Board Involvement, Director Expertise and Executive Incentives[1]

Xiaojing Meng, NYU Stern

Jie Joyce Tian, University of Waterloo

May 30, 2016

[1]We have benefited from discussions with Ramji Balakrishnan, Tim Baldenius, Pingyang Gao, Ilan Guttman, Christian Hofmann, Haresh Sapra and Hao Xue. We would also like to thank seminar participants at NYU Stern, Chicago Booth and LMU for helpful comments and suggestions.

Abstract

Boards of directors have become more involved in monitoring and advising top management. We investigate how board involvement affects executive incentives in a project investment setting. To increase the probability of project success, the CEO engages in a sequence of tasks: first acquiring information to evaluate a potential project, then reporting his assessment of the project to the board, and finally implementing the project if invested. We show that board active involvement, on one hand, weakens the positive incentive spillover effect among the tasks, but on the other hand, provides incremental project-related information, which is also useful to provide incentives. As a consequence of the weakening of spillover effect, investment occurs less frequently than the first-best benchmark. Even in those cases investment efficiency is increased by board involvement, the firm value can still be lowered because board involvement may exacerbate the incentive problems. We find that the level of director expertise determines whether board involvement is beneficial for the firm but director expertise does not always improve executive incentives.

1 Introduction

Over the years, boards of directors have become more active in monitoring firm performance and advising top management. The rising board involvement is partly a response to public pressure after a sequence of corporate frauds and crisis (Adams, Hermalin and Weisbach 2010). Correspondingly, the Securities and Exchange Commission (SEC) enhanced disclosure rules (2010) that require firms to describe in their proxy statements the role of the board of directors in risky projects, particularly the relationship of a firm's compensation policies and practices to project risk management. For similar reasons, the National Association of Corporate Directors (NACD) published "The NACD Blue Ribbon Commission Report on Strategy Development (2014)", emphasizing that corporate directors should not limit their engagement in strategy development to annual discussions with management or crisis situations. It recognizes that board engagement is a continual process.

The common view is that active involvement by boards improves firms' performance. Furthermore, academic studies use the level of board involvement as one measure of board effectiveness. For example, Bebchuk and Fried (2004) argue that insufficient board involvement/attention is contributed to high managerial power, therefore it is necessary to find solutions to limit managerial power. Vafeas (1999) suggests that increasing board meeting frequency can improve firm performance. But does more active involvement by boards always improve firms' performance?

We show that board involvement increases firm value only if board directors' expertise exceeds a threshold. For board directors' expertise lower than that threshold, active board involvement can actually hurt firm value. The main reason is that active board involvement breaks the executives' natural incentive synergy (i.e., positive incentive spillover) in evaluating a risky project and further implementing the project.

Executives typically engage in a sequence of tasks to complete a risky project. For a non-routine project such as acquiring a supplier, entering new geographic market, developing new products, forming strategic alliances, etc., executives are expected to evaluate the profitability of a potential project before an investment decision is made (evaluation task: investigating feasible production technology, potential market demand, etc.). If the project is invested, executives also need to expend effort to implement the project (implementation task: recruiting and training employees, carrying out production plan). We show that there is positive incentive spillover between the two tasks.

We use a "passive" board as a benchmark to demonstrate the spillover effect. Levels of board involvement differs across firms. We stylize a passive board that only designs the executives' incentive contracts at the outset and does not involve in any monitoring or advising activities about the project throughout. The executive decides whether to evaluate the project, make the investment and implement the project. We show that in the presence of evaluation task, motivating implementation task is at no extra cost. Evaluation effort only indirectly increases the probability of project success by improving investment efficiency. Implementation effort, on the other hand, has more direct impact on the project outcome. Without any implementation effort, no production is carried out then the project is surely a failure. As the final project success is more indicative of the CEO's implementation effort than his evaluation effort, the incentives the board sets up-front to motivate the evaluation task are also sufficient to motivate the implementation task.

With active board involvement, however, the positive spillover effect disappears. An active board is involved in both advisory activities related to project decisions and monitoring activities of CEO incentives.[1] Board monitoring activities normally include setting CEO compensation, retaining or firing the CEO, overseeing financial reporting, etc. Such monitoring activities may break the CEO's incentive synergy among tasks, as it normally takes a long period of time to complete a project. We focus on the fact that the board is required to assess the CEO's compensation program periodically to ensure the current incentive plans provides optimal motivation. After the CEO evaluates the project, the board's attention now shifts to motivating the CEO's implementation task. The compensation risk will be lowered (as stated previously, motivating implementation task requires less compensation risk) and not sufficient to motivate the evaluation task ex ante.[2]

We show that board advisory activities can restore the CEO's evaluation incentives. Large-scale projects normally are subject to board assessment and advising. An important feature of board advising is that its usefulness depends on the CEO's

[1]The literature studies two primary functions of boards: monitoring and advising senior management (e.g. Mace [1971], Hermanlin and Weisbach [2003], Adams and Ferreira [2007], Adams, Hermalin and Weisbach [2010], Drymiotes and Sivaramakrishnan [2012], Baldenius, Melumad and Meng [2014], etc.). Recently, Schwartz-Ziv and Weisbach [2013] provided evidence about what boards actually do from the minutes of board meetings. It shows that approximately two-thirds of the issues boards discussed were of a monitoring (surpervisory) nature, and the other one-third is of a advising (managerial) nature.

[2]Alternatively, we could have modeled the board monitoring activity as retaining or firing the CEO before the project is completed. The new CEO will have to be motivated separately and the incentive synergy will be broken as well. But in the middle of a project, firing a CEO seems to be a less frequent event than re-evaluating the CEO compensation plan which is conducted at least once a year.

initial evaluation of the project. Directors are less familiar with the firm's operations than the CEO, due to lack of firm-specific expertise or limited time spent in the firm. Thus the board often relies on the CEO's initial evaluation as a starting point for their own analysis. Only when the CEO exerts effort to evaluate the project and communicates his findings truthfully to the board, the board is able to provide useful advice.[3] Given that both the CEO and the board conduct analysis about the same project, their assessments should be positively correlated, and the more carefully the CEO evaluates the project, the stronger the correlation is. Therefore, such correlation between board advice and the CEO's report can be used to provide incentives for the CEO to evaluate the project.

With board active involvement breaking the incentive synergy between the two tasks, the CEO's evaluation effort is now motivated by referencing the board advice, and his implementation effort is motivated by referencing the project outcome. This separation may render board involvement unappealing: (1) board involvement could reduce investment efficiency in some cases; (2) even in those cases investment efficiency is increased, board involvement can still hurt firm value.

Conventional wisdom suggests that board involvement improve a firm's investment efficiency because board advice can possess useful information about the project. However, board involvement also breaks the CEO's incentive synergy between the tasks. Motivating implementation effort is no long "free". To avoid the extra cost for motivating implementation effort, investment occurs less frequently than the first-best investment policy. When such an under-investment problem occurs, a "passive" board's investment policy that bases on the CEO's information alone may be closer

[3] Adams and Ferreira [2007] and Song and Thakor [2006] also rely on the similar feature of board advising in their models.

to the first-best policy, producing higher investment efficiency.

For situations where board involvement does improve investment efficiency, however, the firm value may still be reduced. The reason is that, board involvement may hurt the CEO's incentives, and this negative effect dominates the investment improvement, rendering the total effect on firm value to be negative. Without the incentive synergy between the tasks, motivating implementation effort will always be more costly with an active board. If the board expertise is not high enough to provide relevant information about the project, motivating evaluation effort will be less efficient as well. Ultimately, board expertise determines whether board active involvement is beneficial for the firm.

In addition, we find that board expertise may not improve the CEO's incentives monotonically. The CEO's incentives to evaluate the project is always enhanced by facing a board that is more capable to assess the project. However, the impact of board expertise on implementation incentives depends on whether it is good enough to influence the investment decision. If the board fails to influence the investment decision, board advice only injects volatility into the perceived project quality, hurting the CEO's incentives to implement the project.

Related Literature Our paper adds to the fast-growing literature studying the board's advising role, e.g., Adams and Ferreira [2007], Baldenius, Melumad and Meng [2014], Harris and Raviv [2008], Raheja [2005], Song and Thakor [2006], etc. Although the focus of each paper is different, they all emphasize that directors' information helps the board to play a more active role in decision-making. Our paper shows that board advising not only plays a role in decision-making, but also serves an incentive (contracting) role to motivate the CEO to acquire information.

Aghion and Tirole [1997] and Burkart, Gromb and Panunzi [1997] also study the CEO's information acquiring activities. But in their incomplete contract setting, the board and the CEO have different preferences regarding investment. When the board has the "formal" authority over investment, its information acquisition will reduce the chances that the CEO exercises the "real" authority (or "effective" control), hence reducing the CEO's incentives to acquire information. In our complete contract setting, however, the board's information enhances the CEO's incentives to acquire information.

Literature further discovers that when the agents engage in multiple sequential actions to improve the final output, different incentive spillover effects can occur. Arya, Glover and Radhakrishnan [1998] study a situation where a team is used to come up with project ideas, with individuals subsequently implementing various components of the project. They show that when the second control problem is severe enough, the incentives provided to motivate the second (individual) action can be sufficient to also motivate the first (team) action. Laux [2006] studies a setting where the second action has a more direct impact on the final output than the first action, then the positive spillover effect is from the first to the second action. Our benchmark case without board involvement replicates Laux [2006]'s result. Lambert [1986] and Demski and Sappington [1987], on the other hand, show that a negative incentive spillover can occur: the second action is distorted in order to provide optimal incentives for the first action.

We show that board active involvement breaks the spillover effect, adding to the literature about the potential downside of board intervention. This line of literature finds that (principal's) ex post intervention dampens (agent's) ex ante incentives (e.g., Cremer [1995], Indjejikian and Nanda [1999], Christensen, Demski and Frimor [2002], Laux [2008]). We show that board intervention also exacerbates the *second* incentive problem which takes place after renegotiation.

Our results also shed some light on the literature on busy directors. Empirical evidence shows that busy directors are more likely to have attendance problems at board meetings, indicating busy directors may be less involved in board activities (Adams and Ferreira [2008]). But there are conflicting findings about the effect of busy directors on firm performance (Core, Holthausen, and Larcker [1999], Ferris, Jagannathan and Pritchard [2003] and Fich and Shivdasani [2006]). Our analysis offers an alternative explanation for the mixed findings.

We show that board expertise may not affect a firm's performance monotonically, consistent with the mixed empirical findings. For example, Wang, Xie and Zhu [2015] find that industry expertise enhances independent directors' ability to perform their monitoring function. But Guner, Malmendier and Tate [2008] find that boards with financial expertise are associated with worse acquisitions. The effect of board expertise is subtle.

2 Model Setup

We consider the interaction between a board of directors and a CEO, where the CEO is motivated to: (1) evaluate a potential project (evaluation task), (2) truthfully report his assessment of the project to the board, and (3) if the project is adopted, implement the project (implementation task).

The CEO's (first-stage) evaluation effort a_1 and (second-stage) implementation effort a_2 are binary: $a_1 \in \{0, 1\}$ and $a_2 \in \{0, 1\}$. The cost of $a_t = 0$ is normalized to zero, and the cost of $a_t = 1$ is $k_t > 0$, where $t = 1, 2$. If the project is adopted, the project returns a cash flow x depending on the realized project quality $\theta \in \{0, 1\}$, the CEO's implementation effort a_2 and the size of the project X:

$$x = \theta \cdot a_2 \cdot X.$$

That is, the project is a success only if the project is of good quality $\theta = 1$ and the CEO has exerted effort to implement it. If the project is rejected, the firm receives 0 and the CEO receives a wage as specified in the contract. The game ends.

The project quality θ is either bad ($\theta = 0$) or good ($\theta = 1$), with ex-ante probability of $\theta = 1$ being 1/2. The CEO can expend effort a_1 to search for information about the project. If the CEO exerts evaluation effort, he receives an informative signal $s \in \{G, B\}$ with precision $i \in (0, 0.5)$ about the project quality. If the CEO does not exert evaluation effort, the signal s is pure noise.

$$Pr[s = G|\theta = 1] = Pr[s = B|\theta = 0] = 0.5 + i \cdot a_1.$$

After the CEO receives the signal privately, he can issue a report $\hat{s}$ about s to the board.

Based on the CEO's report, the board can use its expertise to conduct further analyses and generate an additional signal $m \in \{H, L\}$ about the project quality. We label m as board advice. Directors often provide advice to the CEO through formal board and committee meetings or through informal consultation.[4] We focus on the more formal channel of communication. Note that, in this paper, we use "board advice" to refer specifically to the incremental piece of information generated by the board, rather than the board's total information set (which includes both the CEO's

[4]See The Business Roundtable (2005).

report $\hat{s}$ and its own incremental signal m). Specifically, the informativeness of m depends on the board's expertise i_B, whether the CEO has truthfully reported his signal, and the informativeness of the CEO's signal (which depends on the CEO's evaluation effort):

$$Pr[m = H|\theta = 1, s, \hat{s}] = Pr[m = L|\theta = 0, s, \hat{s}] = 0.5 + i_B \cdot \mathbb{1}_{\hat{s}=s} \cdot a_1$$

Where $\mathbb{1}_{\hat{s}=s}$ is an indicator function that takes the value of 1 if $\hat{s} = s$. This information structure aims to capture an important feature of board advising: as the board is less familiar with the firm's daily operations than the CEO, the quality of board advice depends on the CEO's report about the project.[5] Only when the CEO has taken evaluation effort to collect relevant information and truthfully disclosed those findings in his report, the board's advice is informative.[6] If the CEO failed to take evaluation effort or has misreported his findings, the board cannot learn from the CEO's report and hence the signal m becomes pure noise. In addition, directors holding more relevant expertise (higher i_B) are able to provide more useful advice.

It is worth mentioning that our main results are robust to alternative information structures. For example, if we assume instead that the informativeness of m depends on whether *ex-post* the CEO's report matches with the true state, or the informative-

[5]Adams and Ferreira (2007) also recognize in their model that the quality of advice is higher when the advisee reveals his private information to his advisor.

[6]This information structure captures the feature that the quality of the board advice depends on the informativeness of the CEO's report. Note that, an implicit assumption here is that the board does not know the informativeness of its own advice. Otherwise, the board can infer whether the CEO has taken evaluation effort. Due to their various background in expertise and distance from the firm's everyday operation, directors cannot know perfectly how useful their advice truly is. Behavior researches find that it is common for individuals to be overly confident about their advice quality, even for an expert in the area (Kahneman, Slovic and Tversky[1982]).

ness of m is independent of the CEO's report, qualitatively similar results will continue to hold.[7] In addition, to motivate our particular choice of information structure, note that the CEO's report typically contains not only a recommendation/conclusion but also extensive supporting analysis. In many cases, this analysis is essential for the board to generate additional information about the project. Therefore, if the CEO does not exert evaluation effort but happens to draw the correct conclusion by luck, then the analysis presented in his report is not informative enough for the board to provide useful advice.

The CEO makes the investment decision based on its own reports and the board's advice.[8] The project, if invested, requires an up-front cost $I > 0$. We assume that, ignoring the implementation cost k_2, the investment threshold (i.e., the posterior probability of $\theta = 1$ which leads to investment) is 0.5, or equivalently, $0.5X - I = 0$. Of course, taking into consideration that the project has to be implemented to produce cash flow, the final investment threshold will be strictly higher than 0.5.

Besides providing advice to the CEO, the board also actively monitors the CEO.[9]

[7]The reason is that, the important feature giving rise to our main results is that the CEO's evaluation effort increases the correlation between the CEO's and the board's signals. This feature is very robust and holds for many alternative information structures.

[8]It does not matter which party, the CEO or the board, holds the formal authority over investment because there is no residual decision right in this complete contract setting. See Melumad and Reichelstein [1987].

[9]In practice, there are three major board committees performing monitoring role: audit committee, compensation committee and nominating committee (Faleye, Hoitash and Hoitash [2011]). Correspondingly, the prior literature has modeled board monitoring in different ways, such as deterring earnings management (Laux and Laux [2009]), uncovering the CEO's private information about project quality (Baldenius, Melumad and Meng [2014], Tian [2014]), replacing low-type CEOs (Cremer [1995], Laux [2008]), and appropriating control from the CEO (Adams and Ferreira [2007]).

One of the common responsibilities for the board (specifically, the compensation committee) is to assess the executives' compensation program periodically in order to ensure the current incentive plan provides optimal motivation. To capture this particular aspect of monitoring, we assume that the board will renegotiate the compensation contract with the CEO after the investment is undertaken.

The CEO has CARA utility $-e^{-r(\cdot)}$ with multiplicative cost. If the CEO fulfills both tasks, his utility is

$$U_{CEO} = -e^{-r(w-k_1-k_2)} = e^{r(k_1+k_2)}(-e^{-r\cdot w}),$$

where w is the CEO's wage and r is the CEO's risk aversion. The CEO has a reservation utility $-e^0 = -1$. In contrast, the board is risk-neutral and aims to maximize the investment profit less compensation cost:

$$V = (x - I)d - w,$$

where $d \in \{0, 1\}$ represents the investment decision.

We assume that the size of the project I is large enough so that the board always wants to induce the CEO to exert both efforts. In addition, we focus on pure strategies in the CEO's effort choices and the investment decision.

3 No Board Involvement

As a benchmark, we first consider a "passive" board that is not actively involved in monitoring and advising CEO but only designs the CEO's compensation contract at the outset. The sequence of events in the benchmark case is as follows:

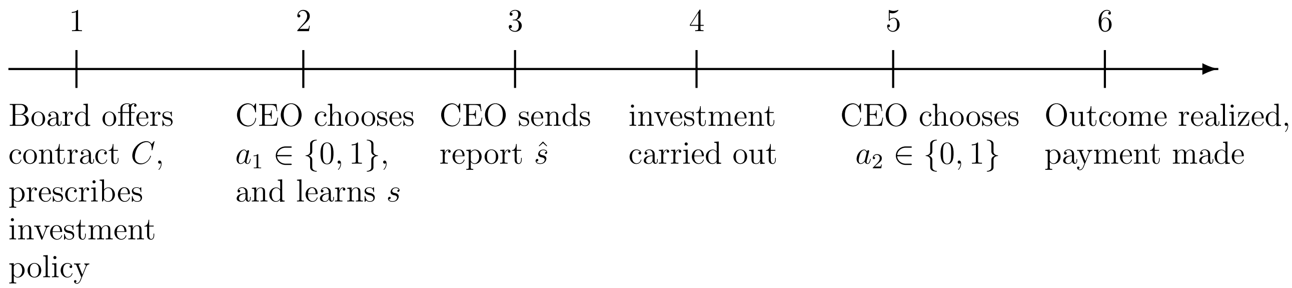

Figure 1: Timeline without board involvement

The board designs the compensation contract to motivate the CEO to (1) evaluate the project, (2) truthfully report, and (3) implement the project if it is undertaken. This is equivalent to a setting where the board does not elicit the CEO's report and leave the investment decision entirely to the CEO.[10] As we discussed in the model setup section, we assume that $X = 2I$ so that the board wants to induce investment if and only if the CEO observes G.

The compensation contract depends on all observable information: the CEO's report $\hat{s}$, the investment decision, and the final project outcome x. Applying the revelation principle in this full commitment setting, we can focus on the truth-telling equilibrium where $\hat{s} = s$. Since the board wants to induce investment if and only if the CEO observes G, in the truth-telling equilibrium, the investment is made only after the CEO reports $\hat{G}$. Therefore, the contract can be written as a triplet $(\overline{W}_{\hat{G}}, W_{\hat{G}}, W_{\hat{B}})$. $\overline{W}_{\hat{G}}$ is paid if the CEO reports $\hat{G}$ and the outcome is a success $(x = X)$, and $W_{\hat{G}}$ is paid if the outcome is a failure $(x = 0)$. When the project is not invested (i.e., when the CEO observes and reports B), the CEO receives $W_{\hat{B}}$.

The corresponding utility terms are $(\overline{U}_{\hat{G}}, U_{\hat{G}}, U_{\hat{B}})$. In the following analysis, we work on the utility space and the wages can be calculated inversely $w = \Phi(U) =$

[10]There is a one-to-one mapping between the CEO's report and the investment decision.

$-\frac{1}{r}ln(-U)$.

Denote EU_p as the CEO's expected utility when he evaluates, truthfully reports, and implements the project as desired. The subscript p represents "passive board" .

$$EU_p = e^{rk_1}\{0.5e^{rk_2}[(0.5+i)\overline{U}_{\hat{G}} + (0.5-i)U_{\hat{G}}] + 0.5U_{\hat{B}}\}.$$

The investment is undertaken with probability 0.5 accordingly, because the ex-ante probability of CEO observing G is 0.5. Conditional on the investment being undertaken and the CEO exerting implementation effort, the project succeeds with $\Pr[\theta = 1|G] = 0.5 + i$, in which case the CEO receives $\overline{W}_{\hat{G}}$ (the corresponding utility is $\overline{U}_{\hat{G}}$). If the project fails, which is with probability $1 - \Pr[\theta = 1|G] = 0.5 - i$, the CEO receives wage $W_{\hat{G}}$ with a corresponding utility $U_{\hat{G}}$. With probability 0.5, no investment is made and the CEO receives $W_{\hat{B}}$.

The board aims to minimize the compensation cost, subject to the CEO's incentive-compatibility constraints and participation constraint:

$$\mathcal{P}_p: \quad \min_{\overline{U}_{\hat{G}}, U_{\hat{G}}, U_{\hat{B}}} \quad CC_p = 0.5[(0.5+i)\Phi(\overline{U}_{\hat{G}}) + (0.5-i)\Phi(U_{\hat{G}})] + 0.5\Phi(U_{\hat{B}})$$

subject to:

$$e^{rk_2}\left[(0.5+i)\overline{U}_{\hat{G}} + (0.5-i)U_{\hat{G}}\right] \geq U_{\hat{G}} \qquad (IC_G - a_2)$$

$$e^{rk_2}\left[(0.5+i)\overline{U}_{\hat{G}} + (0.5-i)U_{\hat{G}}\right] \geq U_{\hat{B}} \qquad (TT_G)$$

$$U_{\hat{B}} \geq \max\left\{U_{\hat{G}},\ e^{rk_2}\left[(0.5-i)\overline{U}_{\hat{G}} + (0.5+i)U_{\hat{G}}\right]\right\} \qquad (TT_B)$$

$$EU_p \geq \max\left\{U_{\hat{B}}, U_{\hat{G}},\ e^{rk_2}\left(0.5\overline{U}_{\hat{G}} + 0.5U_{\hat{G}}\right)\right\} \qquad (IC - a_1)$$

$$EU_p \geq -e^{r\cdot 0} = -1 \qquad (IR)$$

Constraint $(IC_G - a_2)$ ensures that the CEO exerts implementation effort after the project is undertaken upon observing and reporting G. Constraints (TT_G) and (TT_B)

ensure that the CEO truthfully reports his signal.[11] Finally, constraint $(IC - a_1)$ ensures that the CEO prefers the equilibrium actions to not evaluating the project and simply (i) reporting B and forgoing the project, (ii) reporting G, investing but not exerting implementation effort or (iii) reporting G, investing and implementing the project.

Note that both the CEO's evaluation effort and implementation effort affect the final project outcome. The evaluation effort indirectly increases the probability of project success by improving the investment efficiency. Implementation effort, on the other hand, has a more direct impact on project outcome. Without any implementation effort, by construction, the project is surely a failure. That is, the final project success is more indicative of the CEO's implementation effort, so implementation is easier to motivate. Thus, the incentives that ensure the CEO evaluates the project are also sufficient (i.e., $\overline{U}_{\hat{G}} - U_{\hat{G}}$ is large enough) to ensure the CEO's implementation effort. We label it the "positive incentive spillover" effect from motivating evaluation effort to implementation effort. The result can be formally represented by the following Lemma.

Lemma 1 *In the benchmark case without board involvement, the implementation effort constraint $(IC_G - a_2)$ is always slack.*

The next result characterizes the optimal compensation contract for the benchmark case.

Proposition 1 *In the benchmark case without board involvement,*

[11] These truthtelling constraints are equivalent to the constraints that ensure that the CEO follows the efficient investment decisions: invest the project upon observing G and forgo the investment upon observing B.

- *For small implementation cost, $e^{rk_2} \leq 1 + \frac{1-e^{-rk_1}}{i}$, the board's reporting constraints (TT_G) and (TT_B) are slack, and the optimal compensation contract is given by*

$$\begin{aligned} W_{\hat{B}} &= 0, \\ \overline{W}_{\hat{G}} &= k_2 - \frac{1}{r} Ln \left(1 - \frac{1-e^{-rk_1}}{i}\right) > 0, \\ W_{\hat{G}} &= k_2 - \frac{1}{r} Ln \left(1 + \frac{1-e^{-rk_1}}{i}\right) \leq 0. \end{aligned}$$

- *For large implementation cost, $e^{rk_2} > 1 + \frac{1-e^{-rk_1}}{i}$, the board's reporting constraints (TT_G) is slack but (TT_B) is binding, and the optimal compensation contract is given by*

$$\begin{aligned} W_{\hat{B}} &= W_{\hat{G}} = 0, \\ \overline{W}_{\hat{G}} &= -\frac{1}{r} Ln \left(\frac{e^{-rk_2}(2e^{-rk_1} - 1) - (0.5 - i)}{0.5 + i}\right) > 0. \end{aligned}$$

To understand this result, consider first the special case where the implementation effort is costless, i.e., $k_2 = 0$. Then the compensation contract is set to motivate the evaluation effort alone. For that purpose, the board will (i) reward project success by paying $\overline{W}_{\hat{G}} = -\frac{1}{r} Ln \left(1 - \frac{1-e^{-rk_1}}{i}\right) > 0$, (ii) punish project failure by setting $W_{\hat{G}} = -\frac{1}{r} Ln \left(1 + \frac{1-e^{-rk_1}}{i}\right) < 0$, and (iii) pay $W_{\hat{B}} = 0$ to the CEO if the project is rejected.[12]

Recall that the incentives to ensure the CEO's evaluation effort are sufficient to ensure implementation effort (Lemma 1). That implies that, as the implementation

[12] To make sense of this optimal contract, it is helpful to look at the corresponding utility term: $\overline{U}_{\hat{G}} = -1 + \frac{1-e^{-rk_1}}{i}$ and $U_{\hat{G}} = -1 - \frac{1-e^{-rk_1}}{i}$. Note that, when the project is undertaken (with probability 0.5), the CEO's evaluation effort increases the probability of project success by i, hence the board can motivate evaluation effort by setting $\overline{U}_{\hat{G}} - U_{\hat{G}} = \frac{1-e^{-rk_1}}{0.5i}$.

effort becomes costly, i.e., $k_2 > 0$, the board will simply pay the implementation cost k_2 to the CEO when the project is implemented. That is, both $\overline{W}_{\hat{G}}$ and $W_{\hat{G}}$ are increased by k_2 in lock-step.

As k_2 increases further, $W_{\hat{G}}$ will reach $W_{\hat{B}}$ eventually. To ensure the CEO truthfully reports if B is observed, the board has to pay higher wage for a $\hat{B}$ report than for project failure, i.e., $W_{\hat{B}} \geq W_{\hat{G}}$. Therefore, as k_2 increases further, $W_{\hat{G}} = W_{\hat{B}} = 0$, and $\overline{W}_{\hat{G}}$ keeps increasing.

It is worth mentioning that our passive board setting resembles the setup of Laux [2006], hence generates similar results. However, the interpretation of the results and the focus are different. Laux [2006] focuses on finding a sufficient condition that renders a performance measure worthless. Whereas our paper focuses on demonstrating the "positive incentive spillover" effect.

4 With Board Involvement

Board involvement normally takes the form of providing advice to the CEO and actively monitoring the CEO's actions. Here we focus on one of the monitoring activities performed by the compensation committee: adjusting the CEO's compensation contract to ensure the incentive plans provides optimal motivation.[13] The sequence of events in this setting is described in Figure 2:

[13]Our main results would be maintained qualitatively as long as there is any board intervention that breaks the positive incentive spillover effect among tasks. For example, assigning the CEO the role of project search and evaluation only, while leaving the implementation to another agent to complete, such as a COO (Chief Operating Office).

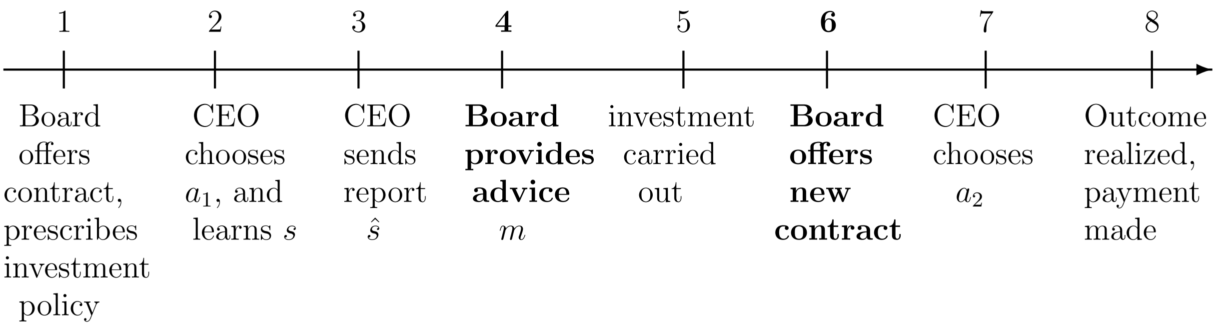

Figure 2: Timeline with board involvement

Compared with Figure 1 (the timeline without board involvement), Figure 2 has two more stages where the board provides advice to the CEO (Date 4) and adjusts the CEO's compensation contract (Date 6).

4.1 Board Monitoring Hurts CEO Incentives

In this section, we examine the role of board monitoring alone. That is, compared with the benchmark case in Section 3, the board only renegotiates the compensation contract with the CEO after investment is undertaken (Date 6).

The board solves the problem backwards. After investment is undertaken, the board can offer a revised contract to the CEO which keeps the CEO no worse off but may reduce the compensation cost. As we have established in Section 3, motivating evaluation effort demands imposing higher risks on the CEO than motivating implementation effort. Now the evaluation effort is sunk, the board can reduce the CEO's compensation risk to the level just sufficient for motivating implementation effort. This is beneficial ex-post for both parties, as the risk-averse CEO is kept no worse off from his initial compensation contract and the board can lower the risk premium paid to the CEO.[14] Note that renegotiation happens only when the investment is

[14] The board has incentives to revise the CEO's contract even if there are inside directors. There

undertaken. If the project is foregone, the CEO does not face any compensation risk so there is no gain for renegotiation.

Instead of describing the actual renegotiation, we can restrict attention to contracts that are renegotiation proof (Fudenberg and Tirole [1990]). A contract is renegotiation proof if the principal will not choose to alter it at the renegotiation stage. That is, the contract is *ex-post* optimal from the board's perspective, for all information events at the renegotiation stage.

Specifically, a contract $C = (\overline{U}_{\hat{G}}, U_{\hat{G}}, U_{\hat{B}})$ is renegotiation-proof if, at the renegotiation stage, it minimizes the board's expected compensation cost subject to the implementation effort constraint $(IC_G - a_2)$ and the interim IR constraint (IR_G):

$$\begin{aligned} e^{rk_2}\left[(0.5+i)\overline{U}_{\hat{G}} + (0.5-i)U_{\hat{G}}\right] &\geq U_{\hat{G}} && (IC_G - a_2) \\ e^{rk_2}\left[(0.5+i)\overline{U}_{\hat{G}} + (0.5-i)U_{\hat{G}}\right] &\geq EU^I_{\hat{G}} && (IR_G) \end{aligned}$$

where $EU^I_{\hat{G}}$ is the CEO's expected utility upon observing and reporting G according to the initial contract.

As is standard in the literature (Fudenberg and Tirole [1990]), both the implementation effort constraint $(IC_G - a_2)$ and the interim IR constraint (IR_G) have to be binding for any renegotiation-proof contract. Hence,

$$\overline{U}_{\hat{G}} = \left(1 - \frac{1-e^{-rk_2}}{0.5+i}\right) U_{\hat{G}}.$$

The pay differential is set just high enough to motivate the CEO to take the implementation effort.

is always a strict gain because the compensation cost is reduced but the CEO's expected utility is kept constant. Thus, the presence of insiders does not reduce the board's willingness to adjust the CEO's compensation plans.

Although the revised contract does not reduce to a risk-free payment as in Fudenberg and Tirole [1990], the same result obtains that the agent's *ex-ante* incentives completely collapse for pure strategies.[15]

Observation 1 *If the board only adjusts the CEO's compensation contract but provides no advice, then motivating the CEO's first-stage evaluation effort is not feasible.*

As Lemma 1 shows, motivating evaluation effort requires higher compensation risk than motivating implementation effort. Hence the renegotiation-proof contract that tailors the compensation risk to implementation effort will not provide sufficient incentive to motivate evaluation effort.

Observation 1 demonstrates that the CEO's *ex-ante* evaluation effort incentive is completely wiped out by the board resetting the CEO's incentive plans, if there were no board advising. In the following sections, we bring back the board's advising role and study how board advising *restores* the CEO's ex-ante evaluation effort incentives and potentially affects investment efficiency.

4.2 Board Advising Restores CEO Incentives

With board advising, there is yet another variable the CEO's compensation contract can be based on: the board's advice m. Now the CEO's compensation can depend on all the contractible information: the CEO's report $\hat{s}$, the board's advice m, the investment decision, and the final project outcome x. Given that the investment decision is fully determined by the CEO's report and the board's advice, it

[15] Fudenberg and Tirole [1990] consider the agent's mixed strategies to prevent ex ante incentive collapse. We show later that board advising can prevent ex ante incentive collapse by providing an additional contracting variable.

becomes redundant as a contracting variable. Hence the wage contract can be written as $(W_{\hat{G}H}, W_{\hat{G}L}, W_{\hat{B}H}, W_{\hat{B}L}, \overline{W}_{\hat{G}H}, \overline{W}_{\hat{G}L}, \overline{W}_{\hat{B}H}, \overline{W}_{\hat{B}L})$, where $W_{\hat{s}m}$ is the CEO's wage when the CEO's report is $\hat{s}$, the board's advice is m, and the final project outcome is zero, i.e. $x = 0$ (either due to project failure or no investment), and $\overline{W}_{\hat{s}m}$ represents the wage for $(\hat{s}, m)$ combination and final project success, i.e., $x = X$. The corresponding utility terms are represented by $(U_{\hat{G}H}, U_{\hat{G}L}, U_{\hat{B}H}, U_{\hat{B}L}, \overline{U}_{\hat{G}H}, \overline{U}_{\hat{G}L}, \overline{U}_{\hat{B}H}, \overline{U}_{\hat{B}L})$. Notice that the $\overline{W}_{\hat{s}m}$ is relevant only when the investment is made after $(\hat{s}, m)$.

After the board provides advice, the CEO makes the investment decision accordingly. Given that truthful disclosure by the CEO is necessary for the board's advice to be useful, for important investment decisions (large I), the board always wants to induce truthful reports by the CEO. Therefore, in the following analysis, we focus on the equilibrium where the CEO is motivated to truthfully report his signal.

After the investment is made, the board's objective then shifts to ensuring that the invested project is implemented efficiently. At this moment (Date 6), the board can offer a revised contract to the CEO which has to keep the CEO no worse off. Following the same logic as in Section 4.1, a contract $C = (\overline{U}_{\hat{s}m}, U_{\hat{s}m})$ is renegotiation-proof for the information event $(\hat{s}, m)$ if it minimizes the board's expected compensation cost subject to the implementation effort constraint $(IC_{\hat{s}m} - a_2)$ and the interim IR constraint $(IR_{\hat{s}m})$.

$$
\begin{aligned}
e^{rk_2}\left(\Pr[\theta = 1|\hat{s}, m]\overline{U}_{\hat{s}m} + (1 - \Pr[\theta = 1|\hat{s}, m])U_{\hat{s}m}\right) &\geq U_{\hat{s}m} && (IC_{\hat{s}m} - a_2) \\
e^{rk_2}\left(\Pr[\theta = 1|\hat{s}, m]\overline{U}_{\hat{s}m} + (1 - \Pr[\theta = 1|\hat{s}, m)U_{\hat{s}m}\right) &\geq EU^I(\hat{s}, m) && (IR_{\hat{s}m})
\end{aligned}
$$

where $EU^I(\hat{s}, m)$ is the CEO's expected (interim) utility before accepting the revised contract. The board proposes the revised contract anticipating that the CEO has exerted evaluation effort and truthfully reported his signals. That is, $\Pr[\theta =$

$1|\hat{s}, m]$ in the above constraints takes the value of $\Pr[\theta = 1|s, m, a_1 = 1, \hat{s} = s]$.

As is standard in the literature, the implementation effort constraint $(IC_{\hat{s}m} - a_2)$ has to be binding for any renegotiation-proof contract, which implies

$$\overline{U}_{\hat{s}m} = \left(1 - \frac{1 - e^{-rk_2}}{\Pr[\theta = 1|\hat{s}, m]}\right) U_{\hat{s}m}. \tag{1}$$

Converting to the wage space,

$$\overline{W}_{\hat{s}m} = W_{\hat{s}m} - \frac{1}{r} Ln \left(1 - \frac{1 - e^{-rk_2}}{\Pr[\theta = 1|\hat{s}, m]}\right). \tag{2}$$

Denote by $B_{\hat{s}m} \equiv \overline{W}_{\hat{s}m} - W_{\hat{s}m} = -\frac{1}{r} Ln \left(1 - \frac{1-e^{-rk_2}}{\Pr[\theta=1|\hat{s},m]}\right)$ the bonus for success for $(\hat{s}, m)$ combination. $B_{\hat{s}m}$ is set just high enough to motivate the CEO (on the equilibrium path) to take the implementation effort.

Recall that Observation 1 shows that the CEO's *ex-ante* evaluation effort incentive is wiped out by board adjusting the CEO's compensation contract, if there were no board advising. We now study whether board advising can restore the CEO's evaluation effort incentive. Note that both the CEO and the board are evaluating the same project, therefore it is likely that both parties will arrive at the same conclusion. That is, the two parties' signals should be positively correlated. If the CEO carefully evaluates the project, his signal is more informative, and consequently, the correlation between the two parties' signals becomes stronger. Therefore, the combination/comparison of board advising and CEO's report is informative about whether the CEO has taken the evaluation effort. This suggests that the board can rely on the comparison of the CEO's report and board advice to motive the CEO's first-stage evaluation effort. The following analysis confirms this intuition.[16]

[16]The timing of renegotiation is crucial. If, instead, we assume that the renegotiation happens before the board provides advice, then as in Fudenberg and Tirole [1990], there will be no pure-strategy equilibrium such that the CEO exerts evaluation effort.

4.2.1 The Board's Optimization Problem

In this subsection, we formulate the board's optimization problem. Recall that renegotiation-proofness implies that the bonus $B_{\hat{s}m}$ is set just high enough to motivate the CEO to implement the project. Hence, for those $(\hat{s}, m)$ combination for which the investment is undertaken, the CEO's expected utility when he exerts implementation effort as desired is the same as $U_{\hat{s}m}$, his utility if he shirks on implementation. On the other hand, for those $(\hat{s}, m)$ combination for which the investment is foregone, the CEO will receive utility $U_{\hat{s}m}$. Therefore, the CEO's *ex ante* expected utility if he takes the equilibrium actions (evaluating the project, truthfully reporting, and implementing the project when invested) is[17]

$$
\begin{aligned}
EU_a &= e^{rk_1}\left[\sum_{\hat{s}\in\{\hat{G},\hat{B}\},m\in\{H,L\}} Pr[\hat{s},m]\cdot U_{\hat{s}m}\right] \\
&= e^{rk_1}\left[\underbrace{(0.25+i_Bi)}_{Pr[\hat{G},H]=Pr[\hat{B},L]}\left(U_{\hat{G}H}+U_{\hat{B}L}\right)+\underbrace{(0.25-i_Bi)}_{Pr[\hat{B},H]=Pr[\hat{G},L]}\left(U_{\hat{B}H}+U_{\hat{G}L}\right)\right].
\end{aligned}
$$

The subscript a indicates "active" board. The board's expected compensation cost is:

$$CC_a = \sum_{\hat{s}\in\{\hat{G},\hat{B}\},m\in\{H,L\}} Pr[\hat{s},m]\cdot EW_{\hat{s},m}$$

where $EW_{\hat{s},m}$ represents the expected wage paid to the CEO when the CEO reports $\hat{s}$ and the board advises m. $EW_{\hat{s},m}$ is calculated as follows:

$$EW_{\hat{s},m} = d_{(\hat{s},m)}\left\{Pr[\theta=1|\hat{s},m]\overline{W}_{\hat{s}m}+(1-Pr[\theta=1|\hat{s},m])W_{\hat{s}m}\right\}+\left[1-d_{(\hat{s},m)}\right]W_{\hat{s}m}$$

[17] With a slight abuse of notation, we use $Pr[\hat{s},m]$ to represent the probability of $(\hat{s},m)$ on the equilibrium path, which equals $Pr[\hat{s},m|a_1=1,\hat{s}=s]=Pr[s,m|a_1=1,\hat{s}=s]$. Similarly, we use $Pr[\theta=1|\hat{s},m]$ to represent the posterior probability of θ on the equilibrium path, which equals $Pr[\theta=1|s,m,a_1=1,\hat{s}=s]$.

$d_{(\hat{s},m)} \in \{0,1\}$ represents the investment decision based on $(\hat{s},m)$. If the investment is undertaken, then the project succeeds with $Pr[\theta = 1|\hat{s},m]$, in which case the board pays out $\overline{W}_{\hat{s}m}$. If the project fails, which is with probability $1 - Pr[\theta = 1|\hat{s},m]$, the CEO receives wage $W_{\hat{s}m}$. If no investment is made after $(\hat{s},m)$, the CEO receives $W_{\hat{s}m}$.

After incorporating the binding implementation effort constraint (2), the active board's compensation cost can be reduced to:[18]

$$\begin{aligned} CC_a &= \underbrace{(0.25 + i_B i)}_{Pr[\hat{G},H]=Pr[\hat{B},L]} (W_{\hat{G}H} + W_{\hat{B}L}) + \underbrace{(0.25 - i_B i)}_{Pr[\hat{B},H]=Pr[\hat{G},L]} (W_{\hat{B}H} + W_{\hat{G}L}) \\ &+ \sum_{\hat{s}\in\{\hat{G},\hat{B}\},m\in\{H,L\}} Pr[\hat{s},m] \cdot d_{(\hat{s},m)} \cdot Pr[\theta = 1|\hat{s},m] \cdot B_{\hat{s}m}. \end{aligned}$$

Specifically, we denote

$$CC_{a_1} \equiv (0.25 + i_B i)\,(W_{\hat{G}H} + W_{\hat{B}L}) + (0.25 - i_B i)\,(W_{\hat{B}H} + W_{\hat{G}L})$$

as the compensation cost to motivate first-stage evaluation effort, and

$$CC_{a_2} \equiv \sum_{\hat{s}\in\{\hat{G},\hat{B}\},m\in\{H,L\}} Pr[\hat{s},m] \cdot \underbrace{d_{(\hat{s},m)} \cdot Pr[\theta = 1|\hat{s},m] \cdot B_{\hat{s}m}}_{CC_{a_2}^{(\hat{s},m)}}$$

as the compensation cost to motivate implementation effort. To understand $CC_{a_2}^{(\hat{s},m)}$, note that only when the investment is made (i.e., $d_{(\hat{s},m)} = 1$) and the project succeeds (with probability $Pr[\theta = 1|\hat{s},m]$), the board needs to pay out the bonus for project success.

The board aims to design the CEO's compensation contract and choose the optimal investment policy to maximize the firm value, denoted by FV_a (which is the

[18] Due to the CEO's CARA utility and renegotiation-proofness, there is a separation of the compensation costs for different efforts.

expected NPV of the project net of the expected compensation cost), subject to the constraints that ensure that the CEO does not deviate from the equilibrium actions. With renegotiation-proofness ensuring that the CEO (on the equilibrium path) does not deviate at the implementation stage, the CEO's potential deviation could occur at the evaluation stage or the reporting stage. Either deviation would lead the board's advice to become pure noise.

If the investment is undertaken after $(\hat{s}, m)$ combination, then the CEO will decide whether to exert implementation effort, given the contract he is facing. Note that renegotiation-proofness implies that the bonus paid to the CEO ($B_{\hat{s}m}$) is based on the board's equilibrium conjecture of the project quality. If the CEO knows that the true project quality is worse than the board's conjectured project quality, he will choose to shirk on implementation and secure $U_{\hat{s}m}$. This is exactly the CEO's choice if (1) he fails to evaluate the project (so the true project quality is 0.5); or (2) he evaluates the project and obtains signal B but misreports $\hat{G}$ (the true project quality in this case is $0.5 - i$).

For the third possible deviation, where the CEO evaluates the project, obtains signal G but misreports $\hat{B}$, the CEO knows that the true project quality is $0.5 + i$. His expected deviation utility is then given by $0.5D_{\hat{B}H} + 0.5D_{\hat{B}L}$, where

$$D_{\hat{B}m} \equiv U_{\hat{B}m} + d_{(\hat{B},m)} \cdot \max\left\{0, e^{rk_2}\left[(0.5+i)\overline{U}_{\hat{B}m} + (0.5-i)U_{\hat{B}m}\right] - U_{\hat{B}m}\right\}, \quad \text{for} \quad m \in \{H, L\}.$$

$D_{\hat{B}m} - U_{\hat{B}m}$ reflects the option value to the CEO at the implementation stage. Only when the investment is undertaken, the CEO has the option to choose implementation effort, and if he indeed chooses to exert such effort, his payoff is $e^{rk_2}\left[(0.5+i)\overline{U}_{\hat{B}m} + (0.5-i)U_{\hat{B}m}\right]$. Note that no investment will be made after $(\hat{B}, L)$ because the conjecture project quality is too low to justify investment, hence

$D_{\hat{B}L} = U_{\hat{B}L}$.

The active board's optimization program can be formally written as below:

$$\mathcal{P}_a : \max_{\{U_{\hat{s}m} \in R, \quad d_{(\hat{s},m)} \in \{0,1\}\}} \quad FV_a \equiv -CC_a + \sum_{\hat{s} \in \{\hat{G},\hat{B}\}, m \in \{H,L\}} Pr[\hat{s}, m] \cdot d_{(\hat{s},m)} \cdot \{Pr[\theta = 1|\hat{s}, m]X - I\}$$

subject to

$$\underbrace{(0.5 + 2i_B i)}_{Pr[H|\hat{s}=\hat{G}, s=G, a_1=1]} U_{\hat{G}H} + \underbrace{(0.5 - 2i_B i)}_{Pr[L|\hat{s}=\hat{G}, s=G, a_1=1]} U_{\hat{G}L} \geq 0.5D_{\hat{B}H} + 0.5U_{\hat{B}L} \qquad (TT_G)$$

$$\underbrace{(0.5 - 2i_B i)}_{Pr[H|\hat{s}=\hat{B}, s=B, a_1=1]} U_{\hat{B}\hat{H}} + \underbrace{(0.5 + 2i_B i)}_{Pr[L|\hat{s}=\hat{B}, s=B, a_1=1]} U_{\hat{B}L} \geq 0.5U_{\hat{G}H} + 0.5U_{\hat{G}L} \qquad (TT_B)$$

$$EU_a \geq \max\{0.5U_{\hat{G}H} + 0.5U_{\hat{G}L}, 0.5U_{\hat{B}H} + 0.5U_{\hat{B}L}\} \qquad (AIC - a_1)$$

$$EU_a \geq -e^{r \cdot 0} = -1 \qquad (IR)$$

The solution to the optimization problem is denoted by $(U^*_{\hat{s}m}, d^*_{(\hat{s},m)})$. Constraints (TT_G) and (TT_B) ensure that the CEO truthfully reports his signals after evaluating the project. Constraint $(AIC - a_1)$ ensures that the CEO prefers the equilibrium actions to not evaluating the project and simply reporting (i) $\hat{G}$ or (ii) $\hat{B}$.

In the following subsections, we solve the board's optimization problem by first examining the cost-minimizing compensation contracts for given investment policies, and then choose the optimal investment policy.

4.2.2 Cost-Minimizing Contracts for Given Investment Policies

A closer examination of the board's optimization problem suggests that the cost-minimization problems for different investment policies are similar except that $D_{\hat{B}H}$ may take different values depending on whether investment is made after $(\hat{B}, H)$. If no investment is made after $(\hat{B}, H)$, i.e., $d_{(\hat{B},H)} = 0$, then $D_{\hat{B}H} = U_{\hat{B}H}$. The cost-minimizing contract is given by the next proposition.

Proposition 2 *Consider an "active" board that provides advice to the CEO and actively adjusts the CEO's compensation contract. If the CEO's report $\hat{G}$ is necessary for investment to be undertaken, then $d_{(\hat{B},H)} = 0$, and the cost-minimizing contract is:*

$$\begin{aligned} U_{\hat{G}H} &= U_{\hat{B}L} = -1 + \frac{1-e^{-rk_1}}{4i_B i}, \\ U_{\hat{B}H} &= U_{\hat{G}L} = -1 - \frac{1-e^{-rk_1}}{4i_B i}. \end{aligned}$$

The two reporting constraints (TT_G) *and* (TT_B) *are slack.*

The board motivates the CEO to exert evaluation effort by paying a higher compensation to the CEO if his reports are consistent with board advice. Only if the CEO has diligently evaluated the project and truthfully disclosed his information, the board's advice is useful. Reflecting the same underlying state θ, the CEO's report is positively correlated with the board advice. Therefore, the more consistent the CEO's report is with board advice, the more likely the CEO has exerted first-stage evaluation effort. Specifically, the CEO's evaluation effort increases the probability of having consistent reports by $2i_B i$ (from 0.5 to $0.5 + 2i_B i$), hence the board can motivate evaluation effort by setting $U_{\hat{B}L} - U_{\hat{G}L} = U_{\hat{G}H} - U_{\hat{B}H} = \frac{1-e^{-rk_1}}{2i_B i}$.

Furthermore, given that the board pays a higher compensation if the CEO's reports are consistent with board advice, the CEO has incentives to truthfully report his signal because by truthful reporting he maximizes the probability of having consistent reports with board advice. That is, the CEO's truthful reporting constraints are slack. Note that, a subtle but important assumption here is that the CEO's report $\hat{G}$ is necessary for investment to be undertaken. If the CEO who observes G but misreport $\hat{B}$, the investment will be forgone and the CEO will then receive a certain payment.

However, if the investment is undertaken even when the CEO reports $\hat{B}$ (which is the case when the investment policy is fully determined by board advice), then the CEO who observes G may have incentives to misreport $\hat{B}$. This is because by doing so he can mislead the board to believe that the project quality is not so good and thus will get higher bonus for implementing the project. The following proposition examines the cost-minimizing contract for such investment policy:

Proposition 3 *Consider an "active" board that provides advice to the CEO and actively adjusts the CEO's compensation contract. If the investment policy is fully determined by the board's advice, i.e.,* $d_{(\hat{s},H)} = 1$ *and* $d_{(\hat{s},L)} = 0$*, then there exists* $Z \equiv \frac{1-e^{-rk_1}}{1+\frac{1-e^{-rk_1}}{4i_B i}} - 0.5(e^{rk_2} - 1) \cdot \max\left\{\frac{0.5+i}{\Pr[\theta=1|B,H,a_1=1,\hat{s}=s]} - 1, 0\right\}$ *such that*

- *If* $Z \geq 0$*, the CEO's truthful reporting constraints* (TT_G) *and* (TT_B) *are slack, and the cost-minimizing contract is the same as characterized in Proposition 2.*

- *If* $Z < 0$*, the CEO's truthful reporting constraint upon observing G signal,* (TT_G)*, is binding.*

As we argued in the previous paragraph, if the investment decision is fully determined by the board's advice, the CEO with signal G may have incentives to misreport. Now there are two countervailing forces regarding the CEO's reporting behavior. On one hand, by Proposition 2, the board motivates the CEO's evaluation effort by paying a higher compensation to the CEO if his reports are consistent with board advice. From this perspective, the CEO has incentives to report truthfully. On the other hand, the CEO who observes favorable signals regarding a project may wish to claim that the project is bad so as to boost his bonus for project success. To examine how the two countervailing forces are influenced by parameters, it is helpful to look at the following breakdown:

$$\begin{aligned} Z &\propto (1-e^{-rk_1}) - 0.5(e^{rk_2}-1)\left(1+\frac{1-e^{-rk_1}}{4i_B i}\right)\cdot \max\left\{\frac{0.5+i}{\Pr[\theta=1|B,H,a_1=1,\hat{s}=s]}-1,0\right\} \\ &= \underbrace{(1-e^{-rk_1})}_{\text{Truthful Force}} \underbrace{-0.5e^{rk_2}\left(\overline{U}_{\hat{B}H}-U_{\hat{B}H}\right)\cdot \max\left\{\left(0.5+i-\Pr[\theta=1|\hat{B},H]\right),0\right\}}_{\text{Misreporting Force}} \end{aligned}$$

If the truthful force dominates, i.e., $Z \geq 0$, the CEO's truthful reporting constraint (TT_G) will be slack. If, on the other hand, the misreporting force dominates, i.e., $Z < 0$, then (TT_G) will have to be binding.

The following corollary examines under what circumstances the CEO's truthful reporting constraint (TT_G) is more likely to be binding.

Corollary 1 $\frac{\partial Z}{\partial i_B} \geq 0$, $\frac{\partial Z}{\partial k_1} \geq 0$ *and* $\frac{\partial Z}{\partial k_2} \leq 0$.

Corollary 1 implies that the CEO's truthful reporting constraint (TT_G) is more likely to be binding if (i) the board's expertise i_B is small; (ii) the CEO's first-stage evaluation effort cost k_1 is small; and/or (iii) the CEO's second-stage evaluation effort cost k_2 is large. Intuitively, the larger the CEO's evaluation effort cost k_1, the higher the rewards for consistent reports, hence, the stronger the incentives for the CEO to truthfully report. On the other hand, for the misreporting force, if the CEO misleads the board to believe that the project is of a lower quality ($\Pr[\theta = 1|\hat{B}, H]$) than its actual quality $(0.5 + i)$, the CEO will receive excessively high expected bonus for success. Specifically, such benefit of misreporting increases in the implementation effort cost k_2 and decreases in board expertise i_B. The reason is that: (1) higher k_2 leads to higher bonus for project success since such bonus is used to motivate implementation effort; (2) higher board expertise i_B leads the board to rely relatively less on the CEO's report to form its conjecture about the project quality, which reduces the CEO's benefit from misreporting.

4.2.3 The Optimal Investment Policy

In this subsection, we examine the optimal investment policy the board wants to prescribe. For that purpose, it is useful to calculate the posterior project quality based on the CEO's report and board's advice (factoring in the CEO's equilibrium behavior):

$$\begin{aligned}
\Pr[\theta=1|\hat{G},H] = \Pr[\theta=1|G,H,a_1=1,\hat{s}=s] &= \frac{1}{1+\frac{(0.5-i)(0.5-i_B)}{(0.5+i)(0.5+i_B)}} > 0.5+i \\
\Pr[\theta=1|\hat{B},H] = \Pr[\theta=1|B,H,a_1=1,\hat{s}=s] &= \frac{1}{1+\frac{(0.5+i)(0.5-i_B)}{(0.5-i)(0.5+i_B)}} > 0.5-i \\
\Pr[\theta=1|\hat{G},L] = \Pr[\theta=1|G,L,a_1=1,\hat{s}=s] &= \frac{1}{1+\frac{(0.5-i)(0.5+i_B)}{(0.5+i)(0.5-i_B)}} < 0.5+i \\
\Pr[\theta=1|\hat{B},L] = \Pr[\theta=1|B,L,a_1=1,\hat{s}=s] &= \frac{1}{1+\frac{(0.5+i)(0.5+i_B)}{(0.5-i)(0.5-i_B)}} < 0.5-i
\end{aligned}$$

If the CEO's report and the board's advice are consistent with each other, then the optimal investment decision is straightforward: to invest if both reports are favorable and to forgo investment if both reports are unfavorable. The more interesting case happens if the parties' reports are contradicting with each other. The optimal investment policy is then summarized by the following Proposition.

Proposition 4 *There exist $\delta_1 > 0$ and $\delta_2 > 0$ such that*

- *If $i_B < i - \delta_1$, then $d^*_{(\hat{G},m)} = 1$ and $d^*_{(\hat{B},m)} = 0$ for $m \in \{H, L\}$. That is, the optimal investment policy is fully determined by the CEO's report.*

- *If $i - \delta_1 \leq i_B \leq i + \delta_2$, then $d^*_{(\hat{G},H)} = 1$ and $d^*_{(\hat{s},m)} = 0$ for other $(\hat{s}, m)$ combinations. That is, the investment is undertaken if and only if both parties' reports are favorable.*

- *If $i_B > i + \delta_2$, then $d^*_{(\hat{s},H)} = 1$ and $d^*_{(\hat{s},L)} = 0$ for $\hat{s} \in \{\hat{G}, \hat{B}\}$. That is, the optimal investment policy is fully determined by the board's advice.*

In case that the parties' reports are contradictory, the optimal investment policy basically goes with the report with higher precision. However, if the precision of the two sources is very similar ($i_B \in [i - \delta_1, i + \delta_2]$), then the optimal investment policy is to forgo investment upon inconsistent reports. The reason is that, if the CEO and the board have similar precision yet deliver contradicting reports, then the posterior project quality is "close to" the prior, 0.5, which is the indifference point of investment ignoring the implementation cost (given that $X = 2I$). However, recall that the implementation effort has to be motivated after the investment is made. Therefore, if the two parties' reports regarding the project quality are contradicting with each other, then the expected NPV of the project might be too small to cover the additional compensation cost, hence rendering the investment to be optimally forgone.

5 The Effect of Board Involvement

In this section, we study the effect of board involvement on investment efficiency, firm value and CEO incentives. To deliver the intuition more clearly, we invoke the following assumption:

Assumption 1 $e^{rk_2} \leq 1 + \frac{1-e^{-rk_1}}{i+\frac{1-e^{-rk_1}}{4i}}$

Assumption 1 ensures that $Z \geq 0$ for $i_B \geq i$, i.e., the reporting constraints in $\mathcal{P}_a$ are slack. At the same time, it is readily verified that Assumption 1 is also sufficient to ensure that the reporting constraints in $\mathcal{P}_p$ are slack. The benefit of having the reporting constraints slack is that the solutions to the optimization programs have a clear and intuitive structure.[19]

Casual intuition suggests that board involvement should (weakly) improve the firm's investment decisions, because it provides incremental information on project quality. Our analysis shows that this intuition does not always hold. To that end, we first examine the "first-best" investment policy in the absence of any incentive problems. Then it is optimal for the board to absorb all of the risk. The CEO will be paid a constant k_1 to acquire information, and if investment is undertaken, the CEO again will be paid a constant k_2 to implement the project. Formally, the board is choosing investment policy to maximize:

$$\mathcal{P}_{FB}: \qquad \max_{d_{(\hat{s},m)}} \qquad FV = \Pr[\hat{s},m]\left(\Pr[\theta=1|\hat{s},m]X - I - k_2\right) - k_1$$

Denote $d^{FB}_{(\hat{s},m)}$ as the solution to $\mathcal{P}_{FB}$.

Proposition 5 ***(The Under-Investment Problem)***

There exists $\varepsilon_1 < \delta_1$ and $\varepsilon_2 < \delta_2$ such that

- *If $i_B < i - \varepsilon_1$, then $d^{FB}_{(\hat{G},m)} = 1$ and $d^{FB}_{(\hat{B},m)} = 0$ for $m \in \{H, L\}$. That is, the first-best investment policy is fully determined by the CEO's report.*

- *If $i - \varepsilon_1 \leq i_B \leq i + \varepsilon_2$, then $d^{FB}_{(\hat{G},H)} = 1$ and $d^{FB}_{(\hat{s},m)} = 0$ for other $(\hat{s},m)$ combinations. That is, the first-best investment policy is to invest when both parties' reports are favorable.*

- *If $i_B > i + \varepsilon_2$, then $d^{FB}_{(\hat{s},H)} = 1$ and $d^{FB}_{(\hat{s},L)} = 0$ for $\hat{s} \in \{\hat{G}, \hat{B}\}$. That is, the first-best investment policy is fully determined by the board's advice.*

[19] Assumption 1 is not necessary for the results. If Assumption 1 doesn't hold, then the following results (Proposition 5, 6 and 7) will be qualitatively the same.

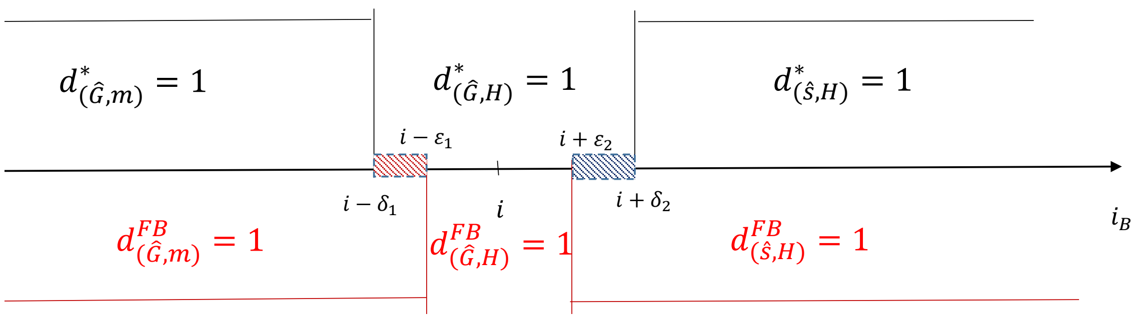

Figure 3: Under-Investment

A comparison of Propositions 4 and 5 suggests that the incentive problems will lead to under-investment. The intuition is simple. Board involvement breaks the CEO's incentive synergy between the two tasks. Now the implementation effort has to be motivated separately. The additional cost for motivating implementation effort makes the project less profitable, therefore, investment occurs less frequently than the "first-best" benchmark. See Figure 3 for illustration: under-investment occurs for $i_B \in (i - \delta_1, i - \varepsilon_1)$ and $i_B \in (i + \varepsilon_2, i + \delta_2)$.

When such an under-investment problem occurs, a "passive" board's investment policy that bases on the CEO's information alone may be closer to the first-best policy. Specifically, for $i_B \in (i - \delta_1, i - \varepsilon_1)$, the first-best policy is to invest as long as the CEO issues favorable report, which is exactly the same as a "passive" board would do. However an "active" board will only invest when both parties issue favorable reports.

Corollary 2 *For $i_B \in (i - \delta_1, i - \varepsilon_1)$, a passive board's investment policy concurs with the first-best policy, while an active board invests less frequently compared with the first-best policy.*

One might think for situations where board involvement does improve the firm's investment decisions, it should also increase firm value. However such reasoning overlooks the effect of board involvement on CEO incentives. The following Proposition shows that it exists a parameter region for which board involvement improves investment decision but nonetheless reduces firm value.

Proposition 6 *Suppose Assumption 1 holds. When $i \leq \frac{1}{4}$, there exists an $i_B^* > i - \varepsilon_1$, such that for $i_B < i_B^*$, an active board reduces firm value, i.e., $FV_a(i_B) < FV_p$, and for $i_B \geq i_B^*$, an active board improves firm value, i.e., $FV_a(i_B) \geq FV_p$.*

We focus on the scenario where the CEO's information is not very precise, i.e., $i \leq 0.25$, so that one would expect that board involvement is more valuable. We show that board expertise has to be higher than a threshold i_B^* in order to increase firm value and such threshold is high enough to improve the investment decision, i.e., $i_B^* > i - \varepsilon_1$. This suggests that for $i_B \in (i - \varepsilon_1, i_B^*)$, even if the board's expertise is high enough to improve the investment efficiency, board involvement may still decrease firm value. The reason is that board involvement may hurt CEO incentives and such negative impact may outweigh its positive impact on investment efficiency, rendering the overall effect on firm value to be negative.

To understand how board involvement affects CEO incentives, note that, board involvement breaks the synergy between motivating the two tasks, rendering it impossible to rely on the final project outcome to motivate the CEO's evaluation effort.

Instead the board has to rely on the interim information to restore/provide evaluation incentives. Board advice complements the CEO's report in serving as the interim information. Now with board involvement, the board relies on the conformity between the CEO's report and the board's advice to motivate the CEO's evaluation effort. The lower the board expertise i_B, the less enhancement in the correlation between the two parties' reports due to the CEO's evaluation effort. Then the evaluation effort is harder to be motivated. Specifically, we show in the appendix that when $i_B < \frac{1}{4}$, board involvement results in a higher compensation cost for evaluation effort (CC_{a_1}) compared with the benchmark case without board involvement.

That is, board involvement hurts CEO incentives if board expertise is low (i.e., $i_B < \frac{1}{4}$). At the same time, board involvement strictly improves investment efficiency if board expertise $i_B > i - \varepsilon_1$. For board expertise slightly higher than $i - \varepsilon_1$, the impact on investment efficiency is dominated by the negative effect on CEO incentives. Therefore, if the CEO's information is not very precise (i.e., $i < \frac{1}{4}$), there exists an non-empty region $i_B \in (i-\varepsilon_1, i_B^*)$ such that board involvement reduces firm value even if it actually improves investment decisions.

We now analyze how board involvement affects CEO incentives.

Proposition 7 *Suppose Assumption 1 holds. Then the impact of board expertise i_B on CEO incentives:*

- *It is always easier to motivate the CEO's* ***evaluation*** *effort when board expertise is higher: $\frac{\partial CC_{a_1}}{\partial i_B} < 0$.*
- *The effect of board expertise on the CEO's* ***implementation*** *effort incentives depends on whether the board's information can influence the investment decision. Specifically,*

(1) when $i_B < i - \delta_1$, i.e., the investment decision depends solely on the CEO's report, board expertise negatively affects the CEO's implementation incentives: $\frac{\partial CC_{a_2}}{\partial i_B} > 0$*;*

(2) when $i_B > i + \delta_2$, i.e., the investment decision depends solely on the board's information, board expertise positively affects the CEO's implementation incentives: $\frac{\partial CC_{a_2}}{\partial i_B} < 0$*.*

(3) when $i - \delta_1 \leq i_B \leq i + \delta_2$, i.e., the investment decision depends jointly on the CEO's and board's information, the effect of board expertise on the CEO's implementation incentives is undetermined.

The CEO's incentives to evaluate the project is always enhanced by facing a board that is more capable to assess the project. When the board has higher expertise in project evaluation, its information will be more correlated with the CEO's report if the CEO has carefully evaluates the project. That is, it is easier for the board to infer whether the evaluation effort has been taken. As a result, the CEO is more willing to exert effort.

The impact of board expertise on implementation incentives is mixed, however. Only if board expertise is high enough to influence the investment decision, may board expertise have a positive impact on the CEO's implementation incentives. If the board fails to influence the investment decision, its expertise can actually hurt the CEO's incentives to implement the project.

To understand why, note that if board advice fails to influence the investment decision, which is the case when $i_B < i - \delta_1$, then board advice only injects volatility into the perceived project quality invested without improving the *average* project quality invested (which is always $Pr[\theta = 1|G]$). Therefore, higher board expertise only increases the volatility of the project quality the CEO needs to implement, hurting the CEO's incentives to implement.

For the opposite case where the investment decision is solely based on board advice, which is the case when $i_B > i + \delta_2$ (part 2), the result is reversed. In this case, the project has *ex-ante* probability of 1/2 to get invested and thus needs to be implemented. Higher board expertise not only improves the *average* project quality invested ($Pr[\theta = 1|H]$), but also reduces the volatility of the project quality faced by the CEO (technically, $Pr[\theta = 1|\hat{G}, H] - Pr[\theta = 1|\hat{B}, H]$ is decreasing in i_B). Therefore, it is easier to motivate the CEO to implement the project when board expertise is higher.

In the intermediate case where $i - \delta_1 \leq i_B \leq i + \delta_2$, the investment decision depends jointly on CEO's and board's information. In this case, higher i_B improves the project quality faced by the CEO ($Pr[\theta = 1|\hat{G}, H]$), but at the same time it also increases the necessity to motivate the CEO's implementation effort since the ex-ante probability of investment increases. Specifically,

$$CC_{a_2}(i - \delta_1 \leq i_B \leq i + \delta_2) = \underbrace{0.5(0.5 + i)(0.5 + i_B)}_{Pr[\theta=1,G,H,a_1=1]} \left[-\frac{1}{r} Ln \left(1 - \frac{1 - e^{-rk_2}}{Pr[\theta = 1|\hat{G}, H]} \right) \right]$$

Increasing i_B lowers the bonus for success (the term inside the squire bracket) but increases the probability of payment occurance ($Pr[\theta = 1, G, H, a_1 = 1]$). The two forces work in opposite direction and depending on which force dominates, higher board expertise can increase or decrease the compensation cost for implementing the project.

6 Conclusion

We study how board involvement affects CEO incentives in a project investment setting. To achieve a higher probability of a success, the CEO engages in a sequence of tasks: first expends effort to evaluate a potential project, then reports the assessment of the project to the board, and finally expends effort to implement the project. We show that there is a positive incentive spillover effect between motivating the evaluation and implementation tasks. That is, if the CEO is motivated to evaluate the project, he is also willing to implement the project properly.

We demonstrate that board active involvement can break the incentive synergy between the evaluation and implementation tasks. As a result, investment occurs less frequently than the first-best investment policy. Even in those cases investment efficiency is increased by board involvement, the firm value can still be lowered because board involvement may exacerbate the incentive problems. We find that the level of director expertise determines whether board involvement is beneficial for the firm but director expertise does not always improve executive incentives.

We argue that the value of board involvement depends on the specifics of the business environments a firm resides in. In this study, we focus on the positive incentive spillover effect among tasks. But there could be negative incentive spillover effects under different circumstances. Then board involvement may help control the negative effect. In addition, along with the CEO whose responsibilities are mainly discovering and evaluating potential projects, the board can assign another agent on the implementation side, for example, a COO (the Chief Operating Officer). When the control of two tasks should be separated is an interesting venue for future research.

Appendix: Proofs

Proof of Lemma 1:

The "passive" board's optimization program reads

$$\mathcal{P}_p: \quad \min_{\overline{U}_{\hat{G}}, U_{\hat{G}}, U_{\hat{B}}} \; CC_p = 0.5[(0.5+i)\Phi(\overline{U}_{\hat{G}}) + (0.5-i)\Phi(U_{\hat{G}})] + 0.5\Phi(U_{\hat{B}})$$

subject to:

$$e^{rk_2}\left[(0.5+i)\overline{U}_{\hat{G}} + (0.5-i)U_{\hat{G}}\right] \geq U_{\hat{G}} \qquad (IC_G - a_2)$$

$$e^{rk_2}\left[(0.5+i)\overline{U}_{\hat{G}} + (0.5-i)U_{\hat{G}}\right] \geq U_{\hat{B}} \qquad (TT_G)$$

$$U_{\hat{B}} \geq \max\left\{U_{\hat{G}},\; e^{rk_2}\left[(0.5-i)\overline{U}_{\hat{G}} + (0.5+i)U_{\hat{G}}\right]\right\} \qquad (TT_B)$$

$$EU_p \geq \max\left\{U_{\hat{B}}, U_{\hat{G}},\; e^{rk_2}\left(0.5\overline{U}_{\hat{G}} + 0.5U_{\hat{G}}\right)\right\} \qquad (IC - a_1)$$

$$EU_p \geq -e^{r\cdot 0} = -1 \qquad (IR)$$

It is straightforward that $(IC_G - a_2)$ is implied by (TT_G) and (TT_B). That is,

$$e^{rk_2}\left[(0.5+i)\overline{U}_{\hat{G}} + (0.5-i)U_{\hat{G}}\right] \geq U_{\hat{B}} \geq U_{\hat{G}}.$$

■

Proof of Proposition 1:

The proof of Proposition 1 follows the following steps: Step (1)-(4) get rid of the redundant/slack conditions; Step (5) solves for the optimization program.

Step (1): As we argued in the proof of Lemma 1, $(IC_G - a_2)$ is always slack.

Step (2): We show that (TT_G) is always slack.

Proof: Suppose (TT_G) is binding, then $EU_p = e^{rk_1}U_{\hat{B}}$, which will be smaller than $U_{\hat{B}}$, contradicting with $(IC - a_1)$.[20]

[20] Note that the CARA utility functions are negative.

Step (3): We show that $U_{\hat{B}} \geq e^{rk_2}\left[(0.5-i)\overline{U}_{\hat{G}} + (0.5+i)U_{\hat{G}}\right]$ always holds and hence (TT_B) can be reduced to $U_{\hat{B}} \geq U_{\hat{G}}$.

Proof: Suppose not, instead $U_{\hat{B}} < e^{rk_2}\left[(0.5-i)\overline{U}_{\hat{G}} + (0.5+i)U_{\hat{G}}\right]$. Then

$$
\begin{aligned}
EU_p &= e^{rk_1}\{0.5e^{rk_2}[(0.5+i)\overline{U}_{\hat{G}} + (0.5-i)U_{\hat{G}}] + 0.5U_{\hat{B}}\} \\
&< e^{rk_1}\ e^{rk_2}\left(0.5\overline{U}_{\hat{G}} + 0.5U_{\hat{G}}\right) \\
&< \ e^{rk_2}\left(0.5\overline{U}_{\hat{G}} + 0.5U_{\hat{G}}\right),
\end{aligned}
$$

which contradicting with $(IC-a_1)$.

Step (4): Since $U_{\hat{B}} \geq U_{\hat{G}}$ is implied by (TT_B), $(IC-a_1)$ is equivalent to the following set of conditions:

$$EU_p \geq U_{\hat{B}} \tag{$IC-a_1-1$}$$

$$EU_p \geq e^{rk_2}(0.5\overline{U}_{\hat{G}} + 0.5U_{\hat{G}}) \tag{$IC-a_1-2$}$$

Step (5): Now the "passive" board's optimization program $\mathcal{P}_p$ can be reduced to:

$$\min_{\overline{U}_{\hat{G}}, U_{\hat{G}}, U_{\hat{B}}} CC_p = 0.5[(0.5+i)\Phi(\overline{U}_{\hat{G}}) + (0.5-i)\Phi(U_{\hat{G}})] + 0.5\Phi(U_{\hat{B}})$$

subject to:

$$U_{\hat{B}} \geq U_{\hat{G}} \tag{TT_B}$$

$$EU_p \geq U_{\hat{B}} \tag{$IC-a_1-1$}$$

$$EU_p \geq e^{rk_2}(0.5\overline{U}_{\hat{G}} + 0.5U_{\hat{G}}) \tag{$IC-a_1-2$}$$

$$EU_p \geq -e^{r\cdot 0} = -1 \tag{IR}$$

Let μ_1, μ_2 and μ_3 denote the Lagrangian Multipliers of constraints (TT_B), $(IC-a_1-1)$ and $(IC-a_1-2)$, respectively, and λ denote the Lagrangian Multiplier of the (IR) constraint. The first-order conditions are:

$$
\begin{aligned}
\Phi'(\overline{U}_{\hat{G}}) &= \mu_3[e^{r(k_1+k_2)} - \frac{e^{rk_2}}{0.5+i}] + \mu_2 e^{r(k_1+k_2)} + \lambda e^{r(k_1+k_2)} \\
\Phi'(U_{\hat{G}}) &= \mu_3[e^{r(k_1+k_2)} - \frac{e^{rk_2}}{0.5-i}] + \mu_2 e^{r(k_1+k_2)} - \mu_1 \frac{1}{0.5(0.5-i)} + \lambda e^{r(k_1+k_2)} \\
\Phi'(U_{\hat{B}}) &= \mu_3 e^{rk_1} + \mu_2[e^{rk_1} - \frac{1}{0.5}] + \mu_1 \frac{1}{0.5} + \lambda e^{rk_1}.
\end{aligned}
$$

First, we show that $\mu_1 > 0$ or $\mu_3 > 0$. Proof by contradiction. Suppose not, instead $\mu_1 = \mu_3 = 0$. Then that implies $\overline{U}_{\hat{G}} = U_{\hat{G}}$, which violates the implementation effort IC constraint $(IC_G - a_2)$.

Secondly, we show that $\mu_2 > 0$. Proof by contradiction. Suppose $\mu_2 = 0$, then

$$
\begin{aligned}
e^{-rk_2}\Phi'(\overline{U}_{\hat{G}}) &= \mu_3[e^{rk_1} - \frac{1}{0.5+i}] + \lambda e^{rk_1} \\
e^{-rk_2}\Phi'(U_{\hat{G}}) &= \mu_3[e^{rk_1} - \frac{1}{0.5-i}] - \mu_1 \frac{1}{e^{rk_2}0.5(0.5-i)} + \lambda e^{rk_1} \\
\Phi'(U_{\hat{B}}) &= \mu_3 e^{rk_1} + \mu_1 \frac{1}{0.5} + \lambda e^{rk_1}.
\end{aligned}
$$

Clearly $e^{-rk_2}\Phi'(\overline{U}_{\hat{G}}) < \Phi'(U_{\hat{B}})$ and $e^{-rk_2}\Phi'(U_{\hat{G}}) < \Phi'(U_{\hat{B}})$. Substituting $\Phi'(U) = -\frac{1}{rU}$, we have $e^{rk_2}\overline{U}_{\hat{G}} < U_{\hat{B}}$ and $e^{rk_2}U_{\hat{G}} < U_{\hat{B}}$, which violates constraint (TT_G).

Last, we argue that (IR) constraint is always binding. Suppose (IR) constraint is slack, then the board can multiply all three utility terms by $e^{-r\varepsilon}$. Then all the constraints will continue to satisfy and the board can save wage cost by ε.

Hence, with only three unknowns, there are two possibilities: (1) constraints (TT_B), $(IC - a_1 - 1)$ and (IR) are binding, and (2) constraints $(IC - a_1 - 1)$, $(IC - a_1 - 2)$ and (IR) are binding. Solve the program and convert the solutions to the wage space, we get:

When $k_2 \leq \frac{1}{r} Ln\left(1+\frac{1-e^{-rk_1}}{i}\right)$,

$$\begin{aligned}
W_{\hat{B}} &= 0, \\
\overline{W}_{\hat{G}} &= k_2 - \frac{1}{r} Ln\left(1-\frac{1-e^{-rk_1}}{i}\right) > 0, \\
W_{\hat{G}} &= k_2 - \frac{1}{r} Ln\left(1+\frac{1-e^{-rk_1}}{i}\right) \leq 0.
\end{aligned}$$

When $k_2 > \frac{1}{r} Ln\left(1+\frac{1-e^{-rk_1}}{i}\right)$,

$$\begin{aligned}
W_{\hat{B}} &= W_{\hat{G}} = 0, \\
\overline{W}_{\hat{G}} &= -\frac{1}{r} Ln\left(\frac{e^{-rk_2}(2e^{-rk_1}-1)-(0.5-i)}{0.5+i}\right) > 0.
\end{aligned}$$

■

Proof of Proposition 2:

If no investment is triggered by $(\hat{B}, H)$, i.e., $d_{(\hat{B},H)} = 0$, then the CEO's off-equilibrium payoff $D_{\hat{B}H} = U_{\hat{B}H}$. The cost-minimizing program $\mathcal{P}_a^{d_{(\hat{B},H)}=0}$ is thus

$$\min_{\{U_{\hat{s}m} \in R\}} \quad CC_a = (0.25 + i_B i)\left[\Phi(U_{\hat{G}H}) + \Phi(U_{\hat{B}L})\right] + (0.25 - i_B i)\left[\Phi(U_{\hat{B}H}) + \Phi(U_{\hat{G}L})\right]$$

subject to

$$(0.5 + 2i_B i)U_{\hat{G}H} + (0.5 - 2i_B i)U_{\hat{G}L} \geq 0.5U_{\hat{B}H} + 0.5U_{\hat{B}L} \qquad (TT_G)$$

$$(0.5 - 2i_B i)U_{\hat{B}\hat{H}} + (0.5 + 2i_B i)U_{\hat{B}L} \geq 0.5U_{\hat{G}H} + 0.5U_{\hat{G}L} \qquad (TT_B)$$

$$EU_a \geq 0.5U_{\hat{G}H} + 0.5U_{\hat{G}L} \qquad (AIC - a_1 - 1)$$

$$EU_a \geq 0.5U_{\hat{B}H} + 0.5U_{\hat{B}L} \qquad (AIC - a_1 - 2)$$

$$EU_a \geq -e^{r \cdot 0} = -1 \qquad (IR)$$

where $EU_a = e^{rk_1}\left[(0.25 + i_B i)(U_{\hat{G}H} + U_{\hat{B}L}) + (0.25 - i_B i)(U_{\hat{B}H} + U_{\hat{G}L})\right]$.

First, we argue that the solution to program $\mathcal{P}_a^{d_{(\hat{B},H)}=0}$ is the same as that to the following program $\mathcal{P}'_a$:

$$\min_{\{U_{\hat{s}m}\in R\}} \quad CC_a = (0.25 + i_B i)\,[\Phi(U_{\hat{G}H}) + \Phi(U_{\hat{B}L})] + (0.25 - i_B i)\,[\Phi(U_{\hat{B}H}) + \Phi(U_{\hat{G}L})]$$

subject to

$$(0.5 + 2i_B i)\,(U_{\hat{G}H} + U_{\hat{B}L}) + (0.5 - 2i_B i)\,(U_{\hat{G}L} + U_{\hat{B}H}) \geq 0.5\,(U_{\hat{B}H} + U_{\hat{B}L} + U_{\hat{G}L} + U_{\hat{G}H}) \tag{TT}$$

$$EU_a \geq 0.25\,(U_{\hat{B}H} + U_{\hat{B}L} + U_{\hat{G}L} + U_{\hat{G}H}) \tag{$AIC' - a_1$}$$

$$EU_a \geq -e^{r\cdot 0} = -1 \tag{IR}$$

Where the constraint (TT) derives from $(TT_G) + (TT_B)$, and $(AIC' - a_1)$ derives from $(AIC - a_1 - 1) + (AIC - a_1 - 2)$. Clearly, the constraints in program $\mathcal{P}'_a$ is more relax than the constraints in the original program $\mathcal{P}_a^{d_{(\hat{B},H)}=0}$.

A close observation of program $\mathcal{P}'_a$ suggests that the optimal solution must entail that $U_{\hat{G}H} = U_{\hat{B}L}$ and $U_{\hat{B}H} = U_{\hat{G}L}$. The reason is that the program is symmetric between $U_{\hat{G}H}$ and $U_{\hat{B}L}$: if we switch $U_{\hat{G}H}$ and $U_{\hat{B}L}$, the program is exactly the same. Similarly, the program is also symmetric between $U_{\hat{G}L}$ and $U_{\hat{B}H}$.

Given $U_{\hat{G}H} = U_{\hat{B}L}$ and $U_{\hat{B}H} = U_{\hat{G}L}$, the constraint (TT) is exactly the same as (TT_G) and (TT_B), and the constraint $(AIC' - a_1)$ is exactly the same as $(AIC - a_1 - 1)$ and $(AIC - a_1 - 2)$. Therefore, the solution to the relaxed program $\mathcal{P}'_a$ will also satisfy the constraints in the original program $\mathcal{P}_a^{d_{(\hat{B},H)}=0}$, and will be the solution to the original program $\mathcal{P}_a^{d_{(\hat{B},H)}=0}$.

Now, let's solve for Program $\mathcal{P}'_a$. With $U_{\hat{G}H} = U_{\hat{B}L}$ and $U_{\hat{B}H} = U_{\hat{G}L}$, and substituting EU_a, Program $\mathcal{P}'_a$ can be reduced to:

$$\min_{\{U_{\hat{s}m}\in R\}} \quad CC_a = (0.5+2i_Bi)\Phi(U_{\hat{G}H}) + (0.5-2i_Bi)\Phi(U_{\hat{G}L})$$

subject to

$$4i_Bi\,(U_{\hat{G}H} - U_{\hat{G}L}) \geq 0 \qquad (TT)$$

$$e^{rk_1}\left[(0.5+2i_Bi)U_{\hat{G}H} + (0.5-2i_Bi)U_{\hat{G}L}\right] \geq 0.5\,(U_{\hat{G}L} + U_{\hat{G}H}) \quad (AIC' - a_1)$$

$$e^{rk_1}\left[(0.5+2i_Bi)U_{\hat{G}H} + (0.5-2i_Bi)U_{\hat{G}L}\right] \geq -e^{r\cdot 0} = -1 \qquad (IR)$$

We first argue that the constraint (TT) is always slack. Prove by contradiction. Suppose (TT) is binding, then $U_{\hat{G}H} = U_{\hat{G}L}$, which will violate constraint $(AIC' - a_1)$.

Let μ and λ denote the Lagrangian Multipliers for evaluation effort constraints $(AIC' - a_1)$ and (IR) constraint respectively. The first-order conditions are

$$\begin{aligned}
\Phi'(U_{\hat{G}H}) &= \mu[e^{rk_1} - \frac{0.5}{0.5+2i_Bi}] + \lambda e^{rk_1} \\
\Phi'(U_{\hat{G}L}) &= \mu[e^{rk_1} - \frac{0.5}{0.5-2i_Bi}] + \lambda e^{rk_1}
\end{aligned}$$

Clearly, $\mu > 0$. Suppose not, instead $\mu = 0$, then it follows that $U_{\hat{G}H} = U_{\hat{G}L}$, which will violate constraint $(AIC' - a_1)$.

At the same time, (IR) constraint is always binding. Suppose (IR) constraint is slack, then the board can multiply all utility terms by $e^{-r\varepsilon}$. Then all the constraints will continue to satisfy and the board can save wage cost by ε.

With the binding $(AIC' - a_1)$ and (IR) constraints, we can solve for the choice variables:

$$\begin{aligned}
U_{\hat{G}H} &= U_{\hat{B}L} = -1 + \frac{1-e^{-rk_1}}{4i_Bi} \\
U_{\hat{B}H} &= U_{\hat{G}L} = -1 - \frac{1-e^{-rk_1}}{4i_Bi}.
\end{aligned}$$

■

Proof of Proposition 3:

If the investment decision is fully determined by the board's advice, then $d_{(\hat{B},H)} = 1$, and the CEO's off-equilibrium payoff is

$$\begin{aligned}
D_{\hat{B}H} &\equiv U_{\hat{B}H} + \max\left\{0, e^{rk_2}\left[(0.5+i)\overline{U}_{\hat{B}H} + (0.5-i)U_{\hat{B}H}\right] - U_{\hat{B}H}\right\} \\
&= U_{\hat{B}H} + \max\left\{0, e^{rk_2}\left[\frac{(0.5+i)(e^{-rk_2}-1)}{\Pr[\theta=1|B,H,a_1=1]}+1\right]U_{\hat{B}H} - U_{\hat{B}H}\right\} \\
&= U_{\hat{B}H} + \max\left\{0, (1-e^{rk_2})\left[\frac{0.5+i}{\Pr[\theta=1|B,H,a_1=1]}-1\right]U_{\hat{B}H}\right\} \\
&= U_{\hat{B}H} + \max\left\{0, \frac{0.5+i}{\Pr[\theta=1|B,H,a_1=1]}-1\right\}(1-e^{rk_2})U_{\hat{B}H}.
\end{aligned}$$

The cost-minimizing program $\mathcal{P}_a^{d_{(\hat{B},H)}=1}$ is thus

$$\min_{\{U_{\hat{s}m}\in R\}} \quad CC_a = (0.25+i_Bi)\left[\Phi(U_{\hat{G}H})+\Phi(U_{\hat{B}L})\right] + (0.25-i_Bi)\left[\Phi(U_{\hat{B}H})+\Phi(U_{\hat{G}L})\right]$$

subject to

$$(0.5+2i_Bi)U_{\hat{G}H} + (0.5-2i_Bi)U_{\hat{G}L} \geq 0.5D_{\hat{B}H} + 0.5U_{\hat{B}L} \qquad (TT_G)$$

$$(0.5-2i_Bi)U_{\hat{B}\hat{H}} + (0.5+2i_Bi)U_{\hat{B}L} \geq 0.5U_{\hat{G}H} + 0.5U_{\hat{G}L} \qquad (TT_B)$$

$$EU_a \geq 0.5U_{\hat{G}H} + 0.5U_{\hat{G}L} \qquad (AIC-a_1-1)$$

$$EU_a \geq 0.5U_{\hat{B}H} + 0.5U_{\hat{B}L} \qquad (AIC-a_1-2)$$

$$EU_a \geq -e^{r\cdot 0} = -1 \qquad (IR)$$

Notice that program $\mathcal{P}_a^{d_{(\hat{B},H)}=1}$ is similar to program $\mathcal{P}_a^{d_{(\hat{B},H)}=0}$ in the proof of Proposition 2, with the only difference being the (TT_G) constraint. Specifically, given $D_{\hat{B}H} \geq U_{\hat{B}H}$, the (TT_G) constraint in $\mathcal{P}_a^{d_{(\hat{B},H)}=1}$ is harder to satisfy than the corresponding constraint in $\mathcal{P}_a^{d_{(\hat{B},H)}=0}$.

Suppose the (TT_G) constraint in $\mathcal{P}_a^{d_{(\hat{B},H)}=1}$ is slack, then the optimal solution of program $\mathcal{P}_a^{d_{(\hat{B},H)}=1}$ is the same as that of program $\mathcal{P}_a^{d_{(\hat{B},H)}=0}$. Substituting the solution in Proposition 2 to the current (TT_G) constraint and rearranging the terms, the current (TT_G) constraint is reduced to

$$1 - e^{-rk_1} \geq 0.5 \max\left\{0, \frac{0.5+i}{\Pr[\theta=1|B,H,a_1=1]} - 1\right\}(e^{rk_2}-1)\left(1+\frac{1-e^{-rk_1}}{4i_B i}\right). \quad (3)$$

Therefore, if $Z \equiv \frac{1-e^{-rk_1}}{1+\frac{1-e^{-rk_1}}{4i_B i}} - 0.5(e^{rk_2}-1)\cdot\max\left\{0, \frac{0.5+i}{\Pr[\theta=1|B,H,a_1=1]} - 1\right\} \geq 0$, the (TT_G) constraint is indeed slack, and the optimal solution of program $\mathcal{P}_a^{d_{(\hat{B},H)}=1}$ is the same as characterized in Proposition 2.

On the other hand, if $Z < 0$, then the (TT_G) constraint in program $\mathcal{P}_a^{d_{(\hat{B},H)}=1}$ must be binding. Prove by contradiction. Suppose the (TT_G) constraint is instead slack, then the optimal solution is the same as characterized in Proposition 2, and the (TT_G) constraint can be reduced to (3). If $Z < 0$, the (TT_G) constraint is violated. A contradiction. ∎

Proof of Proposition 4:

Given the assumption that $0.5X - I = 0$, a necessary condition for the board to induce investment is that $Pr[\theta = 1|\hat{s}, m] \geq 0.5$. We consider the following two cases:

(1) $i_B < i$**.**

If $i_B < i$, the ranking of posteriors is as follows (factoring in that the CEO truthfully reports in equilibrium):

$$Pr[\theta=1|\hat{G},H] > Pr[\theta=1|\hat{G},L] > 0.5 > Pr[\theta=1|\hat{B},H] > Pr[\theta=1|\hat{B},L].$$

Then the board needs to decide between the following two options: (1) invest if and only if both parties' reports are favorable, i.e., $d_{(\hat{G},H)} = 1$; or (2) invest if and only if the CEO reports G, i.e., $d_{(\hat{G},m)} = 1$.

If the investment policy is to invest if and only if both parties' reports are favorable, i.e., $d_{(\hat{G},H)} = 1$, then the firm value is

$$\begin{aligned}
& FV_a(d_{(\hat{G},H)} = 1) \\
= \; & Pr[\hat{G},H]\left\{Pr[\theta = 1|\hat{G},H]X - I\right\} - CC_{a_1}(d_{(\hat{G},H)} = 1) - Pr[\hat{G},H]CC_{a_2}^{(\hat{G},H)} \\
= \; & 0.5(i + i_B)I - CC_{a_1}(d_{(\hat{G},H)} = 1) - Pr[\hat{G},H]CC_{a_2}^{(\hat{G},H)} \qquad (4)
\end{aligned}$$

If the investment policy is to invest if and only if the CEO reports G, i.e., $d_{(\hat{G},m)} = 1$ and $d_{(\hat{B},m)} = 0$, then the firm value is

$$\begin{aligned}
& FV_a(d_{(\hat{G},m)} = 1) \\
= \; & Pr[\hat{G}]\left\{Pr[\theta = 1|\hat{G}]X - I\right\} - CC_{a_1}(d_{(\hat{G},m)} = 1) - Pr[\hat{G},H]CC_{a_2}^{(\hat{G},H)} - Pr[\hat{G},L]CC_{a_2}^{(\hat{G},L)} \\
= \; & i \cdot I - CC_{a_1}(d_{(\hat{G},m)} = 1) - Pr[\hat{G},H]CC_{a_2}^{(\hat{G},H)} - Pr[\hat{G},L]CC_{a_2}^{(\hat{G},L)} \qquad (5)
\end{aligned}$$

Proposition 2 shows that $CC_{a_1}(d_{(\hat{G},H)} = 1) = CC_{a_1}(d_{(\hat{G},m)} = 1)$, therefore,

$$FV_a(d_{(\hat{G},m)} = 1) - FV_a(d_{(\hat{G},H)} = 1) = 0.5(i - i_B)I - Pr[\hat{G},L]CC_{a_2}^{(\hat{G},L)}. \qquad (6)$$

It is immediate that (given I is sufficiently large),

$$\begin{aligned}
\lim_{i_B \to i} FV_a(d_{(\hat{G},m)} = 1) - FV_a(d_{(\hat{G},H)} = 1) &= -Pr[\hat{G},L]CC_{a_2}^{(\hat{G},L)} < 0, \\
\lim_{i_B \to 0} FV_a(d_{(\hat{G},m)} = 1) - FV_a(d_{(\hat{G},H)} = 1) &= 0.5 \cdot i \cdot I - Pr[\hat{G},L]CC_{a_2}^{(\hat{G},L)} > 0, \\
\frac{\partial\left(FV_a(d_{(\hat{G},m)} = 1) - FV_a(d_{(\hat{G},H)} = 1)\right)}{\partial i_B} &= -0.5 \cdot I - \frac{\partial\left(Pr[\hat{G},L]CC_{a_2}^{(\hat{G},L)}\right)}{\partial i_B} < 0.
\end{aligned}$$

Therefore, there exists a $\delta_1 > 0$ such that, for $i_B < i - \delta_1$, $FV_a(d_{(\hat{G},m)} = 1) - FV_a(d_{(\hat{G},H)} = 1) > 0$, i.e., the optimal investment policy is to invest according to the CEO's report; for $i_B > i - \delta_1$, $FV_a(d_{(\hat{G},m)} = 1) - FV_a(d_{(\hat{G},H)} = 1) < 0$, then the optimal investment policy is to invest if and only if both parties' reports are favorable. δ_1 is determined by setting (6) equals 0.

(2) $i_B \geq i$

Similar argument shows that for $i_B \geq i$, the board is deciding between (1) invest if and only if both parties' reports are favorable, i.e., $d_{(\hat{G},H)} = 1$; or (2) invest according to the board's advice, i.e., $d_{(\hat{s},H)} = 1$.

$$\begin{aligned} & FV_a(d_{(\hat{s},H)} = 1) - FV_a(d_{(\hat{G},H)} = 1) \\ = \ & 0.5(i_B - i)I - Pr[\hat{B}, H]CC_{a_2}^{(\hat{B},H)} - CC_{a_1}(d_{(\hat{s},H)} = 1) + CC_{a_1}(d_{(\hat{G},H)} = 1). \end{aligned}$$

Comparing Proposition 2 and Proposition 3, we know that, $CC_{a_1}(d_{(\hat{G},H)} = 1) \leq CC_{a_1}(d_{(\hat{s},H)} = 1)$, therefore, given I sufficiently large,

$$\begin{aligned} & \lim_{i_B \to i} FV_a(d_{(\hat{s},H)} = 1) - FV_a(d_{(\hat{G},H)} = 1) \\ = \ & -Pr[\hat{B}, H]CC_{a_2}^{(\hat{B},H)} - CC_{a_1}(d_{(\hat{s},H)} = 1) + CC_{a_1}(d_{(\hat{G},H)} = 1) < 0, \\ & \lim_{i_B \to 0.5} FV_a(d_{(\hat{s},H)} = 1) - FV_a(d_{(\hat{G},H)} = 1) \\ = \ & 0.5(0.5 - i) \cdot I - Pr[\hat{B}, H]CC_{a_2}^{(\hat{B},H)} - CC_{a_1}(d_{(\hat{s},H)} = 1) + CC_{a_1}(d_{(\hat{G},H)} = 1) > 0, \\ & \frac{\partial \Big(FV_a(d_{(\hat{s},H)} = 1) - FV_a(d_{(\hat{G},H)} = 1) \Big)}{\partial i_B} > 0. \end{aligned}$$

Therefore, there exists a $\delta_2 > 0$ such that, for $i_B < i + \delta_2$, $FV_a(d_{(\hat{s},H)} = 1) - FV_a(d_{(\hat{G},H)} = 1) < 0$, i.e., the optimal investment policy is to invest if and only if both parties' reports are favorable; for $i_B > i + \delta_2$, $FV_a(d_{(\hat{s},H)} = 1) - FV_a(d_{(\hat{G},H)} = 1) > 0$, then the optimal investment policy is to invest according to the board's advice. ■

Proof of Proposition 5:

Absent any incentive problems, the board is choosing investment policy to:

$$\mathcal{P}_{FB}: \quad \max_{d_{(\hat{s},m)}} \quad FV = \Pr[\hat{s}, m] \left(\Pr[\theta = 1 | \hat{s}, m] X - I - k_2 \right) - k_1$$

Similar proof as Proposition 4 shows that there exists a $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ such that, for $i_B < i - \varepsilon_1$, $d^{FB}_{(\hat{G},m)} = 1$; for $i - \varepsilon_1 \leq i_B \leq i + \varepsilon_2$, $d^{FB}_{(\hat{G},H)} = 1$; for $i_B > i + \varepsilon_2$, $d^{FB}_{(\hat{s},H)} = 1$.

Next we need to show that $\varepsilon_1 < \delta_1$ and $\varepsilon_2 < \delta_2$. To that end, note that the indifferent condition giving rise to ε_1 is

$$FV_a^{FB}(d_{(\hat{G},m)} = 1) - FV_a^{FB}(d_{(\hat{G},H)} = 1) = 0.5(i - i_B)I - \Pr[\hat{G}, L]k_2 = 0. \qquad (7)$$

Comparing (6) and (7) suggests that if $CC_{a_2}^{(\hat{s},m)} > k_2$, then $\varepsilon_1 < \delta_1$. The next result shows that indeed $CC_{a_2}^{(\hat{s},m)} > k_2$.

Lemma 2 $\frac{d\ CC_{a_2}^{(\hat{s},m)}}{d\ Pr[\theta=1|\hat{s},m]} < 0.$

Recall that

$$\begin{aligned} CC_{a_2}^{(\hat{s},m)} &= Pr[\theta = 1|\hat{s}, m]\left[-\frac{1}{r}Ln\left(1 - \frac{1 - e^{-rk_2}}{\Pr[\theta = 1|\hat{s}, m]}\right)\right] \\ &= -\frac{1}{r}\left\{Pr[\theta = 1|\hat{s}, m]Ln\left(1 - \frac{1 - e^{-rk_2}}{\Pr[\theta = 1|\hat{s}, m]}\right) + (1 - Pr[\theta = 1|\hat{s}, m])\,Ln(1)\right\} \end{aligned}$$

The term inside the brace can be interpreted as the expected utility of a risk-averse individual with utility function $Ln(\cdot)$ who is facing a lottery

$$\begin{cases} 1 - \frac{1-e^{-rk_2}}{\Pr[\theta=1|\hat{s},m]} & \text{with probability } Pr[\theta = 1|\hat{s}, m] \\ 1 & \text{with probability } 1 - Pr[\theta = 1|\hat{s}, m] \end{cases}$$

The mean of the lottery is e^{-rk_2}. At the same time, as $Pr[\theta = 1|\hat{s}, m]$ increases, the spread becomes smaller. The risk-averse individual always prefer the lottery with the smaller spread. Therefore, $\frac{d\ CC_{a_2}^{(\hat{s},m)}}{d\ Pr[\theta=1|\hat{s},m]} < 0$.

Lemma 2 and $Pr[\theta = 1|\hat{s}, m] < 1$ together imply that $CC_{a_2}^{(\hat{s},m)} > -\frac{1}{r}\cdot(-rk_2) = k_2$.

Similarly, we could show that $\varepsilon_2 < \delta_2$. ∎

Proof of Proposition 6:

Define i_B^* as the indifference point where $FV_a(i_B^*) = FV_p$. We prove that when $i < \frac{1}{4}$, $i_B^* > i - \varepsilon_1$.

Firstly, we show that for $i_B < i - \varepsilon_1$, $FV_a(i_B) < FV_p$. Incorporating the optimal investment policy characterized in Proposition 4, the firm values for active board are:

$$\begin{aligned} FV_a(i_B < i - \delta_1) &= Pr[\hat{G}]\left\{Pr[\theta = 1|\hat{G}]X - I\right\} - CC_{a_1} \\ &\quad - Pr[\hat{G}, H]CC_{a_2}^{(\hat{G},H)} - Pr[\hat{G}, L]CC_{a_2}^{(\hat{G},L)}, \qquad (8) \\ FV_a(i - \delta_1 < i_B < i - \varepsilon_1) &= Pr[\hat{G}, H]\left\{Pr[\theta = 1|\hat{G}, H]X - I\right\} - CC_{a_1} - Pr[\hat{G}, H]CC_{a_2}^{(\hat{G},H)}. \\ & \qquad (9) \end{aligned}$$

By Assumption 1 and Proposition 1,

$$CC_p = \underbrace{-\frac{1}{r}\left\{(0.25 + 0.5i)Ln\left(1 - \frac{1 - e^{-rk_1}}{i}\right) + (0.25 - 0.5i)Ln\left(1 + \frac{1 - e^{-rk_1}}{i}\right)\right\}}_{CC_{p_1}} + \underbrace{0.5k_2}_{CC_{p_2}}$$

The firm value for passive board is given by:

$$FV_p = Pr[\hat{G}]\left\{Pr[\theta = 1|\hat{G}]X - I\right\} - CC_{p_1} - Pr[\hat{G}, H]k_2 - Pr[\hat{G}, L]k_2. \quad (10)$$

The proof for $FV_a(i_B < i - \varepsilon_1) < FV_p$ follows three steps:

Step 1. For $i < \frac{1}{4}$, $CC_{a_1}(i_B < i - \varepsilon_1) > CC_{p_1}$.

Proof: By Proposition 2,

$$CC_{a_1}(i_B < i - \varepsilon_1) = -\frac{1}{r}\left\{(0.5 + 2i_B i)Ln\left(1 - \frac{1 - e^{-rk_1}}{4i_B i}\right) + (0.5 - 2i_B i)Ln\left(1 + \frac{1 - e^{-rk_1}}{4i_B i}\right)\right\}.$$

The term inside the brace

$$\Psi_1 \equiv (0.5 + 2i_B i)Ln\left(1 - \frac{1 - e^{-rk_1}}{4i_B i}\right) + (0.5 - 2i_B i)Ln\left(1 + \frac{1 - e^{-rk_1}}{4i_B i}\right)$$

is the expected utility of a risk-averse individual with utility function $Ln(\cdot)$ who is facing the following lottery:

$$\begin{cases} 1 - \frac{1-e^{-rk_1}}{4i_B i} & \text{with probability } 0.5 + 2i_B i \\ 1 + \frac{1-e^{-rk_1}}{4i_B i} & \text{with probability } 0.5 - 2i_B i \end{cases}$$

The mean of the lottery is e^{-rk_1}, which is independent of i_B. At the same time, as i_B increases, the spread becomes smaller. The risk-averse individual always prefer the lottery with the smaller spread, which is due to a larger i_B. Therefore, Ψ_1 is increasing in i_B, which leads to $\frac{\partial CC_{a_1}(\cdot)}{\partial i_B} < 0$.

For $i < \frac{1}{4}$, $i_B < i - \varepsilon_1 < \frac{1}{4}$. Therefore,

$$\begin{aligned} CC_{a_1}(i_B < i - \varepsilon_1) &> CC_{a_1}(i_B = \frac{1}{4}) \\ &= -\frac{1}{r}\left\{(0.5+0.5i)Ln\left(1 - \frac{1-e^{-rk_1}}{i}\right) + (0.5-0.5i)Ln\left(1 + \frac{1-e^{-rk_1}}{i}\right)\right\} \\ &= CC_{p_1} - \frac{1}{r}\left\{0.25Ln\left(1 - \frac{1-e^{-rk_1}}{i}\right) + 0.25Ln\left(1 + \frac{1-e^{-rk_1}}{i}\right)\right\} \\ &= CC_{p_1} - \frac{1}{4r}Ln\left(1 - \left(\frac{1-e^{-rk_1}}{i}\right)^2\right) \\ &> CC_{p_1}. \end{aligned}$$

Step 2. Lemma 2 implies $CC_{a_2}^{(\hat{G},H)} > k_2$ and $CC_{a_2}^{(\hat{G},L)} > k_2$.

Step 3. Comparing (8) and (10), it is immediate by Step 1 and 2 that

$$FV_a(i_B < i - \delta_1) < FV_p.$$

Similarly, comparing (9) and (10), and by Step 1 and 2,

$$\begin{aligned} FV_a(i - \delta_1 < i_B < i - \varepsilon_1) - FV_p &< Pr[\hat{G}, L]\left\{Pr[\theta = 1|\hat{G}, L]X - I\right\} - Pr[\hat{G}, L]k_2 \\ &< 0. \end{aligned}$$

The last inequality is implied by the first-best investment policy (7).

Secondly, we show that for $i_B \to 0.5$, $FV_a(i_B) > FV_p$. For $i_B \to 0.5$, by Proposition 4, the optimal investment policy is to invest according to the board's advice. Therefore,

$$\begin{aligned}
\lim_{i_B \to 0.5} FV_a(i_B) &= \lim_{i_B \to 0.5} Pr[H]\left\{Pr[\theta = 1|H]X - I\right\} - CC_a = 0.5(X - I) - CC_a \\
FV_p &= Pr[\hat{G}]\left\{Pr[\theta = 1|\hat{G}]X - I\right\} - CC_p = 0.5((0.5 + i)X - I) - CC_p
\end{aligned}$$

Hence, for X sufficiently large,

$$\lim_{i_B \to 0.5} FV_a(i_B) - FV_p = 0.5(0.5 - i)X - CC_a - CC_p > 0.$$

Finally, it is straightforward to show that for $i_B > i - \varepsilon_1$, $FV_a(i_B)$ monotonically increases in i_B with project size X sufficiently large. Therefore, by continuity, there must exist one cutoff $i_B^* > i - \varepsilon_1$ such that $FV_a(i_B^*) = FV_p$.

■

Proof of Proposition 7:

- Assumption 1 ensures that the reporting constraints are slack for $\mathcal{P}_a$. By Proposition 2 and 3,

  $$CC_{a_1}(\cdot) = -\frac{1}{r}\left\{(0.5 + 2i_B i)Ln\left(1 - \frac{1 - e^{-rk_1}}{4i_B i}\right) + (0.5 - 2i_B i)Ln\left(1 + \frac{1 - e^{-rk_1}}{4i_B i}\right)\right\}.$$

  As is shown in the proof of Proposition 6 Step 1, $\frac{\partial CC_{a_1}(\cdot)}{\partial i_B} < 0$.

- Part (1): when $i_B < i - \delta_1$, the optimal investment policy is $\mathbb{1}^*_{(Inv|\hat{G},m)} = 1$ and

$\mathbb{1}^*_{(Inv|\hat{B},m)} = 0$, hence

$$\begin{aligned} CC_{a_2}(i_B < i-\delta_1) = \quad & \underbrace{0.5(0.5+i)(0.5+i_B)}_{Pr[\theta=1,\hat{G},H]}\left[-\frac{1}{r}Ln\left(1-\frac{1-e^{-rk_2}}{Pr[\theta=1|\hat{G},H]}\right)\right] \\ & +\underbrace{0.5(0.5+i)(0.5-i_B)}_{Pr[\theta=1,\hat{G},L]}\left[-\frac{1}{r}Ln\left(1-\frac{1-e^{-rk_2}}{Pr[\theta=1|\hat{G},L]}\right)\right] \\ = \quad & -\frac{1}{r}0.5(0.5+i)\left\{(0.5+i_B)Ln\left(1-\frac{1-e^{-rk_2}}{Pr[\theta=1|\hat{G},H]}\right)\right. \\ & \left.+(0.5-i_B)Ln\left(1-\frac{1-e^{-rk_2}}{Pr[\theta=1|\hat{G},L]}\right)\right\} \end{aligned}$$

The term inside the brace

$$\Omega \equiv (0.5+i_B)Ln\left(1-\frac{1-e^{-rk_2}}{Pr[\theta=1|\hat{G},H]}\right)+(0.5-i_B)Ln\left(1-\frac{1-e^{-rk_2}}{Pr[\theta=1|\hat{G},L]}\right)$$

is the expected utility of a risk-averse individual with utility function $Ln(\cdot)$ who is facing the following lottery:

$$\begin{cases} 1-\frac{1-e^{-rk_2}}{Pr[\theta=1|\hat{G},H]} & \text{with probability } 0.5+i_B \\ 1-\frac{1-e^{-rk_2}}{Pr[\theta=1|\hat{G},L]} & \text{with probability } 0.5-i_B \end{cases}$$

The mean of the lottery is $1-\frac{1-e^{-rk_2}}{0.5+i}$, which is independent of i_B. At the same time, as i_B increases, the spread, $\frac{1-e^{-rk_2}}{Pr[\theta=1|\hat{G},L]}-\frac{1-e^{-rk_2}}{Pr[\theta=1|\hat{G},H]}$, becomes larger. That is, as i_B increases, the lottery becomes a Mean-Preserving-Spread of the original lottery. Therefore, Ω is decreasing in i_B, which leads to $\frac{\partial CC_{a_2}(i_B<i-\delta_1)}{\partial i_B} > 0$.

Part (2): when $i_B > i+\delta_2$, the optimal investment policy is $\mathbb{1}^*_{(Inv|\hat{s},H)} = 1$ and

$\mathbb{1}^*_{(Inv|\hat{s},L)} = 0$, hence

$$CC_{a_2}(i_B > i + \delta_2) = \underbrace{0.5(0.5+i)(0.5+i_B)}_{Pr[\theta=1,\hat{G},H]}\left[-\frac{1}{r}Ln\left(1-\frac{1-e^{-rk_2}}{Pr[\theta=1|\hat{G},H]}\right)\right]$$
$$+\underbrace{0.5(0.5-i)(0.5+i_B)}_{Pr[\theta=1,\hat{B},H]}\left[-\frac{1}{r}Ln\left(1-\frac{1-e^{-rk_2}}{Pr[\theta=1|\hat{B},H]}\right)\right]$$
$$= -\frac{1}{2r}\left\{(0.5+i)(0.5+i_B)Ln\left(1-\frac{1-e^{-rk_2}}{Pr[\theta=1|\hat{G},H]}\right)\right.$$
$$\left.+(0.5-i)(0.5+i_B)Ln\left(1-\frac{1-e^{-rk_2}}{Pr[\theta=1|\hat{B},H]}\right)+(0.5-i_B)Ln(1)\right\}$$

The term inside the brace is the expected utility of a risk-averse individual with utility function $Ln(\cdot)$ who is facing the following lottery:

$$\begin{cases} 1-\frac{1-e^{-rk_2}}{Pr[\theta=1|\hat{G},H]} & \text{with probability } (0.5+i)(0.5+i_B) \\ 1-\frac{1-e^{-rk_2}}{Pr[\theta=1|\hat{B},H]} & \text{with probability } (0.5-i)(0.5+i_B) \\ 1 & \text{with probability } 0.5-i_B \end{cases}$$

It is readily to verify that the mean of the above lottery is e^{-rk_2}, independent of i_B. Furthermore, as i_B increases, both $1-\frac{1-e^{-rk_2}}{Pr[\theta=1|\hat{G},H]}$ and $1-\frac{1-e^{-rk_2}}{Pr[\theta=1|\hat{B},H]}$ move towards 1. At the same time, the distant between the two, $\frac{1-e^{-rk_2}}{Pr[\theta=1|\hat{B},H]} - \frac{1-e^{-rk_2}}{Pr[\theta=1|\hat{G},H]}$, is also decreasing in i_B. That is, as i_B increases, the three mass points get closer to each other. Therefore, a risk-averse individual always prefers the lottery with a higher i_B, which implies $\frac{\partial CC_{a_2}(i_B>i+\delta_2)}{\partial i_B} < 0$.

Part (3): when $i-\delta_1 \leq i_B \leq i+\delta_2$, i.e., the investment decision depends jointly on CEO's and board's information. Then

$$CC_{a_2}(i-\delta_1 \leq i_B \leq i+\delta_2) = \underbrace{0.5(0.5+i)(0.5+i_B)}_{Pr[\theta=1,\hat{G},H]}\left[-\frac{1}{r}Ln\left(1-\frac{1-e^{-rk_2}}{Pr[\theta=1|\hat{G},H]}\right)\right]$$

Taking derivative with respect to i_B and after some manipulation,

$$\frac{\partial CC_{a_2}(\cdot)}{\partial i_B} = \frac{0.5(0.5+i)}{r}\left\{Ln\left(\frac{\Pr[\theta=1|\hat{G},H]}{\Pr[\theta=1|\hat{G},H]-(1-e^{-rk_2})}\right)-\frac{(1-e^{-rk_2})\frac{0.5-i}{0.5+2i_B i}}{\Pr[\theta=1|\hat{G},H]-(1-e^{-rk_2})}\right\}$$

The sign of $\frac{\partial CC_{a_2}(\cdot)}{\partial i_B}$ is undetermined. ■

References

Adams, R. and D. Ferreira. 2007. A theory of friendly boards. *The Journal of Finance* 62 (1): 217-250.

Adams, R. and D. Ferreira. 2008. Do directors perform for pay? *Journal of Accounting and Economics* 46(1): 154–71.

Adams, R., B. Hermalin and M. Weisbach. 2010. The role of boards of directors in corporate governance: a conceptual framework and survey. *Journal of Economic Literature* 48 (1): 58-107.

Aghion, P. and J. Tirole. 1997. Real and formal authority in organizations. *Journal of Political Economy* 105 (1): 1-29.

Arya, A., J. Glover and S. Radhakrishnan. 2007. The Controllability Principle in Responsibility Accounting: Another Look. *Essays in Accounting Theory in Honour of Joel S. Demski*: 183-198. Springer New York.

Baldenius, T., N. Melumad, and X. Meng. 2014. Board composition and CEO power. *Journal of Financial Economics* 112 (1): 53-68.

Bebchuk, L. and J. Fried. 2004. Pay without Performance: the Unfulfilled Promise of Executive Compensation. Cambridge and London: Harvard University Press.

Burkart, M., D. Gromb and F. Panunzi. 1997. Large shareholders, monitoring and the value of the firm. *Quarterly Journal of Economics* 112 (3): 693-728.

Business Roundtable. 2005. Principles of corporate governance.

Christensen, P., J. Demski and H. Frimor. 2002. Accounting policies in agencies with moral hazard and renegotiation. *Journal of Accounting Research* 40(4): 1071-90.

Cremer, J. 1995. Arm's length relationships. *The Quarterly Journal of Economics* 110 (2): 275-295.

Demski, J. and D. Sappington. 1987. Delegated expertise. *Journal of Accounting Research* 25 (1): 68-89.

Drymiotes, G. and K. Sivaramakrishnan. 2012. Board Monitoring, Consulting, and Reward Structures. *Contemporary Accounting Research* 29 (2): 453-486

Fudenberg, D. and J. Tirole. 1990. Moral hazard and renegotiation in agency contracts. *Econometrica* 58 (6): 1279-1319.

Guner, B., U. Malmendier and G. Tate. 2008. Financial expertise of directors. *Journal of Financial Economics* 88 (2): 323-354.

Hermalin, B. and M. Weisbach. 2003. Boards of directors as an endogenously determined institution: a survey of the economic literature. *Economic Policy Review* 9, 7–26.

Holmström, B. 2005. Pay without performance and the managerial power hypothesis: a comment. *Journal of Corporation Law* 30 (4): 703–715.

Indjejikian, R. and D. J. Nanda. 1999. Dynamic incentives and responsibility accounting. *Journal of Accounting and Economics* 27 (2): 177-201.

Kahneman, D., P. Slovic and A. Tversky. 1982. Judgment under uncertainty: Heuristics and biases. Cambridge University Press

Lambert, R. 1986. Executive effort and selection of risky projects. *Rand Journal of Economics* 17 (1): 77-88.

Laux, V. 2006. The ignored performance measure. *Journal of Economics & Management Strategy* 15 (3): 707-733.

Laux, V. 2008. Board independence and CEO turnover. *Journal of Accounting Research* 46 (1): 137-171.

Laux, C. and V. Laux. 2009. Board Committees, CEO Compensation, and Earnings Management. *The Accounting Review* 84 (3): 869-891.

Mace, M. 1971. Directors: Myth and Reality. Harvard Business School Press, Boston.

Melumad, N. and S. Reichelstein. 1987. Centralization vs. decentralization and the value of communication. *Journal of Accounting Research* 25 (3): 1-18.

NACD Blue Ribbon Commission Report on Strategy Development 2014. https://www.nacdonline.org/Strategy.

Schwartz-Ziv,M. and M. Weisbach. 2013. What do boards really do? Evidence from minutes of board meetings. *Journal of Financial Economics* 108(2): 349–366.

Tian, J. 2014. Board monitoring and endogenous information asymmetry. *Contemporary Accounting Research* 31(1): 136–151.

Vafeas, N. (1999). Board meeting frequency and firm performance. *Journal of Financial Economics* 53 (1): 113–142.

Wang, C., F. Xie and M. Zhu. 2015. Industry expertise of independent directors and board monitoring. *Journal of Financial and Quantitative Analysis* 50(5): 929-962.