Why Do We Observe Stockless Operations on the Internet? - Stockless Operations under Competition[*]

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Abstract

Due to the proliferation of electronic commerce and the development of Internet technologies, many firms have considered new pricing-inventory models. In this paper, we study the role of stockless (i.e., zero-inventory) operations in online retailing by considering duopoly competition in which two retailers compete to maximize profit by jointly optimizing their pricing and inventory decisions. In our model, the retailers are allowed to choose either an in-stock policy or stockless operations with a discounted price. We first present the characteristics and properties of the equilibrium. We then demonstrate that the traditional outcome of asymmetric Bertrand competition is observed under head-to-head competition. However, when the two firms choose different operational policies, with corresponding optimal pricing, they can share the market under certain conditions. Finally, we report interesting observations on the interaction between pricing and inventory decisions obtained from an extensive computational study.

1 Introduction

One of the main advantages of electronic retailing, compared to traditional "bricks and mortar" retailing, is the ability to operate the firm with reduced inventory levels. Increasingly, whether through the use of drop shipping or quick response programs, electronic retailers such as buy.com and hardwarestreet.com are choosing to operate with little or no inventory (Karpinski 1999, Vargas 2000). While there are clear cost advantages that can be obtained from such stockless (i.e., zero-inventory) operations, such a strategy can also lead to increased customer dissatisfaction and decreased demand due to longer order fulfillment times. This is especially true under competition, i.e., when consumers have multiple options for obtaining a particular good. Electronic retailing, in particular, allows consumers to quickly compare multiple sources across multiple dimensions, such as price and order fulfillment time. Thus, in this paper we consider the question of when stockless operations will be a feasible and profitable inventory strategy for a firm operating in a competitive e-retailing market.

Specifically, we study how an e-retailer in a competitive market will choose between an in-stock inventory policy, in which he seeks to eliminate all backorders, and a stockless strategy, in which he chooses to hold no inventory. We do so by considering a duopoly model in which two firms sell an identical product and compete on both price and customer waiting time, i.e., the time it takes to fulfill an order. In this model, each firm must make a sequence of decisions. First, at the strategic level, each firm must determine whether it will pursue an in-stock or stockless strategy. Clearly, the stockless policy has the benefit of drastically reducing inventory costs relative to the in-stock policy. This reduction in inventory costs, however, may result in decreased demand for the product since we assume that consumers choose among the products based on both price and waiting time. Thus, in order to compensate for the increased waiting time associated with the stockless policy, a firm may need to decrease its selling price for the product. This strategy of offering a price reduction to customers who

face a delay in order fulfillment is known as stock-out compensation (see Bhargava, Sun, and Xu (2006)).

Once the firm has made a strategic choice between the in-stock and stockless policies, it must make the operational decisions of selling price and inventory policy. We assume deterministic total market demand and the presence of economies of scale in production/ordering, and therefore we use a standard economic order quantity (EOQ) inventory model. Thus, in addition to choosing its selling price, each firm must also choose its inventory reorder interval. For a firm following an in-stock policy, orders will be placed whenever the inventory level reaches zero (thus eliminating backorders). For a firm following a stockless policy, orders will be placed whenever a certain level of backorders is achieved. This order will be just enough to satisfy the existing backorders.

Our model will reflect the reality that on-line consumers have demonstrated great diversity in their willingness to tolerate a delay in order fulfillment. To do so, we assume that the market share seen by a given firm is sensitive to both the price and waiting time offered by that firm. Specifically, customers will choose between the firms by maximizing their net utility, which is modeled as a decreasing function of both price and average customer waiting time. The sensitivity of customers to waiting time is modeled as a random variable whose distribution represents the distribution (i.e., heterogeneity) of waiting time sensitivity across the market.

We develop a game theoretic model to capture the interactions between the firms. The game consists of two stages: the strategic stage, in which each firm chooses between the stockless and in-stock policies, and the operational stage, in which each firm chooses its price and reorder interval. For the latter, we adopt the basic framework of Bertrand price competition. A typical asymmetric Bertrand competition model is a pure price game in which the firm with the lowest price ends up capturing all of the market. Our modeling approach is distinctive in that the firms make joint decisions on pricing and reorder intervals, and that

each firm's market share is determined by the dynamics of price and waiting time. For the former, i.e., the strategic stage, the game can be presented using a standard 2×2 pay-off table, with two strategies (stockless vs. in-stock) available to each firm. Thus, the system equilibrium can be easily determined once each of the four subcases has been analyzed.

The objective of this paper is to investigate the following questions: 1) Can the stockless policy be a feasible business model for electronic retailers in competitive environments? and 2) Under what conditions will each player choose in-stock or stockless retailing? To answer these questions, in §3 we formulate the problem described above. Then, in §4, we analyze the equilibrium behavior of the system for each combination of strategic choices. Specifically, in §4.1 and §4.2, we consider two forms of head-to-head competition (i.e., in-stock vs. in-stock and stockless vs. stockless). We demonstrate that, as in typical asymmetric Bertrand competition, the low-cost firm will serve the market and the high-cost firm will be out of business. In §4.3, we consider the two cases in which the firms choose different strategies (in-stock vs. stockless). We show that the equilibrium solution for these cases is such that either a) both firms will serve the market or b) only the low-cost firm serves the market. Finally, in §4.4 we analyze an alternative incumbent-entrant version of the game.

In order to study the nature of the system equilibrium and to understand when a firm will choose to operate under a stockless policy, we performed a comprehensive numerical study for the strategic level game. The results are presented in §5. We find that, in most cases, both firms will serve the market. However, the firms will avoid head-to-head competition by choosing different inventory strategies, i.e., in all cases one firm will choose a stockless policy while the other chooses an in-stock policy. In addition, we find that the stockless firm will always practice stock-out compensation, i.e., will offer a lower price compared to the in-stock firm in order to compensate for the longer order fulfillment times. Thus, the stockless policy in our model is best referred to as *stockless operations with stock-out compensation.* Finally, §6 provides conclusions and presents future research directions.

2   Literature Review

This paper extends the basic EOQ inventory model to incorporate the impact of pricing and customer waiting time on end-customer demand in a two-firm, competitive environment. We divide the relevant previous literature into two major categories. We first consider single location models which capture the impact of price and waiting time on inventory decisions (for a comprehensive review, see Bhargava, Sun, and Xu (2006)). We then consider multi-location inventory models in which the independent locations compete on the basis of price and/or waiting time.

2.1   Single Location Models

There has been a vast literature considering inventory management when demand is a function of price. See Chan, Shen, Simchi-Levi, and Swann (2004) for a detailed review. For our purposes, the most relevant previous work considers joint pricing and inventory decisions in an EOQ framework, e.g., Whitin (1955) and Kunreuther and Richard (1971).

While many authors have studied the joint inventory and pricing problem, few have considered inventory management when demand is a decreasing function of customer waiting time. Bhargava, Sun, and Xu (2006) consider stock-out compensation for a single firm model similar to the model considered in this paper. They demonstrate that a hybrid operation, i.e., an inventory strategy somewhere in between the stockless and in-stock strategies, is more efficient than either of the pure strategies. Majumder and Groenevelt (2001) investigate the optimality of periodic availability when the demand rate changes due to the waiting period; they do not consider price as a decision variable. Cheung (1998) considers a model in which the retailer can issue a price discount to motivate customers to accept delayed delivery. Numerous authors have considered the related problem of inventory management when demand is a decreasing function of customer service, e.g., Schwartz (1970), Caine and

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Plaut (1976), Ernst and Cohen (1992), and Ernst and Powell (1995).

2.2 Inventory Competition among Multiple Firms

Several authors have considered models in which multiple independent firms compete on inventories, e.g., Parlar (1988), Lippman and McCardle (1997), and Mahajan and van Ryzin (2001). However, all of these papers assume that price is fixed and exogenous, and thus the firms compete only on their inventory levels. Since the current paper considers a duopoly model in which firms compete on both price and customer waiting time, here we focus on those papers in which multiple firms compete on price and/or waiting time (or, alternatively, customer service).

We consider competition between retailers assuming that each uses an EOQ inventory policy. There have been relatively few papers that consider inventory competition in an EOQ setting, the most relevant being Bernstein and Federgruen (2003). However, the authors assume no backordering and thus waiting time is not an issue and the retailers compete only on price. Similarly, Cachon and Harker (2002) consider an EOQ game in which the firms compete only on price.

Another stream of research considers firms which compete on waiting or delivery time. These models generally assume a fixed, exogenously specified price. The most relevant is the work by Li (1992), who considers an *oligopoly racing market*. In this model, customers will choose to purchase from the firm with the earliest delivery time. Price is fixed and not a basis for competition. Sensitivity to delivery time is captured by assuming that the customer's utility decreases linearly in the waiting time. The author demonstrates that firms are more likely to choose a make-to-stock strategy, as opposed to make-to-order, under competition than under monopoly. Gans (2002) considers a model in which customers choose among competing suppliers on the basis of "quality", which may include expected waiting time. Hall and Porteus (2000) consider the impact of stock-outs on customer demand in a multi-

firm environment. Both papers assume, however, that price is exogenous, and hence the firms compete only on quality / stock-outs, but not on price.

A few papers have attempted to capture competition on both price and service level / delivery speed. Li and Lee (1994) consider a duopoly model in which firms compete on price, quality and delivery speed. However, an equilibrium analysis is performed only for the case in which firms compete solely on price, with quality and speed taken as fixed. Dana and Petruzzi (2001) present an inventory model which attempts to capture the impact of both price and stock-out potential on customer demand. However, they consider only a single firm, with competition captured through the introduction of an exogenous outside option for customers. Cachon and Harker (2002) consider competition between two firms with economies of scale in their inventory decisions. The firms compete on both price and operational performance. As a special case, the authors consider a queueing game in which operational performance is defined as expected waiting time. Bernstein and Federgruen (2004) consider an N period model with stochastic demand in which retailers compete on both price and fill rate.

3   Model

In this section, we present our models of the firm and of consumer choice for the problem described above.

3.1   Model of the Firm

We consider duopoly competition in which two on-line retailers compete to attract customers. Recent developments in technology enable a firm to monitor an on-line competitor's price and to react instantaneously to a competitor's price change. Thus, we will use a Bertrand competition model in which firms make decisions on price rather than quantity.

Two firms, $i \in \{l, h\}$, sell an identical product. The firms are homogeneous in the sense that they incur an identical fixed ordering cost (A) and use the economic order quantity model to determine their inventory policy. However, the firms are heterogeneous in the sense that they have different unit purchasing costs (i.e., $c_h > c_l$). Thus, we refer to firm h as the high-cost firm and firm l as the low-cost firm. Inventory holding costs are equal to a fixed fraction of the unit purchasing costs (i.e., $\gamma \cdot c_i$ where $0 < \gamma < 1$). Note that this implies that the two firms incur different inventory holding costs. As in the standard EOQ model, we assume that the constant market demand rate for the product is d. We use d_i to represent the demand rate seen by firm i, $i \in \{l, h\}$, where $d_i \leq d$. Finally, we assume there are no order lead times. However, as in the standard EOQ model, the model can easily be extended to incorporate fixed lead times.

Each firm makes both strategic and operational decisions. At the strategic level, a firm must choose between a stockless policy, in which inventory is never kept on site and all demands are backordered, and an in-stock policy, in which the firm always holds positive inventory and no demands are backordered[1]. The latter case is equivalent to the standard EOQ model in which backorders are not allowed (e.g., Hadley and Whitin (1963), Zipkin (2000)). Thus, if firm i follows an in-stock policy, it will order Q_i units every T_i time, where $Q_i = T_i \, d_i$. In the former case, i.e., the stockless case, firm i will accumulate backorders and place an order for Q_i units only when the backorders reach some fixed level. When the order arrives, all backorders will be filled and the firm will be left with zero inventory. As in the usual EOQ model, the time between orders, T_i, will be fixed and will satisfy $Q_i = T_i \, d_i$.

Given the strategic choice, i.e., stockless or in-stock strategy, each firm must make two key operational decisions: selling price and reorder interval. Firm i will choose its price (p_i) and reorder interval (T_i) in order to maximize its profit per unit time (which we will refer to

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[1]In order to focus our analysis on the feasibility of pure stockless operations, we do not consider the in-between case in which, in each reorder cycle, the firm spends some time with positive inventory followed by sometime with backorders.

as simply the profit), π_i, $i \in \{l, h\}$. For a firm that chooses an in-stock policy, we can write the profit, π_i^I, as a function of the pricing and inventory decisions at both firms, p_i, p_j, T_i, T_j, $i, j \in \{l, h\}$, as follows:

$$\pi_i^I(p_i, p_j, T_i, T_j) = (p_i - c_i)\, d_i - \frac{A}{T_i} - \left(\frac{\gamma c_i\, d_i}{2}\right) T_i. \tag{1}$$

For a firm that chooses a stockless policy, we can write the profit, π_i^S, as a function of the pricing and inventory decisions at both firms, p_i, p_j, T_i, T_j, $i, j \in \{l, h\}$, as follows:

$$\pi_i^S(p_i, p_j, T_i, T_j) = (p_i - c_i)\, d_i - \frac{A}{T_i}. \tag{2}$$

Notice that a firm using a stockless policy will incur no holding costs.

3.2 Consumer Choice Model

Given two firms offering identical products, customers will choose between the firms on the basis of price and the average waiting time for an order to be filled. When a customer places an order, he will not be told the exact waiting time, but instead will be quoted the average waiting time for all orders. For a firm choosing an in-stock policy, the product is always available in-stock and thus the average waiting time is zero. For a firm choosing a stockless policy, orders are placed every T_i time, so the average waiting time is $\frac{T_i}{2}$.[2]

We use a consumer choice model in which customers seek to maximize their net utility. Customers earn some positive utility from purchasing the product, but also experience a decrease in utility, i.e., a disutility, when forced to wait before obtaining the purchased product. We assume that all customers receive an identical utility, $v > 0$, from the purchase of the

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[2]We assume throughout the paper that the actually delivery time for the product, i.e., the time between when the product becomes available to the firm and when that product arrives at the customer, is the same for both firms and is negligible. Hence, we do not include that delivery time in our model.

product. We assume that the disutility associated with waiting consists of two components, one fixed and the other time-based. In addition, we assume heterogeneity in this disutility across the market. Specifically, if a customer expects a wait of $t > 0$ before acquiring a purchased product, then his disutility will be equal to $b\,(t + \alpha)$, where $b\,t$ represents the time-based sensitivity to waiting and $b\,\alpha$ represents the fixed component of the disutility (e.g., due to the hassle or frustration caused by a backorder). In other words, a customer who must wait before obtaining the product will incur a one-time fixed disutility $(b\,\alpha)$ regardless of the length of the wait, plus an additional disutility (b) for each unit of time he expects to wait. The parameter b, which we sometimes refer to as the waiting sensitivity, is modeled as a random variable in order to capture the heterogeneity of the market. We let G denote the cumulative distribution function (cdf) of b. In order to ensure tractability, for all of the analytical results presented in this paper, we will assume that G is Uniform[0,1]. Analogously, without loss of generality, we normalize the scale and set $v = 1$. While we consider a linear disutility function throughout this paper, in the appendix we discuss how our analysis and results might change under more general disutility functions.

Customers will choose to purchase the product from the firm that offers the highest net utility, as long as that net utility is positive. Thus, if firm i offers price p_i and average waiting time t_i, while firm j offers price p_j and average waiting time t_j, $i, j \in \{l, h\}$, then a customer with waiting time sensitivity b will choose to purchase from firm i if

$$v - p_i - b\,[t_i + \alpha I(t_i > 0)] > v - p_j - b\,[t_j + \alpha I(t_j > 0)], \tag{3}$$

and

$$v - p_i - b\,[t_i + \alpha I(t_i > 0)] \geq 0, \tag{4}$$

where $I(\cdot)$ is the indicator function. Thus, if we let θ_i denote the fraction of the market

captured by firm i, for $i \in \{l, h\}$, we can write

$$\theta_i = Pr\{v - p_i - b\,[t_i + \alpha I(t_i > 0)] \geq max\,(0, v - p_j - b\,[t_j + \alpha I(t_j > 0)])\}. \qquad (5)$$

The demand rate seen by firm i is then $d_i = d\,\theta_i$. Finally, we assume that $p_i \leq v$ for $i \in \{l, h\}$, i.e., no firm will set a price higher than the utility earned by the customer from the purchase of the product with no waiting time.

If firm i chooses a stockless policy, the average waiting time for all customers is $t_i = \frac{T_i}{2}$. Thus, the net utility for a customer who purchases from firm i will be $v - p_i - b\,(\frac{T_i}{2} + \alpha)$. On the other hand, if firm i chooses an in-stock policy, the average waiting time for all customers is $t_i = 0$. In this case, the net utility for a customer who purchases from firm i will be $v - p_i$. Thus, the demand rate, d_i, seen by firm i will not be a function of T_i, and firm i can choose T_i to minimize its EOQ inventory costs. Therefore, if firm i uses an in-stock policy, the optimal reorder interval will be $T_i^* = \sqrt{\frac{2A}{d_i \gamma c_i}}$, and Eq. (1) can be rewritten as:

$$\pi_i^I(p_i, p_j, T_i, T_j) = (p_i - c_i)\,d_i - \sqrt{2A\gamma c_i d_i}. \qquad (6)$$

Table 1 summarizes the notation used in this paper.

4   Analysis of Various Competitive Strategies

The game considered in this paper consists of decisions at two levels: strategic and operational. At the strategic level, each firm chooses between the in-stock and stockless policies. This decision is taken as fixed when considering the operational level, in which each firm chooses its price and reorder interval. Thus, at the operational level, there are four possible cases to consider:

Table 1: Notation

i	index for firm, $i \in \{l, h\}$
A	fixed ordering cost per order
γ	holding cost rate, $0 < \gamma < 1$
c_i	unit purchasing cost for firm i, $c_h > c_l$
v	customer reservation for the product
b	disutility of waiting / waiting sensitivity per unit time
$b\,\alpha$	fixed component of the disutility of stock-out
α	degree of fixed disutility of stock-out
$G(\cdot)$	cdf of customer waiting sensitivity, b, with pdf $g(\cdot)$
d	market demand rate per unit time
d_i	firm i's demand rate
θ_i	fraction of the market captured by firm i
p_i	firm i's price for the product
T_i	reorder interval for firm i
t_i	average waiting time offered by firm i
π_i	profit per unit time earned by firm i

- CASE 1: Both firms use an in-stock policy.

- CASE 2: Both firms use a stockless policy.

- CASE 3: The low-cost firm uses an in-stock policy while the high cost firm uses a stockless policy.

- CASE 4: The high-cost firm uses an in-stock policy while the low cost firm uses a stockless policy.

In this section we analyze each of these four cases, discussing what is known about the equilibrium behavior in each case. The results of these four cases can then be used to analyze the strategic game by forming a payoff table showing the expected payoffs for each firm for each possible combination of strategic choices. Given this payoff table, it is easy to compute the equilibrium strategic choices for the firms. We refer to this equilibrium as the *system equilibrium*. In §5 we will present a numerical analysis to determine, for a variety of model parameters, which of these four cases is the system equilibrium. Finally, we will conclude this section with an analysis of an alternative version of the game in which there exists an

incumbent firm and a potential new entrant to the market.

4.1 CASE 1: In-Stock vs. In-Stock

We first consider the case in which both firms choose an in-stock policy. Since both firms provide the product with no wait, customers will choose between the firms solely on the basis of price. Therefore, we have $d_i = d$ if $p_i < p_j$ and $d_i = \frac{d}{2}$ if $p_i = p_j$, for $i, j \in \{l, h\}$. Given d_i, the profit for firm i is given by Eq. (6), $i \in \{l, h\}$.

   The following proposition characterizes the equilibrium behavior of this system.

Proposition 1 *When the two in-stock firms have different cost structures (i.e., $c_l < c_h$), there exists a unique equilibrium in which the low-cost firm (firm l) dominates the market and earns positive profit, while the high-cost firm (firm h) exits the market. The equilibrium is characterized by*

$$p_l^E = c_h + \sqrt{\frac{2A \cdot \gamma \cdot c_h}{d}} \qquad and \qquad \pi_l^E = (\sqrt{c_h} - \sqrt{c_l}) \left[(\sqrt{c_h} + \sqrt{c_l}) d + \sqrt{2A \cdot \gamma \cdot d} \right].$$

   This result is similar to that of a typical asymmetric Bertrand competition, except that an ϵ-policy (see, e.g., Tirole (1988)) is not required for our model setting. This difference is due to the inclusion of inventory costs in our model.

4.2 CASE 2: Stockless vs. Stockless

We next consider the case in which both firms choose a stockless policy. In this case, a customer will make her purchasing decision considering the prices and average waiting times of both firms. Specifically, a customer with waiting sensitivity b will choose firm l if and only if

$$v - p_l - \left(\frac{T_l}{2} + \alpha \right) b \geq max \left(0, v - p_h - \left(\frac{T_h}{2} + \alpha \right) b \right). \tag{7}$$

From Eq. (7), it is easy to see that $\{p_l < p_h, T_l > T_h\}$ and $\{p_l > p_h, T_l < T_h\}$ may result in market sharing, i.e., may have $\theta_i > 0$ for $i \in \{l, h\}$, while $\{p_l < p_h, T_l < T_h\}$ and $\{p_l > p_h, T_l > T_h\}$ imply that all customers who choose to purchase the product will choose firm l (i.e., $\theta_h = 0$) and firm h (i.e., $\theta_l = 0$), respectively.

Next, let $\tilde{b}_i$ denote the time-based sensitivity to waiting at which a customer would be indifferent between purchasing the product from firm i and not purchasing. Similarly, let $\tilde{b}_s$ denote the time-based sensitivity to waiting at which a customer would be indifferent between purchasing the product from firm l and firm h. Solving $v - p_i - \left(\frac{T_i}{2} + \alpha \right) b = 0$, for $i \in \{l, h\}$, and $v - p_l - \left(\frac{T_l}{2} + \alpha \right) b = v - p_h - \left(\frac{T_h}{2} + \alpha \right) b$, we find

$$\tilde{b}_i = \frac{2(v - p_i)}{T_i + 2\alpha} \qquad \text{and} \qquad \tilde{b}_s = \frac{2(p_l - p_h)}{T_h - T_l}.$$

We can now characterize the fraction of the market that will be seen by each firm:

Lemma 1 *If $p_i > p_j$, $T_i < T_j$ and if a solution exists in which $\theta_i > 0$ and $\theta_j > 0$, for $i, j \in \{l, h\}$, then*

1. $\tilde{b}_s < \tilde{b}_j < \tilde{b}_i$,

2. $\theta_j = \tilde{b}_s = \frac{2(p_l - p_h)}{T_h - T_l}$, *and*

3. $\theta_i = \min[1, \tilde{b}_i] - \tilde{b}_s$.

The profit functions of the two firms can then be expressed as

$$\pi_l^S(p_l, p_h, T_l, T_h) = (p_l - c_l) \, d \, \theta_l - \frac{A}{T_l},$$
$$\pi_h^S(p_l, p_h, T_l, T_h) = (p_h - c_h) \, d \, \theta_h - \frac{A}{T_h}.$$

Unfortunately, for the case of stockless vs. stockless competition, an analytical analysis identifying all equilibria is intractable due to the complexity of the profit functions

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and the interactions among the four decision variables. Thus, to investigate the equilibria numerically, we first set up the profit maximization problem for the individual firm as a nonlinear optimization problem using GAMS. The GAMS program adopts a systematical and exhaustive search algorithm. We then use a standard iterative approach to determine the equilibrium:

1. Given firm j's decision variables, i.e., p_j, T_j, firm i determines its optimal decision variables, i.e., $p_i^*(p_j, T_j)$, $T_i^*(p_j, T_j)$. In doing so, two options are considered:

   - Firm i may choose to serve those customers who are more sensitive to waiting time by choosing $p_i > p_j$, $T_i < T_j$.

   - Firm i may choose to serve those customers who are less sensitive to waiting time by choosing $p_i < p_j$, $T_i > T_j$.

2. Given firm i's optimal decision variables, p_i^*, T_i^*, firm j determines its optimal decision variables in a similar manner.

3. The program terminates when a given firm cannot deviate (or improve its profit) from one iteration to the next.

For each set of problem parameters considered, we implemented this iterative approach for a wide range of initial values for p_j and T_j. Specifically, we considered 5 initial prices (i.e., $c_h + \frac{1-c_h}{6} \cdot n$, where $n = 1 \ldots 5$) and 5 initial reorder intervals (i.e., $T_h^* \cdot n$, where $n = 1 \ldots 5$ and T_h^* is the optimal reorder interval for a monopoly firm following a stockless policy). In sum, the GAMS program considers 25 initial starting points for each set of problem parameters.

After conducting a comprehensive numerical study in which we varied numerous model parameters (refer to §5 for further details), we find that there exists only one type of equilibrium for the case of stockless vs. stockless competition. Specifically, in all cases we find that firm l will serve the market, while firm h will exit the market. This result suggests that

the main result of the typical asymmetric Bertrand competition model, in which the firms compete only on price, also applies to the case in which two stockless firms compete on both price and customer waiting time.

4.3 CASE 3: Low-cost Firm with In-stock Policy vs. High-cost Firm with Stockless Policy

We next consider the case in which the two firms choose different inventory policies. We will start by considering the case in which firm l chooses an in-stock policy and firm h chooses a stockless policy. Analogous results, although not presented here, will apply for the reverse case (CASE 4). All of the analytical results for CASE 4 can be derived by interchanging the key subscripts (i.e., l and h) in the propositions in this section.

We first note that, for any set of prices and reorder intervals, there are three possible outcomes: i) all customers prefer the high-cost stockless firm; ii) all customers prefer the low-cost in-stock firm; or iii) the customers' preferences are split between the two firms. We will investigate the equilibrium behavior for each of these cases, starting with the case in which the firms split the market.

4.3.1 High-cost and Low-cost Firm Split the Market

In this section we consider the case in which each firm captures some portion of the market. We are interested in determining whether or not there exists an equilibrium in which both firms will make positive profit (a necessary requirement for both firms to participate in the market). If such an equilibrium exists, we then would like to determine the equilibrium values of p_h, T_h and p_l, which we denote by p_h^E, T_h^E and p_l^E, respectively. Recall that the in-stock firm, i.e., firm l, will always choose $T_l^E = \sqrt{\frac{2A}{d_l \gamma c_l}}$.

Since $p_i \leq v$ for $i \in \{l, h\}$, a customer with waiting time sensitivity b will choose to

purchase the product from firm h if $v - p_h - \left(\frac{T_h}{2} + \alpha\right) b > v - p_l$. Thus, the fraction of the market that will purchase from firm h, θ_h, can be written as

$$\theta_h = Pr\left\{b < \frac{2(p_l - p_h)}{T_h + 2\alpha}\right\} = \max\left[0, \min\left[1, \frac{2(p_l - p_h)}{T_h + 2\alpha}\right]\right]. \tag{8}$$

Since all customers will purchase the product (because $p_l < v$), we have $\theta_l = 1 - \theta_h$.

We want to consider the case in which the firms split the market, i.e., $0 < \theta_i < 1$ for $i \in \{l, h\}$. From Eq. (8), this will hold only under the following condition: $p_h < p_l < p_h + \alpha + \frac{T_h}{2}$. Therefore, we will start our analysis by considering the optimal policy for the high-cost firm given p_l, where p_l satisfies $p_h < p_l < p_h + \alpha + \frac{T_h}{2}$.

Let $p_h^*(p_l)$ and $T_h^*(p_l)$ denote the optimal price and reorder interval for the high-cost firm given the price, p_l, offered by the low-cost firm. Similarly, we define $\theta_h^*(p_l)$ and $\pi_h^{S*}(p_l)$ to be the optimal market share and profit for the high-cost firm given p_l, respectively. The following proposition presents our key results on the behavior of these functions.

Proposition 2 *If* $c_h + \sqrt{\frac{2A}{d}} < p_l < c_h + \sqrt{\frac{2A}{d}} + 2\alpha$, *then*

$$i) \; p_h^*(p_l) = \frac{1}{2}(p_l + c_h), \qquad ii) \; T_h^*(p_l) = \frac{-4\alpha\sqrt{A}}{2\sqrt{A} - \sqrt{2d}\sqrt{(c_h - p_l)^2}} > 0,$$

$$iii) \; 0 < \theta_h^*(p_l) < 1, \qquad iv) \; \pi_h^{S*}(p_l) > 0, \qquad and \qquad v) \; \frac{d\pi_h^{S*}}{dp_l} > 0.$$

We are interested in the profit earned by the low-cost firm given that the high-cost firm uses the optimal parameters $p_h^*(p_l)$ and $T_h^*(p_l)$, as defined above. We use $\pi_l^I(p_l|p_h^*, T_h^*)$ to denote this profit function. We next demonstrate that this profit will be positive when $c_h + \sqrt{\frac{2A}{d}} < p_l < c_h + \sqrt{\frac{2A}{d}} + 2\alpha$.

Proposition 3 *For* p_l *satisfying* $c_h + \sqrt{\frac{2A}{d}} < p_l < c_h + \sqrt{\frac{2A}{d}} + 2\alpha$, $\pi_l^I(p_l|p_h^*, T_h^*) > 0$.

Together, Propositions 2 and 3 present conditions under which, acting optimally, the

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high-cost firm will choose a price and reorder interval such that the two firms will split the market ($0 < \theta_h^*(p_l) < 1$) and both firms will earn positive profits ($\pi_h^{S*}(p_l) > 0$ and $\pi_l^I(p_l | p_h^*, T_h^*) > 0$).

Finally, the following proposition demonstrates the conditions under which there will be a unique equilibrium solution for this case.

Proposition 4 *If $c_h + \sqrt{\frac{2A}{d}} < p_l < c_h + \sqrt{\frac{2A}{d}} + 2\alpha$, when the low-cost firm uses an in-stock policy and the high-cost firm uses a stockless policy, there exists a unique equilibrium.*

Further context for the results of Propositions 2, 3, and 4 is provided in the next section.

4.3.2 Only One Firm Serves the Market

In the previous section, we considered the case in which the low-cost in-stock firm and the high-cost stockless firm split the market. We next consider the two extreme cases: (1) only the high-cost stockless firm serves the market and (2) only the low-cost in-stock firm serves the market.

The first extreme case (all customers prefer the high-cost stockless firm) implies $\theta_h = 1$, which is equivalent to the condition $p_l \geq p_h + \frac{T_h}{2} + \alpha$. For this case, we can show that an equilibrium does not exist.

Proposition 5 *There is no equilibrium that satisfies i) the low-cost firm chooses an in-stock policy, ii) the high-cost firm chooses a stockless policy, and iii) $p_l \geq p_h + \frac{T_h}{2} + \alpha$.*

The second extreme case (all customers prefer the low-cost in-stock firm) implies $\theta_h = 0$, which (since $T_h > 0$) directly implies that $p_l \leq p_h$. In the following proposition, we characterize the equilibrium solution for this case.

Proposition 6 *When i) the low-cost firm chooses an in-stock policy, ii) the high-cost firm chooses a stockless policy, and iii) $p_l \leq p_h$, there exists an equilibrium characterized by*

$$
\begin{cases}
p_l^E = c_h + \sqrt{\frac{2A}{d}} & \text{if } c_h + \sqrt{\frac{2A}{d}} < v, \\
p_l^E = v & \text{otherwise}, \\
T_l^E = \sqrt{\frac{2A}{d_l \gamma c_l}}.
\end{cases}
$$

To summarize, Propositions 2, 3, and 4 characterize an equilibrium in which two firms will serve some fraction of the market and make positive profit, while Propositions 5 and 6 demonstrate that an equilibrium might occur at the extreme point in which only the low-cost firm serves the market, but not at the other extreme in which only the high-cost firm serves the market. The type of equilibrium is determined by the low-cost firm because the price given in Proposition 6 does not allow the firm high-cost firm to take any fraction of the market with positive profit. Specifically, we can determine the nature of the equilibrium for CASE 3 as follows: Using Proposition 2, we can write the profit for the low cost firm as a function of p_l, given that p_h and T_h are chosen optimally. We can then find the value of p_l that maximizes this profit. If this value satisfies $c_h + \sqrt{\frac{2A}{d}} < p_l < c_h + \sqrt{\frac{2A}{d}} + 2\alpha$, then both firms stay in the market; otherwise, only the low cost firm serves the market. In other words, for any set of problem parameters, we can determine whether the equilibrium for CASE 3 will satisfy the conditions of Propositions 2 - 4 or of Proposition 6.

4.4 An Incumbent-Entrant Game

So far in our analysis, we have assumed that each firm may choose either the in-stock policy or the stockless policy. In this subsection, we consider an important alternative formulation in which there exists in the market an incumbent firm, assumed to be the low-cost firm (perhaps due to economies of scale or learning curve effects) which follows a traditional in-

stock policy. A new high-cost firm is considering entering the market and must make several decisions: whether or not to enter the market, whether to use an in-stock or stockless policy, and how to set the price and reorder interval.

We can easily use the analytical results obtained in this section to characterize the equilibrium solution for this problem. From Proposition 1, we know that if the high-cost entrant attempts to enter the market using an in-stock policy, he will be forced from the market by the incumbent low-cost firm. Thus, any new entrant will choose a stockless policy and the results of §4.3 (CASE 3) will apply. Finally, using the approach outlined above (in the final paragraph of §4.3.2), we can easily determine the nature of the resulting equilibrium, i.e., we can determine whether or not the entrant will remain in the market.

5   Analysis of Firms' Strategic Choices

In this section, we conduct a series of numerical analyses in order to better understand how the two firms make their strategic choices, i.e., choose between the in-stock and stockless policies, and how a variety of experimental parameters affect the profits of both firms.

The values of the experimental parameters used in the analysis are summarized as follows: 1) fixed ordering cost per order ($A \in \{1, 3, 5\}$), 2) inventory holding cost rate ($\gamma \in \{0.1, 0.2, 0.3\}$), 3) unit purchasing cost ($c_i \in \{0.2, 0.3, 0.4\}$), 4) demand rate ($d \in \{100, 500, 1000\}$), and 5) degree of fixed disutility of stock-out ($\alpha \in \{0.01, 0.2, 0.4, 0.6, 0.8\}$). This parameter setting results in 405 ($= 3 \times 3 \times 3 \times 3 \times 5$) unique experimental problems.

5.1   Analysis of System Equilibrium

In this section, we present the results of our numerical study on the nature of the system equilibrium, as defined in §4. As noted in §4, we find the system equilibrium by solving a strategic game with a 2×2 payoff table.

We find that 303 out of 405 problems (74.8%) have a unique Nash equilibrium. Among these, only one firm serves the market (refer to Proposition 6) in 87 instances (28.7%), while the two firms split the market in the remaining 72.3% (216 out of 303) of these problems. For the problems in which a single firm serves the market, the surviving firm is the low-cost firm and the strategic choice is an in-stock policy. For the problems in which the two firms split the market, we know from the analysis in Sections 4.1 and 4.2 that an equilibrium does not exist in which both firms serve the market and use the same strategy, i.e., stockless vs. stockless or in-stock vs. in-stock cannot be an equilibrium. Thus, in all of the problems with a single equilibrium in which both firms serve the market, we find that CASE 3 is the equilibrium, i.e., the low-cost firm chooses an in-stock policy and the high-cost firm chooses a stockless policy. Intuitively, the high-cost firm chooses a stockless policy in an attempt to reduce its costs to a competitive level.

In 102 out of 405 problems (25.2%), two Nash equilibria exist; in each, one firm chooses an in-stock policy and the other firm chooses a stockless policy. Two equilibria will exist when the profit earned by the low-cost firm in CASE 4 (low-cost firm is stockless, high-cost firm is in-stock) are greater than those earned in CASE 1 (low-cost firm is in-stock, high-cost firm is in-stock, and only the low cost firm survives in the market), i.e., when the benefits of stockless operations (relative to using an in-stock policy) for the low-cost firm outweigh the benefits of having a monopoly. In this case, in addition to having CASE 3 (low-cost firm is in-stock, high-cost firm is stockless) as an equilibrium, CASE 4 (low-cost firm is stockless, high-cost firm is in-stock) becomes an equilibrium. Intuitively, the low cost firm will prefer CASE 4 to CASE 1 when the holding cost rate is high and the ordering cost is small (these conditions make the stockless policy more attractive relative to the in-stock policy) and when the demand rate is large (this condition enables a firm that splits the market to achieve some of the economies of scale that would be enjoyed by a monopoly). We also find that the low cost firm will prefer CASE 4 to CASE 1 when the degree of fixed disutility (α) is moderate.

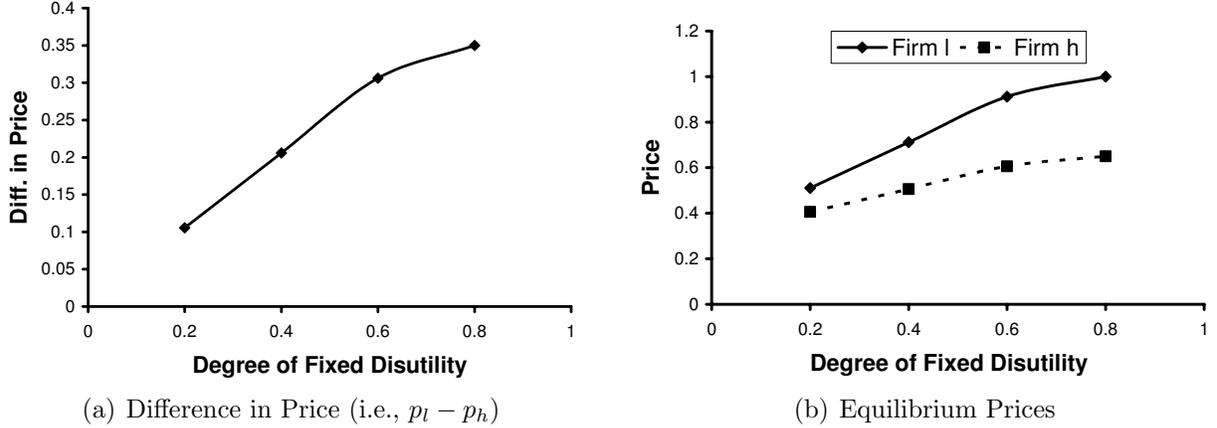(a) Difference in Price (i.e., $p_l - p_h$)   (b) Equilibrium Prices

Figure 1: Effect of Degree of Fixed Disutility ($A = 3$, $\gamma = 0.2$, $c_l = 0.2$, $c_h = 0.3$, $d = 500$)

This behavior will be explained in detail later in this section. Thus, in summary, problems with two equilibria are more likely when the holding cost rate is high, the ordering cost is small, demand is large, and the degree of fixed disutility is moderate.

In the remainder of this section, we will focus our analysis on the case in which two firms remain in the market, with the low-cost firm choosing an in-stock policy and the high-cost firm choosing a stockless policy. As noted above, this is the most common outcome of our model.

5.2   Favorable Conditions for Stockless Operations

We find that the firm that chooses an in-stock policy primarily dominates in the market. In other words, at the Nash equilibria, the firm that chooses an in-stock policy maintains the higher price and captures the greater market share. Thus, a firm which operates under a stockless policy must practice stockout compensation, i.e., must offer a reduced price relative to the competing in-stock firm. This result is demonstrated in Figure 1, which shows that the price charged by the low-cost, in-stock firm is always higher than the price charged by the high-cost, stockless firm. Thus, the stockless policy is in our model is best referred to as *stockless operations with stock-out compensation* since stockless operations by itself, i.e.,

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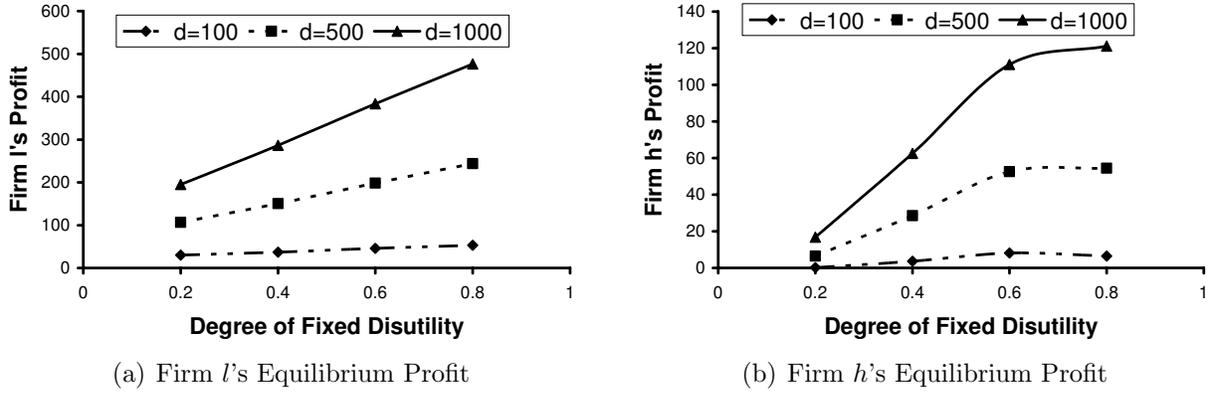(a) Firm l's Equilibrium Profit   (b) Firm h's Equilibrium Profit

Figure 2: Equilibrium Profits ($A = 3$, $\gamma = 0.2$, $c_l = 0.2$, $c_h = 0.3$)

without stock-out compensation, is not a feasible strategy.

This observation leads to a key practical question: Under what environmental conditions can a firm's stockless policy be relatively profitable? From our comprehensive numerical study, we found that a stockless policy can be favorable when there exist a high demand rate, high inventory holding cost, low fixed ordering cost, medium degree of fixed disutility, and a relatively small difference in purchasing costs between the two firms. In the following discussion, we attempt to explain why these conditions are necessary for stockless operations to be profitable by evaluating the impact of the key experimental parameters on both firm's optimal prices and profits.

Figure 1(b) illustrates the main effect of the degree of fixed disutility (α) on the optimal prices. As the disutility increases, both firms' optimal prices increase. In other words, as α increases, customers prefer to purchase from the low-cost firm with the in-stock policy. Under these circumstances, the low-cost firm would increase its price to maximize its profits, which would lead the high-cost firm to increase its price, mimicking the low-cost firm's high price policy.

The interaction between the demand rate (d) and the degree of fixed disutility (α) can be seen in Figure 2. As seen in the figure, the combination of a high rate of demand and a high degree of fixed disutility creates favorable conditions for the low-cost firm. However,
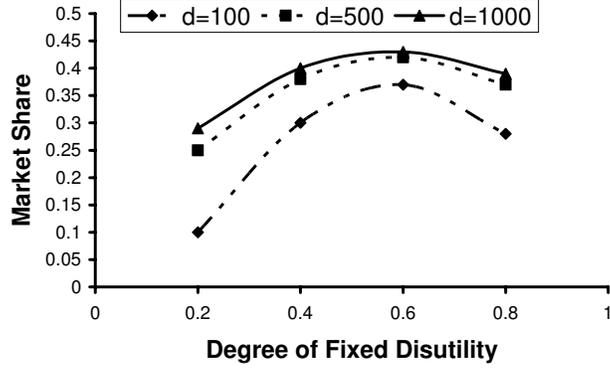
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Figure 3: Equilibrium Market Share of Firm h ($A = 3$, $\gamma = 0.2$, $c_l = 0.2$, $c_h = 0.3$)

the high-cost firm's profit behavior demonstrates a slightly different pattern - increasing at a decreasing rate with respect to degree of fixed disutility (α). In particular, when demand is 100, the high-cost firm's profit is actually lower under $\alpha = 0.8$ than under $\alpha = 0.6$, as shown in Figure 2(b).

The behavior of the high-cost firm's profit can be explained by considering the relationship between the degree of fixed disutility and the high-cost firm's market share. As illustrated in Figure 3, the high-cost firm's market share (θ) is unimodal with respect to the degree of fixed disutility (within the environment defined by the experimental parameters). This implies that there exists a range for the degree of fixed disutility in which the high-cost firm's stockless policy can be more favorable. This result can be explained as follows: When the degree of fixed disutility is small, stock-out has little effect on a customer's choice between the two firms. Thus, the low-cost firm can use a low price to capture significant market share. When the degree of fixed disutility is high, the stockless firm must choose a low price in order to compensate customers for this large disutility. However, because of its cost structure, the high-cost firm is unable to dramatically decrease its price in order to gain significant market share.

The effects of the holding cost rate (γ) and fixed ordering cost (A) on both firms' profit are not surprising. The low-cost and high-cost firm's profits behave in opposing manners

with respect to inventory holding cost and ordering cost. Since the high-cost firm's total inventory cost is just the sum of the fixed ordering costs, its profit decreases as the fixed ordering cost increases. Thus, a high ordering cost is in favor of the low-cost firm with in-stock policy. In contrast, as the holding cost rate increases, the low-cost firm's profit declines and the high-cost firm's profit increases. Because the high-cost firm does not hold inventory, the increased holding cost rate only affects the low-cost firm's profit. Therefore, a higher rate of inventory holding cost favors the high-cost firm.

6   Concluding Remarks

In this paper we have considered a duopoly model in which two electronic retailers compete to capture a larger market share using alternative pricing-inventory strategies (in-stock vs. stockless). The objective of our analysis was to investigate whether the stockless policy can be a viable online business model in the presence of competition. Based on the analytical models developed in this paper, along with the corresponding computational study, we found that, unlike the typical outcome of asymmetric Bertrand price competition in which one firm is always driven out of competition, two retailers can coexist in the market by choosing a policy (for example, a stockless policy) which differs from their competitor's policy (for example, an in-stock policy).

There are several significant contributions made in this paper. First, we show that the stockless policy is a legitimate option for a firm which intends to avoid head-to-head competition with the existing dominant player in the traditional market. In other words, it is possible for a firm to establish a profitable business by creating virtual stores to attract customers who may be less sensitive to waiting time for product delivery. Second, we demonstrate that a stockless firm must offer a reduced price relative to the competing in-stock firm. Thus, in practice, the stockless policy becomes stockless operations with stock-out compen-

sation. Third, we identify a set of conditions that may help stockless policies become more profitable. These conditions include a high demand rate, a medium degree of fixed disutility, a low ordering cost, a high inventory holding cost rate, and a purchasing cost comparable to that of other retailers.

Finally, there are numerous ways in which this research can be extended and improved upon. As discussed above, there are many reasons why a firm may choose to use a zero (or very low) inventory policy. One important reason not captured in this paper is demand uncertainty. It is well known that high demand uncertainty can lead to high inventory requirements and significant inventory costs. Thus, a stockless operation, coupled with stock-out compensation, may prove to be a profitable alternative for firms facing high demand uncertainty. Further research in this direction would be an interesting extension of our work. In addition, in this paper we assume that a firm must choose a single mode of competition (either in-stock or stockless policy) since our objective was to provide clear comparisons between the two policies under investigation. We have also used the Uniform distribution to represent the customers' heterogeneity in their sensitivity to waiting. Future research will improve upon the present study by allowing the possibility that a firm can choose a policy in between the extreme in-stock and stockless policies and by considering more general probability distributions to represent the customers' sensitivity to waiting.

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A   Discussion of Disutility of Waiting

In this paper, we have considered a linear disutility of waiting, i.e., the disutility increases linearly in t, the expected time to have an order filled. The idea that customers are delay-sensitive and that the disutility of waiting is linear has been expressed by several other authors. For example, Van Ackere and Ninios (1993), Hassin (1986), Li (1992), Li and Lee

(1994) and Stidham (1970) model the disutility of waiting in a similar, but more limited, manner compared to our model. Specifically, they assume that all customers have the same unit waiting cost, b, per unit time (i.e., customers are homogeneous and the disutility is linear). In Lederer and Li (1997), the disutility of waiting is heterogeneous (as in our model). However, a linear form is still assumed.

While most of the existing literature has assumed a linear form, it is quite possible that, for some consumers, the disutility would follow a more complex form. For example, one can imagine situations in which the disutility would be increasing and convex, reflecting the fact that consumers will tolerate small waits, but become increasingly distressed by long waits. Unfortunately, we have found that for even quite simple non-linear convex disutility functions, our analytical model becomes intractable. Here we briefly demonstrate the main difficulty in the analysis. Let $g(t)$ be a general function of t (the expected waiting time) so that the disutility of waiting for a customer with waiting sensitivity b is $b(g(t)+\alpha)$. Following the procedures described in the paper, we have

$$\theta_h = Pr\left\{b < \frac{p_l - p_h}{g(t)+\alpha}\right\} = \max\left[0, \min\left[1, \frac{p_l - p_h}{g(t)+\alpha}\right]\right].$$

To analyze CASE 3 (low-cost firm with in-stock policy vs. high-cost firm with stockless policy), we follow the procedures described in the proof of Proposition 2. Specifically, it is easy to see that the condition $\frac{\partial \pi_h^S}{\partial p_h} = 0$ is independent of T_h. Thus, the optimal p_h can be set independently of T_h. Solving $\frac{\partial \pi_h^S}{\partial p_h} = 0$, we obtain $p_h^*(p_l) = \frac{1}{2}(p_l + c_h)$.

We next consider the first order conditions for T_h. Using the fact that $p_h^*(p_l) = \frac{1}{2}(p_l + c_h)$ for any T_h, we have

$$\frac{\partial \pi_h^S}{\partial T_h} = -\frac{(p_l - c_h)^2 d}{4} \frac{g'\left(\frac{T_h}{2}\right)}{\left(g\left(\frac{T_h}{2}\right)+\alpha\right)^2} + \frac{A}{T_h^2} = 0$$

Although this equation yields closed-form solutions when $g\left(\frac{T_h}{2}\right)$ is a linear function, the solutions are intractable when $g\left(\frac{T_h}{2}\right)$ is a convex function. For example, when $g\left(\frac{T_h}{2}\right) = \left(\frac{T_h}{2}\right)^2$, the equation becomes a four degree polynomial which has no easily solvable roots. Alternatively, if we take $g\left(\frac{T_h}{2}\right) = e^{\gamma\left(\frac{T_h}{2}\right)}$, then the above equation is not possible to solve analytically.

While we are unable to perform a complete analysis when the disutility of waiting is increasing and convex, we can comment on how the results of our model would likely change in this case. When a consumer's disutility increases more rapidly, as with an increasing and convex disutility function, we speculate that i) the price charged by the in-stock firm would increase, ii) the price charged by the stockless firm would increase, iii) the difference between the two prices would increase, i.e., the degree of stock-out compensation will increase, and iv) equilibria in which the two firms split the market will become less common. The intuition behind the first three points is based on the fact that, under a rapidly increasing and convex disutility function, customers quickly become very sensitive to the waiting time. Hence, the in-stock firm can increase its price and the stockless firm will likely also increase its price (to compensate for the expected decrease in its market share that would occur due to customers being more time-sensitive). However, to compensate for the customer's higher disutility, the stockless firm should increase the amount of stock-out compensation. We have verified this intuition through a small set of informal numerical tests. The intuition behind the fourth point is that competing with a stockless policy becomes more difficult as consumers become more sensitive to waiting time. Thus, for example, with a rapidly increasing disutility function we would expect the stockless firm to have more difficulty in capturing a significant portion of the market.

Finally, we note that identical results were observed when customers have a higher degree of fixed disutility (α). Therefore, we cautiously speculate that the impact of a rapidly increasing customer disutility would be similar to that of a higher degree of fixed disutility.

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To summarize, while the main findings of this paper are derived assuming a linear disutility of waiting, we believe that our main result, i.e., that using a stockless operation with stock-out compensation can be a profitable alternative in a competitive market, will still hold.

B   Technical Details & Proofs

Proof of Proposition 1.   Let $\pi_i^{I,1}$ and $\pi_i^{I,2}$ be firm i's profit with market domination and market sharing. First, note that

$$
\begin{aligned}
\pi_l^{I,1}(p_i) - \pi_h^{I,1}(p_i) &= (c_h - c_l)d + \sqrt{2A \cdot \gamma \cdot d} \cdot (\sqrt{c_h} - \sqrt{c_l}) > 0, \\
\pi_l^{I,2}(p_i) - \pi_h^{I,2}(p_i) &= (c_h - c_l)\frac{d}{2} + \sqrt{A \cdot \gamma \cdot d} \cdot (\sqrt{c_h} - \sqrt{c_l}) > 0,
\end{aligned}
$$

and that all four of these profit functions are strictly increasing-linear functions of p_i. Therefore, the rest of the proof follows standard proof procedures of Bertrand competition (see, e.g., Tirole (1988)). Here we demonstrate why the equilibrium price does not cause any deviation. Note that at the equilibrium price, firm h's profit is zero with market domination. Since this price is also acceptable to firm l, firm l serves the market. Now, firm h needs to exit the market because it will have negative profit if it shares the market at the equilibrium price. ∎

Proof of Lemma 1.   Let $NS_i(b_k)$ be net surplus of a customer with waiting sensitivity b_k from purchasing a product from firm i (i.e., $v - p_i - \left(\frac{T_i}{2} + \alpha\right) b_k$). First, note that $NS_i(b_k)$ is a linear-decreasing function of b_k. Therefore, combining the precondition ($\{p_l > p_h, T_l < T_h\}$ or $\{p_l < p_h, T_l > T_h\}$) and the linearity implies that there exists at most one intersection (i.e., $\tilde{b}_s$) on b_k. Next, the precondition of strictly positive market share implies $\tilde{b}_s < 1$, which implies that $NS_l(b_k)$ and $NS_h(b_k)$ intersect where $b_k < 1$.

Now consider the case where $p_l > p_h$ and $T_l < T_h$. Under this case, we see that $NS_h(b_k) > NS_l(b_k)$ where $b_k < \tilde{b}_s$ and $NS_h(b_k) < NS_l(b_k)$ where $b_k > \tilde{b}_s$. Therefore, it is easy to see that 1) $\tilde{b}_s < \tilde{b}_h < \tilde{b}_l$, 2) customers whose waiting sensitivity is less than $\tilde{b}_s$ prefer firm h, and 3) the other customers whose waiting sensitivity is greater than $\tilde{b}_s$ AND net surplus is greater than 0 prefer firm l. The proof of the other case $\{p_l < p_h, T_l > T_h\}$ is exactly identical to this procedure. ∎

Proof of Proposition 2. *Part i) & ii):* We start by considering the first order conditions for p_h. We first note that the condition $\frac{\partial \pi_h^S}{\partial p_h} = 0$ is independent of T_h. Thus, the optimal p_h can be set independently of T_h. Solving $\frac{\partial \pi_h^S}{\partial p_h} = 0$, we obtain $p_h^*(p_l) = \frac{1}{2}(p_l + c_h)$.

We next consider the first order conditions for T_h. Solving $\frac{\partial \pi_h^S}{\partial T_h} = 0$, and using the fact that $p_h^*(p_l) = \frac{1}{2}(p_l + c_h)$ for any T_h, we find two possible solutions for $T_h^*(p_l)$. However, one of these solutions is strictly negative and can be ruled out. Thus we are left with $T_h^*(p_l) = \frac{-4\alpha\sqrt{A}}{2\sqrt{A} - \sqrt{2d}\sqrt{(c_h - p_l)^2}}$. Next, it is easy to show that, in order to ensure $T_h^*(p_l) > 0$, we need either $p_l < c_h - \sqrt{\frac{2A}{d}}$ or $p_l > c_h + \sqrt{\frac{2A}{d}}$.

The next step in the proof is to check the second order conditions for the optimality of $p_h^*(p_l)$ and $T_h^*(p_l)$. As noted above, the optimal p_h can be set independently of T_h. Thus, we first verify the second order conditions for p_h. It is easy to show that, for any $T_h > 0$, $\frac{\partial^2 \pi_h^S}{\partial p_h^2} = \frac{-4d}{T_h + 2\alpha} \leq 0$. Thus, the second order conditions for p_h are satisfied under the condition on p_l assumed in the statement of the proposition.

Next, assuming that $p_h = p_h^*(p_l)$ for any T_h, we can verify the second order conditions for T_h. It is straightforward to show that under the conditions on p_l stated in the proposition, we have $\frac{\partial^2 \pi_h^S}{\partial T_h^2} \leq 0$ at the point $T_h^*(p_l)$. Thus, $T_h^*(p_l)$ is a local maximum. Next, we can show that $\frac{\partial \pi_h^S}{\partial p_h} < 0$ for $T_h > T_h^*(p_l)$ and $\frac{\partial \pi_h^S}{\partial p_h} > 0$ for $T_h < T_h^*(p_l)$. Thus, the profit function is always increasing for $T_h < T_h^*(p_l)$ and decreasing for $T_h > T_h^*(p_l)$, i.e., the profit function is unimodal, and thus $T_h^*(p_l)$ must be a global maximum, given that $p_h = p_h^*(p_l)$.

Part iii): We need to ensure that $0 < \theta_h^*(p_l) < 1$, i.e., that $p_h^*(p_l) < p_l < p_h^*(p_l) +$

$\alpha + \frac{T_h^*(p_l)}{2}$. To ensure that $p_l > p_h^*(p_l) = \frac{1}{2}(p_l + c_h)$, we need $p_l > c_h$. To ensure that

$p_l < p_h^*(p_l) + \alpha + \frac{T_h^*(p_l)}{2} = \frac{1}{2}(p_l + c_h) + \alpha + \frac{T_h^*(p_l)}{2}$, we need $p_l < c_h + T_h^*(p_l) + 2\alpha$. If

we plug in $T_h^*(p_l)$, using the fact that $p_l > c_h$, we find that $p_l < p_h^*(p_l) + \alpha + \frac{T_h^*(p_l)}{2}$ will

hold only if $p_l < c_h + \sqrt{\frac{2A}{d}} + 2\alpha$. Thus, we have shown that $0 < \theta_h^*(p_l) < 1$ will hold if

$c_h < p_l < c_h + \sqrt{\frac{2A}{d}} + 2\alpha$.

The conditions required for $T_h^*(p_l) > 0$ are $p_l < c_h - \sqrt{\frac{2A}{d}}$ or $p_l > c_h + \sqrt{\frac{2A}{d}}$, while the

conditions required for $0 < \theta_h^*(p_l) < 1$ are $c_h < p_l < c_h + \sqrt{\frac{2A}{d}} + 2\alpha$. Combining these two

sets of conditions, we obtain $c_h + \sqrt{\frac{2A}{d}} < p_l < c_h + \sqrt{\frac{2A}{d}} + 2\alpha$, as assumed in the statement

of the proposition.

Part iv): We next show that, under the condition on p_l in the statement of the propo-

sition, $\pi_h^{S*}(p_l) > 0$. We can write $\pi_h^{S*}(p_l) = (p_h^*(p_l) - c_h)d\,\theta_h^*(p_l) - \frac{A}{T_h^*(p_l)} = \frac{d}{2}\frac{(p_l - c_h)^2}{T_h^*(p_l) + 2\alpha} - \frac{A}{T_h^*(p_l)}$,

where the last step follows from the fact that $p_l - p_h^*(p_l) = \frac{1}{2}(p_l - c_h)$. The condition

$\pi_h^{S*}(p_l) > 0$ can now be rewritten as $\frac{T_h^*(p_l)}{T_h^*(p_l) + 2\alpha} > \frac{2A}{d(p_l - c_h)^2}$.

Plugging in for $T_h^*(p_l)$ and simplifying, we obtain $\frac{T_h^*(p_l)}{T_h^*(p_l) + 2\alpha} = \sqrt{\frac{2A}{d}}\frac{1}{\sqrt{(c_h - p_l)^2}}$. Thus, the

condition $\pi_h^{S*}(p_l) > 0$ requires that $\sqrt{\frac{2A}{d}}\frac{1}{\sqrt{(c_h - p_l)^2}} > \frac{2A}{d(p_l - c_h)^2}$. Simplifying and using the fact

that $p_l > c_h$, we obtain $p_l > c_h + \sqrt{\frac{2A}{d}}$. Thus, under the condition on p_l in the proposition,

$\pi_h^{S*}(p_l) > 0$.

Part v): We next show that $\pi_h^{S*}(p_l)$ is an increasing function of p_l. Taking the derivative

with respect to p_l, we have:

$$\frac{\partial \pi_h^{S*}}{\partial p_l} = \frac{d(p_l - c_h)}{T_h^*(p_l) + 2\alpha} + \left(\frac{A}{T_h^{*2}(p_l)} - \frac{d}{2}\frac{(p_l - c_h)^2}{(T_h^*(p_l) + 2\alpha)^2}\right)\frac{dT_h^*(p_l)}{dp_l}.$$

Using the proof of **Part iv)**, we can show that $\left(\frac{A}{T_h^{*2}(p_l)} - \frac{d}{2}\frac{(p_l - c_h)^2}{(T_h^*(p_l) + 2\alpha)^2}\right) = 0$. Thus, $p_l > c_h$

implies that $\frac{\partial \pi_h^{S*}}{\partial p_l} > 0$. ∎

Proof of Proposition 3. To prove this proposition, we proceed in two steps: i) we

show that $\pi_l^I(p_l|p_h^*, T_h^*) = 0$ when $p_l = c_h + \sqrt{\frac{2A}{d}}$ and ii) we prove that $\frac{\partial \pi_l^I(p_l|p_h^*, T_h^*)}{\partial p_l} > 0$ if

$c_h + \sqrt{\frac{2A}{d}} < p_l < c_h + \sqrt{\frac{2A}{d}} + 2\alpha$. Together, these two results ensure that $\pi_l^I(p_l|p_h^*, T_h^*) > 0$ for p_l satisfying $c_h + \sqrt{\frac{2A}{d}} < p_l < c_h + \sqrt{\frac{2A}{d}} + 2\alpha$.

To prove Part i), we first write $\pi_l^I(p_l|p_h^*, T_h^*) = (p_l - c_l)d(1 - \theta_h^*(p_l)) - \sqrt{2A\gamma c_l d(1 - \theta_h^*(p_l))}$, where $\theta_h^*(p_l) = \frac{2(p_l - p_h^*)}{T_h^* + 2\alpha}$ implies that $1 - \theta_h^*(p_l) = \frac{1}{2\alpha}\left(1 - \sqrt{\frac{2A}{d}}\frac{1}{p_l - c_h}\right)$. Next, note that $p_l = c_h + \sqrt{\frac{2A}{d}}$ implies that $\frac{1}{p_l - c_h} = \sqrt{\frac{d}{2A}}$. Thus, when $p_l = c_h + \sqrt{\frac{2A}{d}}$, we have $1 - \theta_h^*(p_l) = 0$. Therefore, when $p_l = c_h + \sqrt{\frac{2A}{d}}$, we have $\pi_l^I(p_l|p_h^*, T_h^*) = 0$.

Next, to prove Part ii), we plug $1 - \theta_h^*(p_l)$ into $\pi_l^I(p_l|p_h^*, T_h^*)$, take the derivative with respect to p_l, and simplify to get $\frac{\partial \pi_l^I(p_l|p_h^*, T_h^*)}{\partial p_l} = \frac{d}{2\alpha}\left(1 - \sqrt{\frac{2A}{d}}\frac{1}{p_l - c_h}\left(1 - \frac{p_l - c_l}{p_l - c_h}\right)\right)$. Finally, using the facts that $p_l > c_h$ and $c_h > c_l$, we know that $\frac{1}{p_l - c_h} > 0$ and $(1 - \frac{p_l - c_l}{p_l - c_h}) < 0$. Therefore, $\frac{\partial \pi_l^I(p_l|p_h^*, T_h^*)}{\partial p_l} > \frac{d}{2\alpha} > 0$, which completes the proof. ∎

Proof of Proposition 4. We show that $\pi_l^I(p_l|p_h, T_h)$ is a unimodal function of p_l, where $\pi_l^I(p_l|p_h, T_h) = (p_l - c_l)d(1 - \theta_h(p_l)) - \sqrt{2A\gamma c_l d(1 - \theta_h(p_l))}$, where $\theta_h(p_l) = \frac{2(p_l - p_h)}{T_h + 2\alpha}$. We take the first derivative, set it equal to 0, and rewrite the first order condition as

$$d\left(1 - \frac{2(p_l - p_h)}{T_h + 2\alpha}\right) + (p_l - c_l) \cdot d\left(-\frac{2}{T_h + 2\alpha}\right) = -\frac{2A\gamma c_l d}{(T_h + 2\alpha)\sqrt{\left(1 - \frac{2(p_l - p_h)}{T_h + 2\alpha}\right)}}. \tag{9}$$

The LHS of Eq. (9) is a decreasing linear function of p_l, while the RHS is a decreasing concave function. We evaluate two functions at $p_l = 0$, and see that $LHS|_{p_l=0} > 0$ and $RHS|_{p_l=0} < 0$. Combining these two observations implies that Eq. (9) has at most two real solutions.

When there exists no solution or two identical solutions, it is easy to see that $\frac{\partial \pi_l^I(p_l|p_h, T_h)}{\partial p_l} \geq 0$, which implies that $\pi_l^I(p_l|p_h, T_h)$ is a strictly increasing function. Now, consider the case in which there are two different real solutions. We note that $\frac{\partial \pi_l^I(p_l|p_h, T_h)}{\partial p_l}|_{p_l=0} > 0$ and $\frac{\partial \pi_l^I(p_l|p_h, T_h)}{\partial p_l}|_{p_l=p_l^U} = -\frac{d(T_h + 2\alpha + 2(p_h - c_l))}{T_h + 2\alpha} < 0$, where $p_l^U = \left(p_h + \frac{T_h}{2} + \alpha\right)$ is an upper bound on p_l derived from $\frac{2(p_l - p_h)}{T_h + 2\alpha} \leq 1$. This implies that p_l^U is located between two stationary points. Therefore, $\pi_l^I(p_l|p_h, T_h)$ is a unimodal function of p_l in $[0, p_l^U]$. This completes the

proof. ■

Proof of Proposition 5. We prove this result by contradiction. Suppose that there is an equilibrium that satisfies $p_l^E \geq p_h^E + \frac{T_h^E}{2} + \alpha$ (by implication, $\pi_l^I = 0$), $\pi_h^S > 0$, and the other conditions of Proposition 5. Let $p_l^{L,M}$ be the lower bound of firm l's price under a monopolistic setting with an in-stock policy (i.e., $\pi_l^{I,M}(p_l^{L,M}) = 0$).

First, consider $p_h^E > p_l^{L,M}$. For this case, we construct a pricing policy that generates positive profit for firm l. To this end, note that firm l can set $p_l = p_h^E$ and make positive profit, since all customers prefer firm l (i.e., $\tilde{b}_s = 0$ & $\pi_l^{I,M}(p_l^{L,M} + \epsilon) > 0$).

Second, consider the other case in which $p_h^E \leq p_l^{L,M}$, and suppose that there exists an equilibrium. We show that firm h is unable to generate positive profit. To this end, we solve

$$\max \quad \pi_h^S(p_h, T_h) = (p_h - c_h) \cdot d \cdot \max\left[\frac{2(1 - p_h)}{T_h + 2\alpha}, 1\right] - \frac{A}{T_h}$$

$$s.t. \quad p_h \leq p_l^{L,M} = c_l + \sqrt{\frac{2A\gamma c_l}{d}},$$

$$p_h + \frac{T_h}{2} + \alpha \leq p_l^L = c_h + \sqrt{\frac{2A}{d}}.$$

The second constraint is derived from the fact that if firm h sets $p_h^E + \frac{T_h^E}{2} + \alpha > p_l^L$, then firm l would deviate by setting $p_l = \left(p_h^E + \frac{T_h^E}{2} + \alpha\right) - \epsilon$ and have positive profit (refer to Propositions 2 and 3). First, note that the second constraint implies that $\max\left[\frac{2(1-p_h)}{T_h+2\alpha}, 1\right] = 1$. Hence, $\pi_h^S(p_h, T_h)$ becomes $(p_h - c_h) \cdot d - \frac{A}{T_h}$. In addition, we see that the second constraint is binding, i.e., $p_h = c_h + \sqrt{\frac{2A}{d}} - \left(\frac{T_h}{2} + \alpha\right)$. Now, we rewrite the problem as

$$\max \quad \pi_h^S(p_h, T_h) = \left(\sqrt{\frac{2A}{d}} - \left(\frac{T_h}{2} + \alpha\right)\right) d - \frac{A}{T_h}$$

$$s.t. \quad 2\left[c_h + \sqrt{\frac{2A}{d}} - \left(c_l + \sqrt{\frac{2A\gamma c_l}{d}}\right) - \alpha\right] p_h \leq T_h.$$

Solving for the optimal solutions, the unconstrained problem has two stationary points, i.e.,

$T_i^* = \sqrt{\frac{2A}{d}}$ or $T_i^* = -\sqrt{\frac{2A}{d}}$. It is easy to see that $T_i^* = \sqrt{\frac{2A}{d}}$ is a global maximum. By plugging into the profit function and simplifying, we have $\pi_h^S(p_h, T_h) = -d \cdot \alpha$. Therefore, firm h cannot make positive profit in this range. This contradicts the equilibrium assumption and completes the proof. ■

Proof of Proposition 6. Note that the equilibrium should satisfy $p_l^E \leq p_h^E$ (by implication, $\pi_h^S = 0$ & $\pi_l^I > 0$). Next, let p_l^L and p_l^U be the two bounds of Proposition 2, i.e., $p_l^L = c_h + \sqrt{\frac{2A}{d}}$ and $p_l^U = c_h + \sqrt{\frac{2A}{d}} + 2\alpha$. Since $p_l^L < p_l^U$, we consider three scenarios:

Scenario 1: $p_l^U \leq p_l^E$: Note that there is no equilibrium that satisfies the preconditions of Proposition 6 in this region, because firm h can reduce p_h sufficiently so that it can dominate the market.

Scenario 2: $p_l^L < p_l^E < p_l^U$: There exists a unique solution so that two firms share the market (refer to Proof of Proposition 2), but note that the solution does not satisfy $p_l^E \leq p_h^E$.

Scenario 3: $p_l^E \leq p_l^L$: Suppose that firm l sets $p_l = p_l^L$. Then, firm h can consider two strategies: dominating or sharing the market. Since dominating the market is not feasible (see Proof of Proposition 5), we only need to consider sharing the market. We demonstrate that firm h cannot make positive profit with this strategy. To this end, we solve

$$\max \quad \pi_h^S(p_h, T_h | p_l^L) = (p_h - c_h) \cdot d \cdot \frac{2(p_l^L - p_h)}{T_h + 2\alpha} - \frac{A}{T_h}$$

$$s.t. \quad \frac{2(p_l^L - p_h)}{T_h + 2\alpha} \leq 1,$$

where $p_l^L = c_h + \sqrt{\frac{2A}{d}}$. By taking the first derivative with respect to p_h and setting it equal to 0, we have $p_h^* = \frac{p_l^L + c_h}{2}$. Simplifying the profit function with p_h^* and p_l^L, we have $\pi_h^S(T_h | p_h^*, p_l^L) = -\frac{2A\alpha}{T_h(T_h + 2\alpha)}$, which implies that firm h is unable to make positive profit. ■