) = \frac{D_i}{c(1+k)}$. ■

Proof of Proposition 6. The maximization problem for lobbyist 1 is given by

$$\begin{aligned}\hat{\gamma}_1 &\in \arg\max_{\gamma_1 \in [0,1]} \Delta U_{l_1} - \chi\left(|l_1|\right) \\ &= \arg\max_{\gamma_1 \in [0,1]} \frac{k^2}{2c\left(1+k\right)^2} |l_1| \left(\int_{\gamma_1}^{1} D_i di\right)^2 - \chi\left(|l_1|\right) \end{aligned} \tag{35}$$

and the FOC is

$$-\chi'\left(|l_1|\right) = \frac{k^2}{2c\left(1+k\right)^2}\left(\frac{d}{d\gamma_1}\left[\left(\int_{\gamma_1}^{1} \bar{D}_1 di\right)^2\right]|l_1| - \left(\bar{D}_{l_1}\right)^2\right) \tag{36}$$

$$\chi'\left(\left|\hat{l}_1\right|\right)\frac{2c\left(1+k\right)^2}{k^2} = 2\bar{D}_{l_1} D_{\hat{\gamma}_1}|l_1| + \left(\bar{D}_{l_1}\right)^2 \tag{37}$$

$$\chi'\left(\int_{\hat{\gamma}_1}^{1} di\right)\frac{2c\left(1+k\right)^2}{k^2} = 2\left(\int_{\hat{\gamma}_1}^{1} D_i di\right)\left(\int_{\hat{\gamma}_1}^{1} di\right) D_{\hat{\gamma}_1} + \left(\int_{\hat{\gamma}_1}^{1} D_i di\right)^2. \tag{38}$$

The SOC, assumed to be satisfied, is

$$\frac{d}{d\hat{\gamma}_1}\left[2\bar{D}_{l_1} D_{\hat{\gamma}_1}|l_1| + \left(\bar{D}_{l_1}\right)^2 - \chi'\left(\left|\hat{l}_1\right|\right)\frac{2c\left(1+k\right)^2}{k^2}\right] < 0$$

If $\hat{\gamma}_1$, the solution to (37), is between 0 and 1, then the first lobby is $l_1 = [\hat{\gamma}_1, 1]$. If the right-hand-side of (36) is always positive, for instance because $\chi'(0)$ is high, then no lobbies will be formed (i.e., $\gamma_1 = 1$). Alternatively, if the right-hand-side of (36) is always negative, i.e., if $\chi'(1)$ is low, then the grand coalition, $l_1 = [0,1]$, will be optimal (i.e., $\hat{\gamma}_1 = 0$). Assuming $\hat{\gamma}_1 \in (0,1)$, the second lobbyist will choose the second-highest set of firms, $l_2 = [\hat{\gamma}_2, \hat{\gamma}_1]$. Following the approach for $\hat{\gamma}_1$, the second lobbyist's FOC is

$$\chi'\left(\left|\hat{l}_2\right|\right)\frac{2c\left(1+k\right)^2}{k^2} = 2\bar{D}_{l_2} D_{\hat{\gamma}_2}|l_2| + \left(\bar{D}_{l_2}\right)^2 \tag{39}$$

$$\chi'\left(\int_{\hat{\gamma}_2}^{\hat{\gamma}_1} di\right)\frac{2c\left(1+k\right)^2}{k^2} = 2\left(\int_{\hat{\gamma}_2}^{\hat{\gamma}_1} D_i di\right)\left(\int_{\hat{\gamma}_2}^{\hat{\gamma}_1} di\right) D_{\hat{\gamma}_1} + \left(\int_{\hat{\gamma}_2}^{\hat{\gamma}_1} D_i di\right)^2 \tag{40}$$

In general, we can establish the equilibrium lobbying coalition as follows. First, for a lobby with upper-bound γ_{j-1} and lower-bound γ_j, which we will call lobby j or coalition j, we can write the FOC for the lower-bound, $\hat{\gamma}_j$, as

$$\begin{aligned} 0 &= \frac{k^2}{2c(1+k)^2}\frac{d}{d\hat{\gamma}_j}\left[\left(\int_{\hat{\gamma}_j}^{\gamma_{j-1}} di\right)\left(\int_{\hat{\gamma}_j}^{\gamma_{j-1}} D_i di\right)^2\right] - \frac{d}{d\hat{\gamma}_j}\chi\left(\left|\int_{\hat{\gamma}_j}^{\gamma_{j-1}} di\right|\right) \\ \Leftrightarrow 0 &= -\frac{k^2}{2c(1+k)^2}\left(\left(\int_{\hat{\gamma}_j}^{\gamma_{j-1}} D_i di\right)^2 + 2\left(\int_{\hat{\gamma}_j}^{\gamma_{j-1}} D_i di\right)\left(\int_{\hat{\gamma}_j}^{\gamma_{j-1}} di\right) D_{\hat{\gamma}_j}\right) + \chi'\left(\left|\int_{\hat{\gamma}_j}^{\gamma_{j-1}} di\right|\right) \end{aligned} \tag{41}$$

or, equivalently, as $0 = G\left[\gamma_{j-1}, \hat{\gamma}_j\right]$, where G is defined as

$$G = -\frac{k^2}{2c(1+k)^2}\left(\left(\int_{\hat{\gamma}_j}^{\gamma_{j-1}} D_i di\right)^2 + 2\left(\int_{\hat{\gamma}_j}^{\gamma_{j-1}} D_i di\right)\left(\int_{\hat{\gamma}_j}^{\gamma_{j-1}} di\right) D_{\hat{\gamma}_j}\right) + \chi'\left(\left|\int_{\hat{\gamma}_j}^{\gamma_{j-1}} di\right|\right). \tag{42}$$

If the FOC does not hold for some reasonable $\hat{\gamma}_j \in \left(0, \hat{\gamma}_j\right)$, results (b) and (c) in the proposition follow from similar intuition given for the case of the first lobby above. ■

Proof of Proposition 7. Write the FOC as

$$G = -\frac{k^2}{2c(1+k)^2}\left(\begin{array}{c} \left(\int_{\hat{\gamma}_j}^{\gamma_{j-1}} D_i di\right)^2 \\ +2\left(\int_{\hat{\gamma}_j}^{\gamma_{j-1}} D_i di\right)\left(\int_{\hat{\gamma}_j}^{\gamma_{j-1}} di\right) D_{\hat{\gamma}_j} \end{array}\right) + \chi'\left(\left|\int_{\hat{\gamma}_j}^{\gamma_{j-1}} di\right|\right). \tag{43}$$

Taking the derivative of both sides of (42) with respect to γ_{j-1} yields $\frac{\partial G}{\partial \gamma_{j-1}} + \frac{\partial G}{\partial \hat{\gamma}_j}\frac{d\hat{\gamma}_j}{d\gamma_{j-1}} = 0$, which implies $\frac{d\hat{\gamma}_j}{d\gamma_{j-1}} = -\left(\frac{\partial G}{\partial \gamma_{j-1}}\right) / \left(\frac{\partial G}{\partial \hat{\gamma}_{j-1}}\right)$. We know from the SOC that $\frac{\partial G}{\partial \hat{\gamma}_{j-1}} < 0$, implying that $\frac{d\hat{\gamma}_j}{d\gamma_{j-1}} \propto \frac{\partial G}{\partial \gamma_{j-1}}$. Furthermore, we can write the SOC, $\frac{\partial G}{\partial \hat{\gamma}_j}$, as

$$\begin{aligned} \frac{dG}{d\hat{\gamma}_j} &= \frac{k^2\left(\begin{array}{c} 4D_{\hat{\gamma}_j}\int_{\hat{\gamma}_j}^{\gamma_{j-1}} D_i di + 2D_{\hat{\gamma}_j}^2\left(\int_{\hat{\gamma}_j}^{\gamma_{j-1}} di\right) \\ -2\left(\int_{\hat{\gamma}_j}^{\gamma_{j-1}} D_i di\right)\left(\int_{\hat{\gamma}_j}^{\gamma_{j-1}} di\right)\frac{dD_{\hat{\gamma}_j}}{d\hat{\gamma}_j} \end{array}\right)}{2c(1+k)^2} - \chi''\left(\left|\int_{\hat{\gamma}_j}^{\gamma_{j-1}} di\right|\right) < 0, \text{ or} \\ \frac{dG}{d\hat{\gamma}_j} &= \frac{k^2}{c(1+k)^2}\left(2D_{\hat{\gamma}_j}\bar{D}_{l_j} + D_{\hat{\gamma}_j}^2\left|\hat{l}_j\right| - \bar{D}_{l_j}\left|\hat{l}_j\right|\frac{dD_{\hat{\gamma}_j}}{d\hat{\gamma}_j}\right) - \chi''\left(\left|\hat{l}_j\right|\right) < 0 \end{aligned}$$

We can write $\frac{\partial G}{\partial \gamma_{j-1}}$ as

$$\frac{\partial G}{\partial \gamma_{j-1}} = -\frac{k^2}{2c(1+k)^2}\begin{pmatrix} 2\left(\int_{\hat{\gamma}_j}^{\gamma_{j-1}} D_i di\right) D_{\gamma_{j-1}} \\ +2D_{\gamma_{j-1}}\left(\int_{\hat{\gamma}_x}^{\gamma_{j-1}} di\right) D_{\hat{\gamma}_j} \\ +2\left(\int_{\hat{\gamma}_j}^{\gamma_{j-1}} D_i di\right) D_{\hat{\gamma}_j} \end{pmatrix} + \chi''\left(\left|\int_{\hat{\gamma}_j}^{\gamma_{j-1}} di\right|\right)$$

which implies

$$\frac{\partial G}{\partial \gamma_{j-1}} = -\frac{dG}{d\hat{\gamma}_j} - \frac{k^2}{2c(1+k)^2}\begin{pmatrix} 2\left(D_{\gamma_{j-1}} - D_{\hat{\gamma}_j}\right)\int_{\hat{\gamma}_j}^{\gamma_{j-1}} D_i di \\ +2\left(D_{\gamma_{j-1}} - D_{\hat{\gamma}_j}\right)\left(\int_{\hat{\gamma}_j}^{\gamma_{j-1}} di\right) D_{\hat{\gamma}_x} \\ +2\left(\int_{\hat{\gamma}_j}^{\gamma_{j-1}} D_i di\right)\left(\int_{\hat{\gamma}_j}^{\gamma_{j-1}} di\right)\frac{dD_{\hat{\gamma}_j}}{d\hat{\gamma}_j} \end{pmatrix} \tag{44}$$

From $\left|\hat{l}_j\right| = \gamma_{j-1} - \hat{\gamma}_j$ and (44), $\frac{d\left|\hat{l}_j\right|}{d\gamma_{j-1}}$ can be written as

$$\begin{aligned} \frac{d\left|\hat{l}_j\right|}{d\gamma_{j-1}} &= 1 - \frac{d\hat{\gamma}_j}{d\gamma_{j-1}} \\ &= \frac{k^2}{2c(1+k)^2}\begin{pmatrix} 2\left(D_{\gamma_{j-1}} - D_{\hat{\gamma}_j}\right)\int_{\hat{\gamma}_j}^{\gamma_{j-1}} D_i di \\ +2\left(D_{\gamma_{j-1}} - D_{\hat{\gamma}_j}\right)\left(\int_{\hat{\gamma}_j}^{\gamma_{j-1}} di\right) D_{\hat{\gamma}_j} \\ +2\left(\int_{\hat{\gamma}_j}^{\gamma_{j-1}} D_i di\right)\left(\int_{\hat{\gamma}_j}^{\gamma_{j-1}} di\right)\frac{dD_{\hat{\gamma}_j}}{d\hat{\gamma}_j} \end{pmatrix}. \end{aligned} \tag{45}$$

Therefore, it is the case that $\frac{d\left|\hat{l}_j\right|}{d\gamma_{j-1}} > 0$, which implies that the optimal size of the lobby is increasing in the upper-bound. ■

Proof of Corollary 5. Total lobbying efforts within a lobby, j, and across all firms are, respectively,

$$\bar{B}_{l_j} = \int_{i \in l_j} \hat{B}(i; l_j)\, di = \frac{1}{c(1+k)}\left(\bar{D}_{l_j} + k\left|l_j\right|\bar{D}_{l_j}\right), \text{ and} \tag{46}$$

$$\bar{B} = \int_0^1 \frac{D_i + \bar{D}_{l_j} k}{c(1+k)} di = \frac{\bar{D} + k\sum_j\left[\bar{D}_{l_j}\left|l_j\right|\right]}{c(1+k)} \tag{47}$$

Note that the lobbying effort of firm i increases when it joins a coalition. Specifically, relative to not being in a coalition, firm i increases its lobbying effort by $\bar{D}_{l_j}\frac{k}{c(1+k)}$. When the regulatory system is perfectly uniform, the lobbying from a non-organized firm goes to zero, i.e., $\lim_{k\to\infty}\hat{B}(i;\emptyset)=0$. Equation (47) implies that

$$\begin{aligned}\frac{d\bar{B}}{dk} &= \frac{c(1+k)\left(\frac{d}{dk}\left(k\sum_j\left[\bar{D}_{l_j}\left|l_j\right|\right]\right)\right)-c\left(\bar{D}+k\sum_j\left[\bar{D}_{l_j}\left|l_j\right|\right]\right)}{c^2(1+k)^2}\\ &= \frac{-c\sum_j\left[\bar{D}_{l_j}\left(1-\left|l_j\right|\right)\right]+c(1+k)\left(k\sum_j\left[\left(\left|l_j\right|\frac{d}{dk}\bar{D}_{l_j}+\bar{D}_{l_j}\frac{d}{dk}\left|l_j\right|\right)\right]\right)}{c^2(1+k)^2} \qquad (48)\end{aligned}$$

■

Proof of Proposition 8 and Corollary 6. With the FOC for the first lobby expressed as $G=0$, we have $\frac{\partial G}{\partial k}+\frac{\partial G}{\partial\hat{\gamma}_1}\frac{d\hat{\gamma}_1}{dk}=0\Rightarrow\frac{d\hat{\gamma}_j}{dk}=-\left(\frac{\partial G}{\partial k}\right)/\left(\frac{\partial G}{\partial\hat{\gamma}_1}\right)$. For general coalition j,

$$\begin{aligned}\frac{\partial G}{\partial k} &= -\frac{\partial G}{\partial k}\frac{k^2\begin{pmatrix}\left(\int_{\hat{\gamma}_j}^{\hat{\gamma}_{j-1}}D_i di\right)^2\\ +2\left(\int_{\hat{\gamma}_j}^{\hat{\gamma}_{j-1}}D_i di\right)\left(\int_{\hat{\gamma}_{j-1}}^{\hat{\gamma}_{j-1}}di\right)D_{\hat{\gamma}_j}\end{pmatrix}}{2c(1+k)^2}+\frac{\partial G}{\partial k}\chi'\left(\left|\int_{\hat{\gamma}_j}^{\hat{\gamma}_{j-1}}di\right|\right)\\ &= -\frac{k}{c(k+1)^3}\left(\left(\int_{\hat{\gamma}_j}^{\hat{\gamma}_{j-1}}D_i di\right)^2+2\left(\int_{\hat{\gamma}_j}^{\hat{\gamma}_{j-1}}D_i di\right)\left(\int_{\hat{\gamma}_j}^{\hat{\gamma}_{j-1}}di\right)D_{\hat{\gamma}_j}\right)<0\text{, and}\end{aligned}$$

$$\frac{dG}{d\hat{\gamma}_j}=\frac{k^2\begin{pmatrix}2D_{\hat{\gamma}_j}\int_{\hat{\gamma}_j}^{\hat{\gamma}_{j-1}}D_i di+2D_{\hat{\gamma}_j}^2\left(\int_{\hat{\gamma}_j}^{\hat{\gamma}_{j-1}}di\right)\\ +2\left(\int_{\hat{\gamma}_j}^{\hat{\gamma}_{j-1}}D_i di\right)D_{\hat{\gamma}_j}\\ -2\left(\int_{\hat{\gamma}_j}^{\hat{\gamma}_{j-1}}D_i di\right)\left(\int_{\hat{\gamma}_j}^{\hat{\gamma}_{j-1}}di\right)\frac{dD_{\hat{\gamma}_j}}{d\hat{\gamma}_j}\end{pmatrix}}{2c(1+k)^2}-\chi''\left(\left|\int_{\hat{\gamma}_j}^{\hat{\gamma}_{j-1}}di\right|\right)$$

For the first lobby, the change in the lower-bound with an increase in regulatory uniformity is given by,

$$\begin{aligned}
\frac{d\hat{\gamma}_1}{dk} &= \frac{\frac{1}{(k+1)k}\left(\int_{\hat{\gamma}_1}^1 D_i di\right)\left(\left(\int_{\hat{\gamma}_1}^1 D_i di\right) + 2\left(\int_{\hat{\gamma}_1}^1 di\right) D_{\hat{\gamma}_1}\right)}{\left(\begin{pmatrix} D_{\hat{\gamma}_1}\int_{\hat{\gamma}_1}^1 D_i di + D_{\hat{\gamma}_1}^2\left(\int_{\hat{\gamma}_1}^1 di\right) \\ +\left(\int_{\hat{\gamma}_1}^1 D_i di\right) D_{\hat{\gamma}_1} \\ -\left(\int_{\hat{\gamma}_1}^1 D_i di\right)\left(\int_{\hat{\gamma}_1}^1 di\right)\frac{dD_{\hat{\gamma}_1}}{d\hat{\gamma}_1}\end{pmatrix} - \frac{c(1+k)^2}{k^2}\chi''\left(\left|\int_{\hat{\gamma}_1}^1 di\right|\right)\right)} \\
\Leftrightarrow \frac{d\hat{\gamma}_1}{dk} &= \frac{\frac{1}{(k+1)k}\bar{D}_{l_1}\left(\bar{D}_{l_1} + 2\left|\hat{l}_1\right| D_{\hat{\gamma}_1}\right)}{2D_{\hat{\gamma}_1}\bar{D}_{l_1} + D_{\hat{\gamma}_1}^2\left|\hat{l}_1\right| - \bar{D}_{l_1}\left|\hat{l}_1\right|\frac{dD_{\hat{\gamma}_1}}{d\hat{\gamma}_1} - \frac{c(1+k)^2}{k^2}\chi''\left(\left|\hat{l}_1\right|\right)} < 0
\end{aligned}$$

where the inequality follows from the negativity of the SOC in the denominator. The first lobby's size is increasing in uniformity, as

$$\frac{d\left|\hat{l}_1\right|}{dk} = -\frac{d\hat{\gamma}_1}{dk} = -\frac{\frac{1}{(k+1)k}\bar{D}_{l_1}\left(\bar{D}_{l_1} + 2\left|\hat{l}_1\right| D_{\hat{\gamma}_1}\right)}{2D_{\hat{\gamma}_1}\bar{D}_{l_1} + D_{\hat{\gamma}_1}\left|\hat{l}_1\right| D_{\hat{\gamma}_1} - \bar{D}_{l_1}\left|\hat{l}_1\right|\frac{dD_{\hat{\gamma}_1}}{d\hat{\gamma}_1} - \frac{c(1+k)^2}{k^2}\chi''\left(\left|\hat{l}_1\right|\right)} > 0.$$

For a generic interior lobby, (i.e., with J lobbies, $j \in \{2, J-1\}$,

$$\begin{aligned}
\frac{d\left|\hat{l}_j\right|}{dk} &= \frac{d\hat{\gamma}_{j-1}}{dk}\left(1 - \frac{\partial\hat{\gamma}_j}{\partial\hat{\gamma}_{j-1}}\right) - \frac{\partial\hat{\gamma}_j}{\partial k} \\
&= \frac{d\hat{\gamma}_{j-1}}{dk}\frac{k^2}{c(1+k)^2}\left(\left(D_{\hat{\gamma}_{j-1}} - D_{\hat{\gamma}_j}\right)\left(\bar{D}_{l_j} + \left|\hat{l}_j\right| D_{\hat{\gamma}_j}\right) + \bar{D}_{l_j}\left|\hat{l}_j\right|\frac{dD_{\hat{\gamma}_j}}{d\hat{\gamma}_j}\right) \qquad (49) \\
&\quad - \frac{\frac{1}{(k+1)k}\bar{D}_{l_j}\left(\bar{D}_{l_j} + 2\left|\hat{l}_j\right| D_{\hat{\gamma}_j}\right)}{2\bar{D}_{l_j}D_{\hat{\gamma}_j} + D_{\hat{\gamma}_j}^2\left|\hat{l}_j\right| - \bar{D}_{l_j}\left|\hat{l}_j\right|\frac{dD_{\hat{\gamma}_j}}{d\hat{\gamma}_j} - \frac{c(1+k)^2}{k^2}\chi''\left(\left|\hat{l}_j\right|\right)}
\end{aligned}$$

Where the equality follows from substituting

$$\begin{aligned}
1 - \frac{d\hat{\gamma}_j}{d\hat{\gamma}_{j-1}} &= \frac{k^2}{2c(1+k)^2}\left(2\left(D_{\hat{\gamma}_{j-1}} - D_{\hat{\gamma}_j}\right)\bar{D}_{l_j} + 2\left(D_{\hat{\gamma}_{j-1}} - D_{\hat{\gamma}_j}\right)\left|\hat{l}_j\right| D_{\hat{\gamma}_j} + 2\bar{D}_{l_j}\left|\hat{l}_j\right|\frac{dD_{\hat{\gamma}_j}}{d\hat{\gamma}_j}\right) \\
&= \frac{k^2}{c(1+k)^2}\left(\left(D_{\hat{\gamma}_{j-1}} - D_{\hat{\gamma}_j}\right)\left(\bar{D}_{l_j} + \left|\hat{l}_j\right| D_{\hat{\gamma}_j}\right) + \bar{D}_{l_j}\left|\hat{l}_j\right|\frac{dD_{\hat{\gamma}_j}}{d\hat{\gamma}_j}\right).
\end{aligned}$$

Assume $\frac{d\hat{\gamma}_{j-1}}{dk} < 0$, which is true for $j = 1$, i.e., the first coalition. We want to derive a condition for lobby size to be increasing in regulatory uniformity. For lobby size to be increasing in regulatory uniformity for all but the last lobby, it must be true that the lower threshold shifts down more than the upper threshold, i.e., $\frac{d\hat{\gamma}_j}{dk} < \frac{d\hat{\gamma}_{j-1}}{dk} < 0 \Leftrightarrow \frac{d\left|\hat{l}_j\right|}{dk} > 0 \forall j \in \{2, ..., J\}$. This condition further implies the weaker condition that $\frac{d\hat{\gamma}_j}{dk} < 0 \forall j \in \{2, ..., J\}$. If not, then there is at least one coalition for which $\frac{d\hat{\gamma}_j}{dk} \geq 0$. Let $j^\dagger$ be the earliest interior lobby such that $\frac{d\hat{\gamma}_j}{dk} \geq 0$. Then, $\frac{d\hat{\gamma}_{j^\dagger}}{dk} \geq 0 \Rightarrow \frac{d\left|\hat{l}_{j^\dagger}\right|}{dk} < 0$ because $\hat{\gamma}_{j^\dagger-1}$ shifts down with an increase in k and $\hat{\gamma}_{j^\dagger}$ shifts up with an increase in k. Similarly, $\frac{d\left|\hat{l}_{j-1}\right|}{dk} > 0 \Rightarrow \frac{d\hat{\gamma}_{j-1}}{dk} < 0$ for all but the last coalition. For a generic $j \in \{2, ..., J\}$, assuming $\frac{d\hat{\gamma}_{j-1}}{dk} < 0$ and that the SOC is satisfied for all $\hat{\gamma}_j$, $j \in \{1, ..., J-1\}$, from (49) we can write $\frac{d\left|\hat{l}_j\right|}{dk} > 0$ as equivalent to

$$
\begin{aligned}
& \frac{d\hat{\gamma}_{j-1}}{dk} \frac{k^2}{c\left(1+k\right)^2} \left(\left(D_{\hat{\gamma}_{j-1}} - D_{\hat{\gamma}_j} \right) \left(\bar{D}_{l_j} + \left|\hat{l}_j\right| D_{\hat{\gamma}_j} \right) + \bar{D}_{l_j} \left|\hat{l}_j\right| \frac{dD_{\hat{\gamma}_j}}{d\hat{\gamma}_j} \right) \\
> \ & \frac{\frac{1}{(k+1)k} \bar{D}_{l_j} \left(\bar{D}_{l_j} + 2\left|\hat{l}_j\right| D_{\hat{\gamma}_j} \right)}{2\bar{D}_{l_j} D_{\hat{\gamma}_j} + D_{\hat{\gamma}_j}^2 \left|\hat{l}_j\right| - \bar{D}_{l_j}\left|\hat{l}_j\right| \frac{dD_{\hat{\gamma}_j}}{d\hat{\gamma}_j} - \frac{c(1+k)^2}{k^2} \chi''\left(\left|\hat{l}_j\right|\right)} \\
\Leftrightarrow \ & \frac{d\hat{\gamma}_{j-1}}{dk} \left(2\bar{D}_{l_j} D_{\hat{\gamma}_j} + D_{\hat{\gamma}_j}^2 \left|\hat{l}_j\right| - \bar{D}_{l_j}\left|\hat{l}_j\right| \frac{dD_{\hat{\gamma}_j}}{d\hat{\gamma}_j} - \frac{c\left(1+k\right)^2}{k^2} \chi''\left(\left|\hat{l}_j\right|\right) \right) \\
< \ & \frac{c\frac{k+1}{k^3} \bar{D}_{l_j} \left(\bar{D}_{l_j} + 2\left|\hat{l}_j\right| D_{\hat{\gamma}_j} \right)}{\left(\left(D_{\hat{\gamma}_{j-1}} - D_{\hat{\gamma}_j} \right) \left(\bar{D}_{l_j} + \left|\hat{l}_j\right| D_{\hat{\gamma}_j} \right) + \bar{D}_{l_j} \left|\hat{l}_j\right| \frac{dD_{\hat{\gamma}_j}}{d\hat{\gamma}_j} \right)}.
\end{aligned}
$$

This is equivalent to the following condition on the local curvature of the coalition cost function, χ:

$$
\begin{aligned}
\chi''\left(\left|\hat{l}_j\right|\right) \ < \ & \left(2\bar{D}_{l_j} D_{\hat{\gamma}_j} + D_{\hat{\gamma}_j}^2 \left|\hat{l}_j\right| - \bar{D}_{l_j}\left|\hat{l}_j\right| \frac{dD_{\hat{\gamma}_j}}{d\hat{\gamma}_j} \right) \frac{k^2}{c\left(1+k\right)^2} \\
& - \frac{1}{\frac{d\hat{\gamma}_{j-1}}{dk}} \frac{\frac{1}{k(k+1)} \bar{D}_{l_j} \left(\bar{D}_{l_j} + 2\left|\hat{l}_j\right| D_{\hat{\gamma}_j} \right)}{\left(\left(D_{\hat{\gamma}_{j-1}} - D_{\hat{\gamma}_j} \right) \left(\bar{D}_{l_j} + \left|\hat{l}_j\right| D_{\hat{\gamma}_j} \right) + \bar{D}_{l_j} \left|\hat{l}_j\right| \frac{dD_{\hat{\gamma}_j}}{d\hat{\gamma}_j} \right)}
\end{aligned}
\tag{50}
$$

From (50), coalition size is increasing in regulatory uniformity as long as $\frac{d\hat{\gamma}_{j-1}}{dk} < 0$ and $\chi''\left(\left|\hat{l}_j\right|\right)$ is not too large. Note that $\frac{d\hat{\gamma}_{j-1}}{dk} < 0$ implies that the term on the right-hand side of (50) is positive as long as $2\bar{D}_{l_j}D_{\hat{\gamma}_j}+D_{\hat{\gamma}_j}\left|\hat{l}_j\right|D_{\hat{\gamma}_j}-\bar{D}_{l_j}\left|\hat{l}_j\right|\frac{dD_{\hat{\gamma}_j}}{d\hat{\gamma}_j}>0$. The SOC implies,

$$\frac{k^2}{c\left(1+k\right)^2}\left(2\bar{D}_{l_j}D_{\hat{\gamma}_j}+D_{\hat{\gamma}_j}\left|\hat{l}_j\right|D_{\hat{\gamma}_j}-\bar{D}_{l_j}\left|\hat{l}_j\right|\frac{dD_{\hat{\gamma}_j}}{d\hat{\gamma}_j}\right)<\chi''\left(\left|\hat{l}_j\right|\right). \tag{51}$$

Combining (50) and (51) gives

$$\begin{aligned} 0 &< \chi''\left(\left|\hat{l}_j\right|\right)-\frac{k^2}{c\left(1+k\right)^2}\left(2\bar{D}_{l_j}D_{\hat{\gamma}_j}+D_{\hat{\gamma}_j}\left|\hat{l}_j\right|D_{\hat{\gamma}_j}-\bar{D}_{l_j}\left|\hat{l}_j\right|\frac{dD_{\hat{\gamma}_j}}{d\hat{\gamma}_j}\right) \\ &< -\frac{1}{\frac{d\hat{\gamma}_{j-1}}{dk}}\frac{\frac{1}{k(k+1)}\bar{D}_{l_j}\left(\bar{D}_{l_j}+2\left|\hat{l}_j\right|D_{\hat{\gamma}_j}\right)}{\left(\left(D_{\hat{\gamma}_{j-1}}-D_{\hat{\gamma}_j}\right)\left(\bar{D}_{l_j}+\left|\hat{l}_j\right|D_{\hat{\gamma}_j}\right)+\bar{D}_{l_j}\left|\hat{l}_j\right|\frac{dD_{\hat{\gamma}_j}}{d\hat{\gamma}_j}\right)} \end{aligned} \tag{52}$$

where the first inequality is the SOC. As $\frac{dD_{\hat{\gamma}_j}}{d\hat{\gamma}_j}$ increases, i.e., as firm heterogeneity driven by local differences in D_i increases, the SOC will remain positive, but the second inequality in (52) will be harder to satisfy, because higher $\frac{dD_{\hat{\gamma}_j}}{d\hat{\gamma}_j}$ increases the middle term while shrinking the right-most term.

Considering (50) inductively, we have, first, that $\frac{d\left|\hat{l}_1\right|}{dk}>0\Rightarrow\frac{d\hat{\gamma}_1}{dk}<0$, then

$$\chi''\left(\left|\hat{l}_2\right|\right) < \left(\begin{array}{c}\left(2\bar{D}_{l_2}D_{\hat{\gamma}_2}+D_{\hat{\gamma}_2}^2\left|\hat{l}_2\right|-\bar{D}_{l_2}\left|\hat{l}_2\right|\frac{dD_{\hat{\gamma}_2}}{d\hat{\gamma}_2}\right)\frac{k^2}{c(1+k)^2} \\ -\frac{1}{\frac{d\hat{\gamma}_1}{dk}}\frac{\frac{1}{k(k+1)}\bar{D}_{l_2}\left(\bar{D}_{l_2}+2\left|\hat{l}_2\right|D_{\hat{\gamma}_2}\right)}{\left(\left(D_{\hat{\gamma}_1}-D_{\hat{\gamma}_2}\right)\left(\bar{D}_{l_2}+\left|\hat{l}_2\right|D_{\hat{\gamma}_2}\right)+\bar{D}_{l_2}\left|\hat{l}_2\right|\frac{dD_{\hat{\gamma}_2}}{d\hat{\gamma}_2}\right)}\end{array}\right)\Rightarrow\frac{d\hat{\gamma}_2}{dk}<0,$$

$$\chi''\left(\left|\hat{l}_3\right|\right) < \left(\begin{array}{c}\left(2\bar{D}_{l_3}D_{\hat{\gamma}_3}+D_{\hat{\gamma}_3}^2\left|\hat{l}_3\right|-\bar{D}_{l_3}\left|\hat{l}_3\right|\frac{dD_{\hat{\gamma}_3}}{d\hat{\gamma}_3}\right)\frac{k^2}{c(1+k)^2} \\ -\frac{1}{\frac{d\hat{\gamma}_2}{dk}}\frac{\frac{1}{k(k+1)}\bar{D}_{l_3}\left(\bar{D}_{l_3}+2\left|\hat{l}_3\right|D_{\hat{\gamma}_3}\right)}{\left(\left(D_{\hat{\gamma}_2}-D_{\hat{\gamma}_3}\right)\left(\bar{D}_{l_3}+\left|\hat{l}_3\right|D_{\hat{\gamma}_3}\right)+\bar{D}_{l_3}\left|\hat{l}_3\right|\frac{dD_{\hat{\gamma}_3}}{d\hat{\gamma}_3}\right)}\end{array}\right)\Rightarrow\frac{d\hat{\gamma}_3}{dk}<0,$$

and so on until the last coalition. Given the J coalitions in equilibrium, if the coalition structure is dense, then the size of the last coalition depends only on its upper-bound, implying $\frac{d\left|\hat{l}_J\right|}{dk}=\frac{d\hat{\gamma}_{J-1}}{dk}<0$. If the coalition structure is not dense then $\frac{d\left|\hat{l}_J\right|}{dk}=\frac{d\hat{\gamma}_{J-1}}{dk}-\frac{d\hat{\gamma}_J}{dk}$ and the above analysis implies $\frac{d\left|\hat{l}_J\right|}{dk}>0$ if (50) holds. Applying induction to (50), we see that (50) holding for all $j\in\{2,...,J-1\}$ is a sufficient set of conditions for $\frac{d\left|\hat{l}_j\right|}{dk}>0\forall j\in\{1,...,J-1\}$, with $\frac{d\left|\hat{l}_J\right|}{dk}<0$. It is also possible that a slight increase in k in the presence of a non-dense coalition structure will lead to an increase in the number of coalitions, since the lobbyist may find it optimal to organize a set of firms with $D_i < D_{\hat{\gamma}_J}$ into a new coalition. ■

Proof of Proposition 9. We can write the marginal effect of an increase in regulatory uniformity on total lobbying as

$$\begin{aligned}\frac{d\bar{B}}{dk} &= \frac{c(1+k)\left(\frac{d}{dk}\left(k\sum_j\left[\bar{D}_{l_j}|l_j|\right]\right)\right) - c\left(\bar{D} + k\sum_j\left[\bar{D}_{l_j}|l_j|\right]\right)}{c^2(1+k)^2} \\ &= \frac{-\sum_j\left[\bar{D}_{l_j}(1-|l_j|)\right] + k(1+k)\left(\sum_j\left[\left(|l_j|\frac{d}{dk}\bar{D}_{l_j} + \bar{D}_{l_j}\frac{d}{dk}|l_j|\right)\right]\right)}{c(1+k)^2}\end{aligned} \tag{53}$$

The first term in the sum is positive, while the remaining terms, $\frac{d\gamma_j}{dk}|l_j|k(1+k)\left(D_{\gamma_{j-1}} - D_{\gamma_j}\right) - \bar{D}_{l_j}(1-|l_j|)$, are negative. From this expression, if there is one grand lobby in equilibrium, then $\frac{d\bar{B}}{dk} = 0$, as $|l_1| = 1$ and $\bar{D}_{l_j} = \bar{D}$. This is straightforward to see from (47), as, when there is only one grand lobby, $\bar{B} = \bar{D}/c$. When there are no lobbies, $|l_j| = \bar{D}_{\gamma_j} = 0\forall j$, implying that $\bar{B} = \frac{\bar{D}}{c(1+k)}$ and $\frac{d\bar{B}}{dk} < 0$. From the Fundamental Theorem of Calculus, $\frac{d\bar{D}_{l_j}}{dk} = D_{\gamma_{j-1}}\frac{d\gamma_{j-1}}{dk} - D_{\gamma_j}\frac{d\gamma_j}{dk}$ and $\frac{d|l_j|}{dk} = \frac{d\gamma_{j-1}}{dk} - \frac{d\gamma_j}{dk}$, so we can write $\frac{d\bar{B}}{dk}$ from (48) as

$$\begin{aligned}&\frac{k(1+k)}{c(1+k)^2}\begin{pmatrix}\sum_j\left[|l_j|\left(\frac{d\gamma_{j-1}}{dk}D_{\gamma_{j-1}} - \frac{d\gamma_j}{dk}D_{\gamma_j}\right)\right] \\ +\sum_j\left[\bar{D}_{l_j}\left(\left(\frac{d\gamma_{j-1}}{dk} - \frac{d\gamma_j}{dk}\right)\right)\right] - \sum_j\left[\bar{D}_{l_j}\frac{(1-|l_j|)}{k(1+k)}\right]\end{pmatrix} \\ = &\frac{k(1+k)}{c(1+k)^2}\sum_j\left[\bar{D}_{l_j}\begin{pmatrix}\left(\frac{D_{\gamma_j}}{\bar{D}_{l_j}/|l_j|} + 1\right)\left(\frac{d\gamma_{j-1}}{dk} - \frac{d\gamma_j}{dk}\right) \\ -\frac{(1-|l_j|)}{k(1+k)} + \frac{d\gamma_{j-1}}{dk}\frac{D_{\gamma_{j-1}} - D_{\gamma_j}}{\bar{D}_{l_j}/|l_j|}\end{pmatrix}\right]\end{aligned} \tag{54}$$

From above, we can write $\frac{d\gamma_j}{dk} = \frac{\partial\gamma_j}{\partial k} + \frac{\partial\gamma_{j-1}}{\partial k}\frac{d\gamma_j}{d\gamma_{j-1}}$, with

$$\begin{aligned}\frac{\partial\hat{\gamma}_j}{\partial\hat{\gamma}_{j-1}} &= 1 - \frac{k^2}{c(1+k)^2}\left(\left(D_{\hat{\gamma}_{j-1}} - D_{\hat{\gamma}_j}\right)\left(\bar{D}_{l_j} + \left|\hat{l}_j\right|D_{\hat{\gamma}_j}\right) + \bar{D}_{l_j}\left|\hat{l}_j\right|\frac{dD_{\hat{\gamma}_j}}{d\hat{\gamma}_j}\right), \text{ and} \\ \frac{\partial\hat{\gamma}_j}{\partial k} &= \frac{\frac{1}{(k+1)k}\bar{D}_{l_j}\left(\bar{D}_{l_j} + 2\left|\hat{l}_j\right|D_{\hat{\gamma}_j}\right)}{2\bar{D}_{l_j}D_{\hat{\gamma}_j} + D^2_{\hat{\gamma}_j}\left|\hat{l}_j\right| - \bar{D}_{l_j}\left|\hat{l}_j\right|\frac{dD_{\hat{\gamma}_j}}{d\hat{\gamma}_j} - \frac{c(1+k)^2}{k^2}\chi''\left(\left|\hat{l}_j\right|\right)}\end{aligned}$$

implying that

$$\begin{aligned}
\frac{d\hat{\gamma}_j}{dk} &= \frac{d\hat{\gamma}_{j-1}}{dk}\frac{\partial\hat{\gamma}_j}{\partial\hat{\gamma}_{j-1}} + \frac{\partial\hat{\gamma}_j}{\partial k} \\
&= \frac{d\hat{\gamma}_{j-1}}{dk}\left(1 - \frac{k^2}{c\left(1+k\right)^2}\left(\left(D_{\hat{\gamma}_{j-1}} - D_{\hat{\gamma}_j}\right)\left(\bar{D}_{l_j} + \left|\hat{l}_j\right| D_{\hat{\gamma}_j}\right) + \bar{D}_{l_j}\left|\hat{l}_j\right|\frac{dD_{\hat{\gamma}_j}}{d\hat{\gamma}_j}\right)\right) \\
&\quad + \frac{\frac{1}{(k+1)k}\bar{D}_{l_j}\left(\bar{D}_{l_j} + 2\left|\hat{l}_j\right| D_{\hat{\gamma}_j}\right)}{2\bar{D}_{l_j}D_{\hat{\gamma}_j} + D^2_{\hat{\gamma}_j}\left|\hat{l}_j\right| - \bar{D}_{l_j}\left|\hat{l}_j\right|\frac{dD_{\hat{\gamma}_j}}{d\hat{\gamma}_j} - \frac{c(1+k)^2}{k^2}\chi''\left(\left|\hat{l}_j\right|\right)}
\end{aligned} \tag{55}$$

From (55), $\frac{d\gamma_{j-1}}{dk} - \frac{d\gamma_j}{dk} =$

$$\begin{array}{c}
\frac{d\hat{\gamma}_{j-1}}{dk}\left(\frac{k^2}{c\left(1+k\right)^2}\left(\begin{array}{c}\left(D_{\hat{\gamma}_{j-1}} - D_{\hat{\gamma}_j}\right)\left(\bar{D}_{l_j} + \left|\hat{l}_j\right| D_{\hat{\gamma}_j}\right) \\ +\bar{D}_{l_j}\left|\hat{l}_j\right|\frac{dD_{\hat{\gamma}_j}}{d\hat{\gamma}_j}\end{array}\right)\right) \\
-\frac{\frac{1}{(k+1)k}\bar{D}_{l_j}\left(\bar{D}_{l_j} + 2\left|\hat{l}_j\right| D_{\hat{\gamma}_j}\right)}{2\bar{D}_{l_j}D_{\hat{\gamma}_j} + D^2_{\hat{\gamma}_j}\left|\hat{l}_j\right| - \bar{D}_{l_j}\left|\hat{l}_j\right|\frac{dD_{\hat{\gamma}_j}}{d\hat{\gamma}_j} - \frac{c(1+k)^2}{k^2}\chi''\left(\left|\hat{l}_j\right|\right)}
\end{array}$$

Substitute this into the term in parentheses in the expression for $\frac{d\bar{B}}{dk}$ in (54), yielding $\frac{d\bar{B}}{dk} = \frac{k(1+k)}{c(1+k)^2}\sum_j\left[\bar{D}_{l_j}\hat{Y}_j\right]$, where

$$\begin{aligned}
\hat{Y}_j &= \frac{d\gamma_{j-1}}{dk}\left(\frac{D_{\gamma_{j-1}} - D_{\gamma_j}}{\bar{D}_{l_j}/\left|l_j\right|}\right) - \frac{\left(1 - \left|l_j\right|\right)}{k\left(1+k\right)} \\
&\quad + \left(\frac{D_{\hat{\gamma}_j}}{\bar{D}_{l_j}/\left|l_j\right|} + 1\right)\left(\begin{array}{c}
\frac{d\gamma_{j-1}}{dk}\frac{k^2}{c(1+k)^2}\left(\begin{array}{c}\left(D_{\gamma_{j-1}} - D_{\gamma_j}\right)\left(\bar{D}_{l_j} + \left|\hat{l}_j\right| D_{\hat{\gamma}_j}\right) \\ +\bar{D}_{l_j}\left|\hat{l}_j\right|\frac{dD_{\hat{\gamma}_j}}{d\hat{\gamma}_j}\end{array}\right) \\
-\frac{\frac{1}{(k+1)k}\bar{D}_{l_j}\left(\bar{D}_{l_j} + 2\left|\hat{l}_j\right| D_{\hat{\gamma}_j}\right)}{2\bar{D}_{l_j}D_{\hat{\gamma}_j} + D^2_{\hat{\gamma}_j}\left|\hat{l}_j\right| - \bar{D}_{l_j}\left|\hat{l}_j\right|\frac{dD_{\hat{\gamma}_j}}{d\hat{\gamma}_j} - \frac{c(1+k)^2}{k^2}\chi''\left(\left|\hat{l}_j\right|\right)}
\end{array}\right)
\end{aligned}$$

For $\frac{d\bar{B}}{dk} > 0$, it is sufficient to have $\hat{Y}_j > 0 \forall j$ as long as there is a positive number of lobbies and $\left|\hat{l}_1\right| < 1$, implying

$$
\begin{aligned}
& \left(\frac{D_{\hat{\gamma}_j} + \bar{D}_{l_j} / |l_j|}{\bar{D}_{l_j} / |l_j|}\right) \frac{\frac{1}{(k+1)k} \bar{D}_{l_j} \left(\bar{D}_{l_j} + 2\left|\hat{l}_j\right| D_{\hat{\gamma}_j}\right)}{2\bar{D}_{l_j} D_{\hat{\gamma}_j} + D^2_{\hat{\gamma}_j}\left|\hat{l}_j\right| - \bar{D}_{l_j}\left|\hat{l}_j\right| \frac{dD_{\hat{\gamma}_j}}{d\hat{\gamma}_j} - \frac{c(1+k)^2}{k^2}\chi''\left(\left|\hat{l}_j\right|\right)} \\
< \ & \frac{d\gamma_{j-1}}{dk}\left(\frac{D_{\gamma_{j-1}} - D_{\gamma_j}}{\bar{D}_{l_j} / |l_j|}\right) - \frac{(1 - |l_j|)}{k(1+k)} \\
& + \left(\frac{D_{\hat{\gamma}_j} + \bar{D}_{l_j} / |l_j|}{\bar{D}_{l_j} / |l_j|}\right) \frac{d\gamma_{j-1}}{dk} \frac{k^2}{c(1+k)^2}\left(\left(D_{\gamma_{j-1}} - D_{\gamma_j}\right)\left(\bar{D}_{l_j} + \left|\hat{l}_j\right| D_{\hat{\gamma}_j}\right) + \bar{D}_{l_j}\left|\hat{l}_j\right| \frac{dD_{\hat{\gamma}_j}}{d\hat{\gamma}_j}\right) \\
\Leftrightarrow \ & \chi''\left(\left|\hat{l}_j\right|\right) < \frac{k^2}{c(1+k)^2}\left(2\bar{D}_{l_j} D_{\hat{\gamma}_j} + D^2_{\hat{\gamma}_j}\left|\hat{l}_j\right| - \bar{D}_{l_j}\left|\hat{l}_j\right| \frac{dD_{\hat{\gamma}_j}}{d\hat{\gamma}_j}\right. \\
& - \frac{\frac{1}{(k+1)k} \bar{D}_{l_j}\left(\bar{D}_{l_j} + 2\left|\hat{l}_j\right| D_{\hat{\gamma}_j}\right)}{\left(\begin{array}{c} \frac{c(1+k)^2}{k^2} \frac{d\gamma_{j-1}}{dk}\left(\frac{D_{\gamma_{j-1}} - D_{\gamma_j}}{D_{\hat{\gamma}_j} + \bar{D}_{l_j}/|l_j|}\right) - (1 - |l_j|) \frac{c(1+k)}{k^3} \frac{\bar{D}_{l_j}/|l_j|}{D_{\hat{\gamma}_j} + \bar{D}_{l_j}/|l_j|} \\ + \frac{d\gamma_{j-1}}{dk}\left(\left(D_{\gamma_{j-1}} - D_{\gamma_j}\right)\left(\bar{D}_{l_j} + \left|\hat{l}_j\right| D_{\hat{\gamma}_j}\right) + \bar{D}_{l_j}\left|\hat{l}_j\right| \frac{dD_{\hat{\gamma}_j}}{d\hat{\gamma}_j}\right) \end{array}\right)}
\end{aligned}
$$

which is the condition given in (24). This is a more restrictive condition than (52), as

$$
\begin{aligned}
& \frac{k^2}{c(1+k)^2}\left(2\bar{D}_{l_j} D_{\hat{\gamma}_j} + D^2_{\hat{\gamma}_j}\left|\hat{l}_j\right| - \bar{D}_{l_j}\left|\hat{l}_j\right| \frac{dD_{\hat{\gamma}_j}}{d\hat{\gamma}_j}\right) \\
& - \frac{\frac{1}{k(k+1)} \bar{D}_{l_j}\left(\bar{D}_{l_j} + 2\left|\hat{l}_j\right| D_{\hat{\gamma}_j}\right)}{\frac{d\hat{\gamma}_{j-1}}{dk}\left(\left(D_{\hat{\gamma}_{j-1}} - D_{\hat{\gamma}_j}\right)\left(\bar{D}_{l_j} + \left|\hat{l}_j\right| D_{\hat{\gamma}_j}\right) + \bar{D}_{l_j}\left|\hat{l}_j\right| \frac{dD_{\hat{\gamma}_j}}{d\hat{\gamma}_j}\right)} \\
> \ & \frac{k^2}{c(1+k)^2}\left(2\bar{D}_{l_j} D_{\hat{\gamma}_j} + D^2_{\hat{\gamma}_j}\left|\hat{l}_j\right| - \bar{D}_{l_j}\left|\hat{l}_j\right| \frac{dD_{\hat{\gamma}_j}}{d\hat{\gamma}_j}\right) \\
& - \frac{\frac{1}{(k+1)k} \bar{D}_{l_j}\left(\bar{D}_{l_j} + 2\left|\hat{l}_j\right| D_{\hat{\gamma}_j}\right)}{\left(\begin{array}{c} \frac{c(1+k)^2}{k^2} \frac{d\gamma_{j-1}}{dk}\left(\frac{D_{\gamma_{j-1}} - D_{\gamma_j}}{D_{\hat{\gamma}_j} + \bar{D}_{l_j}/|l_j|}\right) - (1 - |l_j|) \frac{c(1+k)}{k^3} \frac{\bar{D}_{l_j}/|l_j|}{D_{\hat{\gamma}_j} + \bar{D}_{l_j}/|l_j|} \\ + \frac{d\gamma_{j-1}}{dk}\left(\left(D_{\gamma_{j-1}} - D_{\gamma_j}\right)\left(\bar{D}_{l_j} + \left|\hat{l}_j\right| D_{\hat{\gamma}_j}\right) + \bar{D}_{l_j}\left|\hat{l}_j\right| \frac{dD_{\hat{\gamma}_j}}{d\hat{\gamma}_j}\right) \end{array}\right)}
\end{aligned}
$$

$$
\Leftrightarrow 0 > \frac{c(1+k)^2}{k^2} \frac{d\gamma_{j-1}}{dk}\left(\frac{D_{\gamma_{j-1}} - D_{\gamma_j}}{D_{\hat{\gamma}_j} + \bar{D}_{l_j} / |l_j|}\right) - (1 - |l_j|) \frac{c(1+k)}{k^3} \frac{\bar{D}_{l_j} / |l_j|}{D_{\hat{\gamma}_j} + \bar{D}_{l_j} / |l_j|}.
$$

■